Online Appendix to:
Trade, Domestic Frictions, and Scale Effects
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A Proof of Proposition 1

Using (7), labor market clearing (3) implies that for $m \in \Omega_i$ we have

$$v_m = \left(\frac{t_m}{l_m}\right)^{1/(1+\theta)} \Delta_i, \tag{A1}$$

where

$$\Delta_i^{1+\theta} = \sum_n \sum_{k \in \Omega_n} \frac{\tau_{ni}^{-\theta}}{\sum_j \sum_{l \in \Omega_j} t_l v_l^{-\theta} \tau_{nj}^{-\theta}} v_k l_k. \tag{A2}$$

Plugging ($A1$) back into (A2) and using the definition of T_i in (6), after simplifications , a system of equations in Δ_i for $i = 1, ..., N$,

$$\Delta_i^{1+\theta} = \sum_n \frac{\tau_{ni}^{-\theta}}{\sum_j T_j^{1/(1+\theta)} L_j^{\theta/(1+\theta)} \Delta_j^{-\theta} \tau_{nj}^{-\theta}} T_n^{1/(1+\theta)} L_n^{\theta/(1+\theta)} \Delta_n.$$

Plugging ($A1$) into (1), and using (7), $\tau_{nn} = 1$ for all n, as well as the definition of X_{ni}, yield, after simplifications,

$$X_{ni} = \frac{T_i^{1/(1+\theta)} L_i^{\theta/(1+\theta)} \Delta_i^{-\theta} \tau_{nj}^{-\theta}}{\sum_j T_j^{1/(1+\theta)} L_j^{\theta/(1+\theta)} \Delta_j^{-\theta} \tau_{nj}^{-\theta}} T_n^{1/(1+\theta)} L_n^{\theta/(1+\theta)} \Delta_n. \tag{A3}$$

Also note that

$$\frac{X_i}{L_i} = \frac{\sum_n X_{ni}}{L_i} = \sum_n \frac{T_i^{1/(1+\theta)} L_i^{\theta/(1+\theta)} \Delta_i^{-\theta} \tau_{nj}^{-\theta}}{\sum_j T_j^{1/(1+\theta)} L_j^{\theta/(1+\theta)} \Delta_j^{-\theta} \tau_{nj}^{-\theta}} T_n^{1/(1+\theta)} L_n^{\theta/(1+\theta)} \Delta_n = (T_i/L_i)^{1/(1+\theta)} \Delta_i.$$

Our definition $w_i \equiv X_i/L_i$ implies that

$$w_i = (T_i/L_i)^{1/(1+\theta)} \Delta_i. \tag{A4}$$

Plugging this expression for w_i into ($A3$) yields (4). From ($A1$), and (2) and (7), with $\tau_{nn} = 1$ for all n, we have for region $m \in \Omega_n$

$$p_m = \gamma^{-1} \left(\sum_{k \in \Omega_i} t_k^{1/(1+\theta)} l_k^{\theta/(1+\theta)} \Delta_i^{-\theta} \tau_{ni}^{-\theta} \right)^{-1/\theta}.$$

Using ($A4$) and the expression for T_i in (6), we get (5). Finally, combining expressions (4) and (5), and using $\lambda_{nn} \equiv X_{nn}/X_n$ and $\tau_{nn} = 1$, we get the expression in (8) for real wages under frictionless internal trade.

B Proof of Proposition 2

Replacing (1) in the paper into $X_{ni} \equiv \sum_{m \in \Omega_n} \sum_{k \in \Omega_i} x_{mk}$, and using A2, (7), and $t_k = T_i/M_i$ for $k \in \Omega_i$, and $x_m = X_n/M_n$ for $m \in \Omega_n$, we get, for $n \neq l$,

$$X_{ni} = \sum_{m \in \Omega_n} \frac{T_i w_i^{-\theta} \tau_{ni}^{-\theta}}{\sum_{j \neq n} T_j w_j^{-\theta} \tau_{nj}^{-\theta} + (M_n - 1)(T_n/M_n) w_n^{-\theta} \delta_n^{-\theta} + (T_n/M_n) w_n^{-\theta}} \frac{X_n}{M_n},$$

while

$$X_{nn} = \sum_{m \in \Omega_n} \frac{(M_n - 1)(T_n/M_n) w_n^{-\theta} \delta_n^{-\theta} + (T_n/M_n) w_n^{-\theta}}{\sum_{j \neq n} T_j w_j^{-\theta} \tau_{nj}^{-\theta} + (M_n - 1)(T_n/M_n) w_n^{-\theta} \delta_n^{-\theta} + (T_n/M_n) w_n^{-\theta}} \frac{X_n}{M_n}.$$

Turning to the price index, we know that for $m \in \Omega_n$ we have $p_m = P_n$. Hence,

$$P_n = \gamma^{-1} \left(\sum_{j \neq n} T_j w_j^{-\theta} \tau_{nj}^{-\theta} + (M_n - 1) \frac{T_n}{M_n} w_n^{-\theta} \delta_n^{-\theta} + \frac{T_n}{M_n} w_n^{-\theta} \right)^{-1/\theta}.$$

Collecting terms and using (11), we get (4) and (5) as in the paper. Finally, combining expressions (4) and (5), and using $\lambda_{nn} \equiv X_{nn}/X_n$, we get the expression for real wages under symmetry in (12), and under A1.

C Proof of Proposition 3

Assumptions A1 and A2 imply that equilibrium wages are determined by the system

$$w_i L_i = \sum_n \frac{L_i w_i^{-\theta} \tau_{ni}^{-\theta}}{\sum_j L_j w_j^{-\theta} \tau_{nj}^{-\theta}} w_n L_n,$$

with

$$\tau_{nn}^{-\theta} = \frac{1}{M_n} + \frac{M_n - 1}{M_n}\delta_n^{-\theta}.$$

Given A3 and letting $\Phi \equiv \sum_j M_j w_j^{-\theta}\tau^{-\theta}$, we then have

$$\begin{aligned} w_i M_i &= \frac{w_i^{-\theta}\left(1-\delta^{-\theta}\right) + w_i^{-\theta} M_i \delta^{-\theta}}{\Phi + w_i^{-\theta}\left(1-\delta^{-\theta} + M_i\left(\delta^{-\theta} - \tau^{-\theta}\right)\right)} w_i M_i \\ &\quad + \sum_{n \neq i} \frac{M_i w_i^{-\theta} \tau^{-\theta}}{\Phi + w_n^{-\theta}\left(1-\delta^{-\theta} + M_n\left(\delta^{-\theta} - \tau^{-\theta}\right)\right)} w_n M_n, \end{aligned}$$

and hence,

$$\frac{w_i^{1+\theta}}{\Phi + w_i^{-\theta}\left(1-\delta^{-\theta} + M_i\left(\delta^{-\theta} - \tau^{-\theta}\right)\right)} = \frac{\tau^{-\theta}\Gamma}{\Phi}, \tag{A5}$$

where $\Gamma \equiv \sum_n \frac{w_n M_n}{\Phi + w_n^{-\theta}\left[1-\delta^{-\theta}+M_n\left(\delta^{-\theta}-\tau^{-\theta}\right)\right]}$. Since $\tau > \delta$, then $\delta^{-\theta} > \tau^{-\theta}$, so that the left-hand side is decreasing in M_i and increasing in w_i. This implies that if $M_i > M_j$ then necessarily $w_i > w_j$: larger countries have higher wages. In contrast, if $\tau = \delta$, then the left-hand side is invariant to M_i and hence w must be common across countries.

To compare import shares across countries in a given equilibrium, note that domestic trade shares are given by

$$\lambda_{ii} = \frac{1 + \left(M_i - 1\right)\delta^{-\theta}}{\Phi w_i^{\theta} + 1 - \delta^{-\theta} + M_i\left(\delta^{-\theta} - \tau^{-\theta}\right)}.$$

Plugging this expression into (A5) and rearranging yields

$$w_i^{1+\theta}\left(1 - \frac{1-\delta^{-\theta} + M_i\left(\delta^{-\theta} - \tau^{-\theta}\right)}{1 + \left(M_i - 1\right)\delta^{-\theta}}\lambda_{ii}\right) = \tau^{-\theta}\Gamma. \tag{A6}$$

Since $w_i > w_j$ when $M_i > M_j$,

$$\frac{1-\delta^{-\theta} + M_i(\delta^{-\theta} - \tau^{-\theta})}{1-\delta^{-\theta} + M_i\delta^{-\theta}}\lambda_{ii} > \frac{1-\delta^{-\theta} + M_j(\delta^{-\theta} - \tau^{-\theta})}{1-\delta^{-\theta} + M_j\delta^{-\theta}}\lambda_{jj}.$$

But since $\frac{1-\delta^{-\theta}+x(\delta^{-\theta}-\tau^{-\theta})}{1-\delta^{-\theta}+x\delta^{-\theta}}$ is decreasing in x, then $M_i > M_j$ also implies that

$$\frac{1-\delta^{-\theta} + M_i(\delta^{-\theta} - \tau^{-\theta})}{1-\delta^{-\theta} + M_i\delta^{-\theta}} < \frac{1-\delta^{-\theta} + M_j(\delta^{-\theta} - \tau^{-\theta})}{1-\delta^{-\theta} + M_j\delta^{-\theta}},$$

and hence $\lambda_{ii} > \lambda_{jj}$.

For price indices, note that

$$(\gamma P_n)^{-\theta} = \sum_j M_j w_j^{-\theta} \tau_{nj}^{-\theta} = \Phi + w_n^{-\theta}\left(1 - \delta^{-\theta} + M_n\left(\delta^{-\theta} - \tau^{-\theta}\right)\right).$$

Hence, ($A5$) implies that

$$w_n^{1+\theta} P_n^{\theta} = \frac{\gamma^{-\theta}\tau^{-\theta}\Gamma}{\Phi}. \tag{A7}$$

Again, since $w_i > w_j$ when $M_i > M_j$, then $P_i < P_j$. Combining the results for wages and price indices, real wages are also increasing in size. Moreover, if $\tau = \delta$, then the result that wages are the same across countries immediately follows from ($A7$), which also implies that the price index is the same across countries.

D Proof of Proposition 4

The result trivially follows from replacing assumptions A4, A4′, and A4″, subsequently, into the expressions in the paper for real wages, trade flows, and price indices in (12), (4) and (5), respectively. The nominal wage follows from multiplying the real wage by the price index.

E Equivalence with Melitz (2003) Model

Assume that productivity draws in each region z_m are from a Pareto distribution with shape parameter θ and lower bound b_m. Replacing the expression for regional trade flows in (1) in the paper into $X_{ni} \equiv \sum_{m\in\Omega_n}\sum_{k\in\Omega_i} x_{mk}$, we get

$$X_{nl} = \sum_{m\in\Omega_n}\sum_{k\in\Omega_l} l_k b_k^{\theta} v_k^{-\theta} d_{mk}^{-\theta} \sum_{k'} l_{k'} b_{k'}^{\theta} v_{k'}^{-\theta} d_{mk'}^{-\theta} x_m.$$

The equivalent of A2 here would be $b_m = b_{m'} = b_n$ for all $m, m' \in \Omega_n$. Replacing, we get

$$X_{nl} = \frac{L_l b_l^{\theta} w_l^{-\theta} \tau_{nl}^{-\theta}}{\sum_j L_j b_j^{\theta} w_j^{-\theta} \tau_{nj}^{-\theta}} X_n,$$

for all n, l, and τ_{nn} defined as in (11). Analogously to the results in Melitz (2003)'s, the productivity cut-off for a region $m \in \Omega_n$ is given by:

$$z_{km}^* = C_0 \left(\frac{f_m}{l_m}\right)^{1/(\sigma-1)} \frac{v_k d_{mk}}{p_m},$$

where C_0 is a constant. Turning to the price index, we get

$$\begin{aligned}
P_n^{1-\sigma} &= \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \sum_j \sum_{k\in\Omega_j} l_k \left(v_k d_{mk}\right)^{1-\sigma} \int_{z_{km}^*}^{\infty} z^{\sigma-1} b_k^\theta z^{-\theta-1} dz \\
&= C_1 \sum_j \sum_{k\in\Omega_j} l_k b_k^\theta \left(v_k d_{mk}\right)^{1-\sigma} \left(z_{mk}^*\right)^{\sigma-1-\theta} \\
&= C_1 \sum_j \sum_{k\in\Omega_j} l_k b_k^\theta \left(v_k d_{mk}\right)^{1-\sigma} \left(\left(\frac{f_m}{l_m}\right)^{1/(\sigma-1)} \frac{v_k d_{mk}}{p_m}\right)^{\sigma-1-\theta},
\end{aligned}$$

where C_1 is a constant. Further, assumption A2 in this case also implies that $f_m = f_n$. Hence, for $m \in \Omega_n$, $P_n = p_m$. Replacing and after some algebra, we get

$$\begin{aligned}
P_n^{-\theta} &= C_2 \sum_{j\neq n} L_j b_j^\theta \left(w_j \tau_{nj}\right)^{-\theta} \left(\frac{f_n}{L_n/M_n}\right)^{1-\frac{\theta}{\sigma-1}} + C_2 (L_n/M_n) b_n^\theta w_n^{-\theta} \left(\frac{f_n}{L_n/M_n}\right)^{1-\frac{\theta}{\sigma-1}} \left(\left(M_n-1\right)\delta_n^{-\theta}+1\right) \\
&= C_2 \left(\frac{f_n}{L_n/M_n}\right)^{1-\frac{\theta}{\sigma-1}} \sum_j L_j b_j^\theta \left(w_j \tau_{nj}\right)^{-\theta},
\end{aligned}$$

where C_2 is a constant. Thus,

$$\sum_j L_j b_j^\theta \left(w_j \tau_{nj}\right)^{-\theta} = C_2^{-1} P_n^{-\theta} \left(\frac{f_n}{L_n/M_n}\right)^{-[1-\theta/(\sigma-1)]},$$

and hence,

$$\lambda_{nn} = \frac{L_n b_n^\theta w_n^{-\theta} \tau_{nn}^{-\theta}}{C_2^{-1} P_n^{-\theta} \left(\frac{f_n}{L_n/M_n}\right)^{-\left(1-\frac{\theta}{\sigma-1}\right)}},$$

so that the real wage for country n

$$U_n = C_2^{-1/\theta} L_n^{1/\theta} b_n \tau_{nn}^{-1} \lambda_{nn}^{-1/\theta} \left(\frac{f_n}{L_n/M_n}\right)^{1/\theta-1/(\sigma-1)},$$

and

$$\begin{aligned} U_n &= C_2^{-1/\theta} L_n^{1/\theta} b_n \tau_{nn}^{-1} \lambda_{nn}^{-1/\theta} \left(\frac{U_n f_n}{L_n/M_n} \right)^{1/\theta - 1/(\sigma-1)} \\ &= C_2^{-1/\theta} M_n^{1/\theta} (L_n/M_n)^{1/(\sigma-1)} b_n \tau_{nn}^{-1} \lambda_{nn}^{-1/\theta} f_n^{1/\theta - 1/(\sigma-1)}. \end{aligned}$$

Thus, if f_n does not vary with L_n/M_n, the growth rate would be $g_L/\left(\sigma-1\right)$. To have the growth rate be g_L/θ, we need to assume that f_n scales up with L_n/M_m proportionally, or $\theta \approx \sigma - 1$, in which case

$$U_n \sim b_n \times L_n^{1/\theta} \times \tau_{nn}^{-1} \times \lambda_{nn}^{-1/\theta}.$$

F Additional Table: Decomposition of the General Model

Table A.1: The Role of Domestic Frictions and Real Wages.

	Real Wage					Gains from trade
	Scale effects	Int' trade	Domestic frictions	Full model	Data	Full model
	(1)	(2)	(3)	(4)	(5)	(6)
Australia	0.47	0.47	0.65	0.65	0.97	1.028
Austria	0.33	0.38	0.59	0.73	1.12	1.259
Benelux	0.47	0.51	0.79	0.92	1.16	1.185
Canada	0.53	0.54	0.72	0.78	0.86	1.108
Switzerland	0.37	0.42	0.64	0.81	0.88	1.303
Denmark	0.33	0.38	0.67	0.80	0.94	1.220
Spain	0.44	0.45	0.63	0.69	1.14	1.122
Finland	0.39	0.42	0.77	0.84	0.84	1.116
France	0.57	0.59	0.82	0.91	1.07	1.137
Great Britain	0.56	0.58	0.87	0.93	1.00	1.092
Germany	0.65	0.67	0.90	0.97	0.92	1.101
Greece	0.29	0.31	0.53	0.59	0.90	1.143
Hungary	0.28	0.32	0.56	0.66	0.65	1.193
Ireland	0.26	0.31	0.52	0.65	1.32	1.275
Iceland	0.17	0.22	0.35	0.50	1.17	1.457
Italy	0.45	0.47	0.64	0.70	1.20	1.130
Japan	0.83	0.83	1.20	1.19	0.72	1.016
Korea	0.52	0.53	0.88	0.90	0.63	1.047
Mexico	0.31	0.31	0.40	0.43	0.78	1.116
Norway	0.35	0.39	0.71	0.79	1.11	1.148
New Zealand	0.28	0.29	0.56	0.57	0.74	1.036
Poland	0.41	0.43	0.61	0.69	0.50	1.153
Portugal	0.29	0.31	0.51	0.58	0.97	1.162
Sweden	0.41	0.44	0.67	0.78	0.81	1.192
Turkey	0.28	0.29	0.43	0.46	0.61	1.096
United States	1.00	1.00	1.00	1.00	1.00	1.022
Avg all	0.43	0.46	0.68	0.75	0.92	1.148
Avg 6 smallest	0.30	0.33	0.60	0.69	0.95	1.209
Avg 6 largest	0.56	0.57	0.80	0.85	1.02	1.099

Column 1 refers to the model with only scale effects, column 2 to the model with scale effects and international trade, column 3 to the model with scale effects and domestic trade costs, and column 4 to the model with scale effects, international trade, and domestic trade costs. The real wage in the data (column 5) is the real GDP (PPP-adjusted) per unit of equipped labor. Column 6 shows the gains from trade (i.e. change in the real wage from autarky to the one with the observed trade levels) computed using the calibrated model. All variables are calculated relative to the United States. The six smallest countries (with respect to R&D-adjusted size) are Iceland, Ireland, New Zealand, Finland, Norway, and Denmark, while the six largest countries are Italy, France, Great Britain, Germany, Japan, and the United States.

G Additional Table: Summary Statistics. Data and Model.

Table A.2: Calibrated Model and Data: Summary Statistics.

	Average			Size elasticity
	full sample	6 largest countries	6 smallest countries	
Real Wage				
data	0.92	0.95	1.02	-0.01 (0.03)
no dom.fric.	0.49	0.59	0.38	0.20 (0.01)
sym. dom.fric.	0.75	0.75	0.78	0.09 (0.02)
gral. dom.fric	0.75	0.85	0.69	0.13 (0.02)
Nominal Wage				
data	0.83	0.91	1.01	0.07 (0.06)
no dom.fric.	0.67	0.72	0.63	0.10 (0.02)
sym. dom.fric.	0.82	0.80	0.88	0.06 (0.02)
gral. dom.fric	0.82	0.85	0.83	0.07 (0.02)
Price Index				
data	0.88	0.97	1.00	0.07 (0.04)
no dom.fric.	1.45	1.25	1.71	-0.09 (0.01)
sym. dom.fric.	1.11	1.10	1.14	-0.03 (0.01)
gral. dom.fric	1.11	1.02	1.21	-0.05 (0.01)
Import Share				
data	2.59	1.83	3.35	-0.23 (0.07)
no dom.fric.	10.6	6.03	16.0	-0.39 (0.09)
sym. dom.fric.	3.68	3.57	3.697	-0.15 (0.08)
gral. dom.fric	4.79	3.66	5.96	-0.28 (0.07)

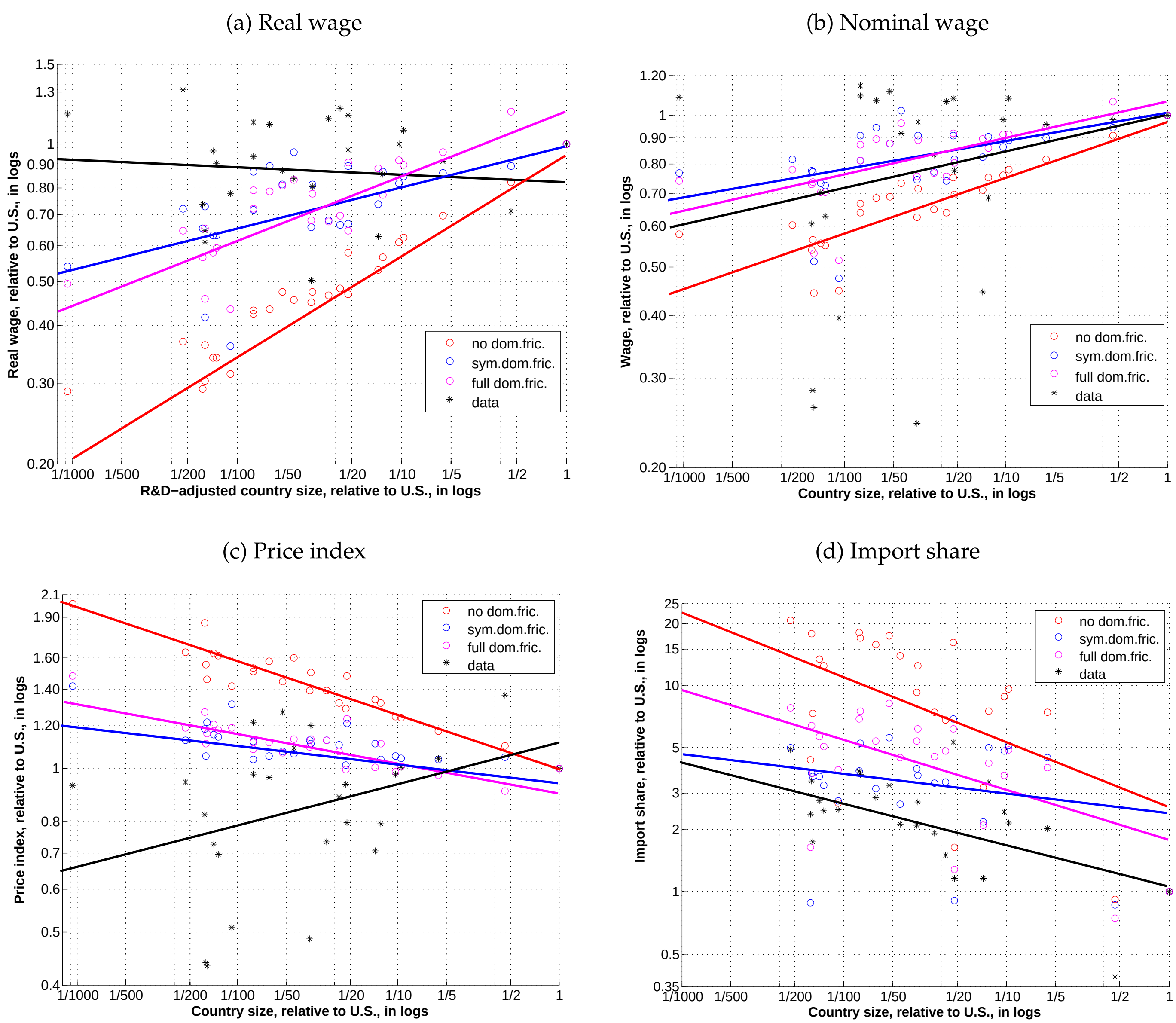
′′gral. dom.fric.", ′′sym.dom.fric.", and ′′no dom.fric." refer, respectively, to the calibrated general and symmetric model with domestic trade costs, and the model with no domestic trade costs. Variables are calculated relative to the United States. The size elasticity of each variable is from an OLS regressions with a constant and robust standard errors (in parenthesis). The six smallest countries (with respect to R&D-adjusted size) are Iceland, Ireland, New Zealand, Finland, Norway, and Denmark, while the six largest countries are Italy, France, Great Britain, Germany, Japan, and the United States.

H Calibration with Symmetric Regions

As mentioned in the paper, we need to calibrate the matrix of international trade costs to calculate the equilibrium nominal wages, prices, and trade shares, under A2.

We parametrize international trade costs as $\tau_{ni} = \beta_1 dist_{ni}^{\beta_3}$, for $i \neq n$, and $dist_{ni}$ the geographical distance between country i and n (i.e., distance between the most populated cities from *Centre d'Etudes Prospectives et Informations Internationales*). Since the model under A2 delivers country-level gravity, we can directly impose $\beta_3 = 0.27$ and choose β_1 to match the average bilateral international trade share observed in the data, as before. Figure A.1 shows the results for the model with symmetric regions (blue) and compare them with the general model (pink) and the model without domestic trade costs (red); the data are also shown (black).

Figure A.1: Calibrated Model and Data.



‘‘No dom.fric." refers to the model without domestic trade costs; ‘‘sym. dom.fric." refers to the symmetric model with domestic trade costs; ‘‘full dom.fric." refers to the model without A2 but domestic trade costs. In the data: the real wage is computed as real GDP (PPP-adjusted) divided by equipped labor, L_n; the nominal wage is calculated as GDP at current prices divided by equipped labor, L_n; the price index is calculated as the nominal wage divided by the real wage; and import shares refer to total imports, as share of absorption, in the manufacturing sector. R&D-adjusted country size refers to $\phi_n L_n$, where ϕ_n is the share of R&D employment. Solid lines fitted through the dots.

I Multinational Production and Non-Tradable Goods

In the model of Section 1, international trade was the only channel through which countries could gain from openness. But, arguably, the activity of multinational firms could be even more important. We now incorporate multinational production as an extra channel for the gains from openness. To such end, we extend the model of Section 1 by allowing technologies to be used outside of the region where they originate; whenever this happens we say that there is multinational production (MP). To proceed we assume that countries are a collection of symmetric regions.

We follow Ramondo and Rodríguez-Clare (2013) and assume that a technology has a productivity z_n in each country $n = 1, ..., N$. To introduce frictions to the "movement of ideas" within countries, analogously to the way we introduced domestic frictions for trade, we assume that each technology has a "home region" in each country. Using a technology originated in country i for production outside of the technology's home region in country i entails an iceberg-type efficiency loss, or "MP cost," of $h_{ii} \geq 1$. Moreover, using a technology originated in country i in the technology's home region in country $l \neq i$ entails an MP cost of $\gamma_{li} \geq 1$. Finally, the total MP cost associated with using a technology from country i outside of the technology's home region in country $l \neq i$ is $\gamma_{li} h_{ll}$.[1]

In sum, each technology is characterized by three elements: first, the country i from which it originates; second, a vector that specifies the technology's productivity parameter in each country, $\boldsymbol{z} = (z_1, ..., z_N)$; and third, a vector that specifies the technology's home region in each country, $\boldsymbol{m} = (m_1, ..., m_N)$. The effective productivity of a technology $(i, \boldsymbol{z}, \boldsymbol{m})$ is z_i if used in region m_i, z_i/h_{ii} if used in region $m \in \Omega_i$ with $m \neq m_i$, z_l/γ_{li} if used in region m_l for $l \neq i$, and $z_l/\gamma_{li} h_{ll}$ if used in region $m \in \Omega_l$ for $l \neq i$ and $m \neq m_l$.

We assume that productivity levels in $\boldsymbol{z}$, for technologies originating in country i, are independently drawn from the Fréchet distribution with parameters $\bar{T}_i$ and θ, and we assume that m_n is uniformly and independently drawn from the set Ω_n.

In the model with MP, we introduce both tradable and non-tradable goods, since around half of MP flows in the data occur in non-tradable goods. We assume that tradable goods are intermediate goods while non-tradable goods are final goods. There is a con-

[1] The assumption that technologies have a home region in each country is made to keep the treatment of domestic and foreign technologies consistent. We assume that technologies originated in country i are "born" in a particular region and then face an MP cost h_{ii} to be used in another region of country i. The analogous assumption for the use of technologies from i in country $n \neq i$ is that they also have a region in country n where they are "reincarnated" (their home region), and then face an MP cost h_{nn} to be used in another region of country n.

tinuum of final goods and a continuum of intermediate goods, both in the interval $[0, 1]$. Preferences over final goods are CES with elasticity of substitution $\sigma > 0$. Intermediate goods are used to produce a composite intermediate good with a CES aggregator with elasticity $\sigma > 0$. The composite intermediate together with labor are used, via a Cobb-Douglas production function, to produce final and intermediate goods with labor shares α and β, respectively.

We assume that MP is possible in both the final and intermediate goods, and that the MP costs are the same in both cases. Further, we assume that $1 \leq d_{nn} = h_{nn}$. Consider a particular intermediate good whose home region is m_n. The price of this good in other regions of country n ($m \in \Omega_n$, $m \neq m_n$) is determined by z/d_{nn} if traded and z/h_{nn} if produced locally via MP. Our assumption that $d_{nn} = h_{nn}$ implies that there is indifference between these two options. We assume that the indifference is broken in favor of trade, which implies that there is no MP across regions within countries for intermediate goods. Summing up, there is "domestic" MP in final, but not intermediate, goods, whereas trade is feasible in intermediate, but not final, goods, within countries. Across countries, MP is feasible in both types of goods, while trade is only possible in intermediate goods.

Our object of interest is the equilibrium real wage in each country n, which we compare with the real wage in the data. In the model with trade, MP, and domestic frictions, analogously to the baseline model, equilibrium wages can be written as

$$\frac{w_n}{P_n} = \mu^M \times \underbrace{\phi_n^{\frac{1+\eta}{\theta}}}_{\text{R\&D Intensity}} \times \underbrace{L_n^{\frac{1+\eta}{\theta}}}_{\text{Pure Scale Effect}} \times \underbrace{\gamma_{nn}^{-1} \tau_{nn}^{-\eta}}_{\text{Domestic Frictions}} \times \underbrace{\lambda_{nn}^{-\frac{\eta}{\theta}}}_{\text{Gains Trade}} \times \underbrace{\pi_{nn}^{-\frac{1+\eta}{\theta}}}_{\text{Gains MP}}, \tag{A8}$$

where μ^M is a positive constant,

$$\eta \equiv \frac{1-\alpha}{\beta}, \tag{A9}$$

and

$$\gamma_{nn} \equiv \left[\frac{1}{M_n} + \frac{M_n - 1}{M_n} h_{nn}^{-\theta} \right]^{-1/\theta}, \tag{A10}$$

and π_{nn} is the domestic MP share.[2] There are several points to be made about the result in ($A8$). First, the pure scale effect now has elasticity $(1 + \eta)/\theta$ rather than $1/\theta$. The reason is that there are scale effects operating in both the final and intermediate goods sectors. The scale effect elasticity in the final goods sector is $1/\theta$, as in the baseline model, but this elasticity is η/θ in the intermediate goods sector. The term η captures the amplification of gains by the factor $1/\beta$ in the intermediate goods sector because of the input-output loop

[2] Formally, $\pi_{li} \equiv Y_{li}/Y_l$, where Y_{li} is value of production in country l with technologies originated in country i, and $Y_l \equiv \sum_i Y_{li}$.

and the weakening of the overall effect due to intermediate goods being only used with share $1-\alpha$ in the production of final goods. Second, the real wage is now affected by frictions to domestic trade and to "domestic" MP. The impact of domestic trade frictions is $\tau_{nn}^{-\eta}$, while the impact of domestic MP frictions is γ_{nn}^{-1}. Third, the gains from trade are now captured by $\lambda_{nn}^{-\eta/\theta}$, rather than $\lambda_{nn}^{-1/\theta}$. Finally, the term $\pi_{nn}^{-(1+\eta)/\theta}$ captures the gains from MP (i.e., the change in the real wage from a situation with no MP to the observed equilibrium), for both final and intermediate goods. The gains from openness are just the product of the gains from trade and the gains from MP.

As the last term in (A8) indicates, the gains from MP can be expressed as a function of observed flows, in the same way the gains from trade are. Data on the gross value of production for multinational affiliates from i in n are used as the empirical counterpart of bilateral MP flows in the model, which in turn are used to compute the MP shares, π_{nn}, from Ramondo, Rodriguez-Clare, and Tintelnot (2015). The labor shares α and β are set to 0.75 and 0.50, respectively, following Alvarez and Lucas (2007), while the parameter θ is set to a value of 6 following the different approaches described in the paper. It is worth noting here that $(1+\eta)/\theta = 1/4$ so that the strength of scale effects is the same as in the baseline calibration. Our calibration of domestic frictions for trade in goods is equivalent to the procedure described for the symmetric model in the paper. For $\theta = 6$, we get $d = 1.81$, and we assume that $d = h$.

Columns 2 to 7 in Table A.3 show each term in the right-hand side of (A8), relative to the United States. Given our assumption that $h = d$, $\gamma_{nn} = \tau_{nn}$; still, these frictions are different across countries due to differences in M_n. Together with $(1+\eta)/\theta = 1/4$ and the (re)calibration of d to satisfy (11), there is no difference in the role of domestic frictions here with respect to the symmetric model. But the gains from trade are now $\lambda_{nn}^{-\eta/\theta}$, with $\eta/\theta = 1/12$, rather than $\lambda_{nn}^{-1/\theta}$ with $1/\theta = 1/4$. Consequently, the gains from trade have a smaller role now, as shown in column 6 of Table A.3, although the gains from openness also include the gains from MP. But as column 3 indicates, MP does not help much to increase real wages, relative to the United States, for small countries because the United States has large gains from MP. While only Japan has lower gains from trade than the United States (column 2), several countries have lower gains from MP than the United States.

The result in the paper still holds: the existence of domestic frictions, rather than openness, remains the dominant channel to bring the calibrated model closer to the data. For instance, for Denmark, adding MP does not help much quantitatively to bring the relative real wage in the calibrated model closer to the one observed in the data: the implied relative real wage is 0.76, against 0.94 in the data, and 0.86 in the baseline model. More generally, looking at the average for the six smallest countries in the sample, trade and MP openness together help to close around three percent of the gap between the standard model with only scale effects and the data on relative real wages, while domestic frictions close almost 50 percent of the gap.

As a final remark, suppose that there is no MP, but we add non tradable final goods to the baseline model of Section 1. This would require setting $\pi_{nn} = 1$ in (A8), and taking a stand about the nature of non-tradable goods. If these goods were local at the region level, then $h_{nn} \to \infty$, and $\gamma_{nn} = (1/M_n)^{-1/\theta}$; if they were local at the country level, then $h_{nn} = 1$ and $\gamma_{nn} = 1$. The question is then: how much would the baseline results change by just adding non tradable goods? In the first case ($h_{nn} \to \infty$), our baseline results would be reinforced: a country like Denmark would reach a real wage (relative to U.S.) of 0.93, and domestic frictions would explain almost 90 percent of the gap between the data and the model with only scale effects. A lower bound would be obtained if, instead, non-tradable goods were national ($h_{nn} = 1$): for Denmark, the real wage would be half the United States's (versus 0.85 in our baseline calibration). Still, domestic frictions, as opposed to openness to trade, would have the largest role in bringing the model closer to the data.

I.1 Equilibrium Analysis

The following Proposition characterizes trade and MP flows for the model of trade and MP with domestic frictions presented in Section I.

We introduce the following notation: $c_l^f \equiv A w_l^{\alpha} \left(P_l^g\right)^{1-\alpha}$, $c_l^g \equiv A w_l^{\beta} \left(P_l^g\right)^{1-\beta}$ and $Y_l^s \equiv \sum_i Y_{li}^s$, where P_l^g is the price index of intermediate goods and where Y_{li}^f and Y_{li}^g denote the value of production of final and intermediate goods, respectively. It is easy to show that $Y_l^g = \eta w_l L_l$ while $Y_l^f = w_l L_l$.

Proposition 1. Country-level trade flows are

$$X_{nl} = \frac{\Gamma_l \left(\tau_{nl} c_l^g\right)^{-\theta}}{\sum_{l'} \Gamma_{l'} \left(\tau_{nl'} c_{l'}^g\right)^{-\theta}} X_n, \tag{A11}$$

while country-level MP flows in intermediate and final goods are

$$Y_{li}^s = \frac{T_i \gamma_{li}^{-\theta}}{\Gamma_l} Y_l^s \text{ and } Y_{ll}^s = \frac{T_l}{\Gamma_l} Y_l^s \text{ for } s = g, f, \tag{A12}$$

and price indices at the country-level are

$$P_n^g = \mu^{-1}\left(\sum_l \Gamma_l \left(c_l^g\right)^{-\theta} \tau_{nl}^{-\theta}\right)^{-1/\theta}, \tag{A13}$$

and

$$P_n^f = \mu^{-1} c_n^f \left(\gamma_{nn}^{-\theta}\Gamma_n\right)^{-1/\theta}, \tag{A14}$$

where

$$\gamma_{ll} \equiv \left(\frac{1}{M_l} + \frac{M_l - 1}{M_l} h_{ll}^{-\theta}\right)^{-1/\theta}. \tag{A15}$$

Proof: First, note that no intermediate goods will be produced with technologies outside of their home region. This is because of our assumption that $h_{nn} = d_{nn}$, with the indifference broken in favor of trade rather than MP. Now, for $k \in \Omega_l$, we have an analogous result as in (1)m except that now, instead of $\widetilde{T}_k$, we have $\sum_{i \neq l} \frac{M_i \bar{T}_i}{M_l} \gamma_{il}^{-\theta} + \widetilde{T}_k$. Country-level trade flows are then

$$X_{nl} = \frac{\left(\sum_{i \neq l} T_i \gamma_{ll}^{-\theta} + T_l\right) w_l^{-\theta} \tau_{nl}^{-\theta}}{\sum_j \left(\sum_{i \neq j} T_i \gamma_{ji}^{-\theta} + T_j\right) w_j^{-\theta} \tau_{nj}^{-\theta}} X_i = \frac{\Gamma_l w_l^{-\theta} \tau_{nl}^{-\theta}}{\sum_j \Gamma_j w_j^{-\theta} \tau_{nj}^{-\theta}} X_i.$$

MP shares are simply given by the contribution of each source to Γ_l, hence

$$Y_{li}^s / Y_l^s = T_i \gamma_{li}^{-\theta} / \Gamma_l \text{ and } Y_{ll}^s / Y_l^s = T_l / \Gamma_l \text{ for } s = f, g.$$

The price index for intermediate goods is simply $\gamma^{-1}\left(\sum_j \Gamma_j w_j^{-\theta} \tau_{nj}^{-\theta}\right)^{-1/\theta}$, while for final goods we have

$$\begin{aligned}
\left(\mu P_n^f\right)^{-\theta} &= \sum_{i \neq n} \frac{M_i \bar{T}_i}{M_n} \gamma_{ni}^{-\theta}\left(1 + (M_n - 1) h_{nn}^{-\theta}\right) + (M_n - 1) \bar{T}_n h_{nn}^{-\theta} + \bar{T}_n \\
&= \sum_{i \neq n} T_i \gamma_{ni}^{-\theta} \gamma_{nn}^{-\theta} + T_n \gamma_{nn}^{-\theta} = \gamma_{nn}^{-\theta} \Gamma_n.
\end{aligned}$$

□

The results for trade flows are very similar to those in a model with only trade, except that now technology levels are augmented because of the possibility of using technologies from other countries, appropriately discounted by the efficiency costs: $\Gamma_l \equiv \sum_{i \neq l} T_i \gamma_{li}^{-\theta} + T_l$. Note that if MP costs go to infinity, then $\Gamma_l \to T_l$, as in the model with no MP of Section 1.

We now derive an expression for real wages. First, from ($A11$) and ($A13$), we get

$$\frac{c_n^g}{P_n^g} = \mu \Gamma_n^{1/\theta} \tau_{nn}^{-1} \lambda_{nn}^{-1/\theta}.$$

Using ($A12$),

$$\frac{c_n^g}{P_n^g} = \mu T_n^{1/\theta} \tau_{nn}^{-1} \lambda_{nn}^{-1/\theta} \left(\frac{Y_{nn}^g}{Y_n^g} \right)^{-1/\theta}.$$

Using $c_n^g = B w_n^\beta \left(P_n^g\right)^{1-\beta}$,

$$\frac{w_n}{P_n^g} = B^{-1/\beta} \mu^{1/\beta} T_n^{1/\beta\theta} \tau_{nn}^{-1/\beta} \lambda_{nn}^{-1/\beta\theta} \left(\frac{Y_{nn}^g}{Y_n^g} \right)^{-1/\beta\theta}. \tag{A16}$$

From ($A14$) and ($A12$), we get

$$P_n^f = c_n^f \mu^{-1} \gamma_{nn} T_n^{-1/\theta} \left(\frac{Y_{nn}^f}{Y_n^f} \right)^{1/\theta}.$$

Using $c_n^f = A w_n^\alpha \left(P_n^g\right)^{1-\alpha}$ and ($A16$) yield

$$P_n^f = A B^\eta w_n \mu^{-(1+\eta)} T_n^{-\frac{1+\eta}{\theta}} \gamma_{nn} \tau_{nn}^\eta \lambda_{nn}^{\eta/\theta} \left(\frac{Y_{nn}^g}{Y_n^g} \right)^{\eta/\theta} \left(\frac{Y_{nn}^f}{Y_n^f} \right)^{1/\theta}.$$

Further rearranging yields

$$\frac{w_n}{P_n^f} = A^{-1} B^{-\eta} \mu^{(1+\eta)} T_n^{\frac{1+\eta}{\theta}} \gamma_{nn}^{-1} \tau_{nn}^{-\eta} \lambda_{nn}^{-\eta/\theta} \left(\frac{Y_{nn}^g}{Y_n^g} \right)^{-\eta/\theta} \left(\frac{Y_{nn}^f}{Y_n^f} \right)^{-1/\theta}.$$

Using $Y_{nn}^g / Y_n^g = Y_{nn}^f / Y_n^f = T_l / \Gamma_l = \pi_{ll}$ and $T_n = \phi_n L_n$, and setting $\mu^M \equiv A^{-1} B^{-\eta} \mu^{(1+\eta)}$ yields ($A8$).

References

[1] **Ramondo, Natalia, Andrés Rodríguez-Clare, and Felix Tintelnot.** 2015. "Multinational Production: Data and Stylized Facts." *American Economic Review Papers and Proceedings*, 105(5): 530-36.

Table A.3: The Symmetric Model with Multinational Production.

	Size	GT	GMP	GO	Dom.Fric.	Real Wage				
	(1)	(2)	(3)	(4)	(5)	(1)x(2)	(1)x(3)	(1)x(4)	(1)x(4)x(5)	data
Australia	0.47	1.00	1.02	1.03	1.55	0.47	0.48	0.48	0.74	0.97
Austria	0.33	1.06	1.04	1.10	1.81	0.37	0.35	0.37	0.67	1.11
Benelux	0.47	1.13	1.09	1.22	1.55	0.59	0.51	0.57	0.88	1.16
Canada	0.53	1.05	1.07	1.12	1.41	0.58	0.56	0.59	0.84	0.86
Switzerland	0.37	1.04	1.06	1.11	1.81	0.40	0.39	0.41	0.74	0.88
Denmark	0.33	1.06	1.00	1.06	2.14	0.38	0.33	0.35	0.76	0.94
Spain	0.44	1.01	1.01	1.02	1.47	0.45	0.44	0.45	0.66	1.14
Finland	0.39	1.02	1.01	1.03	2.14	0.40	0.39	0.40	0.85	0.84
France	0.57	1.02	1.01	1.03	1.33	0.59	0.57	0.58	0.78	1.07
Great Britain	0.56	1.02	1.06	1.09	1.30	0.59	0.59	0.61	0.79	1.00
Germany	0.65	1.02	1.03	1.05	1.20	0.67	0.67	0.68	0.81	0.92
Greece	0.29	1.03	0.98	1.01	1.81	0.31	0.29	0.30	0.54	0.90
Hungary	0.28	1.05	1.12	1.17	2.14	0.31	0.32	0.33	0.71	0.65
Ireland	0.26	1.10	1.08	1.19	2.14	0.32	0.28	0.31	0.67	1.32
Iceland	0.17	1.06	0.97	1.04	2.14	0.20	0.17	0.18	0.39	1.17
Italy	0.45	1.01	0.99	1.00	1.37	0.46	0.45	0.45	0.62	1.20
Japan	0.83	0.99	0.97	0.96	1.08	0.81	0.80	0.80	0.86	0.71
Korea	0.52	1.00	0.98	0.98	1.37	0.52	0.51	0.51	0.69	0.63
Mexico	0.31	1.03	1.01	1.04	1.37	0.33	0.31	0.32	0.44	0.78
Norway	0.35	1.03	1.00	1.04	2.14	0.38	0.35	0.37	0.79	1.11
New Zealand	0.28	1.02	1.03	1.06	2.14	0.29	0.29	0.30	0.64	0.74
Poland	0.41	1.02	1.03	1.05	1.55	0.42	0.42	0.43	0.66	0.50
Portugal	0.29	1.03	1.08	1.12	2.14	0.31	0.31	0.32	0.70	0.97
Sweden	0.41	1.03	1.04	1.07	1.81	0.44	0.43	0.44	0.80	0.81
Turkey	0.28	1.01	0.98	0.99	1.47	0.29	0.28	0.28	0.41	0.61
United States	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Avg all	0.43	1.03	1.03	1.06	1.67	0.46	0.44	0.45	0.71	0.92
Avg 6 smallest	0.30	1.05	1.02	1.07	2.14	0.33	0.30	0.32	0.68	1.02
Avg 6 largest	0.68	1.01	1.01	1.02	1.21	0.69	0.68	0.69	0.81	0.98

Column 1 refers to the first term (size), column 2 to the third term (gains from trade), column 3 to the fourth term (gains from MP), and column 5 (domestic frictions) to the second term, respectively, on the right-hand side of (A8). Column 4 are the gains from openness, $GO_n = GT_n \times GMP_n$. The real wage in the data is the real GDP (PPP-adjusted) per unit of equipped labor. All variables are calculated relative to the United States. The six smallest countries (with respect to R&D-adjusted size) are Iceland, Ireland, New Zealand, Finland, Norway, and Denmark, while the six largest countries are Italy, France, Great Britain, Germany, Japan, and the United States.