Online Appendix for "Collective Self-Control"

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Abstract

In this Online Appendix, we discuss in detail several extensions to the model described in the body of the text, and their implications.

1 Naive Agents

In the literature, when modeling time inconsistent agents, an assumption of naivete is sometimes made in contrast to the sophistication we have assumed throughout the body of the paper.[1] Naive agents have $\beta - \delta$ preferences, but believe that they will have standard geometric preferences in any future period. Sometimes agents are assumed to be partially naive. This is modeled as agents having beliefs about their future selves that are intermediate between full sophistication and full naivete.

Most of our analysis would go through, with some modifications, if agents were partially naive. However, it is useful to comment on the qualitative impact of such agents in the electorate. To simplify our discussion, suppose that some agents in the population are fully naive.

In our model naive agents behave like time consistent (high β) individuals in period 1: they do not have any demand for commitment because they are unaware of their time inconsistency problem. Therefore, the higher the mass of naive agents in the economy, the lower the investment in commitment in equilibrium. However, once period 2 arrives, these agents are tempted by immediate consumption, lowering the effective pivotal β in the centralized consumption scenario. Overall, the presence of these naive agents reduces welfare

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[1] See, for instance, O'Donoughue and Rabin (1999).

for the sophisticated agents. However, the naive agents make "worse" individual choices than sophisticated agents so they are more likely to benefit from centralization. If the naive agents constitute a majority and the median β in the second period is such that $\beta < \frac{v_2}{v_3}$, then full decentralization is best: the political outcomes of any centralized decisions would be bad so decentralization would at least deliver good choices for the relatively high β, sophisticated agents.

If the naive agents are a minority, then there are opposing forces in favor and against centralization: the presence of the naive agents worsens the choices but the naive agents benefit more from centralization.[2]

2 Commitment Subsidies

Instead of considering a centralized commitment scenario where the elected government chooses the amount of commitment in period 1, one could consider a scenario where candidates propose subsidies to commitment. If a voter receives a subsidy s, the choice of commitment in period 1 can be obtained by maximizing

$$U_1\left(x\left(c,\beta\right),c,\beta,s\right) = \beta v_3 + x\left(c,\beta\right)\left(\beta v_2 - \beta v_3\right) - \beta k\left(x\left(c,\beta\right),c\right) - I\left(c,s\right)$$

where $\frac{\partial I(c,s)}{\partial c}$ is decreasing in s. Thus, the amount of commitment chosen by each individual is increasing in s. However, the voting decision between two candidates who offer different levels of subsidies needs to take into account the budgetary impact of the subsidies and how the corresponding expenses are distributed in the population. The total amount of subsidies depends on the aggregate amount of commitment. Consider then a setting in which subsidies are chosen collectively, and consumption is chosen in a decentralized fashion. If the burden is shared equally across the electorate,[3] it can be shown that the pivotal agent remains that with a preference parameter β^{CD} (the pivotal agent in our baseline centralized commitment-decentralized consumption setting absent subsidies). If this agent invests relatively little in commitment, the value of subsidies for her is lower than her contribution to the collective pool

[2] Hiedhues and Koszegi (2010) suggested how commitment policieis in the credit card market might be beneficial for naive consumers from a welfare perspective. In our setting, whenever choices are made collectively, there are additional forces due to externalities, which alters the calculus of political influence.

[3] Formally, for any profile of commitment $c(\beta)$, the overall budgetary consequence of a subsidy level s is given by:

$$\int_0^1 I(c(\beta),s)dG(\beta) - \int_0^1 I(c(\beta))dG(\beta),$$

which is shared equally within the population.

covering overall subsidies in the population. In this case, the outcome of the election would generate zero subsidies. On the other hand, if this agent has a relatively high investment in commitment, so that she is a net beneficiary of the subsidies, she will support fairly high subsidies. In this case, the outcome would lead to higher investment in commitment by all agents relative to that chosen under the fully decentralized scenario. Note, however, that from the perspective of period 1, commitment subsidies generate lower welfare than a laissez-faire economy.

3 Supplementing Commitment

Another natural extension pertains to agents' potential ability to supplement commitment investments that are chosen by the government.

Suppose public and private commitments are governed by the same technology. That is, for any government choice of commitment c_g, each agent experiences a period 1 cost of $I(c_g)$, while additional private commitment of c_p leads the agent to experience an overall period 1 cost of $I(c_g + c_p)$. That is, the cost of supplementing public investment in commitment is incremental. Our equilibrium characterization changes only in the centralized commitment, decentralized consumption setting. Since commitment costs are convex, the government's commitment technology is not inferior to the private technology, and the amount of commitment chosen by the government is given by our Proposition 1. Individuals who seek greater commitment will then supplement the collective commitment privately. From a welfare perspective, this setting still generates lower welfare levels than the fully decentralized one as agents can emulate the generated outcomes privately.

Suppose instead that public and private commitment technologies are independent, so that a choice of government commitment c_g and private commitment c_p generate a period 1 cost of $I_g(c_g)+I_p(c_p)$, where I_g and I_p satisfy our assumptions on the underlying commitment technology that were made in Section 3. In this case, when commitment is subject to collective action agents will typically mix private and public investment. The precise formulation of the equilibrium characterization in the relevant two settings depends more intricately on the functional forms of our model. In such settings, centralizing commitment alone may be beneficial relative to full decentralization as that setting effectively provides individuals access to an aggregate commitment cost technology that is more efficient: individuals can smooth the cost of commitment by splitting their commitment investments between public and private ones.

4 Endogenous Turnout

There are few models of endogenous turnout when the alternatives are themselves endogenous and determined via campaign competition. One obstacle is that most models of this sort have an inherent implausibility—when candidate positions converge (as in many models of campaign competition), there is absolutely no reason for voters to turn out in equilibrium regardless of the assumption about voters' participation motives, be it selfish, expressive, or ethical.

Nevertheless, it is interesting to consider how the forces in our model are modified by the presence of endogenous turnout. To this end, we use the model of Ledyard (1984), who studies equilibrium candidate platforms in a spatial model in which candidates are office motivated, and voters have i.i.d. draws of costs of participation, independent of their policy preferences. The main result in Ledyard (1984) is that, in equilibrium, candidates converge to the position that maximizes the utilitarian surplus of voters, and no voters turn out. The result is driven by the fact that, should a candidate deviate, more extreme voters would have a higher incentive to participate, leading to preference intensity being incorporated into candidates' objectives. This model can be immediately adapted to our environment. For any given distribution G of voters' present bias parameters, the equilibrium can therefore easily be computed and compared to the case of exogenous turnout that we have studied so far.

Let us first consider what happens if only commitment is centralized. The equilibrium level of commitment with endogenous turnout will maximize voters' surplus given the subsequent choices of consumption induced in period 2. Whether this level is higher or lower than the equilibrium choice with exogenous turnout depends on the distribution of preferences.

Similarly, full centralization will lead to the choice of commitment and subsequent choice of consumption that maximize voters' surplus, and, again, these choices may be lower or higher than the one preferred by the median voter that results under exogenous turnout. Of course, under endogenous turnout, full centralization becomes much more appealing from a welfare perspective.

5 Linear Commitment Costs and Single-Peaked Preferences

Throughout the paper, we have often assumed that $\frac{\partial k(1,c)}{\partial x} > v_2$. In that case, individual preferences for commitment are single peaked. When preferences are not single peaked, our analysis needs to be modified, especially for the case of centralized commitment-decentralized consumption.

We will now outline what happens when preferences are not single peaked by considering the special case of linear costs (and dropping the requirement that $\frac{\partial k(x,0)}{\partial x} = 0$). This case is useful since its structure is particularly simple. We first emphasize that the main welfare results still hold in this case. However, the equilibrium construction is more complex.

When consumption costs are linear, we can normalize parameters so that $k(x, c) = cx$. Furthermore, the optimal choice in the second period is generically either $x = 0$ or $x = 1$. In case of indifference, we will assume that an agent breaks the indifference to favor her "commitment self," i.e., she chooses $x = 0$.[4]

Suppose that in period 1 a cost c was chosen, and consider the period 2 choice problem of a voter of type β. She will consume in period 3 if and only if

$$U_2 = v_2 - c \leq \beta v_3. \tag{1}$$

Thus, as before, agents with $\beta > \frac{v_2}{v_3}$ are not willing to pay for commitment: they do not find it necessary.

Commitment is perceived beneficial in period 1 if the delay in consumption due to commitment is worth its costs $I(c)$. That is, whenever there is a commitment parameter c such that:

$$\beta v_3 - I(c) \geq \beta v_2 \iff \beta (v_3 - v_2) \geq I(c). \tag{2}$$

How do investment incentives now vary with β? It is very difficult (and costly) to make low β agents wait until period 3 to consume. On the other side of the spectrum, high β agents are virtuous and will wait till period 3 even with no commitment instruments. Therefore, investment only pays for intermediate β's.

Thus, as in the case studied previously, incentives to invest are not monotonic in β since both low- and high-β agents dislike investment (for different reasons). However, unlike the previous case, utilities are *not single peaked* with respect to the commitment c: for intermediate β's payoffs are first decreasing in c because we violate condition (1) and so commitment initially affects utility only through its costs, but carries no benefits in terms

[4] This setting can fit a special case of Gul and Pesendorfer (2001, 2004, 2007) type of preferences. Namely, suppose that two functions govern an individual's utility from consumption: $u(x)$ is the direct utility of x, while $v(y)$ is the temptation cost of not having consumed y available at the time of choice. In such a setting, in order to delay consumption in period 2, $u(v_3) - v(v_2) \geq u(v_2)$. Suppose $u(x) = x$ and $v(y) = \alpha y$, where $\alpha > 0$. Then delayed consumption in period 3 occurs when $v_3 \geq v_2(1 + \alpha)$, which is analogous to our linear costs case when taking $\beta = \frac{1}{1+\alpha}$.

of the timing of consumption, until we reach a level of commitment c^* such that condition (1) is satisfied, so that $c = 0$ and $c = c^*$ are both local optima.

Consider now the case of collective commitment accompanied by decentralized choice. For all agents of preference parameter $\beta \geq \frac{v_2}{v_3}$, there is no willingness to pay for commitment no matter what the commitment technology is. Recall that $\beta^* = \frac{v_2}{v_3}$. If $1 - G(\beta^*) \geq 1/2$, there is a majority supporting no commitment and, as before, there is a unique equilibrium in which both candidates offer commitment $c^{CD} = 0$. Suppose there is a substantial fraction of the population that is moderate, $1 - G(\beta^*) < 1/2$. Now note that by raising c we obtain an increasing mass of β's for which $\beta v_3 \geq v_2 - c$. Let $\beta(c) \equiv \frac{v_2 - c}{v_3}$. The mass is given by $G(\beta(c))$. Define c_L such that

$$G(\beta^*) - G(\beta(c_L)) = \frac{1}{2}$$

and let $\beta_L \equiv \beta(c_L)$.

Let $\tilde{c}$ be the unique commitment level such that[5]

$$\beta(\tilde{c})(v_3 - v_2) = I(\tilde{c}).$$

The next result characterizes the equilibria in this environment.

Proposition 1 *Assume that* $k(x, c) = cx$. *When only commitment decisions are centralized,*

1. *If* $\beta_L(v_3 - v_2) \leq I(c_L)$, *there exists a unique equilibrium with investment of zero in commitment instruments.*

2. *If* $\beta_L(v_3 - v_2) > I(c_L)$, *there is no pure strategy equilibrium. In this case, there is a continuum of equilibria in mixed strategies. All symmetric profiles having a two-point support* $c_1 < c_2$ *with equal probability on* c_1 *and* c_2, *where* $c_2 \in [c_L, \tilde{c}]$, *constitute part of an equilibrium.*

The intuition for the non existence of positive commitment, pure strategy equilibria is the following. Assume $c > 0$ is part of an equilibrium. A deviation to a slightly lower commitment level attracts votes from two groups of voters: all agents with (low) β's such that c is not sufficient to generate delay and so a lower c is preferable, and all agents with (high) β's such that c is more than enough. Thus, support for the deviating candidate is overwhelming, with the extremes "squeezing" the middle. Zero commitment is an equilibrium

[5] Note that $\beta(\tilde{c})(v_3 - v_2)$ is decreasing in $\tilde{c}$. Since $\beta(0)(v_3 - v_2) > I(0) = 0$ and $0 = \beta(v_2)(v_3 - v_2) < I(v_2)$, the existence of a unique $\tilde{c} \in (0, v_2)$ satisfying the equality is guaranteed.

if the commitment technology is not "too efficient." If, however, investment is very cheap ($I(c)$ is very low), then zero commitment cannot be an equilibrium because a "global" deviation to a large commitment would attract a majority of support. The proposition describes the mixed strategy equilibria in this case.

When only consumption choices are mandated (but commitment is chosen individually), the same analysis as in the general case holds and equilibrium is characterized by the entire electorate choosing not to invest in commitment.

Consider, last, the case in which both commitment and choices are mandated. Incentives to vote for investment in the first period may be high for high-β individuals. The optimal commitment parameter c is either 0 or the c^* that is just sufficient to make the median-β individual choose consumption at period 3, i.e., the minimal level of cost that solves

$$v_2 - c^* \geq \beta_M v_3 \text{ or } c^* = \max\{v_2 - \beta_M v_3, 0\}.$$

In period 1, all voters such that $\beta(v_3 - v_2) \geq I(c^*)$ or equivalently such that $\beta \geq \frac{I(c^*)}{(v_3 - v_2)}$ prefer c^* to 0; all agents with lower β's prefer 0. Thus, there can be a broad consensus in favor of investing.

Proposition 2 *Suppose $k(x,c) = cx$. When both commitment and consumption decisions are centralized, there exist $\check{\beta}, \hat{\beta}$ such that if $\beta_M \leq \check{\beta}$ or $\beta_M \geq \hat{\beta}$, there is a unique equilibrium with $c = 0$, and if $\beta_M \in \left(\check{\beta}, \hat{\beta}\right)$, there is an equilibrium with positive commitment.*

Now that we have characterized equilibria in this environment, it can easily be seen that the main forces behind our welfare results from Section 6 in the body of the paper are still in place: either full centralization or full decentralization are best, and the comparison between these two institutions depends on how virtuous the median voter is. In fact, the proof of Proposition 6 remains intact.

6 Proof of Proposition 1

We first show that with linear costs there is no pure strategy equilibrium with positive commitment. Assume by way of contradiction that candidate 1 chooses $c > 0$ with probability 1. Then candidate 2 can win with probability 1 by choosing $c - \varepsilon$ for ε sufficiently small. All voters with preference parameter β such that $\beta v_3 \geq v_2 - (c - \varepsilon)$ prefer candidate 2 because they still get to consume in period 3 but the lower investment in commitment is sufficient to do so. Furthermore, all voters with β such $\beta v_3 < v_2 - c$ prefer candidate 2 because they consume in period 2 with both levels of commitment, so prefer the candidate who offers the lower level. The only voters who may prefer c over $c-\varepsilon$ are those whose preference parameter β is such that $\beta v_3 \geq v_2 - c$ and $\beta v_3 < v_2 - (c - \varepsilon)$. However, because the distribution G is continuous, the mass of these voters can be made arbitrarily small by choosing ε small enough.

If $\beta_L (v_3 - v_2) \leq I(c_L)$, then all agents with preference parameter β such that $\beta \leq \beta_L$ prefer $c = 0$ to c_L. Since $I(c)$ is convex, they prefer $c = 0$ to all $c > c_L$. Furthermore, any $0 < c < c_L$ is also worse than $c = 0$ for these agents because $\beta v_3 < v_2 - c$ by the definition of c_L and β_L. Since $(1 - G(\beta^*)) + G(\beta_L) = \frac{1}{2}$, there is a majority in favor of $c = 0$ against all other c's.

If $\beta_L (v_3 - v_2) > I(c_L)$, then all β's between β^* and β_L strictly prefer $c_L + \varepsilon$ to $c = 0$. Furthermore, some β's slightly higher than β_L also prefer $c_L + \varepsilon$ to $c = 0$. Since there half the mass of voters is concentrated between β_L and β^*, $c_L + \varepsilon$ defeats $c = 0$. As shown above, there is no pure strategy equilibrium with positive commitment. This establishes that when $\beta_L (v_3 - v_2) > I(c_L)$, there is no pure strategy equilibrium.

We now show that when $\beta_L (v_3 - v_2) > I(c_L)$ the mixed-strategy profiles in the statement of the proposition constitute equilibria. Note first that c_1 and c_2 as defined in the proposition tie. Consider now a policy $\hat{c} > c_2$. This policy may win against c_1. However, $\hat{c}$ loses against c_2 because all agents of preference parameter $\beta > \beta(c_2) - \delta$ (for some δ) would vote for c_2 over $\hat{c}$. Since $G(\beta(c_1)) - G(\beta(c_2)) = \frac{1}{2}$, there is more than 50% of the voters supporting c_2. Thus, $\hat{c}$ wins with probability 1/2. Consider now a policy $c_1 < \hat{c} < c_2$. Such a policy may win against c_2. However, against c_1, the only potential supporters are agents with preference parameters within $[\beta(\hat{c}), \beta(c_1))$, which by construction entails less than 50% of the population. In particular, c_L is a policy that would lose against c_1. Last, consider a policy $\hat{c} < c_1$. This policy may win against c_1. Against c_2, its only potential supporters are agents with preference parameters $\beta \leq \beta(c_2)$ or $\beta \geq \beta(\hat{c})$, which from the definition of the pair (c_1, c_2) account for less than 50% of the voters. Thus, the candidate equilibrium strategy profile wins with probability at least 1/2 against all possible deviations and no deviation is strictly beneficial. ■

References

Gul, F. and W. Pesendorfer (2001), "Temptation and Self-Control," *Econometrica*, 69, 1403-1436.

Gul, F. and W. Pesendorfer (2004), "Self-Control and the Theory of Consumption," *Econometrica,* 72, 119-158.

Gul, F. and W. Pesendorfer (2007), "Harmful Addiction," *Review of Economic Studies*, 74(1), 147-172.

Hiedhues, P. and B. Koszegi (2010), "Exploiting Naïvete about Self-Control in the Credit Market," *The American Economic Review*, 100, 2279-2303.

Ledyard, J. (1984), "The Pure Theory of Large Two-Candidate Elections," *Public Choice*, 44, 7-43.

O'Donoghue, T. and M. Rabin (1999), "Doing It Now or Later," *The American Economic Review*, 89(1), 103-124.