Online Appendix for

"Welfare Consequences of Information Aggregation and Optimal Market Size"

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This online appendix contains all the proofs and background analyses to the main text. We characterize a price-taking equilibrium and a strategic equilibrium simultaneously, with general $\mu \in (0, 1]$ and $\omega \in (0, 1]$. We label endogenous variables in a strategic equilibrium with "st" (e.g., Π_i^{st}, G_i^{st}, and G^{st} etc). Whenever it is necessary to do so, "pt" is used for a price-taking equilibrium.

The rest of this appendix is organized as follows:

1. Proofs for the main text.
2. Background analysis.
    - 2.1 Equilibrium with $\tau_\varepsilon > 0$ (Lemma A1).
        - Information aggregation (Lemma A2).
        - Trade volume, hedging effectiveness, price impact (Lemma A3).
        - Equilibrium as $n \to \infty$ (Lemma A4).
    - 2.2 Equilibrium with $\tau_\varepsilon = 0$ (Lemma A5).
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        - Ex ante characterization (Lemma A7 through A10).
    - 2.4 Optimal market size.
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        - For $\mu\omega < 1$ (Lemma A12).

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1 Proofs for the main text

Proof of Lemma 1

Set $\mu = \omega = 1$ in **Lemma A1(a)** to obtain the equilibrium demand function $q_i(p)$. See **Lemma A1(c)** for the expression of the price informativeness φ. The limit result follows from $p^* = \frac{\beta_s}{\beta_p}(v + \bar{\varepsilon}) - \frac{\beta_e}{\beta_p}\bar{e}$, $q_i^* = \beta_s(\varepsilon_i - \bar{\varepsilon}) - \beta_e(e_i - \bar{e})$, and the expression of $(\beta_s, \beta_e, \beta_p)$. ■ (**L1**)

Proof of Lemma 2

See **Lemma A6(a)** for the derivation and the decomposition of the interim profit Π_i. ■ (**L2**)

Proof of Proposition 1

The results immediately follow from the expression of $\exp(2\rho\Pi)$ shown after Proposition 1 in the main text. This expression of $\exp(2\rho\Pi)$ is derived in **Lemma A10** (substitute $X = \frac{n}{1+n}(1-\varphi)$, $\exp(2\rho\Pi^{nt}) = 1 - \alpha$, and $\alpha = \frac{\rho^2}{\tau_e\tau_v}$ to obtain the exact expression shown in the main text). From the expression of the lower bound for n^* derived in **Lemma A11(a)** (i.e., $\sqrt{\frac{1}{\varphi}\left(1 + \frac{\tau_v}{\tau_\varepsilon}\right)}$), any comparative statics that implies $\varphi = \left(1 + \frac{\rho^2}{\tau_e\tau_\varepsilon}\right)^{-1} \to 0$ also implies $n^* \to \infty$. ■ (**P1**)

Proof of Lemma 3

See **Lemma A3(a,b)** for trade volume and hedging effectiveness. See **Lemma A5** for the characterization of equilibrium with $\tau_\varepsilon = 0$. ■ (**L3**)

Proof of Lemma 4

See **Lemma A4(c)**. ■ (**L4**)

Proof of Proposition 2

See **Lemma A12(b)** for the ex ante profit. See **Lemma A3(a,b)** for trade volume and hedging effectiveness. ■ (**P2**)

Proof of Lemma 5

See **Lemma A3(a)** for trade volume. See **Lemma A10** for the ex ante profit. ■ (**L5**)

Proof of Proposition 3

See **Lemma A11(b)** for the ex ante gains from trade. See **Lemma A3(c)** for price impact. See **Lemma A3(b)** for hedging effectiveness. ■ (**P3**)

2 Background analysis

This section presents a background analysis for the main text. We use the following notations throughout this section:

$$\alpha \equiv \frac{\rho^2}{\tau_v\tau_e}, \qquad d_\varepsilon \equiv \frac{\tau_\varepsilon}{\tau_\varepsilon + \tau_v}, \qquad \alpha_\varepsilon \equiv \frac{\rho^2}{\tau_\varepsilon\tau_e}.$$

Our main objective is to characterize the ex ante payoff Π and gains from trade (henceforth GFT) G, defined as below.

Definition 1 (ex ante profits)
The ex ante profit is $\Pi \equiv -\log\left(E[\exp\left(-\rho\pi_i\right)]\right)$.
The ex ante no-trade profit is $\Pi^{nt} \equiv -\log\left(E[\exp\left(-\rho v e_i\right)]\right)$.
The ex ante gains from trade is $G \equiv \Pi - \Pi^{nt}$.

Definition 2 (interim profits)
The interim profit is $\Pi_i \equiv -\log\left(E_i[\exp\left(-\rho\pi_i\right)]\right)$.
The interim no-trade profit is $\Pi_i^{nt} \equiv -\log\left(E_i[\exp\left(-\rho v e_i\right)]\right)$.
The interim gains from trade is $G_i \equiv \Pi_i - \Pi_i^{nt}$.

Note that Π is the right ex ante welfare measure because $\exp\left(-\Pi\right) = E[\exp\left(-\rho\pi_i\right)]$. We use interim profits and interim gains from trade only for the intermediate step in the characterization of ex ante profits. We also define $\widetilde{G} \equiv -\log\left(E[\exp\left(-\rho G_i\right)]\right)$. Due to risk aversion, $E\left[\Pi_i\right] = \Pi$ does *not* hold.[1] For the same reason, G and $\widetilde{G}$ are *not* equivalent.

2.1 Equilibrium with $\tau_\varepsilon > 0$

We characterize the equilibrium where traders submit the order

$$q_i\left(p\right) = \beta_s s_i - \beta_e e_i - \beta_p p. \tag{1}$$

We define the balance of motives by $B \equiv \frac{\tau_\varepsilon}{\rho}\frac{\beta_e}{\beta_s}$.

Lemma A1 (equilibrium with $\tau_\varepsilon > 0$)
(a) *A price-taking equilibrium exists for all $n \geq 1$ and the optimal order has coefficients*

$$\begin{aligned}
\beta_s^{pt} &= \frac{1-\varphi}{1+\left(1-\omega\right)\left(\omega n-1\right)\varphi+\left(1-\mu\right)\frac{\tau_\varepsilon}{\tau_v}\left\{1+\left(\omega n-\left(1-\omega\right)\right)\varphi\right\}}\sqrt{\mu}\frac{\tau_\varepsilon}{\rho}, \\
\beta_e^{pt} &= \frac{1-\varphi}{1-\left(1-\omega\right)\varphi}, \\
\beta_p^{pt} &= \frac{1-\varphi}{1+\left\{\omega n-\left(1-\omega\right)\right\}\varphi}\frac{\tau}{\rho},
\end{aligned}$$

where $\tau \equiv \left(Var_i\left[v\right]\right)^{-1}$ and $\varphi \in (0,1)$ are characterized in the proof.
(b) *A strategic equilibrium exists if and only if*

$$0 < \frac{n+1}{n-1} < \frac{1}{\omega}\frac{1-\varphi}{\varphi}. \tag{2}$$

The optimal order has coefficients $\beta_x^{st} = \frac{\frac{n-1}{n}-\left(1+\omega-\frac{1-\omega}{n}\right)\varphi}{1-\varphi}\beta_x^{pt}$ *for* $x \in \{s,e,p\}$.
(c) *B, φ and traders' beliefs are the same in both equilibria.*

[1] Similarly, $E_i\left[\pi_i\right] = \Pi_i$ does *not* hold.

If $\mu\omega = 1$, *then* $B = 1$ *and* $\varphi = (1 + \alpha_\varepsilon)^{-1}$.

Remark. If $\mu\omega < 1$, we show below that φ decreases in n and $\lim_{n\to\infty} \varphi = 0$ (**Lemma A2**). Hence, the condition (2) implicitly defines a unique $\underline{n} > 1$ such that a strategic equilibrium exists for all $n > \underline{n}$. If $\mu\omega = 1$, then part **(c)** implies that this $\underline{n}$ is determined by $\frac{\underline{n}+1}{\underline{n}-1} = \alpha_\varepsilon$.

Proof.
(a,b,c) We proceed in three steps:

1) Characterize beliefs $E_i[\widetilde{v}]$, $\widetilde{\tau} \equiv (Var_i[\widetilde{v}])^{-1}$, $E_i[v]$, and $\tau \equiv (Var_i[v])^{-1}$.
2) Derive the optimal order $q_i(p)$.
    - a price-taking equilibrium and a strategic equilibrium.
3) Characterize the balance of motives $B \equiv \frac{\tau_\varepsilon}{\rho}\frac{\beta_e}{\beta_s}$ and the price informativeness φ.

[Step 1] Characterize $E_i[\widetilde{v}]$, $\widetilde{\tau}$, $E_i[v]$ and τ.

From the conjectured order (1) and the market-clearing condition, information in p from trader i's perspective is summarized by

$$h_i \equiv \frac{n\beta_p p - q_i}{n\beta_s} = \widetilde{v} + \left(\overline{\varepsilon}_{-i} - \frac{\beta_e}{\beta_s}\overline{e}_{-i}\right), \tag{3}$$

where $\overline{\varepsilon}_{-i} = \sqrt{1-\omega}\epsilon_0 + \sqrt{\omega}\overline{\epsilon}_{-i}$. Hence, $[\widetilde{v}, s_i, e_i, h_i]^\top$ is jointly normal with mean zero and a covariance matrix

$$\begin{bmatrix} \frac{1}{\tau_v} & \frac{1}{\tau_v} & 0 & \frac{1}{\tau_v} \\ & \frac{1}{\tau_v} + \frac{1}{\tau_\varepsilon} & 0 & \frac{1}{\tau_v} + (1-\omega)\frac{1}{\tau_\varepsilon} \\ & & \frac{1}{\tau_e} & 0 \\ & & & \frac{1}{\tau_v} + \frac{1}{n\tau_\varepsilon}\left\{\omega + n(1-\omega) + \left(\frac{\beta_e}{\beta_s}\right)^2 \frac{\tau_\varepsilon}{\tau_e}\right\} \end{bmatrix}.$$

Let Σ be the variance-covariance matrix of $[s_i, e_i, h_i]^\top$. By Bayes' rule,

$$\begin{aligned} E_i[\widetilde{v}] &= \left[\frac{1}{\tau_v}, 0, \frac{1}{\tau_v}\right] \Sigma^{-1} [s_i, e_i, h_i]^\top, \\ \widetilde{\tau}^{-1} &= \tau_v^{-1} - \left[\frac{1}{\tau_v}, 0, \frac{1}{\tau_v}\right] \Sigma^{-1} \left[\frac{1}{\tau_v}, 0, \frac{1}{\tau_v}\right]^\top. \end{aligned}$$

Define

$$\varphi \equiv \left\{1 + \left(\frac{\beta_e}{\beta_s}\right)^2 \frac{\tau_\varepsilon}{\tau_e}\right\}^{-1} \tag{4}$$

to write the variance of the second term in (3) as

$$Var\left[\overline{\varepsilon}_{-i} - \frac{\beta_e}{\beta_s}\overline{e}_{-i}\right] = \frac{1}{n\tau_\varepsilon}\left\{\omega + n(1-\omega) + \left(\frac{\beta_e}{\beta_s}\right)^2 \frac{\tau_\varepsilon}{\tau_e}\right\} = \frac{1}{n\tau_\varepsilon}\left\{\frac{1}{\varphi} + (1-\omega)(n-1)\right\}.$$

Computing $E_i[\widetilde{v}]$ and $\widetilde{\tau}$ using this φ,

$$E_i[\widetilde{v}] = \frac{\tau_\varepsilon}{\widetilde{\tau}} \frac{(1-\varphi)\, s_i + \omega\varphi \left\{ \frac{\beta_e}{\beta_s} e_i + \frac{\beta_p}{\beta_s} (n+1)\, p \right\}}{1 + (1-\omega)(\omega n - 1)\, \varphi}, \tag{5}$$

and

$$\widetilde{\tau} = \tau_v + \tau_\varepsilon \frac{1 + (\omega n - (1-\omega))\, \varphi}{1 + (1-\omega)(\omega n - 1)\, \varphi}. \tag{6}$$

Note that φ is the right measure of price informativeness, because setting $\varphi = 0$ attains the lower bound $\tau_v + \tau_\varepsilon$ for $\widetilde{\tau}$ (i.e., with only one signal), while setting $\varphi = 1$ attains the upper bound $\tau_v + \tau_\varepsilon \frac{1+n}{1+(1-\omega)n}$ for $\widetilde{\tau}$ (with $1+n$ signals).

Write $E_i[v] = \sqrt{\mu} E_i[\widetilde{v}] = \gamma_s s_i + \gamma_e e_i + \gamma_p p$, so that

$$\begin{aligned}
\gamma_s &= \sqrt{\mu} \frac{\tau_\varepsilon}{\widetilde{\tau}} \frac{1-\varphi}{1 + (1-\omega)(\omega n - 1)\, \varphi}, \\
\gamma_e &= \sqrt{\mu} \frac{\tau_\varepsilon}{\widetilde{\tau}} \frac{\omega\varphi}{1 + (1-\omega)(\omega n - 1)\, \varphi} \frac{\beta_e}{\beta_s}, \\
\gamma_p &= \sqrt{\mu} \frac{\tau_\varepsilon}{\widetilde{\tau}} \frac{\omega\varphi\, (n+1)}{1 + (1-\omega)(\omega n - 1)\, \varphi} \frac{\beta_p}{\beta_s}.
\end{aligned} \tag{7}$$

Next,

$$\begin{aligned}
\tau^{-1} &\equiv Var_i[v] \\
&= (1-\mu) \frac{1}{\tau_v} + \mu \frac{1}{\widetilde{\tau}} \\
&= \frac{1}{\widetilde{\tau}} \left\{ \mu + (1-\mu) \frac{\widetilde{\tau}}{\tau_v} \right\} \\
&= \frac{1}{\tau_v} \frac{1 + (1-\omega)(\omega n - 1)\, \varphi}{1 + (1-\omega)(\omega n - 1)\, \varphi + \frac{\tau_\varepsilon}{\tau_v} \{1 + (\omega n - (1-\omega))\, \varphi\}} \times \\
&\quad \left\{ \mu + (1-\mu) \frac{1 + (1-\omega)(\omega n - 1)\, \varphi + \frac{\tau_\varepsilon}{\tau_v} \{1 + (\omega n - (1-\omega))\, \varphi\}}{1 + (1-\omega)(\omega n - 1)\, \varphi} \right\} \\
&= \frac{1}{\tau_v} \frac{\mu \{1 + (1-\omega)(\omega n - 1)\, \varphi\} + (1-\mu) \left\{ 1 + (1-\omega)(\omega n - 1)\, \varphi + \frac{\tau_\varepsilon}{\tau_v} \{1 + (\omega n - (1-\omega))\, \varphi\} \right\}}{1 + (1-\omega)(\omega n - 1)\, \varphi + \frac{\tau_\varepsilon}{\tau_v} \{1 + (\omega n - (1-\omega))\, \varphi\}} \\
&= \frac{1}{\tau_v} \frac{1 + (1-w)(\omega n - 1)\, \varphi + (1-\mu) \frac{\tau_\varepsilon}{\tau_v} \{1 + (\omega n - (1-\omega))\, \varphi\}}{1 + (1-w)(\omega n - 1)\, \varphi + \frac{\tau_\varepsilon}{\tau_v} \{1 + (\omega n - (1-\omega))\, \varphi\}}.
\end{aligned}$$

Thus, the belief updating with respect to variance is summarized by

$$\frac{\tau_v}{\tau} = \frac{1 + (1-\omega)(\omega n - 1)\, \varphi + (1-\mu) \frac{\tau_\varepsilon}{\tau_v} \{1 + (\omega n - (1-\omega))\, \varphi\}}{1 + (1-\omega)(\omega n - 1)\, \varphi + \frac{\tau_\varepsilon}{\tau_v} \{1 + (\omega n - (1-\omega))\, \varphi\}}. \tag{8}$$

From (4), (6) and (7), the equilibrium beliefs depend on the strategy (1) only through the ratios $\frac{\beta_e}{\beta_s}$ and $\frac{\beta_p}{\beta_s}$. Using the definition of the balance of motive $B \equiv \frac{\tau_\varepsilon}{\rho}\frac{\beta_e}{\beta_s}$, φ in (4) can be written as

$$\varphi = \left(1 + \alpha_\varepsilon B^2\right)^{-1}, \qquad \text{where } \alpha_\varepsilon \equiv \frac{\rho^2}{\tau_\varepsilon \tau_e}. \tag{9}$$

Finally,

$$\begin{aligned}
\frac{\tau}{\widetilde{\tau}} &= \frac{1}{\widetilde{\tau}}\left\{(1-\mu)\frac{1}{\tau_v} + \mu\frac{1}{\widetilde{\tau}}\right\}^{-1} = \frac{1}{\mu + (1-\mu)\frac{\widetilde{\tau}}{\tau_v}} \\
&= \frac{1}{1 + (1-\mu)\frac{\tau_\varepsilon}{\tau_v}\frac{1+(\omega n-(1-\omega))\varphi}{1+(1-\omega)(\omega n-1)\varphi}} \\
&= \frac{1 + (1-\omega)(\omega n - 1)\varphi}{1 + (1-\omega)(\omega n - 1)\varphi + (1-\mu)\frac{\tau_\varepsilon}{\tau_v}\left\{1 + (\omega n - (1-\omega))\varphi\right\}}.
\end{aligned} \tag{10}$$

[Step 2] Derive $q_i(p; e_i, s_i)$.

We derive the optimal order given the belief $E_i[\widetilde{v}]$ and $\widetilde{\tau}$ derived above. From the conjecture (1) and the market-clearing condition $\sum_{j\neq i} q_j + q_i = 0$,

$$-q_i = \sum_{j\neq i} q_j = \beta_s \sum_{j\neq i} s_j - \beta_e \sum_{j\neq i} e_j - n\beta_p p.$$

Solving for the price, we obtain

$$p = p_i + \lambda q_i, \tag{11}$$

where

$$p_i \equiv \frac{\beta_s}{\beta_p}\overline{s}_{-i} - \frac{\beta_e}{\beta_p}\overline{e}_{-i} \text{ and } \lambda \equiv \frac{1}{n\beta_p}.$$

Trader i maximizes $E_i[-\exp(-\rho\pi_i)] = -\exp(-\rho\Pi_i)$. Because of the normality of v conditional on each trader's information, the objective becomes

$$\Pi_i = E_i[v](q_i + e_i) - \frac{\rho}{2} Var_i[v](q_i + e_i)^2 - pq_i \tag{12}$$

subject to (11). The first-order condition is

$$E_i[v] - \frac{\rho}{\tau}(q_i + e_i) = p_i + 2\lambda q_i,$$

which, by (11), becomes

$$E_i[v] - \frac{\rho}{\tau}(q_i + e_i) = p + \lambda q_i. \tag{13}$$

The second-order condition is

$$2\lambda + \frac{\rho}{\tau} > 0. \tag{14}$$

From (13), we obtain

$$q_i(p) = \frac{E_i[v] - p - \frac{\rho}{\tau} e_i}{\lambda + \frac{\rho}{\tau}}. \tag{15}$$

By substituting $E_i[v] = \gamma_s s_i - \gamma_e e_i - \gamma_p p$ into (15),

$$q_i(p) = \frac{\gamma_s s_i - \left(\frac{\rho}{\tau} - \gamma_e\right) e_i - \left(1 - \gamma_p\right) p}{\lambda + \frac{\rho}{\tau}}.$$

By substituting (7), we have three best response coefficients:

$$\widehat{\beta}_s = \frac{\tau_\varepsilon}{\lambda\tau + \rho} \frac{1 - \varphi}{1 + (1-\omega)(\omega n - 1)\varphi} \frac{\tau}{\widetilde{\tau}} \sqrt{\mu}, \tag{16}$$

$$\widehat{\beta}_e = \frac{\rho}{\lambda\tau + \rho} \left(1 - \frac{\omega\varphi}{1 + (1-\omega)(\omega n - 1)\varphi} \frac{\tau_\varepsilon}{\rho} \frac{\beta_e}{\beta_s} \frac{\tau}{\widetilde{\tau}} \sqrt{\mu}\right), \tag{17}$$

$$\widehat{\beta}_p = \frac{\tau}{\lambda\tau + \rho} \left(1 - \frac{\omega\varphi}{1 + (1-\omega)(\omega n - 1)\varphi} \frac{(n+1)\tau_\varepsilon}{\widetilde{\tau}} \frac{\beta_p}{\beta_s} \sqrt{\mu}\right). \tag{18}$$

An important observation is that the value of λ affects the level of coefficients $\left(\widehat{\beta}_s, \widehat{\beta}_e, \widehat{\beta}_p\right)$, but not their ratios. Since the equilibrium price $p^* = \frac{\beta_s}{\beta_p}\overline{s} - \frac{\beta_e}{\beta_p}\overline{e}$ and the associated information aggregation depend only on the ratios $\left(\frac{\beta_s}{\beta_p}, \frac{\beta_e}{\beta_p}\right)$, equilibrium beliefs (i.e. φ, $E_i[v]$, $\widetilde{\tau}$, τ) are identical in a strategic equilibrium and in a price-taking equilibrium. This proves that B, φ and traders' beliefs are the same in both equilibria (the first claim in part (c)).

For both types of equilibria, using (16) and (18), solving the fixed point problem $\frac{\widehat{\beta}_p}{\widehat{\beta}_s} = \frac{\beta_p}{\beta_s}$ yields

$$\frac{\beta_p}{\beta_s} = \frac{\widetilde{\tau}}{\sqrt{\mu}\tau_\varepsilon} \frac{1 + (1-\omega)(\omega n - 1)\varphi}{1 + \{\omega n - (1-\omega)\}\varphi}. \tag{19}$$

Substituting $\widetilde{\tau}$ given in (6),

$$\begin{aligned} \frac{\beta_p}{\beta_s} &= \frac{1}{\sqrt{\mu}} \frac{\tau_v \{1 + (1-\omega)(\omega n - 1)\varphi\} + \tau_\varepsilon \{1 + (\omega n - (1-\omega))\varphi\}}{\tau_\varepsilon \{1 + \{\omega n - (1-\omega)\}\varphi\}} \\ &= \frac{1}{\sqrt{\mu}} \left(\frac{\tau_v}{\tau_\varepsilon} \frac{1 + (1-\omega)(\omega n - 1)\varphi}{1 + \{\omega n - (1-\omega)\}\varphi} + 1\right) > 1. \end{aligned} \tag{20}$$

[A price-taking equilibrium]

By setting $\lambda = 0$, (13)-(15) characterize a price-taking equilibrium. Hence (14) is satisfied in a price-taking equilibrium. From (16) with $\lambda = 0$ and (19),

$$\begin{aligned} \beta_s^{pt} &= \frac{\tau_\varepsilon}{\rho} \frac{1 - \varphi}{1 + (1-\omega)(\omega n - 1)\varphi} \frac{\tau}{\widetilde{\tau}} \sqrt{\mu}, \\ \beta_p^{pt} &= \frac{\tau}{\rho} \frac{1 - \varphi}{1 + \{\omega n - (1-\omega)\}\varphi}. \end{aligned}$$

Combining this with the balance of motive $B \equiv \frac{\tau_\varepsilon}{\rho}\frac{\beta_e}{\beta_s}$, and (10), $\left(\beta_s^{pt}, \beta_e^{pt}, \beta_p^{pt}\right)$ is obtained. Therefore, the optimal order in a price-taking equilibrium has coefficients

$$\begin{aligned}
\beta_s^{pt} &= \frac{1-\varphi}{1+(1-\omega)(\omega n-1)\varphi}\sqrt{\mu}\frac{\tau_\varepsilon}{\rho}\frac{\tau}{\widetilde{\tau}}, \\
\beta_e^{pt} &= \frac{\rho}{\tau_\varepsilon}B\beta_s^{pt}, \\
\beta_p^{pt} &= \frac{\tau}{\rho}\frac{1-\varphi}{1+\{\omega n-(1-\omega)\}\varphi}.
\end{aligned} \tag{21}$$

Using these results, (p^*, q_i^*) can be computed by

$$\begin{aligned}
p^* &= \frac{\beta_s}{\beta_p}\overline{s} - \frac{\beta_e}{\beta_p}\overline{e}, \\
q_i(p^*) &= \beta_s(s_i - \overline{s}) - \beta_e(e_i - \overline{e}).
\end{aligned}$$

Using (10) in (21),

$$\begin{aligned}
q_i^{pt}(p) &= \frac{\sqrt{\mu}(1-\varphi)}{1+(1-\omega)(\omega n-1)\varphi+(1-\mu)\frac{\tau_\varepsilon}{\tau_v}\{1+(\omega n-(1-\omega))\varphi\}} \\
&\quad\times\left\{\frac{\tau_\varepsilon}{\rho}s_i - Be_i - \frac{\widetilde{\tau}}{\sqrt{\mu}\rho}\frac{1+(1-\omega)(\omega n-1)\varphi}{1+\{\omega n-(1-\omega)\}\varphi}p\right\}.
\end{aligned} \tag{22}$$

Substituting $\widetilde{\tau}$ given in (6), coefficients can be written as

$$\begin{aligned}
\beta_s^{pt} &= \frac{1-\varphi}{1+(1-\omega)(\omega n-1)\varphi+(1-\mu)\frac{\tau_\varepsilon}{\tau_v}\{1+(\omega n-(1-\omega))\varphi\}}\sqrt{\mu}\frac{\tau_\varepsilon}{\rho}, \\
\beta_e^{pt} &= \frac{1-\varphi}{1+(1-\omega)(\omega n-1)\varphi+(1-\mu)\frac{\tau_\varepsilon}{\tau_v}\{1+(\omega n-(1-\omega))\varphi\}}\sqrt{\mu}B, \\
\beta_p^{pt} &= \frac{1+(1-\omega)(\omega n-1)\varphi+\frac{\tau_\varepsilon}{\tau_v}\{1+(\omega n-(1-\omega))\varphi\}}{1+(1-\omega)(\omega n-1)\varphi+(1-\mu)\frac{\tau_\varepsilon}{\tau_v}\{1+(\omega n-(1-\omega))\varphi\}}\frac{1-\varphi}{1+(\omega n-(1-\omega))\varphi}\frac{\tau_v}{\rho}.
\end{aligned}$$

The expression of β_e^{pt} will be simplified in Step 3 after characterizing B.

[A strategic equilibrium]
From (19),

$$\lambda = \frac{1}{n\beta_p^{st}} = \frac{1}{n\beta_s^{st}}\frac{\tau_\varepsilon\sqrt{\mu}}{\widetilde{\tau}}\frac{1+\{\omega n-(1-\omega)\}\varphi}{1+(1-\omega)(\omega n-1)\varphi}.$$

Combine this and (16) to solve for β_s^{st}:

$$\beta_s^{st} = \frac{\frac{n-1}{n} - \left(1+\omega-\frac{1-\omega}{n}\right)\varphi}{1+(1-\omega)(\omega n-1)\varphi}\frac{\tau}{\rho}\sqrt{\mu}\frac{\tau_\varepsilon}{\widetilde{\tau}}.$$

From (19),

$$\beta_p^{st} = \frac{\tau}{\rho} \frac{\frac{n-1}{n} - \left(1 + \omega - \frac{1-\omega}{n}\right) \varphi}{1 + \{\omega n - (1-\omega)\} \varphi}.$$

Notice that β_s^{st} is $\frac{\frac{n-1}{n} - \left(1+\omega-\frac{1-\omega}{n}\right)\varphi}{1-\varphi}$ times β_s^{pt}, and β_p^{st} is also $\frac{\frac{n-1}{n} - \left(1+\omega-\frac{1-\omega}{n}\right)\varphi}{1-\varphi}$ times β_p^{pt}. Because the balance of trading motives $B \equiv \frac{\tau_\varepsilon}{\rho} \frac{\beta_e}{\beta_s}$ is the same in both equilibria, β_e^{st} is also $\frac{\frac{n-1}{n} - \left(1+\omega-\frac{1-\omega}{n}\right)\varphi}{1-\varphi}$ times β_e^{pt}.

Next, we check the second order condition for a strategic equilibrium. Substitute $\lambda = \frac{1}{n\beta_p^{st}}$ into (14) to obtain $\frac{2}{n\beta_p^{st}} + \frac{\rho}{\tau} > 0 \Leftrightarrow 0 < 1 + \frac{\tau}{\rho} \frac{2}{n\beta_p^{st}}$. Substituting the expression of β_p^{st},

$$\begin{aligned}
& 1 + \frac{2}{n} \frac{\tau}{\rho} \frac{1}{\beta_p^{st}} \\
= \;& 1 + \frac{2}{n} \frac{1 + \{\omega n - (1-\omega)\} \varphi}{\frac{n-1}{n} - \left(1 + \omega - \frac{1-\omega}{n}\right) \varphi} \\
= \;& 1 + \frac{2\left\{1 + (\omega n - (1-\omega)) \varphi\right\}}{n - 1 - \{(1+\omega) n - (1-\omega)\} \varphi} \\
= \;& \frac{n + 1 - \{(1+\omega) n - (1-\omega)\} \varphi + 2 (\omega n - (1-\omega)) \varphi}{n - 1 - \{(1+\omega) n - (1-\omega)\} \varphi} \\
= \;& \frac{n + 1 - \{(1+\omega) n - (1-\omega) - 2 (\omega n - (1-\omega))\} \varphi}{n - 1 - \{(1+\omega) n - (1-\omega)\} \varphi} \\
= \;& \frac{n + 1 - \{(1-\omega) n + (1-\omega)\} \varphi}{n - 1 - \{(1+\omega) n - (1-\omega)\} \varphi} \\
= \;& \frac{(n+1) \{1 - (1-\omega) \varphi\}}{n - 1 - \{(1+\omega) n - (1-\omega)\} \varphi} \\
= \;& \frac{(n+1) \{1 - (1-\omega) \varphi\}}{\{1 - (1+\omega) \varphi\} n - \{1 - (1-\omega) \varphi\}}.
\end{aligned}$$

Because $1 - (1-\omega) \varphi > 0$,

$$(14) \Leftrightarrow 0 < \frac{n+1}{n-1} < \frac{1}{\omega} \frac{1-\varphi}{\varphi}.$$

[Step 3] Characterize $B \equiv \frac{\tau_\varepsilon}{\rho} \frac{\beta_e}{\beta_s}$ and φ.

In both equilibria, solving a fixed point problem $\frac{\widehat{\beta}_e}{\widehat{\beta}_s} = \frac{\beta_e}{\beta_s}$ from (16) and (17) yields

$$\sqrt{\mu} \frac{\tau_\varepsilon}{\rho} \frac{\widehat{\beta}_e}{\widehat{\beta}_s} = \frac{\widetilde{\tau}}{\tau} \frac{1 + (1-\omega)(\omega n - 1) \varphi}{1 - (1-\omega)\varphi}. \tag{23}$$

Using $B \equiv \frac{\tau_\varepsilon}{\rho}\frac{\beta_e}{\beta_s}$, this becomes

$$\widehat{B} = \frac{\widetilde{\tau}}{\sqrt{\mu}\tau}\frac{1+(1-\omega)(\omega n-1)\varphi}{1-(1-\omega)\varphi}, \tag{24}$$

where φ depends on B through the expression given in (9). Combining (9) and (24) defines a cubic equation in B:

$$F(B) \equiv \left(\alpha_\varepsilon B^2+\omega\right)\left\{\sqrt{\mu}B - \left(1+(1-\mu)\frac{\tau_\varepsilon}{\tau_v}\right)\right\} - \omega\left(1-\omega+(1-\mu)\frac{\tau_\varepsilon}{\tau_v}\right)n = 0.$$

Use $1+(1-\mu)\frac{\tau_\varepsilon}{\tau_v} = \frac{1-\mu d_\varepsilon}{1-d_\varepsilon}$ and $1-\omega+(1-\mu)\frac{\tau_\varepsilon}{\tau_v} = \frac{1-\mu d_\varepsilon}{1-d_\varepsilon}-\omega$ to write this as

$$F(B) \equiv \left(\alpha_\varepsilon B^2+\omega\right)\left(\sqrt{\mu}B - \frac{1-\mu d_\varepsilon}{1-d_\varepsilon}\right) - \ \omega\left(\frac{1-\mu d_\varepsilon}{1-d_\varepsilon}-\omega\right)n = 0. \tag{25}$$

Because $\lim_{B\to-\infty} F(B) = -\infty$, $\lim_{B\to\infty} F(B) = \infty$ and $F(0) < 0$, the cubic equation (25) has at least one and at most three positive solutions. Moreover, because $\frac{1-\mu d_\varepsilon}{1-d_\varepsilon} - \omega > 0$, any solution must satisfy $\sqrt{\mu}B \geq \frac{1-\mu d_\varepsilon}{1-d_\varepsilon}$. The uniqueness follows because $F'(B) > 0$ for all B that satisfies $\sqrt{\mu}B \geq \frac{1-\mu d_\varepsilon}{1-d_\varepsilon}$. This unique solution to (25) characterizes $B \equiv \frac{\tau_\varepsilon}{\rho}\frac{\beta_e}{\beta_s}$. Substituting this back into (9), we obtain the price informativeness φ.

We simplify the expression of β_e^{pt} using the property of B. Because B is a solution to (25),

$$\sqrt{\mu}B = \omega\frac{\frac{1-\mu d_\varepsilon}{1-d_\varepsilon}-\omega}{\alpha_\varepsilon B^2+\omega}n + \frac{1-\mu d_\varepsilon}{1-d_\varepsilon} = \omega\frac{\frac{1-\mu d_\varepsilon}{1-d_\varepsilon}-\omega}{1-(1-\omega)\varphi}\varphi n + \frac{1-\mu d_\varepsilon}{1-d_\varepsilon},$$

where the second equality follows from $\varphi = \left(1+\alpha_\varepsilon B^2\right)^{-1} \Leftrightarrow \alpha_\varepsilon B^2 + \omega = \frac{1-(1-\omega)\varphi}{\varphi}$.

Recall that $1+(1-\mu)\frac{\tau_\varepsilon}{\tau_v} = \frac{1-\mu d_\varepsilon}{1-d_\varepsilon}$. Using these expression,

$$\begin{aligned}
\beta_e^{pt} &= \frac{1-\varphi}{1+(1-\omega)(\omega n-1)\varphi+(1-\mu)\frac{\tau_\varepsilon}{\tau_v}\left\{1+(\omega n-(1-\omega))\varphi\right\}}\sqrt{\mu}B \\
&= (1-\varphi)\frac{\omega\frac{\frac{1-\mu d_\varepsilon}{1-d_\varepsilon}-\omega}{1-(1-\omega)\varphi}\varphi n + \frac{1-\mu d_\varepsilon}{1-d_\varepsilon}}{\omega n\varphi\left\{1-\omega+(1-\mu)\frac{\tau_\varepsilon}{\tau_v}\right\} + \left\{1-(1-\omega)\varphi\right\}\left\{1+(1-\mu)\frac{\tau_\varepsilon}{\tau_v}\right\}} \\
&= \frac{1-\varphi}{1-(1-\omega)\varphi}\frac{\omega n\varphi\frac{\frac{1-\mu d_\varepsilon}{1-d_\varepsilon}-\omega}{1-(1-\omega)\varphi} + \frac{1-\mu d_\varepsilon}{1-d_\varepsilon}}{\omega n\varphi\frac{\frac{1-\mu d_\varepsilon}{1-d_\varepsilon}-\omega}{1-(1-\omega)\varphi} + \frac{1-\mu d_\varepsilon}{1-d_\varepsilon}} \\
&= \frac{1-\varphi}{1-(1-\omega)\varphi}.
\end{aligned}$$

Finally, with $\mu = \omega = 1$, the cubic equation (25) becomes

$$F(B; \mu = \omega = 1) \equiv \left(\alpha_\varepsilon B^2 + 1\right)(B-1) = 0.$$

It is immediate that $B = 1$ is the unique solution, and $\varphi = (1 + \alpha_\varepsilon)^{-1}$ follows from (9). ■
(A1)

2.1.1 Information aggregation

Lemma A2 (information aggregation)
(a) *If* $\mu\omega = 1$, *then* $\widetilde{\tau} = \tau_v + \tau_\varepsilon (1 + n\varphi)$ *and* $\frac{\tau_v}{\widetilde{\tau}}$ *converges to zero at the rate* n^{-1}.
(b) *If* $\mu\omega < 1$, *then* φ decreases in n *at the rate* $n^{-\frac{2}{3}}$ and $n\varphi$ *and* B *increase in* n *at the rate* $n^{\frac{1}{3}}$. τ *increases in* n *and* $\lim_{n\to\infty} \frac{\tau_v}{\tau} > 0$.

Proof.
(a) From (8) with $\mu = \omega = 1$, $\widetilde{\tau} = \tau_v + \tau_\varepsilon (1 + n\varphi)$, where $\varphi = (1 + \alpha_\varepsilon)^{-1}$ from **Lemma A1(c)**.

(b) We proceed in four steps:
1) characterize B by solving the cubic equation (25).
2) characterize φ,
3) characterize $n\varphi$,
4) characterize τ.

[**Step 1**] Characterize B.
Because (25) is linear in n, it can be written as

$$F(B) = \frac{\partial F}{\partial n} n + \left(\alpha_\varepsilon B^2 + \omega\right) \left(\sqrt{\mu} B - \frac{1 - \mu d_\varepsilon}{1 - d_\varepsilon}\right), \tag{26}$$

where $\frac{\partial F}{\partial n} = -\omega \left(\frac{1-\mu d_\varepsilon}{1-d_\varepsilon} - \omega\right)$. First, we show that the solution B increases in n. From (25), the solution B must satisfy $\sqrt{\mu} B > \frac{1-\mu d_\varepsilon}{1-d_\varepsilon}$. Let $\frac{\partial F}{\partial n}\big|_B$ denote $\frac{\partial F}{\partial n}$ evaluated at the solution B. From (26), $\frac{\partial F}{\partial n}\big|_B < 0$ because $F(B) = 0$ and the second term is positive. Because $F'(B) > 0$, by the implicit function theorem, B increases in n.

[**Step 2**] Characterize φ.
Because B increases in n, $\varphi = (1 + \alpha_\varepsilon B^2)^{-1}$ decreases in n. The unique B solves

$$F(B) = \left(\alpha_\varepsilon B^2 + \omega\right) \left(\sqrt{\mu} B - \frac{1 - \mu d_\varepsilon}{1 - d_\varepsilon}\right) - n\omega \left(\frac{1 - \mu d_\varepsilon}{1 - d_\varepsilon} - \omega\right) = 0.$$

Therefore, $\sqrt{\mu} B > \frac{1-\mu d_\varepsilon}{1-d_\varepsilon}$ and B increases in n without a bound at the rate $n^{\frac{1}{3}}$. Hence, $\varphi = (1 + \alpha_\varepsilon B^2)^{-1}$ decreases in n at the rate $n^{-\frac{2}{3}}$.

[**Step 3**] Characterize $n\varphi$.
$F(B) = 0$ implies

$$\frac{1}{\omega n} \left(\omega + \alpha_\varepsilon B^2\right) = \frac{\frac{1-\mu d_\varepsilon}{1-d_\varepsilon} - \omega}{\sqrt{\mu} B - \frac{1-\mu d_\varepsilon}{1-d_\varepsilon}}.$$

Using this,

$$\frac{1}{n}\frac{1}{\varphi} = \frac{1}{n}\left(1+\alpha_\varepsilon B^2\right)$$
$$= \frac{1}{n}\left(\omega+\alpha_\varepsilon B^2\right)+\frac{1-\omega}{n}$$
$$= \omega\frac{\frac{1-\mu d_\varepsilon}{1-d_\varepsilon}-\omega}{\sqrt{\mu}B-\frac{1-\mu d_\varepsilon}{1-d_\varepsilon}}+\frac{1-\omega}{n}.$$

This decreases in n because B increases in n. Hence $n\varphi$ increases in n. The rate of $n\varphi$ follows from the rate of φ.

[Step 4] Characterize τ.
From (6),

$$\widetilde{\tau} = \tau_v + \tau_\varepsilon \frac{1-(1-\omega)\varphi \qquad\qquad + \omega n\varphi}{1-(1-\omega)\varphi+(1-\omega)\,\omega n\varphi}.$$

This increases in n and $\lim_{n\to\infty}\widetilde{\tau} = \tau_v + \tau_\varepsilon\frac{1}{1-\omega}$. From $\tau = \left(\frac{1-\mu}{\tau_v}+\frac{\mu}{\widetilde{\tau}}\right)^{-1}$, τ increases in n and has a finite limit. ■ **(A2)**

2.1.2 Trade volume, hedging effectiveness, price impact

Lemma A3 (trade volume, hedging effectiveness, price impact)

(a) *Trade volume is smaller in a strategic equilibrium than in a price-taking equilibrium. Trade volume increases in n in both equilibria.*

(b) *Hedging effectiveness is identical in both equilibria*
Hedging effectiveness decreases in n for sufficiently large n.
Suppose $\mu = \omega = 1$.
If $\varphi \geq \frac{1}{2}$, then hedging effectiveness decreases in n.
Otherwise, it is hump-shaped in n and maximized at $n = \widehat{n} \equiv \frac{1}{\varphi} - 2$.

(c) *In a strategic equilibrium, price impact decreases in n and converges to zero as $n \to \infty$.*

Proof.

(a) To compute trade volume $\frac{1}{2}E\left[|q_i^*|\right] = \frac{1}{2}\sqrt{\frac{2}{\pi}Var\left[q_i^*\right]}$, recall $q_i^* = \beta_s\left(\varepsilon_i - \overline{\varepsilon}\right) - \beta_e\left(e_i - \overline{e}\right) = \frac{n}{n+1}\left\{\beta_s\sqrt{\omega}\left(\epsilon_i - \overline{\epsilon}_{-i}\right) - \beta_e\left(e_i - \overline{e}_{-i}\right)\right\}$. From **Lemma A1(b)**, $\frac{\beta_x^{st}}{\beta_x^{pt}} = \frac{\frac{n-1}{n}-\left(1+\omega-\frac{1-\omega}{n}\right)\varphi}{1-\varphi} < 1$ for $x \in \{s,e,p\}$. This implies that $Var\left[q_i^*\right]$ is smaller in a strategic equilibrium than in a price-taking equilibrium.

To do comparative statics of trade volume with respect to n, compute $Var\left[q_i^*\right]$:

$$\begin{aligned}
Var\left[q_i^*\right] &= \frac{n}{n+1}\left(\frac{\omega}{\tau_\varepsilon}\beta_s^2+\frac{1}{\tau_e}\beta_e^2\right) \\
&= \frac{n}{n+1}\frac{1}{\tau_e}\beta_e^2\left\{\omega\frac{\tau_e}{\tau_\varepsilon}\left(\frac{\beta_s}{\beta_e}\right)^2+1\right\} \\
&= \frac{n}{n+1}\frac{1}{\tau_e}\beta_e^2\left\{\omega\frac{\tau_e\tau_\varepsilon}{\rho^2}\frac{1}{B^2}+1\right\} \\
&= \frac{n}{n+1}\frac{1}{\tau_e}\beta_e^2\frac{\omega+\alpha_\varepsilon B^2}{\alpha_\varepsilon B^2}.
\end{aligned}$$

Using $\varphi=\left(1+\alpha_\varepsilon B^2\right)^{-1}$,

$$\frac{\omega+\alpha_\varepsilon B^2}{\alpha_\varepsilon B^2}=\frac{\frac{1}{\varphi}-1+\omega}{\frac{1}{\varphi}-1}=\frac{1-(1-\omega)\varphi}{1-\varphi}=\frac{1}{\beta_e^{pt}}.$$

Therefore, for a price-taking equilibrium,

$$Var\left[q_i^*\right]=\frac{n}{n+1}\frac{1}{\tau_e}\beta_e^{pt}=\frac{n}{n+1}\frac{1}{\tau_e}\frac{1-\varphi}{1-(1-\omega)\varphi}.$$

This increases in n, because from **Lemma A2** φ is either independent of n (for $\mu\omega=1$) or decreases in n (for $\mu\omega<1$).

For a strategic equilibrium,

$$\begin{aligned}
Var\left[q_i^*\right] &= \frac{n}{n+1}\frac{1}{\tau_e}\left(\beta_e^{st}\right)^2\frac{1}{\beta_e^{pt}} \\
&= \frac{n}{n+1}\frac{1}{\tau_e}\frac{\frac{n-1}{n}-\left(1+\omega-\frac{1-\omega}{n}\right)\varphi}{1-(1-\omega)\varphi}\frac{\frac{n-1}{n}-\left(1+\omega-\frac{1-\omega}{n}\right)\varphi}{1-\varphi} \\
&= \left(\frac{\frac{n-1}{n}-\left(1+\omega-\frac{1-\omega}{n}\right)\varphi}{1-\varphi}\right)^2\frac{n}{n+1}\frac{1}{\tau_e}\frac{1-\varphi}{1-(1-\omega)\varphi}.
\end{aligned}$$

Because φ is the same in both equilibria, we already know that the term $\frac{n}{n+1}\frac{1}{\tau_e}\frac{1-\varphi}{1-(1-\omega)\varphi}$ increases in n. The other term in the above expression also increases in n because $\frac{\frac{n-1}{n}-\left(1+\omega-\frac{1-\omega}{n}\right)\varphi}{1-\varphi}=\frac{n-1}{n}\frac{\frac{1}{\varphi}-\omega\frac{n+1}{n-1}}{\frac{1}{\varphi}-1}$ increases in n.

(b) To compute the hedging effectiveness $Corr\left[v-p,v\right]=\frac{Cov[v-p,v]}{\sqrt{Var[v-p]Var[v]}}$, recall $v=\sqrt{1-\mu}v_0+\sqrt{\mu}\widetilde{v}$ and the market-clearing price

$$p=\frac{\beta_s}{\beta_p}\overline{s}-\frac{\beta_e}{\beta_p}\overline{e}=\frac{\beta_s}{\beta_p}\left(\widetilde{v}+\sqrt{1-\omega}\epsilon_0+\sqrt{\omega}\overline{\epsilon}\right)-\frac{\beta_e}{\beta_p}\overline{e}.$$

Hence,

$$v-p=\sqrt{1-\mu}v_0+\left(\sqrt{\mu}-\frac{\beta_s}{\beta_p}\right)\widetilde{v}-\frac{\beta_s}{\beta_p}\left(\sqrt{1-\omega}\epsilon_0+\sqrt{\omega}\overline{\epsilon}\right)+\frac{\beta_e}{\beta_p}\overline{e}$$

Because only the ratios $\frac{\beta_s}{\beta_p}$ and $\frac{\beta_e}{\beta_p}$ are relevant, $Corr\left[v-p,v\right]$ is the same in a price-taking equilibrium and in a strategic equilibrium.

Computing $Cov\left[v-p,v\right]$,

$$Cov\left[v-p,v\right]=\left\{1-\mu+\left(\sqrt{\mu}-\frac{\beta_s}{\beta_p}\right)\sqrt{\mu}\right\}\frac{1}{\tau_v}=\left(1-\sqrt{\mu}\frac{\beta_s}{\beta_p}\right)\frac{1}{\tau_v}.$$

Computing $Var\left[v-p\right]$,

$$\begin{aligned}
Var\left[v-p\right] &= \left\{1-2\sqrt{\mu}\frac{\beta_s}{\beta_p}+\left(\frac{\beta_s}{\beta_p}\right)^2\right\}\frac{1}{\tau_v}+\left(\frac{\beta_s}{\beta_p}\right)^2\left(1-\omega+\frac{\omega}{n+1}\right)\frac{1}{\tau_\varepsilon}+\left(\frac{\beta_e}{\beta_p}\right)^2\frac{1}{n+1}\frac{1}{\tau_e} \\
&= \left\{1-2\sqrt{\mu}\frac{\beta_s}{\beta_p}+\left(\frac{\beta_s}{\beta_p}\right)^2\right\}\frac{1}{\tau_v}+\left(\frac{\beta_s}{\beta_p}\right)^2\frac{1}{n+1}\frac{1}{\tau_\varepsilon}\left\{\left(1-\omega\right)\left(n+1\right)+\omega+\left(\frac{\beta_e}{\beta_s}\right)^2\frac{\tau_\varepsilon}{\tau_e}\right\} \\
&= \left\{1-2\sqrt{\mu}\frac{\beta_s}{\beta_p}+\left(\frac{\beta_s}{\beta_p}\right)^2\right\}\frac{1}{\tau_v}+\left(\frac{\beta_s}{\beta_p}\right)^2\frac{1}{n+1}\frac{1}{\tau_\varepsilon}\left\{\left(1-\omega\right)n+1+\alpha_\varepsilon B^2\right\} \\
&= \left\{1-2\sqrt{\mu}\frac{\beta_s}{\beta_p}+\left(\frac{\beta_s}{\beta_p}\right)^2\right\}\frac{1}{\tau_v}+\left(\frac{\beta_s}{\beta_p}\right)^2\frac{1+\left(1-\omega\right)n\varphi}{\tau_\varepsilon\left(n+1\right)\varphi},
\end{aligned}$$

where the last equality used $1+\alpha_\varepsilon B^2=\frac{1}{\varphi}$. Note that $1-2\sqrt{\mu}\frac{\beta_s}{\beta_p}+\left(\frac{\beta_s}{\beta_p}\right)^2=\left(1-\frac{\beta_s}{\beta_p}\right)^2+2\left(1-\sqrt{\mu}\right)\frac{\beta_s}{\beta_p}$. Combining $Cov\left[v-p,v\right]$ and $Var\left[v-p\right]$,

$$\begin{aligned}
\frac{Cov\left[v-p,v\right]}{\sqrt{Var\left[v-p\right]Var\left[v\right]}} &= \frac{\left(1-\sqrt{\mu}\frac{\beta_s}{\beta_p}\right)\frac{1}{\tau_v}}{\sqrt{\left[\left\{\left(1-\frac{\beta_s}{\beta_p}\right)^2+2\left(1-\sqrt{\mu}\right)\frac{\beta_s}{\beta_p}\right\}\frac{1}{\tau_v}+\left(\frac{\beta_s}{\beta_p}\right)^2\frac{1+(1-\omega)n\varphi}{\tau_\varepsilon(n+1)\varphi}\right]\frac{1}{\tau_v}}} \\
&= \frac{1-\frac{\beta_s}{\beta_p}+\left(1-\sqrt{\mu}\right)\frac{\beta_s}{\beta_p}}{\sqrt{\left(1-\frac{\beta_s}{\beta_p}\right)^2+2\left(1-\sqrt{\mu}\right)\frac{\beta_s}{\beta_p}+\left(\frac{\beta_s}{\beta_p}\right)^2\frac{\tau_v}{\tau_\varepsilon}\frac{1+(1-\omega)n\varphi}{(n+1)\varphi}}}
\end{aligned}$$

By dividing by $1-\frac{\beta_s}{\beta_p}$ and using $\chi \equiv \frac{\frac{\beta_s}{\beta_p}}{1-\frac{\beta_s}{\beta_p}} = \frac{1}{\frac{\beta_p}{\beta_s}-1}$ and $\chi+1 = \frac{1}{1-\frac{\beta_s}{\beta_p}}$,

$$
\begin{aligned}
& \frac{1+\left(1-\sqrt{\mu}\right)\chi}{\sqrt{1+2\left(1-\sqrt{\mu}\right)\chi\left(1+\chi\right)+\chi^2\frac{\tau_v}{\tau_\varepsilon}\frac{1+(1-\omega)n\varphi}{(n+1)\varphi}}} \\
= & \frac{1+\left(1-\sqrt{\mu}\right)\chi}{\sqrt{1+2\left(1-\sqrt{\mu}\right)\chi+\chi^2\left\{\frac{\tau_v}{\tau_\varepsilon}\frac{1+(1-\omega)n\varphi}{(n+1)\varphi}+2\left(1-\sqrt{\mu}\right)\right\}}} \\
= & \frac{1+\left(1-\sqrt{\mu}\right)\chi}{\sqrt{\left\{1+\left(1-\sqrt{\mu}\right)\chi\right\}^2+\chi^2\left\{\frac{\tau_v}{\tau_\varepsilon}\frac{1+(1-\omega)n\varphi}{(n+1)\varphi}+2\left(1-\sqrt{\mu}\right)-\left(1-\sqrt{\mu}\right)^2\right\}}} \\
= & \frac{1+\left(1-\sqrt{\mu}\right)\chi}{\sqrt{\left\{1+\left(1-\sqrt{\mu}\right)\chi\right\}^2+\chi^2\left\{\frac{\tau_v}{\tau_\varepsilon}\frac{1+(1-\omega)n\varphi}{(n+1)\varphi}+1-\mu\right\}}} \\
= & \frac{1}{\sqrt{1+\left\{\frac{\tau_v}{\tau_\varepsilon}\frac{1+(1-\omega)n\varphi}{(n+1)\varphi}+1-\mu\right\}\left(\frac{\chi}{1+\left(1-\sqrt{\mu}\right)\chi}\right)^2}}.
\end{aligned}
$$

From (20) in the proof of **Lemma A1**,

$$
\begin{aligned}
\frac{\beta_p}{\beta_s} &= \frac{1}{\sqrt{\mu}}\left(\frac{\tau_v}{\tau_\varepsilon}\frac{1+(1-\omega)(\omega n-1)\varphi}{1+\{\omega n-(1-\omega)\}\varphi}+1\right) \\
&= \frac{1}{\sqrt{\mu}}\left(\frac{\tau_v}{\tau_\varepsilon}\frac{1-(1-\omega)\varphi+(1-\omega)\omega n\varphi}{1-(1-\omega)\varphi \qquad\qquad +\omega n\varphi}+1\right) \\
&= \frac{1}{\sqrt{\mu}}\left(\frac{\tau_v}{\tau_\varepsilon}\frac{\frac{1-(1-\omega)\varphi}{\omega n\varphi}+1-\omega}{\frac{1-(1-\omega)\varphi}{\omega n\varphi} \qquad +1}+1\right).
\end{aligned}
$$

From (25), $\frac{1-(1-\omega)\varphi}{\omega n\varphi} = \frac{\frac{1-\mu d_\varepsilon}{1-d_\varepsilon}-\omega}{\sqrt{\mu}B-\frac{1-\mu d_\varepsilon}{1-d_\varepsilon}}$. This decreases in n because B increases in n (from **Lemma A2(b)**). Therefore, $\frac{\beta_p}{\beta_s}$ decreases in n with $\lim_{n\to\infty}\frac{\beta_p}{\beta_s} = \frac{1}{\sqrt{\mu}}\left\{\frac{\tau_v}{\tau_\varepsilon}(1-\omega)+1\right\}$. Therefore, $\chi = \frac{1}{\frac{\beta_p}{\beta_s}-1}$ increases in n with $\lim_{n\to\infty}\chi = \frac{\sqrt{\mu}}{\frac{\tau_v}{\tau_\varepsilon}(1-\omega)+1-\sqrt{\mu}}$.

To show that $Corr\left[v-p,v\right]$ decreases in n for sufficiently large n, we show that

$$
\left\{\frac{\tau_v}{\tau_\varepsilon}\frac{1+(1-\omega)n\varphi}{(n+1)\varphi}+1-\mu\right\}\left(\frac{\chi}{1+\left(1-\sqrt{\mu}\right)\chi}\right)^2
$$

increases in n for sufficiently large n. First,

$$
\begin{aligned}
\frac{\chi}{1+\left(1-\sqrt{\mu}\right)\chi} &= \frac{1}{\frac{\beta_p}{\beta_s}-1+1-\sqrt{\mu}} = \frac{1}{\frac{\beta_p}{\beta_s}-\sqrt{\mu}} \\
&= \frac{\sqrt{\mu}}{\frac{\tau_v}{\tau_\varepsilon}\frac{1-(1-\omega)\varphi+(1-\omega)\omega n\varphi}{1-(1-\omega)\varphi+\omega n\varphi}+1-\mu} \\
&= \frac{\frac{\tau_\varepsilon}{\tau_v}\sqrt{\mu}}{\frac{1-(1-\omega)\varphi+(1-\omega)\omega n\varphi}{1-(1-\omega)\varphi+\omega n\varphi}+\frac{\tau_\varepsilon}{\tau_v}\left(1-\mu\right)}.
\end{aligned}
$$

Therefore,

$$
\left(\frac{\chi}{1+\left(1-\sqrt{\mu}\right)\chi}\right)^2 = \frac{\left(\frac{\tau_\varepsilon}{\tau_v}\right)^2\mu}{\left\{\frac{1-(1-\omega)\varphi+(1-\omega)\omega n\varphi}{1-(1-\omega)\varphi+\omega n\varphi}+\frac{\tau_\varepsilon}{\tau_v}\left(1-\mu\right)\right\}^2}.
$$

Combining this with $\frac{\tau_v}{\tau_\varepsilon}\frac{1+(1-\omega)n\varphi}{(n+1)\varphi}+1-\mu = \frac{\tau_v}{\tau_\varepsilon}\left\{\frac{1+(1-\omega)n\varphi}{(n+1)\varphi}+\frac{\tau_\varepsilon}{\tau_v}\left(1-\mu\right)\right\}$,

$$
\begin{aligned}
&\left\{\frac{\tau_v}{\tau_\varepsilon}\frac{1+\left(1-\omega\right)n\varphi}{\left(n+1\right)\varphi}+1-\mu\right\}\left(\frac{\chi}{1+\left(1-\sqrt{\mu}\right)\chi}\right)^2 \\
= &\frac{\frac{1+(1-\omega)n\varphi}{(n+1)\varphi}+\frac{\tau_\varepsilon}{\tau_v}\left(1-\mu\right)}{\frac{1-(1-\omega)\varphi+(1-\omega)\omega n\varphi}{1-(1-\omega)\varphi+\omega n\varphi}+\frac{\tau_\varepsilon}{\tau_v}\left(1-\mu\right)}\frac{\frac{\tau_\varepsilon}{\tau_v}\mu}{\frac{1-(1-\omega)\varphi+(1-\omega)\omega n\varphi}{1-(1-\omega)\varphi+\omega n\varphi}+\frac{\tau_\varepsilon}{\tau_v}\left(1-\mu\right)}.
\end{aligned}
$$

Note that

$$
\frac{\frac{1+(1-\omega)n\varphi}{(n+1)\varphi}+\frac{\tau_\varepsilon}{\tau_v}\left(1-\mu\right)}{\frac{1-(1-\omega)\varphi+(1-\omega)\omega n\varphi}{1-(1-\omega)\varphi+\omega n\varphi}+\frac{\tau_\varepsilon}{\tau_v}\left(1-\mu\right)} = 1+\frac{\frac{1+(1-\omega)n\varphi}{(n+1)\varphi}-\frac{1-(1-\omega)\varphi+(1-\omega)\omega n\varphi}{1-(1-\omega)\varphi+\omega n\varphi}}{\frac{1-(1-\omega)\varphi+(1-\omega)\omega n\varphi}{1-(1-\omega)\varphi+\omega n\varphi}+\frac{\tau_\varepsilon}{\tau_v}\left(1-\mu\right)}.
$$

Computing $\frac{1+(1-\omega)n\varphi}{(n+1)\varphi}-\frac{1-(1-\omega)\varphi+(1-\omega)\omega n\varphi}{1-(1-\omega)\varphi+\omega n\varphi}$ yields

$$
\begin{aligned}
&\frac{\left\{\frac{1}{\varphi}+\left(1-\omega\right)n\right\}\left[1-\left(1-\omega\right)\varphi+\omega n\varphi\right]-\left(n+1\right)\left\{1-\left(1-\omega\right)\varphi+\left(1-\omega\right)\omega n\varphi\right\}}{\left(n+1\right)\left[1-\left(1-\omega\right)\varphi+\omega n\varphi\right]} \\
= &\frac{\omega\frac{1-\varphi}{\varphi}}{\left(n+1\right)\left[1+\left\{n\omega-\left(1-\omega\right)\right\}\varphi\right]}.
\end{aligned}
$$

All in all,

$$
\begin{aligned}
&\left\{\frac{\tau_v}{\tau_\varepsilon}\frac{1+\left(1-\omega\right)n\varphi}{\left(n+1\right)\varphi}+1-\mu\right\}\left(\frac{\chi}{1+\left(1-\sqrt{\mu}\right)\chi}\right)^2 \\
= &\left[1+\frac{\omega\frac{1-\varphi}{\varphi}\frac{1}{n+1}\frac{1}{1-(1-\omega)\varphi+\omega n\varphi}}{\frac{1-(1-\omega)\varphi+(1-\omega)\omega n\varphi}{1-(1-\omega)\varphi+\omega n\varphi}+\frac{\tau_\varepsilon}{\tau_v}\left(1-\mu\right)}\right]\frac{\frac{\tau_\varepsilon}{\tau_v}\mu}{\frac{1-(1-\omega)\varphi+(1-\omega)\omega n\varphi}{1-(1-\omega)\varphi+\omega n\varphi}+\frac{\tau_\varepsilon}{\tau_v}\left(1-\mu\right)}.
\end{aligned}
$$

$[\mu\omega < 1]$ Because $\varphi \sim n^{-\frac{2}{3}}$ and $n\varphi \sim n^{\frac{1}{3}}$, for sufficiently large n, the terms in the square bracket approaches one from above as $\omega\frac{1-\varphi}{\varphi}\frac{1}{n+1}\frac{1}{1-(1-\omega)\varphi+\omega n\varphi}$ converges to zero at the rate $n^{-\frac{2}{3}}$. The term after the square bracket approaches $\frac{\frac{\tau_\varepsilon}{\tau_v}\mu}{1-\omega+\frac{\tau_\varepsilon}{\tau_v}(1-\mu)}$ from below as $\frac{1-(1-\omega)\varphi}{1-(1-\omega)\varphi+\omega n\varphi}$ converges zero at the rate $n^{-\frac{1}{3}}$. Therefore, $\left\{\frac{\tau_v}{\tau_\varepsilon}\frac{1+(1-\omega)n\varphi}{(n+1)\varphi}+1-\mu\right\}\left(\frac{\chi}{1+\left(1-\sqrt{\mu}\right)\chi}\right)^2$ approaches its limit from below.

$[\mu = \omega = 1]$

First, $\frac{\beta_p}{\beta_s} = \frac{\tau_v}{\tau_\varepsilon}\frac{1}{1+n\varphi} + 1$ and $\chi = \frac{\tau_\varepsilon}{\tau_v}(1+n\varphi)$, where φ is independent of n. The hedging effectiveness is

$$Corr\,[v-p,v] = \frac{1}{\sqrt{1+\frac{\tau_v}{\tau_\varepsilon}\frac{1}{(n+1)\varphi}\left\{\frac{\tau_\varepsilon}{\tau_v}(1+n\varphi)\right\}^2}} = \frac{1}{\sqrt{1+\frac{\tau_\varepsilon}{\tau_v}\frac{1}{\varphi}\frac{1}{n+1}(1+n\varphi)^2}}.$$

This is inversely related to $\frac{(1+n\varphi)^2}{n+1}$. Taking the derivative of $\frac{(1+n\varphi)^2}{n+1}$ with respect to n,

$$\frac{2(1+n\varphi)\varphi(n+1)-(1+n\varphi)^2}{(n+1)^2} = \frac{\varphi^2 n^2 + 2\varphi^2 n - (1-2\varphi)}{(n+1)^2}.$$

If $1-2\varphi \le 0$, $\frac{(1+n\varphi)^2}{n+1}$ always increases in n and hence $Corr\,[v-p,v]$ decreases in n.

If $1-2\varphi > 0$, $\varphi^2 n^2 + 2\,\varphi^2 n - (1-2\varphi) = 0$ has two solutions

$$\frac{-\varphi^2 \pm \sqrt{\varphi^4+\varphi^2(1-2\varphi)}}{\varphi^2} = -1 \pm \frac{1-\varphi}{\varphi} = \left\{-\frac{1}{\varphi}, \frac{1-2\varphi}{\varphi}\right\}.$$

It remains to show that $\frac{\varphi^2 n^2+2\varphi^2 n-(1-2\varphi)}{(n+1)^2}$ is increasing in n at $n = \frac{1-2\varphi}{\varphi}$. Taking the derivative of $\frac{\varphi^2 n^2+2\varphi^2 n-(1-2\varphi)}{(n+1)^2}$ with respect to n,

$$\begin{aligned}
& \frac{2\varphi^2(n+1)(n+1)^2 - 2(n+1)\{\varphi^2 n^2 + 2\varphi^2 n - (1-2\varphi)\}}{(n+1)^4} \\
= & \frac{2\left[\varphi^2(n+1)^2 - \{\varphi^2 n^2 + 2\varphi^2 n - (1-2\varphi)\}\right]}{(n+1)^3} \\
= & \frac{2}{(n+1)^3}\left[\varphi^2+1-2\varphi\right] = \frac{2(1-\varphi)^2}{(n+1)^3} > 0.
\end{aligned}$$

Therefore, $\frac{(1+n\varphi)^2}{n+1}$ is uniquely minimized at $n = \widehat{n} \equiv \frac{1}{\varphi} - 2$ and hence $Corr\,[v-p,v]$ is uniquely maximized at $\widehat{n}$.

(c) The price impact is $\lambda = \frac{1}{n\beta_p^{st}}$. Using the expression of β_p^{st} given in **Lemma A1**,

$$n\beta_p^{st} = \frac{n(1-\varphi)}{1+\{\omega n-(1-\omega)\}\varphi}\frac{\tau}{\rho}.$$

$[\mu\omega = 1]$ From **Lemma A2(a)**, $\tau = \tau_v + \tau_\varepsilon (1 + n\varphi)$ and φ is constant. Hence $n\beta_p^{st} = (1-\varphi)\frac{n}{1+n\varphi}\frac{\tau_v+\tau_\varepsilon(1+n\varphi)}{\rho}$ goes to infinity as $n \to \infty$. Also,

$$n\beta_p^{st} = \frac{\tau_\varepsilon}{\rho}(1-\varphi)\, n \frac{\frac{\tau_v}{\tau_\varepsilon} + 1 + n\varphi}{1 + n\varphi}.$$

Taking the derivative of $n\frac{\frac{\tau_v}{\tau_\varepsilon}+1+n\varphi}{1+n\varphi}$ with respect to n,

$$\frac{\left(\frac{\tau_v}{\tau_\varepsilon} + 1 + 2n\varphi\right)(1+n\varphi) - n\varphi\left(\frac{\tau_v}{\tau_\varepsilon} + 1 + n\varphi\right)}{(1+n\varphi)^2}.$$

The numerator is $(n\varphi)^2 + \left\{3 + \frac{\tau_v}{\tau_\varepsilon} - \left(\frac{\tau_v}{\tau_\varepsilon} + 1\right)\right\} n\varphi + \frac{\tau_v}{\tau_\varepsilon} + 1 = \varphi^2 n^2 + 2\varphi n + \frac{\tau_v}{\tau_\varepsilon} + 1 > 0$. This implies $n\beta_p^{st}$ is strictly increasing in n.

$[\mu\omega < 1]$ From **Lemma A2(b)**, $\lim_{n\to\infty} \tau < \infty$, $\lim_{n\to\infty} \varphi = 0$, and $\lim_{n\to\infty} n\varphi = \infty$. This implies $\lim_{n\to\infty} n\beta_p^{st} = \infty$. To show that λ decreases in n, it suffices to show that $\frac{\lambda\tau}{\rho+\lambda\tau}$ decreases in n, because τ increases in n (**Lemma A2(b)**) and $\lambda = \frac{1}{n\beta_p^{st}} > 0$ in equilibrium. First we show that $\frac{\lambda\tau}{\rho+\lambda\tau} = \frac{\omega\varphi + \frac{1-(1-\omega)\varphi}{n}}{1-\varphi}$. Using $\lambda = \frac{1}{n\beta_p^{st}}$, $\frac{\lambda\tau}{\rho+\lambda\tau} = \frac{1}{\rho\beta_p^{st}\frac{n}{\tau}+1}$. Recalling $\beta_p^{st} = \beta_p^{pt}\frac{\frac{n-1}{n} - \left(1+\omega-\frac{1-\omega}{n}\right)\varphi}{1-\varphi}$ and $\beta_p^{pt} = \frac{\tau}{\rho}\frac{1-\varphi}{1+\{\omega n-(1-\omega)\}\varphi}$,

$$\rho\beta_p^{st}\frac{n}{\tau} = \frac{\rho}{\tau}\beta_p^{pt} n \frac{\frac{n-1}{n} - \left(1+\omega-\frac{1-\omega}{n}\right)\varphi}{1-\varphi} = \frac{n-1-\{(1+\omega)n-(1-\omega)\}\varphi}{1+\{\omega n-(1-\omega)\}\varphi}.$$

Hence,

$$\begin{aligned}\rho\beta_p^{st}\frac{n}{\tau} + 1 &= \frac{n-1-\{(1+\omega)n-(1-\omega)\}\varphi + 1 + \{\omega n-(1-\omega)\}\varphi}{1+\{\omega n-(1-\omega)\}\varphi} \\ &= \frac{(1-\varphi)n}{1+\{\omega n-(1-\omega)\}\varphi},\end{aligned}$$

which implies

$$\frac{\lambda\tau}{\rho+\lambda\tau} = \frac{\left(\omega - \frac{1-\omega}{n}\right)\varphi + \frac{1}{n}}{1-\varphi} = \frac{\omega\varphi + \frac{1-(1-\omega)\varphi}{n}}{1-\varphi}. \tag{27}$$

Next, we show that $\frac{1-(1-\omega)\varphi}{n}$ decreases in n. From (25), $\frac{1-(1-\omega)\varphi}{n} = \frac{\omega\left(\frac{1-\mu d_\varepsilon}{1-d_\varepsilon}-\omega\right)\varphi}{\sqrt{\mu}B - \frac{1-\mu d_\varepsilon}{1-d_\varepsilon}}$. This decreases in n because φ decreases in n and B increases in n (from **Lemma A2(b)**). Therefore, $\frac{\lambda\tau}{\rho+\lambda\tau}$ decreases in n. ■ **(A3)**

2.1.3 Equilibrium as $n \to \infty$

Lemma A4 (equilibrium as $n \to \infty$)

(a) *If* $\mu < 1$ *or* $\omega < 1$*, then there is* $\underline{n} \in (1, \infty)$ *such that* (2) *is satisfied for all* $n > \underline{n}$*.*
If $\mu = \omega = 1$*, then the same holds if* $\alpha_\varepsilon > 1$*.*

(b) *Suppose* $\mu = \omega = 1$*. For a strategic equilibrium, additionally assume* $\alpha_\varepsilon > 1$*.*

$$\lim_{n\to\infty} \left(\beta_s, \beta_e, \beta_p\right) = \begin{cases} \left(\frac{\rho}{\tau_e}\varphi, 1-\varphi, \frac{\rho}{\tau_e}\varphi\right) & \text{in a price-taking equilibrium} \\ \left(\frac{\rho}{\tau_e}\left(1-2\varphi\right), 1-2\varphi, \frac{\rho}{\tau_e}\left(1-2\varphi\right)\right) & \text{in a strategic equilibrium} \end{cases},$$

where $\varphi = \left(1+\alpha_\varepsilon\right)^{-1}$*.*

(c) *Suppose* $\mu < 1$ *or* $\omega < 1$*. In both equilibria:*
β_s *and* β_p *converge to zero at the rate* $n^{-\frac{1}{3}}$*,*
$1-\beta_e$ *decreases in* n *and converges to zero at the rate* $n^{-\frac{2}{3}}$*, and*
the allocation approaches the average endowment.

(d) $\lim_{n\to\infty} p^* = \frac{\sqrt{\mu}d_\varepsilon}{(1-\omega)(1-d_\varepsilon)+d_\varepsilon}\left(\widetilde{v} + \sqrt{1-\omega}\epsilon_0\right)$ *for all* μ, ω *in both equilibria.*

(e) *The price impact* λ *converges to zero at the rate* n^{-1} *if* $\mu = \omega = 1$*,*
and at the rate $n^{-\frac{2}{3}}$ *if* $\mu\omega < 1$*.*

Proof.

(a) From **Lemma A2**, $\lim_{n\to\infty}\varphi = 0$ for $\mu < 1$ or $\omega < 1$. There exists a unique $\underline{n} > 1$ such that $\frac{\underline{n}+1}{\underline{n}-1} = \frac{1-\varphi}{\varphi}$, because $\frac{n+1}{n-1}$ increases in n with $\lim_{n\searrow 1}\frac{n+1}{n-1} = \infty$ and $\lim_{n\to\infty}\frac{n+1}{n-1} = 1$ while φ decreases in n and $\lim_{n\to\infty}\frac{1-\varphi}{\varphi} = \infty$. Clearly, (2) is satisfied if and only if $n > \underline{n}$.

If $\mu = \omega = 1$, (2) becomes $\frac{n+1}{n-1} < \alpha_\varepsilon$. As $\frac{n+1}{n-1} > 1$ but $\lim_{n\to\infty}\frac{n+1}{n-1} = 1$, the result follows.

(b) This follows from **Lemma A2** and the expression of coefficients in **Lemma A1**. Note that $\alpha_\varepsilon > 1$ implies $1-2\varphi = \frac{\alpha_\varepsilon - 1}{\alpha_\varepsilon + 1} > 0$.

(c) First, recall $\beta_x^{st} = \frac{\frac{n-1}{n} - \left(1+\omega-\frac{1-\omega}{n}\right)\varphi}{1-\varphi}\beta_x^{pt}$ for $x \in \{s, e, p\}$ (from **Lemma A1**) and note that $\frac{\frac{n-1}{n} - \left(1+\omega-\frac{1-\omega}{n}\right)\varphi}{1-\varphi} \to 1$ because $\varphi \to 0$ (from **Lemma A2**). Therefore, it suffices to show the result for a price-taking equilibrium. We drop the superscript "*pt*".

For β_s and β_e, from their expressions given in **Lemma A1**, $\lim_{n\to\infty}\varphi = 0$ and $\lim_{n\to\infty} n\varphi = \infty$ directly imply $\lim_{n\to\infty}\beta_s = 0$ and $\lim_{n\to\infty}\beta_e = 1$.

For $\beta_p = \frac{1-\varphi}{1+\{n\omega-(1-\omega)\}\varphi}\frac{\tau}{\rho}$, note that τ is bounded. Hence, β_p converges zero at the rate of $\frac{1}{n\varphi}$, i.e., $n^{-\frac{1}{3}}$. Using the results from **Lemma A2** for φ, $n\varphi$, and τ given in (8),

$$\lim_{n\to\infty}\frac{\beta_s}{\beta_p} = \frac{\sqrt{\mu}\tau_\varepsilon}{\left(1-\omega\right)\tau_v + \tau_\varepsilon} = \frac{\sqrt{\mu}d_\varepsilon}{\left(1-\omega\right)\left(1-d_\varepsilon\right)+d_\varepsilon} \in (0, \infty).$$

Hence, β_s converges zero also at the rate $n^{-\frac{1}{3}}$. The rate at which $1-\beta_e$ converges to zero is obvious from

$$1-\beta_e = \frac{1-\left(1-\omega\right)\varphi - \left(1-\varphi\right)}{1-\left(1-\omega\right)\varphi} = \frac{\omega\varphi}{1-\left(1-\omega\right)\varphi}.$$

The result on the allocation follows from $q_i^* = \beta_s (s_i - \overline{s}) - \beta_e (e_i - \overline{e})$ and $(\beta_s, \beta_e) \to (0,1)$.

(d) We compute the limit of $p^* = \frac{\beta_s}{\beta_p}\overline{s} - \frac{\beta_e}{\beta_p}\overline{e}$. First, from **Lemma A1**,

$$\frac{\beta_s}{\beta_p} = \frac{1+\{\omega n-(1-\omega)\}\varphi}{1+(1-\omega)(\omega n-1)\varphi+(1-\mu)\frac{\tau_\varepsilon}{\tau_v}\{1+(\omega n-(1-\omega))\varphi\}}\sqrt{\mu}\frac{\tau_\varepsilon}{\tau},$$

$$\frac{\beta_e}{\beta_p} = \frac{1+\{\omega n-(1-\omega)\}\varphi}{1-(1-\omega)\varphi}\frac{\rho}{\tau}. \tag{28}$$

Therefore, $\lim_{n\to\infty}\frac{\beta_s}{\beta_p}\overline{s} = \frac{\sqrt{\mu}d_\varepsilon}{(1-\omega)(1-d_\varepsilon)+d_\varepsilon}\left(\widetilde{v}+\sqrt{1-\omega}\epsilon_0\right)$. It remains to show $\lim_{n\to\infty}\frac{\beta_e}{\beta_p}\overline{e} \to 0$.

First, consider the case $\mu=\omega=1$. In this case, $\tau = \tau_v + \tau_\varepsilon(1+n\varphi)$ and

$$\frac{\beta_e}{\beta_p} = \frac{(1+n\varphi)\rho}{\tau_v+\tau_\varepsilon(1+n\varphi)},$$

where φ is independent of n. Thus, $\lim_{n\to\infty}\frac{\beta_e}{\beta_p} = \frac{\rho}{\tau_\varepsilon}$ and $\lim_{n\to\infty}\frac{\beta_e}{\beta_p}\overline{e} \to 0$.

Next, consider the case $\mu<1$ or $\omega<1$. In this case, (28) is unbounded in n and increases in n at the same rate with $n\varphi$. From **Lemma A2**, this rate is $n^{\frac{1}{3}} = n^{-\frac{1}{6}}n^{\frac{1}{2}}$. Because $n^{\frac{1}{2}}\overline{e}$ converges in distribution to a normal random variable, $\lim_{n\to\infty}\frac{\beta_e}{\beta_p}\overline{e} \to 0$.

(e) This is immediate from the result for β_p in **(c)** and **(d)**. ■ **(A4)**

2.2 Equilibrium with $\tau_\varepsilon = 0$

Lemma A5 (equilibrium with $\tau_\varepsilon = 0$)

(a) *A price-taking equilibrium exists for all $n \geq 1$ and the optimal order is $q_i^{pt}(p) = -e_i - \frac{\tau_v}{\rho}p$.*

(b) *A strategic equilibrium exists if and only if $1 < n$. The optimal order has coefficients $\beta_x^{st} = \frac{n-1}{n}\beta_x^{pt}$ for $x \in \{e,p\}$.*

(c) *Trade volume and hedging effectiveness increase in n, while price impact decreases in n.*

Proof.

(a) Conjecture $q_i(p) = \beta_e e_i - \beta_p p$. Step 1 of **Lemma A1** becomes $E_i[v] = 0$ and $\tau = \tau_v$. Step 2 becomes $q_i(p) = \frac{-p-\frac{\rho}{\tau_v}e_i}{\lambda+\frac{\rho}{\tau_v}}$. Hence, $\widehat{\beta}_e = \frac{\rho}{\lambda\tau_v+\rho}$ and $\widehat{\beta}_p = \frac{\tau_v}{\lambda\tau_v+\rho}$. Price-taking or strategic, $\frac{\beta_e}{\beta_p} = \frac{\rho}{\tau_v}$. By setting $\lambda = 0$, the optimal order in a price-taking equilibrium has $\beta_e^{pt} = 1$ and $\beta_p^{pt} = \frac{\rho}{\tau_v}$. Note that the second order condition $\frac{\rho}{\tau_v} > 0$ is always satisfied.

(b) For a strategic equilibrium, solve a fixed point problem in λ defined by $\widehat{\lambda} = \frac{1}{n\widehat{\beta}_e} = \frac{\lambda\tau_v+\rho}{n\tau_v}$. Solving $\widehat{\lambda} = \lambda$, obtain $\lambda = \frac{1}{n-1}\frac{\rho}{\tau_v}$, $\beta_p^{st} = \frac{\tau_v}{\frac{1}{n-1}\frac{\rho}{\tau_v}\tau_v+\rho} = \frac{n-1}{n}\frac{\rho}{\tau_v}$ and $\beta_e^{st} = \frac{\rho}{\tau_v}\beta_p^{st} = \frac{n-1}{n}$. Finally, the second order condition is $2\lambda + \frac{\rho}{\tau_v} > 0 \Leftrightarrow \frac{2}{n-1}+1 > 0 \Leftrightarrow n > 1$. Note that $\lim_{n\searrow 1} q_i^{st}(p) = 0$.

(c) The quantity traded is $q_i^*(p^*) = \beta_e(e_i - \overline{e}) = \beta_e \frac{n}{n+1}(e_i - \overline{e}_i)$. Trade volume is

$$\begin{aligned}\frac{1}{2}E\left[\left|\beta_e \frac{n}{n+1}(e_i - \overline{e}_i)\right|\right] &= \frac{1}{2}Var\left[\beta_e \frac{n}{n+1}(e_i - \overline{e}_i)\right] \\ &= \frac{1}{2}\beta_e^2 \frac{n}{n+1}\frac{1}{\tau_e}.\end{aligned}$$

This increases in n in both equilibria, because $\beta_e^{pt} = 1$ and $\beta_e^{st} = \frac{n-1}{n}$ both (weakly) increase in n. The price impact $\lambda = \frac{1}{n-1}\frac{\rho}{\tau_v}$ clearly decreases in n. Finally, the market-clearing price $p = -\frac{\beta_e}{\beta_p}\overline{e}$ is uncorrelated with v and $v - p = v + \frac{\rho}{\tau_v}\overline{e}$. Therefore, the hedging effectiveness is

$$Corr[v-p, v] = \frac{\frac{1}{\tau_v}}{\sqrt{\left(\frac{1}{\tau_v} + \left(\frac{\rho}{\tau_v}\right)^2 \frac{1}{n+1}\frac{1}{\tau_e}\right)\frac{1}{\tau_v}}} = \frac{1}{\sqrt{1 + \frac{\rho^2}{\tau_v \tau_e}\frac{1}{n+1}}}.$$

This increases in n and $\lim_{n\to\infty} Corr[v-p, v] = 1$. ■ **(A5)**

2.3 Ex ante profits

2.3.1 Interim characterization

We first characterize the interim GFT. Recall that the interim payoff, the interim GFT, and the ex ante GFT in a strategic equilibrium are denoted with superscript "*st*", i.e. Π_i^{st}, G_i^{st}, and G^{st}. We drop "*pt*" for the price-taking case for brevity.

Lemma A6 (interim characterization)
(a) $\Pi_i = \frac{\tau}{2\rho}(a_i^2 + b_i^2 - c_i^2)$ *and* $\Pi_i^{nt} = \frac{\tau}{2\rho}(b_i^2 - c_i^2)$, *where*

$$a_i \equiv E_i[v] - p - \frac{\rho}{\tau}e_i, \qquad b_i \equiv E_i[v], \qquad c_i \equiv E_i[v] - \frac{\rho}{\tau}e_i.$$

(b) $\Pi_i^{st} = \frac{\tau}{2\rho}\left(\left(1 - \widetilde{\lambda}\right)a_i^2 + b_i^2 - c_i^2\right)$ *and* $G_i^{st} = \left(1 - \widetilde{\lambda}\right)G_i$,
where $\widetilde{\lambda} \in (0,1)$ *defined below decreases in* n.

$$\widetilde{\lambda} \equiv \left(\frac{\lambda\tau}{\rho + \lambda\tau}\right)^2 = \left(\frac{\left(\omega - \frac{1-\omega}{n}\right)\varphi + \frac{1}{n}}{1 - \varphi}\right)^2. \tag{29}$$

If $\mu = \omega = 1$, *then* $\lim_{n\to\infty}\widetilde{\lambda} = \left(\frac{\varphi}{1-\varphi}\right)^2 > 0$ *with* $\varphi = (1 + \alpha_\varepsilon)^{-1}$.
Otherwise, $\lim_{n\to\infty}\widetilde{\lambda} = 0$ *at the rate* $n^{-\frac{4}{3}}$.

Proof.

(a) By plugging the optimal demand function (15) into the interim profit (12), obtain

$$\Pi_i^{st} = \left(1 - \left(\frac{\lambda\tau}{\rho + \lambda\tau}\right)^2\right)\left\{\frac{\tau}{2\rho}\left(E_i[v] - p\right)^2 + pe_i\right\} + \left(\frac{\lambda\tau}{\rho + \lambda\tau}\right)^2\left(E_i[v]e_i - \frac{\rho}{2\tau}e_i^2\right). \quad (30)$$

By setting $q_i = 0$ in (12), the interim no-trade profit is

$$\begin{aligned}\Pi_i^{nt} &= E_i[v]e_i - \frac{\rho}{2\tau}e_i^2 \\ &= \frac{\tau}{2\rho}\left(E_i[v]\right)^2 - \frac{\rho}{2\tau}\left(\frac{\tau}{\rho}E_i[v] - e_i\right)^2 \\ &= \frac{\tau}{2\rho}\left\{\left(E_i[v]\right)^2 - \left(E_i[v] - \frac{\rho}{\tau}e_i\right)^2\right\} \\ &= \frac{\tau}{2\rho}\left(b_i^2 - c_i^2\right).\end{aligned}$$

By setting, $\lambda = 0$ in (30), the interim profit in the price-taking equilibrium is

$$\Pi_i = \frac{\tau}{2\rho}(E_i[v] - p)^2 + pe_i.$$

Because $G_i \equiv \Pi_i - \Pi_i^{nt} = \frac{\tau}{2\rho}\left(E_i[v] - p - \frac{1}{\tau}e_i\right)^2 = \frac{\tau}{2}a_i^2$,

$$\Pi_i = G_i + \Pi_i^{nt} = \frac{\tau}{2\rho}\left(a_i^2 + b_i^2 - c_i^2\right).$$

(b) From (30),

$$\Pi_i^{st} = \left(1 - \left(\frac{\lambda\tau}{\rho + \lambda\tau}\right)^2\right)\Pi_i + \left(\frac{\lambda\tau}{\rho + \lambda\tau}\right)^2\Pi_i^{nt} = \left(1 - \widetilde{\lambda}\right)\Pi_i + \widetilde{\lambda}\Pi_i^{nt}. \quad (31)$$

Using the result above,

$$\begin{aligned}\Pi_i^{st} &= \left(1 - \widetilde{\lambda}\right)\left(G_i + \Pi_i^{nt}\right) + \widetilde{\lambda}\Pi_i^{nt} = \left(1 - \widetilde{\lambda}\right)G_i + \Pi_i^{nt} \\ &= \frac{\tau}{2\rho}\left(1 - \widetilde{\lambda}\right)a_i^2 + \Pi_i^{nt} = \frac{\tau}{2\rho}\left(\left(1 - \widetilde{\lambda}\right)a_i^2 + b_i^2 - c_i^2\right).\end{aligned}$$

This implies $G_i^{st} \equiv \Pi_i^{st} - \Pi_i^{nt} = \left(1 - \widetilde{\lambda}\right)G_i$. Recall that $\frac{\lambda\tau}{\rho+\lambda\tau} = \frac{\left(\omega - \frac{1-\omega}{n}\right)\varphi + \frac{1}{n}}{1-\varphi}$ decreases in n (see (27) in the proof of **Lemma A3(c)**). Accordingly, $\widetilde{\lambda} = \left(\frac{\lambda\tau}{\rho+\lambda\tau}\right)^2$ decreases in n.

If $\mu = \omega = 1$, then $\lim_{n\to\infty}\frac{\lambda\tau}{\rho+\lambda\tau} = \lim_{n\to\infty}\frac{\left(\omega - \frac{1-\omega}{n}\right)\varphi + \frac{1}{n}}{1-\varphi} = \frac{\varphi}{1-\varphi}$ with $\varphi = (1 + \alpha_\varepsilon)^{-1}$. Therefore, $\lim_{n\to\infty}\widetilde{\lambda} = \left(\frac{\varphi}{1-\varphi}\right)^2$.

If $\mu < 1$ or $\omega < 1$, then from **Lemma A2** φ decreases in n at the rate $n^{-\frac{2}{3}}$. Therefore,

$\lim_{n\to\infty} \frac{\left(\omega - \frac{1-\omega}{n}\right)\varphi + \frac{1}{n}}{1-\varphi} = 0$ and hence $\lim_{n\to\infty} \widetilde{\lambda} = 0$ at the rate $n^{-\frac{4}{3}}$. ■ **(A6)**

2.3.2 Ex ante characterization

Denote the covariance matrix of (a_i, b_i, c_i) by

$$\Sigma_{abc} \equiv Var\left[[a_i, b_i, c_i]\right] = \begin{bmatrix} V_a & V_{ab} & V_{ac} \\ & V_b & V_{bc} \\ & & V_c \end{bmatrix}.$$

Lemma A7 (ex ante #1)

$$\begin{aligned} \exp(2\rho\Pi) &= (1 + \tau V_a)\exp\left(2\rho\Pi^{nt}\right) + \Delta, \\ \exp\left(2\rho\Pi^{st}\right) &= \left(1 + \left(1 - \widetilde{\lambda}\right)\tau V_a\right)\exp\left(2\rho\Pi^{nt}\right) + \left(1 - \widetilde{\lambda}\right)\Delta, \end{aligned} \tag{32}$$

$$\textit{where } \exp\left(2\rho\Pi^{nt}\right) = (1 + \tau V_b)(1 - \tau V_c) + (\tau V_{bc})^2$$
$$\textit{and } \Delta \equiv \tau^2\left(V_{ac}^2 - V_{ab}^2\right) + \tau^3\left(V_{ac}^2 V_b + V_{ab}^2 V_c - 2V_{ab}V_{bc}V_{ac}\right).$$

Remark. **Lemma A7** immediately implies:

$$\begin{aligned} \exp(2\rho G) &= 1 + \tau V_a + \Delta\exp\left(-2\rho\Pi^{nt}\right), \\ \exp\left(2\rho G^{st}\right) &= 1 + \left(1 - \widetilde{\lambda}\right)\left\{\tau V_a + \Delta\exp\left(-2\rho\Pi^{nt}\right)\right\}. \end{aligned} \tag{33}$$

Proof. We apply the following fact to $(G_i, \Pi_i, \Pi_i^{nt}, G_i^{st}, \Pi_i^{st})$.

__**Fact 1.**__ *Given the n-dimensional random vector z that is normally distributed with mean zero and variance-covariance matrix Σ,*

$$E[-\exp(-\rho(zCz^\top))] = -\left\{\det\left(I_n + 2\rho\Sigma C\right)\right\}^{-\frac{1}{2}},$$

where I_n is the n-dimensional identity matrix and C is an n-by-n matrix.

Since (a_i, b_i, c_i) have zero means, we can apply **Fact 1** to $\Pi_i = \frac{\tau}{2\rho}\left(a_i^2 + b_i^2 - c_i^2\right)$:

$$\begin{aligned} E\left[-\exp\left(-\rho\Pi_i\right)\right] &= E\left[-\exp\left(-\rho\left([a_i, b_i, c_i]\, C\, [a_i, b_i, c_i]^\intercal\right)\right)\right] = -\left\{\det\left(I_3 + 2\rho\Sigma_{abc}C\right)\right\}^{-\frac{1}{2}}, \\ \text{where } C &\equiv \frac{\tau}{2\rho}\begin{bmatrix} 1 & & \\ & 1 & \\ & & -1 \end{bmatrix}. \end{aligned}$$

Similarly,

$$
\begin{aligned}
E\left[-\exp\left(-\rho\Pi_i^{st}\right)\right] &= -\left\{\det\left(I_n+2\rho\Sigma_{abc}C^{st}\right)\right\}^{-\frac{1}{2}}, \\
\text{where } C^{st} &\equiv \frac{\tau}{2\rho}\begin{bmatrix} 1-\widetilde{\lambda} & & \\ & 1 & \\ & & -1 \end{bmatrix}
\end{aligned}
$$

Because off-diagonal elements of C and C^{st} are zeros, we have

$$
\begin{aligned}
I_3+2\rho\Sigma_{abc}C &= \begin{bmatrix} 1+\tau V_a & \tau V_{ab} & -\tau V_{ac} \\ \tau V_{ab} & 1+\tau V_b & -\tau V_{bc} \\ \tau V_{ac} & \tau V_{bc} & 1-\tau V_c \end{bmatrix}, \\
I_3+2\rho\Sigma_{abc}C^{st} &= \begin{bmatrix} 1+\left(1-\widetilde{\lambda}\right)\tau V_a & \tau V_{ab} & -\tau V_{ac} \\ \left(1-\widetilde{\lambda}\right)\tau V_{ab} & 1+\tau V_b & -\tau V_{bc} \\ \left(1-\widetilde{\lambda}\right)\tau V_{ac} & \tau V_{bc} & 1-\tau V_c \end{bmatrix}.
\end{aligned}
$$

Because $\Pi_i^{nt}=\frac{\tau}{2\rho}\left(b_i^2-c_i^2\right)$, the 2-by-2 matrix on the bottom-right of the above two matrices corresponds to the ex ante no-trade profit. Using $|\cdot|$ as determinant operator,

$$
\exp\left(2\rho\Pi^{nt}\right)=\begin{vmatrix} 1+\tau V_b & -\tau V_{bc} \\ \tau V_{bc} & 1-\tau V_c \end{vmatrix}=\left(1+\tau V_b\right)\left(1-\tau V_c\right)+\left(\tau V_{bc}\right)^2. \tag{34}
$$

Also, from $\widetilde{G}\equiv-\frac{1}{\rho}\log\left(E[\exp\left(-\rho G_i\right)]\right)$ and $\widetilde{G}^{st}\equiv-\frac{1}{\rho}\log\left(E[\exp\left(-\rho G_i^{st}\right)]\right)$,

$$
\exp\left(2\rho\widetilde{G}\right)=1+\tau V_a \text{ and } \exp\left(2\rho\widetilde{G}^{st}\right)=1+\left(1-\widetilde{\lambda}\right)\tau V_a.
$$

Therefore,

$$
\begin{aligned}
\exp\left(2\rho\Pi\right) &= \left(1+\tau V_a\right)\begin{vmatrix} 1+\tau V_b & -\tau V_{bc} \\ \tau V_{bc} & 1-\tau V_c \end{vmatrix}-\tau V_{ab}\begin{vmatrix} \tau V_{ab} & -\tau V_{ac} \\ \tau V_{bc} & 1-\tau V_c \end{vmatrix}+\tau V_{ac}\begin{vmatrix} \tau V_{ab} & -\tau V_{ac} \\ 1+\tau V_b & -\tau V_{bc} \end{vmatrix} \\
&= \exp\left(2\rho\widetilde{G}\right)\exp\left(2\rho\Pi^{nt}\right) \\
&\quad -\tau V_{ab}\left\{\tau V_{ab}\left(1-\tau V_c\right)+\tau^2V_{ac}V_{bc}\right\}+\tau V_{ac}\left\{\tau V_{ac}\left(1+\tau V_b\right)-\tau^2V_{ab}V_{bc}\right\} \\
&= \exp\left(2\rho\widetilde{G}\right)\exp\left(2\rho\Pi^{nt}\right) \\
&\quad +\tau^2\left(V_{ac}^2-V_{ab}^2\right)+\tau^3\left(V_{ac}^2V_b+V_{ab}^2V_c-2V_{ab}V_{bc}V_{ac}\right) \\
&= \exp\left(2\rho\widetilde{G}\right)\exp\left(2\rho\Pi^{nt}\right)+\Delta.
\end{aligned}
$$

Computing $\exp\left(2\rho\Pi^{st}\right)$ is similar and omitted. ■ **(A7)**

We need to characterize Σ_{abc}. This is done in two lemmas below. Recall $E_i[v]=\gamma_s s_i+$

$\gamma_e e_i + \gamma_p p$, where $\gamma_s, \gamma_e, \gamma_p$ are given in (7). With these coefficients,

$$\begin{aligned} a_i &= \gamma_s s_i - \left(\frac{\rho}{\tau} - \gamma_e\right) e_i - \left(1 - \gamma_p\right) p, \\ b_i &= \gamma_s s_i + \gamma_e e_i + \gamma_p p, \\ c_i &= \gamma_s s_i - \left(\frac{\rho}{\tau} - \gamma_e\right) e_i + \gamma_p p. \end{aligned}$$

Lemma A8 ($\gamma_s, \gamma_e, \gamma_p$)
(a) $\gamma_e = \frac{\rho}{\tau}\frac{\omega\varphi}{1-(1-\omega)\varphi}$, $\gamma_p = \frac{\omega\varphi(n+1)}{1+\{n\omega-(1-\omega)\}\varphi}$, *and* $\gamma_s = \frac{\tau_\varepsilon}{\rho}\frac{1-\varphi}{\omega\varphi}\frac{1}{B}\gamma_e$.
(b) $\frac{\rho}{\tau} - \gamma_e = \frac{1-\varphi}{\omega\varphi}\gamma_e$ *and* $\frac{\rho}{\tau} - 2\gamma_e = \frac{1-(1+\omega)\varphi}{\omega\varphi}\gamma_e$.
$1 - \gamma_p = \frac{1-\varphi}{1+\{n\omega-(1-\omega)\}\varphi}$, $1 - 2\gamma_p = \frac{1-\{1+\omega(n+1)\}\varphi}{1+\{n\omega-(1-\omega)\}\varphi}$, *and* $\frac{1-\gamma_p}{\gamma_p} = \frac{1}{n+1}\frac{1-\varphi}{\omega\varphi}$.

Proof.
(a) First, from (7) and (21),

$$\begin{aligned} \gamma_s &= \sqrt{\mu}\frac{\tau_\varepsilon}{\widetilde{\tau}}\frac{1-\varphi}{1+(1-\omega)(\omega n-1)\varphi}, \\ \gamma_e &= \sqrt{\mu}\frac{\tau_\varepsilon}{\widetilde{\tau}}\frac{\omega\varphi}{1+(1-\omega)(\omega n-1)\varphi}\frac{\beta_e}{\beta_s}, \\ \gamma_p &= \sqrt{\mu}\frac{\tau_\varepsilon}{\widetilde{\tau}}\frac{\omega\varphi(n+1)}{1+(1-\omega)(\omega n-1)\varphi}\frac{\beta_p}{\beta_s}. \end{aligned}$$

Use (23) for γ_e to obtain

$$\gamma_e = \left(\sqrt{\mu}\frac{\tau_\varepsilon}{\rho}\frac{\beta_e}{\beta_s}\right)\frac{\tau_\varepsilon}{\widetilde{\tau}}\frac{\rho}{\tau_\varepsilon}\frac{\omega\varphi}{1+(1-\omega)(\omega n-1)\varphi} = \frac{\rho}{\tau}\frac{\omega\varphi}{1-(1-\omega)\varphi}.$$

Similarly, use (19) for γ_p to obtain

$$\gamma_p = \sqrt{\mu}\frac{\tau_\varepsilon}{\widetilde{\tau}}\frac{\omega\varphi(n+1)}{1+(1-\omega)(\omega n-1)\varphi}\frac{\widetilde{\tau}}{\sqrt{\mu}\tau_\varepsilon}\frac{1+(1-\omega)(\omega n\omega-1)\varphi}{1+\{\omega n-(1-\omega)\}\varphi} = \frac{\omega\varphi(n+1)}{1+\{\omega n-(1-\omega)\}\varphi}.$$

Finally, using $\frac{\beta_e}{\beta_s} = \frac{\rho}{\tau_\varepsilon}B$, $\frac{\gamma_s}{\gamma_e} = \frac{\tau_\varepsilon}{\rho}\frac{1-\varphi}{\omega\varphi}\frac{1}{B}$.

(b) Using the results from **(a)**,

$$\frac{\rho}{\tau} - \gamma_e = \frac{\rho}{\tau}\left(1 - \frac{\omega\varphi}{1-(1-\omega)\varphi}\right) = \frac{1-\varphi}{\omega\varphi}\gamma_e,$$

$$\frac{\rho}{\tau} - 2\gamma_e = \left(\frac{1-\varphi}{\omega\varphi} - 1\right)\gamma_e = \frac{1-(1+\omega)\varphi}{\omega\varphi}\gamma_e,$$

$$1-\gamma_p = 1-\frac{\omega\varphi(n+1)}{1+\{\omega n-(1-\omega)\}\varphi} = \frac{1+\{\omega n-(1-\omega)\}\varphi - \omega\varphi(n+1)}{1+\{\omega n-(1-\omega)\}\varphi} = \frac{1-\varphi}{1+\{\omega n-(1-\omega)\}\varphi},$$

$$1-2\gamma_p = \frac{1-\varphi}{1+\{\omega n-(1-\omega)\}\varphi} - \frac{\omega\varphi(n+1)}{1+\{\omega n-(1-\omega)\}\varphi} = \frac{1-\{1+\omega(n+1)\}\varphi}{1+\{\omega n-(1-\omega)\}\varphi},$$

$$\frac{1-\gamma_p}{\gamma_p} = \frac{1}{n+1}\frac{1-\varphi}{\omega\varphi}. \qquad \blacksquare \text{ (A8)}$$

Lemma A9 (Σ_{abc})
(a) $V_b = V_{bc} = \frac{1}{\tau_v} - \frac{1}{\tau}$ *and* $V_c = V_b + \frac{\alpha}{\tau}\frac{\tau_v}{\tau}$.
(b) $V_a = V_{ac} = \frac{\alpha}{\tau}\frac{\tau_v}{\tau}\frac{n}{1+n}\frac{1-\varphi}{1-(1-\omega)\varphi}$.
(c) $V_{ab} = 0$.

Proof.
(a) First,

$$\begin{aligned} V_b &= Var\left[E[v|s_i,e_i,p]\right] \\ &= Var\left[v\right] - Var\left[v|s_i,e_i,p\right] = \frac{1}{\tau_v} - \frac{1}{\tau}. \end{aligned}$$

Because $c_i = b_i - \frac{\rho}{\tau}e_i$,

$$\begin{aligned} V_{bc} &= V_b - \frac{\rho}{\tau}Cov\left[b_i,e_i\right], \\ V_c &= V_b + \left(\frac{\rho}{\tau}\right)^2\frac{1}{\tau_x} - 2\frac{\rho}{\tau}Cov\left[b_i,e_i\right] \\ &= V_b + \frac{\alpha}{\tau}\frac{\tau_v}{\tau} - 2\frac{\rho}{\tau}Cov\left[b_i,e_i\right]. \end{aligned}$$

Thus, showing $Cov\left[b_i,e_i\right]=0$ proves the results. We first characterize $\Sigma_{sep} \equiv Var[[s_i,e_i,p]] = \begin{bmatrix} V_s & 0 & V_{sp} \\ & V_e & V_{ep} \\ & & V_p \end{bmatrix}$. First, $V_e = Var\left[e_i\right] = \frac{1}{\tau_e}$ and

$$V_s = Var\left[s_i\right] = \frac{1}{\tau_v} + \frac{1}{\tau_\varepsilon} = \frac{\tau_v+\tau_\varepsilon}{\tau_v\tau_\varepsilon} = \frac{1}{d_\varepsilon\tau_v}.$$

Using $p^* = \frac{\beta_s}{\beta_p}\overline{s} - \frac{\beta_e}{\beta_p}\overline{e}$, (7) and (21), we have

$$\begin{aligned} V_{sp} &= \frac{\gamma_s}{\gamma_p}\frac{\omega\varphi}{1-\varphi}\left\{(1+d_\varepsilon n)V_s + (1-\omega)\frac{n}{\tau_\varepsilon}\right\}, \\ V_{ep} &= -\frac{\gamma_e}{\gamma_p}V_e. \\ V_p &= (1+n)\left(\frac{\gamma_s}{\gamma_p}\frac{\omega\varphi}{1-\varphi}V_{sp} - \frac{\gamma_e}{\gamma_p}V_{ep}\right). \end{aligned}$$

Then,

$$\begin{aligned} Cov\left[b_i, e_i\right] &= Cov\left[\gamma_e e_i + \gamma_p p, e_i\right] \\ &= \gamma_e V_e + \gamma_p\left(-\frac{\gamma_e}{\gamma_p}V_e\right) = 0. \end{aligned}$$

(b) Using **Lemma A8** and the expression of V_p obtained in the proof of part **(a)**,

$$\begin{aligned} V_a &= Var\left[\gamma_s s_i - \left(\frac{\rho}{\tau} - \gamma_e\right)e_i - \left(1-\gamma_p\right)p\right] \\ &= \gamma_s^2 V_s + \left(\frac{\rho}{\tau} - \gamma_e\right)^2 V_e + \left(1-\gamma_p\right)^2(1+n)\left(\frac{\gamma_s}{\gamma_p}\frac{\omega\varphi}{1-\varphi}V_{sp} - \frac{\gamma_e}{\gamma_p}V_{ep}\right) \\ &\quad -2\left(1-\gamma_p\right)\left\{\gamma_s V_{sp} - \left(\frac{\rho}{\tau} - \gamma_e\right)V_{ep}\right\} \\ &= \gamma_s^2 V_s + \left(\frac{1-\varphi}{\omega\varphi}\gamma_e\right)^2 V_e + \left(1-\gamma_p\right)\gamma_s\left\{\frac{1-\gamma_p}{\gamma_p}(1+n)\frac{\omega\varphi}{1-\varphi} - 2\right\}V_{sp} \\ &\quad -\left(1-\gamma_p\right)\gamma_e\left\{\frac{1-\gamma_p}{\gamma_p}(1+n) - 2\frac{1-\varphi}{\omega\varphi}\right\}V_{ep}. \end{aligned}$$

Using $\frac{1-\gamma_p}{\gamma_p} = \frac{1}{n+1}\frac{1-\varphi}{\omega\varphi}$,

$$\begin{aligned} V_a &= \gamma_s^2 V_s + \left(\frac{1-\varphi}{\omega\varphi}\gamma_e\right)^2 V_e - \left(1-\gamma_p\right)\left(\gamma_s V_{sp} - \frac{1-\varphi}{\omega\varphi}\gamma_e V_{ep}\right) \\ &= \gamma_s^2\left\{1 - \frac{1-\gamma_p}{\gamma_p}\frac{\omega\varphi}{1-\varphi}(1+d_\varepsilon n)\right\}V_s - \gamma_s^2\frac{1-\gamma_p}{\gamma_p}\frac{\omega\varphi}{1-\varphi}\left(1-\omega\right)\frac{n}{\tau_\varepsilon} \\ &\quad +\gamma_e^2\left\{\left(\frac{1-\varphi}{\omega\varphi}\right)^2 - \frac{1-\varphi}{\omega\varphi}\frac{1-\gamma_p}{\gamma_p}\right\}V_e \\ &= \gamma_s^2\left(1 - \frac{1+d_\varepsilon n}{n+1}\right)\frac{1}{d_\varepsilon\tau_v} - \gamma_s^2\frac{n}{n+1}\frac{1-\omega}{\tau_\varepsilon} + \left(\frac{1-\varphi}{\omega\varphi}\right)^2\gamma_e^2\frac{n}{n+1}\frac{1}{\tau_e} \\ &= \left(\frac{\tau_\varepsilon}{\rho}\frac{1-\varphi}{\omega\varphi}\frac{1}{B}\gamma_e\right)^2\frac{n}{n+1}\frac{\omega}{\tau_\varepsilon} + \left(\frac{1-\varphi}{\omega\varphi}\right)^2\gamma_e^2\frac{n}{n+1}\frac{1}{\tau_e} \\ &= \left(\frac{1-\varphi}{\omega\varphi}\right)^2\gamma_e^2\frac{n}{n+1}\frac{1}{\tau_e}\left\{\frac{\tau_\varepsilon\tau_x}{\rho^2}\frac{1}{B^2}\omega + 1\right\} \\ &= \left(\frac{1-\varphi}{\omega\varphi}\right)^2\gamma_e^2\frac{n}{n+1}\frac{1}{\tau_e}\left\{\frac{\omega\varphi}{1-\varphi} + 1\right\} \\ &= \left(\frac{\gamma_e}{\omega\varphi}\right)^2\left(1-\varphi\right)\frac{n}{n+1}\frac{1}{\tau_e}\left(1-\left(1-\omega\right)\varphi\right). \end{aligned}$$

Use $\gamma_e = \frac{\rho}{\tau}\frac{\omega\varphi}{1-(1-\omega)\varphi}$ to obtain

$$V_a = \left(\frac{\rho}{\tau}\right)^2 \frac{1}{\tau_e}\frac{n}{n+1}\frac{1-\varphi}{1-(1-\omega)\varphi} = \frac{\alpha}{\tau}\frac{\tau_v}{\tau}\frac{n}{n+1}\frac{1-\varphi}{1-(1-\omega)\varphi}.$$

Next, we show $V_{ac} = V_a$. Because $c_i = b_i - \frac{\rho}{\tau}e_i$,

$$V_{ac} = Cov\left[a_i, b_i - \frac{\rho}{\tau}e_i\right] = V_{ab} - \frac{\rho}{\tau}Cov\left[a_i, e_i\right].$$

Because $V_{ab} = 0$ is proved in part **(c)** below, it suffices to show $-\frac{\rho}{\tau}Cov\left[a_i, e_i\right] = V_a$.

$$\begin{aligned}
-\frac{\rho}{\tau}Cov\left[a_i, e_i\right] &= -\frac{\rho}{\tau}Cov\left[\gamma_s s_i - \left(\frac{\rho}{\tau} - \gamma_e\right)e_i - \left(1-\gamma_p\right)p, e_i\right] \\
&= -\frac{\rho}{\tau}\left\{-\left(\frac{\rho}{\tau} - \gamma_e\right)V_e - \left(1-\gamma_p\right)V_{ep}\right\} \\
&= \frac{\rho}{\tau}\gamma_e\left\{\frac{1-\varphi}{\omega\varphi} - \frac{1-\gamma_p}{\gamma_p}\right\}V_e \\
&= \frac{\rho}{\tau}\gamma_e\frac{1-\varphi}{\omega\varphi}\left(1 - \frac{1}{n+1}\right)V_e \\
&= \left(\frac{\rho}{\tau}\right)^2\frac{1-\varphi}{1-(1-w)\varphi}\frac{n}{n+1}\frac{1}{\tau_e} = V_a.
\end{aligned}$$

(c)

$$\begin{aligned}
V_{ab} &= Cov\left[\gamma_s s_i - \left(\frac{\rho}{\tau} - \gamma_e\right)e_i - \left(1-\gamma_p\right)p, \gamma_s s_i + \gamma_e e_i + \gamma_p p\right] \\
&= \gamma_s^2 V_s - \left(\frac{\rho}{\tau} - \gamma_e\right)\gamma_e V_e - \left(1-\gamma_p\right)\gamma_p V_p \\
&\quad -\gamma_s\left(1-2\gamma_p\right)V_{sp} - \left\{\left(\frac{\rho}{\tau} - \gamma_e\right)\gamma_p + \gamma_e\left(1-\gamma_p\right)\right\}V_{ep} \\
&= \gamma_s^2 V_s - \left(\frac{\rho}{\tau} - \gamma_e\right)\gamma_e V_e - \left(1-\gamma_p\right)\gamma_p(1+n)\left(\frac{\gamma_s}{\gamma_p}\frac{w\varphi}{1-\varphi}V_{sp} - \frac{\gamma_e}{\gamma_p}V_{ep}\right) \\
&\quad -\gamma_s\left(1-2\gamma_p\right)V_{sp} - \left\{\left(\frac{\rho}{\tau} - 2\gamma_e\right)\gamma_p + \gamma_e\right\}V_{ep}.
\end{aligned}$$

Using $\frac{\rho}{\tau} - \gamma_e = \frac{1-\varphi}{\omega\varphi}\gamma_e$ and $\frac{\rho}{\tau} - 2\gamma_e = \frac{1-(1+\omega)\varphi}{\omega\varphi}\gamma_e$ (by **Lemma A8**),

$$\begin{aligned}
V_{ab} &= \gamma_s^2 V_s - \frac{1-\varphi}{\omega\varphi}\gamma_e^2 V_e - \left(1-\gamma_p\right)(1+n)\left(\gamma_s\frac{\omega\varphi}{1-\varphi}V_{sp} - \gamma_e V_{ep}\right) \\
&\quad -\gamma_s\left(1-2\gamma_p\right)V_{sp} - \gamma_e\left\{\frac{1-(1+\omega)\varphi}{\omega\varphi}\gamma_p + 1\right\}V_{ep} \\
&= \gamma_s^2 V_s - \frac{1-\varphi}{\omega\varphi}\gamma_e^2 V_e - \gamma_s V_{sp}\left\{\left(1-\gamma_p\right)(1+n)\frac{\omega\varphi}{1-\varphi} + \left(1-2\gamma_p\right)\right\} \\
&\quad +\gamma_e V_{ep}\left\{\left(1-\gamma_p\right)(1+n) - \left\{\frac{1-(1+\omega)\varphi}{\omega\varphi}\gamma_p + 1\right\}\right\}.
\end{aligned}$$

Using $1-\gamma_p = \frac{1-\varphi}{1+\{n\omega-(1-\omega)\}\varphi}$ and $1-2\gamma_p = \frac{1-\varphi(1+\omega)-\omega\varphi n}{1+\{n\omega-(1-\omega)\}\varphi}$,

$$
\begin{aligned}
V_{ab} &= \gamma_s^2 V_s - \frac{1-\varphi}{\omega\varphi}\gamma_e^2 V_e - \frac{\gamma_s V_{sp}}{1+\{n\omega-(1-\omega)\}\varphi}\{(1+n)\omega\varphi + 1 - \varphi(1+\omega) - n\omega\varphi\} \\
&\quad - \frac{\gamma_e V_{ep}}{1+\{n\omega-(1-\omega)\}\varphi}\left\{\frac{1-(1+\omega)\varphi}{\omega\varphi}\omega\varphi(n+1) + 1 + \{n\omega-(1-\omega)\}\varphi - (1+n)(1-\varphi)\right\} \\
&= \gamma_s^2 V_s - \frac{1-\varphi}{w\varphi}\gamma_e^2 V_e - \frac{1-\varphi}{1+\{n\omega-(1-\omega)\}\varphi}(\gamma_s V_{sp} + \gamma_e V_{ep}).
\end{aligned}
$$

Substituting V_{sp} and V_{ep},

$$
\begin{aligned}
V_{ab} &= \gamma_s^2 V_s \left\{1 - \frac{1-\varphi}{1+\{n\omega-(1-\omega)\}\varphi}\frac{1}{\gamma_p}\frac{\omega\varphi}{1-\varphi}(1+d_\varepsilon n)\right\} \\
&\quad -\gamma_s^2 \frac{1-\varphi}{1+\{n\omega-(1-\omega)\}\varphi}\frac{1}{\gamma_p}\frac{\omega\varphi}{1-\varphi}(1-\omega)\frac{n}{\tau_\varepsilon} \\
&\quad -\gamma_e^2 V_e \left\{\frac{1-\varphi}{\omega\varphi} - \frac{1-\varphi}{1+\{n\omega-(1-\omega)\}\varphi}\frac{1}{\gamma_p}\right\} \\
&= \gamma_s^2 V_s \left\{1 - \frac{1+d_\varepsilon n}{n+1}\right\} - \gamma_s^2 \frac{n}{n+1}\frac{1-\omega}{\tau_\varepsilon} - \gamma_e^2 V_e \frac{1-\varphi}{\omega\varphi}\left\{1 - \frac{1}{n+1}\right\}.
\end{aligned}
$$

Using $\gamma_s = \frac{\tau_\varepsilon}{\rho}\frac{1-\varphi}{\omega\varphi}\frac{1}{B}\gamma_e$, $V_s = \frac{1}{d_\varepsilon \tau_v}$, and $V_e = \frac{1}{\tau_e}$,

$$
\begin{aligned}
V_{ab} &= \frac{n}{n+1}\gamma_e^2 \left\{\left(\frac{\tau_\varepsilon}{\rho}\frac{1-\varphi}{\omega\varphi}\frac{1}{B}\right)^2 \left\{\frac{1}{\tau_\varepsilon} - \frac{1-\omega}{\tau_\varepsilon}\right\} - \frac{1-\varphi}{\omega\varphi}\frac{1}{\tau_e}\right\} \\
&= \frac{n}{n+1}\gamma_e^2 \frac{1-\varphi}{\omega\varphi}\left\{\frac{\tau_\varepsilon}{\rho^2}\frac{1}{B^2}\frac{1-\varphi}{\varphi} - \frac{1}{\tau_e}\right\} = 0.
\end{aligned}
$$

The last equality follows from $\frac{1-\varphi}{\varphi} = \frac{\rho^2}{\tau_\varepsilon \tau_e}B^2$ by (9). ■ **(A9)**

Lemma A10 (ex ante #2)
Given $\alpha < 1$, $\exp(2\rho\Pi^{nt}) = 1-\alpha$ *and*

$$
\exp(2\rho\Pi) = 1-\alpha+\alpha\frac{\tau_v}{\tau}X(1-\alpha+\alpha X) > \exp\left(2\rho\Pi^{st}\right) = 1-\alpha+\alpha\frac{\tau_v}{\tau}X^{st}(1-\alpha+\alpha X),
$$

$$
\textit{where } \frac{\tau_v}{\tau} = \frac{1-\mu d_\varepsilon + \frac{\omega n\varphi}{1-(1-\omega)\varphi}(1-\mu d_\varepsilon - \omega(1-d_\varepsilon))}{1+\frac{\omega n\varphi}{1-(1-\omega)\varphi}(1-\omega(1-d_\varepsilon))} < 1-\mu d_\varepsilon, \tag{35}
$$

$$
X \equiv \frac{n}{1+n}\frac{1-\varphi}{1-(1-\omega)\varphi} < 1, \tag{36}
$$

$$
X^{st} \equiv \frac{n-1}{n} - \frac{n+1}{n}\frac{\omega\varphi}{1-\varphi} < 1. \tag{37}
$$

Also, $\frac{X^{st}}{X} = 1-\widetilde{\lambda}$ *increases in* n.

Remark. $\alpha < 1$ is necessary for Π^{nt} to be well-defined. Given this condition, **Lemma A10** immediately implies

$$\exp\left(2G\right) = 1+\alpha\frac{\tau_v}{\tau}X\left(1+\frac{\alpha}{1-\alpha}X\right) \qquad \text{and} \qquad \exp\left(2G^{st}\right) = 1+\alpha\frac{\tau_v}{\tau}X^{st}\left(1+\frac{\alpha}{1-\alpha}X\right).$$

Proof.
By **Lemma A9(a)**, $1+\tau V_b = \frac{\tau}{\tau_v}$. Applying **Lemma A9** to Δ and (34),

$$\begin{aligned}
\Delta &\equiv \tau^2\left(V_{ac}^2 - V_{ab}^2\right) + \tau^3\left(V_{ac}^2 V_b + V_{ab}^2 V_c - 2V_{ab}V_{bc}V_{ac}\right) \\
&= \left(\tau V_a\right)^2\left(1+\tau V_b\right) \\
&= \left(\tau V_a\right)^2\frac{\tau}{\tau_v},
\end{aligned}$$

$$\begin{aligned}
\exp\left(2\rho\Pi^{nt}\right) &= \left(1+\tau V_b\right)\left(1-\tau V_c\right) + \left(\tau V_{bc}\right)^2 \\
&= \left(1+\tau V_b\right)\left(1-\tau\left(V_b + \left(\frac{\rho}{\tau}\right)^2\frac{1}{\tau_x}\right)\right) + \left(\tau V_b\right)^2 \\
&= 1-\left(1+\tau V_b\right)\frac{\rho^2}{\tau\tau_e} \\
&= 1-\frac{\rho^2}{\tau_v\tau_e} = 1-\alpha.
\end{aligned}$$

From (32) in **Lemma A7**,

$$\begin{aligned}
\exp\left(2\rho\Pi\right) &= \left(1-\alpha\right)\left(1+\tau V_a\right) + \left(\tau V_a\right)^2\frac{\tau}{\tau_v} = 1-\alpha+\tau V_a\left(1-\alpha+\tau V_a\frac{\tau}{\tau_v}\right), \\
\exp\left(2\rho\Pi^{st}\right) &= 1-\alpha+\left(1-\widetilde{\lambda}\right)\tau V_a\left(1-\alpha+\tau V_a\frac{\tau}{\tau_v}\right).
\end{aligned}$$

From **Lemma A9(b)**,

$$\tau V_a = \alpha\frac{n}{n+1}\frac{1-\varphi}{1-\left(1-\omega\right)\varphi}\frac{\tau_v}{\tau} = \alpha\frac{\tau_v}{\tau}X.$$

Therefore,

$$\exp\left(2\rho\Pi\right) = 1-\alpha+\alpha\frac{\tau_v}{\tau}X\left(1-\alpha+\alpha X\right).$$

Using $1+(1-\mu)\frac{\tau_\varepsilon}{\tau_v}=\frac{1-\mu d_\varepsilon}{1-d_\varepsilon}$ and $1+\frac{\tau_\varepsilon}{\tau_v}=\frac{1}{1-d_\varepsilon}$ in (8),

$$\begin{aligned}\frac{\tau_v}{\tau} &= \frac{\frac{1-\mu d_\varepsilon}{1-d_\varepsilon}\left(1-(1-\omega)\varphi\right)+\omega n\varphi\left(\frac{1-\mu d_\varepsilon}{1-d_\varepsilon}-\omega\right)}{\frac{1}{1-d_\varepsilon}\left(1-(1-\omega)\varphi\right)+\omega n\varphi\left(\frac{1}{1-d_\varepsilon}-\omega\right)} \\ &= \frac{1-\mu d_\varepsilon+\frac{\omega n\varphi}{1-(1-\omega)\varphi}\left(1-\mu d_\varepsilon-\omega\left(1-d_\varepsilon\right)\right)}{1+\frac{\omega n\varphi}{1-(1-\omega)\varphi}\left(1-\omega\left(1-d_\varepsilon\right)\right)}.\end{aligned}$$

To derive Π^{st}, recall from **Lemma A6(b)** that $\widetilde{\lambda}=\left(\frac{\left(\omega-\frac{1-\omega}{n}\right)\varphi+\frac{1}{n}}{1-\varphi}\right)^2$ decreases in n. Hence, $1-\widetilde{\lambda}$ decreases in n. Computing $1-\widetilde{\lambda}$,

$$\begin{aligned}1-\widetilde{\lambda} &= \left(1-\left(\frac{\left(\omega-\frac{1-\omega}{n}\right)\varphi+\frac{1}{n}}{1-\varphi}\right)^2\right) \\ &= \frac{1}{(1-\varphi)^2}\left(1-\varphi-\left(\omega-\frac{1-\omega}{n}\right)\varphi-\frac{1}{n}\right)\left(1-\varphi+\left(\omega-\frac{1-\omega}{n}\right)\varphi+\frac{1}{n}\right) \\ &= \frac{1}{(1-\varphi)^2}\left(\frac{n-1}{n}-\left(\frac{n-1}{n}+\frac{n+1}{n}\omega\right)\varphi\right)\left(\frac{n+1}{n}-(1-\omega)\frac{n+1}{n}\varphi\right) \\ &= \frac{1}{(1-\varphi)^2}\left\{\frac{n-1}{n}(1-\varphi)-\frac{n+1}{n}\omega\varphi\right\}\left(1-(1-\omega)\varphi\right)\frac{n+1}{n} \\ &= \frac{1-(1-\omega)\varphi}{1-\varphi}\frac{n+1}{n}\left(\frac{n-1}{n}-\frac{n+1}{n}\frac{\omega\varphi}{1-\varphi}\right)=\frac{X^{st}}{X}<1.\end{aligned}$$

Therefore,

$$\begin{aligned}\exp\left(2\rho\Pi^{st}\right) &= 1-\alpha+\left(1-\widetilde{\lambda}\right)\alpha\frac{\tau_v}{\tau}X\left(1-\alpha+\alpha X\right) \\ &= 1-\alpha+\alpha\frac{\tau_v}{\tau}X^{st}\left(1-\alpha+\alpha X\right) \\ &< \exp\left(2\rho\Pi\right). \qquad \blacksquare \textbf{ (A10)}\end{aligned}$$

2.4 Optimal market size

Recall from **Lemma A10** that

$$\exp\left(2\rho\Pi\right)=1-\alpha+\alpha\frac{\tau_v}{\tau}X\left(1-\alpha+\alpha X\right) \qquad \text{and} \qquad \exp\left(2\rho\Pi^{st}\right)=1-\alpha+\alpha\frac{\tau_v}{\tau}X^{st}\left(1-\alpha+\alpha X\right).$$

The optimal market size maximizes $\frac{X(1-\alpha+\alpha X)}{\tau}$ in a price-taking equilibrium, and $\frac{X^{st}(1-\alpha+\alpha X)}{\tau}$ in a strategic equilibrium.

2.4.1 The case with $\mu = \omega = 1$

Lemma A11 (optimal market size with $\mu\omega = 1$)

(a) $\lim_{n\to\infty} \Pi = \Pi^{nt}$ *and there is unique market size* $n^* > \sqrt{\frac{1}{d_\varepsilon \varphi}}$ *that maximizes* Π.

(b) $\lim_{n\to\infty} \Pi^{st} = \Pi^{nt}$ *and the optimal market size* n^*_{st} *is greater than* n^*.

(c) *For sufficiently large* τ_v, $n^* > \widehat{n}$, *where* $\widehat{n} \equiv \frac{1}{\varphi} - 2$ *is the market size which maximizes hedging effectiveness.*

Proof.

From **Lemma A2**, $B = 1$ and

$$\varphi = (1+\alpha_\varepsilon)^{-1} = \frac{1}{1+\frac{\rho^2}{\tau_\varepsilon \tau_e}} = \frac{\tau_\varepsilon}{\tau_\varepsilon + \frac{\rho^2}{\tau_e}}.$$

Also, $\frac{\tau_v}{\tau}$, X, X^{st} defined by (35)-(37) become

$$\frac{\tau_v}{\tau} = \frac{1-d_\varepsilon}{1+d_\varepsilon n\varphi}, \qquad X = \frac{n}{1+n}(1-\varphi), \qquad X^{st} = \frac{n-1}{n} - \frac{n+1}{n\alpha_\varepsilon}.$$

Note that we used $\frac{\varphi}{1-\varphi} = \frac{1}{\alpha_\varepsilon}$ for X^{st}.

(a) Using the expression of $\frac{\tau_v}{\tau}$ and X above, we have

$$\exp(2\rho\Pi) = 1 - \alpha + \alpha^2 (1-d_\varepsilon)(1-\varphi) \frac{1}{1+d_\varepsilon n\varphi} \frac{n}{1+n} \left(\frac{1-\alpha}{\alpha} + \frac{n}{1+n}(1-\varphi) \right). \quad (38)$$

Because the right hand side converges $1-\alpha$ as $n \to \infty$, $\lim_{n\to\infty} \Pi = \Pi^{nt}$. From above, the optimal market maximizes

$$O_n \equiv \frac{1}{1+d_\varepsilon \varphi n} \frac{n}{1+n} \left\{ \frac{1-\alpha}{\alpha(1-\varphi)} + \frac{n}{1+n} \right\}. \quad (39)$$

O_n is increasing (decreasing) in n if and only if $0 < (>)$

$$\begin{aligned} & -\frac{d_\varepsilon \varphi}{(1+d_\varepsilon \varphi n)^2} \frac{n}{1+n} \left\{ \frac{1-\alpha}{\alpha(1-\varphi)} + \frac{n}{1+n} \right\} + \frac{1}{1+d_\varepsilon \varphi n} \left\{ \frac{2n}{1+n} + \frac{1-\alpha}{\alpha(1-\varphi)} \right\} \frac{1}{(1+n)^2} \\ = \; & \frac{1}{(1+d_\varepsilon \varphi n)^2 (1+n)} \left[-d_\varepsilon \varphi n \left\{ \frac{1-\alpha}{\alpha(1-\varphi)} + \frac{n}{1+n} \right\} + \left\{ \frac{1-\alpha}{\alpha(1-\varphi)} + \frac{2n}{1+n} \right\} \frac{1+d_\varepsilon \varphi n}{1+n} \right]. \end{aligned}$$

Note that

$$\frac{1-\alpha}{\alpha(1-\varphi)} + \frac{n}{1+n} = \frac{(1-\alpha)(1+n) + \alpha(1-\varphi)n}{\alpha(1-\varphi)(1+n)} = \frac{1-\alpha+(1-\alpha\varphi)n}{\alpha(1-\varphi)(1+n)},$$

and

$$\frac{1-\alpha}{\alpha(1-\varphi)} + \frac{2n}{1+n} = \frac{(1-\alpha)(1+n) + 2\alpha(1-\varphi)n}{\alpha(1-\varphi)(1+n)} = \frac{1-\alpha+(1+\alpha(1-2\varphi))n}{\alpha(1-\varphi)(1+n)}.$$

Therefore, the sign of terms in the square bracket is determined by the sign of

$$\begin{aligned} & -d_\varepsilon \varphi n \{1-\alpha+(1-\alpha\varphi)n\}(1+n)+\{1-\alpha+(1+\alpha(1-2\varphi))n\}(1+d_\varepsilon\varphi n) \\ = & -\left[d_\varepsilon\varphi(1-\alpha\varphi)n^3+d_\varepsilon\varphi\{1-\alpha(2-\varphi)\}n^2-\{1+\alpha(1-2\varphi)\}n-(1-\alpha)\right]. \end{aligned}$$

Defining

$$\Gamma(n) \equiv d_\varepsilon\varphi(1-\alpha\varphi)n^3+d_\varepsilon\varphi\{1-\alpha(2-\varphi)\}n^2-\{1+\alpha(1-2\varphi)\}n-(1-\alpha),$$

$$(39) \text{ is } \begin{matrix}\text{increasing}\\ \text{decreasing}\end{matrix} \text{ in } n \text{ if and only if } \Gamma(n) \begin{matrix}<\\>\end{matrix} 0.$$

First, $\Gamma(n)$ can be written as

$$\Gamma(n) = d_\varepsilon\varphi n^2\left[(1-\alpha\varphi)n+1-\alpha(2-\varphi)\right]-\left[\{1+\alpha(1-2\varphi)\}n+1-\alpha\right]. \tag{40}$$

Consider $(1-\alpha\varphi)n+1-\alpha(2-\varphi)$ and $\{1+\alpha(1-2\varphi)\}n+1-\alpha$. Both are positive and linearly increasing in $n \geq 1$. Therefore,

$$\Gamma(n) \lesseqgtr 0 \Leftrightarrow d_\varepsilon\varphi n^2 \lesseqgtr \frac{\{1+\alpha(1-2\varphi)\}n+1-\alpha}{(1-\alpha\varphi)n+1-\alpha(2-\varphi)}.$$

Since $1-\alpha\varphi < 1+\alpha(1-2\varphi)$ and $1-\alpha(2-\varphi) < 1-\alpha$, the former is strictly smaller than the latter for any $n \geq 1$. Because $\frac{\{1+\alpha(1-2\varphi)\}n+1-\alpha}{(1-\alpha\varphi)n+1-\alpha(2-\varphi)} > 1$ for all n, $\Gamma(n) < 0$ for all $n \leq \sqrt{\frac{1}{d_\varepsilon\varphi}}$. Because the first term in (40) is strictly convex and cuts the second linear term from below, there is a unique $n^* > \sqrt{\frac{1}{d_\varepsilon\varphi}}$ for which $\Gamma(n^*) = 0$ and

$$\Gamma(n) \lesseqgtr 0 \Leftrightarrow n \lesseqgtr n^*.$$

Thus, Π is uniquely maximized at $n = n^*$.

(b) For a strategic equilibrium,

$$\exp\left(2\rho\Pi^{st}\right) = 1-\alpha+\alpha^2(1-d_\varepsilon)(1-\varphi)\left(\frac{n-1}{n}-\frac{n+1}{n}\frac{\varphi}{1-\varphi}\right)\left(\frac{1-\alpha}{\alpha}+\frac{n}{1+n}(1-\varphi)\right). \tag{41}$$

Thus, $\lim_{n\to\infty}\Pi^{st} = \Pi^{nt}$. The optimal market size maximizes

$$O_n \frac{\frac{n-1}{n}-\frac{n+1}{n}\frac{\varphi}{1-\varphi}}{\frac{n}{1+n}}, \tag{42}$$

where O_n is given in (39). Taking the derivative with respect to n,

$$\left(\frac{d}{dn}O_n\right)\frac{\frac{n-1}{n}-\frac{n+1}{n}\frac{\varphi}{1-\varphi}}{\frac{n}{1+n}}+O_n\frac{d}{dn}\left\{\frac{\frac{n-1}{n}-\frac{n+1}{n}\frac{\varphi}{1-\varphi}}{\frac{n}{1+n}}\right\}. \tag{43}$$

Because

$$\frac{\frac{n-1}{n} - \frac{n+1}{n}\frac{\varphi}{1-\varphi}}{\frac{n}{1+n}} = \frac{n^2-1}{n^2} - \frac{n^2+2n+1}{n^2}\frac{\varphi}{1-\varphi}$$

increases in n, and $\frac{d}{dn}O_n = 0$ at n^*, (43) is strictly positive for all $n \leq n^*$. Therefore, the optimal market size n^*_{st} is greater than n^*.

(c) We find the condition that implies $\sqrt{\frac{1}{d_\varepsilon \varphi}} > \frac{1}{\varphi} - 2$, which in turn implies $n^* > \widehat{n}$.

Using $\frac{1}{d_\varepsilon} = 1 + \frac{\tau_v}{\tau_\varepsilon}$,

$$\begin{aligned}
\sqrt{\frac{1}{\varphi}\left(1+\frac{\tau_v}{\tau_\varepsilon}\right)} &> \frac{1}{\varphi} - 2 \Leftrightarrow \frac{1}{\varphi}\left(1+\frac{\tau_v}{\tau_\varepsilon}\right) > \left(\frac{1}{\varphi}\right)^2 - 4\frac{1}{\varphi} + 4 \\
&\Leftrightarrow \left(\frac{1}{\varphi}\right)^2 - \left(\frac{\tau_v}{\tau_\varepsilon}+5\right)\frac{1}{\varphi} + 4 < 0 \\
&\Leftrightarrow 4\varphi^2 - \left(\frac{\tau_v}{\tau_\varepsilon}+5\right)\varphi + 1 < 0.
\end{aligned}$$

Therefore, we need $\varphi \in \left(\frac{\frac{\tau_v}{\tau_\varepsilon}+5-\sqrt{9+10\frac{\tau_v}{\tau_\varepsilon}+\left(\frac{\tau_v}{\tau_\varepsilon}\right)^2}}{8}, \frac{\frac{\tau_v}{\tau_\varepsilon}+5+\sqrt{9+10\frac{\tau_v}{\tau_\varepsilon}+\left(\frac{\tau_v}{\tau_\varepsilon}\right)^2}}{8}\right)$, where $\frac{\frac{\tau_v}{\tau_\varepsilon}+5+\sqrt{9+10\frac{\tau_v}{\tau_\varepsilon}+\left(\frac{\tau_v}{\tau_\varepsilon}\right)^2}}{8} > 1$ and

$$\begin{aligned}
\frac{\frac{\tau_v}{\tau_\varepsilon}+5-\sqrt{9+10\frac{\tau_v}{\tau_\varepsilon}+\left(\frac{\tau_v}{\tau_\varepsilon}\right)^2}}{8} &= \frac{\left(\frac{\tau_v}{\tau_\varepsilon}+5\right)^2 - \left(9+10\frac{\tau_v}{\tau_\varepsilon}+\left(\frac{\tau_v}{\tau_\varepsilon}\right)^2\right)}{8\left(\frac{\tau_v}{\tau_\varepsilon}+5+\sqrt{9+10\frac{\tau_v}{\tau_\varepsilon}+\left(\frac{\tau_v}{\tau_\varepsilon}\right)^2}\right)} \\
&= \frac{2}{\frac{\tau_v}{\tau_\varepsilon}+5+\sqrt{9+10\frac{\tau_v}{\tau_\varepsilon}+\left(\frac{\tau_v}{\tau_\varepsilon}\right)^2}} \in \left(0, \frac{1}{4}\right).
\end{aligned}$$

For any fixed $\varphi \in (0,1)$, sufficiently large τ_v implies $\frac{2}{\frac{\tau_v}{\tau_\varepsilon}+5+\sqrt{9+10\frac{\tau_v}{\tau_\varepsilon}+\left(\frac{\tau_v}{\tau_\varepsilon}\right)^2}} < \varphi$ and hence $\sqrt{\frac{1}{d_\varepsilon\varphi}} > \frac{1}{\varphi} - 2$. ■ **(A11)**

2.4.2 The case with $\mu\omega < 1$

Lemma A12 (optimal market size with $\mu\omega < 1$)

(a) $\lim_{n\to\infty} \exp(2\rho\Pi) = \lim_{n\to\infty} \exp(2\rho\Pi^{st}) = 1 - \alpha + \alpha\frac{1-\omega+(1-\mu)\frac{d_\varepsilon}{1-d_\varepsilon}}{1-\omega+\frac{d_\varepsilon}{1-d_\varepsilon}}$.

(b) *Π and Π^{st} decrease in n for sufficiently large n, and $n^*_{st} > n^*$.*

Proof.

(a) From **Lemma A2**, $\lim_{n\to\infty} n\varphi = \infty$ while $\lim_{n\to\infty} \varphi = 0$. Thus, $\lim_{n\to\infty} X = \lim_{n\to\infty} X^{st} = 1$ and

$$\begin{aligned}
\lim_{n\to\infty} \frac{\tau_v}{\tau} &= \frac{1-\omega+\omega d_\varepsilon-\mu d_\varepsilon}{1-\omega+\omega d_\varepsilon} \\
&= \frac{(1-\omega)(1-d_\varepsilon)+d_\varepsilon-\mu d_\varepsilon}{(1-\omega)(1-d_\varepsilon)+d_\varepsilon} \\
&= \frac{1-\omega+(1-\mu)\frac{d_\varepsilon}{1-d_\varepsilon}}{1-\omega+\frac{d_\varepsilon}{1-d_\varepsilon}}.
\end{aligned}$$

(b) First,

$$\begin{aligned}
\frac{\tau_v}{\tau} &= \frac{(1-\mu d_\varepsilon)\frac{\alpha_\varepsilon B^2+\omega}{\omega n}+1-\mu d_\varepsilon-\omega(1-d_\varepsilon)}{\frac{\alpha_\varepsilon B^2+\omega}{\omega n}+1-\omega(1-d_\varepsilon)} \\
&= \frac{(1-\mu d_\varepsilon)\left(\frac{\alpha_\varepsilon B^2+\omega}{\omega n}+1\right)-\omega(1-d_\varepsilon)}{\frac{\alpha_\varepsilon B^2+\omega}{\omega n}+1-\omega(1-d_\varepsilon)} \\
&= \frac{\frac{1-\mu d_\varepsilon}{1-d_\varepsilon}\left(1+\frac{\alpha_\varepsilon B^2+\omega}{\omega n}\right)-\omega}{\frac{1}{1-d_\varepsilon}\left(1+\frac{\alpha_\varepsilon B^2+\omega}{\omega n}\right)-\omega} \\
&= \frac{\frac{1-\mu d_\varepsilon}{1-d_\varepsilon}-\omega+\frac{1-\mu d_\varepsilon}{1-d_\varepsilon}\frac{\alpha_\varepsilon B^2+\omega}{\omega n}}{\frac{1}{1-d_\varepsilon}-\omega+\frac{1}{1-d_\varepsilon}\frac{\alpha_\varepsilon B^2+\omega}{\omega n}} \in \left(\frac{\frac{1-\mu d_\varepsilon}{1-d_\varepsilon}-\omega}{\frac{1}{1-d_\varepsilon}-\omega}, 1-\mu d_\varepsilon\right).
\end{aligned}$$

This decreases in n and $\lim_{n\to\infty} \frac{\tau_v}{\tau} = \frac{\frac{1-\mu d_\varepsilon}{1-d_\varepsilon}-\omega}{\frac{1}{1-d_\varepsilon}-\omega}$, because $\lim_{n\to\infty} \frac{\alpha_\varepsilon B^2+\omega}{\omega n} = 0$ at the rate $n^{-\frac{1}{3}}$.

Next,

$$\begin{aligned}
X &= \frac{n}{1+n}\frac{1-\varphi}{1-(1-\omega)\varphi} \\
&= \frac{n}{1+n}\frac{\varphi}{1-(1-\omega)\varphi}\frac{1-\varphi}{\varphi} \\
&= \frac{n}{1+n}\frac{\alpha_\varepsilon B^2}{\frac{1}{\varphi}-(1-\omega)} \\
&= \frac{n}{1+n}\frac{\alpha_\varepsilon B^2}{\alpha_\varepsilon B^2+\omega} \\
&= \frac{1}{1+n^{-1}}\frac{\alpha_\varepsilon}{\alpha_\varepsilon+\omega B^{-2}} \in (0,1).
\end{aligned}$$

This increases in n and $\lim_{n\to\infty} X = 1$, because $\lim_{n\to\infty} B^{-2} = 0$ at the rate $n^{-\frac{2}{3}}$. Because both $\frac{\tau_v}{\tau}$ and X monotonically converge to positive limits, whether $\frac{\tau_v}{\tau} X(1-\alpha+\alpha X)$ decreases in n for sufficiently large n depends on which force (increasing or decreasing) converges faster. We use the following fact:

<u>**Fact 2.**</u> *Consider $n \geq 1$ and $\{a_j, d_j\}_{j=1}^J, b, c, e, f > 0$. Let $\underline{a} \equiv \min_j a_j$.*

$\left[\prod_{j=1}^{J}\frac{d_j}{n^{-a_j}+d_j}\right] \times \frac{n^{-b}+e}{n^{-b}+f} \times (1-n^{-c})$ *decreases in* n *for sufficiently large* n *if* $b < \min\{\underline{a}, c\}$ *and* $e < f$.

Apply **Fact 2** to $\frac{\tau_v}{\tau}X$ and $\frac{\tau_v}{\tau}X^2$, where $b = \frac{1}{3}$, $\underline{a} = \frac{2}{3}$. Thus, Π decreases in n for sufficiently large n.

For Π^{st}, note that

$$\begin{aligned}
X^{st} &= \frac{n-1}{n} - \frac{n+1}{n}\frac{\omega}{\alpha_\varepsilon B^2} \\
&= 1 - \frac{1}{n} - \left(1+\frac{1}{n}\right)\frac{\omega}{\alpha_\varepsilon B^2} \\
&= 1 - \frac{\omega}{\alpha_\varepsilon B^2} - \frac{1}{n}\left(1+\frac{\omega}{\alpha_\varepsilon B^2}\right)
\end{aligned}$$

approaches its upper bound 1 at the rate at which $\frac{1}{B^2}$ approaches zero, which is $n^{-\frac{2}{3}}$. Apply **Fact 2** to $\frac{\tau_v}{\tau}X^{st}$ and $\frac{\tau_v}{\tau}X^{st}X$, where $b = \frac{1}{3}$, $\underline{a} = c = \frac{2}{3}$.

Finally, from **Lemma A10**, $\frac{X^{st}}{X} = 1 - \widetilde{\lambda} < 1$ increases in n. Therefore, Π^{st} still increases in n at n^* and $n^* < n^*_{st}$. ■ **(A12)**

Proof of Fact 2. Take log to obtain

$$\sum_{j=1}^{J}\left\{\ln d_j - \ln\left(n^{-a_j}+d_j\right)\right\} + \ln\left(n^{-b}+e\right) - \ln\left(n^{-b}+f\right) + \ln\left(1-n^{-c}\right).$$

Taking the derivative with respect to n,

$$\begin{aligned}
&\sum_{j=1}^{J}\frac{a_j n^{-a_j-1}}{n^{-a_j}+d_j} - \frac{bn^{-b-1}}{n^{-b}+e} + \frac{bn^{-b-1}}{n^{-b}+f} + \frac{cn^{-c-1}}{1-n^{-c}} \\
= &\frac{1}{(n^{-b}+e)(n^{-b}+f)(1-n^{-c})\prod_{j=1}^{J}(n^{-a_j}+d_j)} \times \\
&\left[\left(n^{-b}+e\right)\left(n^{-b}+f\right)\left(1-n^{-c}\right)\sum_{j=1}^{J}\left\{a_j n^{-a_j-1}\prod_{k\neq j}\left(n^{-a_k}+d_k\right)\right\}\right. \\
&\left. - \left[bn^{-b-1}\left\{\left(n^{-b}+f\right)-\left(n^{-b}+e\right)\right\}\left(1-n^{-c}\right) - cn^{-c-1}\left(n^{-b}+e\right)\left(n^{-b}+f\right)\right]\prod_{j=1}^{J}\left(n^{-a_j}+d_j\right)\right] \\
= &\frac{\left(n^{-b}+e\right)\left(n^{-b}+f\right)(1-n^{-c})\sum_{j=1}^{J}\frac{a_j n^{-a_j}}{n^{-a_j}+d_j} - \left\{bn^{-b}(f-e)(1-n^{-c}) - cn^{-c}\left(n^{-b}+e\right)\left(n^{-b}+f\right)\right\}}{n(n^{-b}+e)(n^{-b}+f)(1-n^{-c})} \\
= &\frac{\left(n^{-b}+e\right)\left(n^{-b}+f\right)(1-n^{-c})\sum_{j=1}^{J}a_j\frac{n^{\underline{a}}}{1+d_j n^{a_j}} - \left\{bn^{\underline{a}-b}(f-e)(1-n^{-c}) - cn^{\underline{a}-c}\left(n^{-b}+e\right)\left(n^{-b}+f\right)\right\}}{n(n^{-b}+e)(n^{-b}+f)(1-n^{-c})}.
\end{aligned}$$

If $b < \min\{\underline{a}, c\}$ and $e < f$, the numerator is negative for sufficiently large n. $\blacksquare$ **(F2)**