Online Appendix to "State dependent government spending multipliers: Downward Nominal Wage Rigidity and Sources of Business Cycle Fluctuations"

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A.1 Appendix: Analytics of state-dependent government spending multipliers

An equilibrium is a set of stochastic processes $\{\lambda_t, c_t, w_t, mc_t, R_t, \pi_t, x_t^1, x_t^2, y_t, n_t, n_t^s, u_t, s_t, p_t^*\}_{t=0}^{\infty}$ satisfying:

$$\lambda_t = (c_t - \chi n_t^{\varphi})^{-\sigma} \tag{A.1}$$

$$\chi\varphi n_t^{s\varphi-1} = w_t \tag{A.2}$$

$$\lambda_t = R_t \mathbb{E}_t \frac{\beta_{t+1}\lambda_{t+1}}{\pi_{t+1}} \tag{A.3}$$

$$W_t \geq \gamma W_{t-1}; w_t \geq \gamma \frac{w_{t-1}}{\pi_t} \tag{A.4}$$

$$(n_t^s - n_t)(w_t - \gamma \frac{w_{t-1}}{\pi_t}) = 0 \tag{A.5}$$

When DNWR does not bind ($w_t > \gamma \frac{w_{t-1}}{\pi_t}$), full employment is achieved, $n_t^s = n_t$ and $u_t = 0$. As opposed, if DNWR binds, that is, $w_t = \gamma \frac{w_{t-1}}{\pi_t}$, there is an excess supply of labor, $n_t^s > n_t$ and $u_t > 0$.

$$u_t = \frac{n_t^s - n_t}{n_t^s} \tag{A.6}$$

$$p_t^* = \frac{\theta}{\theta - 1} \frac{x_t^1}{x_t^2} \tag{A.7}$$

$$x_t^1 = \lambda_t y_t mc_t + \omega \mathbb{E}_t \beta_{t+1} \pi_{t+1}^{\theta} x_{t+1}^1 \tag{A.8}$$

$$x_t^2 = \lambda_t y_t + \omega \mathbb{E}_t \beta_{t+1} \pi_{t+1}^{\theta-1} x_{t+1}^2 \tag{A.9}$$

$$mc_t = \frac{w_t}{A_t} \tag{A.10}$$

$$\pi_t = \left[\frac{1}{\omega} - \frac{1-\omega}{\omega} p_t^{*1-\theta}\right]^{\frac{1}{\theta-1}} \tag{A.11}$$

$$y_t = A_t n_t / s_t \tag{A.12}$$

$$y_t = c_t + g_t \tag{A.13}$$

$$s_t = (1-\omega) p_t^{*-\theta} + \omega \pi_t^{\theta} s_{t-1} \tag{A.14}$$

$$\frac{R_t}{R} = (\frac{\pi_t}{\pi})^{\alpha_\pi} (\frac{y_t}{y})^{\alpha_y} \tag{A.15}$$

, given exogenous stochastic processes $\{g_t, \beta_t, A_t\}_{t=o}^{\infty}$, which are following AR(1) processes specified as below:

$$\ln \frac{g_t}{g} = \rho^g \ln \frac{g_{t-1}}{g} + \epsilon_t^g \tag{A.16}$$

$$\ln \frac{\beta_t}{\beta} = \rho^\beta \ln \frac{\beta_{t-1}}{\beta} + \epsilon_t^\beta \tag{A.17}$$

$$\ln \frac{A_t}{A} = \rho^A \ln \frac{A_{t-1}}{A} + \epsilon_t^A \tag{A.18}$$

A.1.1 Derivation of IS-PC curves

We derive the IS and the Phillips curve (PC) summarizing equilibrium conditions, (A.1) ~ (A.15). To derive the IS equation, log-linearize both the monetary policy rule (A.15) and the household's intertemporal optimization equation (A.3). Combining the previous two equations yields

$$\widehat{\lambda_t} = \mathbb{E}_t \widehat{\lambda}_{t+1} + \alpha_\pi \widehat{\pi}_t - \mathbb{E}_t \widehat{\pi}_{t+1} + \mathbb{E}_t \widehat{\beta}_{t+1}. \tag{A.19}$$

, where hat variables stand for log-deviations from the steady state and the variable without time subscript represents its steady-state value. Find $\widehat{\lambda}_t$ by log-linearizing the marginal utility of consumption (A.1),

$$\widehat{\lambda_t} = -\frac{\sigma c}{c - \chi n^\varphi} \widehat{c}_t + \frac{\sigma \chi \varphi n^\varphi}{c - \chi n^\varphi} \widehat{n}_t. \tag{A.20}$$

Now let's find the steady-state values of variables. From the production function (A.12), we know that the steady state level of output y=A. Note that the steady-state value of s is zero under the zero inflation steady-state (Galí (2008)). By the market clearing condition (A.13), we find the steady-state consumption is then $c = y - g$. Define the steady-state government spending-to-output ratio as $s_g \equiv \frac{g}{y}$. Then, $c = (1 - s_g)A$. Assume the steady-state labor n equals to labor supply, n^s, which equals to 1. Using Equation (A.2) and (A.10), solve for the model-implied parameter χ assuring $n = 1$ as

$$\chi = \frac{w}{\varphi} = \frac{1}{\varphi} \times A \times mc = \frac{A}{\varphi}\frac{\theta - 1}{\theta}.$$

Substituting the steady-state values to the Equation (A.20) yields

$$\widehat{\lambda_t} = -\frac{\theta(1-s_g)}{\Psi}\widehat{c}_t + \frac{(\theta-1)}{\Psi}\widehat{n}_t, \tag{A.21}$$

where $\Psi = \frac{\theta\varphi(1-s_g)-(\theta-1)}{\sigma\varphi}$. The log linearization of the market clearing condition (A.13) and the production function (A.12) leads

$$\widehat{c}_t = \frac{1}{1-s_g}\widehat{y}_t - \frac{s_g}{1-s_g}\widehat{g}_t \tag{A.22}$$

$$\widehat{y}_t = \widehat{a}_t + (\widehat{n}_t - \widehat{s}_t). \tag{A.23}$$

Galí (2008) shows that $\widehat{s}_t$ equals to zero up to a first-order approximation. Combining (A.19), (A.21), (A.22), and (A.23) yields the NKIS equation:

$$\widehat{y}_t = \mathbb{E}_t\widehat{y}_{t+1} - (\theta-1)(\widehat{a}_t - \mathbb{E}_t\widehat{a}_{t+1}) + \theta s_g(\widehat{g}_t - \mathbb{E}_t\widehat{g}_{t+1}) - \Psi(\alpha_\pi\widehat{\pi}_t - \mathbb{E}_t\widehat{\pi}_{t+1}) - \Psi\mathbb{E}_t\widehat{\beta}_{t+1} \tag{A.24}$$

where $\Psi = \frac{\theta\varphi(1-s_g)-\theta+1}{\sigma\varphi}$.

Now let's derive Phillips curve (PC). The PC can be written in two ways, depending upon whether DNWR binds or not. The first-order approximation of Equation (A.7) and (A.11) yields

$$\widehat{\pi}_t = \frac{(1-\omega)(1-\omega\beta)}{\omega}\widehat{mc}_t + \beta\mathbb{E}_t\widehat{\pi}_{t+1}, \tag{A.25}$$

where $\widehat{mc}_t$ takes two forms. When DNWR does not bind, full employment is achieved ($\widehat{n}_t = \widehat{n}^s_t$). Log-linearization of the Equation (A.2) under the full employment regime yields $\widehat{w}_t = (\varphi - 1)\widehat{n}_t$. From the Equation (A.10), we know that $\widehat{mc}_t = \widehat{w}_t - \widehat{a}_t$. Combining previous two equations with Equation (A.23) leads

$$\widehat{mc}_t = (\varphi - 1)\widehat{y}_t - \varphi\widehat{a}_t. \tag{A.26}$$

Substituting (A.26) into (A.25) yields the PC curve under the full employment equilibrium:

$$\widehat{\pi}_t = \triangle(\varphi - 1)\widehat{y}_t - \triangle\varphi\widehat{a}_t + \beta\mathbb{E}_t\widehat{\pi}_{t+1}, \tag{A.27}$$

where $\triangle = \frac{(1-\omega)(1-\omega\beta)}{\omega}$. When DNWR binds ($\gamma = 1$), we can re-write $\widehat{w}_t = \widehat{w}_{t-1} - \widehat{\pi}_t$. Then,

$$\widehat{mc}_t = \widehat{w}_{t-1} - \widehat{\pi}_t - \widehat{a}_t. \tag{A.28}$$

Substituting (A.28) into (A.25) yields the modified PC curve under the binding DNWR

$$(1 + \triangle)\widehat{\pi}_t = \triangle[\widehat{w}_{t-1} - \widehat{a}_t] + \beta\mathbb{E}_t\widehat{\pi}_{t+1}, \tag{A.29}$$

or

$$\widehat{\pi}_t = \frac{(1-\omega)(1-\omega\beta)}{\omega + (1-\omega)(1-\omega\beta)}[\widehat{w}_{t-1} - \widehat{a}_t] + \frac{\omega\beta}{\omega + (1-\omega)(1-\omega\beta)}\mathbb{E}_t\widehat{\pi}_{t+1}. \tag{A.30}$$

A.1.2 Analytical results

Now let's consider the two sources of business cycle fluctuations - a preference shock ($\widehat{\beta}_{t+1}$) and a productivity shock ($\widehat{a}_t$). For tractability, we assume complete DNWR, with $\gamma = 1$. Additionally, nominal interest rates respond to deviations in the inflation rate from its steady state but not to output deviations ($\alpha_\pi > 0$ and$\alpha_y = 0$).The sequences of the preference shock ($\mathbb{E}_t\widehat{\beta}_{t+1} = b_L < 0$ and $\mathbb{E}_t\widehat{\beta}_{t+2} = 0$) and ($\mathbb{E}_t\widehat{\beta}_{t+1} = b_H > 0$ and $\mathbb{E}_t\widehat{\beta}_{t+2} = 0$) cause a demand-driven expansion and recession, respectively, in period t. The sequence of the technology shock ($\widehat{a}_t = a_H > 0$, $\mathbb{E}_t\widehat{a}_{t+1}$=$\rho_a a_H$, and $\mathbb{E}_t\widehat{a}_{t+2}$=$a_L$) and ($\widehat{a}_t = a_L < 0$, $\mathbb{E}_t\widehat{a}_{t+1}$=$\rho_a a_L$, and $\mathbb{E}_t\widehat{a}_{t+2}$=$a_H$) drive a supply-driven expansion and recession, respectively, in period t.

Proposition A.1. In response to a preference shock, output ($\widehat{y}_t$) and inflation ($\widehat{\pi}_t$) co-move, and in response to a technology shock, output and inflation move in the opposite direction. That is,

$$\frac{\partial\widehat{y}_t}{\partial\widehat{\beta}_{t+1}} < 0; \frac{\partial\widehat{\pi}_t}{\partial\widehat{\beta}_{t+1}} < 0, \text{ and } \frac{\partial\widehat{y}_t}{\partial\widehat{a}_t} > 0; \frac{\partial\widehat{\pi}_t}{\partial\widehat{a}_t} < 0.$$

Proof. Let's consider two independent shock processes. The demand-driven business cycles follow ($\mathbb{E}_t\widehat{\beta}_{t+1} = \widehat{\beta}_{t+1}$, and $\mathbb{E}_t\widehat{\beta}_{t+2} = 0$) where $\mathbb{E}_t\widehat{\beta}_{t+1}$ is β_H in a demand-driven recession and $\mathbb{E}_t\widehat{\beta}_{t+1}$ is β_L in a demand shock-boom. The supply-driven business cycles are to follow ($\widehat{a}_t$=$\widehat{a}_t$, $\mathbb{E}_t\widehat{a}_{t+1}$=$\rho_a\widehat{a}_t$,and $\mathbb{E}_t\widehat{a}_{t+2}$=$\widehat{a}_{t+2}$) where $(\widehat{a}_t, \widehat{a}_{t+2}) = (a_H, a_L)$ in a supply-driven boom and $(\widehat{a}_t, \widehat{a}_{t+2}) = (a_L, a_H)$ in a supply-driven recession. Suppose that the market clearing solution takes the form:

$$\widehat{y}_t = A_y\widehat{g}_t + B_y\mathbb{E}_t\widehat{\beta}_{t+1} + C_y\widehat{a}_t + D_y\mathbb{E}_t\widehat{a}_{t+1} = A_y\widehat{g}_t + B_y\mathbb{E}_t\widehat{\beta}_{t+1} + C_y\widehat{a}_t + \rho_a D_y\widehat{a}_t$$

$$\widehat{\pi}_t = A_\pi \widehat{g}_t + B_\pi \mathbb{E}_t \widehat{\beta}_{t+1} + C_\pi \widehat{a}_t + D_\pi \mathbb{E}_t \widehat{a}_{t+1} = A_\pi \widehat{g}_t + B_\pi \mathbb{E}_t \widehat{\beta}_{t+1} + C_\pi \widehat{a}_t + \rho_a D_\pi \widehat{a}_t.$$

Given the assumptions on shock processes and government spending, the expected output and inflation are

$$\mathbb{E}_t \widehat{y}_{t+1} = A_y \mathbb{E}_t \widehat{g}_{t+1} + B_y \mathbb{E}_t \widehat{\beta}_{t+2} + C_y \mathbb{E}_t \widehat{a}_{t+1} + D_y \mathbb{E}_t \widehat{a}_{t+2} = \rho_a C_y \widehat{a}_t + D_y \widehat{a}_{t+2}$$

$$\mathbb{E}_t \widehat{\pi}_{t+1} = A_\pi \mathbb{E}_t \widehat{g}_{t+1} + B_\pi \mathbb{E}_t \widehat{\beta}_{t+2} + C_\pi \mathbb{E}_t \widehat{a}_{t+1} + D_\pi \mathbb{E}_t \widehat{a}_{t+2} = \rho_a C_\pi \widehat{a}_t + D_\pi \widehat{a}_{t+2}$$

Plug the projected solution into the IS curve (A.24) and Phillips curve (A.27) and solve for coefficients using the method of undetermined coefficients,

$$A_y = \frac{\theta s_g}{1 + \Psi \alpha_\pi \triangle(\varphi - 1)} > 0$$

$$A_\pi = \frac{\triangle(\varphi - 1)\theta s_g}{1 + \Psi \alpha_\pi \triangle(\varphi - 1)} > 0$$

$$B_y = \frac{\partial \widehat{y}_t}{\partial \widehat{\beta}_{t+1}} = -\frac{\Psi}{[1 + \Psi \alpha_\pi \triangle(\varphi - 1)]} < 0$$

$$B_\pi = \frac{\partial \widehat{\pi}_t}{\partial \widehat{\beta}_{t+1}} = -\frac{\Psi \triangle(\varphi - 1)}{[1 + \Psi \alpha_\pi \triangle(\varphi - 1)]} < 0$$

$\mathbb{I}(\mathrm{H}(\pi_{it}))$ indicates high inflation US-states-years,

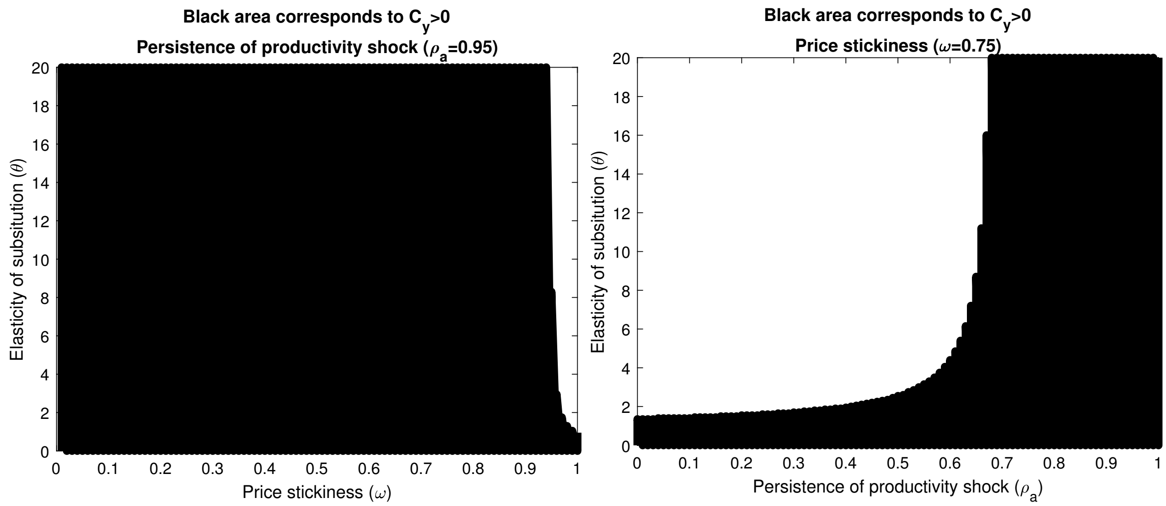
$$D_\pi = 0$$

$$D_y = 0$$

$$C_\pi = \frac{\partial \widehat{\pi}_t}{\partial \widehat{a}_t} = \frac{-\triangle}{(1 - \beta\rho_a)} [\frac{(\varphi - 1)(\theta - 1)(1 - \rho_a)(1 - \beta\rho_a) + \varphi(1 - \rho_a)(1 - \beta\rho_a)}{(1 - \rho_a)(1 - \beta\rho_a) + \Psi(\alpha_\pi - \rho_a)\triangle(\varphi - 1)}] < 0$$

$$C_y = \frac{\partial \widehat{y}_t}{\partial \widehat{a}_t} = \frac{-(\theta - 1)(1 - \rho_a)(1 - \beta\rho_a) + \frac{\theta\varphi(1 - s_g) - (\theta - 1)}{\sigma\varphi}(\alpha_\pi - \rho_a)\frac{(1-\omega)(1-\omega\beta)}{\omega}\varphi}{(1 - \rho_a)(1 - \beta\rho_a) + \frac{\theta\varphi(1 - s_g) - (\theta - 1)}{\sigma\varphi}(\alpha_\pi - \rho_a)\frac{(1-\omega)(1-\omega\beta)}{\omega}(\varphi - 1)}.$$

Figure A.1: Parameter space corresponding to positive C_y



Notes: The left panel shows the parameter space (θ, ω) that corresponds to positive C_y given the persistence of productivity shock is 0.95. The right panel shows the combination of (θ, ρ_a) that ensures positive C_y.

The sign of all coefficients except C_y is determinant under common parameter values.[1] However, depending on the parameter values, the sign of C_y changes. For example, for a high enough elasticity of substitution (θ) and price-stickiness parameter (ω) or a low enough persistence of productivity shock (ρ_a), C_y can be negative. To determine the sign of C_y, we fix the typical parameter values – the discount factor (β) is 0.99, the Frisch elasticity $(\frac{1}{\varphi-1})$ is 0.5, and coefficient on inflation in the monetary policy rule (α_π) is 1.5. The steady-state government spending to output ratio s_g is calibrated to 0.2. The left panel of Figure A.1 shows the parameter space of θ and ω that corresponds to positive C_y, under the persistence productivity shock (ρ_a) being 0.95. C_y is positive for plausible parameter space. In New Keynesian literature, it is common to set ω as 0.75. The price rigidity of posted prices varies from 0.45 to 0.73 from microdata literature (see Nakamura and Steinsson (2013)). The right panel of Figure A.1 shows the combination of θ and ρ_a that ensures positive C_y, when the price stickiness parameter, ω, is 0.75. For a high enough persistent productivity, we find that C_y is positive. To summarize, C_y is positive under the plausible parameter space. □

Proposition A.2. In a model without DNWR, the government spending multiplier takes the same value M_y in expansion and recession states, i.e. is acyclical.

[1]The elasticity of substitution parameter θ is greater than 1, the discount factor β is less than 1 and greater than zero. The government spending share in output, s_g is less than one. The intertemporal elasticity of substitution σ is assumed to be greater than one, while the frequency of price adjustment is ω is less than one. The coefficient on inflation in the monetary policy rule is assumed to be higher than one.

Proof. From the proof of Proposition A.1, the government spending multiplier is

$$M_y \equiv \frac{dy}{dg} = \frac{\partial \widehat{y}_t}{\partial \widehat{g}_t}\frac{\overline{y}}{\overline{g}} = \frac{A_y}{s_g} = \frac{\omega\theta}{\omega + \Psi\alpha_\pi(1-\omega)(1-\omega\beta)(\varphi-1)} \geq 0$$

regardless of the shock processes and the state of the economy. □

When DNWR binds in period t under the expectation of achieving full employment in period $(t+1)$, the spending multiplier is M_{DNWR} , which is bigger than M_y – the multiplier when DNWR does not bind.

Proof. Guess the solution that satisfies both IS curve (Equation (A.24)) and the modified Phillips curve (Equation (A.29)). Note that the binding DNWR constraint leaves IS curve unchanged while PC changes. Let's first consider the demand-driven business cycle – $(\mathbb{E}_t\widehat{\beta}_{t+1} = \widehat{\beta}_{t+1}$, and $\mathbb{E}_t\widehat{\beta}_{t+2} = 0)$. Then, the projected solution becomes

$$\widehat{y}_t = F_y\widehat{w}_{t-1} + H_y\widehat{g}_t + I_y\mathbb{E}_t\widehat{\beta}_{t+1} \tag{A.31}$$

$$\widehat{\pi}_t = F_\pi\widehat{w}_{t-1} + H_\pi\widehat{g}_t + I_\pi\mathbb{E}_t\widehat{\beta}_{t+1}. \tag{A.32}$$

Under the assumption that DNWR does not bind in period $(t+1)$, the expected output and inflation $\mathbb{E}_t\widehat{y}_{t+1}$ and $\mathbb{E}_t\widehat{\pi}_{t+1}$ become zero. Plug in suggested solutions (A.31) and (A.32) into IS curve (A.24) and the modified Phillips curves (Equation (A.29)) and find the coefficients using the method of undetermined coefficients,

$$F_y\widehat{w}_{t-1} + H_y\widehat{g}_t + I_y\widehat{\beta}_{t+1} = \theta s_g\widehat{g}_t - \Psi\alpha_\pi(F_\pi\widehat{w}_{t-1} + H_\pi\widehat{g}_t + I_\pi\widehat{\beta}_{t+1}) - \Psi\widehat{\beta}_{t+1}$$

$$(1+\triangle)(F_\pi\widehat{w}_{t-1} + H_\pi\widehat{g}_t + I_\pi\widehat{\beta}_{t+1}) = \triangle\widehat{w}_{t-1}$$

The multiplier in the demand-driven business cycle is

$$M^D_{DNWR} = \frac{dy}{dg} = \frac{\partial \widehat{y}_t}{\partial \widehat{g}_t}\frac{y}{g} = H_y\frac{1}{s_g} = \theta$$

, which is bigger than $M_y = \frac{\omega\theta}{\omega+\Psi\alpha_\pi(1-\omega)(1-\omega\beta)(\varphi-1)}$.

Now, let's consider the supply-driven business cycles following ($\widehat{a}_t$=$\widehat{a}_t$, $\mathbb{E}_t\widehat{a}_{t+1}$=$\rho_a\widehat{a}_t$,and $\mathbb{E}_t\widehat{a}_{t+2}$=$\widehat{a}_{t+2}$). Conjecture solution as,

$$\widehat{y}_t = O_y\widehat{w}_{t-1} + S_y\widehat{g}_t + U_y\widehat{a}_t + V_y\rho_a\widehat{a}_t \tag{A.33}$$

$$\widehat{\pi}_t = O_\pi\widehat{w}_{t-1} + S_\pi\widehat{g}_t + U_\pi\widehat{a}_t + V_\pi\rho_a\widehat{a}_t. \tag{A.34}$$

Under the assumption that DNWR does not bind in period $(t+1)$, the expected output and inflation are given by the full employment solution shown in the proof of Proposition A.1, as below.

$$\mathbb{E}_t\widehat{y}_{t+1} = C_y\rho_a\widehat{a}_t + D_y\widehat{a}_{t+2} \tag{A.35}$$

$$\mathbb{E}_t\widehat{\pi}_{t+1} = C_\pi\rho_a\widehat{a}_t + D_\pi\widehat{a}_{t+2} \tag{A.36}$$

Combining the suggested solution ((A.33) and (A.34)) with the expected output and inflation ((A.35) and (A.36)) into the IS curve (A.24) and the modified Phillips curves (Equation (A.29)) brings

$$\begin{aligned} O_y\widehat{w}_{t-1} + S_y\widehat{g}_t + U_y\widehat{a}_t + V_y\rho_a\widehat{a}_t = C_y\rho_a\widehat{a}_t + D_y\widehat{a}_{t+2} - (\theta-1)(\widehat{a}_t - \rho_a\widehat{a}_t) \\ + \theta s_g\widehat{g}_t - \Psi\alpha_\pi(O_\pi\widehat{w}_{t-1} + S_\pi\widehat{g}_t + U_\pi\widehat{a}_t + V_\pi\rho_a\widehat{a}_t) + \Psi(C_\pi\rho_a\widehat{a}_t + D_\pi\widehat{a}_{t+2}) \end{aligned}$$

$$(1+\triangle)[O_\pi\widehat{w}_{t-1} + S_\pi\widehat{g}_t + U_\pi\widehat{a}_t + V_\pi\rho_a\widehat{a}_t] = \triangle[\widehat{w}_{t-1} - \widehat{a}_t] + \beta(C_\pi\rho_a\widehat{a}_t + D_\pi\widehat{a}_{t+2})$$

Using the undetermined coefficients method, we find $S_\pi = 0$ and $S_y = \theta s_g$. The output multiplier in the supply-driven business cycle is

$$M^S_{DNWR} = \frac{\partial\widehat{y}}{\partial\widehat{g}}\frac{y}{g} = S_y\frac{1}{s_g} = \theta.$$

Thus, we have shown that the multiplier is θ when DNWR binds (M_{DNWR}), regardless of the source of fluctuation. □

Lemma A.1. Assume the economy is at the steady-state in period $t-1$, $\widehat{w}_{t-1} = 0$. In the presence of the DNWR constraint ($\gamma = 1$), a positive discount factor shock or a negative productivity shock triggers the DNWR constraint to bind and induces unemployment in period t.

Proof. Log-linearized DNWR constraint (Equation (A.4)) can be expressed as follows.

$$\widehat{w}_t \geq \gamma(\widehat{w}_{t-1} - \widehat{\pi}_t). \tag{A.37}$$

To show the DNWR constraint binds in period t under the assumption that $\widehat{w}_{t-1} = 0$ and $\gamma = 1$, we have to show

$$\widehat{w}_t + \widehat{\pi}_t < 0. \tag{A.38}$$

Let's conjecture DNWR does not bind and $\widehat{n}_t = \widehat{n}^s_t$. Now check whether the conjecture holds, that is, Equation (A.37) is true. First, we obtain $\widehat{w}_t$ by combining two log-linearized Equation (A.2) and (A.12):

$$\widehat{w}_t = (\varphi - 1)(\widehat{y}_t - \widehat{a}_t).$$

From the proof of Proposition A.1, we know that we can write $\widehat{y}_t$ and $\widehat{\pi}_t$ as follows.

$$\widehat{y} = A_y \widehat{g}_t + B_y \mathbb{E}_t \widehat{\beta}_{t+1} + C_y \widehat{a}_t + D_y \mathbb{E}_t \widehat{a}_{t+1} = B_y \mathbb{E}_t \widehat{\beta}_{t+1} + C_y \widehat{a}_t + \rho_a D_y \widehat{a}_t$$

$$\widehat{\pi} = A_\pi \widehat{g}_t + B_\pi \mathbb{E}_t \widehat{\beta}_{t+1} + C_\pi \widehat{a}_t + D_\pi \mathbb{E}_t \widehat{a}_{t+1} = B_\pi \mathbb{E}_t \widehat{\beta}_{t+1} + C_\pi \widehat{a}_t + \rho_a D_\pi \widehat{a}_t$$

Plug in $\widehat{y}_t$ and $\widehat{\pi}_t$ into the left-hand-side of inequality constraint (A.38)

$$\widehat{w}_t + \widehat{\pi}_t = (\varphi - 1)(B_y \mathbb{E}_t \widehat{\beta}_{t+1} + (C_y + \rho_a D_y - 1)\widehat{a}_t) + B_\pi \mathbb{E}_t \widehat{\beta}_{t+1} + (C_\pi + \rho_a D_\pi)\widehat{a}_t$$

In a demand-driven recession, where $\mathbb{E}_t \widehat{\beta}_{t+1} = \beta_H$ and $\widehat{a}_t = 0$,

$$\widehat{w}_t + \widehat{\pi}_t = ((\varphi - 1)B_y + B_\pi)\beta_H.$$

From the proof of Proposition A.1, we know coefficients B_y and B_π are negative. Thus, for any positive discount factor shock, we know that

$$\widehat{w}_t + \widehat{\pi}_t < 0,$$

which contradicts the conjecture. Thus, we conclude that DNWR binds in response to a positive discount factor shock.

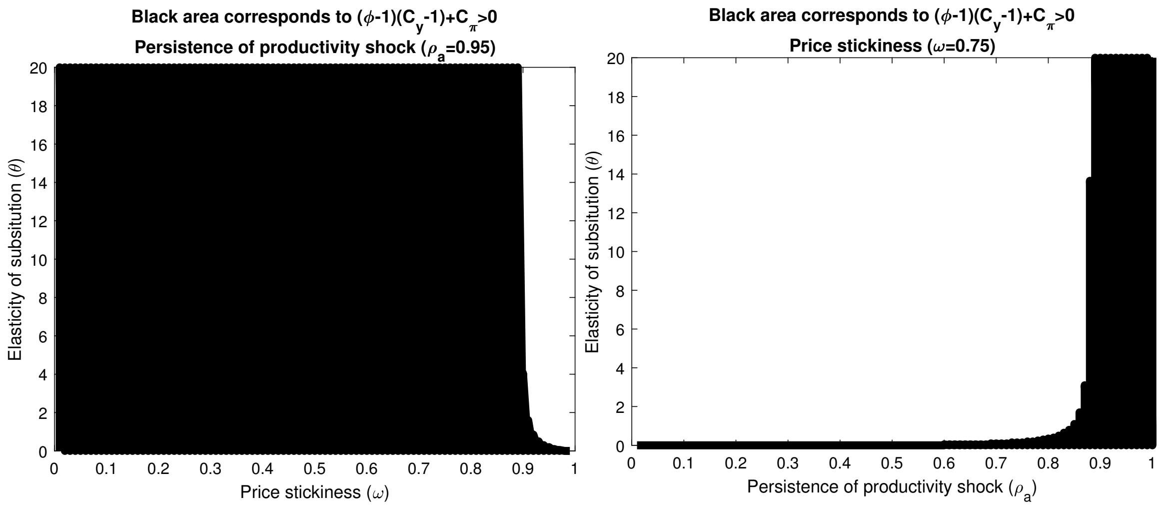
In a supply-driven recession, where $\widehat{a}_t = a_L$ and $\mathbb{E}_t \widehat{\beta}_{t+1} = 0$,

$$\widehat{w}_t + \widehat{\pi}_t = (\varphi - 1)((C_y + \rho_a D_y - 1)a_L) + (C_\pi + \rho_a D_\pi)a_L.$$

As $D_y = D_\pi = 0$, the conjecture that DNWR does not bind is not true if

$$\widehat{w}_t + \widehat{\pi}_t = [(\varphi - 1)(C_y - 1) + C_\pi]a_L < 0.$$

Figure A.2: Parameter space corresponding to positive $(\varphi-1)(C_y-1)+C_\pi$



Notes: The left panel shows the parameter space (θ,ω) that gives positive $(\varphi-1)(C_y-1)+C_\pi$ given the persistence of productivity shock is 0.95. The right panel shows the combination of (θ,ρ_a) that ensures positive $(\varphi-1)(C_y-1)+C_\pi$.

Based on the baseline parameter values[2], the black area in the left panel of Figure A.2 shows the combination of the elasticity of substitution (θ) and the price stickiness (ω) that satisfies

$$[(\varphi-1)(C_y-1)+C_\pi]>0. \tag{A.39}$$

, where the persistence of the productivity shock ρ_a is 0.95. The right panel of FigureA.2 shows the combination of θ and ρ_a that satisfies Equation (A.39), when the price stickiness parameter, ω, is 0.75. Under the assumption of highly persistent productivity shock, we conclude that DNWR condition binds. □

Lemma A.2. Assume the economy is at steady-state in period $t-1$, $\widehat{w}_{t-1}=0$. In a demand-driven recession, if government spending is less than $\frac{\Psi}{\theta s_g}\beta_H \equiv c_d(\beta_H)$, the DNWR constraint binds, and unemployment is greater than zero. Otherwise, DNWR is no longer a binding constraint, and unemployment is zero. In a supply-driven recession, if government spending is less than $c_s(a_L)$, the DNWR constraint binds, and unemployment is greater than zero. Otherwise, DNWR is no longer a binding constraint, and unemployment is zero.

Proof. Find the upper bound of nonzero $\widehat{g}_t$ that still violates DNWR condition, that is, $\widehat{w}_t<\gamma(\widehat{w}_{t-1}-\widehat{\pi}_t)$, or $\widehat{w}_t+\widehat{\pi}_t<0$. With the nonzero government spending $\widehat{g}_t$, we can guess

[2]The discount factor (β) is 0.99, the Frisch elasticity $(\frac{1}{\varphi-1})$ is 0.5, and the Taylor coefficient on inflation (α_π) is 1.5.

the solution as

$$\widehat{y} = A_y\widehat{g}_t + B_y\mathbb{E}_t\widehat{\beta}_{t+1} + C_y\widehat{a}_t + D_y\mathbb{E}_t\widehat{a}_{t+1} = A_y\widehat{g}_t + B_y\mathbb{E}_t\widehat{\beta}_{t+1} + C_y\widehat{a}_t + \rho_a D_y\widehat{a}_t$$

$$\widehat{\pi} = A_\pi\widehat{g}_t + B_\pi\mathbb{E}_t\widehat{\beta}_{t+1} + C_\pi\widehat{a}_t + D_\pi\mathbb{E}_t\widehat{a}_{t+1} = A_\pi\widehat{g}_t + B_\pi\mathbb{E}_t\widehat{\beta}_{t+1} + C_\pi\widehat{a}_t + \rho_a D_\pi\widehat{a}_t.$$

Then we can rewrite the left-hand-side of the DNWR constraint (A.38)

$$\widehat{w}_t + \widehat{\pi}_t = (\varphi - 1)(A_y\widehat{g}_t + B_y\mathbb{E}_t\widehat{\beta}_{t+1} + (C_y + \rho_a D_y - 1)\widehat{a}_t) + A_\pi\widehat{g}_t + B_\pi\mathbb{E}_t\widehat{\beta}_{t+1} + (C_\pi + \rho_a D_\pi)\widehat{a}_t$$

In a demand-driven recession, where $\mathbb{E}_t\widehat{\beta}_{t+1} = \beta_H$ and $\widehat{a}_t = 0$,

$$\widehat{w}_t + \widehat{\pi}_t = ((\varphi - 1)A_y + A_\pi)\widehat{g}_t + ((\varphi - 1)B_y + B_\pi)\beta_H$$

Using the coefficients that we find from the proof of Proposition A.1, we can rewrite the above equation as

$$\widehat{w}_t + \widehat{\pi}_t = (\frac{(\varphi-1)\theta s_g(1+\triangle)}{1+\Psi\alpha_\pi\triangle(\varphi-1)})\widehat{g}_t + (-\frac{\Psi(1+\triangle)(\varphi-1)}{[1+\Psi\alpha_\pi\triangle(\varphi-1)]})\beta_H$$

DNWR binds with non-zero government spending if $\widehat{w}_t + \widehat{\pi}_t < 0$, that is,

$$\frac{(\varphi-1)\theta s_g(1+\triangle)}{1+\Psi\alpha_\pi\triangle(\varphi-1)}\widehat{g}_t < \frac{\Psi(1+\triangle)(\varphi-1)}{1+\Psi\alpha_\pi\triangle(\varphi-1)}\beta_H$$

$$\widehat{g}_t < \frac{\Psi}{\theta s_g}\beta_H \equiv c_d(\beta_H)$$

In a supply-driven recession, where $\widehat{a}_t = a_L$ and $\mathbb{E}_t\widehat{\beta}_{t+1} = 0$, the left-hand-side of the inequality constraint (A.38) is

$$\widehat{w}_t + \widehat{\pi}_t = [(\varphi - 1)A_y + A_\pi]\widehat{g}_t + [(\varphi - 1)(C_y - 1) + C_\pi]a_L.$$

DNWR binds with non-zero government spending if $\widehat{w}_t + \widehat{\pi}_t < 0$, or, equivalently,

$$\widehat{g}_t < \frac{[(\varphi-1)(C_y-1)+C_\pi]}{[(\varphi-1)A_y+A_\pi]}(-a_L) \equiv c_s(a_L) \tag{A.40}$$

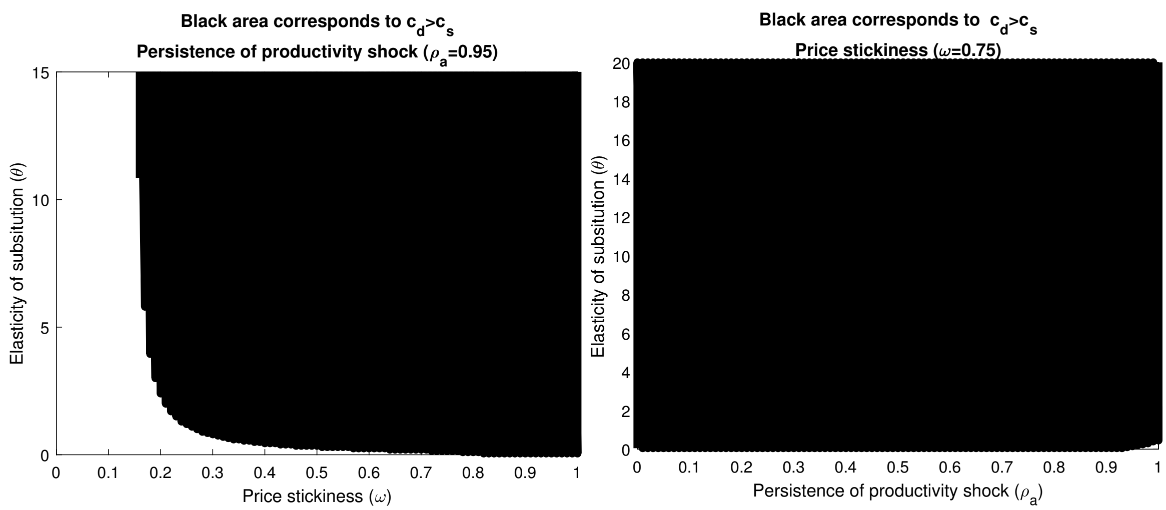
Given the negative productivity shock, the right hand side of Equation (A.40) is positive. Note that we show both A_y and A_π are positive in the proof of Proposition A.1 and $[(\varphi - 1)(C_y - 1) + C_\pi]$ is positive from the proof of Lemma A.1. □

Lemma A.3. Under the assumption that $|\beta_H| = |a_L|$, it can be shown that $0 < c_s(a_L) < c_d(\beta_H)$. In other words, the government spending required to ensure DNWR is no longer binding is smaller in a supply driven recession than a demand driven recession.

Proof. For given $|\beta_H| = |a_L|$, we want to show that

$$\frac{[(\varphi - 1)(C_y - 1) + C_\pi]}{[(\varphi - 1)A_y + A_\pi]} < \frac{\Psi}{\theta s_g} \tag{A.41}$$

Figure A.3: Parameter space corresponding to $c_s(a_L) < c_d(\beta_H)$



Notes: The left panel shows the parameter space (θ, ω) that satisfies $c_s(a_L) < c_d(\beta_H)$ given the persistence of productivity shock is 0.95. The right panel shows the combination of (θ, ρ_a) that ensures $c_s(a_L) < c_d(\beta_H)$.

Based on the baseline parameter values, the black area in the left panel of Figure A.3 shows the combination of the elasticity of substitution (θ) and the price stickiness (ω) that satisfies Equation (A.41), where the persistence of the productivity shock ρ_a is 0.95. The right panel of Figure A.3 shows the combination of θ and ρ_a that satisfies Equation (A.41), when the price stickiness parameter, ω, is 0.75. We find the Equation (A.41) holds for most cases. □

Proposition 1. Under the assumption that $|\beta_H| = |a_L|$, i.e. equal sized business cycle fluctuations,

the spending multiplier in a demand-driven recession $\geq$
the spending multiplier in a supply-driven recession $\geq$
the spending multiplier in an expansion,

for a given size of government spending shock.

Proof. In the absence of DNWR, the multipliers are the same regardless of the state of the economy or the source of fluctuation. In the presence of DNWR ($\gamma = 1$), if government spending (g) satisfies $g < c_s(a_L)$, the DNWR constraint still binds for both recessions (Lemma A.2), thus, the spending multiplier in a demand-driven recession (M^D_{DNWR}) is the same as the spending multiplier in a supply-driven recession (M^S_{DNWR}), which is greater than the spending multiplier in an expansion (M_y). If $c_s(a_L) < g < c_d(\beta_H)$, DNWR condition binds in a demand-driven recession but not in a supply-driven recession. In this case, the spending multiplier in a demand-driven recession is M^D_{DNWR} , which is higher than the spending multiplier in a supply-driven recession when DNWR is not a binding constraint, equal to the spending multiplier in an expansion, M_y. If $c_s(a_L) < c_d(\beta_H) < g$, government spending is large enough to raise nominal wages and achieve full employment, the spending multiplier would be M_y regardless of the source of fluctuation and the state of the business cycle. □

A.2 Appendix: Quantitative Model

A.2.1 Business Cycle fluctuations under supply and demand shocks in the quantitative model

We begin by considering the impulse responses to both contractionary and expansionary supply and demand shocks. The size of the shock is normalized to match the average output gap during the Great Recession. According to the Congressional Budget Office estimates,[3] the average output gap from 2008 to 2010 was 4%. We consider productivity and discount factor shocks to match this impact on output in a recession. This results in considering 1.7% deviations from the steady-state value of the discount factor and 2.9% deviations from the steady-state value of productivity. Both shock processes follow AR(1) process, following Equation (4) and Equation (5).[4]

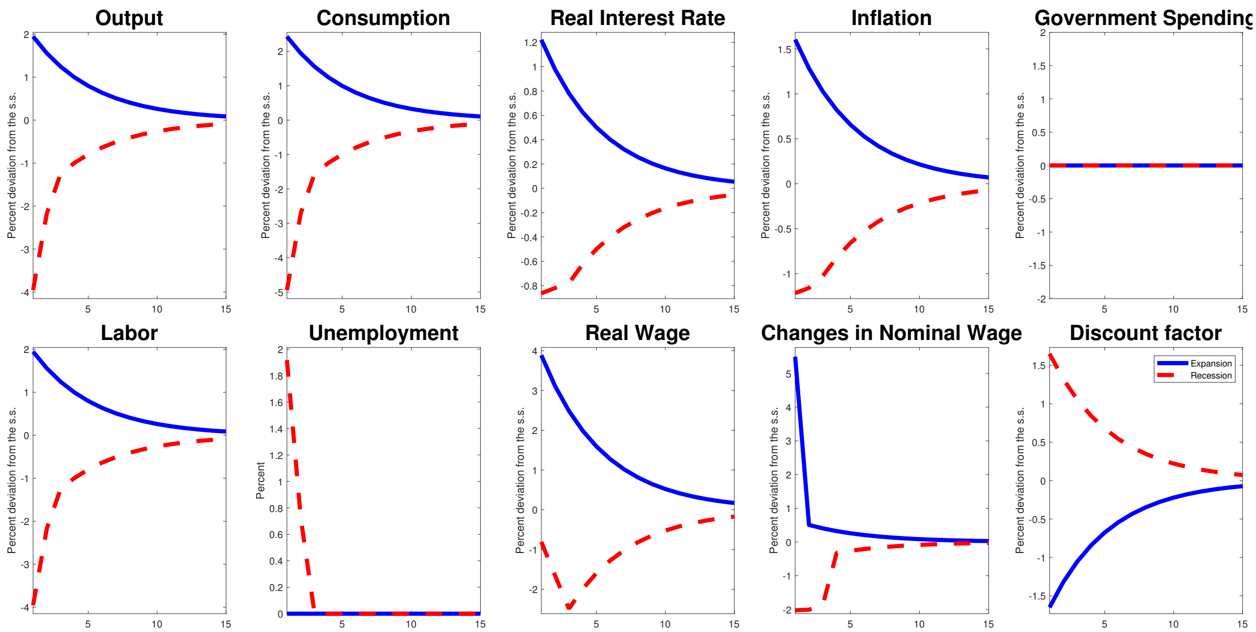
Figure A.4 displays impulse response in a demand-driven expansion and recession, without government spending. In response to a negative discount factor shock (shown with solid blue lines), consumers spend more in the current period leading to a demand-driven expansion. An increase in demand raises inflation and equilibrium labor. As there are no frictions in adjusting nominal wages upward, the labor market always clear, and the unemployment rate is zero.

In response to a positive discount factor shock (shown with dashed red lines in Figure A.4), consumers postpone current consumption, which causes a recession. As labor

[3]Source: https://fred.stlouisfed.org/graph/?g=f1cZ.

[4]We determine the size of the shock based on the average size of the output gap during the Great Recession, however, the slow recovery during the Great Recession was not matched in the following exercises.

Figure A.4: Demand-driven business cycle

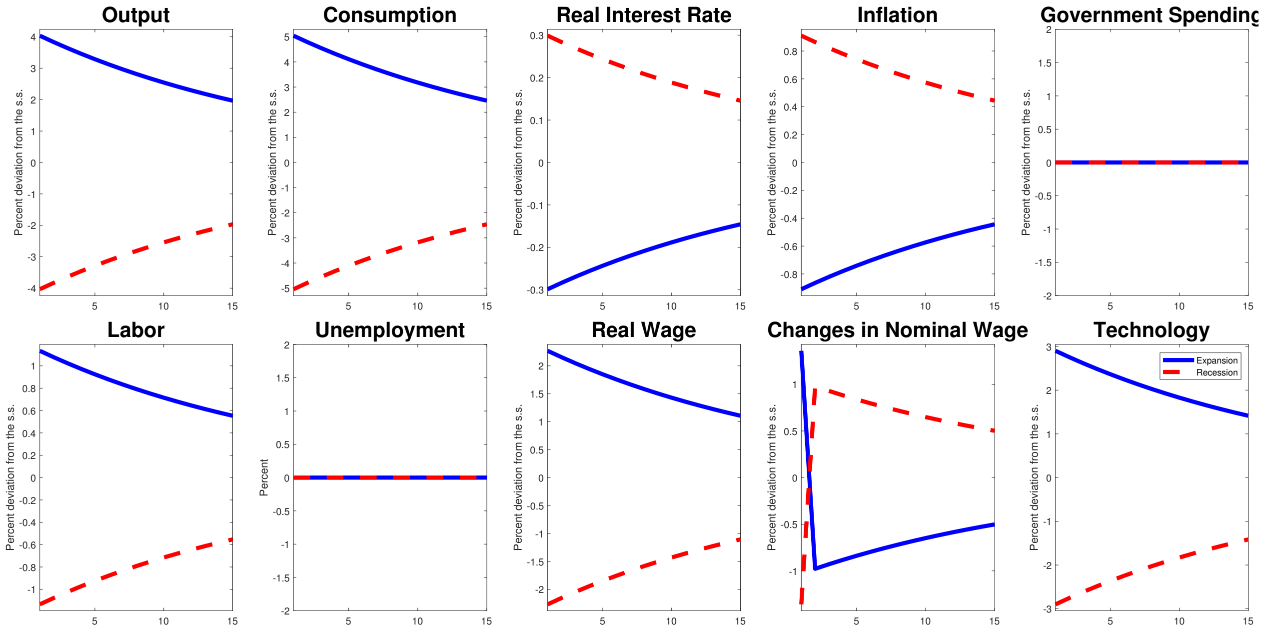


Notes: This graph shows impulse responses to a positive and a negative discount factor shock. The solid blue lines correspond to a negative discount factor shock (a demand-driven expansion), and the dashed red line represents impulse responses to a negative discount factor shock (a demand-driven recession). $\pm 1.7\%$ deviations of the discount factor shocks are imposed. All graph is drawn in terms of the percent deviations from its steady-state except the unemployment rate. The y-axis of the unemployment rate is percent.

demand decreases, there is downward pressure on wages. Although real wage goes up more than 4% in an expansion, the downward adjustment of real wage is about 1% at the beginning of the recession due to deflation and the binding of the DNWR constraint. The DNWR constraint allows at most 2% downward adjustment of real wage. At the same time, there is deflation that drives the real wages upward. The comovement of inflation and output, shown in Proposition A.1, exacerbates the labor market outcome and raises unemployment. Overall, the binding DNWR constraint generates an asymmetric business cycle.

Figure A.5 shows a supply-driven business cycle. As shown in Proposition A.1, inflation and output move in the opposite directions in a supply-driven recession. In a recession (dashed red lines), the marginal product of labor goes down, and firms hire less labor. Accordingly, nominal wage goes down about 1.5%. As we allow the downward adjustment of nominal wage up to 2%, the DNWR constraint does not bind. Consequently, the labor market clears, and the unemployment rate is zero. Unlike the demand-driven recession, the downward adjustment of real wage is greater than that of nominal wage in the supply-driven recession due to inflation. This is also highlighted in the analytical

Figure A.5: Supply-driven business cycle



Notes: This graph displays impulse responses to a positive and a negative productivity shock. The solid blue lines correspond to a positive productivity shock (a supply-driven expansion). The dashed red lines represent impulse responses to a negative productivity shock(a supply-driven recession). $\pm 2.9\%$ deviations of the technology shocks are imposed. All graph is drawn in terms of the percent deviations from its steady-state except the unemployment rate. The y-axis of the unemployment rate is percent.

section. The supply-driven business cycle is fully symmetric as DNWR does not bind.[5]

A.2.2 Robustness results

Our baseline model considers GHH (Greenwood, Hercowitz, and Huffman (1988)) preferences which do not allow a wealth effect on labor supply. We relax this assumption and allow for wealth effects on labor supply by introducing KPR (King, Plosser, and Rebelo (1988)) preferences commonly used in the literature. In particular, the preferences take the following form,

$$U(c_t, n_t) = \frac{[c_t(1 - \chi n_t^{\varphi})]^{1-\sigma}}{1-\sigma},$$

where we calibrate φ to ensure the same degree of Frisch elasticity of labor supply as in our baseline model.

As Table 5 shows, the multipliers under these preferences are smaller across the board

[5]When we consider King, Plosser, and Rebelo (1988) preferences in Section A.2.2, we find asymmetric business cycles in response to a supply shock as well. Once we allow for a wealth effect on labor supply in response to a technology shock, nominal wages fall more than in our baseline case and are bound below by DNWR in a supply-driven recession as well.

Table A.1: Cumulative output and consumption multipliers under the KPR preference

		Demand-driven business cycle				Supply-driven business cycle		
		Impact	4 quarters	20 quarters		Impact	4 quarters	20 quarters
				KPR preference				
Output Multiplier	Expansion	0.485	0.485	0.485	Expansion	0.485	0.485	0.485
	Recession	0.668	0.621	0.564	Recession	0.528	0.500	0.494
Consumption Multiplier	Expansion	-0.515	-0.515	-0.515	Expansion	-0.515	-0.515	-0.515
	Recession	-0.332	-0.379	-0.436	Recession	-0.472	-0.500	-0.506

Notes. This table reports the cumulative output and consumption multipliers in an expansion and a recession depending on the source of fluctuation. The cumulative output and consumption multipliers are calculated as $\Sigma_{i=0}^{i=k-1} \frac{\Delta y_{t+i}}{(1+r_{t+i})} \Big/ \Sigma_{i=0}^{i=k-1} \frac{\Delta g_{t+i}}{(1+r_{t+i})}$ and $\Sigma_{i=0}^{i=k-1} \frac{\Delta c_{t+i}}{(1+r_{t+i})} \Big/ \Sigma_{i=0}^{i=k-1} \frac{\Delta g_{t+i}}{(1+r_{t+i})}$, respectively, where Δ denotes the level differences of each variable with and without government spending and r_t is the real interest rate.

relative to GHH preferences.[6] An increase in government spending under KPR preferences leads to negative wealth effects on the labor supply, as agents internalize higher taxes now or in the future. Earlier studies, such as Monacelli and Perotti (2008), have shown that the degree of complementarity between hours and consumption is inversely related to this wealth effect on labor. Thus, this complementarity is largest with GHH preferences, and declines as we move towards KPR preferences, and leads to a larger response of output under GHH preferences. Under KPR preferences, the multiplier in a demand-driven recession is larger than in an expansion (0.67 in a recession and 0.49 in an expansion under KPR preferences), but the difference is much smaller in magnitude relative to under GHH preferences, (1.74 in a recession and 0.54 in an expansion under GHH preferences). This is because the labor supply curve shifts to the right, and overall weakens the effects of increased spending in reducing unemployment. Under these preferences, DNWR binds in a supply-driven recession as well, leading to a larger output multiplier in a recession relative to an expansion.[7] The multiplier in a supply-driven recession is thus smaller than the multiplier in a demand-driven recession (0.53 versus 0.67, respectively), although this difference across states is small. The intuition follows from Proposition 1 shown in Section 3. We also show robustness results respect to trend inflation (Appendix A.2.3), and alternative degree of price and wage rigidity (Appendix A.2.4).

[6]Under these preferences, we need to adjust the size of both the discount factor and productivity shock in order to generate the same size recession state.

[7]DNWR is more likely to bind in this case in response to a technology shock, since wages have a relatively larger response and labor has a smaller response with KPR preferences as the wealth effects from a technology shock shift the labor supply curve, an effect missing with GHH preferences.

A.2.3 Robustness to trend inflation

Table A.2: Cumulative output and consumption multipliers with nonzero steady-state inflation

		Demand-driven business cycle				Supply-driven business cycle		
		Impact	4 quarters	20 quarters		Impact	4 quarters	20 quarters
A. 2% steady-state annual inflation with GHH preference								
Output	Expansion	0.345	0.314	0.236	Expansion	0.345	0.314	0.236
Multiplier	Recession	1.067	0.653	0.439	Recession	0.345	0.314	0.236
Consumption	Expansion	-0.655	-0.686	-0.764	Expansion	-0.655	-0.686	-0.764
Multiplier	Recession	0.067	-0.347	-0.561	Recession	-0.655	-0.686	-0.764
A. 2% steady-state annual inflation with KPR preference								
Output	Expansion	0.477	0.467	0.446	Expansion	0.477	0.467	0.446
Multiplier	Recession	0.646	0.598	0.529	Recession	0.512	0.480	0.455
Consumption	Expansion	-0.523	-0.533	-0.554	Expansion	-0.523	-0.533	-0.554
Multiplier	Recession	-0.354	-0.402	-0.471	Recession	-0.488	-0.520	-0.545

Notes. This table reports the cumulative output and consumption multipliers in an expansion and a recession with 2% steady-state annual inflation under GHH and KPR preferences.. The cumulative output and consumption multipliers are calculated as $\Sigma_{i=0}^{i=k-1} \frac{\Delta y_{t+i}}{(1+r_{t+i})} \Big/ \Sigma_{i=0}^{i=k-1} \frac{\Delta g_{t+i}}{(1+r_{t+i})}$ and $\Sigma_{i=0}^{i=k-1} \frac{\Delta c_{t+i}}{(1+r_{t+i})} \Big/ \Sigma_{i=0}^{i=k-1} \frac{\Delta g_{t+i}}{(1+r_{t+i})}$, respectively, where Δ denotes the level differences of each variable with and without government spending and r_t is the real interest rate.

While demand and supply shocks generate deviations from steady-state inflation in opposite direction, we also consider the importance of the level of steady-state inflation. Table A.2 shows the cumulative output and consumption multipliers when we consider a 2% annual steady-state inflation. The main results hold qualitatively: notably that the output and consumption multipliers are higher in a demand-driven recession compared to an expansion for both GHH and KPR preferences. For KPR preferences, shown in the bottom panel, similar to the zero steady-state inflation case, the multiplier in a supply-driven recession, while larger than in an expansion, is smaller than in a demand-driven recession.

The positive steady-state inflation multipliers are overall smaller than the zero steady-state inflation multipliers. This is because non-zero steady-state inflation in a New Keynesian model leads to a rise in price dispersion and a loss in labor efficiency in response to exogenous shocks. With GHH preferences, the increase in inefficient price dispersion due to an increase in government spending limits the expansionary effects on output significantly. As a result, the government spending multipliers are much smaller with a 2% steady-state annual inflation rate. However, with KPR preference, labor supply also responds to government spending shocks which partially offsets the impact on aggregate demand due to a change in price dispersion. Consequently, there are smaller differences in the size of the multipliers across zero and non-zero steady-state inflation.

A.2.4 Robustness to alternative degree of nominal rigidity

We also consider an alternative degree of downward nominal wage rigidity and price rigidity. Once we assume a more downwardly flexible wage ($\gamma = 0.96$), it results in a lower multiplier in a demand-driven recession, as shown in Panel A of Table A.3. In contrast, a more rigid wage rigidity assumption ($\gamma = 0.99$) leads to higher multipliers in both recessions than the baseline case, reported in Panel B of Table A.3. The main results still hold that the multiplier in a demand-driven recession is higher than the multipliers in a supply-driven recession and expansion. These results confirm that the extent of binding DNWR is one of the key determinants of the size of multipliers.

Table A.3: Cumulative output and consumption multipliers by the source of fluctuation under alternative degree of wage and price rigidity

		Demand-driven business cycle				Supply-driven business cycle		
		Impact	4 quarters	20 quarters		Impact	4 quarters	20 quarters
A. Less rigid DNWR ($\gamma = 0.96$)								
Output	Expansion	0.535	0.535	0.535	Expansion	0.535	0.535	0.535
Multiplier	Recession	1.124	0.733	0.649	Recession	0.535	0.535	0.535
Conumption	Expansion	-0.465	-0.465	-0.465	Expansion	-0.465	-0.465	-0.465
Multiplier	Recession	0.124	-0.267	-0.351	Recession	-0.465	-0.465	-0.465
B. More rigid DNWR ($\gamma = 0.99$)								
Output	Expansion	0.535	0.535	0.535	Expansion	0.535	0.535	0.535
Multiplier	Recession	3.046	2.683	1.849	Recession	1.128	0.734	0.650
Conumption	Expansion	-0.465	-0.465	-0.465	Expansion	-0.465	-0.465	-0.465
Multiplier	Recession	2.046	1.683	0.849	Recession	0.128	-0.266	-0.350
C. Less rigid prices ($\omega = 0.65$)								
Output	Expansion	0.259	0.259	0.259	Expansion	0.259	0.259	0.259
Multiplier	Recession	1.868	1.067	0.727	Recession	0.259	0.259	0.259
Conumption	Expansion	-0.741	-0.741	-0.741	Expansion	-0.741	-0.741	-0.741
Multiplier	Recession	0.868	0.067	-0.273	Recession	-0.741	-0.741	-0.741
D. More rigid prices ($\omega = 0.85$)								
Output	Expansion	1.269	1.269	1.269	Expansion	1.269	1.269	1.269
Multiplier	Recession	2.397	1.968	1.674	Recession	1.269	1.269	1.269
Conumption	Expansion	0.269	0.269	0.269	Expansion	0.269	0.269	0.269
Multiplier	Recession	1.397	0.968	0.674	Recession	0.269	0.269	0.269

Notes. This table reports the cumulative output and consumption multipliers in an expansion and a recession depending on the source of fluctuation. The cumulative output and consumption multipliers are calculated as $\Sigma_{i=0}^{i=k-1} \frac{\Delta y_{t+i}}{(1+r_{t+i})} \Big/ \Sigma_{i=0}^{i=k-1} \frac{\Delta g_{t+i}}{(1+r_{t+i})}$ and $\Sigma_{i=0}^{i=k-1} \frac{\Delta c_{t+i}}{(1+r_{t+i})} \Big/ \Sigma_{i=0}^{i=k-1} \frac{\Delta g_{t+i}}{(1+r_{t+i})}$, respectively, where Δ denotes the level differences of each variable with and without government spending and r_t is the real interest rate.

When considering the dynamics of real wages in a recession, the degree of price rigidity also matters in determining the government spending multiplier. Panel C and D of Table A.3 reports the government spending multipliers with less and more rigid prices than the benchmark case, respectively. Overall, the government spending multipliers are larger in an economy with higher price rigidity, which is seen in a standard New Keynesian model also. With higher price rigidity, an increase in spending raises prices less and labor demand shifts out more due to increased public spending demand, leading to a larger increase in output. This increased price rigidity combined with GHH preferences in Panel C lead to an output multiplier larger than 1, even in an expansion. However, in our specific simulations, price rigidities also matter for the real wage dynamics with DNWR binding. Our results that the spending multipliers are higher in a demand-driven recession are robust for different degrees of price rigidity. In our baseline case, the multiplier in a demand recession is close to 70% larger than in a supply driven recession. With less rigid prices, the multiplier is over 85% larger, since a lower degree of price rigidity further amplifies the difference in real wage response when the DNWR is binding or not. The difference in the multipliers across the two types of recessions shrinks when we have more rigid prices, where the demand recession multiplier is about 45% larger than a supply recession/ expansion multiplier.

A.3 Appendix: Time series empirical evidence

Our data set constitutes of quarterly data for the U.S. spanning 1889Q1-2015Q4. We define inflation as year-over-year growth of the GDP deflator, and use data for GDP, unemployment rate, government spending and GDP deflator from Ramey and Zubairy (2018). Our baseline measure of narrative military news variables also comes from Ramey and Zubairy (2018). When we use taxes as a control variable, we use tax revenues as a share of GDP as a control. This series also comes from Ramey and Zubairy (2018).

When we consider real interest rate as a control variable, we construct the real rate as3 month T-bill rate minus YoY GDP deflator inflation. Since the interpolated series for the T-bill rate is available only from 1915q1 onward, the regressions with real interest rates as controls are run on a shorter sample than our baseline results.

Table A.4: State-dependent fiscal multipliers for output: military news shocks (Robustness to potential GDP)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	2-year integral output				4-year integral output			
Σg_t	0.67				0.71			
	(0.05)				(0.03)			
$\Sigma g_t \times \mathbb{I}(L(u_{t-1}))$		0.64		0.64		0.68		0.68
		(0.08)		(0.08)		(0.10)		(0.10)
$\Sigma g_t \times \mathbb{I}(H(u_{t-1}))$		0.61				0.69		
		(0.09)				(0.05)		
$\Sigma g_t \times \mathbb{I}(L(\pi_{t-1}))$			0.66				0.63	
			(0.09)				(0.07)	
$\Sigma g_t \times \mathbb{I}(H(\pi_{t-1}))$			0.65				0.75	
			(0.05)				(0.04)	
$\Sigma g_t \times \mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1}))$				0.96				0.78
				(0.18)				(0.05)
$\Sigma g_t \times \mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				-0.13				0.28
				(0.11)				(0.06)
P-value from the test								
$\mathbb{I}(L(u_{t-1})) = \mathbb{I}(H(u_{t-1}))$		0.83				0.95		
$\mathbb{I}(L(\pi_{t-1})) = \mathbb{I}(H(\pi_{t-1}))$			0.88				0.14	
$\mathbb{I}(L(u_{t-1})) = \mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1}))$				0.13				0.47
$\mathbb{I}(L(u_{t-1})) = \mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				0.00				0.00
$\mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1})) = \mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				0.00				0.00
Effective first-stage F statistics								
Linear	20.59				11.55			
$\mathbb{I}(L(u_{t-1}))$		10.23		8.82		15.01		14.58
$\mathbb{I}(H(u_{t-1}))$		382.96				122.45		
$\mathbb{I}(L(\pi_{t-1}))$			10.62				5.24	
$\mathbb{I}(H(\pi_{t-1}))$			72.43				46.82	
$\mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1}))$				74.53				305.02
$\mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				52.23				75.99
Observations	493	493	493	493	485	485	485	485

Notes: The top panel reports the 2 and 4 year cumulative multiplier along with associated heteroskedasticity and autocorrelation robust standard errors below. $\mathbb{I}(L(u_{t-1}))$ and $\mathbb{I}(H(u_{t-1}))$ indicate the state where lagged unemployment is low and high, respectively. Potential GDP is constructed using the CBO measure of potential GDP with an interpolated measure of potential GDP for early years. The second panel shows p-values testing whether the multipliers are statistically significantly different across states. The last panel reports the Olea and Pflueger (2013) effective first-stage F statistics for military news as an instrument at 2 and 4 year horizons for the relevant subsample. We use the threshold for the 5 percent critical value for testing the null hypothesis that the TSLS bias exceeds 10 percent of the worst-case TSLS bias. For one instrument, this threshold is always 23.1.

Table A.5: State-dependent fiscal multipliers: military news shocks (Robustness to inflation thresholds: 3%)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	2-year integral output				4-year integral output			
Σg_t	0.66 (0.07)				0.71 (0.04)			
$\Sigma g_t \times \mathbb{I}(L(u_{t-1}))$		0.59 (0.09)		0.59 (0.09)		0.67 (0.12)		0.67 (0.12)
$\Sigma g_t \times \mathbb{I}(H(u_{t-1}))$		0.60 (0.09)				0.68 (0.05)		
$\Sigma g_t \times \mathbb{I}(L(\pi_{t-1}))$			0.83 (0.12)				0.73 (0.08)	
$\Sigma g_t \times \mathbb{I}(H(\pi_{t-1}))$			0.61 (0.05)				0.71 (0.04)	
$\Sigma g_t \times \mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1}))$				1.07 (0.22)				0.76 (0.06)
$\Sigma g_t \times \mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				0.42 (0.13)				0.59 (0.07)
P-value from the test								
$\mathbb{I}(L(u_{t-1})) = \mathbb{I}(H(u_{t-1}))$		0.95				0.92		
$\mathbb{I}(L(\pi_{t-1})) = \mathbb{I}(H(\pi_{t-1}))$			0.10				0.84	
$\mathbb{I}(L(u_{t-1})) = \mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1}))$				0.07				0.55
$\mathbb{I}(L(u_{t-1})) = \mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				0.25				0.56
$\mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1})) = \mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				0.01				0.06
Effective first-stage F statistics								
Linear	19.38				11.22			
$\mathbb{I}(L(u_{t-1}))$		8.44		8.23		10.85		10.55
$\mathbb{I}(H(u_{t-1}))$		403.28				130.20		
$\mathbb{I}(L(\pi_{t-1}))$			4.90				4.26	
$\mathbb{I}(H(\pi_{t-1}))$			109.31				38.65	
$\mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1}))$				350.03				228.48
$\mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				279.98				330.42
Observations	493	493	493	493	485	485	485	485

Notes: The top panel reports the 2 and 4 year cumulative multiplier along with associated heteroskedasticity and autocorrelation robust standard errors below. $\mathbb{I}(L(\pi_{t-1}))$ and $\mathbb{I}(H(\pi_{t-1}))$ represent the state where lagged inflation is lower and higher than 3%, respectively. The second panel shows p-values testing whether the multipliers are statistically significantly different across states. The last panel reports the Olea and Pflueger (2013) effective first-stage F statistics for military news as an instrument at 2 and 4 year horizons for the relevant subsample. We use the threshold for the 5 percent critical value for testing the null hypothesis that the TSLS bias exceeds 10 percent of the worst-case TSLS bias. For one instrument, this threshold is always 23.1.

Table A.6: State-dependent fiscal multipliers for output: military news shocks (Robustness to time-varying thresholds)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	2-year integral output				4-year integral output			
Σg_t	0.66 (0.07)				0.71 (0.04)			
$\Sigma g_t \times \mathbb{I}(L(u_{t-1}))$		0.66 (0.18)		0.66 (0.18)		0.75 (0.26)		0.75 (0.26)
$\Sigma g_t \times \mathbb{I}(H(u_{t-1}))$		0.52 (0.08)				0.56 (0.08)		
$\Sigma g_t \times \mathbb{I}(L(\pi_{t-1}))$			0.78 (0.11)				0.69 (0.08)	
$\Sigma g_t \times \mathbb{I}(H(\pi_{t-1}))$			0.58 (0.04)				0.73 (0.04)	
$\Sigma g_t \times \mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1}))$				0.78 (0.26)				0.58 (0.15)
$\Sigma g_t \times \mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				0.26 (0.07)				0.52 (0.15)
P-value from the test								
$\mathbb{I}(L(u_{t-1})) = \mathbb{I}(H(u_{t-1}))$		0.45				0.51		
$\mathbb{I}(L(\pi_{t-1})) = \mathbb{I}(H(\pi_{t-1}))$			0.10				0.67	
$\mathbb{I}(L(u_{t-1})) = \mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1}))$				0.71				0.57
$\mathbb{I}(L(u_{t-1})) = \mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				0.01				0.45
$\mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1})) = \mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				0.04				0.80
Effective first-stage F statistics								
Linear	19.38				11.22			
$\mathbb{I}(L(u_{t-1}))$		15.36		14.93		5.06		4.92
$\mathbb{I}(H(u_{t-1}))$		3.80				2.75		
$\mathbb{I}(L(\pi_{t-1}))$			6.17				4.40	
$\mathbb{I}(H(\pi_{t-1}))$			131.60				38.59	
$\mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1}))$				22.81				9.82
$\mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				19.25				17.83
Observations	493	493	493	493	485	485	485	485

Notes: The top panel reports the 2 and 4 year cumulative multiplier along with associated heteroskedasticity and autocorrelation robust standard errors below. $\mathbb{I}(L(u_{t-1}))$ and $\mathbb{I}(H(u_{t-1}))$ indicate the state where lagged unemployment is low and high, respectively. In this robustness check, we consider time-varying thresholds for both the unemployment rate and inflation. The time-varying trend is based on the HP filter with $\lambda = 10^6$, over a split sample, 1889–1929 and 1947–2015 and linearly interpolated for the small gap in trend unemployment between 1929 and 1947, in order to capture the evolution of the natural rate. The high inflation regime is one where inflation is above a HP filtered trend based on $\lambda = 1600$. The top panel panel reports the 2 and 4 year cumulative multiplier along with associated standard errors below. The second panel shows p-values testing whether the multipliers are statistically significantly different across states. The last panel reports the Olea and Pflueger (2013) effective first-stage F statistics for military news as an instrument at 2 and 4 year horizons for the relevant subsample. We use the threshold for the 5 percent critical value for testing the null hypothesis that the TSLS bias exceeds 10 percent of the worst-case TSLS bias. For one instrument, this threshold is always 23.1.

Table A.7: State-dependent fiscal multipliers for output: military news shocks (Robustness to time varying thresholds for inflation)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	2-year integral output				4-year integral output			
Σg_t	0.66				0.71			
	(0.07)				(0.04)			
$\Sigma g_t \times \mathbb{I}(L(u_{t-1}))$		0.59		0.59		0.67		0.67
		(0.09)		(0.09)		(0.12)		(0.12)
$\Sigma g_t \times \mathbb{I}(H(u_{t-1}))$		0.60				0.68		
		(0.09)				(0.05)		
$\Sigma g_t \times \mathbb{I}(L(\pi_{t-1}))$			0.78				0.69	
			(0.11)				(0.08)	
$\Sigma g_t \times \mathbb{I}(H(\pi_{t-1}))$			0.58				0.73	
			(0.04)				(0.04)	
$\Sigma g_t \times \mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1}))$				1.26				0.82
				(0.27)				(0.08)
$\Sigma g_t \times \mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				0.23				0.58
				(0.07)				(0.06)
P-value from the test								
$\mathbb{I}(L(u_{t-1})) = \mathbb{I}(H(u_{t-1}))$		0.95				0.92		
$\mathbb{I}(L(\pi_{t-1})) = \mathbb{I}(H(\pi_{t-1}))$			0.10				0.67	
$\mathbb{I}(L(u_{t-1})) = \mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1}))$				0.03				0.34
$\mathbb{I}(L(u_{t-1})) = \mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				0.00				0.56
$\mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1})) = \mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				0.00				0.01
Effective first-stage F statistics								
Linear	19.38				11.22			
$\mathbb{I}(L(u_{t-1}))$		8.44		8.23		10.85		10.56
$\mathbb{I}(H(u_{t-1}))$		403.28				130.20		
$\mathbb{I}(L(\pi_{t-1}))$			6.17				4.40	
$\mathbb{I}(H(\pi_{t-1}))$			131.60				38.59	
$\mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1}))$				139.80				90.16
$\mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				619.63				722.32
Observations	493	493	493	493	485	485	485	485

Notes: The top panel reports the 2 and 4 year cumulative multiplier along with associated heteroskedasticity and autocorrelation robust standard errors below. $\mathbb{I}(L(u_{t-1}))$ and $\mathbb{I}(H(u_{t-1}))$ indicate the state where lagged unemployment is low and high, respectively. In this robustness check, we consider time-variant thresholds for only inflation. The high inflation regime is one where inflation is above a HP filtered trend based on $\lambda = 1600$. The second panel shows p-values testing whether the multipliers are statistically significantly different across states. The last panel reports the Olea and Pflueger (2013) effective first-stage F statistics for military news as an instrument at 2 and 4 year horizons for the relevant subsample. We use the threshold for the 5 percent critical value for testing the null hypothesis that the TSLS bias exceeds 10 percent of the worst-case TSLS bias. For one instrument, this threshold is always 23.1.

Table A.8: State-dependent fiscal multipliers for output: both military news and Blanchard-Perotti (2002) as instruments (Robustness to time varying thresholds for inflation)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	2-year integral output				4-year integral output			
Σg_t	0.42				0.56			
	(0.10)				(0.08)			
$\Sigma g_t \times \mathbb{I}(L(u_{t-1}))$		0.33				0.39		
		(0.11)				(0.11)		
$\Sigma g_t \times \mathbb{I}(H(u_{t-1}))$		0.62				0.68		
		(0.09)				(0.05)		
$\Sigma g_t \times \mathbb{I}(L(\pi_{t-1}))$			0.47				0.49	
			(0.09)				(0.10)	
$\Sigma g_t \times \mathbb{I}(H(\pi_{t-1}))$			0.48				0.67	
			(0.07)				(0.07)	
$\Sigma g_t \times \mathbb{I}(L(u_{t-1}))\mathbb{I}(L(\pi_{t-1}))$				0.43				0.30
				(0.15)				(0.20)
$\Sigma g_t \times \mathbb{I}(L(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				0.42				0.55
				(0.07)				(0.12)
$\Sigma g_t \times \mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1}))$				0.92				0.81
				(0.23)				(0.07)
$\Sigma g_t \times \mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				0.56				0.80
				(0.20)				(0.13)
P-value from the test								
$\mathbb{I}(L(u_{t-1})) = \mathbb{I}(H(u_{t-1}))$		0.10				0.02		
$\mathbb{I}(L(\pi_{t-1})) = \mathbb{I}(H(\pi_{t-1}))$			0.94				0.09	
$\mathbb{I}(L(u_{t-1}))\mathbb{I}(L(\pi_{t-1})) = \mathbb{I}(L(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				0.93				0.21
$\mathbb{I}(L(u_{t-1}))\mathbb{I}(L(\pi_{t-1})) = \mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1}))$				0.60				0.06
$\mathbb{I}(L(u_{t-1}))\mathbb{I}(H(\pi_{t-1})) = \mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1}))$				0.05				0.06
$\mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1})) = \mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				0.22				0.96
Effective first-stage F statistics								
Linear	36.70				14.44			
	13.19				15.46			
$\mathbb{I}(L(u_{t-1}))$		48.92				39.63		
		[17.04]				[17.05]		
$\mathbb{I}(H(u_{t-1}))$		61.56				69.75		
		[19.27]				[10.83]		
$\mathbb{I}(L(\pi_{t-1}))$			36.59				18.75	
			[20.81]				[15.56]	
$\mathbb{I}(H(\pi_{t-1}))$			80.18				24.85	
			[15.24]				[18.13]	
$\mathbb{I}(L(u_{t-1}))\mathbb{I}(L(\pi_{t-1}))$				51.67				20.15
				[17.49]				[15.63]
$\mathbb{I}(L(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				54.60				23.38
				[7.57]				[10.05]
$\mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1}))$				48.93				33.64
				[16.48]				[17.50]
$\mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				53.41				46.18
				[20.99]				[21.11]
Observations	493	493	493	493	485	485	485	485

Notes: The top panel reports the 2 and 4 year cumulative multiplier along with associated heteroskedasticity and autocorrelation robust standard errors below. $\mathbb{I}(L(u_{t-1}))$ and $\mathbb{I}(H(u_{t-1}))$ indicate the state where lagged unemployment is low and high, respectively. In this robustness check, we consider time-variant thresholds for only inflation. The high inflation regime is one where inflation is above a HP filtered trend based on $\lambda = 1600$. The second panel shows p-values testing whether the multipliers are statistically significantly different across states. The last panel reports the Olea and Pflueger (2013) effective first-stage F statistics for military news and Blanchard and Perotti (2002) shocks jointly as instruments at 2 and 4 year horizons for the relevant subsample.The numbers in brackets provide the 5 percent critical value used to

Table A.9: State-dependent fiscal multipliers for output: military news shocks (Controlling for taxes)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	2-year integral output				4-year integral output			
Σg_t	0.66				0.72			
	(0.07)				(0.05)			
$\Sigma g_t \times \mathbb{I}(L(u_{t-1}))$		0.54		0.54		0.60		0.60
		(0.11)		(0.11)		(0.15)		(0.15)
$\Sigma g_t \times \mathbb{I}(H(u_{t-1}))$		0.67				0.69		
		(0.12)				(0.08)		
$\Sigma g_t \times \mathbb{I}(L(\pi_{t-1}))$			0.79				0.68	
			(0.14)				(0.10)	
$\Sigma g_t \times \mathbb{I}(H(\pi_{t-1}))$			0.54				0.61	
			(0.11)				(0.10)	
$\Sigma g_t \times \mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1}))$				0.99				0.80
				(0.20)				(0.06)
$\Sigma g_t \times \mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				-0.12				0.24
				(0.12)				(0.08)
P-value from the test								
$\mathbb{I}(L(u_{t-1})) = \mathbb{I}(H(u_{t-1}))$		0.47				0.64		
$\mathbb{I}(L(\pi_{t-1})) = \mathbb{I}(H(\pi_{t-1}))$			0.18				0.56	
$\mathbb{I}(L(u_{t-1})) = \mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1}))$				0.07				0.27
$\mathbb{I}(L(u_{t-1})) = \mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				0.00				0.03
$\mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1})) = \mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				0.00				0.00
Effective first-stage F statistics								
Linear	18.73				11.56			
$\mathbb{I}(L(u_{t-1}))$		8.41		8.12		15.98		15.51
$\mathbb{I}(H(u_{t-1}))$		118.25				72.91		
$\mathbb{I}(L(\pi_{t-1}))$			8.07				4.59	
$\mathbb{I}(H(\pi_{t-1}))$			58.30				100.57	
$\mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1}))$				72.26				222.68
$\mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				42.84				75.31
Observations	493	493	493	493	485	485	485	485

Notes: The top panel reports the 2 and 4 year cumulative multiplier along with associated heteroskedasticity and autocorrelation robust standard errors below. $\mathbb{I}(L(u_{t-1}))$ and $\mathbb{I}(H(u_{t-1}))$ indicate the state where lagged unemployment is low and high, respectively. These are defined as our baseline specification, shown in Table 2. We include average tax rates as additional controls on the right hand side.

Table A.10: State-dependent fiscal multipliers for output: military news shocks (1915-2015)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	2-year integral output				4-year integral output			
Σg_t	0.70				0.74			
	(0.08)				(0.05)			
$\Sigma g_t \times \mathbb{I}(L(u_{t-1}))$		0.69		0.69		0.68		0.68
		(0.08)		(0.08)		(0.14)		(0.14)
$\Sigma g_t \times \mathbb{I}(H(u_{t-1}))$		0.55				0.68		
		(0.12)				(0.07)		
$\Sigma g_t \times \mathbb{I}(L(\pi_{t-1}))$			0.74				0.70	
			(0.15)				(0.12)	
$\Sigma g_t \times \mathbb{I}(H(\pi_{t-1}))$			0.61				0.69	
			(0.07)				(0.05)	
$\Sigma g_t \times \mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1}))$				0.63				0.73
				(0.30)				(0.12)
$\Sigma g_t \times \mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				-0.62				0.03
				(0.46)				(0.30)
P-value from the test								
$\mathbb{I}(L(u_{t-1})) = \mathbb{I}(H(u_{t-1}))$		0.31				0.98		
$\mathbb{I}(L(\pi_{t-1})) = \mathbb{I}(H(\pi_{t-1}))$			0.46				0.94	
$\mathbb{I}(L(u_{t-1})) = \mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1}))$				0.84				0.80
$\mathbb{I}(L(u_{t-1})) = \mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				0.00				0.04
$\mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1})) = \mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				0.02				0.03
Effective first-stage F statistics								
Linear	12.68				9.94			
$\mathbb{I}(L(u_{t-1}))$		6.40		6.17		7.92		7.54
$\mathbb{I}(H(u_{t-1}))$		242.24				76.60		
$\mathbb{I}(L(\pi_{t-1}))$			5.53				4.10	
$\mathbb{I}(H(\pi_{t-1}))$			37.96				45.29	
$\mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1}))$				61.74				109.23
$\mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				31.37				41.40
Observations	393	393	393	393	385	385	385	385

Notes: The top panel panel reports the 2 and 4 year cumulative multiplier along with associated heteroskedasticity and autocorrelation robust standard errors below. $\mathbb{I}(L(u_{t-1}))$ and $\mathbb{I}(H(u_{t-1}))$ indicate the state where lagged unemployment is low and high, respectively. These are defined as our baseline specification, shown in Table 2. The sample under consideration is shorter than our baseline to make comparison with the specification with real interest rates as additional control easier, and spans 1915-2015.

Table A.11: State-dependent fiscal multipliers for output: military news shocks (1915-2015, Controlling for real interest rates)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	2-year integral output				4-year integral output			
Σg_t	0.70				0.75			
	(0.08)				(0.07)			
$\Sigma g_t \times \mathbb{I}(L(u_{t-1}))$		0.69		0.69		0.77		0.77
		(0.14)		(0.12)		(0.14)		(0.14)
$\Sigma g_t \times \mathbb{I}(H(u_{t-1}))$		0.51				0.64		
		(0.13)				(0.09)		
$\Sigma g_t \times \mathbb{I}(L(\pi_{t-1}))$			0.73				0.68	
			(0.16)				(0.13)	
$\Sigma g_t \times \mathbb{I}(H(\pi_{t-1}))$			0.65				0.74	
			(0.05)				(0.03)	
$\Sigma g_t \times \mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1}))$				0.54				0.67
				(0.28)				(0.13)
$\Sigma g_t \times \mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				-0.28				0.11
				(0.37)				(0.22)
P-value from the test								
$\mathbb{I}(L(u_{t-1})) = \mathbb{I}(H(u_{t-1}))$		0.34				0.36		
$\mathbb{I}(L(\pi_{t-1})) = \mathbb{I}(H(\pi_{t-1}))$			0.66				0.65	
$\mathbb{I}(L(u_{t-1})) = \mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1}))$				0.61				0.59
$\mathbb{I}(L(u_{t-1})) = \mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				0.01				0.01
$\mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1})) = \mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				0.08				0.03
Effective first-stage F statistics								
Linear	14.53				10.23			
$\mathbb{I}(L(u_{t-1}))$		4.29		5.59		7.72		7.46
$\mathbb{I}(H(u_{t-1}))$		283.90				14.13		
$\mathbb{I}(L(\pi_{t-1}))$			4.83				4.20	
$\mathbb{I}(H(\pi_{t-1}))$			67.13				42.64	
$\mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1}))$				57.33				142.65
$\mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				34.98				41.72
Observations	392	392	392	392	384	384	384	384

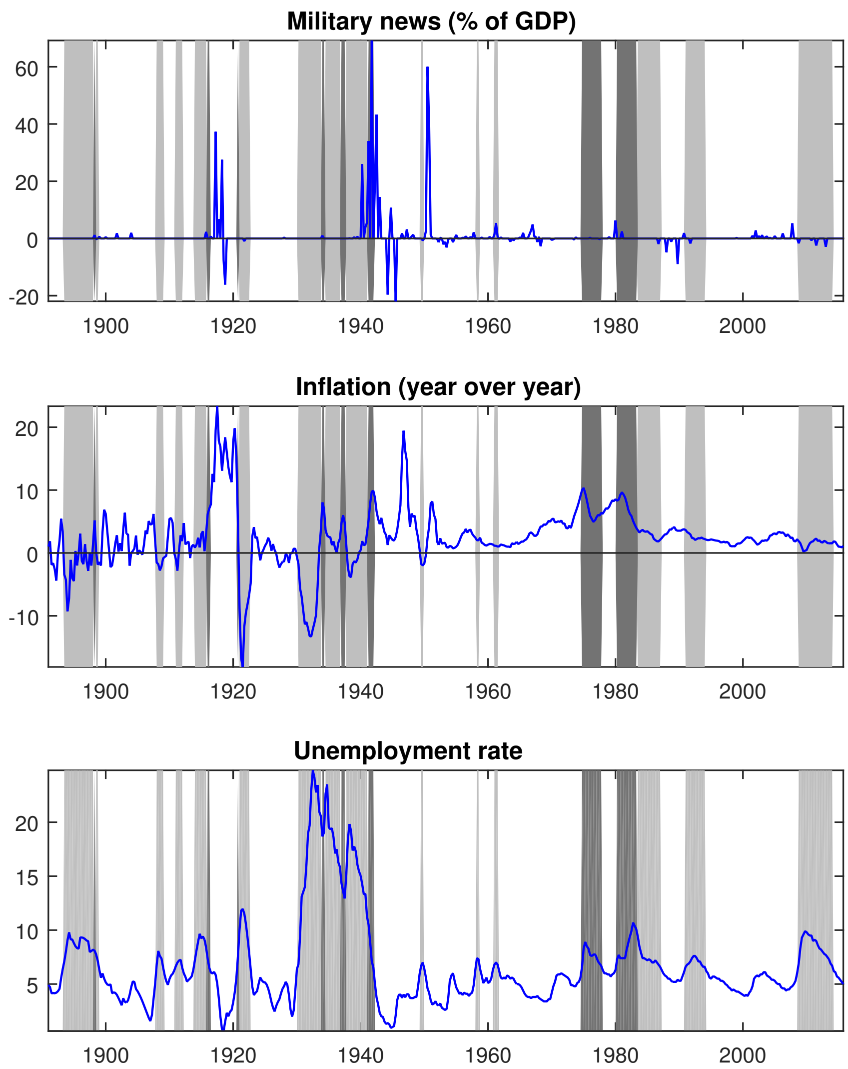
Notes: The top panel reports the 2 and 4 year cumulative multiplier along with associated heteroskedasticity and autocorrelation robust standard errors below. $\mathbb{I}(L(u_{t-1}))$ and $\mathbb{I}(H(u_{t-1}))$ indicate the state where lagged unemployment is low and high, respectively. These are defined as our baseline specification, shown in Table 2. The sample under consideration is shorter than our baseline and spans 1915-2015. We include real interest rates as additional controls on the right hand side.

Table A.12: Response of unemployment, inflation and interest rate

	2 year horizon response			
	Unemployment	Inflation	Tbill rate	Real rate
$\Sigma g_t \times \mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1}))$	-0.1971	0.4699	-0.7946	-0.5609
	(0.1465)	(0.1176)	(3.6384)	(0.1791)
$\Sigma g_t \times \mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$	0.0784	0.1599	8.5678	-0.000
	(0.0833)	(0.2370)	(19.788)	(0.3399)
P-value from test $\mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1})) = \mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$	0.12	0.23	0.70	0.15
Observation	493	493	389	389
	4 year horizon response			
	Unemployment	Inflation	Tbill rate	Real rate
$\Sigma g_t \times \mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1}))$	-0.1880	0.1553	0.6424	-0.1716
	(0.0458)	(0.0455)	(1.7783)	(0.0458)
$\Sigma g_t \times \mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$	0.0133	0.0963	-7.5802	0.2179
	(0.0530)	(0.1479)	(10.947)	(0.1634)
P-value from test $\mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1})) = \mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$	0.01	0.69	0.46	0.02
Observation	485	485	381	381

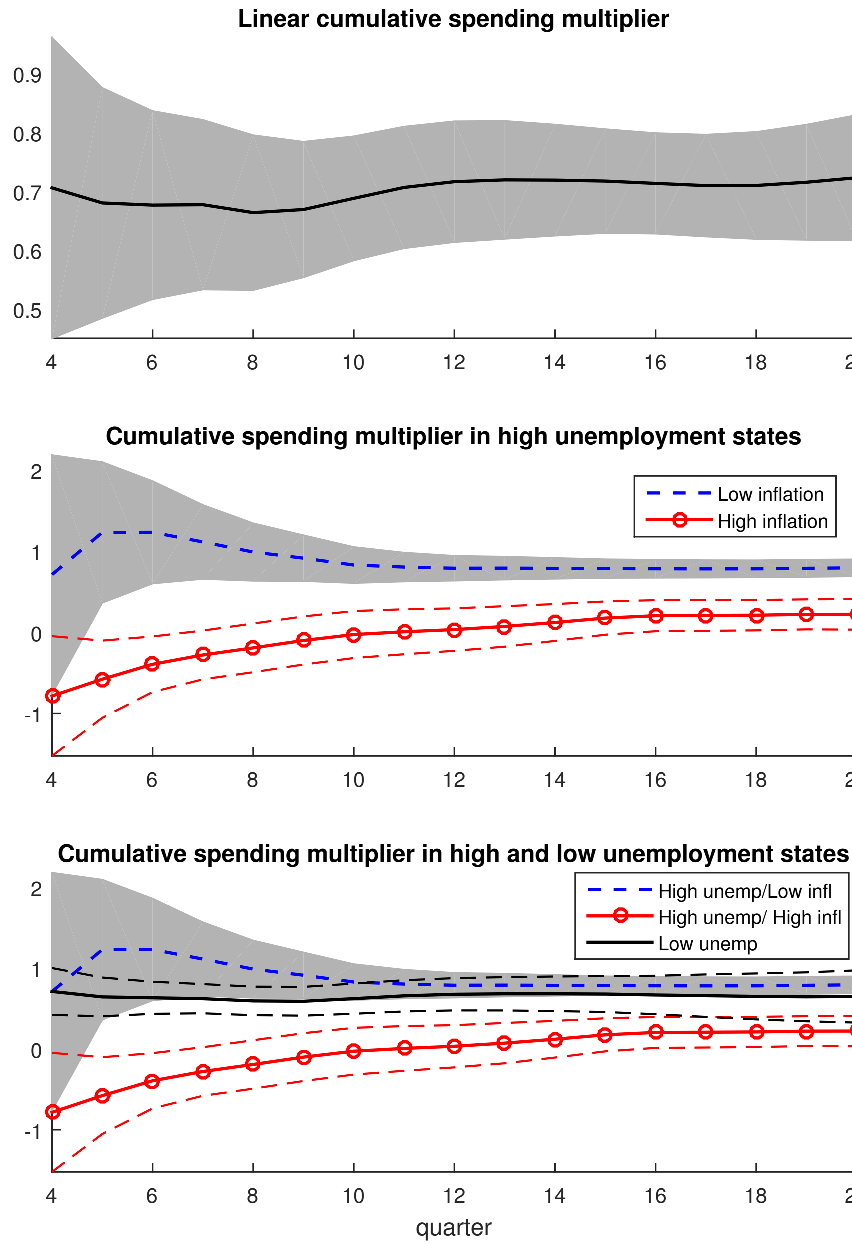
Notes: The top panel reports the 2 and 4 year cumulative responses of unemployment, inflation, Tbill rate, and real interest rate along with associated heteroskedasticity and autocorrelation robust standard errors below. $\mathbb{I}(L(u_{t-1}))$ and $\mathbb{I}(H(u_{t-1}))$ indicate the state where lagged unemployment is low and high, respectively. These are defined as our baseline specification, shown in Table 2.

Figure A.6: Inflation and unemployment states for U.S. historical data



Notes: Military spending news, year over year GDP deflator inflation rate and the unemployment rate. The shaded areas indicate periods when the unemployment rate is above the threshold of 6.5 percent. The light and dark gray areas correspond with periods where inflation is below and above a threshold of 4%, which corresponds to the 75th percentiles of inflation for our full sample.

Figure A.7: State dependent fiscal multipliers: military news shocks



Notes: These figures show the cumulative multiplier for output in response to a military news shock from 4 quarters after the shock hits the economy. The top panel shows the cumulative multiplier in a linear model. The middle panel shows the state-dependent multiplier in high unemployment/ low inflation (blue dashed) and high unemployment/ high inflation (red circles) states. The bottom panel shows the state dependent multipliers in low unemployment (black solid), high unemployment/ low inflation (blue dashed) and high unemployment/ high inflation (red circles) states. 95 percent confidence intervals are shown in all cases.

A.4 Appendix: US state-level empirical evidence

The state-level annual data sample starts in 1969 and ends in 2018. State-level nominal GDP is from the US Bureau of Economic Analysis (BEA). In calculating real GDP, we use the US aggregate Consumer Price Index (CPI) to deflate nominal GDP followed by BEA - calculating state-level GDP by applying national price deflator to state-level nominal GDP. State-level employment is from Current Employment Statistics (CES) by the Bureau of Labor Statistics (BLS) and the state-level population is available from the US Census Bureau. We use state-level inflation data constructed by Nakamura and Steinsson (2014) from 1969 to 2008 and later by Zidar (2019) up to 2014. We further extend the state-level inflation from Regional Price Parity (RPP) from Census until 2018.[8]

For state-level military spending, we use data from prime military contracts awarded by the Department of Defense (DOD). Each individual contractor of DOD reports their contract details using DD Form 350, including the service or product supplies, date awarded, principal place of performance, and information about the DOD agency. For each fiscal year between 1966 and 2000, we rely on state-level military prime contract data constructed by Nakamura and Steinsson (2014) For the remaining sample period from 2001 until 2018, we use electronic DD Form 350 data available from www.USAspending.gov.

In order to identify the state-dependent spending multipliers, we add state-level changes in military spending interacted with indicator variables ($\mathbb{I}(\cdot)$), which provide information on US-states-years corresponding to the state of the economy. Note that the estimated multipliers with regional data are not directly comparable to the closed economy aggregate multipliers from the time series evidence in Section 5. The estimates from the regional analysis, open economy relative multipliers, measure the effect of an increase in government spending in one state relative to another state. However, these regional multipliers are useful in testing whether the effectiveness of fiscal policy depends on the US-state-differential conditions of the economy.

[8]Before 1995, Nakamura and Steinsson (2014) use state-level price indices constructed by Del Negro (1998) from 1969. After 1995, both papers by Nakamura and Steinsson (2014) and Zidar (2019) use county and metro level Cost of Living Index (COLI) published by the American Chamber of Commerce Researchers Association (ACCRA), later renamed as Council for Community and Economic Research (C2ER). As regional level COLI is designed to capture differences in price levels across regions within a year, Nakamura and Steinsson (2014) computed the state-level price indices by multiplying population-weighted COLI from the ACCRA for each state with the US aggregate CPI. We applied for the same procedure to calculate the state-level price indices using the state-level COLI provided by Zidar (2019) and RPP from Census. There are a few missing US state-level inflation observations from Hawaii, Maine, New Hampshire, New Jersey, Rhode Island, and Vermont. We drop those US state-year observations if inflation data is missing.

We divide the state of economy based on the level of employment, inflation, and DNWR. The indicator variable for low employment, $\mathbb{I}(\mathrm{L}(e_{it}))$ is one when the HP-filtered cyclical component of state-level employment to population ratio (e_{it}) is lower than 25th percentile of its distribution across US-states-and-years and zero otherwise. In addition, $\mathbb{I}(\mathrm{H}(\pi_{it}))$ indicates high inflation US-states-years, which takes the value of one if biannual state-level inflation (π_{it}) is greater than 75th percentile of its distribution and zero otherwise. Lastly, the dummy variable $\mathbb{I}(\mathrm{H}(\mathrm{DNWR}_{it}))$ indicates US-states-years when more workers have the binding DNWR constraints. $\mathbb{I}(\mathrm{H}(\mathrm{DNWR}_{it}))$ is one when the biannual changes in the state-level differences between the share of workers with zero wage and the share of workers with wage cut is higher than 75% percentile from its distribution across states and years from 1979 to 2018.[9]

[9]Note that the regression specification does not include the level of state-level inflation and the cyclical component of employment themselves but contains dummy variables indicating a high inflation period or a low employment period. This specification is useful to avoid potential measurement errors in state-level measures of inflation, employment, and DNWR. For example, the level of inflation from our data set differs slightly from the one from Hazell, Herreño, Nakamura, and Steinsson (2020) but the indicator of high inflation is almost the same across the two. Our main results are also robust to using the Hazell, Herreño, Nakamura, and Steinsson (2020) state-level inflation data set.

Online Appendix to "State dependent government spending multipliers: Downward Nominal Wage Rigidity and Sources of Business Cycle Fluctuations"

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January, 2024

A.1 Appendix: Analytics of state-dependent government spending multipliers

An equilibrium is a set of stochastic processes $\{\lambda_t, c_t, w_t, mc_t, R_t, \pi_t, x_t^1, x_t^2, y_t, n_t, n_t^s, u_t, s_t, p_t^*\}_{t=0}^{\infty}$ satisfying:

$$\lambda_t = (c_t - \chi n_t^{\varphi})^{-\sigma} \tag{A.1}$$

$$\chi\varphi n_t^{s\varphi-1} = w_t \tag{A.2}$$

$$\lambda_t = R_t \mathbb{E}_t \frac{\beta_{t+1}\lambda_{t+1}}{\pi_{t+1}} \tag{A.3}$$

$$W_t \geq \gamma W_{t-1}; w_t \geq \gamma \frac{w_{t-1}}{\pi_t} \tag{A.4}$$

$$(n_t^s - n_t)(w_t - \gamma \frac{w_{t-1}}{\pi_t}) = 0 \tag{A.5}$$

When DNWR does not bind ($w_t > \gamma \frac{w_{t-1}}{\pi_t}$), full employment is achieved, $n_t^s = n_t$ and $u_t = 0$. As opposed, if DNWR binds, that is, $w_t = \gamma \frac{w_{t-1}}{\pi_t}$, there is an excess supply of labor, $n_t^s > n_t$ and $u_t > 0$.

$$u_t = \frac{n_t^s - n_t}{n_t^s} \tag{A.6}$$

$$p_t^* = \frac{\theta}{\theta - 1} \frac{x_t^1}{x_t^2} \tag{A.7}$$

$$x_t^1 = \lambda_t y_t mc_t + \omega \mathbb{E}_t \beta_{t+1} \pi_{t+1}^{\theta} x_{t+1}^1 \tag{A.8}$$

$$x_t^2 = \lambda_t y_t + \omega \mathbb{E}_t \beta_{t+1} \pi_{t+1}^{\theta-1} x_{t+1}^2 \tag{A.9}$$

$$mc_t = \frac{w_t}{A_t} \tag{A.10}$$

$$\pi_t = \left[\frac{1}{\omega} - \frac{1-\omega}{\omega} p_t^{*1-\theta}\right]^{\frac{1}{\theta-1}} \tag{A.11}$$

$$y_t = A_t n_t / s_t \tag{A.12}$$

$$y_t = c_t + g_t \tag{A.13}$$

$$s_t = (1-\omega) p_t^{*-\theta} + \omega \pi_t^{\theta} s_{t-1} \tag{A.14}$$

$$\frac{R_t}{R} = (\frac{\pi_t}{\pi})^{\alpha_\pi} (\frac{y_t}{y})^{\alpha_y} \tag{A.15}$$

, given exogenous stochastic processes $\{g_t, \beta_t, A_t\}_{t=o}^{\infty}$, which are following AR(1) processes specified as below:

$$\ln \frac{g_t}{g} = \rho^g \ln \frac{g_{t-1}}{g} + \epsilon_t^g \tag{A.16}$$

$$\ln \frac{\beta_t}{\beta} = \rho^\beta \ln \frac{\beta_{t-1}}{\beta} + \epsilon_t^\beta \tag{A.17}$$

$$\ln \frac{A_t}{A} = \rho^A \ln \frac{A_{t-1}}{A} + \epsilon_t^A \tag{A.18}$$

A.1.1 Derivation of IS-PC curves

We derive the IS and the Phillips curve (PC) summarizing equilibrium conditions, (A.1) ~ (A.15). To derive the IS equation, log-linearize both the monetary policy rule (A.15) and the household's intertemporal optimization equation (A.3). Combining the previous two equations yields

$$\widehat{\lambda_t} = \mathbb{E}_t \widehat{\lambda}_{t+1} + \alpha_\pi \widehat{\pi}_t - \mathbb{E}_t \widehat{\pi}_{t+1} + \mathbb{E}_t \widehat{\beta}_{t+1}. \tag{A.19}$$

, where hat variables stand for log-deviations from the steady state and the variable without time subscript represents its steady-state value. Find $\widehat{\lambda}_t$ by log-linearizing the marginal utility of consumption (A.1),

$$\widehat{\lambda_t} = -\frac{\sigma c}{c - \chi n^\varphi} \widehat{c}_t + \frac{\sigma \chi \varphi n^\varphi}{c - \chi n^\varphi} \widehat{n}_t. \tag{A.20}$$

Now let's find the steady-state values of variables. From the production function (A.12), we know that the steady state level of output y=A. Note that the steady-state value of s is zero under the zero inflation steady-state (Galí (2008)). By the market clearing condition (A.13), we find the steady-state consumption is then $c = y - g$. Define the steady-state government spending-to-output ratio as $s_g \equiv \frac{g}{y}$. Then, $c = (1 - s_g)A$. Assume the steady-state labor n equals to labor supply, n^s, which equals to 1. Using Equation (A.2) and (A.10), solve for the model-implied parameter χ assuring $n = 1$ as

$$\chi = \frac{w}{\varphi} = \frac{1}{\varphi} \times A \times mc = \frac{A}{\varphi}\frac{\theta - 1}{\theta}.$$

Substituting the steady-state values to the Equation (A.20) yields

$$\widehat{\lambda_t} = -\frac{\theta(1-s_g)}{\Psi}\widehat{c}_t + \frac{(\theta - 1)}{\Psi}\widehat{n}_t, \tag{A.21}$$

where $\Psi = \frac{\theta\varphi(1-s_g)-(\theta-1)}{\sigma\varphi}$. The log linearization of the market clearing condition (A.13) and the production function (A.12) leads

$$\widehat{c}_t = \frac{1}{1 - s_g}\widehat{y}_t - \frac{s_g}{1 - s_g}\widehat{g}_t \tag{A.22}$$

$$\widehat{y}_t = \widehat{a}_t + (\widehat{n}_t - \widehat{s}_t). \tag{A.23}$$

Galí (2008) shows that $\widehat{s}_t$ equals to zero up to a first-order approximation. Combining (A.19), (A.21), (A.22), and (A.23) yields the NKIS equation:

$$\widehat{y}_t = \mathbb{E}_t\widehat{y}_{t+1} - (\theta - 1)(\widehat{a}_t - \mathbb{E}_t\widehat{a}_{t+1}) + \theta s_g(\widehat{g}_t - \mathbb{E}_t\widehat{g}_{t+1}) - \Psi(\alpha_\pi\widehat{\pi}_t - \mathbb{E}_t\widehat{\pi}_{t+1}) - \Psi\mathbb{E}_t\widehat{\beta}_{t+1} \tag{A.24}$$

where $\Psi = \frac{\theta\varphi(1-s_g)-\theta+1}{\sigma\varphi}$.

Now let's derive Phillips curve (PC). The PC can be written in two ways, depending upon whether DNWR binds or not. The first-order approximation of Equation (A.7) and (A.11) yields

$$\widehat{\pi}_t = \frac{(1 - \omega)(1 - \omega\beta)}{\omega}\widehat{mc}_t + \beta\mathbb{E}_t\widehat{\pi}_{t+1}, \tag{A.25}$$

where $\widehat{mc}_t$ takes two forms. When DNWR does not bind, full employment is achieved ($\widehat{n}_t = \widehat{n}^s_t$). Log-linearization of the Equation (A.2) under the full employment regime yields $\widehat{w}_t = (\varphi - 1)\widehat{n}_t$. From the Equation (A.10), we know that $\widehat{mc}_t = \widehat{w}_t - \widehat{a}_t$. Combining previous two equations with Equation (A.23) leads

$$\widehat{mc}_t = (\varphi - 1)\widehat{y}_t - \varphi\widehat{a}_t. \tag{A.26}$$

Substituting (A.26) into (A.25) yields the PC curve under the full employment equilibrium:

$$\widehat{\pi}_t = \triangle(\varphi - 1)\widehat{y}_t - \triangle\varphi\widehat{a}_t + \beta\mathbb{E}_t\widehat{\pi}_{t+1}, \tag{A.27}$$

where $\triangle = \frac{(1-\omega)(1-\omega\beta)}{\omega}$. When DNWR binds ($\gamma = 1$), we can re-write $\widehat{w}_t = \widehat{w}_{t-1} - \widehat{\pi}_t$. Then,

$$\widehat{mc}_t = \widehat{w}_{t-1} - \widehat{\pi}_t - \widehat{a}_t. \tag{A.28}$$

Substituting (A.28) into (A.25) yields the modified PC curve under the binding DNWR

$$(1 + \triangle)\widehat{\pi}_t = \triangle[\widehat{w}_{t-1} - \widehat{a}_t] + \beta\mathbb{E}_t\widehat{\pi}_{t+1}, \tag{A.29}$$

or

$$\widehat{\pi}_t = \frac{(1-\omega)(1-\omega\beta)}{\omega + (1-\omega)(1-\omega\beta)}[\widehat{w}_{t-1} - \widehat{a}_t] + \frac{\omega\beta}{\omega + (1-\omega)(1-\omega\beta)}\mathbb{E}_t\widehat{\pi}_{t+1}. \tag{A.30}$$

A.1.2 Analytical results

Now let's consider the two sources of business cycle fluctuations - a preference shock ($\widehat{\beta}_{t+1}$) and a productivity shock ($\widehat{a}_t$). For tractability, we assume complete DNWR, with $\gamma = 1$. Additionally, nominal interest rates respond to deviations in the inflation rate from its steady state but not to output deviations ($\alpha_\pi > 0$ and$\alpha_y = 0$).The sequences of the preference shock ($\mathbb{E}_t\widehat{\beta}_{t+1} = b_L < 0$ and $\mathbb{E}_t\widehat{\beta}_{t+2} = 0$) and ($\mathbb{E}_t\widehat{\beta}_{t+1} = b_H > 0$ and $\mathbb{E}_t\widehat{\beta}_{t+2} = 0$) cause a demand-driven expansion and recession, respectively, in period t. The sequence of the technology shock ($\widehat{a}_t = a_H > 0$, $\mathbb{E}_t\widehat{a}_{t+1}$=$\rho_a a_H$, and $\mathbb{E}_t\widehat{a}_{t+2}$=$a_L$) and ($\widehat{a}_t = a_L < 0$, $\mathbb{E}_t\widehat{a}_{t+1}$=$\rho_a a_L$, and $\mathbb{E}_t\widehat{a}_{t+2}$=$a_H$) drive a supply-driven expansion and recession, respectively, in period t.

Proposition A.1. In response to a preference shock, output ($\widehat{y}_t$) and inflation ($\widehat{\pi}_t$) co-move, and in response to a technology shock, output and inflation move in the opposite direction. That is,

$$\frac{\partial\widehat{y}_t}{\partial\widehat{\beta}_{t+1}} < 0; \frac{\partial\widehat{\pi}_t}{\partial\widehat{\beta}_{t+1}} < 0, \text{ and } \frac{\partial\widehat{y}_t}{\partial\widehat{a}_t} > 0; \frac{\partial\widehat{\pi}_t}{\partial\widehat{a}_t} < 0.$$

Proof. Let's consider two independent shock processes. The demand-driven business cycles follow ($\mathbb{E}_t\widehat{\beta}_{t+1} = \widehat{\beta}_{t+1}$, and $\mathbb{E}_t\widehat{\beta}_{t+2} = 0$) where $\mathbb{E}_t\widehat{\beta}_{t+1}$ is β_H in a demand-driven recession and $\mathbb{E}_t\widehat{\beta}_{t+1}$ is β_L in a demand shock-boom. The supply-driven business cycles are to follow ($\widehat{a}_t$=$\widehat{a}_t$, $\mathbb{E}_t\widehat{a}_{t+1}$=$\rho_a\widehat{a}_t$,and $\mathbb{E}_t\widehat{a}_{t+2}$=$\widehat{a}_{t+2}$) where $(\widehat{a}_t, \widehat{a}_{t+2}) = (a_H, a_L)$ in a supply-driven boom and $(\widehat{a}_t, \widehat{a}_{t+2}) = (a_L, a_H)$ in a supply-driven recession. Suppose that the market clearing solution takes the form:

$$\widehat{y}_t = A_y\widehat{g}_t + B_y\mathbb{E}_t\widehat{\beta}_{t+1} + C_y\widehat{a}_t + D_y\mathbb{E}_t\widehat{a}_{t+1} = A_y\widehat{g}_t + B_y\mathbb{E}_t\widehat{\beta}_{t+1} + C_y\widehat{a}_t + \rho_a D_y\widehat{a}_t$$

$$\widehat{\pi}_t = A_\pi \widehat{g}_t + B_\pi \mathbb{E}_t \widehat{\beta}_{t+1} + C_\pi \widehat{a}_t + D_\pi \mathbb{E}_t \widehat{a}_{t+1} = A_\pi \widehat{g}_t + B_\pi \mathbb{E}_t \widehat{\beta}_{t+1} + C_\pi \widehat{a}_t + \rho_a D_\pi \widehat{a}_t.$$

Given the assumptions on shock processes and government spending, the expected output and inflation are

$$\mathbb{E}_t \widehat{y}_{t+1} = A_y \mathbb{E}_t \widehat{g}_{t+1} + B_y \mathbb{E}_t \widehat{\beta}_{t+2} + C_y \mathbb{E}_t \widehat{a}_{t+1} + D_y \mathbb{E}_t \widehat{a}_{t+2} = \rho_a C_y \widehat{a}_t + D_y \widehat{a}_{t+2}$$

$$\mathbb{E}_t \widehat{\pi}_{t+1} = A_\pi \mathbb{E}_t \widehat{g}_{t+1} + B_\pi \mathbb{E}_t \widehat{\beta}_{t+2} + C_\pi \mathbb{E}_t \widehat{a}_{t+1} + D_\pi \mathbb{E}_t \widehat{a}_{t+2} = \rho_a C_\pi \widehat{a}_t + D_\pi \widehat{a}_{t+2}$$

Plug the projected solution into the IS curve (A.24) and Phillips curve (A.27) and solve for coefficients using the method of undetermined coefficients,

$$A_y = \frac{\theta s_g}{1 + \Psi \alpha_\pi \triangle(\varphi - 1)} > 0$$

$$A_\pi = \frac{\triangle(\varphi - 1)\theta s_g}{1 + \Psi \alpha_\pi \triangle(\varphi - 1)} > 0$$

$$B_y = \frac{\partial \widehat{y}_t}{\partial \widehat{\beta}_{t+1}} = -\frac{\Psi}{[1 + \Psi \alpha_\pi \triangle(\varphi - 1)]} < 0$$

$$B_\pi = \frac{\partial \widehat{\pi}_t}{\partial \widehat{\beta}_{t+1}} = -\frac{\Psi \triangle(\varphi - 1)}{[1 + \Psi \alpha_\pi \triangle(\varphi - 1)]} < 0$$

$\mathbb{I}(\mathrm{H}(\pi_{it}))$ indicates high inflation US-states-years,

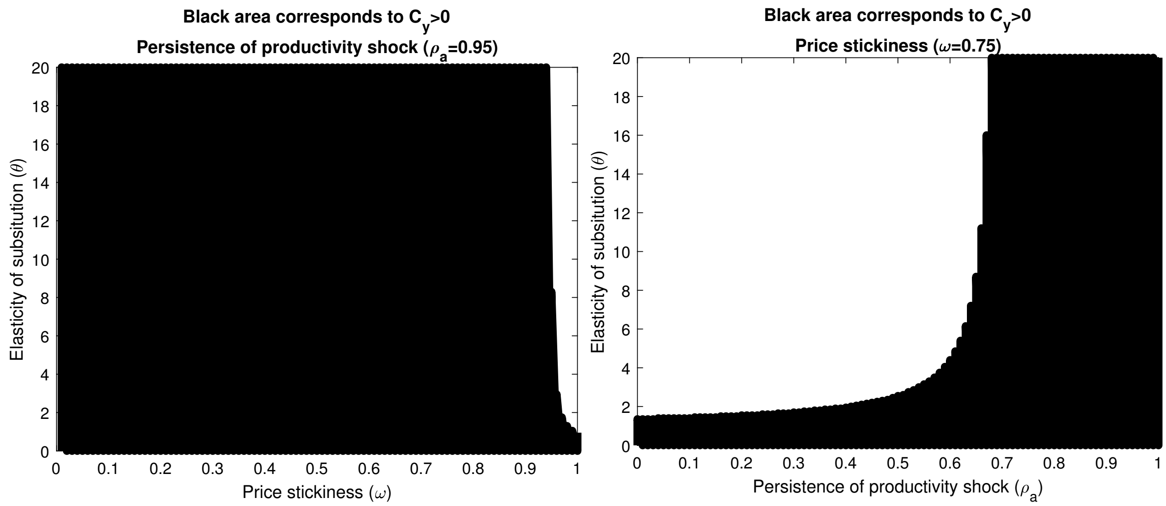
$$D_\pi = 0$$

$$D_y = 0$$

$$C_\pi = \frac{\partial \widehat{\pi}_t}{\partial \widehat{a}_t} = \frac{-\triangle}{(1 - \beta\rho_a)} [\frac{(\varphi - 1)(\theta - 1)(1 - \rho_a)(1 - \beta\rho_a) + \varphi(1 - \rho_a)(1 - \beta\rho_a)}{(1 - \rho_a)(1 - \beta\rho_a) + \Psi(\alpha_\pi - \rho_a)\triangle(\varphi - 1)}] < 0$$

$$C_y = \frac{\partial \widehat{y}_t}{\partial \widehat{a}_t} = \frac{-(\theta - 1)(1 - \rho_a)(1 - \beta\rho_a) + \frac{\theta\varphi(1 - s_g) - (\theta - 1)}{\sigma\varphi}(\alpha_\pi - \rho_a)\frac{(1-\omega)(1-\omega\beta)}{\omega}\varphi}{(1 - \rho_a)(1 - \beta\rho_a) + \frac{\theta\varphi(1 - s_g) - (\theta - 1)}{\sigma\varphi}(\alpha_\pi - \rho_a)\frac{(1-\omega)(1-\omega\beta)}{\omega}(\varphi - 1)}.$$

Figure A.1: Parameter space corresponding to positive C_y



Notes: The left panel shows the parameter space (θ, ω) that corresponds to positive C_y given the persistence of productivity shock is 0.95. The right panel shows the combination of (θ, ρ_a) that ensures positive C_y.

The sign of all coefficients except C_y is determinant under common parameter values.[1] However, depending on the parameter values, the sign of C_y changes. For example, for a high enough elasticity of substitution (θ) and price-stickiness parameter (ω) or a low enough persistence of productivity shock (ρ_a), C_y can be negative. To determine the sign of C_y, we fix the typical parameter values – the discount factor (β) is 0.99, the Frisch elasticity $(\frac{1}{\varphi-1})$ is 0.5, and coefficient on inflation in the monetary policy rule (α_π) is 1.5. The steady-state government spending to output ratio s_g is calibrated to 0.2. The left panel of Figure A.1 shows the parameter space of θ and ω that corresponds to positive C_y, under the persistence productivity shock (ρ_a) being 0.95. C_y is positive for plausible parameter space. In New Keynesian literature, it is common to set ω as 0.75. The price rigidity of posted prices varies from 0.45 to 0.73 from microdata literature (see Nakamura and Steinsson (2013)). The right panel of Figure A.1 shows the combination of θ and ρ_a that ensures positive C_y, when the price stickiness parameter, ω, is 0.75. For a high enough persistent productivity, we find that C_y is positive. To summarize, C_y is positive under the plausible parameter space. □

Proposition A.2. In a model without DNWR, the government spending multiplier takes the same value M_y in expansion and recession states, i.e. is acyclical.

[1]The elasticity of substitution parameter θ is greater than 1, the discount factor β is less than 1 and greater than zero. The government spending share in output, s_g is less than one. The intertemporal elasticity of substitution σ is assumed to be greater than one, while the frequency of price adjustment is ω is less than one. The coefficient on inflation in the monetary policy rule is assumed to be higher than one.

Proof. From the proof of Proposition A.1, the government spending multiplier is

$$M_y \equiv \frac{dy}{dg} = \frac{\partial \widehat{y}_t}{\partial \widehat{g}_t}\frac{\overline{y}}{\overline{g}} = \frac{A_y}{s_g} = \frac{\omega\theta}{\omega + \Psi\alpha_\pi(1-\omega)(1-\omega\beta)(\varphi-1)} \geq 0$$

regardless of the shock processes and the state of the economy. □

When DNWR binds in period t under the expectation of achieving full employment in period $(t+1)$, the spending multiplier is M_{DNWR} , which is bigger than M_y – the multiplier when DNWR does not bind.

Proof. Guess the solution that satisfies both IS curve (Equation (A.24)) and the modified Phillips curve (Equation (A.29)). Note that the binding DNWR constraint leaves IS curve unchanged while PC changes. Let's first consider the demand-driven business cycle – $(\mathbb{E}_t\widehat{\beta}_{t+1} = \widehat{\beta}_{t+1}$, and $\mathbb{E}_t\widehat{\beta}_{t+2} = 0)$. Then, the projected solution becomes

$$\widehat{y}_t = F_y\widehat{w}_{t-1} + H_y\widehat{g}_t + I_y\mathbb{E}_t\widehat{\beta}_{t+1} \tag{A.31}$$

$$\widehat{\pi}_t = F_\pi\widehat{w}_{t-1} + H_\pi\widehat{g}_t + I_\pi\mathbb{E}_t\widehat{\beta}_{t+1}. \tag{A.32}$$

Under the assumption that DNWR does not bind in period $(t+1)$, the expected output and inflation $\mathbb{E}_t\widehat{y}_{t+1}$ and $\mathbb{E}_t\widehat{\pi}_{t+1}$ become zero. Plug in suggested solutions (A.31) and (A.32) into IS curve (A.24) and the modified Phillips curves (Equation (A.29)) and find the coefficients using the method of undetermined coefficients,

$$F_y\widehat{w}_{t-1} + H_y\widehat{g}_t + I_y\widehat{\beta}_{t+1} = \theta s_g\widehat{g}_t - \Psi\alpha_\pi(F_\pi\widehat{w}_{t-1} + H_\pi\widehat{g}_t + I_\pi\widehat{\beta}_{t+1}) - \Psi\widehat{\beta}_{t+1}$$

$$(1+\triangle)(F_\pi\widehat{w}_{t-1} + H_\pi\widehat{g}_t + I_\pi\widehat{\beta}_{t+1}) = \triangle\widehat{w}_{t-1}$$

The multiplier in the demand-driven business cycle is

$$M^D_{DNWR} = \frac{dy}{dg} = \frac{\partial\widehat{y}_t}{\partial\widehat{g}_t}\frac{y}{g} = H_y\frac{1}{s_g} = \theta$$

, which is bigger than $M_y = \frac{\omega\theta}{\omega+\Psi\alpha_\pi(1-\omega)(1-\omega\beta)(\varphi-1)}$.

Now, let's consider the supply-driven business cycles following ($\widehat{a}_t$=$\widehat{a}_t$, $\mathbb{E}_t\widehat{a}_{t+1}$=$\rho_a\widehat{a}_t$,and $\mathbb{E}_t\widehat{a}_{t+2}$=$\widehat{a}_{t+2}$). Conjecture solution as,

$$\widehat{y}_t = O_y\widehat{w}_{t-1} + S_y\widehat{g}_t + U_y\widehat{a}_t + V_y\rho_a\widehat{a}_t \tag{A.33}$$

$$\widehat{\pi}_t = O_\pi\widehat{w}_{t-1} + S_\pi\widehat{g}_t + U_\pi\widehat{a}_t + V_\pi\rho_a\widehat{a}_t. \tag{A.34}$$

Under the assumption that DNWR does not bind in period $(t+1)$, the expected output and inflation are given by the full employment solution shown in the proof of Proposition A.1, as below.

$$\mathbb{E}_t\widehat{y}_{t+1} = C_y\rho_a\widehat{a}_t + D_y\widehat{a}_{t+2} \tag{A.35}$$

$$\mathbb{E}_t\widehat{\pi}_{t+1} = C_\pi\rho_a\widehat{a}_t + D_\pi\widehat{a}_{t+2} \tag{A.36}$$

Combining the suggested solution ((A.33) and (A.34)) with the expected output and inflation ((A.35) and (A.36)) into the IS curve (A.24) and the modified Phillips curves (Equation (A.29)) brings

$$O_y\widehat{w}_{t-1} + S_y\widehat{g}_t + U_y\widehat{a}_t + V_y\rho_a\widehat{a}_t = C_y\rho_a\widehat{a}_t + D_y\widehat{a}_{t+2} - (\theta-1)(\widehat{a}_t - \rho_a\widehat{a}_t)$$
$$+ \theta s_g\widehat{g}_t - \Psi\alpha_\pi(O_\pi\widehat{w}_{t-1} + S_\pi\widehat{g}_t + U_\pi\widehat{a}_t + V_\pi\rho_a\widehat{a}_t) + \Psi(C_\pi\rho_a\widehat{a}_t + D_\pi\widehat{a}_{t+2})$$

$$(1+\triangle)[O_\pi\widehat{w}_{t-1} + S_\pi\widehat{g}_t + U_\pi\widehat{a}_t + V_\pi\rho_a\widehat{a}_t] = \triangle[\widehat{w}_{t-1} - \widehat{a}_t] + \beta(C_\pi\rho_a\widehat{a}_t + D_\pi\widehat{a}_{t+2})$$

Using the undetermined coefficients method, we find $S_\pi = 0$ and $S_y = \theta s_g$. The output multiplier in the supply-driven business cycle is

$$M^S_{DNWR} = \frac{\partial\widehat{y}}{\partial\widehat{g}}\frac{y}{g} = S_y\frac{1}{s_g} = \theta.$$

Thus, we have shown that the multiplier is θ when DNWR binds (M_{DNWR}), regardless of the source of fluctuation. □

Lemma A.1. Assume the economy is at the steady-state in period $t-1$, $\widehat{w}_{t-1} = 0$. In the presence of the DNWR constraint ($\gamma = 1$), a positive discount factor shock or a negative productivity shock triggers the DNWR constraint to bind and induces unemployment in period t.

Proof. Log-linearized DNWR constraint (Equation (A.4)) can be expressed as follows.

$$\widehat{w}_t \geq \gamma(\widehat{w}_{t-1} - \widehat{\pi}_t). \tag{A.37}$$

To show the DNWR constraint binds in period t under the assumption that $\widehat{w}_{t-1} = 0$ and $\gamma = 1$, we have to show

$$\widehat{w}_t + \widehat{\pi}_t < 0. \tag{A.38}$$

Let's conjecture DNWR does not bind and $\widehat{n}_t = \widehat{n}^s_t$. Now check whether the conjecture holds, that is, Equation (A.37) is true. First, we obtain $\widehat{w}_t$ by combining two log-linearized Equation (A.2) and (A.12):

$$\widehat{w}_t = (\varphi - 1)(\widehat{y}_t - \widehat{a}_t).$$

From the proof of Proposition A.1, we know that we can write $\widehat{y}_t$ and $\widehat{\pi}_t$ as follows.

$$\widehat{y} = A_y \widehat{g}_t + B_y \mathbb{E}_t \widehat{\beta}_{t+1} + C_y \widehat{a}_t + D_y \mathbb{E}_t \widehat{a}_{t+1} = B_y \mathbb{E}_t \widehat{\beta}_{t+1} + C_y \widehat{a}_t + \rho_a D_y \widehat{a}_t$$

$$\widehat{\pi} = A_\pi \widehat{g}_t + B_\pi \mathbb{E}_t \widehat{\beta}_{t+1} + C_\pi \widehat{a}_t + D_\pi \mathbb{E}_t \widehat{a}_{t+1} = B_\pi \mathbb{E}_t \widehat{\beta}_{t+1} + C_\pi \widehat{a}_t + \rho_a D_\pi \widehat{a}_t$$

Plug in $\widehat{y}_t$ and $\widehat{\pi}_t$ into the left-hand-side of inequality constraint (A.38)

$$\widehat{w}_t + \widehat{\pi}_t = (\varphi - 1)(B_y \mathbb{E}_t \widehat{\beta}_{t+1} + (C_y + \rho_a D_y - 1)\widehat{a}_t) + B_\pi \mathbb{E}_t \widehat{\beta}_{t+1} + (C_\pi + \rho_a D_\pi)\widehat{a}_t$$

In a demand-driven recession, where $\mathbb{E}_t \widehat{\beta}_{t+1} = \beta_H$ and $\widehat{a}_t = 0$,

$$\widehat{w}_t + \widehat{\pi}_t = ((\varphi - 1)B_y + B_\pi)\beta_H.$$

From the proof of Proposition A.1, we know coefficients B_y and B_π are negative. Thus, for any positive discount factor shock, we know that

$$\widehat{w}_t + \widehat{\pi}_t < 0,$$

which contradicts the conjecture. Thus, we conclude that DNWR binds in response to a positive discount factor shock.

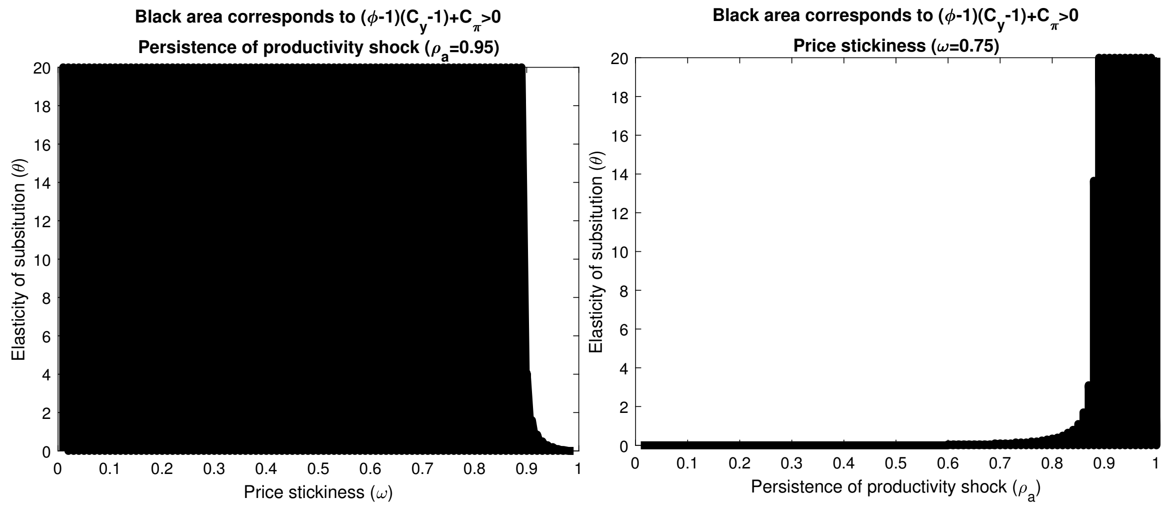
In a supply-driven recession, where $\widehat{a}_t = a_L$ and $\mathbb{E}_t \widehat{\beta}_{t+1} = 0$,

$$\widehat{w}_t + \widehat{\pi}_t = (\varphi - 1)((C_y + \rho_a D_y - 1)a_L) + (C_\pi + \rho_a D_\pi)a_L.$$

As $D_y = D_\pi = 0$, the conjecture that DNWR does not bind is not true if

$$\widehat{w}_t + \widehat{\pi}_t = [(\varphi - 1)(C_y - 1) + C_\pi]a_L < 0.$$

Figure A.2: Parameter space corresponding to positive $(\varphi - 1)(C_y - 1) + C_\pi$



Notes: The left panel shows the parameter space (θ, ω) that gives positive $(\varphi - 1)(C_y - 1) + C_\pi$ given the persistence of productivity shock is 0.95. The right panel shows the combination of (θ, ρ_a) that ensures positive $(\varphi - 1)(C_y - 1) + C_\pi$.

Based on the baseline parameter values[2], the black area in the left panel of Figure A.2 shows the combination of the elasticity of substitution (θ) and the price stickiness (ω) that satisfies

$$[(\varphi - 1)(C_y - 1) + C_\pi] > 0. \tag{A.39}$$

, where the persistence of the productivity shock ρ_a is 0.95. The right panel of FigureA.2 shows the combination of θ and ρ_a that satisfies Equation (A.39), when the price stickiness parameter, ω, is 0.75. Under the assumption of highly persistent productivity shock, we conclude that DNWR condition binds. □

Lemma A.2. Assume the economy is at steady-state in period $t-1$, $\widehat{w}_{t-1} = 0$. In a demand-driven recession, if government spending is less than $\frac{\Psi}{\theta s_g}\beta_H \equiv c_d(\beta_H)$, the DNWR constraint binds, and unemployment is greater than zero. Otherwise, DNWR is no longer a binding constraint, and unemployment is zero. In a supply-driven recession, if government spending is less than $c_s(a_L)$, the DNWR constraint binds, and unemployment is greater than zero. Otherwise, DNWR is no longer a binding constraint, and unemployment is zero.

Proof. Find the upper bound of nonzero $\widehat{g}_t$ that still violates DNWR condition, that is, $\widehat{w}_t < \gamma(\widehat{w}_{t-1} - \widehat{\pi}_t)$, or $\widehat{w}_t + \widehat{\pi}_t < 0$. With the nonzero government spending $\widehat{g}_t$, we can guess

[2] The discount factor (β) is 0.99, the Frisch elasticity $(\frac{1}{\varphi-1})$ is 0.5, and the Taylor coefficient on inflation (α_π) is 1.5.

the solution as

$$\widehat{y} = A_y\widehat{g}_t + B_y\mathbb{E}_t\widehat{\beta}_{t+1} + C_y\widehat{a}_t + D_y\mathbb{E}_t\widehat{a}_{t+1} = A_y\widehat{g}_t + B_y\mathbb{E}_t\widehat{\beta}_{t+1} + C_y\widehat{a}_t + \rho_a D_y\widehat{a}_t$$

$$\widehat{\pi} = A_\pi\widehat{g}_t + B_\pi\mathbb{E}_t\widehat{\beta}_{t+1} + C_\pi\widehat{a}_t + D_\pi\mathbb{E}_t\widehat{a}_{t+1} = A_\pi\widehat{g}_t + B_\pi\mathbb{E}_t\widehat{\beta}_{t+1} + C_\pi\widehat{a}_t + \rho_a D_\pi\widehat{a}_t.$$

Then we can rewrite the left-hand-side of the DNWR constraint (A.38)

$$\widehat{w}_t + \widehat{\pi}_t = (\varphi - 1)(A_y\widehat{g}_t + B_y\mathbb{E}_t\widehat{\beta}_{t+1} + (C_y + \rho_a D_y - 1)\widehat{a}_t) + A_\pi\widehat{g}_t + B_\pi\mathbb{E}_t\widehat{\beta}_{t+1} + (C_\pi + \rho_a D_\pi)\widehat{a}_t$$

In a demand-driven recession, where $\mathbb{E}_t\widehat{\beta}_{t+1} = \beta_H$ and $\widehat{a}_t = 0$,

$$\widehat{w}_t + \widehat{\pi}_t = ((\varphi - 1)A_y + A_\pi)\widehat{g}_t + ((\varphi - 1)B_y + B_\pi)\beta_H$$

Using the coefficients that we find from the proof of Proposition A.1, we can rewrite the above equation as

$$\widehat{w}_t + \widehat{\pi}_t = (\frac{(\varphi - 1)\theta s_g(1 + \triangle)}{1 + \Psi\alpha_\pi\triangle(\varphi - 1)})\widehat{g}_t + (-\frac{\Psi(1 + \triangle)(\varphi - 1)}{[1 + \Psi\alpha_\pi\triangle(\varphi - 1)]})\beta_H$$

DNWR binds with non-zero government spending if $\widehat{w}_t + \widehat{\pi}_t < 0$, that is,

$$\frac{(\varphi - 1)\theta s_g(1 + \triangle)}{1 + \Psi\alpha_\pi\triangle(\varphi - 1)}\widehat{g}_t < \frac{\Psi(1 + \triangle)(\varphi - 1)}{1 + \Psi\alpha_\pi\triangle(\varphi - 1)}\beta_H$$

$$\widehat{g}_t < \frac{\Psi}{\theta s_g}\beta_H \equiv c_d(\beta_H)$$

In a supply-driven recession, where $\widehat{a}_t = a_L$ and $\mathbb{E}_t\widehat{\beta}_{t+1} = 0$, the left-hand-side of the inequality constraint (A.38) is

$$\widehat{w}_t + \widehat{\pi}_t = [(\varphi - 1)A_y + A_\pi]\widehat{g}_t + [(\varphi - 1)(C_y - 1) + C_\pi]a_L.$$

DNWR binds with non-zero government spending if $\widehat{w}_t + \widehat{\pi}_t < 0$, or, equivalently,

$$\widehat{g}_t < \frac{[(\varphi - 1)(C_y - 1) + C_\pi]}{[(\varphi - 1)A_y + A_\pi]}(-a_L) \equiv c_s(a_L) \tag{A.40}$$

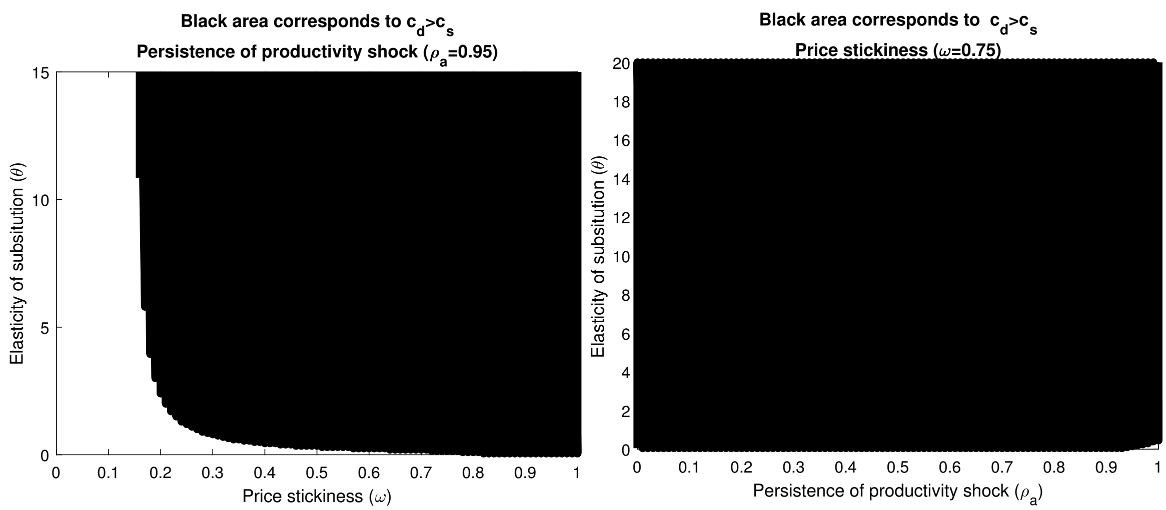
Given the negative productivity shock, the right hand side of Equation (A.40) is positive. Note that we show both A_y and A_π are positive in the proof of Proposition A.1 and $[(\varphi - 1)(C_y - 1) + C_\pi]$ is positive from the proof of Lemma A.1. □

Lemma A.3. Under the assumption that $|\beta_H| = |a_L|$, it can be shown that $0 < c_s(a_L) < c_d(\beta_H)$. In other words, the government spending required to ensure DNWR is no longer binding is smaller in a supply driven recession than a demand driven recession.

Proof. For given $|\beta_H| = |a_L|$, we want to show that

$$\frac{[(\varphi - 1)(C_y - 1) + C_\pi]}{[(\varphi - 1)A_y + A_\pi]} < \frac{\Psi}{\theta s_g} \tag{A.41}$$

Figure A.3: Parameter space corresponding to $c_s(a_L) < c_d(\beta_H)$



Notes: The left panel shows the parameter space (θ, ω) that satisfies $c_s(a_L) < c_d(\beta_H)$ given the persistence of productivity shock is 0.95. The right panel shows the combination of (θ, ρ_a) that ensures $c_s(a_L) < c_d(\beta_H)$.

Based on the baseline parameter values, the black area in the left panel of Figure A.3 shows the combination of the elasticity of substitution (θ) and the price stickiness (ω) that satisfies Equation (A.41), where the persistence of the productivity shock ρ_a is 0.95. The right panel of Figure A.3 shows the combination of θ and ρ_a that satisfies Equation (A.41), when the price stickiness parameter, ω, is 0.75. We find the Equation (A.41) holds for most cases. □

Proposition 1. Under the assumption that $|\beta_H| = |a_L|$, i.e. equal sized business cycle fluctuations,

the spending multiplier in a demand-driven recession $\geq$
the spending multiplier in a supply-driven recession $\geq$
the spending multiplier in an expansion,

for a given size of government spending shock.

Proof. In the absence of DNWR, the multipliers are the same regardless of the state of the economy or the source of fluctuation. In the presence of DNWR ($\gamma = 1$), if government spending (g) satisfies $g < c_s(a_L)$, the DNWR constraint still binds for both recessions (Lemma A.2), thus, the spending multiplier in a demand-driven recession (M^D_{DNWR}) is the same as the spending multiplier in a supply-driven recession (M^S_{DNWR}), which is greater than the spending multiplier in an expansion (M_y). If $c_s(a_L) < g < c_d(\beta_H)$, DNWR condition binds in a demand-driven recession but not in a supply-driven recession. In this case, the spending multiplier in a demand-driven recession is M^D_{DNWR} , which is higher than the spending multiplier in a supply-driven recession when DNWR is not a binding constraint, equal to the spending multiplier in an expansion, M_y. If $c_s(a_L) < c_d(\beta_H) < g$, government spending is large enough to raise nominal wages and achieve full employment, the spending multiplier would be M_y regardless of the source of fluctuation and the state of the business cycle. □

A.2 Appendix: Quantitative Model

A.2.1 Business Cycle fluctuations under supply and demand shocks in the quantitative model

We begin by considering the impulse responses to both contractionary and expansionary supply and demand shocks. The size of the shock is normalized to match the average output gap during the Great Recession. According to the Congressional Budget Office estimates,[3] the average output gap from 2008 to 2010 was 4%. We consider productivity and discount factor shocks to match this impact on output in a recession. This results in considering 1.7% deviations from the steady-state value of the discount factor and 2.9% deviations from the steady-state value of productivity. Both shock processes follow AR(1) process, following Equation (**??**) and Equation (**??**).[4]

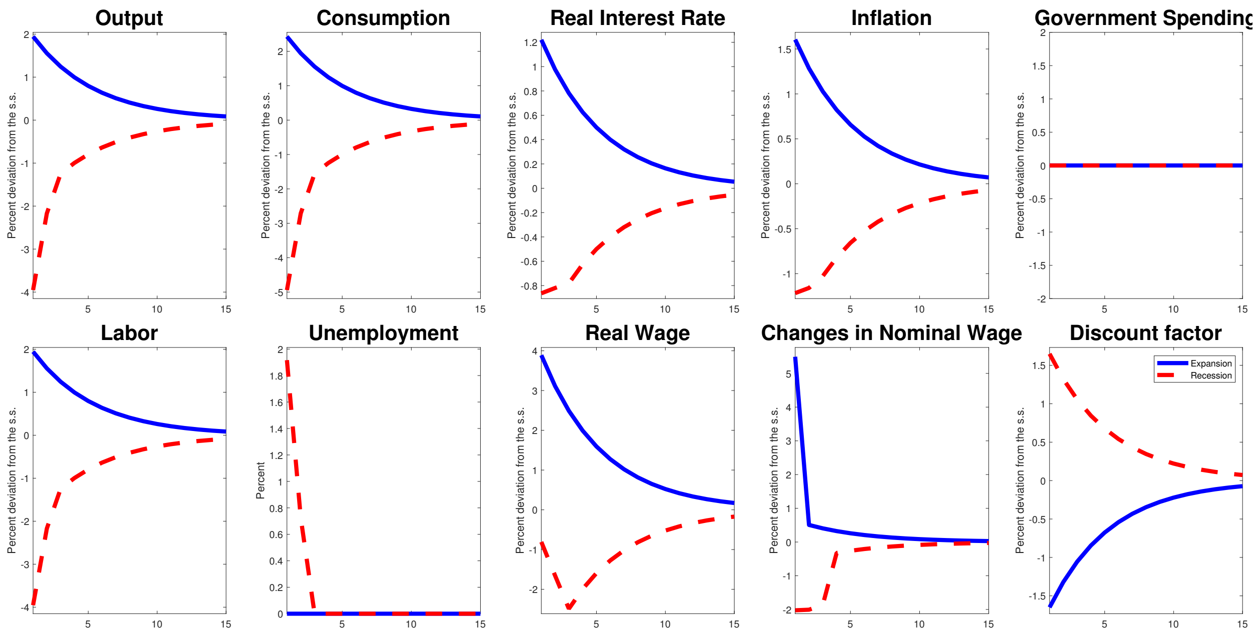
Figure A.4 displays impulse response in a demand-driven expansion and recession, without government spending. In response to a negative discount factor shock (shown with solid blue lines), consumers spend more in the current period leading to a demand-driven expansion. An increase in demand raises inflation and equilibrium labor. As there are no frictions in adjusting nominal wages upward, the labor market always clear, and the unemployment rate is zero.

In response to a positive discount factor shock (shown with dashed red lines in Figure A.4), consumers postpone current consumption, which causes a recession. As labor

[3]Source: https://fred.stlouisfed.org/graph/?g=f1cZ.

[4]We determine the size of the shock based on the average size of the output gap during the Great Recession, however, the slow recovery during the Great Recession was not matched in the following exercises.

Figure A.4: Demand-driven business cycle

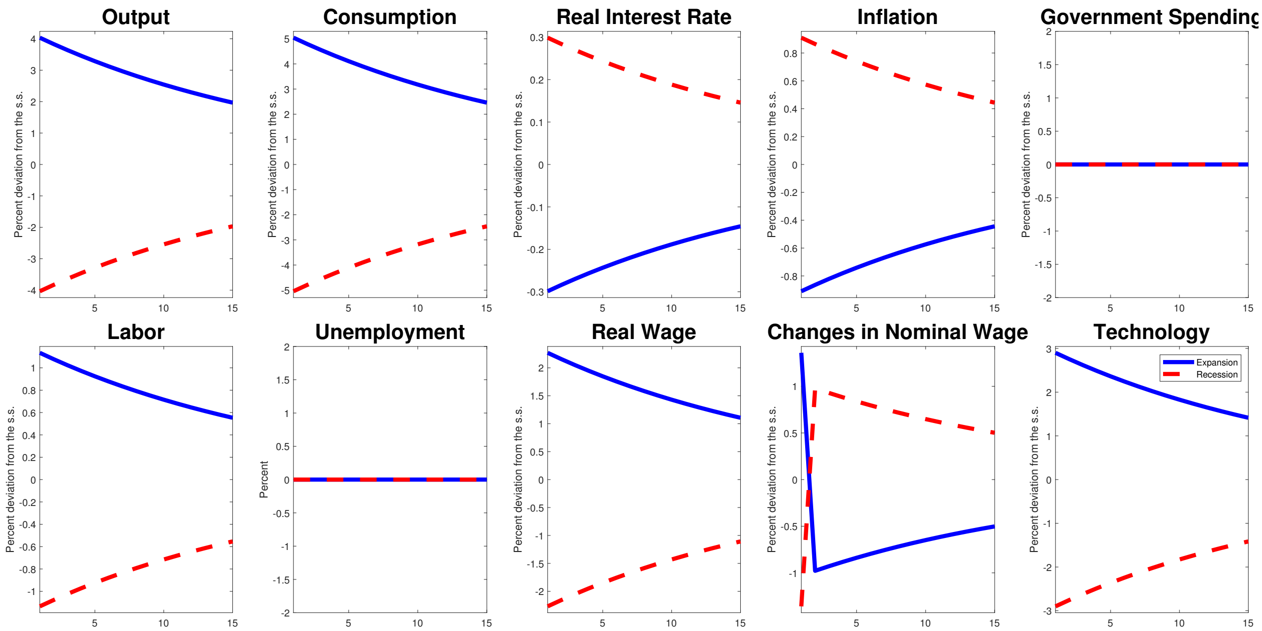


Notes: This graph shows impulse responses to a positive and a negative discount factor shock. The solid blue lines correspond to a negative discount factor shock (a demand-driven expansion), and the dashed red line represents impulse responses to a negative discount factor shock (a demand-driven recession). $\pm 1.7\%$ deviations of the discount factor shocks are imposed. All graph is drawn in terms of the percent deviations from its steady-state except the unemployment rate. The y-axis of the unemployment rate is percent.

demand decreases, there is downward pressure on wages. Although real wage goes up more than 4% in an expansion, the downward adjustment of real wage is about 1% at the beginning of the recession due to deflation and the binding of the DNWR constraint. The DNWR constraint allows at most 2% downward adjustment of real wage. At the same time, there is deflation that drives the real wages upward. The comovement of inflation and output, shown in Proposition A.1, exacerbates the labor market outcome and raises unemployment. Overall, the binding DNWR constraint generates an asymmetric business cycle.

Figure A.5 shows a supply-driven business cycle. As shown in Proposition A.1, inflation and output move in the opposite directions in a supply-driven recession. In a recession (dashed red lines), the marginal product of labor goes down, and firms hire less labor. Accordingly, nominal wage goes down about 1.5%. As we allow the downward adjustment of nominal wage up to 2%, the DNWR constraint does not bind. Consequently, the labor market clears, and the unemployment rate is zero. Unlike the demand-driven recession, the downward adjustment of real wage is greater than that of nominal wage in the supply-driven recession due to inflation. This is also highlighted in the analytical

Figure A.5: Supply-driven business cycle



Notes: This graph displays impulse responses to a positive and a negative productivity shock. The solid blue lines correspond to a positive productivity shock (a supply-driven expansion). The dashed red lines represent impulse responses to a negative productivity shock(a supply-driven recession). $\pm 2.9\%$ deviations of the technology shocks are imposed. All graph is drawn in terms of the percent deviations from its steady-state except the unemployment rate. The y-axis of the unemployment rate is percent.

section. The supply-driven business cycle is fully symmetric as DNWR does not bind.[5]

A.2.2 Robustness results

Our baseline model considers GHH (Greenwood, Hercowitz, and Huffman (1988)) preferences which do not allow a wealth effect on labor supply. We relax this assumption and allow for wealth effects on labor supply by introducing KPR (King, Plosser, and Rebelo (1988)) preferences commonly used in the literature. In particular, the preferences take the following form,

$$U(c_t, n_t) = \frac{[c_t(1 - \chi n_t^{\varphi})]^{1-\sigma}}{1-\sigma},$$

where we calibrate φ to ensure the same degree of Frisch elasticity of labor supply as in our baseline model.

As Table **??** shows, the multipliers under these preferences are smaller across the board

[5]When we consider King, Plosser, and Rebelo (1988) preferences in Section A.2.2, we find asymmetric business cycles in response to a supply shock as well. Once we allow for a wealth effect on labor supply in response to a technology shock, nominal wages fall more than in our baseline case and are bound below by DNWR in a supply-driven recession as well.

Table A.1: Cumulative output and consumption multipliers under the KPR preference

		Demand-driven business cycle				Supply-driven business cycle		
		Impact	4 quarters	20 quarters		Impact	4 quarters	20 quarters
				KPR preference				
Output Multiplier	Expansion	0.485	0.485	0.485	Expansion	0.485	0.485	0.485
	Recession	0.668	0.621	0.564	Recession	0.528	0.500	0.494
Consumption Multiplier	Expansion	-0.515	-0.515	-0.515	Expansion	-0.515	-0.515	-0.515
	Recession	-0.332	-0.379	-0.436	Recession	-0.472	-0.500	-0.506

Notes. This table reports the cumulative output and consumption multipliers in an expansion and a recession depending on the source of fluctuation. The cumulative output and consumption multipliers are calculated as $\Sigma_{i=0}^{i=k-1} \frac{\Delta y_{t+i}}{(1+r_{t+i})} \Big/ \Sigma_{i=0}^{i=k-1} \frac{\Delta g_{t+i}}{(1+r_{t+i})}$ and $\Sigma_{i=0}^{i=k-1} \frac{\Delta c_{t+i}}{(1+r_{t+i})} \Big/ \Sigma_{i=0}^{i=k-1} \frac{\Delta g_{t+i}}{(1+r_{t+i})}$, respectively, where Δ denotes the level differences of each variable with and without government spending and r_t is the real interest rate.

relative to GHH preferences.[6] An increase in government spending under KPR preferences leads to negative wealth effects on the labor supply, as agents internalize higher taxes now or in the future. Earlier studies, such as Monacelli and Perotti (2008), have shown that the degree of complementarity between hours and consumption is inversely related to this wealth effect on labor. Thus, this complementarity is largest with GHH preferences, and declines as we move towards KPR preferences, and leads to a larger response of output under GHH preferences. Under KPR preferences, the multiplier in a demand-driven recession is larger than in an expansion (0.67 in a recession and 0.49 in an expansion under KPR preferences), but the difference is much smaller in magnitude relative to under GHH preferences, (1.74 in a recession and 0.54 in an expansion under GHH preferences). This is because the labor supply curve shifts to the right, and overall weakens the effects of increased spending in reducing unemployment. Under these preferences, DNWR binds in a supply-driven recession as well, leading to a larger output multiplier in a recession relative to an expansion.[7] The multiplier in a supply-driven recession is thus smaller than the multiplier in a demand-driven recession (0.53 versus 0.67, respectively), although this difference across states is small. The intuition follows from Proposition **??** shown in Section **??**. We also show robustness results respect to trend inflation (Appendix A.2.3), and alternative degree of price and wage rigidity (Appendix A.2.4).

[6]Under these preferences, we need to adjust the size of both the discount factor and productivity shock in order to generate the same size recession state.

[7]DNWR is more likely to bind in this case in response to a technology shock, since wages have a relatively larger response and labor has a smaller response with KPR preferences as the wealth effects from a technology shock shift the labor supply curve, an effect missing with GHH preferences.

A.2.3 Robustness to trend inflation

Table A.2: Cumulative output and consumption multipliers with nonzero steady-state inflation

		Demand-driven business cycle				Supply-driven business cycle		
		Impact	4 quarters	20 quarters		Impact	4 quarters	20 quarters
		A. 2% steady-state annual inflation with GHH preference						
Output	Expansion	0.345	0.314	0.236	Expansion	0.345	0.314	0.236
Multiplier	Recession	1.067	0.653	0.439	Recession	0.345	0.314	0.236
Consumption	Expansion	-0.655	-0.686	-0.764	Expansion	-0.655	-0.686	-0.764
Multiplier	Recession	0.067	-0.347	-0.561	Recession	-0.655	-0.686	-0.764
		A. 2% steady-state annual inflation with KPR preference						
Output	Expansion	0.477	0.467	0.446	Expansion	0.477	0.467	0.446
Multiplier	Recession	0.646	0.598	0.529	Recession	0.512	0.480	0.455
Consumption	Expansion	-0.523	-0.533	-0.554	Expansion	-0.523	-0.533	-0.554
Multiplier	Recession	-0.354	-0.402	-0.471	Recession	-0.488	-0.520	-0.545

Notes. This table reports the cumulative output and consumption multipliers in an expansion and a recession with 2% steady-state annual inflation under GHH and KPR preferences.. The cumulative output and consumption multipliers are calculated as $\Sigma_{i=0}^{i=k-1} \frac{\Delta y_{t+i}}{(1+r_{t+i})} \big/ \Sigma_{i=0}^{i=k-1} \frac{\Delta g_{t+i}}{(1+r_{t+i})}$ and $\Sigma_{i=0}^{i=k-1} \frac{\Delta c_{t+i}}{(1+r_{t+i})} \big/ \Sigma_{i=0}^{i=k-1} \frac{\Delta g_{t+i}}{(1+r_{t+i})}$, respectively, where Δ denotes the level differences of each variable with and without government spending and r_t is the real interest rate.

While demand and supply shocks generate deviations from steady-state inflation in opposite direction, we also consider the importance of the level of steady-state inflation. Table A.2 shows the cumulative output and consumption multipliers when we consider a 2% annual steady-state inflation. The main results hold qualitatively: notably that the output and consumption multipliers are higher in a demand-driven recession compared to an expansion for both GHH and KPR preferences. For KPR preferences, shown in the bottom panel, similar to the zero steady-state inflation case, the multiplier in a supply-driven recession, while larger than in an expansion, is smaller than in a demand-driven recession.

The positive steady-state inflation multipliers are overall smaller than the zero steady-state inflation multipliers. This is because non-zero steady-state inflation in a New Keynesian model leads to a rise in price dispersion and a loss in labor efficiency in response to exogenous shocks. With GHH preferences, the increase in inefficient price dispersion due to an increase in government spending limits the expansionary effects on output significantly. As a result, the government spending multipliers are much smaller with a 2% steady-state annual inflation rate. However, with KPR preference, labor supply also responds to government spending shocks which partially offsets the impact on aggregate demand due to a change in price dispersion. Consequently, there are smaller differences in the size of the multipliers across zero and non-zero steady-state inflation.

A.2.4 Robustness to alternative degree of nominal rigidity

We also consider an alternative degree of downward nominal wage rigidity and price rigidity. Once we assume a more downwardly flexible wage ($\gamma = 0.96$), it results in a lower multiplier in a demand-driven recession, as shown in Panel A of Table A.3. In contrast, a more rigid wage rigidity assumption ($\gamma = 0.99$) leads to higher multipliers in both recessions than the baseline case, reported in Panel B of Table A.3. The main results still hold that the multiplier in a demand-driven recession is higher than the multipliers in a supply-driven recession and expansion. These results confirm that the extent of binding DNWR is one of the key determinants of the size of multipliers.

Table A.3: Cumulative output and consumption multipliers by the source of fluctuation under alternative degree of wage and price rigidity

		Demand-driven business cycle				Supply-driven business cycle		
		Impact	4 quarters	20 quarters		Impact	4 quarters	20 quarters
				A. Less rigid DNWR ($\gamma = 0.96$)				
Output	Expansion	0.535	0.535	0.535	Expansion	0.535	0.535	0.535
Multiplier	Recession	1.124	0.733	0.649	Recession	0.535	0.535	0.535
Conumption	Expansion	-0.465	-0.465	-0.465	Expansion	-0.465	-0.465	-0.465
Multiplier	Recession	0.124	-0.267	-0.351	Recession	-0.465	-0.465	-0.465
				B. More rigid DNWR ($\gamma = 0.99$)				
Output	Expansion	0.535	0.535	0.535	Expansion	0.535	0.535	0.535
Multiplier	Recession	3.046	2.683	1.849	Recession	1.128	0.734	0.650
Conumption	Expansion	-0.465	-0.465	-0.465	Expansion	-0.465	-0.465	-0.465
Multiplier	Recession	2.046	1.683	0.849	Recession	0.128	-0.266	-0.350
				C. Less rigid prices ($\omega = 0.65$)				
Output	Expansion	0.259	0.259	0.259	Expansion	0.259	0.259	0.259
Multiplier	Recession	1.868	1.067	0.727	Recession	0.259	0.259	0.259
Conumption	Expansion	-0.741	-0.741	-0.741	Expansion	-0.741	-0.741	-0.741
Multiplier	Recession	0.868	0.067	-0.273	Recession	-0.741	-0.741	-0.741
				D. More rigid prices ($\omega = 0.85$)				
Output	Expansion	1.269	1.269	1.269	Expansion	1.269	1.269	1.269
Multiplier	Recession	2.397	1.968	1.674	Recession	1.269	1.269	1.269
Conumption	Expansion	0.269	0.269	0.269	Expansion	0.269	0.269	0.269
Multiplier	Recession	1.397	0.968	0.674	Recession	0.269	0.269	0.269

Notes. This table reports the cumulative output and consumption multipliers in an expansion and a recession depending on the source of fluctuation. The cumulative output and consumption multipliers are calculated as $\Sigma_{i=0}^{i=k-1} \frac{\Delta y_{t+i}}{(1+r_{t+i})} \Big/ \Sigma_{i=0}^{i=k-1} \frac{\Delta g_{t+i}}{(1+r_{t+i})}$ and $\Sigma_{i=0}^{i=k-1} \frac{\Delta c_{t+i}}{(1+r_{t+i})} \Big/ \Sigma_{i=0}^{i=k-1} \frac{\Delta g_{t+i}}{(1+r_{t+i})}$, respectively, where Δ denotes the level differences of each variable with and without government spending and r_t is the real interest rate.

When considering the dynamics of real wages in a recession, the degree of price rigidity also matters in determining the government spending multiplier. Panel C and D of Table A.3 reports the government spending multipliers with less and more rigid prices than the benchmark case, respectively. Overall, the government spending multipliers are larger in an economy with higher price rigidity, which is seen in a standard New Keynesian model also. With higher price rigidity, an increase in spending raises prices less and labor demand shifts out more due to increased public spending demand, leading to a larger increase in output. This increased price rigidity combined with GHH preferences in Panel C lead to an output multiplier larger than 1, even in an expansion. However, in our specific simulations, price rigidities also matter for the real wage dynamics with DNWR binding. Our results that the spending multipliers are higher in a demand-driven recession are robust for different degrees of price rigidity. In our baseline case, the multiplier in a demand recession is close to 70% larger than in a supply driven recession. With less rigid prices, the multiplier is over 85% larger, since a lower degree of price rigidity further amplifies the difference in real wage response when the DNWR is binding or not. The difference in the multipliers across the two types of recessions shrinks when we have more rigid prices, where the demand recession multiplier is about 45% larger than a supply recession/ expansion multiplier.

A.3 Appendix: Time series empirical evidence

Our data set constitutes of quarterly data for the U.S. spanning 1889Q1-2015Q4. We define inflation as year-over-year growth of the GDP deflator, and use data for GDP, unemployment rate, government spending and GDP deflator from Ramey and Zubairy (2018). Our baseline measure of narrative military news variables also comes from Ramey and Zubairy (2018). When we use taxes as a control variable, we use tax revenues as a share of GDP as a control. This series also comes from Ramey and Zubairy (2018).

When we consider real interest rate as a control variable, we construct the real rate as3 month T-bill rate minus YoY GDP deflator inflation. Since the interpolated series for the T-bill rate is available only from 1915q1 onward, the regressions with real interest rates as controls are run on a shorter sample than our baseline results.

Table A.4: State-dependent fiscal multipliers for output: military news shocks (Robustness to potential GDP)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	2-year integral output				4-year integral output			
Σg_t	0.67				0.71			
	(0.05)				(0.03)			
$\Sigma g_t \times \mathbb{I}(L(u_{t-1}))$		0.64		0.64		0.68		0.68
		(0.08)		(0.08)		(0.10)		(0.10)
$\Sigma g_t \times \mathbb{I}(H(u_{t-1}))$		0.61				0.69		
		(0.09)				(0.05)		
$\Sigma g_t \times \mathbb{I}(L(\pi_{t-1}))$			0.66				0.63	
			(0.09)				(0.07)	
$\Sigma g_t \times \mathbb{I}(H(\pi_{t-1}))$			0.65				0.75	
			(0.05)				(0.04)	
$\Sigma g_t \times \mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1}))$				0.96				0.78
				(0.18)				(0.05)
$\Sigma g_t \times \mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				-0.13				0.28
				(0.11)				(0.06)
P-value from the test								
$\mathbb{I}(L(u_{t-1})) = \mathbb{I}(H(u_{t-1}))$		0.83				0.95		
$\mathbb{I}(L(\pi_{t-1})) = \mathbb{I}(H(\pi_{t-1}))$			0.88				0.14	
$\mathbb{I}(L(u_{t-1})) = \mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1}))$				0.13				0.47
$\mathbb{I}(L(u_{t-1})) = \mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				0.00				0.00
$\mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1})) = \mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				0.00				0.00
Effective first-stage F statistics								
Linear	20.59				11.55			
$\mathbb{I}(L(u_{t-1}))$		10.23		8.82		15.01		14.58
$\mathbb{I}(H(u_{t-1}))$		382.96				122.45		
$\mathbb{I}(L(\pi_{t-1}))$			10.62				5.24	
$\mathbb{I}(H(\pi_{t-1}))$			72.43				46.82	
$\mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1}))$				74.53				305.02
$\mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				52.23				75.99
Observations	493	493	493	493	485	485	485	485

Notes: The top panel reports the 2 and 4 year cumulative multiplier along with associated heteroskedasticity and autocorrelation robust standard errors below. $\mathbb{I}(L(u_{t-1}))$ and $\mathbb{I}(H(u_{t-1}))$ indicate the state where lagged unemployment is low and high, respectively. Potential GDP is constructed using the CBO measure of potential GDP with an interpolated measure of potential GDP for early years. The second panel shows p-values testing whether the multipliers are statistically significantly different across states. The last panel reports the Olea and Pflueger (2013) effective first-stage F statistics for military news as an instrument at 2 and 4 year horizons for the relevant subsample. We use the threshold for the 5 percent critical value for testing the null hypothesis that the TSLS bias exceeds 10 percent of the worst-case TSLS bias. For one instrument, this threshold is always 23.1.

Table A.5: State-dependent fiscal multipliers: military news shocks (Robustness to inflation thresholds: 3%)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	2-year integral output				4-year integral output			
Σg_t	0.66				0.71			
	(0.07)				(0.04)			
$\Sigma g_t \times \mathbb{I}(L(u_{t-1}))$		0.59		0.59		0.67		0.67
		(0.09)		(0.09)		(0.12)		(0.12)
$\Sigma g_t \times \mathbb{I}(H(u_{t-1}))$		0.60				0.68		
		(0.09)				(0.05)		
$\Sigma g_t \times \mathbb{I}(L(\pi_{t-1}))$			0.83				0.73	
			(0.12)				(0.08)	
$\Sigma g_t \times \mathbb{I}(H(\pi_{t-1}))$			0.61				0.71	
			(0.05)				(0.04)	
$\Sigma g_t \times \mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1}))$				1.07				0.76
				(0.22)				(0.06)
$\Sigma g_t \times \mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				0.42				0.59
				(0.13)				(0.07)
P-value from the test								
$\mathbb{I}(L(u_{t-1})) = \mathbb{I}(H(u_{t-1}))$		0.95				0.92		
$\mathbb{I}(L(\pi_{t-1})) = \mathbb{I}(H(\pi_{t-1}))$			0.10				0.84	
$\mathbb{I}(L(u_{t-1})) = \mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1}))$				0.07				0.55
$\mathbb{I}(L(u_{t-1})) = \mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				0.25				0.56
$\mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1})) = \mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				0.01				0.06
Effective first-stage F statistics								
Linear	19.38				11.22			
$\mathbb{I}(L(u_{t-1}))$		8.44		8.23		10.85		10.55
$\mathbb{I}(H(u_{t-1}))$		403.28				130.20		
$\mathbb{I}(L(\pi_{t-1}))$			4.90				4.26	
$\mathbb{I}(H(\pi_{t-1}))$			109.31				38.65	
$\mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1}))$				350.03				228.48
$\mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				279.98				330.42
Observations	493	493	493	493	485	485	485	485

Notes: The top panel reports the 2 and 4 year cumulative multiplier along with associated heteroskedasticity and autocorrelation robust standard errors below. $\mathbb{I}(L(\pi_{t-1}))$ and $\mathbb{I}(H(\pi_{t-1}))$ represent the state where lagged inflation is lower and higher than 3%, respectively. The second panel shows p-values testing whether the multipliers are statistically significantly different across states. The last panel reports the Olea and Pflueger (2013) effective first-stage F statistics for military news as an instrument at 2 and 4 year horizons for the relevant subsample. We use the threshold for the 5 percent critical value for testing the null hypothesis that the TSLS bias exceeds 10 percent of the worst-case TSLS bias. For one instrument, this threshold is always 23.1.

Table A.6: State-dependent fiscal multipliers for output: military news shocks (Robustness to time-varying thresholds)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	2-year integral output				4-year integral output			
Σg_t	0.66				0.71			
	(0.07)				(0.04)			
$\Sigma g_t \times \mathbb{I}(L(u_{t-1}))$		0.66		0.66		0.75		0.75
		(0.18)		(0.18)		(0.26)		(0.26)
$\Sigma g_t \times \mathbb{I}(H(u_{t-1}))$		0.52				0.56		
		(0.08)				(0.08)		
$\Sigma g_t \times \mathbb{I}(L(\pi_{t-1}))$			0.78				0.69	
			(0.11)				(0.08)	
$\Sigma g_t \times \mathbb{I}(H(\pi_{t-1}))$			0.58				0.73	
			(0.04)				(0.04)	
$\Sigma g_t \times \mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1}))$				0.78				0.58
				(0.26)				(0.15)
$\Sigma g_t \times \mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				0.26				0.52
				(0.07)				(0.15)
P-value from the test								
$\mathbb{I}(L(u_{t-1})) = \mathbb{I}(H(u_{t-1}))$		0.45				0.51		
$\mathbb{I}(L(\pi_{t-1})) = \mathbb{I}(H(\pi_{t-1}))$			0.10				0.67	
$\mathbb{I}(L(u_{t-1})) = \mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1}))$				0.71				0.57
$\mathbb{I}(L(u_{t-1})) = \mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				0.01				0.45
$\mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1})) = \mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				0.04				0.80
Effective first-stage F statistics								
Linear	19.38				11.22			
$\mathbb{I}(L(u_{t-1}))$		15.36		14.93		5.06		4.92
$\mathbb{I}(H(u_{t-1}))$		3.80				2.75		
$\mathbb{I}(L(\pi_{t-1}))$			6.17				4.40	
$\mathbb{I}(H(\pi_{t-1}))$			131.60				38.59	
$\mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1}))$				22.81				9.82
$\mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				19.25				17.83
Observations	493	493	493	493	485	485	485	485

Notes: The top panel reports the 2 and 4 year cumulative multiplier along with associated heteroskedasticity and autocorrelation robust standard errors below. $\mathbb{I}(L(u_{t-1}))$ and $\mathbb{I}(H(u_{t-1}))$ indicate the state where lagged unemployment is low and high, respectively. In this robustness check, we consider time-varying thresholds for both the unemployment rate and inflation. The time-varying trend is based on the HP filter with $\lambda = 10^6$, over a split sample, 1889–1929 and 1947–2015 and linearly interpolated for the small gap in trend unemployment between 1929 and 1947, in order to capture the evolution of the natural rate. The high inflation regime is one where inflation is above a HP filtered trend based on $\lambda = 1600$. The top panel panel reports the 2 and 4 year cumulative multiplier along with associated standard errors below. The second panel shows p-values testing whether the multipliers are statistically significantly different across states. The last panel reports the Olea and Pflueger (2013) effective first-stage F statistics for military news as an instrument at 2 and 4 year horizons for the relevant subsample. We use the threshold for the 5 percent critical value for testing the null hypothesis that the TSLS bias exceeds 10 percent of the worst-case TSLS bias. For one instrument, this threshold is always 23.1.

Table A.7: State-dependent fiscal multipliers for output: military news shocks (Robustness to time varying thresholds for inflation)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	2-year integral output				4-year integral output			
Σg_t	0.66				0.71			
	(0.07)				(0.04)			
$\Sigma g_t \times \mathbb{I}(L(u_{t-1}))$		0.59		0.59		0.67		0.67
		(0.09)		(0.09)		(0.12)		(0.12)
$\Sigma g_t \times \mathbb{I}(H(u_{t-1}))$		0.60				0.68		
		(0.09)				(0.05)		
$\Sigma g_t \times \mathbb{I}(L(\pi_{t-1}))$			0.78				0.69	
			(0.11)				(0.08)	
$\Sigma g_t \times \mathbb{I}(H(\pi_{t-1}))$			0.58				0.73	
			(0.04)				(0.04)	
$\Sigma g_t \times \mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1}))$				1.26				0.82
				(0.27)				(0.08)
$\Sigma g_t \times \mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				0.23				0.58
				(0.07)				(0.06)
P-value from the test								
$\mathbb{I}(L(u_{t-1})) = \mathbb{I}(H(u_{t-1}))$		0.95				0.92		
$\mathbb{I}(L(\pi_{t-1})) = \mathbb{I}(H(\pi_{t-1}))$			0.10				0.67	
$\mathbb{I}(L(u_{t-1})) = \mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1}))$				0.03				0.34
$\mathbb{I}(L(u_{t-1})) = \mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				0.00				0.56
$\mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1})) = \mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				0.00				0.01
Effective first-stage F statistics								
Linear	19.38				11.22			
$\mathbb{I}(L(u_{t-1}))$		8.44		8.23		10.85		10.56
$\mathbb{I}(H(u_{t-1}))$		403.28				130.20		
$\mathbb{I}(L(\pi_{t-1}))$			6.17				4.40	
$\mathbb{I}(H(\pi_{t-1}))$			131.60				38.59	
$\mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1}))$				139.80				90.16
$\mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				619.63				722.32
Observations	493	493	493	493	485	485	485	485

Notes: The top panel reports the 2 and 4 year cumulative multiplier along with associated heteroskedasticity and autocorrelation robust standard errors below. $\mathbb{I}(L(u_{t-1}))$ and $\mathbb{I}(H(u_{t-1}))$ indicate the state where lagged unemployment is low and high, respectively. In this robustness check, we consider time-variant thresholds for only inflation. The high inflation regime is one where inflation is above a HP filtered trend based on $\lambda = 1600$. The second panel shows p-values testing whether the multipliers are statistically significantly different across states. The last panel reports the Olea and Pflueger (2013) effective first-stage F statistics for military news as an instrument at 2 and 4 year horizons for the relevant subsample. We use the threshold for the 5 percent critical value for testing the null hypothesis that the TSLS bias exceeds 10 percent of the worst-case TSLS bias. For one instrument, this threshold is always 23.1.

Table A.8: State-dependent fiscal multipliers for output: both military news and Blanchard-Perotti (2002) as instruments (Robustness to time varying thresholds for inflation)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	2-year integral output				4-year integral output			
Σg_t	0.42				0.56			
	(0.10)				(0.08)			
$\Sigma g_t \times \mathbb{I}(L(u_{t-1}))$		0.33				0.39		
		(0.11)				(0.11)		
$\Sigma g_t \times \mathbb{I}(H(u_{t-1}))$		0.62				0.68		
		(0.09)				(0.05)		
$\Sigma g_t \times \mathbb{I}(L(\pi_{t-1}))$			0.47				0.49	
			(0.09)				(0.10)	
$\Sigma g_t \times \mathbb{I}(H(\pi_{t-1}))$			0.48				0.67	
			(0.07)				(0.07)	
$\Sigma g_t \times \mathbb{I}(L(u_{t-1}))\mathbb{I}(L(\pi_{t-1}))$				0.43				0.30
				(0.15)				(0.20)
$\Sigma g_t \times \mathbb{I}(L(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				0.42				0.55
				(0.07)				(0.12)
$\Sigma g_t \times \mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1}))$				0.92				0.81
				(0.23)				(0.07)
$\Sigma g_t \times \mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				0.56				0.80
				(0.20)				(0.13)
P-value from the test								
$\mathbb{I}(L(u_{t-1})) = \mathbb{I}(H(u_{t-1}))$		0.10				0.02		
$\mathbb{I}(L(\pi_{t-1})) = \mathbb{I}(H(\pi_{t-1}))$			0.94				0.09	
$\mathbb{I}(L(u_{t-1}))\mathbb{I}(L(\pi_{t-1})) = \mathbb{I}(L(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				0.93				0.21
$\mathbb{I}(L(u_{t-1}))\mathbb{I}(L(\pi_{t-1})) = \mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1}))$				0.60				0.06
$\mathbb{I}(L(u_{t-1}))\mathbb{I}(H(\pi_{t-1})) = \mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1}))$				0.05				0.06
$\mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1})) = \mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				0.22				0.96
Effective first-stage F statistics								
Linear	36.70				14.44			
	13.19				15.46			
$\mathbb{I}(L(u_{t-1}))$		48.92				39.63		
		[17.04]				[17.05]		
$\mathbb{I}(H(u_{t-1}))$		61.56				69.75		
		[19.27]				[10.83]		
$\mathbb{I}(L(\pi_{t-1}))$			36.59				18.75	
			[20.81]				[15.56]	
$\mathbb{I}(H(\pi_{t-1}))$			80.18				24.85	
			[15.24]				[18.13]	
$\mathbb{I}(L(u_{t-1}))\mathbb{I}(L(\pi_{t-1}))$				51.67				20.15
				[17.49]				[15.63]
$\mathbb{I}(L(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				54.60				23.38
				[7.57]				[10.05]
$\mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1}))$				48.93				33.64
				[16.48]				[17.50]
$\mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				53.41				46.18
				[20.99]				[21.11]
Observations	493	493	493	493	485	485	485	485

Notes: The top panel reports the 2 and 4 year cumulative multiplier along with associated heteroskedasticity and autocorrelation robust standard errors below. $\mathbb{I}(L(u_{t-1}))$ and $\mathbb{I}(H(u_{t-1}))$ indicate the state where lagged unemployment is low and high, respectively. In this robustness check, we consider time-variant thresholds for only inflation. The high inflation regime is one where inflation is above a HP filtered trend based on $\lambda = 1600$. The second panel shows p-values testing whether the multipliers are statistically significantly different across states. The last panel reports the Olea and Pflueger (2013) effective first-stage F statistics for military news and Blanchard and Perotti (2002) shocks jointly as instruments at 2 and 4 year horizons for the relevant subsample.The numbers in brackets provide the 5 percent critical value used to

Table A.9: State-dependent fiscal multipliers for output: military news shocks (Controlling for taxes)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	2-year integral output				4-year integral output			
Σg_t	0.66				0.72			
	(0.07)				(0.05)			
$\Sigma g_t \times \mathbb{I}(L(u_{t-1}))$		0.54		0.54		0.60		0.60
		(0.11)		(0.11)		(0.15)		(0.15)
$\Sigma g_t \times \mathbb{I}(H(u_{t-1}))$		0.67				0.69		
		(0.12)				(0.08)		
$\Sigma g_t \times \mathbb{I}(L(\pi_{t-1}))$			0.79				0.68	
			(0.14)				(0.10)	
$\Sigma g_t \times \mathbb{I}(H(\pi_{t-1}))$			0.54				0.61	
			(0.11)				(0.10)	
$\Sigma g_t \times \mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1}))$				0.99				0.80
				(0.20)				(0.06)
$\Sigma g_t \times \mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				-0.12				0.24
				(0.12)				(0.08)
P-value from the test								
$\mathbb{I}(L(u_{t-1})) = \mathbb{I}(H(u_{t-1}))$		0.47				0.64		
$\mathbb{I}(L(\pi_{t-1})) = \mathbb{I}(H(\pi_{t-1}))$			0.18				0.56	
$\mathbb{I}(L(u_{t-1})) = \mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1}))$				0.07				0.27
$\mathbb{I}(L(u_{t-1})) = \mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				0.00				0.03
$\mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1})) = \mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				0.00				0.00
Effective first-stage F statistics								
Linear	18.73				11.56			
$\mathbb{I}(L(u_{t-1}))$		8.41		8.12		15.98		15.51
$\mathbb{I}(H(u_{t-1}))$		118.25				72.91		
$\mathbb{I}(L(\pi_{t-1}))$			8.07				4.59	
$\mathbb{I}(H(\pi_{t-1}))$			58.30				100.57	
$\mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1}))$				72.26				222.68
$\mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				42.84				75.31
Observations	493	493	493	493	485	485	485	485

Notes: The top panel reports the 2 and 4 year cumulative multiplier along with associated heteroskedasticity and autocorrelation robust standard errors below. $\mathbb{I}(L(u_{t-1}))$ and $\mathbb{I}(H(u_{t-1}))$ indicate the state where lagged unemployment is low and high, respectively. These are defined as our baseline specification, shown in Table **??**. We include average tax rates as additional controls on the right hand side.

Table A.10: State-dependent fiscal multipliers for output: military news shocks (1915-2015)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	2-year integral output				4-year integral output			
Σg_t	0.70				0.74			
	(0.08)				(0.05)			
$\Sigma g_t \times \mathbb{I}(L(u_{t-1}))$		0.69		0.69		0.68		0.68
		(0.08)		(0.08)		(0.14)		(0.14)
$\Sigma g_t \times \mathbb{I}(H(u_{t-1}))$		0.55				0.68		
		(0.12)				(0.07)		
$\Sigma g_t \times \mathbb{I}(L(\pi_{t-1}))$			0.74				0.70	
			(0.15)				(0.12)	
$\Sigma g_t \times \mathbb{I}(H(\pi_{t-1}))$			0.61				0.69	
			(0.07)				(0.05)	
$\Sigma g_t \times \mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1}))$				0.63				0.73
				(0.30)				(0.12)
$\Sigma g_t \times \mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				-0.62				0.03
				(0.46)				(0.30)
P-value from the test								
$\mathbb{I}(L(u_{t-1})) = \mathbb{I}(H(u_{t-1}))$		0.31				0.98		
$\mathbb{I}(L(\pi_{t-1})) = \mathbb{I}(H(\pi_{t-1}))$			0.46				0.94	
$\mathbb{I}(L(u_{t-1})) = \mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1}))$				0.84				0.80
$\mathbb{I}(L(u_{t-1})) = \mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				0.00				0.04
$\mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1})) = \mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				0.02				0.03
Effective first-stage F statistics								
Linear	12.68				9.94			
$\mathbb{I}(L(u_{t-1}))$		6.40		6.17		7.92		7.54
$\mathbb{I}(H(u_{t-1}))$		242.24				76.60		
$\mathbb{I}(L(\pi_{t-1}))$			5.53				4.10	
$\mathbb{I}(H(\pi_{t-1}))$			37.96				45.29	
$\mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1}))$				61.74				109.23
$\mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				31.37				41.40
Observations	393	393	393	393	385	385	385	385

Notes: The top panel panel reports the 2 and 4 year cumulative multiplier along with associated heteroskedasticity and autocorrelation robust standard errors below. $\mathbb{I}(L(u_{t-1}))$ and $\mathbb{I}(H(u_{t-1}))$ indicate the state where lagged unemployment is low and high, respectively. These are defined as our baseline specification, shown in Table **??**. The sample under consideration is shorter than our baseline to make comparison with the specification with real interest rates as additional control easier, and spans 1915-2015.

Table A.11: State-dependent fiscal multipliers for output: military news shocks (1915-2015, Controlling for real interest rates)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	2-year integral output				4-year integral output			
Σg_t	0.70				0.75			
	(0.08)				(0.07)			
$\Sigma g_t \times \mathbb{I}(L(u_{t-1}))$		0.69		0.69		0.77		0.77
		(0.14)		(0.12)		(0.14)		(0.14)
$\Sigma g_t \times \mathbb{I}(H(u_{t-1}))$		0.51				0.64		
		(0.13)				(0.09)		
$\Sigma g_t \times \mathbb{I}(L(\pi_{t-1}))$			0.73				0.68	
			(0.16)				(0.13)	
$\Sigma g_t \times \mathbb{I}(H(\pi_{t-1}))$			0.65				0.74	
			(0.05)				(0.03)	
$\Sigma g_t \times \mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1}))$				0.54				0.67
				(0.28)				(0.13)
$\Sigma g_t \times \mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				-0.28				0.11
				(0.37)				(0.22)
P-value from the test								
$\mathbb{I}(L(u_{t-1})) = \mathbb{I}(H(u_{t-1}))$		0.34				0.36		
$\mathbb{I}(L(\pi_{t-1})) = \mathbb{I}(H(\pi_{t-1}))$			0.66				0.65	
$\mathbb{I}(L(u_{t-1})) = \mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1}))$				0.61				0.59
$\mathbb{I}(L(u_{t-1})) = \mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				0.01				0.01
$\mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1})) = \mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				0.08				0.03
Effective first-stage F statistics								
Linear	14.53				10.23			
$\mathbb{I}(L(u_{t-1}))$		4.29		5.59		7.72		7.46
$\mathbb{I}(H(u_{t-1}))$		283.90				14.13		
$\mathbb{I}(L(\pi_{t-1}))$			4.83				4.20	
$\mathbb{I}(H(\pi_{t-1}))$			67.13				42.64	
$\mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1}))$				57.33				142.65
$\mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$				34.98				41.72
Observations	392	392	392	392	384	384	384	384

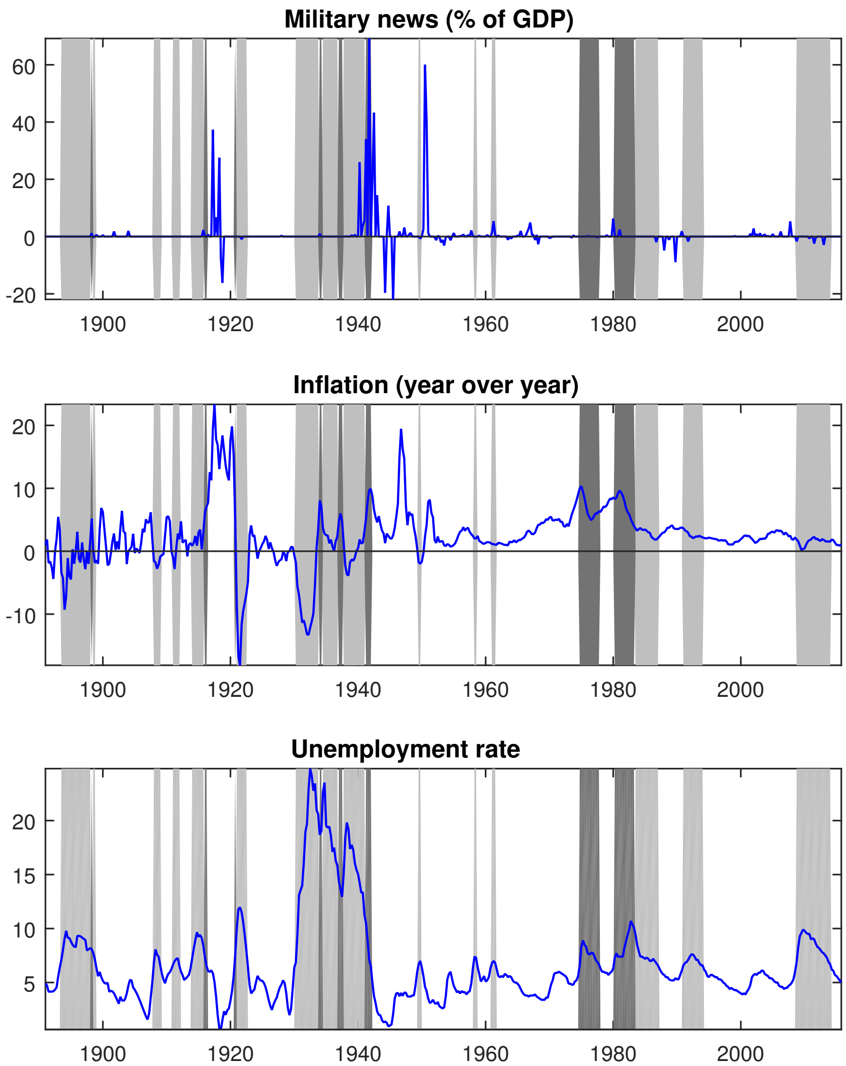
Notes: The top panel reports the 2 and 4 year cumulative multiplier along with associated heteroskedasticity and autocorrelation robust standard errors below. $\mathbb{I}(L(u_{t-1}))$ and $\mathbb{I}(H(u_{t-1}))$ indicate the state where lagged unemployment is low and high, respectively. These are defined as our baseline specification, shown in Table **??**. The sample under consideration is shorter than our baseline and spans 1915-2015. We include real interest rates as additional controls on the right hand side.

Table A.12: Response of unemployment, inflation and interest rate

	2 year horizon response			
	Unemployment	Inflation	Tbill rate	Real rate
$\Sigma g_t \times \mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1}))$	-0.1971	0.4699	-0.7946	-0.5609
	(0.1465)	(0.1176)	(3.6384)	(0.1791)
$\Sigma g_t \times \mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$	0.0784	0.1599	8.5678	-0.000
	(0.0833)	(0.2370)	(19.788)	(0.3399)
P-value from test $\mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1})) = \mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$	0.12	0.23	0.70	0.15
Observation	493	493	389	389
	4 year horizon response			
	Unemployment	Inflation	Tbill rate	Real rate
$\Sigma g_t \times \mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1}))$	-0.1880	0.1553	0.6424	-0.1716
	(0.0458)	(0.0455)	(1.7783)	(0.0458)
$\Sigma g_t \times \mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$	0.0133	0.0963	-7.5802	0.2179
	(0.0530)	(0.1479)	(10.947)	(0.1634)
P-value from test $\mathbb{I}(H(u_{t-1}))\mathbb{I}(L(\pi_{t-1})) = \mathbb{I}(H(u_{t-1}))\mathbb{I}(H(\pi_{t-1}))$	0.01	0.69	0.46	0.02
Observation	485	485	381	381

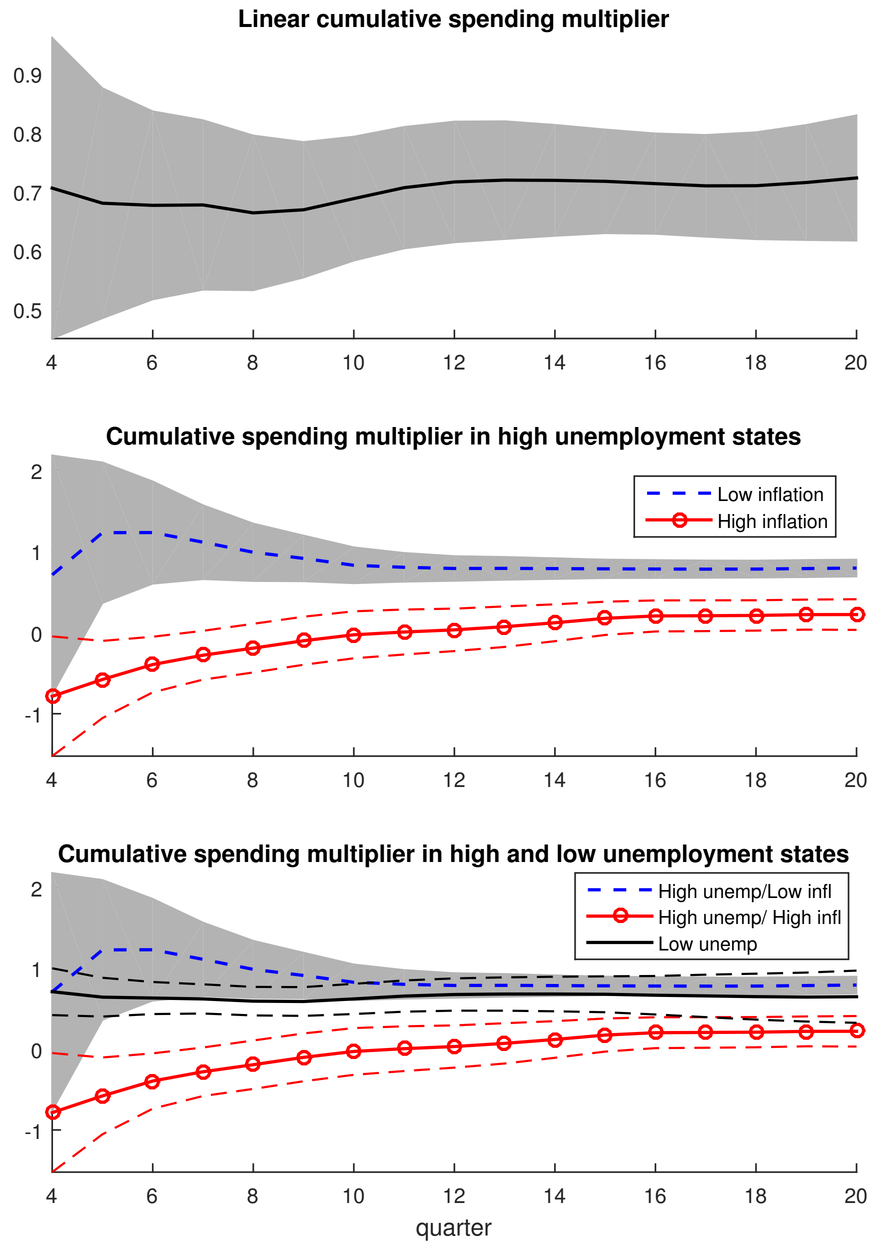
Notes: The top panel reports the 2 and 4 year cumulative responses of unemployment, inflation, Tbill rate, and real interest rate along with associated heteroskedasticity and autocorrelation robust standard errors below. $\mathbb{I}(L(u_{t-1}))$ and $\mathbb{I}(H(u_{t-1}))$ indicate the state where lagged unemployment is low and high, respectively. These are defined as our baseline specification, shown in Table **??**.

Figure A.6: Inflation and unemployment states for U.S. historical data



Notes: Military spending news, year over year GDP deflator inflation rate and the unemployment rate. The shaded areas indicate periods when the unemployment rate is above the threshold of 6.5 percent. The light and dark gray areas correspond with periods where inflation is below and above a threshold of 4%, which corresponds to the 75th percentiles of inflation for our full sample.

Figure A.7: State dependent fiscal multipliers: military news shocks



Notes: These figures show the cumulative multiplier for output in response to a military news shock from 4 quarters after the shock hits the economy. The top panel shows the cumulative multiplier in a linear model. The middle panel shows the state-dependent multiplier in high unemployment/ low inflation (blue dashed) and high unemployment/ high inflation (red circles) states. The bottom panel shows the state dependent multipliers in low unemployment (black solid), high unemployment/ low inflation (blue dashed) and high unemployment/ high inflation (red circles) states. 95 percent confidence intervals are shown in all cases.

A.4 Appendix: US state-level empirical evidence

The state-level annual data sample starts in 1969 and ends in 2018. State-level nominal GDP is from the US Bureau of Economic Analysis (BEA). In calculating real GDP, we use the US aggregate Consumer Price Index (CPI) to deflate nominal GDP followed by BEA - calculating state-level GDP by applying national price deflator to state-level nominal GDP. State-level employment is from Current Employment Statistics (CES) by the Bureau of Labor Statistics (BLS) and the state-level population is available from the US Census Bureau. We use state-level inflation data constructed by Nakamura and Steinsson (2014) from 1969 to 2008 and later by Zidar (2019) up to 2014. We further extend the state-level inflation from Regional Price Parity (RPP) from Census until 2018.[8]

For state-level military spending, we use data from prime military contracts awarded by the Department of Defense (DOD). Each individual contractor of DOD reports their contract details using DD Form 350, including the service or product supplies, date awarded, principal place of performance, and information about the DOD agency. For each fiscal year between 1966 and 2000, we rely on state-level military prime contract data constructed by Nakamura and Steinsson (2014) For the remaining sample period from 2001 until 2018, we use electronic DD Form 350 data available from www.USAspending.gov.

In order to identify the state-dependent spending multipliers, we add state-level changes in military spending interacted with indicator variables ($\mathbb{I}(\cdot)$), which provide information on US-states-years corresponding to the state of the economy. Note that the estimated multipliers with regional data are not directly comparable to the closed economy aggregate multipliers from the time series evidence in Section **??**. The estimates from the regional analysis, open economy relative multipliers, measure the effect of an increase in government spending in one state relative to another state. However, these regional multipliers are useful in testing whether the effectiveness of fiscal policy depends on the US-state-differential conditions of the economy.

[8]Before 1995, Nakamura and Steinsson (2014) use state-level price indices constructed by Del Negro (1998) from 1969. After 1995, both papers by Nakamura and Steinsson (2014) and Zidar (2019) use county and metro level Cost of Living Index (COLI) published by the American Chamber of Commerce Researchers Association (ACCRA), later renamed as Council for Community and Economic Research (C2ER). As regional level COLI is designed to capture differences in price levels across regions within a year, Nakamura and Steinsson (2014) computed the state-level price indices by multiplying population-weighted COLI from the ACCRA for each state with the US aggregate CPI. We applied for the same procedure to calculate the state-level price indices using the state-level COLI provided by Zidar (2019) and RPP from Census. There are a few missing US state-level inflation observations from Hawaii, Maine, New Hampshire, New Jersey, Rhode Island, and Vermont. We drop those US state-year observations if inflation data is missing.

We divide the state of economy based on the level of employment, inflation, and DNWR. The indicator variable for low employment, $\mathbb{I}(\mathrm{L}(e_{it}))$ is one when the HP-filtered cyclical component of state-level employment to population ratio (e_{it}) is lower than 25th percentile of its distribution across US-states-and-years and zero otherwise. In addition, $\mathbb{I}(\mathrm{H}(\pi_{it}))$ indicates high inflation US-states-years, which takes the value of one if biannual state-level inflation (π_{it}) is greater than 75th percentile of its distribution and zero otherwise. Lastly, the dummy variable $\mathbb{I}(\mathrm{H}(\mathrm{DNWR}_{it}))$ indicates US-states-years when more workers have the binding DNWR constraints. $\mathbb{I}(\mathrm{H}(\mathrm{DNWR}_{it}))$ is one when the biannual changes in the state-level differences between the share of workers with zero wage and the share of workers with wage cut is higher than 75% percentile from its distribution across states and years from 1979 to 2018.[9]

[9]Note that the regression specification does not include the level of state-level inflation and the cyclical component of employment themselves but contains dummy variables indicating a high inflation period or a low employment period. This specification is useful to avoid potential measurement errors in state-level measures of inflation, employment, and DNWR. For example, the level of inflation from our data set differs slightly from the one from Hazell, Herreño, Nakamura, and Steinsson (2022) but the indicator of high inflation is almost the same across the two. Our main results are also robust to using the Hazell, Herreño, Nakamura, and Steinsson (2022) state-level inflation data set.

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