An Online Appendix to
"Multidimensional Platform Design"*

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January 31, 2017

Abstract

This appendix provides formal details to accompany the forthcoming article "Multidimensional Platform Design" in the 2017 *American Economic Review Papers and Proceedings* and available at `https://ssrn.com/abstract=2891806`.

1 Model Setup

A monopolistic platform chooses a vector of characteristics $\rho \in \mathbb{R}^n$ and a special "distinguished" characteristic $p \in \mathbb{R}$. p is viewed as harmful by all users and is always beneficial to the platform; it may represent price but need not. There is a unit mass of potential users each characterized by a type vector $\theta \in \mathbb{R}^m$ distributed according to a full-support, strictly positive density, $f : \mathbb{R}^m \to \mathbb{R}_{++}$. The type vector θ determines both the contribution that users make to the platform's value and their taste for using the platform. Let $\phi(\theta)$ be a vector of user characteristics that impact both the platform's profit and its perceived value to users at large. We assume that the profits of the platform and its value to users depend only on the aggregate value of these features. That is, if the set of users participating in the platform is Θ, then platform profits and user utilities are functions of ρ and $\Phi \equiv \int_\Theta \phi(\theta) f(\theta)\, \mathrm{d}\theta$. Moreover, given the platform's characteristics, a user's utility also depends on the particular user's type. Assume that a function u exists such that users participate on the platform exactly if $u(\rho, \Phi; \theta) \geq p$. The platform earns profits given by $\pi(\rho, \Phi, p)$ that are strictly increasing in p everywhere. Then we have $\Theta = \{\theta : u(\rho, \Phi; \theta) \geq p\}$. Assume that f, ϕ and π are continuously differentiable in all of their arguments. Finally, let $\partial\Theta = \{\theta : u(\rho, \Phi; \theta) = p\}$ denote the set of marginal users.

*We thank Chang "Leo" Liu for excellent research assistance. All errors are ours.

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2 Proposition 1

Proof. The platform faces the following maximization problem:

$$\begin{array}{ll} \max\limits_{\rho,p} & \pi(\rho,\Phi,p) \\ \text{s.t.} & \int_\Theta \phi(\theta)\, f(\theta)\, \mathrm{d}\theta = \Phi. \end{array}$$

The Lagrangian associated with this problem is defined by

$$\mathcal{L}(\rho,\Phi,p,\lambda) = \pi(\rho,\Phi,p) + \lambda^\top \left(\int_\Theta \phi(\theta)\, f(\theta)\, \mathrm{d}\theta - \Phi \right). \tag{1}$$

It is convenient to decompose $\theta = (\zeta,\tau)$, where ζ has one less dimension than θ, and $\tau \in (\underline{\tau},\overline{\tau}) \subseteq \mathbb{R}$. We assume a dimension of type τ exists such that $\frac{\partial u(\rho,\Phi;\zeta,\tau)}{\partial\tau} > 0$ and $\forall(\rho,\Phi,\zeta)$, $\lim\limits_{\tau\to\overline{\tau}} u(\rho,\Phi;\zeta,\tau) = \infty$ and $\lim\limits_{\tau\to\underline{\tau}} u(\rho,\Phi;\zeta,\tau) \leq 0$. Given this assumption, a unique function $\widetilde{\tau} = \widetilde{\tau}(\rho,\Phi,p;\zeta)$ exists such that $u(\rho,\Phi;\zeta,\widetilde{\tau}(\rho,\Phi,p;\zeta)) = p$.

First, we use the implicit function theorem on the equation that defines the margin, $u(\rho,\Phi;\zeta,\widetilde{\tau}(\rho,\Phi,p;\zeta)) - p = 0$, to obtain

$$\frac{\partial\widetilde{\tau}}{\partial p} = \frac{1}{\frac{\partial u}{\partial\tau}(\rho,\Phi;\zeta,\widetilde{\tau})}, \quad \frac{\partial\widetilde{\tau}}{\partial\rho_i} = -\frac{\frac{\partial u}{\partial\rho_i}(\rho,\Phi;\zeta,\widetilde{\tau})}{\frac{\partial u}{\partial\tau}(\rho,\Phi;\zeta,\widetilde{\tau})}, \quad \frac{\partial\widetilde{\tau}}{\partial\Phi_j} = -\frac{\frac{\partial u}{\partial\Phi_j}(\rho,\Phi;\zeta,\widetilde{\tau})}{\frac{\partial u}{\partial\tau}(\rho,\Phi;\zeta,\widetilde{\tau})}.$$

Now we have more convenient expressions of Θ and $\partial\Theta$:

$$\Theta = \{\theta : \tau \geq \widetilde{\tau}(\rho,\Phi,p;\zeta)\}, \quad \partial\Theta = \{\theta : \tau = \widetilde{\tau}(\rho,\Phi,p;\zeta)\}.$$

Thus, the density of users in this marginal set M is given by

$$M \equiv -\frac{\partial}{\partial p}\int_\Theta f(\theta)\,\mathrm{d}\theta = -\frac{\partial}{\partial p}\int_\zeta \int_{\widetilde{\tau}(\rho,\Phi,p;\zeta)}^{\overline{\tau}} f(\zeta,\tau)\,\mathrm{d}\tau\,\mathrm{d}\zeta = \int_\zeta \left(\frac{\partial\widetilde{\tau}}{\partial p}\right) f(\zeta,\widetilde{\tau})\,\mathrm{d}\zeta = \int_\zeta \frac{f(\zeta,\widetilde{\tau})}{\frac{\partial u}{\partial\tau}(\rho,\Phi;\zeta,\widetilde{\tau})}\,\mathrm{d}\zeta.$$

In light of this expression, for an arbitrary smooth function $z(x,\theta)$, the expectation conditional on $\partial\Theta$ is *defined as* (not standard)

$$\mathbb{E}\left[z(x,\theta)\middle|\,\partial\Theta\right] \equiv \frac{1}{M}\int_\zeta z(x,\zeta,\widetilde{\tau})\,\frac{f(\zeta,\widetilde{\tau})}{\frac{\partial u}{\partial\tau}(\rho,\Phi;\zeta,\widetilde{\tau})}\,\mathrm{d}\zeta.$$

From

$$\int_\Theta \phi(\theta)\, f(\theta)\,\mathrm{d}\theta = \int_\zeta \int_{\widetilde{\tau}(\rho,\Phi,p;\zeta)}^{\overline{\tau}} \phi(\zeta,\tau)\, f(\zeta,\tau)\,\mathrm{d}\tau\,\mathrm{d}\zeta,$$

we obtain the first-order derivatives:

$$\frac{\partial}{\partial p}\int_\Theta \phi(\theta)\, f(\theta)\,\mathrm{d}\theta = \int_\zeta \left(-\frac{\partial\widetilde{\tau}}{\partial p}\right)\phi(\zeta,\widetilde{\tau})\, f(\zeta,\widetilde{\tau})\,\mathrm{d}\zeta = -\int_\zeta \frac{\phi(\zeta,\widetilde{\tau})\, f(\zeta,\widetilde{\tau})}{\frac{\partial u}{\partial\tau}(\rho,\Phi;\zeta,\widetilde{\tau})}\,\mathrm{d}\zeta = -M\mathbb{E}\left[\phi\middle|\,\partial\Theta\right],$$

and similarly,

$$\frac{\partial}{\partial\rho_i}\int_\Theta \phi(\theta)\, f(\theta)\,\mathrm{d}\theta = M\mathbb{E}\left[\frac{\partial u}{\partial\rho_i}\phi\middle|\,\partial\Theta\right], \quad \frac{\partial}{\partial\Phi_j}\int_\Theta \phi(\theta)\, f(\theta)\,\mathrm{d}\theta = M\mathbb{E}\left[\frac{\partial u}{\partial\Phi_j}\phi\middle|\,\partial\Theta\right].$$

Note that the first-order necessary conditions of the Lagrangian (1) give

$$0 = \frac{\partial \mathcal{L}}{\partial p} = \frac{\partial \pi}{\partial p} + \lambda^\top \left(-M\mathbb{E}\left[\phi \middle| \partial\Theta\right]\right), \tag{2}$$

$$0 = \frac{\partial \mathcal{L}}{\partial \rho_i} = \frac{\partial \pi}{\partial \rho_i} + \lambda^\top \left(M\mathbb{E}\left[\frac{\partial u}{\partial \rho_i}\phi \middle| \partial\Theta\right]\right), \tag{3}$$

$$0 = \frac{\partial \mathcal{L}}{\partial \Phi_j} = \frac{\partial \pi}{\partial \Phi_j} + \lambda^\top \left(M\mathbb{E}\left[\frac{\partial u}{\partial \Phi_j}\phi \middle| \partial\Theta\right]\right) - \lambda_j. \tag{4}$$

From equation (2), we obtain

$$\frac{\partial \pi}{\partial p} = M\lambda^\top \mathbb{E}\left[\phi \middle| \partial\Theta\right], \tag{5}$$

which proves the first equation in Proposition 1.

From equation (3) and (5), we obtain

$$\begin{aligned} -\frac{\partial \pi}{\partial \rho_i} &= M\lambda^\top \mathbb{E}\left[\frac{\partial u}{\partial \rho_i}\phi \middle| \partial\Theta\right] = M\sum_j \lambda_j \left(\mathrm{Cov}\left[\frac{\partial u}{\partial \rho_i}, \phi_j \middle| \partial\Theta\right] + \mathbb{E}\left[\frac{\partial u}{\partial \rho_i}\middle| \partial\Theta\right]\mathbb{E}\left[\phi_j \middle| \partial\Theta\right]\right) \\ &= \left(\sum_j \sigma_{\rho_i\phi_j}\lambda_j\right) + \frac{\partial \pi}{\partial p}\mathbb{E}\left[\frac{\partial u}{\partial \rho_i}\middle| \partial\Theta\right], \end{aligned}$$

which implies that

$$-\frac{\partial \pi}{\partial \rho} = \Sigma_{\rho\phi}\lambda + \frac{\partial \pi}{\partial p}\mathbb{E}\left[\frac{\partial u}{\partial \rho}\middle| \partial\Theta\right],$$

fulfilling the proof of the second equation in Proposition 1. □

Remark. *From equation* (4) *and* (5), *we obtain*

$$\begin{aligned} \lambda_j &= \frac{\partial \pi}{\partial \Phi_j} + M\lambda^\top \mathbb{E}\left[\frac{\partial u}{\partial \Phi_j}\phi \middle| \partial\Theta\right] = \frac{\partial \pi}{\partial \Phi_j} + M\sum_{j'} \lambda_j \left(Cov\left[\frac{\partial u}{\partial \Phi_j}, \phi_{j'} \middle| \partial\Theta\right] + \mathbb{E}\left[\frac{\partial u}{\partial \Phi_j}\middle| \partial\Theta\right]\mathbb{E}\left[\phi_{j'} \middle| \partial\Theta\right]\right) \\ &= \frac{\partial \pi}{\partial \Phi_j} + \left(\sum_{j'} \sigma_{\Phi_j\phi_{j'}}\lambda_j\right) + \frac{\partial \pi}{\partial p}\mathbb{E}\left[\frac{\partial u}{\partial \Phi_{j'}}\middle| \partial\Theta\right], \end{aligned}$$

which implies that

$$\lambda = \frac{\partial \pi}{\partial \Phi} + \Sigma_{\Phi\phi}\lambda + \frac{\partial \pi}{\partial p}\mathbb{E}\left[\frac{\partial u}{\partial \Phi}\middle| \partial\Theta\right] \;\Leftrightarrow\; \lambda = (I - \Sigma_{\Phi\phi})^{-1}\left(\frac{\partial \pi}{\partial \Phi} + \frac{\partial \pi}{\partial p}\mathbb{E}\left[\frac{\partial u}{\partial \Phi}\middle| \partial\Theta\right]\right).$$

3 Insulation

In this section we use a series of examples to explore the issue, discussed in subsection II.B. of the main text, of when insulation and related strategies are possible.

3.1 When is insulation possible?

Consider a setup in which users are heterogeneous along two dimensions $\theta = (\theta_1, \theta_2)$ with a smooth distribution on $\mathbb{R}^2_{++}$ whose first moment exists. In addition to the price, p, there is a single dimension of exogenous quality, ρ. There are two dimensions of endogenous quality, Φ_1 and Φ_2. All users contribute equally to Φ_1, i.e., $\phi_1(\theta) = 1$, for all θ, but they contribute to Φ_2 in proportion to θ_2, i.e., $\phi_2(\theta) = \theta_2$. Users have gross utility $u(\rho) = \theta_2(\Phi_2 + \rho) + \alpha\Phi_1 + \theta_1$. This specification captures, roughly speaking, an "undirected heterogeneity in usage intensity" setup; users who contribute more to the platform also derive more value from both its socially-oriented and non-socially oriented features.

A *strategy* of the platform is a vector function mapping from the Φ-space of endogenous quality to the (p, ρ)-space of exogenous quality (that the platform controls directly). Let $\widetilde{\Phi} = \left(\widetilde{\Phi}_1, \widetilde{\Phi}_2\right)$ denote users' expectation and let $\widehat{\Phi} = \left(\widehat{\Phi}_1, \widehat{\Phi}_2\right)$ denote the endogenous quality at the outcome targeted by the platform.

Definition 1 (Insulating Strategy)**.** *A strategy is said to be* insulating *if for any expectation, $\widetilde{\Phi}$, held commonly among users, the aggregate endogenous quality of users who optimally join is $\widehat{\Phi}$.*

Case 1 (Endogenous Quality of Lower Dimension than Exogenous Quality)**.** Let us begin by considering the case where ρ can take on any value in $\mathbb{R}$. Suppose that Φ_1 is suppressed: $\alpha = 0$ and profits do not depend on Φ_1, so we can treat it as if it does not exist. In this case, endogenous quality is unidimensional and the platform has two exogenous instruments with full range: p and ρ. The platform can thus achieve any desired level of Φ_2 that is feasible (that is, less than $\overline{\Phi}_2 \equiv \mathbb{E}[\theta]$, the maximum achievable value of Φ_2) in many ways. Raising ρ and/or lowering p both lead to a monotonic increase in Φ_2. Increasing p to ∞ drives out all users, forcing Φ_2 to 0, and decreasing ρ to $-\infty$ does the same. Doing the reverse achieves the maximum possible Φ_2, namely $\overline{\Phi}_2 \equiv \mathbb{E}[\theta_2]$. Thus, for any given expectation of endogenous quality $\widetilde{\Phi}_2$, and for any target outcome with endogenous quality $\widehat{\Phi}_2 \in \left(0, \overline{\Phi}_2\right)$, there exists a curve of values in the (p, ρ)-space, all of which lead to endogenous quality $\widehat{\Phi}_2$, when users behave optimally, given their expectation. Therefore, in this case, for any $\widehat{\Phi}$, there exist many insulating strategies.

Case 2 (Endogenous Quality of Higher Dimension than Exogenous Quality)**.** In an opposing case, suppose that $\alpha > 0$ so Φ_1 matters but that ρ takes on a fixed value that cannot be adjusted by the platform and is thus effectively irrelevant. Now consider an arbitrary expectation $\widetilde{\Phi}$, and consider a target $\widehat{\Phi}$ such that $\widehat{\Phi}_1$ is sufficiently large but $\widehat{\Phi}_2$ is sufficiently small. In this case, the platform can set p to achieve $\widehat{\Phi}_1$ *or* $\widehat{\Phi}_2$, but not both. This is because the unidimensional price can be used to attract any given number of users ($\widehat{\Phi}_1$) or any given average quality of users ($\widehat{\Phi}_2$) but, typically, not both at the same time. Therefore, under this setup, for some targets, there exists no insulating strategy.

Case 3 (Endogenous Quality of Equal Dimension to Exogenous Quality)**.** Now consider the case where $\alpha > 0$ and both p and ρ can each be set over the full range of $\mathbb{R}$. Note that, under this specification, the endogenous avergage quality of participating users, Φ_2, and the platform's exogenous quality, ρ, are perfect substitutes from the perspective of users considering whether to join. Hence, it is straightforward to see that, for any "incorrect" expectation of endogenous quality $\widetilde{\Phi}_2 \neq \widehat{\Phi}_2$, there is a unique way to adjust exogenous quality, ρ, to maintain the desired sum of the two. Similar logic implies that, for any "incorrect" expectation of total user participation, $\widetilde{\Phi}_1 \neq \widehat{\Phi}_1$, there is a unique way to adjust p in order to attract the desired demand level. Moreover, due to the full range assumption, for any given expectation, the implied values of p and ρ are feasible. Therefore, under this setup, for any target, $\widehat{\Phi}$, that is consistent with basic demand requirements,[1] there exists a unique insulating strategy.

Now consider a variation on case 3, above, in which the feasible range of exogenous quality is limited to an interval $\left[\underline{\rho}, \overline{\rho}\right]$, where $-\infty < \underline{\rho} < \overline{\rho} < \infty$. Here, then insulation may once again be infeasible. The trouble is that the set of values of $\Phi_2 + \rho$ that are achievable depends on the expectation $\widetilde{\Phi}_2$. In an extreme case, suppose that $\overline{\Phi}_2 \gg \overline{\rho} = \underline{\rho} + 1$. Then, for some target $\widehat{\Phi}_2$ high enough, if expectation $\widetilde{\Phi}_2$ is low, then adjusting exogenous quality up to its maximum value, $\overline{\rho}$, is not sufficient to achieve the required sum. Thus, an insulating strategy cannot be guaranteed to exist. Essentially, if the feasible range of ρ is limited, so too may be its usefulness as a tool for insulation.

We conjecture that the basic pattern illustrated above applies much more broadly than to just the environment we have assumed in this subsection. We expect that, in a significantly broader class of models,

(i) if the dimensionality of exogenous quality (including price) is lower than that of endogenous quality, insulation is impossible,

(ii) if the dimensionality of endogenous quality is lower than that of exogenous quality and the feasible range of exogenous quality instruments is sufficiently large, then many insulating strategies exist, and

(iii) if exogenous quality and endogenous quality have the same dimension, and the feasible range of exogenous quality instruments is sufficiently large, then there exists a unique insulating strategy.

Formalizing this conjecture in a way with meaningful operational and/or intuitive content is an interesting project for future research.

3.2 Quasi-insulation in a social network

To simplify exposition and provide the cleanest example, we now depart somewhat for the technical boundaries of our model. This example can be embedded as a limit case of our model, but it is simpler to exposit it in slightly different terms.

Suppose that there are two distinct groups of users each with a type uniformly drawn from the interval $[0, 1]$ and a single dimension of exogenous quality ρ that the platform can set to any level in $[-1, 1]$; there is no "price" p. Endogenous quality dimension i, Φ_i, for $i = 1, 2$ is the fraction of users from group i that join the platform. Users in group 2 join if and only if their expectation, $\widetilde{\Phi}_1$, is weakly greater than their type; these users do not care about exogenous quality. In group 1, users join if $\widetilde{\Phi}_1 + \rho$ is weakly greater than their type. The platform's profit depends only on the level of exogenous quality, ρ, and the level of Φ_2.

[1]Note that some vectors (Φ_1, Φ_2) are not valid target outcomes, such as one in which all users participate and the average quality of user is "only the best", i.e., $\overline{\Phi}_2$. Insulation provides a robust way to implement outcomes that are valid as targets, not a way to enlarge the set of such outcomes. A valid outcome can be defined as one that is part of a Nash equilibrium in the subgame played by users for *some* constant vector (p, ρ).

In this setting, we extend the notion of user expectations in the following manner: members of each group share the same expectation, but expectations can be different among groups, thus forming a *profile of expectations*. Under this setup, for a given target $\widehat{\Phi}$, there may exist no insulating strategy. For example, if group 2 users' expectation is for minimal participation among group 1, there is no way for the platform to counteract this.

However, if group 2 users *know* that the platform employs an insulating strategy, with a particular target, *vis-à-vis* group 1, then it seems reasonable to exclude the expectation on the part of group 2 that some other, non-targeted outcome will arise in group 1. We formalize this idea by defining allowable profiles of user expectations as follows. All profiles of expectations are *allowable*, except, if the platform's strategy satisfies the criterion of insulation for a particular group, i, then all other groups' expectation for group i must be consistent with this target. Using this restriction on expectations, we now define *quasi-insulating strategies*.

Definition 2 (Quasi-insulating Strategy)**.** *A strategy is said to be* quasi-insulating *if for any allowable profile of expectations, $\widetilde{\Phi}$, the aggregate endogenous quality of users who optimally join is $\widehat{\Phi}$.*

In the current setting, the platform *can* quasi-insulate Φ_1. To achieve $\widehat{\Phi}_1 \in [0,1]$, it sets $\rho = \widehat{\Phi}_1 - \widetilde{\Phi}_1$. Having thus targeted $\widehat{\Phi}_1$ using an insulating strategy, group 2 users expectations are set to the targeted level, thus guaranteeing $\widehat{\Phi}_2$.

The above discussion can be extended to the case of $K > 2$ groups. To do so, however, the set of allowable expectations may need to take into account higher-order considerations. For instance, suppose $K = 3$, and that ρ directly influences only group 1, group 2 cares only about group 1 participation, and group 3 is concerned only about group 2 participation. Then, group 2's expectation regarding group 1 could be pinned down by the platform's insulation of group 1, as in the 2-group case. However, group 3's expectation regarding group 2 must be pinned down by appealing to the fact that group 3 users know that group 2 users know that the platform insulates group 1. Thus, quasi-insulation might be said to combine the logic of insulation with that of backwards induction, although the analogy to the latter is not perfect, given that we assume the different groups to move simultaneously.

More broadly, we suspect that there are many richer and less toy-like cases in which quasi-insulation or something close to it may be feasible. Exogenous quality (e.g. the ability to view high quality images of and find classmates) may be used to make a platform robustly attractive to a core group of "high prestige" users (e.g., Harvard undergraduates) who then are attractive to a somewhat lower prestige group of users (e.g., Yale undergraduates) who are then attractive to an again larger and slightly lower prestige group (e.g., Columbia and Princeton undergraduates), etc. Deriving conditions on primitives that allow for this sort of quasi-insulation, in richer settings, seems like a useful direction for future research. This seems particularly true, because, as we describe in Subsection III.B of the main text, such strategies of "cascading" quasi-insulation may provide a more insightful model, relative to full insulation, of the dynamic strategies platform start-ups use (or can benefit from using) in the real world.