Procurement design with corruption
Online Appendix

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Section 2.3

First, we show that concavity of $B(m)$ is sufficienty for the optimal choice of m to be zero.

Lemma 1 *For any IC, IR direct mechanism, (p, q, m), there exists an IC, IR mechanism with $m(\theta) = 0 \ \forall \theta \in [\underline{\theta}, \overline{\theta}]$, so that $E[q(\theta) - m(\theta) - p(\theta)]$ is higher for the latter.*

Proof. Assume $m(\theta) > 0$ for some value θ, and consider a change in the mechanism so that $q'(\theta) = q(\theta) - m(\theta)$, $m'(\theta) = 0$, and $p'(\theta) = p(\theta) - B(m(\theta))$. The profits of type θ do not change. Also, a type θ' imitating type θ could achieve

$$p(\theta) - \min_{z \in [0, q(\theta)]} \left\{ C\left(q(\theta) - z; \theta'\right) + B(z) \right\},$$

with the original mechanism, whereas with the modified mechanism she can obtain

$$\begin{aligned}
& p'(\theta) - \min_{z \in [0, q'(\theta)]} \left\{ C\left(q'(\theta) - z; \theta'\right) + B(z) \right\} \\
= \; & p(\theta) - B(m(\theta)) - \min_{z \in [0, q(\theta) - m(\theta)]} \left\{ C\left(q(\theta) - m(\theta) - z; \theta'\right) + B(z) \right\} \\
= \; & p(\theta) - \min_{z \in [0, q(\theta) - m(\theta)]} C\left(q(\theta) - m(\theta) - z; \theta'\right) + B(z) + B(m(\theta)) \\
= \; & p(\theta) - \min_{h \in [m(\theta), q(\theta)]} C\left(q(\theta) - h; \theta'\right) + B(h - m(\theta)) + B(m(\theta)).
\end{aligned}$$

where we have used the change of variable $h = z + m(\theta)$. This expression is smaller since B is concave and the choice set of h is smaller than the choice set of z in the original mechanism. The profits of θ' imitating any other type have not changed, and the profits of θ imitating any other type are not larger. ■

Next, we prove the claim that the results in Proposition 3 extend to the concave case, provided assumptions A1, A2, and A3 are satisfied

Claim 2 *Under concavity of $B(m)$, A1, A2, and A3, if $q^{NB}(\theta)$ violates (12) then there exist θ^a and θ^c, with $\underline{\theta} < \theta^a \leq \theta^c < \overline{\theta}$ such that at the optimal mechanism; (i) $q(\theta) = 0$ if $\theta > \theta^c$; (ii) $q(\theta) = q^{NB}(\theta)$ if $\theta \in (\theta^a, \theta^c)$; and (iii) $q(\theta) = q^{NB}(\theta^a)$ if $\theta < \theta^a$.*

Proof. Given an exogenous $q(\underline{\theta})$, the result is proved exactly as Proposition 3. Thus, we need only show that the sponsor's surplus is maximized for $q(\underline{\theta}) < q^{NB}(\underline{\theta})$. The sponsor's objective is still given by (22), and so its derivative at $q^{NB}(\underline{\theta})$ is also given by (23). Then, we only need show that $\frac{d\theta^c}{dq(\underline{\theta})} < 0$. Totally differentiatin the equivalent now to (21),

$$B(q(\underline{\theta})) - C(q(\underline{\theta}); \underline{\theta}) - \int_{\underline{\theta}}^{\theta^a} C_\theta(q(\underline{\theta}); z)dz - \int_{\theta^a}^{\theta^c} C_\theta(q^{NB}(z); z)dz = 0,$$

we have

$$\frac{d\theta^c}{dq(\underline{\theta})} = \frac{B'(q(\underline{\theta})) - C_q(q(\underline{\theta}); \underline{\theta}) - \int_{\underline{\theta}}^{\theta^a} C_{\theta q}(q(\underline{\theta}); z)dz}{C_\theta(q^{NB}(\theta^c); \theta^c)} < 0,$$

where the inequality follows from A2 and the fact that $C_{\theta q}(q; \theta) > 0$. ■