Online Appendix
"Centralized decision making and informed lobbying"

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This appendix online is divided in four parts: (I) the conclusion of the proof of Proposition 1; (II) detailed analysis of the examples; (III) the mechanism design approach; and (IV) the characterization of pooling equilibria.

I. Computing the expressions for the equilibrium policy

PROOF OF PROPOSITION 1 (CONTINUATION):

Here we compute the full expression for the policy in the centralized structure under private information, given by (12) and (13) in the paper. This proof also presents the necessary steps to obtain the closed form expressions for the policy. In order to compute these expressions, we must solve a system of first-order conditions of best responses.

Aligned preferences

The first- and second-order conditions of the lobbies' best responses are given by two expressions of (17) in the paper, one for each lobby and the first-order condition for the policymaker (9) in the paper. The uniform distribution of type over $\left[\underline{\theta},\overline{\theta}\right]$ leads to the following expressions for the hazard rate and the welfare function.

$$\begin{aligned}\frac{1-F\left(\theta_j\right)}{f\left(\theta_j\right)} &= \overline{\theta}-\theta_j,\\ \lambda W'\left(p\right) &= \lambda\left(\alpha_A+\alpha_B-2p\right).\end{aligned}$$

Substituting these into the first-order condition (17) in the paper gives

$$-\left(p-\theta_i\right)+\frac{\partial C}{\partial p}\left(p,\theta_j\right)-\lambda 2b\left(p-p^e\right)=\left(\overline{\theta}-\theta_j\right)\frac{\partial^2 C}{\partial\theta_j\partial p}\left(p,\theta_j\right).$$

For the aligned setting the first-order conditions (11) and (9) in the paper become

$$\begin{aligned}
\theta_i - p + \frac{\partial C}{\partial p}(p,\theta_j) - \lambda(2p - \alpha_A - \alpha_B) &= \left(\overline{\theta} - \theta_j\right)\frac{\partial^2 C}{\partial \theta_j \partial p}(p,\theta_j),\\
\theta_j - p + \frac{\partial C}{\partial p}(p,\theta_i) - \lambda(2p - \alpha_A - \alpha_B) &= \left(\overline{\theta} - \theta_i\right)\frac{\partial^2 C}{\partial \theta_i \partial p}(p,\theta_j),\\
\frac{\partial C}{\partial p}(p,\theta_i) + \frac{\partial C}{\partial p}(p,\theta_j) - \lambda(2p - \alpha_A - \alpha_B) &= 0.
\end{aligned}$$

To compute the explicit expressions for the policies, we follow Martimort and Moreira (2010) by conjecturing that the optimal contribution is a quadratic function when the distribution of types is uniform. We then postulate the following expression for the contribution:

$$C(p,\theta) = \frac{g}{2}p^2 + (e\theta + f)p + C^0(\theta),$$

which has linear marginal contribution in p and is separable in its arguments. With this expression, we re-write the above system of equations as

$$\theta_i - p + gp + e\theta_j + f - \lambda(2p - \alpha_A - \alpha_B) = \left(\overline{\theta} - \theta_j\right)e, \tag{A.1}$$

$$\theta_j - p + gp + e\theta_i + f - \lambda(p - \alpha_A - \alpha_B) = \left(\overline{\theta} - \theta_i\right)e, \tag{A.2}$$

$$2gp + 2f + e(\theta_i + \theta_j) - \lambda(2p - \alpha_A - \alpha_B) = 0. \tag{A.3}$$

We can re-write (A.1) and (A.2) as

$$\begin{aligned}
&\theta_i + 2e\theta_j = (1 - g + 2\lambda)p - f - \lambda(\alpha_A + \alpha_B) + e\overline{\theta} \text{ and}\\
&\theta_j + 2e\theta_i = (1 - g + 2\lambda)p - f - \lambda(\alpha_A + \alpha_B) + e\overline{\theta}.
\end{aligned}$$

From these equations we have

$$\theta_i + 2e\theta_j = \theta_j + 2e\theta_i,$$

which can only be true for any given (θ_i, θ_j) if $e = \frac{1}{2}$. Combining (A.1) and (A.3) gives

$$\frac{\theta_i}{2} - p - gp - f = \frac{\overline{\theta} - \theta_j}{2}.$$

This expression can be rearranged to

$$f + (1+g)\, p = \frac{\theta_i + \theta_j - \overline{\theta}}{2}. \tag{A.4}$$

Substituting (A.4) back into (A.3) gives

$$\theta_i + \theta_j - \overline{\theta} + \frac{1}{2}(\theta_i + \theta_j) - 2p = \lambda\,(2p - \alpha_A - \alpha_B)$$

and simple rearrangement gives

$$p = \frac{1}{2(1+\lambda)}\left(\frac{3}{2}(\theta_i + \theta_j) - \bar{\theta} + \lambda\,(\alpha_A + \alpha_B)\right), \tag{A.5}$$

which defines the equilibrium policy. Moreover, to obtain the explicit form for f and g we substitute (A.5) back into (A.4) and get

$$f + \frac{1+g}{2(1+\lambda)}\left(\frac{3}{2}(\theta_i + \theta_j) - \bar{\theta} + \lambda\,(\alpha_A + \alpha_B)\right) = \frac{\theta_i + \theta_j - \overline{\theta}}{2}.$$

There are many combinations of f and g that may solve this equation. However, by definition, f and g do not depend on θ. Hence, the only (constant) value of g that ensures this equation holds for all realization of types is $g = (2\lambda - 1)/3$, which results in $f = -\left(\overline{\theta} + 2\lambda\,(\alpha_A + \alpha_B)\right)/6$. Therefore, the contribution is given by

$$C(p, \theta_i) = \frac{2\lambda - 1}{6}p^2 + \left(\frac{\theta_i}{2} - \frac{\overline{\theta}}{6} - \frac{\lambda\,(\alpha_A + \alpha_B)}{3}\right)p + C^0(\theta_i).$$

Finally, we must compute the constant term $C^0(\theta_i)$. It is computed from the policymaker's binding participation constraint. When the rival lobby is $\underline{\theta}$ the lobby does not leave information rents to the policymaker. Therefore, from the policymaker's binding participation constraint and the expression for the contributions we have

$$\begin{aligned}
C(p, \underline{\theta}) + C(p, \theta_i) - \frac{\lambda}{2}(p - \alpha_A)^2 - \frac{\lambda}{2}(p - \alpha_B)^2 &= -\frac{\lambda}{4}(\Delta\alpha)^2, \\
\frac{2\lambda - 1}{6}p^2 + \frac{1}{3}\left(\frac{3\underline{\theta} - \bar{\theta}}{2} - \lambda\,(\alpha_A + \alpha_B)\right)p + C^0(\underline{\theta}) &= C(p, \underline{\theta}) \text{ and} \\
\frac{2\lambda - 1}{6}p^2 + \frac{1}{3}\left(\frac{3\theta_i - \bar{\theta}}{2} - \lambda\,(\alpha_A + \alpha_B)\right)p + C^0(\theta_i) &= C(p, \theta_i).
\end{aligned}$$

We begin computing $C^0(\underline{\theta})$ for the symmetric case when both lobbies have the lowest type. In this case we must have

$$\begin{aligned}\frac{2\lambda-1}{3}p^2+\frac{1}{3}\left(3\underline{\theta}-\bar{\theta}-2\lambda\left(\alpha_A+\alpha_B\right)\right)p+2C^0\left(\underline{\theta}\right)&\\ -\frac{\lambda}{2}\left[\left(p^2-2p\alpha_A+\alpha_A^2\right)+\left(p^2-2p\alpha_B+\alpha_B^2\right)\right] &= -\frac{\lambda}{4}\left(\alpha_A^2-2\alpha_A\alpha_B+\alpha_B^2\right),\end{aligned}$$

where $p=p(\underline{\theta},\underline{\theta})$. This expression simplifies to

$$-\frac{\lambda+1}{3}p^2+\frac{1}{3}\left(3\underline{\theta}-\bar{\theta}+\lambda\left(\alpha_A+\alpha_B\right)\right)p+2C^0\left(\underline{\theta}\right)=\frac{\lambda}{4}\left(\alpha_A+\alpha_B\right)^2,$$

and substituting $p=p(\underline{\theta},\underline{\theta})$ into (A.5), we rearrange the last expression to

$$C^0\left(\underline{\theta}\right)=-\frac{\lambda+1}{6}\left(\frac{3\underline{\theta}-\bar{\theta}+\lambda\left(\alpha_A+\alpha_B\right)}{2\left(1+\lambda\right)}\right)^2+\frac{\lambda}{8}\left(\alpha_A+\alpha_B\right)^2.$$

Now we can compute $C^0(\theta_i)$ from the binding participation constraint, for non-symmetric realization of types. We have

$$\begin{aligned}\frac{2\lambda-1}{3}p^2+\frac{1}{3}\left(\frac{3\theta_i-\bar{\theta}}{2}-\lambda\left(\alpha_A+\alpha_B\right)\right)p&\\ +\frac{1}{3}\left(\frac{3\underline{\theta}-\bar{\theta}}{2}-\lambda\left(\alpha_A+\alpha_B\right)\right)p+C^0\left(\underline{\theta}\right)+C^0\left(\theta_i\right)&\\ -\frac{\lambda}{2}\left[\left(p^2-2p\alpha_A+\alpha_A^2\right)+\left(p^2-2p\alpha_B+\alpha_B^2\right)\right] &= -\frac{\lambda}{4}\Delta\alpha^2,\end{aligned}$$

where $p=p(\theta_i,\underline{\theta})$. This expression simplifies to

$$\begin{aligned}-\frac{\lambda+1}{3}p^2+\left(\frac{\frac{3}{2}\left(\theta_i+\underline{\theta}\right)-\bar{\theta}}{3}+\frac{\lambda\left(\alpha_A+\alpha_B\right)}{3}\right)p&\\ -\frac{\lambda+1}{6}\left(\frac{3\underline{\theta}-\bar{\theta}+\lambda\left(\alpha_A+\alpha_B\right)}{2\left(1+\lambda\right)}\right)^2+\frac{\lambda}{8}\left(\alpha_A+\alpha_B\right)^2+C^0\left(\theta_i\right) &= \frac{\lambda}{4}\left(\alpha_A+\alpha_B\right),\end{aligned}$$

which gives

$$\begin{aligned}C^0\left(\theta_i\right)= &-\tfrac{\lambda+1}{3}\left[\left(\tfrac{\frac{3}{2}\left(\theta_i+\underline{\theta}\right)-\bar{\theta}+\lambda\left(\alpha_A+\alpha_B\right)}{2(1+\lambda)}\right)^2-\tfrac{1}{2}\left(\tfrac{3\underline{\theta}-\bar{\theta}+\lambda\left(\alpha_A+\alpha_B\right)}{2(1+\lambda)}\right)^2\right]\\ &+\tfrac{\lambda}{8}\left(\alpha_A+\alpha_B\right)^2.\end{aligned}$$

Now we can check that the full expression for the contribution is increasing with respect to the lobby's own type, since

$$\frac{\partial C}{\partial \theta_i} = \frac{1}{2}\left[p\left(\theta_i, \theta_j\right) - p\left(\theta_i, \underline{\theta}\right)\right] \geq 0,$$

and the policy is increasing in the lobby's type. Additionally, this contribution satisfies the Spence-Mirrlees condition, since

$$\frac{\partial^2 C}{\partial \theta \partial p} = \frac{1}{2} > 0.$$

Polarized preferences

With polarized preferences and uniform distribution, we have

$$\begin{aligned} \frac{F\left(\theta_j\right)}{f\left(\theta_j\right)} &= \theta_j - \underline{\theta}, \\ \lambda W'\left(p\right) &= -\lambda\left(2p - \left(\alpha_A + \alpha_B\right)\right). \end{aligned}$$

Substituting into the first-order condition gives

$$-\left(p - \theta_i\right) + \frac{\partial C}{\partial p}\left(p, \theta_j\right) - \lambda\left(2p - \Delta\alpha\right) = -\left(\theta_j - \underline{\theta}\right)\frac{\partial^2 C}{\partial \theta_j \partial p}.$$

Thus the system turns out to be similar to the aligned case. It is given by

$$\begin{aligned} \theta_i - p + \frac{\partial C}{\partial p}\left(p, \theta_j\right) - \lambda\left(2p - \Delta\alpha\right) &= -\left(\theta_j - \underline{\theta}\right)\frac{\partial^2 C}{\partial \theta_j \partial p}, \\ -\theta_j - p + \frac{\partial C}{\partial p}\left(p, \theta_i\right) - \lambda\left(2p - \Delta\alpha\right) &= -\left(\theta_i - \underline{\theta}\right)\frac{\partial^2 C}{\partial \theta_i \partial p}, \\ \frac{\partial C}{\partial p}\left(p, \theta_i\right) + \frac{\partial C}{\partial p}\left(p, \theta_j\right) - \lambda\left(2p - \Delta\alpha\right) &= 0. \end{aligned}$$

The main difference lies in the expressions for the contributions. Since lobbies want to push policies in opposite directions, the contributions are no longer symmetric. Yet, they are still quadratic

$$\begin{aligned} C_A\left(\theta, p\right) &= \frac{g}{2}p^2 + p\left(e\theta + f + h\right) + C_A^0\left(\theta\right), \\ C_B\left(\theta, p\right) &= \frac{g}{2}p^2 - p\left(e\theta + f - h\right) + C_B^0\left(\theta\right). \end{aligned}$$

Given these contributions, the system of equations is given by

$$\begin{aligned}
\theta_i - p + gp - e\theta_j - f + h - \lambda\left(2p - \Delta\alpha\right) &= \left(\theta_j - \underline{\theta}\right) e, \\
-\theta_j - p + gp + e\theta_i + f + h - \lambda\left(2p - \Delta\alpha\right) &= -\left(\theta_i - \underline{\theta}\right) e, \\
2gp + 2h + e\left(\theta_i - \theta_j\right) - \lambda\left(2p - \Delta\alpha\right) &= 0.
\end{aligned}$$

Again, the following steps to compute the policies are similar to the aligned case. We omit them for the sake of exposition. The equilibrium policy is given by

$$p = \frac{\frac{3}{2}\left(\theta_i - \theta_j\right) + \lambda\Delta\alpha}{2\left(1+\lambda\right)}.$$

The final expression for the contributions are given by

$$\begin{aligned}
C_A &= \frac{2\lambda - 1}{3} p^2 + \left(\frac{\theta_i + \underline{\theta}}{2} - \frac{2\lambda\Delta\alpha}{3}\right) p + C_A^0\left(\theta_i\right) \text{ and} \\
C_B &= \frac{2\lambda - 1}{3} p^2 + \left(-\frac{\theta_i + \underline{\theta}}{2} - \frac{2\lambda\Delta\alpha}{3}\right) p + C_B^0\left(\theta_i\right),
\end{aligned}$$

where

$$\begin{aligned}
C_A^0\left(\theta_i\right) = \; & -\tfrac{\lambda(\Delta\alpha)^2}{8} + \tfrac{3\left(\bar{\theta} - \theta_i\right)}{4(1+\lambda)^2}\left[\tfrac{(\theta_i + \underline{\theta})(1+\lambda)}{2} + \lambda\Delta\alpha + \tfrac{(2\lambda-1)\left(\bar{\theta} - \theta_i\right)}{4}\right] \\
& -\tfrac{(\theta_i + \underline{\theta})\lambda\Delta\alpha}{4(1+\lambda)}
\end{aligned}$$

and

$$\begin{aligned}
C_B^0\left(\theta_i\right) = \; & -\tfrac{\lambda(\Delta\alpha)^2}{8} + \tfrac{3\left(\bar{\theta} - \theta_i\right)}{4(1+\lambda)^2}\left[-\tfrac{(\theta_i + \underline{\theta})(1+\lambda)}{2} + \lambda\Delta\alpha + (2\lambda - 1)\left(\bar{\theta} - \theta_i\right)\right] \\
& -\tfrac{(\theta_i + \underline{\theta})\lambda\Delta\alpha}{4(1+\lambda)}.
\end{aligned}$$

II. Examples

A. *Local public goods provision*

We adapt the model of provision of local public goods to our framework and consider three alternative institutional structures. In the first one the decision is decentralized and public budgets across districts are separate. In the second structure the district budgets are still separate but the policy decision is centralized and constrained to be uniform. Finally, we study a centralized decision structure without policy uniformity, while there is a two-stage budgeting process that is integrated across districts (a common pool financing).[1]

Two districts, $i \in \{A, B\}$, have to decide how much of a local public good to provide. We assume the following utility function[2] for consumers in district i

$$u_i(p) = \left(\alpha_i - 1 - \frac{p_i}{2}\right) p_i + y_i, \tag{A.6}$$

where, for consistency, we denote the amount of public good by p_i and income y_i, which is exogenous in this problem. Notice that this is a transformed version of the utility function presented in Section 2 of the paper.

The lobbies represent organized members of the society with higher valuation for the public good.[3] For example, lobbies could be interpreted as organized elites with preferences not aligned with the average citizen. Specifically, we assume that these elite preferences are given by

$$V(\theta_i, p_i, C_i) = \left(\theta_i - 1 - \frac{p_i}{2}\right) p_i - C_i,$$

where $\theta_i > \alpha_i$ and C_i denotes the money contribution to be paid to influence the policymaker(s). It is straightforward to see that these preferences are also transformations of the lobbies' preferences and correspond to the "aligned preferences" case.[4] We are now in a position to reinterpret our previous results comparing centralization and decentralization.

Decentralization When the public good decision is decentralized and budgets are separate, the results from the decentralized structure apply. The model is solved as a principal-agent game in which the policymaker only takes into consideration the welfare of his own district. The public good is provided up to the point where the lobby's marginal benefit equals the marginal cost of provision, and simple computations provide the policy implemented in this structure as given by

$$p_i = \frac{\theta_i + \lambda\alpha_i}{1 + \lambda} - 1,$$

[1] An alternative structure would combine a centralized decision without uniform policy and with a decentralized budget. In absence of inter-district externalities, this last is identical to a decentralized structure.

[2] We derive such utility function departing from a quasi-linear preference specification on public goods (p_i) and money (m_i) represented by

$$u_i(p_i, m_i) = \left(\alpha_i - \frac{p_i}{2}\right) p_i + m_i.$$

Consumers' income is denoted by y_i. The public good is provided by the government and financed through lump-sum income taxes τ_i. It is produced from income on a one-to-one basis. If the budgets are separate, then $\tau_i y_i = p_i$ and the consumer's budget constraint is given by $(1-\tau_i)y_i = m_i$, or $y_i = p_i + m_i$. This allow us to write the consumer's utility as presented below.

[3] Our results would be similar if lobbies had lower valuation for the public good.

[4] An alternative explanation for the difference between θ and α would be a difference in the marginal value of money for the fraction of society organized as a lobby.

which is just a re-parametrization of (4) in the paper.

Centralization: uniform public good provision and separate budgets Under uniform public good provision with separate district budgets, the lobbies offer contributions to the same policymaker, as in the "aligned preferences" case. Under perfect information, the level of public good is given by

$$\overline{p} = \frac{\theta_A + \theta_B + \lambda(\alpha_A + \alpha_B)}{2(1+\lambda)} - 1,$$

which is a simple re-parametrization of equation (6) from the manuscript. Under asymmetric information, when θ_i is private information of lobby i, and it is drawn from a uniform distributions in $[\underline{\theta}, \bar{\theta}]$, the policy under private information is given by

$$p^* = \frac{\frac{3}{2}(\theta_A + \theta_B) - \underline{\theta} + \lambda(\alpha_A + \alpha_B)}{2(1+\lambda)} - 1,$$

which is a simple re-parametrization of equation (12) from the manuscript.

Centralization: non-uniform public good provision and two-stage integrated budget Consider now a centralized system where the income tax is fixed at $\bar{\tau}$ as a result of a two-stage budget process and that the budget is a common pool such that $\overline{\tau}(y_A + y_B) = p_A + p_B$. Consequently, we have that $p_i = R - p_{-i}$, where $R = \overline{\tau}(y_A + y_B)$ is the amount of resources available for the provision of local public goods. Now an increase in the public good provided for district A decreases the amount of resources available for district B. The district social preferences in this case write as[5]

$$\begin{aligned} W_A(p_A) &= (1-\overline{\tau})\, y_A + \frac{1}{2}(\alpha_A - p_A)\, p_A, \\ W_B(p_A) &= (1-\overline{\tau})\, y_B + \frac{1}{2}(\alpha_B - R + p_A)(R - p_A). \end{aligned}$$

The lobby's preferences are similar to these and are given by

$$\begin{aligned} V_A(\theta_A, p_A, C_A) &= (1-\overline{\tau})\, y_A + \frac{1}{2}(\theta_A - p_A)\, p_A - C_A(p_A), \\ V_B(\theta_B, p_A, C_B) &= (1-\overline{\tau})\, y_B + \frac{1}{2}(\theta_B - R + p_A)(R - p_A) - C_B(p_A). \end{aligned}$$

[5]Notice, however, that the preferences are not directly comparable to the preferences derived in (A.6). The indirect utility form derived in (A.6) considers that the income tax is not fixed, so that consumers pay the marginal cost of production of the local public good. In the current institutional setting, the marginal cost of producing one good is the reduction in the other district's good.

With a non-uniform public good decision and a two-stage integrated budget process, lobbies have polarized preferences. The reason is that the budget is now already set when lobbies offer contributions to the policymaker, so what a lobby can get in terms of his district-specific public good is just what the other lobby cannot get for his own district. The policy that emerges under perfect information within this structure is given by

$$p_A = \frac{R}{2} + \frac{\theta_A - \theta_B + \lambda \Delta\alpha}{2(1+\lambda)},$$

which is a re-parametrization of equation (7) from the manuscript. Under private information the policy that solves the political game is given by[6]

$$p_A = \frac{R}{2} + \frac{\frac{3}{2}(\theta_A - \theta_B) + \lambda \Delta\alpha}{2(1+\lambda)},$$

which is a re-parametrization of equation (13) from the manuscript.

B. *Tariff protection in customs unions*

We consider a simple partial equilibrium model with a good x that can be imported by both countries A and B.[7] When the domestic price of good x in country $i \in \{A, B\}$ is p_i, the domestic demand for good x is given by

$$x_i(p_i) = a - bp_i,$$

with $a, b > 0$. In each country, good x is produced using labor and a specific factor that is in limited supply. Consequently, producers have capacity constraints. To simplify the analysis, the marginal cost of production is set to zero for production below the output capacity. Therefore, the sector's competitive profits are given by $\pi_i(p_i) = \gamma p_i$, where γ is the capacity constraint. For simplicity we set $\gamma = 1$.

Each government collects import taxes with a tariff revenue given by

$$TR = (p_i - p^e)(x(p_i) - y(p_i)),$$

where p^e is the international price of good x, $y(p)$ is the home supply of x which, by the envelope theorem, is equal to γ. With such specifications, the sum of the firm's profits, consumer surplus, and the government's tariff revenue gives the

[6]Notice that the expression depends on the amount of resources R available for the public good. Under this particular institutional setting, we assume that this variable is exogenously chosen prior to the realization of the lobbies types. It is not difficult, however, to show that this expression will never reproduce the level of public good achieved in a centralized decision with separate budgets, no matter how much resources are available, provided the amount of resources is not itself a function of the lobbies' types.

[7]There is also a numeraire good produced from labor only in a one-to-one rate of transformation.

welfare of the society, which takes the following quadratic form

$$W_i\left(p_i\right) = \overline{w} - \frac{b}{2}\left(p_i - p^e\right)^2,$$

where $\bar{w}$ is a constant that is a function of the parameters. Notice this is a rescaled version of the welfare function presented in Section 2 in the paper.

A political influence game takes place within each economy. The lobby of each country offers contributions C_i to the policymaker in order to influence the tariff decision. Each economy has a lobby that represents the producers of good x. Lobbies are "principals" of the political game. Their utility function is given by[8]

$$V\left(\theta_i, p_i, C_i\right) = \theta_i p_i - C_i.$$

Policymakers are agents in the political game. The two countries may form a customs union or retain a non-coordinated trade policy. Without the agreement, the policy decision is decentralized. Each country delegates its trade policy decision to a national policymaker who chooses the import tariff of the economy or, equivalently, the economy's domestic price p_i. In country i, the policymaker's preferences are given by

$$U_i\left(p_i, C_i\right) = C_i + \lambda W_i\left(p_i\right),$$

where λ is the relative preference between contributions and welfare.

If the two countries sign a customs union agreement, they delegate the policy choice to a single policymaker who is restricted to setting a uniform policy (the tariffs of the two economies are the same). This is a centralized decision making setting. In this case the policymaker's preferences are given by

$$U\left(p, C_A, C_B\right) = \Sigma_i C_i + 2\lambda W\left(p\right).$$

Notice that the lobby's preference is linear in the policy p_i. This is a little different from the quadratic function[9] presented in Section 2 in the paper. Yet, this is a case of aligned preferences, and the results are similar to our baseline model,[10] with $\alpha_A = \alpha_B = p^e$. Assuming that θ's are drawn from an i.i.d. uniform distribution over $\left[\underline{\theta}, \overline{\theta}\right]$, with $3\underline{\theta} > \overline{\theta}$, we can apply the results from Section 3 in

[8]This utility function comes from the fact that lobbies care about the sector's profits and dislike giving money contributions. We assume the production function has a capacity constraint given by one. Then profits are given by p_i. Plus, lobbies have an organization cost of providing contributions so that one dollar put in the lobby turns into $1/\theta_i$. This allows us to represent the lobbies preferences by the given utility function.

[9]Here, the utility function has an infinite bliss point. The value θ now measures the constant marginal benefit of the policy.

[10]We have not developed the model with a linear objective function, but all the results remain, except that expressions are slightly different.

the paper. Domestic prices without a trade agreement (with perfect and with asymmetric information) are therefore given by

$$\check{p}\left(\theta_i\right) = p^e + \frac{\theta_i}{\lambda b}.$$

Under a customs union with perfect information these prices are given by

$$\overline{p}\left(\theta_i, \theta_j\right) = p^e + \frac{\theta_i + \theta_j}{2\lambda b},$$

where it is easy to see that the customs union implements the average tariff. Under a customs union with privately informed lobbies, domestic prices become

$$p^*\left(\theta_i, \theta_j\right) = p^e + \frac{1}{2\lambda b}\left(\frac{3}{2}\left(\theta_i + \theta_j\right) - \overline{\theta}\right) = \overline{p}(\theta_i, \theta_j) - \frac{1}{2\lambda b}\left(\overline{\theta} - \frac{\theta_i + \theta_j}{2}\right).$$

It is simple matter to see that $\overline{p}\left(\theta_i, \theta_j\right) - p^*\left(\theta_i, \theta_j\right) \geq 0$. Therefore, there is less protection in a customs union agreement when lobbies have private information.

From a social welfare perspective it is important to notice that the two countries' optimal policy is free trade. Consequently, this model is similar to a linear version of our baseline model with $\alpha_A = \alpha_B$. Thus, under perfect information, customs union agreements are always welfare superior to the decentralized protectionist game in each country. Therefore, there is no *uniformization* effect associated with centralized decision making. Only the *preference dilution* effect remains, which promotes the customs union regime (i.e., centralized decision making). When lobbies have private information, the *information transmission* effect provides an additional boost in favor of the customs union mechanism.[11]

III. Mechanism design approach

In this section we compute the equilibrium policies with reversed bargaining power. The setting is the same of the lobbying game, except that now the policymaker proposes the mechanism. Since lobbies have private information, the model becomes a standard principal-agent model under decentralization and a multi-agent model under centralization.

We use the MDA to solve these agency problems. Under decentralized structure,

[11] Obviously, this model is extremely simple and the results must be viewed as illustrative of how lobbies' private information may interact with the trade policy mechanisms discussed in the literature. Direct trade effects (trade diversion, trade creation, terms of trade) are important to qualify the potential gains from a customs union agreement. However, when lobbies have private information, centralization of decision making gives policymakers additional bargaining power to negotiate with other rent-seekers. Hence, the *information transmission* effect is still likely to have a stake at the decision to form a customs union.

the policymaker is the principal with utility defined by

$$U_i(p, C) = C + \lambda W_i(p),$$

where $W_i(p)$ is defined by (1) in the paper. He contracts with one agent (lobby group) with utility function given by

$$V(\theta, p, C) = -\frac{1}{2}(p - \theta)^2 - C,$$

where θ is the lobby's type. Under centralized structure, the policymaker is the principal with utility defined by

$$U(p, C_A, C_B) = \sum_i U_i(p, C_i)$$

who contracts with two agents (lobby groups) with utility functions $V_i(\theta_i, p, C_i)$, where θ_i is the lobby i's type, $i \in \{A, B\}$.

A. *Decentralized structure and aligned preferences*

Perfect information The equilibrium policy is given by (4) in the paper. Under decentralized structure with perfect information and reversed bargaining power, the equilibrium policy does not change.

Asymmetric information In the decentralized structure each policymaker offers a contract to the lobby of its own district. The contract is a pair $(C(\theta_i), p(\theta_i))$ of contribution and policy contingent on the "announced" type by the lobby.

In what follows we use the standard approach to solving principal-agent problems. Defining the lobby's rent function as $\mathcal{V}(\theta) = V(\theta, p(\theta), C(\theta))$ and solving for $C(\theta)$, we have that the policymaker's maximization problem is given by

$$\max_{\mathcal{V}(\cdot), p(\cdot)} E\left[-\frac{1}{2}(p(\theta) - \theta)^2 - \frac{\lambda}{2}(p(\theta) - \alpha_i)^2 - \mathcal{V}(\theta)\right]$$

subject to

$$\begin{array}{rcl} \dot{\mathcal{V}}(\theta) & = & p(\theta) - \theta, \\ p(\theta) & \text{is} & \text{non-decreasing}, \\ \mathcal{V}(\theta) & \geq & \bar{V} \end{array}$$

where the first constraint is the envelope condition, the second is the monotonicity condition and the third is the participation constraint; $\bar{V}$ represents the lobby's outside option and $E[\cdot]$ is the expectation operator. Substituting

$$\mathcal{V}(\theta) - \mathcal{V}(\underline{\theta}) = \int_{\underline{\theta}}^{\theta} (p(x) - x)\, dx,$$

and assuming that the participation constraint binds at $\underline{\theta}$, the problem is equivalent to

$$\max_{\mathcal{V}(\cdot),p(\cdot)} E\left[-\frac{1}{2}(p(\theta)-\theta)^2 - \frac{\lambda}{2}(p(\theta)-\alpha_i)^2 - \int_{\underline{\theta}}^{\theta}(p(x)-x)\,dx\right].$$

After an integration by parts, we finally have

$$\max_{\mathcal{V}(\cdot),p(\cdot)} E\left[-\frac{1}{2}(p(\theta)-\theta)^2 - \frac{\lambda}{2}(p(\theta)-\alpha_i)^2 - \frac{1-F(\theta)}{f(\theta)}(p(\theta)-\theta)\right].$$

The first-order condition for the pointwise maximization under the uniform distribution is

$$-p(\theta)(1+\lambda) + \theta + \lambda\alpha_i - \left(\bar{\theta}-\theta\right) = 0,$$

which gives

$$\tilde{p}_i^{md}(\theta) = \tilde{p}_i(\theta) - \frac{\bar{\theta}-\theta}{1+\lambda},$$

where md stands for "mechanism design" and $\tilde{p}_i(\theta)$ is defined by (4) in the paper. Hence, the lobby's influence decreases and policies are closer to the policymaker's bliss point α_i.

B. *Decentralized structure and polarized preferences*

Perfect information Again the equilibrium policy is given by (4) in the paper.

Asymmetric information In this case, lobby B's preference is slightly different and the policy is given by

$$\tilde{p}_B^{md}(\theta_B) = \tilde{p}_B(\theta_B) + \frac{\bar{\theta}-\theta_B}{1+\lambda},$$

with the same interpretation as above.

C. *Centralized structure and aligned preferences*

Perfect information The policymaker must solve

$$\max_{C_A, C_B, p} U(p, C_A, C_B)$$

subject to

$$V(\theta_i, p, C_i) \geq \underline{V}_i, \ i = A, B,$$

where $\underline{V}_i$ is the lobby i's outside option. Therefore, the policy that solves this program is identical to the one derived in the common agency game, i.e., $\overline{p}(\theta_A, \theta_B)$ defined by (6) in the paper.

Asymmetric information The policymaker offers a menu of contracts conditional on the lobbies' type report. Since reports are simultaneous, each lobby chooses her contract without knowing the rival's choice. And since lobbies' types are not correlated, the proposed contract to one lobby is similar to the one that would be proposed in case the policymaker were dealing with a single lobby.

A mechanism proposed by the policymaker is a pair of contributions and policies conditional on reported types: $(C(\theta_i, \theta_j), p(\theta_i, \theta_j))$. The lobby i's rent function on the proposed mechanism is

$$\mathcal{V}_i(\theta) = -E\left[\frac{1}{2}(p(\theta, \cdot) - \theta_i)^2 + C(\theta, \cdot)\right],$$

where $E[\cdot]$ is the expectation operator with respect to the rival type.

Incentive compatibility is equivalent to

$$\mathcal{V}_i(\theta) \geq -E\left[\frac{1}{2}\left(p(\tilde{\theta}, \cdot) - \theta\right)^2 + C(\tilde{\theta}, \cdot)\right]$$

for all $\theta, \tilde{\theta}$. The envelope (first-order) condition gives

$$\dot{\mathcal{V}}_i(\theta) = E[p(\theta, \cdot)] - \theta, \tag{A.7}$$

where dot represents the derivative with respect to θ, and the monotonicity (second-order) condition is equivalent to[12,13]

$$\left(E[p(\theta, \cdot)] - E\left[p(\tilde{\theta}, \cdot)\right]\right)(\theta - \tilde{\theta}) \geq 0, \tag{A.8}$$

i.e., $E[p(\theta, \cdot)]$ is non-decreasing. Reciprocally, suppose that (A.7) and (A.8) hold.

[12] To derive this condition we only need to interchange θ and $\tilde{\theta}$ and combine the resulting inequalities of the IC constraints.

[13] If contracts are differentiable, this condition is equivalent to $E\left[\frac{\partial p}{\partial \theta}(\theta, \cdot)\right] \geq 0$.

Hence, (A.7) gives

$$\mathcal{V}_i(\theta) - \mathcal{V}_i(\tilde{\theta}) = \int_{\tilde{\theta}}^{\theta} \left(E\left[p(x,\cdot)\right] - x\right) dx$$

and (A.8) implies

$$\mathcal{V}_i(\theta) \geq \mathcal{V}_i(\tilde{\theta}) + (\theta - \tilde{\theta}) \left(E\left[p(\tilde{\theta},\cdot)\right] - \frac{\theta + \tilde{\theta}}{2} \right),$$

which is equivalent to incentive compatibility.

Notice that expected transfers can be expressed in terms of the lobby's rent function as

$$E\left[C(\theta,\cdot)\right] = -\frac{1}{2}E\left[(p(\theta,\cdot) - \theta)^2\right] - \mathcal{V}_i(\theta).$$

The policymaker's maximization problem is

$$\max_{\mathcal{V}_i(\theta_i), p(\theta_i,\theta_j)} E\left[\sum_i \left(-\frac{1}{2}E\left[(p(\theta_i,\cdot) - \theta_i)^2 + \lambda\left(p(\theta_i,\cdot) - \alpha_i\right)^2\right] - \mathcal{V}_i(\theta_i)\right)\right]$$

subject to

$$\begin{array}{lll} \dot{\mathcal{V}}_i(\theta_i) & = & E\left[p(\theta_i,\cdot)\right] - \theta_i \\ E\left[p(\theta_i,\cdot)\right] & \text{is} & \text{non-decreasing} \\ \mathcal{V}_i(\theta_i) & \geq & \underline{V}_i,\ i = A, B. \end{array}$$

Ignoring the monotonicity condition and assuming that the participation constraint binds only at $\underline{\theta}$ and after an integration by parts, the problem becomes:

$$\max_{p(\theta_i,\theta_j)} E\left[\sum_i \left(-\frac{1}{2}E\left[(p(\theta_i,\cdot) - \theta_i)^2 + \lambda\left(p(\theta_i,\cdot) - \alpha_i\right)^2 - \frac{1 - F(\theta_i)}{f(\theta_i)}(p(\theta_i,\cdot) - \theta_i)\right]\right)\right].$$

The first-order condition is given by

$$-2p + \theta_i + \theta_j - 2\lambda p + \lambda\left(\alpha_A + \alpha_B\right) - \frac{1 - F\left(\theta_i\right)}{f\left(\theta_i\right)} - \frac{1 - F\left(\theta_j\right)}{f\left(\theta_j\right)} = 0,$$

where, for convenience, we are omitting the dependence of p on (θ_i, θ_j). Under the uniform distribution this first-order condition becomes

$$-2p + \theta_i + \theta_j - 2\lambda p + \lambda\left(\alpha_A + \alpha_B\right) - \left(\bar{\theta} - \theta_i\right) - \left(\bar{\theta} - \theta_j\right) = 0$$

and its solution is given by

$$p^{*md}(\theta_i, \theta_j) = \bar{p}(\theta_i, \theta_j) - \frac{2\bar{\theta} - (\theta_i + \theta_j)}{2(1+\lambda)},$$

where $\bar{p}(\theta_i, \theta_j)$ is defined by (6) in the paper. While the solution of the centralized common agency game is given by

$$p^{*}(\theta_i, \theta_j) = \bar{p}(\theta_A, \theta_B) - \frac{2\bar{\theta} - (\theta_i + \theta_j)}{4(1+\lambda)}$$

(see (12) in the paper). One can easily compute the contributions as we did under the PEA. To avoid countervailing incentive problem, we are implicitly assuming that the participation constraint only binds at the bottom type $\underline{\theta}$.

Compared to the common agency equilibrium policies, information distortions are greater under the MDA and policies closer to the welfare optimum. This is because the policymaker is able to extract more rents from lobbies privately informed about their preferred policy. The lobby's private information is directly relevant for the policymaker. In the common agency game, lobbies only care about the policymaker's endogenous cost of providing a favorable policy. This cost is an endogenous function of the rival's type realization. So the rival lobby's private information is only indirectly relevant, which results in a greater stake for information distortion under the MDA.

D. *Centralized structure and polarized preferences*

Perfect information — The solution here is again identical to the common agency game solution, i.e., the equilibrium policy is given by $\overline{p}(\theta_i, \theta_j)$ defined by (7) in the paper.

Asymmetric information — Incentive compatibility is exactly the same as in the aligned preferences case for lobby A. The lobby B's rent function on the proposed mechanism is now

$$\mathcal{V}_i(\theta) = -E\left[\frac{1}{2}\left(p\left(\theta,\cdot\right)+\theta\right)^2 + C\left(\theta,\cdot\right)\right]$$

and incentive compatibility is equivalent to

$$\mathcal{V}_i(\theta) \geq -E\left[\frac{1}{2}\left(p\left(\tilde{\theta},\cdot\right)+\theta\right)^2 + C\left(\tilde{\theta},\cdot\right)\right]$$

for all $\theta, \tilde{\theta}$. The envelope (first-order) condition gives

$$\dot{\mathcal{V}}_i(\theta) = -\left(E\left[p(\theta,\cdot)\right] + \theta\right) \tag{A.9}$$

and the monotonicity (second-order) condition is equivalent to[14]

$$\left(E\left[p(\theta,\cdot)\right] - E\left[p(\tilde{\theta},\cdot)\right]\right)(\tilde{\theta} - \theta) \geq 0, \tag{A.10}$$

i.e., $E\left[p(\theta,\cdot)\right]$ is non-increasing. Reciprocally, suppose that conditions (A.9) and (A.10) hold. Hence, condition (A.9) implies

$$\mathcal{V}_i(\theta) - \mathcal{V}_i(\tilde{\theta}) = -\int_{\tilde{\theta}}^{\theta} \left(E\left[p(x,\cdot)\right] + x\right) dx$$

and condition (A.10) implies

$$\mathcal{V}_i(\theta) \geq \mathcal{V}_i(\tilde{\theta}) - (\theta - \tilde{\theta})\left(E\left[p(\tilde{\theta},\cdot)\right] + \frac{\theta + \tilde{\theta}}{2}\right),$$

which is equivalent to the incentive compatibility constraint.

Notice that expected transfers can be expressed in terms of the lobby B's rent function:

$$E\left[C(\theta,\cdot)\right] = -\frac{1}{2}E\left[(p(\theta,\cdot)+\theta)^2\right] - \mathcal{V}(\theta).$$

The policymaker's maximization problem is

$$\max_{\mathcal{V}_A(\theta_A),\mathcal{V}_B(\theta_B),p(\theta_A,\theta_B)} E\left[-\frac{1}{2}\left((p-\theta_A)^2 + (p+\theta_B)^2\right) - \sum_i \left(\frac{\lambda}{2}E\left[(p(\theta_i,\cdot)-\alpha_i)^2\right] + \mathcal{V}_i(\theta_i)\right)\right]$$

subject to

$$\begin{array}{ll}
\dot{\mathcal{V}}_A(\theta_A) = E\left[p(\theta_A,\cdot)\right] - \theta_A, & \dot{\mathcal{V}}_B(\theta_B) = -\left(E\left[p(\cdot,\theta_B)\right] + \theta_B\right) \\
E\left[p(\theta_A,\cdot)\right] \text{ is non-decreasing in } \theta_A & E\left[p(\cdot,\theta_B)\right] \text{ is non-increasing in } \theta_B \\
\mathcal{V}_A(\theta_A) \geq 0, & \mathcal{V}_B(\theta_B) \geq 0.
\end{array}$$

Ignoring the monotonicity condition and assuming that the participation con-

[14] If contracts are differentiable, this condition is equivalent to $E\left[\frac{\partial p}{\partial \theta}(\theta,\cdot)\right] \leq 0$.

straints bind at $\underline{\theta}$, the problem after an integration by parts becomes:

$$\max_{p(\theta_i,\theta_j)} E\left[\begin{array}{l} -\frac{1}{2}\left((p-\theta_A)^2+(p+\theta_B)^2\right)-\frac{\lambda}{2}\sum_i E\left[(p-\alpha_i)^2\right] \\ +\frac{F(\theta_A)}{f(\theta_A)}(p-\theta_A)-\frac{F(\theta_B)}{f(\theta_B)}(p+\theta_B) \end{array}\right].$$

The first-order condition is given by

$$-2p+\theta_A-\theta_B-2\lambda p+\lambda\left(\alpha_A+\alpha_B\right)+\frac{F\left(\theta_A\right)}{f\left(\theta_A\right)}-\frac{F\left(\theta_B\right)}{f\left(\theta_B\right)}=0.$$

Under the uniform distribution we have

$$-2p+\theta_A-\theta_B-2\lambda p+\lambda\left(\alpha_A+\alpha_B\right)+(\theta_A-\underline{\theta})-(\theta_B-\underline{\theta})=0$$

and its solution is given by

$$p^{*md}(\theta_A,\theta_B)=\bar{p}(\theta_A,\theta_B)+\frac{\theta_A-\theta_B}{2\left(1+\lambda\right)},$$

while the solution of the centralized common agency game is given by

$$p^{*}(\theta_A,\theta_B)=\bar{p}(\theta_A,\theta_B)+\frac{\theta_A-\theta_B}{4\left(1+\lambda\right)},$$

where p^* is defined by (13) in the paper. One can easily compute the contributions as we did under the PEA. To avoid countervailing incentive problem, we are implicitly assuming that the participation constraint only binds at the top type $\overline{\theta}$.

The information distortions are also greater in the polarized case. However, in this setting the information distortion is detrimental to lobbying competition and welfare, which results in more distortionary policies.

The obtained results may be then summarized in the following proposition:

PROPOSITION 1: *Compared to the Political Economy Approach (PEA) model described in Section 2 of the paper, under the Mechanism Design Approach (MDA) in which the policymaker is now the principal, we have the following:*
(i) information distortions are greater;
(ii) in decentralized and centralized settings with aligned preferences, society's welfare is higher and equilibrium policies are closer to the welfare optimum;
(iii) in a centralized setting with polarized preferences, society's welfare is lower and the equilibrium policy is further away from the welfare optimum.

IV. Pooling equilibria

Consider the forcing contribution that targets policy p^0:

$$C^0(p) = \begin{cases} -\frac{1}{2}W(p) & \text{if } p = p^0 > 0 \\ 0 & \text{otherwise} \end{cases},$$

where $W(p) = W_A(p) + W_B(p)$ is the utilitarian welfare of both entities. When both lobbies offer this contract, they share equally the cost of p^0. When both lobbies offer this contract, they share equally the cost of p^0.

For the aligned preference case, let $\tilde{\mathcal{W}}(\theta_i) = \max_p -\frac{1}{2}(p - \theta_i)^2 + W(p)$ be the aggregate welfare of a bilateral coalition between a lobby i with type θ_i and the policymaker, and the corresponding optimal (increasing in type) policy $\tilde{p}(\theta_i) = \frac{\theta_i + \lambda(\alpha_A + \alpha_B)}{1+2\lambda}$. The following proposition is analogous to Proposition 2 of Martimort and Moreira (2010).

PROPOSITION 2: *(Aligned preferences) Suppose that* $-(p^0 - \underline{\theta})^2 + W(p^*) \geq 2\tilde{\mathcal{W}}(\underline{\theta})$ *and* $p^0 \geq \tilde{p}(\overline{\theta})$. *There exists a pooling equilibria in which both lobbies offer* $C^0(p)$ *whatever their types are.*

PROOF:

Suppose that lobby B offers $C^0(p)$ whatever his own type. The policymaker learns nothing from this offer and has no endogenous private information. Consider lobby A's best response. Two possibilities arises. First, he may agree with lobby B and induce the policymaker to target p^0. This is done by offering also $C^0(p)$ whatever his type is. This yields welfare

$$\tilde{\mathcal{W}}^0(\theta_A) = -\frac{1}{2}(p^0 - \theta_A)^2 + \frac{1}{2}W(p^0).$$

The second possibility is that lobby A deviates and induces another policy. The best of such deviation should solve

$$\max_{p, C_A(\cdot,\theta_A)} -\frac{1}{2}(p - \theta_A)^2 - C_A(p, \theta_A)$$

subject to

$$C_A(p, \theta_A) + W(p) \geq \max \left\{0, \frac{1}{2}W(p^0)\right\} = 0,$$

where this last condition is the agent's participation constraint. This best deviation implements the policy $\tilde{p}(\theta_A)$ with a forcing contract

$$C(p, \theta_A) = \begin{cases} -W(\tilde{p}(\theta_A)) > 0 & \text{for } p = \tilde{p}(\theta_A) \\ 0 & \text{for } p \neq \tilde{p}(\theta_A) \end{cases}$$

which gives payoff $\tilde{\mathcal{W}}(\theta_A)$ to the deviating lobby. This deviation is unprofitable when

$$\tilde{\mathcal{W}}^0(\theta_A) = -\frac{1}{2}(p^0 - \theta_A)^2 + \frac{1}{2}W(p^0) \geq \tilde{\mathcal{W}}(\theta_A), \text{ for all } \theta_A \in \Theta.$$

Since $\tilde{\mathcal{W}}^{0\prime}(\theta_A) = p^0 - \theta_A \geq \tilde{p}(\overline{\theta}) - \theta_A \geq \tilde{p}(\theta_A) - \theta_A = \tilde{\mathcal{W}}'(\theta_A)$, the previous inequality holds everywhere if it holds also at $\underline{\theta}$.

Hence, offering $C^0(p)$ is the best response for all θ_A under the assumptions of the proposition.

REMARK 1: *(Interpretation) If preferences are aligned, from the characterization of $\bar{p}(\cdot,\cdot)$ ((6) in the paper) and the definition of p^0 in Proposition 2, we have that*

$$p^0 \geq \tilde{p}(\overline{\theta}) \geq \bar{p}(\overline{\theta}, \overline{\theta})$$

if and only if $\overline{\theta} \leq \alpha_A + \alpha_B$, i.e., the common policymaker has greater bliss point the sum of the lobbies highest type's bliss point. In this case, since $\tilde{p}(\overline{\theta}) \in [\overline{\theta}, \alpha_A + \alpha_B]$, if we choose a pooling policy $p^0 \in [\tilde{p}(\overline{\theta}), \overline{\theta}]$, the pooling equilibrium leads to lower distortion than the separating equilibrium characterized by (12) in the paper. In this case the pooling equilibrium may be harmful for the lobbies and welfare improving. Otherwise, the pooling may have more or less distortion than the separating equilibrium and there is no clear welfare ranking between the pooling and the separating equilibria.

If preferences are polarized, the same conclusions can be obtained with only one main difference. Because of the competition effect, pooling can now be beneficial for the lobbies when compared with the separating equilibrium in the same fashion we have discussed in the main text when we compared aligned with polarized preferences.

REMARK 2: *(Bunching) In the same fashion of Martimort and Moreira (2010) - see p. 183 for their discussion about bunching -, we can also define equilibria that combine ranges of equilibrium policy with separating and pooling regions.*

REMARK 3: *(Existence of pooling equilibria) Notice that condition $-(p^0 - \underline{\theta})^2 + W(p^0) \geq 2\tilde{\mathcal{W}}(\underline{\theta})$ in Proposition 2 is equivalent to*

(A.11)
$$-(p^0 - \underline{\theta})^2 - \frac{\lambda}{2}\left[(p^0 - \alpha_A)^2 + (p^0 - \alpha_B)^2\right] \geq -(\tilde{p}(\underline{\theta}) - \underline{\theta})^2 - \lambda\left[(\tilde{p}(\underline{\theta}) - \alpha_A)^2 + (\tilde{p}(\underline{\theta}) - \alpha_B)^2\right].$$

By the definition of $\tilde{p}(\underline{\theta})$, the right hand side of (A.11) is the maximum value of that quadratic expression as a function of variable $x = \tilde{p}(\underline{\theta})$. Taking $x = p^0 = \tilde{p}(\underline{\theta})$, (A.11) becomes

$$-(x - \underline{\theta})^2 - \frac{\lambda}{2}\left[(x - \alpha_A)^2 + (x - \alpha_B)^2\right] \geq -(x - \underline{\theta})^2 - \lambda\left[(x - \alpha_A)^2 + (x - \alpha_B)^2\right]$$

or

$$x^2 - (\alpha_A + \alpha_B)x + \frac{1}{2}(\alpha_A^2 + \alpha_B^2) \geq 0,$$

which is always true. Hence, there exists $\bar{p}^0 > \tilde{p}(\underline{\theta})$ *solution of the quadratic equation in the variable* p^0 *defined by substituting the inequality in (A.11) by an equality. Now it is easy to check that we can find* $p^0 \geq \tilde{p}(\overline{\theta})$ *satisfying (A.11) if and only if* $\tilde{p}(\overline{\theta}) \in (\tilde{p}(\underline{\theta}), \bar{p}^0)$*. If* $\Delta\theta$ *is sufficiently small, this last condition is satisfied. Therefore, if* $\Delta\theta$ *is sufficiently small, there exists a pooling equilibrium.*