Online Appendix for "Sovereign Bank Diabolic Loop and ESBies"

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A1. Proof of Proposition 2

To prove claim (i), note that if the space (α^s, F^s) is split into a subset in which the diabolic loop occurs and one, $\mathcal{N}$, in which it does not, identifying the boundary of $\mathcal{N}$ will enable us to characterize the diabolic loop region. To do so, we compute senior bond prices under the diabolic-loop equilibrium and require that the losses associated with the sunspot reduce bank equity exactly to zero.

If the sunspot is not observed, debt trades at its no default-value $\underline{S}$, and the same holds for the senior tranche, which trades at F^s. If the sunspot is observed and banks require a recapitalization, the cost to the government is $C^s \equiv \tau\psi L_0 - \alpha(B_1^s - B_0^s) - E_0$, where B_t^s denotes the price of the senior tranche. If the surplus at $t = 3$ is $\overline{S}$, the government can repay its debt in full after incurring the cost C^s because of A4, so that the senior tranche pays its face value F^s; if instead the surplus is $\underline{S}$, the government can only pay $\underline{S} - C^s$ and the senior tranche yields $F^s - [C^s - (\underline{S} - F^s)]$, where $\underline{S} - F^s$ is the loss absorbed by the junior tranche. Hence, the price of the senior tranche at $t = 1$ is $B_1^s = F^s - \pi[C^s - (\underline{S} - F^s)]$. So the analysis is the same as in the case of no tranching except that C is replaced by $C^s - (\underline{S} - F^s)$. This amounts to replacing $\tau\psi L_0$ in Equation (2) by $\tau\psi L_0 - (\underline{S} - F^s)$. In other words, the bailout is avoided if

$$E_0 \geq \alpha^s \pi(1-p)\left[\tau\psi L_0 - (\underline{S} - F^s)\right] =: \underline{E}_0^s. \quad (4)$$

This proves claim (i).

Claim (ii) follows by noticing that a diabolic loop cannot occur if banks' equity is $E_0 > \underline{E}_0^s$, so that the junior bond is also risk-free.

To prove claim (iii) note that for pairs (α^s, F^s) on the boundary of the no-diabolic-loop subset $\mathcal{N}$, the inequality (4) holds with equality. The right-hand side of (4) is increasing in both α^s and F^s, which means that at the boundary if banks hold a larger fraction of the senior tranche α^s, this tranche must have a lower face value F^s, and vice versa. We want to find the pair $(\alpha^{s*}, F^{s*}) \in \mathcal{N}$ that maximizes the total value of safe assets available to the banking system:

$$\max_{(\alpha^s, F^s)\in\mathcal{N}} \alpha^s F^s = \max_{(\alpha^s, F^s)\in\mathcal{N}} \frac{E_0 F^s}{\pi(1-p)\left[\tau\psi L_0 - (\underline{S} - F^s)\right]}.$$

The maximand is decreasing in F^s, because $\underline{S} > \tau\psi L_0$. Therefore, the maximization requires setting the optimal face value F^{s*} at the lowest possible value that meets (4) with equality. In turn, this requires setting α^s at its upper bound $\alpha^{s*} = 1$, so that

$$F^{s*} = \underline{S} + \frac{E_0}{\pi(1-p)} - \tau\psi L_0 < \underline{S}, \quad (5)$$

where the inequality follows from A3. Since the solution for $\max_{(\alpha^s, F^s)\in\mathcal{N}} \alpha^s F^s$ differs from the no-tranching solution, tranching allows the economy to generate a larger amount of safe assets for the banking system. QED

A2. Proof of Proposition 3

As in the case where tranching occurs in a single country, we wish to characterize the set $\mathcal{N}$ of pairs $(\alpha^{\mathcal{E}}, F^{\mathcal{E}})$ that rule out the diabolic-loop equilibrium. To do so, we initially compute prices of ESBies for a given $(\alpha^{\mathcal{E}}, F^{\mathcal{E}})$ under a diabolic-loop equilibrium and require that bank equity remains non-negative. Consider the parameter region in which the senior tranche incurs losses when the (union-wide) sunspot is observed. There are two scenarios to be considered:

First, suppose equity E_0 is large enough that ESBies incur losses only in the worse-case outcome at $t = 3$, in which both countries have primary surplus $\underline{S}$ realization. In this scenario, which occurs with probability π^2, the pooled asset pays $\underline{S} - C^{\mathcal{E}}$, and the senior tranche pays $F^{\mathcal{E}} - [C^{\mathcal{E}} - (\underline{S} - F^{\mathcal{E}})]$. Hence, junior bond holders are wiped out. Clearly, ESBies are better protected than a single country senior bond, where the low surplus realization occurs with probability π.

Second, for lower equity levels E_0 the diabolic loop might be so large that ESBies might incur losses if only one of the two countries has a low primary surplus realization. In this case the pooled asset pays $\underline{S} - \frac{1}{2}C^{\mathcal{E}}$ and the junior bond holder will be wiped out in three of the four possible surplus realizations. This case occurs with probability $2\pi(1-\pi)$.

In the first scenario, in which ESBies only default in the state where surplus realization is $\underline{S}$ for both governments, the following inequality must hold

$$\underline{S} - \frac{1}{2}C^{\mathcal{E}} \geq F^{\mathcal{E}}. \tag{6}$$

If (6) holds, then the price of the senior tranche in period 1 is $B_1^{\mathcal{E}} = F^{\mathcal{E}} - \pi^2[C^{\mathcal{E}} - (\underline{S} - F^{\mathcal{E}})]$. The analysis is the same as in the one-country case with tranching except that π is replaced by π^2. A recapitalization is not needed if

$$E_0 \geq \alpha^{\mathcal{E}}\pi^2(1-p)\left[\tau\psi L_0 - (\underline{S} - F^{\mathcal{E}})\right]. \tag{7}$$

In the second scenario, where (6) is violated, if one country has surplus $\overline{S}$ and the other $\underline{S}$, the senior tranche receives $F^{\mathcal{E}} - [\frac{1}{2}C^{\mathcal{E}} - (\underline{S} - F^{\mathcal{E}})]$ and its price at $t = 1$ is

$$\begin{aligned} B_1^{\mathcal{E}} &= F^{\mathcal{E}} - \left[\frac{1}{2}2\pi(1-\pi) + \pi^2\right]C^{\mathcal{E}} + \left[2\pi(1-\pi) + \pi^2\right](\underline{S} - F^{\mathcal{E}}) \\ &= F^{\mathcal{E}} - \pi\left[C^{\mathcal{E}} - (2-\pi)(\underline{S} - F^{\mathcal{E}})\right]. \end{aligned}$$

The analysis is the same as in the one-country case with tranching except that we must replace $\underline{S} - F^{\mathcal{E}}$ by $(2-\pi)(\underline{S} - F^{\mathcal{E}})$. A recapitalization is not needed if

$$E_0 \geq \alpha^{\mathcal{E}}\pi(1-p)\left[\tau\psi L_0 - (2-\pi)(\underline{S} - F^{\mathcal{E}})\right]. \tag{8}$$

Setting $\alpha^{\mathcal{E}} = \alpha^s$ and $F^{\mathcal{E}} = F^s$ in (7) and (8) and comparing them with (4), it follows that the lower bound on equity to sovereign exposure ratio is less stringent with ESBies than with single country tranching. This completes part (i) of the proof.

The claim in part (ii) follows directly from the Equations (7) and (8) which rule out the diabolic loop equilibrium.

To prove the claim in part (iii), note that in the first scenario the pair $(\alpha^{\mathcal{E}*}, F^{\mathcal{E}*})$ that maximizes the value of the safe asset available to the banks satisfies (7) with equality, and $\alpha^{\mathcal{E}*} = 1$ by the same argument as in the one-country case. The resulting value of the senior tranche is analogous to (5) in the one-country case with tranching:

$$F^{\mathcal{E}*} = \underline{S} + \frac{E_0}{\pi^2(1-p)} - \tau\psi L_0. \tag{9}$$

Since π is now replaced by π^2, we have $F^{\mathcal{E}*} > F^{s*}$: pooling and tranching generates a larger supply of the safe asset than tranching in each country separately.

We must finally check that ESBies suffer no losses even in the next to worst-case scenario, i.e. (6) is satisfied. Noting that

$$C^{\mathcal{E}*} = \tau\psi L_0 - \alpha(B_1^{\mathcal{E}*} - B_0^{\mathcal{E}*}) - E_0 = \tau\psi L_0 + \alpha^{\mathcal{E}*}(1-p)\Delta_1^{\mathcal{E}*} - E_0,$$

and that in the two-country case with tranching $\Delta_1^{\mathcal{E}}$ is given by an equation analogous to (1) where $\tau\psi L_0$ is replaced by $\tau\psi L_0 - (\underline{S} - F^{\mathcal{E}})$ and π by π^2, the no-loss condition (6) can be rewritten as

$$\underline{S} - F^{\mathcal{E}*} - \frac{1}{2}\left[\tau\psi L_0 + \alpha^{\mathcal{E}*}(1-p)\frac{\pi^2(\tau\psi L_0 - E_0 - (\underline{S} - F^{\mathcal{E}*}))}{1 - \alpha^{\mathcal{E}*}\pi^2(1-p)} - E_0\right] \geq 0. \tag{10}$$

We next set $\alpha^{\mathcal{E}*} = 1$ and $F^{\mathcal{E}*}$ equal to its value in (9). Because these values satisfy (7) with equality, the sum of the second and third term in the square bracket of (10) is zero, so using (9), (10) becomes

$$E_0 \leq \frac{1}{2}\pi^2(1-p)\tau\psi L_0, \tag{11}$$

which is part of the parameter space for E_0 we consider under A3.

For the second scenario, in which (6) holds, going through the same steps as for the first scenario, we find that the pair $(\alpha^{\mathcal{E}*}, F^{\mathcal{E}*})$ that maximizes the value of safe investment available to the banks satisfies $\alpha^{\mathcal{E}*} = 1$ and

$$F^{\mathcal{E}*} = \underline{S} - \frac{1}{(2-\pi)}\left[\tau\psi L_0 - \frac{E_0}{\pi(1-p)}\right]. \tag{12}$$

The face value $F^{\mathcal{E}*}$ is larger than in the one-country case because by A3 the term in square brackets is positive. We are in the second scenario, i.e. (6) is violated, if equity is in the region

$$\frac{1}{2}\pi^2(1-p)\tau\psi L_0 < E_0 < \pi(1-p)\tau\psi L_0.$$

QED