Online Appendix to: News or Noise? The Missing Link

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A Proofs

Proof of Proposition (2)**.** By rational expectations, $\mathcal{H}_t(x) \subseteq \mathcal{H}_t(\hat{x})$, and the fact that $\{\hat{x}_{i,t}\}$ forms a Gaussian system, it follows that agents' date$-t$ information is fully summarized by the random variables $\hat{x}_{i,\tau}$ across all i and $\tau \leq t$.

We can let $\mathcal{F}_t(\hat{x})$ denote the smallest σ-algebra generated by these variables. That is, $\mathcal{F}_t(\hat{x})$ is generated by cylinder sets of the form

$$\mathcal{A}_t \equiv \{\omega \in \Omega : \hat{x}_{i_1,t_1} \in \mathcal{G}_1, \ldots, \hat{x}_{i_n,t_n} \in \mathcal{G}_n\},$$

where Ω denotes the space of elementary events, $\mathcal{G}_1, \ldots, \mathcal{G}_n$ are arbitrary Borel sets in $\mathbb{R}$, the indices $t_1, \ldots, t_n$ assume values in the set $\{\tau \in \mathbb{Z} : \tau \leq t\}$, and the indices $i_1, \ldots, i_n$ assume values in $\mathbb{Z}$. By construction, the sequence of σ-algebras $\{\mathcal{F}_t(\hat{x})\}$ is uniquely determined by the forecasts $\{\hat{x}_{i,t}\}$. If two representations of fundamentals and beliefs imply the same dynamics for $\{\hat{x}_{i,t}\}$, they imply the same information structure $\{\mathcal{F}_t(\hat{x})\}$. Therefore, the conditional distribution function of any stochastic process $\{c_t\}$, such that c_t is measurable with respect to $\mathcal{F}_t(\hat{x})$ for each $t \in \mathbb{Z}$, is also the same. $\square$

Proof of Proposition (3)**.** As in the proof of Proposition (1), we can equate the spectral density of $\{d_t\}$ with $d_t \equiv (x_t, \hat{x}_t)'$ under each representation. In this case,

$$f_d(\lambda) = \underbrace{\frac{1}{2\pi}\begin{bmatrix} \sigma_\eta^2 + \sigma_\mu^2 & \left(\frac{\sigma_\mu^4}{\sigma_\mu^2+\sigma_\xi^2}\right)e^{-i\lambda} \\ \left(\frac{\sigma_\mu^4}{\sigma_\mu^2+\sigma_\xi^2}\right)e^{i\lambda} & \frac{\sigma_\mu^4}{\sigma_\mu^2+\sigma_\xi^2} \end{bmatrix}}_{\text{system (6)}} = \underbrace{\frac{1}{2\pi}\begin{bmatrix} \sigma_x^2 & \left(\frac{\sigma_x^4}{\sigma_x^2+\sigma_v^2}\right)e^{-i\lambda} \\ \left(\frac{\sigma_x^4}{\sigma_x^2+\sigma_v^2}\right)e^{i\lambda} & \frac{\sigma_x^4}{\sigma_x^2+\sigma_v^2} \end{bmatrix}}_{\text{noise}}.$$

This equality holds if and only if the relations in Proposition (3) are satisfied. $\square$

Proof of Proposition (4)**.** Consider an arbitrary noise representation of fundamentals and beliefs and an arbitrary endogenous process $\{c_t\}$. Using the structure of signals in a noise representation, $\mathcal{H}(s) = \mathcal{H}(m) \oplus \mathcal{H}(v)$. Because $v_{i,t} \in \mathcal{H}(s) \ominus \mathcal{H}(x)$ for all $i \in \mathcal{I}_s$, the uniqueness of orthogonal decompositions implies that $\mathcal{H}(m) = \mathcal{H}(x)$. Therefore, $\mathcal{H}(s) = \mathcal{H}(x) \oplus \mathcal{H}(v)$. Furthermore, the definition of noise shocks implies that $\mathcal{H}(\epsilon^v) = \mathcal{H}(v)$, so

$$\mathcal{H}(s) = \mathcal{H}(x) \oplus \mathcal{H}(\epsilon^v). \tag{15}$$

By the endogeneity of $\{c_t\}$ and the rationality of expectations, $c_t \in \mathcal{H}(s)$ for all $t \in \mathbb{Z}$. Combining this with equation (15), it follows that for each c_t, there exist two unique elements $a_t \in \mathcal{H}(x)$ and $b_t \in \mathcal{H}(\epsilon^v)$ such that

$$c_t = a_t + b_t. \tag{16}$$

To consider variance decompositions at different frequencies, let $f_y(\lambda)$ denote the spectral density function of a stochastic process $\{y_t\}$. Then because $a_t \perp b_t$ for all $t \in \mathbb{Z}$, it follows that

$$f_c(\lambda) = f_a(\lambda) + f_b(\lambda),$$

where the functions $f_a(\lambda)$ and $f_b(\lambda)$ are uniquely determined by the processes $\{a_t\}$ and $\{b_t\}$. These functions in turn uniquely determine the share of the variance of $\{c_t\}$ due to noise shocks in any frequency range $\underline{\lambda} < \lambda < \overline{\lambda}$, which is equal to

$$\frac{\int_{\underline{\lambda}}^{\overline{\lambda}} f_b(\lambda) d\lambda}{\int_{\underline{\lambda}}^{\overline{\lambda}} f_c(\lambda) d\lambda}.$$

The share due to fundamentals is equal to one minus this expression. $\square$

Proof of Proposition (5)**.** Beginning with the decomposition of $\mathcal{H}(s)$ in equation (15), we can further decompose $\mathcal{H}(x)$ uniquely into the sum of subspaces $\mathcal{D}_t(x) \equiv \mathcal{H}_t(x) \ominus \mathcal{H}_{t-1}(x)$,

$$\mathcal{H}(s) = \left(\bigoplus_{j=-\infty}^{\infty} \mathcal{D}_{t-j}(x) \right) \oplus \mathcal{H}(\epsilon^v).$$

By definition, each fundamental shock $\epsilon_t^x \equiv x_t - E[x_t | \mathcal{H}_{t-1}(x)]$ forms a basis in the space $\mathcal{D}_t(x)$. Since $c_t \in \mathcal{H}(s)$ for all $t \in \mathbb{Z}$, it follows that for each c_t, there exists a unique sequence of projection coefficients $\{\alpha_j\}$ such that

$$c_t = \sum_{j=-\infty}^{\infty} \alpha_j \epsilon^x_{t-j} + b_t,$$

where $\alpha_j \equiv E[c_t \epsilon^x_{t-j}]/\text{var}[\epsilon^x_t]$ and $b_t \perp \mathcal{H}(x)$. The shares of the variance of $\{c_t\}$ due to past, present, and future fundamental shocks are therefore uniquely determined, and are given by

$$\underbrace{\frac{\sum_{j=1}^{\infty} \alpha_j^2 \text{var}[\epsilon^x_t]}{\text{var}[c_t]}}_{\text{past}}, \quad \underbrace{\frac{\alpha_0^2 \text{var}[\epsilon^x_t]}{\text{var}[c_t]}}_{\text{present}}, \quad \text{and} \quad \underbrace{\frac{\sum_{j=-\infty}^{-1} \alpha_j^2 \text{var}[\epsilon^x_t]}{\text{var}[c_t]}}_{\text{future}}.$$

□

Proof of Corollary (1)**.** Consider an arbitrary noise representation of fundamentals and beliefs, and an endogenous process $\{c_t\}$. By the rationality of expectations, agents' best forecast of c_{t+h} as of date t is equal to

$$\hat{c}_{h,t} = E[c_{t+h}|\mathcal{H}_t(s)] = E[c_{t+h}|\mathcal{H}_t(\hat{x})].$$

Therefore, $\hat{c}_{h,t} \in \mathcal{H}_t(\hat{x})$. This means that the forecast error $w^h_t \equiv c_t - \hat{c}_{h,t-h}$ also satisfies $w_{h,t} \in \mathcal{H}_t(\hat{x})$. Therefore, $\{w^h_t\}$ is an endogenous process. By Proposition (4), the variance decomposition of this process in terms of noise and fundamentals is uniquely determined over any frequency range. Moreover, this result is true for any forecast horizon $h \in \mathbb{Z}$ because h was chosen arbitrarily. □

Lemma 1. *Any news representation in which each process $\{a_{i,t}\}$ is i.i.d. over time is observationally equivalent to a noise representation with $x_t \overset{iid}{\sim} \mathcal{N}(0, \sigma_x^2)$ and*

$$s_{i,t} = x_{t+i} + v_{i,t}, \quad v_{i,t} \overset{iid}{\sim} \mathcal{N}(0, \sigma^2_{v,i}),$$

where $v_{i,t} \perp x_\tau$ and $v_{i,t} \perp v_{j,\tau}$ for any $i \neq j \in \mathcal{I}_s$ and $t, \tau \in \mathbb{Z}$, if and only if

$$\sigma_x^2 = \sum_{i\in\mathcal{I}_s} \sigma^2_{a,i} \quad \text{and} \quad \sigma^2_{v,i} = \frac{1}{\sigma^2_{a,i}} \left(\sum_{j<i} \sigma^2_{a,j}\right)\left(\sum_{j\leq i} \sigma^2_{a,j}\right) \quad \text{for all } i \in \mathcal{I}_s.$$

Proof of Lemma (1)**.** The proof of this result is a straightforward generalization of the proof of Proposition (1). In a news representation with i.i.d. news processes, the joint spectral density of any two forecast processes $\{\hat{x}_{j,t}\}$ and $\{\hat{x}_{k,t}\}$ for $j, k \in \mathbb{Z}_+$ is equal to

$$f_{j,k}(\lambda) = \frac{1}{2\pi} \sum_{m \in \mathcal{M}} \sigma^2_{a,m} e^{-i\lambda(k-j)}, \tag{17}$$

where $\mathcal{M}$ is defined as the set of indices $m \in \mathcal{I}_a$ such that $m \geq |k-j| + j$. In a noise representation of the type described in the proposition, the joint spectral density of any two forecast processes $\{\hat{x}_{j,t}\}$ and $\{\hat{x}_{k,t}\}$ for $j, k \in \mathbb{Z}_+$ is equal to

$$\begin{aligned} f_{0,0}(\lambda) &= \frac{1}{2\pi}\sigma^2_x \\ f_{j,k}(\lambda) &= \frac{1}{2\pi}\sigma^2_x \left[1 + \frac{1/\sigma^2_x}{\sum_{m\in\mathcal{M}} 1/\sigma^2_{v,m}}\right]^{-1} e^{-i\lambda(k-j)} \quad \text{for } j,k > 0. \end{aligned} \tag{18}$$

Equating the densities in (17) with those in (18), and recursively solving for the parameters of the noise representation delivers the relations stated in the lemma. □

Proof of Proposition (6)**.** Define the composite shock

$$\epsilon^x_t \equiv \epsilon^a_{0,t} + \epsilon^a_{4,t-4} + \epsilon^a_{8,t-8}. \tag{19}$$

The process $\{\epsilon^x_t\}$ is i.i.d. because $\{\epsilon^a_{i,t}\}$ is i.i.d. for each $i \in \mathcal{I}_a \equiv \{0, 4, 8\}$. agents' date-$t$ information set in representation (11) is $\mathcal{H}_t(\epsilon^a)$. But based on this information set, equation (19) defines a news representation for $\{\epsilon^x_t\}$ with i.i.d. news processes. Therefore, we can apply Lemma (1) to the composite shock process, which gives the relations stated in the proposition. □

Proof of Proposition (7)**.** According to representation (12), the two signals observed by agents in the economy are $s_{0,t} \equiv x_t$ and $s_{1,t} \equiv \mu_t + \xi_t$. Because $\mathcal{H}(x) \subset \mathcal{H}(s)$, there exist two unique elements $m_t \in \mathcal{H}(x)$ and $v_t \perp \mathcal{H}(x)$ such that:

$$s_{1,t} = m_t + v_t \quad \text{for all } t \in \mathbb{Z}. \tag{20}$$

The spectral density of $\{x_t\}$ is non-zero for almost all $\lambda \in [-\pi, \pi]$, which means that $\{m_t\}$ can be obtained from $\{x_t\}$ by a linear transformation of the form

$$m_t = \int_{-\pi}^{\pi} e^{i\lambda t} \varphi(\lambda) \Phi_x(d\lambda), \tag{21}$$

where Φ_x is the random spectral measure of $\{x_t\}$, and $\varphi(\lambda) = f_{s,x}(\lambda)/f_x(\lambda)$ is the spectral characteristic of the transformation. Using the restrictions in the system (12), we have

$$\varphi(\lambda) = \frac{\sigma_\mu^2 e^{i\lambda}}{\sigma_\mu^2 + \sigma_\eta^2|1-\rho e^{-i\lambda}|^2} = \frac{\delta\sigma_\mu^2 e^{i\lambda}}{\rho\sigma_\eta^2|1-\delta e^{-i\lambda}|^2},$$

where $|\delta| < 1$ is equal to the expression stated in the proposition. Combining $\varphi(\lambda)$ with the spectral density of $\{x_t\}$, we can use equation (21) to obtain the spectral density of $\{m_t\}$,

$$f_m(\lambda) = \frac{1}{2\pi}\frac{\delta\sigma_\mu^4}{\rho\sigma_\eta^2}\left|\frac{1}{(1-\rho e^{-i\lambda})(1-\delta e^{-i\lambda})}\right|^2.$$

This corresponds to the law of motion presented in the proposition. From equation (21), it follows that the fundamental process $\{x_t\}$ can be obtained from $\{m_t\}$ by a linear transformation with spectral characteristic $\varphi(\lambda)^{-1}$. Finally, the definition of the noise process $\{v_t\}$ in equation (20) implies that

$$f_v(\lambda) = \frac{1}{2\pi}\frac{\sigma_\mu^2\sigma_\eta^2}{\sigma_\mu^2 + \sigma_\eta^2|1-\rho e^{-i\lambda}|^2} + \sigma_\xi^2 = \frac{1}{2\pi}\sigma_\xi^2\frac{\delta}{\beta}\left|\frac{1-\beta e^{-i\lambda}}{1-\delta e^{-i\lambda}}\right|^2,$$

where $|\beta| < 1$ is equal to the expression stated in the proposition. Because $\mathcal{H}_t(s)$ is unchanged from representation (12) for all $t \in \mathbb{Z}$, it follows that $\hat{x}_{j,t} \equiv E[x_{t+j}|\mathcal{H}_t(s)]$ is also unchanged for any $j \in \mathbb{Z}$. Therefore these two representations are observationally equivalent. $\square$

Proof of Proposition (8)**.** A complication in this case is that both fundamentals and the signal of future fundamentals are difference-stationary, rather than stationary processes. As a result, they do not have finite second moments, which is a prerequisite for working in $\mathcal{L}^2$. We handle this complication by defining a new processes $\{\tilde{x}_t(\theta)\}$ as the solution to the difference equation

$$\tilde{x}_t(\theta) = \theta\tilde{x}_{t-1}(\theta) + \Delta x_t, \quad \text{for all } t \in \mathbb{Z}, \tag{22}$$

where Δ is the first-difference operator; $\Delta x_t \equiv x_t - x_{t-1}$. This new process is stationary for each value of $\theta \in [0,1)$, and admits the spectral representation

$$\tilde{x}_t(\theta) = \int_\pi^\pi e^{i\lambda t}(1-\theta e^{-i\lambda})^{-1}\Phi_{\Delta x}(d\lambda),$$

where $\Phi_{\Delta x}$ is the random spectral measure of $\{\Delta x_t\}$. We define a new signal process $\{\tilde{s}_t(\theta)\}$ analogously, derive the noise representation in terms of $\{\tilde{x}_t(\theta)\}$ and $\{\tilde{s}_t(\theta)\}$ for an arbitrary value of θ, and then take limits as θ approaches one from below.

The two signals observed by agents in the economy are $\tilde{s}_{0,t} \equiv \tilde{x}_t(\theta)$ and $\tilde{s}_{1,t} \equiv \tilde{s}_t(\theta)$. Because $\mathcal{H}(\tilde{x}) \subset \mathcal{H}(\tilde{s})$, there exist two unique elements $\tilde{m}_t(\theta) \in \mathcal{H}(\tilde{x})$ and $\tilde{v}_t(\theta) \perp \mathcal{H}(\tilde{x})$ such that:

$$\tilde{s}_t(\theta) = \tilde{m}_t(\theta) + \tilde{v}_t(\theta) \quad \text{for all } t \in \mathbb{Z}. \tag{23}$$

The spectral density of $\{\tilde{x}_t(\theta)\}$ is non-zero for almost all $\lambda \in [-\pi, \pi]$, which means that $\{\tilde{m}_t(\theta)\}$ can be obtained from $\{\tilde{x}_t(\theta)\}$ by a linear transformation of the form in equation (21), where in this case the spectral characteristic $\varphi(\lambda)$ is

$$\varphi(\lambda) = \rho \frac{\sigma_\mu^2}{\sigma_\eta^2} \frac{1}{|1 - \rho e^{-i\lambda}|^2}.$$

Combining this with the spectral density of $\{\tilde{x}_t(\theta)\}$, it follows that the spectral density of $\{\tilde{m}_t(\theta)\}$ is

$$f_{\tilde{m}}(\lambda; \theta) = \frac{1}{2\pi} \rho \frac{\sigma_\mu^4}{\sigma_\eta^2} \left| \frac{1}{(1 - \theta e^{-i\lambda})(1 - \rho e^{-i\lambda})^2} \right|^2.$$

By writing out the corresponding law of motion for $\{\tilde{m}_t(\theta)\}$ and then taking limits as θ approaches one from below, we obtain the law of motion for $\{m_t\}$ stated in the proposition. In a similar manner, we can obtain the law of motion for $\{x_t\}$ in terms of $\{m_t\}$ by using the spectral characteristic $\varphi(\lambda)^{-1}$. Finally, the definition of the noise process $\{\tilde{v}_t(\theta)\}$ in equation (23) implies that

$$f_v(\lambda; \theta) = \frac{1}{2\pi} \frac{\rho^2 \sigma_v^2}{|\delta|^2} \left| \frac{(1 - e^{-i\lambda})(1 - \delta e^{-i\lambda})(1 - \bar{\delta} e^{-i\lambda})}{(1 - \theta e^{-i\lambda})(1 - \rho e^{-i\lambda})^2} \right|^2,$$

where $|\delta| < 1$ is equal to the expression stated in the proposition. By letting θ tend to one from below, we obtain the law of motion for $\{v_t\}$. Because $\mathcal{H}_t(\tilde{s})$ is unchanged from representation (13) for each $\theta \in [0, 1)$ and all $t \in \mathbb{Z}$, it follows that

$$\hat{x}_{j,t} \equiv \lim_{\theta \to 1^-} E_t[\tilde{x}_{t+j}(\theta) | \mathcal{H}_t(\tilde{s})]$$

is also unchanged for any $j \in \mathbb{Z}$. Therefore these two representations are observationally equivalent. □

B Quantitative Models

The following subsections provide a sketch of each of the three quantitative models considered in this paper. For more details, we refer the reader to the original articles and their supplementary material.

B.1 Model of Schmitt-Grohé and Uribe (2012)

A representative household chooses consumption $\{C_t\}$, labor supply $\{h_t\}$, investment $\{I_t\}$, and the utilization rate of existing capital $\{u_t\}$ to maximizes its lifetime utility,

$$E\left[\sum_{t=0}^{\infty} \beta^t \zeta_t \frac{(C_t - bC_{t-1} - \psi h_t^\theta S_t)^{1-\sigma}}{1-\sigma}\right],$$

subject to a standard sequence of constraints,

$$\begin{aligned} S_t &= (C_t - bC_{t-1})^\gamma S_{t-1}^{1-\gamma} \\ C_t + A_t I_t + G_t &= \frac{W_t}{\mu_t} h_t + r_t u_t K_t + P_t \\ K_{t+1} &= (1-\delta(u_t))K_t + z_t^I I_t \left[1 - \Phi\left(\frac{I_t}{I_{t-1}}\right)\right] \end{aligned}$$

Relative to the standard real business cycle model, this model features investment adjustment costs $\Phi(I_t/I_{t-1})$; variable capacity utilization, which increases the return on capital $r_t u_t$ at the cost of increasing its rate of depreciation through $\delta(u_t)$; one period internal habit formation in consumption, controlled by $0 < b < 1$; a potentially low wealth effect on labor supply, when $0 < \gamma < 1$ approaches its lower limit; and monopolistic labor unions, which effectively reduce the wage rate by an amount μ_t each period but rebate profits lump sum to the household through P_t.

Output is produced by a representative firm, which combines capital K_t, labor h_t, and a fixed factor of production L using a (potentially) decreasing returns to scale production function:

$$Y_t = z_t(u_t K_t)^{\alpha_k}(X_t h_t)^{\alpha_h}(X_t L)^{1-\alpha_k-\alpha_h}.$$

Market clearing requires that the goods and labor markets clear so that the aggregate resource constraint is satisfied: $C_t + A_t I_t + G_t = Y_t$. The seven fundamental processes capture exogenous variation in permanent and transitory neutral productivity $\{X_t, z_t\}$, permanent and transitory investment-specific productivity $\{A_t, z_t^I\}$, government spending $\{G_t\}$, wage markups $\{\mu_t\}$, and preferences $\{\zeta_t\}$.

B.2 Model of Barsky and Sims (2012)

A representative household chooses consumption $\{C_t\}$, labor supply $\{N_t\}$, and real holdings of riskless one-period bonds $\{B_t\}$ to maximize its lifetime utility,

$$E\left[\sum_{t=0}^{\infty}\beta^t\left(\ln(C_t-\kappa C_{t-1})-\frac{N_t^{1+1/\eta}}{1+1/\eta}\right)\right]$$

subject to a standard flow budget constraint,

$$C_t + B_t = w_t N_t - T_t + (1+r_{t-1})B_{t-1} + \Pi_t,$$

where r_t is the net nominally risk-free interest rate, w_t is the wage, T_t denotes lump-sum taxes, and Π_t is aggregate profits.

Final goods producers are competitive and take the price of intermediate goods, $P_t(j)$, as given and each have a production function of the form:

$$Y_t = \left[\int_0^1 Y_t(j)^{\frac{\xi-1}{\xi}}\right]^{\frac{\xi}{\xi-1}}$$

Intermediate goods firms are monopolistically competitive and take the demands of final goods firms as given. They each have a production function of the form $Y_t(j) = A_t K_t(j)^{\alpha} N_t(j)^{1-\alpha}$. Each intermediate firm chooses a price for its own good, subject to the constraint that it will only be able to re-optimize its price each period with constant probability $1-\theta$.

A continuum of capital producers produce new capital (to sell to intermediate firms) according to the production function

$$Y_t^k(\nu) = \phi\left(\frac{I_t(\nu)}{K_t(\nu)}\right)K_t(\nu),$$

where ϕ is an increasing and concave function. The aggregate capital stock evolves according to $K_t = \phi(I_t/K_t)K_{t-1} + (1-\delta)K_{t-1}$, where $0 < \delta < 1$ is the depreciation rate. The aggregate resource constraint is $Y_t = C_t + I_t + G_t$ (ignoring resources lost due to inefficient price dispersion). The monetary authority sets the one-period nominally risk-free rate of return according to a feedback rule of the (log-linear approximate) form:

$$i_t = \rho_i i_{t-1} + (1-\rho_i)\phi_\pi(\pi_t - \pi^*) + (1-\rho_i)\phi_y(\Delta Y_t - \Delta Y^*) + \varepsilon_{i,t}.$$

The three fundamental processes capture exogenous variation in permanent neutral productivity $\{A_t\}$, government spending $\{G_t\}$, and monetary policy $\{\varepsilon_{i,t}\}$

B.3 Model of Blanchard, L'Huillier, and Lorenzoni (2013)

Each household $j \in (0,1)$ chooses consumption $\{C_{j,t}\}$, investment $\{I_{j,t}\}$, nominally risk-free bond holdings $\{B_{j,t}\}$, and the rate of capital utilization $\{U_{j,t}\}$ to maximize its lifetime utility

$$E\left[\sum_{t=0}^{\infty}\beta^t\left(\ln(C_{j,t} - hC_{j,t-1}) - \frac{N_{j,t}^{1+\zeta}}{1+\zeta}\right)\right]$$

subject to a standard flow budget constraint. Each household is the monopoly supplier of labor type j, and chooses wages $\{W_{j,t}\}$ subject to the constraint that it can only re-optimize its wage each period with constant probability $1-\theta_w$. Risk-sharing among households results in a common budget constraint, which is the same as if each household were to receive its pro rata share of the economy's total wage bill:

$$P_tC_t + P_tI_t + T_t + P_t\mathcal{C}(U_t)\bar{K}_{t-1} + B_t = R_{t-1}B_{t-1} + \Upsilon_t + \int_0^1 W_{j,t}N_{j,t}dj + R_t^k U_t \bar{K}_{t-1},$$
$$\bar{K}_t = (1-\delta)\bar{K}_{t-1} + D_t[1 - \mathcal{G}(I_t/I_{t-1})]I_t.$$

P_t is the price level, T_t is a lump sum tax, R_t is the gross nominally risk-free rate, Υ_t is aggregate profits, R_t^k is the capital rental rate, $0 < \delta < 1$ is the rate of depreciation, $\mathcal{G}(I_t/I_{t-1})$ represents investment adjustment costs, $\mathcal{C}(U_t)$ represents the marginal cost of increasing capacity utilization.

Final goods producers are competitive and take the price of intermediate goods as given, P_{jt}, and each have a production function of the form

$$Y_t = \left[\int_0^1 Y_{jt}^{\frac{1}{1+\mu_{pt}}} dj \right]^{1+\mu_{pt}}.$$

Intermediate goods firms are monopolistically competitive, each with a production function of the form $Y_{jt} = (K_{jt})^\alpha (A_t L_{jt})^{1-\alpha}$. Each intermediate firm chooses a price for its own good, subject to a $1 - \theta_p$ probability of re-optimization each period.

Labor services are supplied to intermediate goods producers by competitive labor agencies that take wages as given, W_{jt}, and have a production function of the form

$$N_t = \left[\int_0^1 N_{jt}^{\frac{1}{1+\mu_{wt}}} dj \right]^{1+\mu_{wt}}.$$

Market clearing in the final goods market requires that $C_t + I_t + \mathcal{C}(U_t)\bar{K}_{t-1} + G_t = Y_t$, and in the labor market that $\int_0^1 L_{jt} dj = N_t$. Monetary policy follows the rule:

$$r_t = \rho_r r_{t-1} + (1 - \rho_r)(\gamma_\pi \pi_t + \gamma_y \hat{y}_t) + q_t.$$

The six fundamental processes capture exogenous variation in permanent neutral productivity $\{A_t\}$, transitory investment-specific productivity $\{D_t\}$, price markups $\{\mu_{pt}\}$, wage markups $\{\mu_{wt}\}$, government spending $\{G_t\}$, and monetary policy $\{q_t\}$.

C Estimation Details

We estimate each model using quarterly data on log growth rates of real per-capita output, consumption, and hours, along with the log-levels of inflation and the nominal interest rate. The data span 1960:Q1 to 2017:Q2, with observations from 1954:Q3 to 1959:Q4 used to initialize the Kalman filter. Real variables are deflated by the implicit GDP price deflator, and put in per-capita terms using civilian non-institutional population age 16 and above. Consumption includes expenditure on non-durable goods and services. Inflation is measured by the log-change in the GDP deflator while the nominal interest rate is given by the effective federal funds rate. Data were downloaded from the St. Louis Federal Reserve Database, FRED, on October 25, 2017. Original downloaded data and data transformations can be seen in the online code accompanying this appendix.

For each model, we allow for shocks to the same four exogenous processes: productivity, noise, monetary policy, and government spending. The productivity and information blocks are described in Sections (4.2) and (4.3) of the main text. We allow both the government spending process and the exogenous component of monetary policy to follow first-order autoregressive laws of motion. We follow Barsky and Sims (2012) in fixing the parameters for the government spending process, $\rho_g = 0.95$ and $\sigma_g = 0.25$. (The original estimates of Blanchard et al. (2013) for these parameters are quite similar.) We estimate the parameters of the monetary policy process.

Since our approach targets five measured variables with only four fundamental shocks, we allow for small independent and identically distributed measurement error shocks in the observation of each series. In our estimation, we bound the variance of measurement error for each variable at 2.5% of that variable's unconditional variance in the data. Since this bound is attained in all six of our estimations, we do not report those parameters here. Our results are not sensitive to changing this bound.

For each combination of economic environment and information structure, we re-estimate the model using the method of maximum likelihood. Specifically, we search for the set of parameters that maximizes the log-likelihood function of the data using a robust global optimization routine that combines a genetic algorithm to discover many good initial parameter combinations with a hill-climbing routine that ensures our final answer is (at least) a local optimum. All of our results are robust to changing the random seed that underlies the initial points.

The following tables summarize our parameter estimates for each of the estimated models. In these tables, an asterisk indicates that the estimated parameter lies at, or very close, to the boundary of the parameter space, which we define before maximizing the likelihood function.

Economic parameters		BS info	BLL info
κ	habit	0.3145	0.0252
η	Frisch elasticity	4.9976*	4.9999*
γ	capital adj. cost	5.2093	3.4670
θ	Calvo price	0.9420	0.9309
ϕ_π	Taylor inflation	4.8073	4.8897
ϕ_y	Taylor output growth	0.0042	0.0484
ρ_i	interest smoothing	0.5072	0.4074
σ_i	s.d. policy shock	0.1343	0.1629
ρ_{ε_i}	autocorr. policy	0.9989*	0.9892
BS info parameters			
ρ	autocorr. growth	0.9231	
σ_μ	s.d. growth shock	0.2190	
σ_η	s.d. surprise* shock	0.8716	
σ_ξ	s.d. noise* shock	0.0001*	
BLL info parameters			
ρ	autocorr. growth		0.8581
σ_μ	s.d. growth shock		1.3638
σ_ξ	s.d. noise* shock		0.0010

Table 5: Estimated parameters for alternative versions of the Barsky and Sims (2012) model.

	Economic parameters	BS info	BLL info	BS info + flex wage	BLL info + flex wage
h	habit	0.8145	0.7066	0.6209	0.4726
ζ	inverse Frisch elasticity	0.2000*	0.2000*	0.2000*	0.2000*
ξ	cap. util. cost	0.5023	0.0079	0.4628	0.0010*
χ	inv. adj. cost	15.0000*	15.0000*	15.0000*	15.0000*
θ_p	Calvo price	0.8771	0.8645	0.8654	0.8929*
θ_w	Calvo wage	0.9013	0.8708	-	-
γ_π	Taylor inflation	4.2259	3.8640	1.0100*	1.0100*
γ_y	Taylor output gap	0.0010*	0.0010*	0.4742	0.4135
ρ_r	interest smoothing	0.4686	0.4540	0.2813	0.1127
σ_q	s.d. policy shock	0.3394	0.2792	0.3089	0.4112
ρ_q	autocorr. policy	0.9990*	0.9990*	0.9425	0.9481
	BS info parameters				
ρ	autocorr. growth	0.9166		0.8980	
σ_μ	s.d. growth shock	0.2553		0.4430	
σ_η	s.d. surprise* shock	0.9762		1.2358	
σ_ξ	s.d. noise* shock	0.0001*		0.0001*	
	BLL info parameters				
ρ	autocorr. growth		0.8911		0.8068
σ_μ	s.d. growth shock		1.3025		1.8876
σ_ξ	s.d. noise* shock		0.0001*		0.0001*

Table 6: Estimated parameters for alternative versions of the Blanchard et al. (2013) model.