A Model of Trading in the Art Market

Online Appendix

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Appendix A Prices, Holding Periods, and Returns as Function of Buyer Type

Some results in this Appendix are obtained by setting $\kappa = 0$. It is important to note that our qualitative implications extend to small $\kappa > 0$, because of the continuity of of the bidding function b in the model parameters.

Purchase prices. The purchase price of a type-e buyer in state ω is necessarily included in the range $[b^N_\omega(x^D_\omega(0)), b^N_\omega(e)]$, as $x^D_\omega(0)$ is the bidder type targeted by the reserve price of a type-0 owner in distress meaning that the item will never sell for less than $b^N_\omega(x^D_\omega(0))$, the lowest possible reserve price. When the seller is of type z and has distress status $\zeta \in \{D, N\}$, the average price paid by the buyer equals:

$$P_\omega(e, z, \zeta) := E[\max\{b^N_\omega(\tilde{e}^{(2)}), b^N_\omega(x^\zeta_\omega(z))\} | \tilde{e}^{(2)}, x^\zeta_\omega(z) < e], \tag{A1}$$

which is the expected maximum of the second-highest type's valuation and the reserve price, conditional on the bidder being able to buy. Keeping the type and status of the seller fixed, the expected purchase price increases with the type of the buyer. Moreover, because higher-type buyers bid more aggressively, they are able to purchase from higher-type sellers who set a higher reserve price. This implies that:

Corollary A1 *The average purchase price is strictly increasing in buyer type at any purchase date.*

Holding periods. How long will a buyer hold the artwork? Higher-type owners are less likely to consign their artwork. Moreover, when they do, they set reserve prices that are at least as high as those that lower-type owners with the same distress status would choose. A buyer's ex-ante expected holding period is therefore increasing in his type e.

Corollary A2 *The expected holding period is weakly increasing in buyer type at any purchase date.*

We can be more precise about how the ex-ante expected holding period depends on buyer type in a stationary economy, allowing us to drop the subscripts ω. Noting that e^N then denotes the threshold type below which a non-distressed owner will consign to auction, we can state the following results:

Corollary A3 *In a stationary economy, if $\kappa = 0$, the expected holding period for a type-e buyer can be explicitly computed as follows:*

$$T(e) := \frac{1 - (1-d)F(x^D(e))^n}{(1 - F(x^D(e))^n)(1 - (1-d)F(x^N(e))^n)},$$

which is increasing in e for $e < e^N$ and maximum for $e > e^N$.

Resale prices. At which price will a buyer sell the artwork? To analyze this, let us first denote by $s^{\sigma}_{\omega}(e)$ the price of a type-e owner who is successfully selling when his distress status equals σ and when the macroeconomic state is ω. This price falls in the range $[b^N_{\omega}(x^{\sigma}_{\omega}(e)), b^N_{\omega}(1)]$, with the average value increasing in owner type e:

$$s^{\sigma}_{\omega}(e) := E[\max\{b^N_{\omega}(\tilde{e}^{(2)}), b^N_{\omega}(x^{\sigma}_{\omega}(e))\}|\tilde{e}^{(1)} > x^{\sigma}_{\omega}(e)] > b^{\sigma}_{\omega}(e), \tag{A2}$$

where the inequality follows from the fact that the owner would choose not to auction the artwork if $s^{\sigma}_{\omega}(e) < b^{\sigma}_{\omega}(e)$. Because $x^D_{\omega}(e) < x^N_{\omega}(e)$, we can state the following results:

Corollary A4 *The average (resp. dispersion) of the hammer prices realized by a seller in status σ is increasing (resp. decreasing) in the seller's type e. The hammer prices realized by a distressed seller have a lower average and a larger dispersion than those realized by a non-distressed seller of the same type.*

Of course, at the time of purchase, an individual does not know with certainty what will be his distress status at the time of resale. In a stationary economy, we can denote by $S(e, \tau)$ the expected sale price, conditional on selling after τ periods, for a type-e buyer. This value is decreasing in τ for buyers with $e < e^N$ because the probability of a distressed sale grows with time. Next, we denote by $S(e)$ the unconditional ex-ante expected resale price. Crucially, $S(e)$ is not necessarily monotonically increasing in e for $e < e^N$, even though a non-distressed owner sets a reserve price that is strictly increasing in his type. This is because a higher reserve has two opposite effects. On the one hand, it increases the expected revenues in the event of a successful sale. On the other hand, it increases the probability of a buy-in, and thus the holding period—and therefore also the probability that the owner will eventually sell in distress for a low price.

Corollary A5 *In a stationary economy, if $\kappa = 0$:*

(i) Conditional on selling τ periods after purchase, the probability of selling in distress is increasing in τ for $e < e^N$ and equal to 1 for $e > e^N$. Therefore, the expected sale price $S(e, \tau)$ is decreasing in τ for $e < e^N$ and equal to $s^D(e)$ for $e > e^N$.

(ii) The expected sale price for a type-e buyer equals:

$$S(e) := \frac{s^D(e)d + s^N(e)(1-d)(1 - F(x^N(e))^n)}{1 - (1-d)F(x^N(e))^n},$$

which is increasing in e for e close to 0 and minimum for $e > e^N$.

Returns. We are now ready to analyze the financial returns realized by art buyers. Let us define the (gross total) return on a resale as the ratio of the sale price to the purchase price, and let ω again be the state at the sale date. Observe that collectors only sell in distress—and hence at low prices—whereas they purchase at relatively high prices. Compared to such high-type owners, flippers and investors pay less on average and moreover can expect higher resale revenues as some resales occur when not in distress. More generally, we have:

Corollary A6 *If κ is small, then for any given purchase and sale date, and for any state ω at the time of sale, the average return is lower for owners of type $e \in [e_\omega^N, e_\omega^D]$ than for owners of type $e < e_\omega^N$. The average return will moreover be decreasing in e over $e \in [e_\omega^N, e_\omega^D]$.*

Appendix B Proofs

Proof of Proposition 1

Let us define $\rho^N_\omega(e) := \rho_\omega e$ and $\rho^D_\omega(e) := c$. Furthermore, let $\pi^N := d$ and $\pi^D := (1-\kappa)$, meaning that π^σ is the probability that an owner with status $\sigma \in \{D, N\}$ at time t will have status D at time $t+1$. Then we obtain the following equations for $e \in [0,1]$ and $\sigma \in \{D, N\}$:

$$\begin{aligned} R_\omega(x) &= (1-\gamma)E[\max\{b^N_\omega(\tilde{e}^{(2)}), b^N_\omega(x)\}|\tilde{e}^{(1)} \geq x, \omega]\Pr[\tilde{e}^{(1)} \geq x|\omega] - \gamma_{BI} r \Pr[\tilde{e}^{(1)} < x|\omega] \\ &= n_\omega(1-\gamma)\left(\int_x^1 (n_\omega - 1)b^N_\omega(z)f(z)F(z)^{n_\omega-2}(1-F(z))dz + b^N_\omega(x)F(x)^{n_\omega-1}(1-F(x))\right) \\ &- \gamma_{BI} b^N_\omega(x)F(x)^{n_\omega}, \qquad \text{(B1)} \\ V^\sigma_\omega(e) &= \max\{b^\sigma_\omega(e), \max_{x\in[0,1]} R_\omega(x) + b^\sigma_\omega(e)F(x)^{n_\omega}\}, \qquad \text{(B2)} \\ b^\sigma_\omega(e) &= \rho^\sigma_\omega(e) + \delta \sum_{\omega'\in\Omega} p_\omega(\omega')\left(\pi^\sigma V^D_{\omega'}(e) + (1-\pi^\sigma)V^N_{\omega'}(e)\right). \qquad \text{(B3)} \end{aligned}$$

Let B be the space of bounded, continuous, and non-decreasing functions $g : [0,1] \to \mathbb{R}$, with the sup norm $\eta : B \to \mathbb{R}$, i.e., $\eta(g) := \sup_{e\in[0,1]} |g(e)|$. A state/status-contingent function $b : \Omega \times \{D, N\} \times [0,1] \to \mathbb{R}$ is a point in the product space $\mathcal{B} := \Pi_{\omega\in\Omega\times\{N,D\}} B$ with the sup norm $||b|| := \max_\omega \eta(g_\omega)$. Fix some time t and a state/status-contingent fuction $b \in \mathcal{B}$, and consider a time-t owner of type e in status σ who believes that in any future period $\tau > t$: (i) bidders' bids will follow $b^N_{\omega_\tau}$; (ii) his continuation payoff from not selling in period τ and status σ' is $b^{\sigma'}_{\omega_\tau}(e)$. Given these beliefs, denote by $R_{\omega_\tau}(x)[b]$ and $V^{\sigma'}_{\omega_\tau}(e)[b]$ the expected resale revenues from auctioning using a reserve price $b^N_{\omega_\tau}(x)$ and the expected continuation payoff from owning the artwork at some future time $\tau > t$ in status σ' given a type e, respectively. The expected utility for such an owner if he does not sell in t is then equal to:

$$(\mathcal{T}_{\omega_t} b)(e) := \rho^\sigma(e) + \delta \sum_{\omega'\in\Omega} p_{\omega_t}(\omega')\left(\pi^\sigma V^D_{\omega'}(e)[b] + (1-\pi^\sigma)V^N_{\omega'}(e)[b]\right), \sigma \in \{D, N\}. \qquad \text{(B4)}$$

Let $\mathcal{T}$ be the operator associating to any $b \in \mathcal{B}$ the state/status-contingent function $\{\mathcal{T}_\omega b\}_{\omega\in\Omega^2}$. Clearly an STM equilibrium is a fixed point of the operator $\mathcal{T}$. In what follows, we will use the contraction mapping fixed-point theorem to prove existence and uniqueness of an STM equilibrium. Namely we will prove that there exists a $\mu \in (0,1)$ such that for any $b, b' \in \mathcal{B}$, $||\mathcal{T}b - \mathcal{T}b'|| \leq \mu||b - b'||$.

Let us introduce some notation. For any $g, g' \in B$, if $g(e) \leq g'(e)$ for all $e \in [0,1]$, we write $g \leq g'$. For any status-contingent function $g = \{g^D, g^N\} \in B^2$ and $n \in \mathbb{N}$, let us define the function $R_{g,n}(x)$ as the expected cashflow for an owner who auctions the artwork with a reserve price equal to $g^N(x)$ and who believes that there are n bidders who are bidding following the function g^N. Formally:

$$\begin{aligned} R_{g,n}(x) &:= (1-\gamma)E[\max\{g^N(\tilde{e}^{(2)}), g^N(x)\}|\tilde{e}^{(1)} \geq x]\Pr(\tilde{e}^{(1)} \geq x) - \Pr(\tilde{e}^{(1)} < x)\gamma_{BI} g^N(x) \\ &= n(1-\gamma)\left(\int_x^1 (n-1)g^N(z)f(z)F(z)^{n-2}(1-F(z))dz + g^N(x)F(x)^{n-1}(1-F(x)))\right) \\ &- \gamma_{BI} g^N(x)F^n(x). \qquad \text{(B5)} \end{aligned}$$

Let $Q^\sigma_{g,n}(e,x)$ denote such an owner's expected continuation payoff if he values the asset at $g^\sigma(e)$ and decides to auction it with reserve price $g^N(x)$. Formally:

$$Q^\sigma_{g,n}(e,x) := R_{g,n}(x) + F(x)^n g^\sigma(e). \tag{B6}$$

Let $\mathcal{K}$ be the operator that associates to a function $g \in B^2$ the function $\mathcal{K}g : [0,1] \to \mathbb{R}$ as follows:

$$\mathcal{K}^\sigma g(e) := \max\{g^\sigma(e), \max_{x\in[0,1]} Q^\sigma_{g,n}(e,x)\}, \sigma \in \{D,N\}. \tag{B7}$$

The proof includes two lemmas.

Lemma B1 *For any couple g, g' in B^2 and $\sigma \in \{D,N\}$, it results that $\eta(\mathcal{K}^\sigma g - \mathcal{K}^\sigma g') \leq \eta(g-g')$ and $\{\mathcal{K}^D g, \mathcal{K}^N g\} \in B^2$.*

Proof. Take any $\sigma \in \{D,N\}$. We first show that $\mathcal{K}^\sigma$ satisfies the following two conditions:

a. Monotonicity: for any two functions $g, g' \in B$, $g \leq g'$ implies $(\mathcal{K}^\sigma g)(e) \leq (\mathcal{K}^\sigma g')(e)$ for all $e \in [0,1]$.

b. Scaling: for any $g \in B$ and constant $a \geq 0$:

$$\eta(\mathcal{K}^\sigma(g+a) - \mathcal{K}^\sigma g) \leq a,$$

where $(g+a)(e) := g(e) + a$.

Fix $\sigma \in \{D,N\}$. Let us consider monotonicity. Take any $g, g' \in B$ such that $g \leq g'$ and suppose that there exists an e such that $(\mathcal{K}^\sigma g)(e) > (\mathcal{K}^\sigma g')(e)$. Because $g(e) \leq g'(e)$, it cannot be that $(\mathcal{K}^\sigma g)(e) = g^\sigma(e)$ and $(\mathcal{K}^\sigma g')(e) = g'^\sigma(e)$. Nor can it be that $(\mathcal{K}^\sigma g')(e) \neq g'^\sigma(e)$ and $(\mathcal{K}^\sigma g)(e) = g^\sigma(e)$, because in this case $g'^\sigma(e) \geq g^\sigma(e) = (\mathcal{K}^\sigma g)(e) > (\mathcal{K}^\sigma g')(e)$ would contradict the relation $(\mathcal{K}^\sigma g')(e) \geq g'^\sigma(e)$ implied by Eq. (B7). Hence it must be that $(\mathcal{K} g^\sigma)(e) \neq g^\sigma(e)$, implying that $(\mathcal{K}^\sigma g)(e) = \max_{x\in[0,1]} Q^\sigma_{g,n}(e,x)$. Let $\hat{x}$ be such that $(\mathcal{K}^\sigma g)(e) = Q^\sigma_{g,n}(e,\hat{x})$ and let $\overline{r} := g^N(\hat{x})$. Observe that:

$$g^\sigma(e) < Q^\sigma_{g,n}(e,\hat{x}) = (1-\gamma)E[\max\{g^N(\tilde{e}^{(2)}), \overline{r}\} | g^N(\tilde{e}^{(1)}) \geq \overline{r}] \Pr(g^N(\tilde{e}^{(1)}) \geq \overline{r}) + \Pr(g^N(\tilde{e}^{(1)}) < \overline{r})(g^\sigma(e) - \gamma_{BI}\overline{r}),$$

where the equality comes from Eqs. (B1), (B6), and (B7), whereas the inequality comes from the fact that $(\mathcal{K}^\sigma g)(e) \neq g^\sigma(e)$. This implies that:

$$(1-\gamma)E[\max\{g^N(\tilde{e}^{(2)}), \overline{r}\} | g^N(\tilde{e}^{(1)}) \geq \overline{r}] > (g^\sigma(e) - \gamma_{BI}\overline{r}). \tag{B8}$$

Let x' be such that $g'^N(x') = \overline{r}$. Because $g' \geq g$, we have that:

$$\Pr(g^N(\tilde{e}^{(1)}) \geq \overline{r}) \leq \Pr(g'^N(\tilde{e}^{(1)}) \geq \overline{r}) \tag{B9}$$
$$\Pr(g^N(\tilde{e}^{(1)}) < \overline{r}) \geq \Pr(g'^N(\tilde{e}^{(1)}) < \overline{r}). \tag{B10}$$

Inequalities (B8), (B9), and (B10) imply the first inequality below, whereas the second and third inequalities follow from $g \le g'$:

$$\begin{aligned}
(\mathcal{K}^\sigma g)(e) &= \\
Q^\sigma_{g,n}(e,\hat{x}) &\le (1-\gamma)E[\max\{g^N(\tilde{e}^{(2)}),\overline{r}\}|g^N(\tilde{e}^{(1)})\ge\overline{r}]\Pr({g'}^N(\tilde{e}^{(1)})\ge\overline{r})+\Pr({g'}^N(\tilde{e}^{(1)})<\overline{r})(g^\sigma(e)-\gamma_{BI}\overline{r}) \\
&\le (1-\gamma)E[\max\{g^N(\tilde{e}^{(2)}),\overline{r}\}|g^N(\tilde{e}^{(1)})\ge\overline{r}]\Pr({g'}^N(\tilde{e}^{(1)})\ge\overline{r})+\Pr({g'}^N(\tilde{e}^{(1)})<\overline{r})({g'}^\sigma(e)-\gamma_{BI}\overline{r}) \\
&\le (1-\gamma)E[\max\{{g'}^N(\tilde{e}^{(2)}),\overline{r}\}|{g'}^N(\tilde{e}^{(1)})\ge\overline{r}]\Pr({g'}^N(\tilde{e}^{(1)})\ge\overline{r})+\Pr({g'}^N(\tilde{e}^{(1)})<\overline{r})({g'}^\sigma(e)-\gamma_{BI}\overline{r}) \\
&= Q^\sigma_{g'}(e,x')\le(\mathcal{K}^\sigma g')(e).
\end{aligned}$$

Hence a contradiction that $(\mathcal{K}^\sigma g)(e) > (\mathcal{K}^\sigma g')(e)$.

Let us now consider scaling. Let $g, g' \in B$ such that for any $e \in [0,1]$, $g'(e) = g(e) + a$, where a is a positive constant. For any quadruple $e^1 \ge e^2$, e and x in $[0,1]$, let $Q^\sigma_{g,n}(e,x)(e^1,e^2)$ be the value of $Q^\sigma_{g,n}(e,x)$ when the highest bidder type is e^1 and the second-highest bidder type is e^2. Namely:

$$Q^\sigma_{g,n}(e,x)(e^1,e^2) = \begin{cases} -\gamma_{BI}g^N(x)+g^\sigma(e) \text{ if } e^1<x \\ (1-\gamma)g^N(x) \text{ if } e^2\le x<e^1 \\ (1-\gamma)g^N(e^2) \text{ if } x<e^2. \end{cases}$$

Note that $g'(e) = g(e) + a$ implies that:

$$Q^\sigma_{g',n}(e,x)(e^1,e^2) = \begin{cases} Q^\sigma_{g,n}(e,x)(e^1,e^2)+(1-\gamma_{BI})a \text{ if } e^1<x \\ Q^\sigma_{g,n}(e,x)(e^1,e^2)+(1-\gamma)a \text{ if } x\le e^1. \end{cases} \tag{B11}$$

Because $Q^\sigma_{g,n}(e,x) = E[Q^\sigma_{g,n}(e,x)(\tilde{e}^{(1)},\tilde{e}^{(2)})]$, it follows from Eq. (B11) that for any e and x, $Q^\sigma_{g',n}(e,x) = Q^\sigma_{g,n}(e,x) + a(1-\gamma_{BI}\Pr(\tilde{e}^{(1)} < x) - \gamma\Pr(\tilde{e}^{(1)} \ge x)) \le Q^\sigma_{g,n}(e,x)+a$, and hence $\eta(Q^\sigma_{g,n}(e,x) - Q^\sigma_{g',n}(e,x)) \le a$ for any e and x. As a consequence:

$$\eta(\max_{x\in[0,1]} Q^\sigma_{g,n}(e,x) - \max_{x\in[0,1]} Q^\sigma_{g',n}(e,x)) \le a. \tag{B12}$$

Considering that $\eta(g-g') = a$ and the definition of $\mathcal{K}^\sigma g$ in (B7), we can conclude that $\eta(\mathcal{K}^\sigma g' - \mathcal{K}^\sigma g) \le a$.

We can now prove the first statement of the lemma. Note that for any $g, g' \in B$, we have $g \le g' + \eta(g-g')$. Monotonicity and scaling of $\mathcal{K}^\sigma$ imply:

$$\mathcal{K}^\sigma g \le \mathcal{K}^\sigma(g'+\eta(g-g')) \le \mathcal{K}^\sigma g' + \eta(g-g').$$

Because $g' \le g + \eta(g-g')$, we also have that:

$$\mathcal{K}^\sigma g' \le \mathcal{K}^\sigma g + \eta(g-g').$$

Combining these two inequalities, we find that for any $e \in [0,1]$, $|\mathcal{K}^\sigma g(e) - \mathcal{K}^\sigma g'(e)| \le \eta(g-g')$, or $\eta(g-g') \ge \sup_e |\mathcal{K}^\sigma g(e) - \mathcal{K}^\sigma g'(e)| = \eta(\mathcal{K}^\sigma g - \mathcal{K}^\sigma g')$, as was to be shown. To prove the second statement, we need to show that if $g \in B^2$, then $\mathcal{K}^\sigma g$ is finite-valued, continuous, and non-decreasing in e. Note first that $Q^\sigma_{g,n}(e,x)$ is finite-valued for any $e, x \in [0,1]$. It is also continuous in both

x and e and increasing in e, and therefore $\max_x Q^\sigma_{g,n}(e,x)$ is also continuous and increasing in e. Hence $\max_x Q^\sigma_{g,n}(e,x) \in B$. Because $\mathcal{K}^\sigma g$ is the maximum of two functions in B, it must be in B. ■

Lemma B2 *For any couple b, b' in $\mathcal{B}$, it results that $\eta(\mathcal{T}_\omega b - \mathcal{T}_\omega b') \leq \delta||(b-b')||$.*

Proof. Note first that $V^\sigma_\omega[b] = \mathcal{K}^\sigma b_\omega$. Hence Lemma B1 implies that $\eta(V^\sigma_\omega[b] - V^\sigma_\omega[b']) \leq \eta(b^\sigma_\omega - b'^\sigma_\omega)$. Thus:

$$\begin{aligned}\eta(\mathcal{T}_\omega b - \mathcal{T}_\omega b') &= \sup_e |\mathcal{T}_\omega b(e) - \mathcal{T}_\omega b'(e)| \\ &\leq \delta \sup_{e,\sigma} \sum_{\omega' \in \Omega} p_\omega(\omega')(\pi^\sigma |V^D_{\omega'}(e)[b] - V^D_{\omega'}(e)[b']| + (1-\pi^\sigma)|V^N_{\omega'}(e)[b] - V^N_{\omega'}(e)[b']|) \\ &\leq \max_\sigma \delta \sum_{\omega' \in \Omega} p_\omega(\omega')(\pi^\sigma \eta(b^\sigma_{\omega'} - b'^\sigma_{\omega'}) + (1-\pi^\sigma)\eta(b^\sigma_{\omega'} - b'^\sigma_{\omega'})) = \max_\sigma \delta \sum_{\omega' \in \Omega} p_\omega(\omega')\eta(b_{\omega'} - b'_{\omega'}) \\ &\leq \max_\sigma \delta \sum_{\omega' \in \Omega} p_\omega(\omega') \max_{\omega''} \eta(b^\sigma_{\omega''} - b'^\sigma_{\omega''}) = \delta||b - b'||,\end{aligned}$$

because $\sum_{\omega' \in \Omega} p_\omega(\omega') = 1$. ■

Because $||\mathcal{T}b - \mathcal{T}b'|| = \max_{\omega \in \Omega} \eta(\mathcal{T}_\omega b - \mathcal{T}_\omega b')$, Lemma B2 implies that $||\mathcal{T}b - \mathcal{T}b'|| \leq \delta||b-b'||$. Thus $\mathcal{T}$ is a contraction mapping because $\delta < 1$. Note that $\mathcal{T}_\omega b \in B$, because it is a positive linear combination of functions in B. Hence, $\mathcal{T}b \in \mathcal{B}$. Therefore $\mathcal{T}$ is a contraction mapping from $\mathcal{B}$ into $\mathcal{B}$ and has a unique fixed point.

Take $\{b^D_\omega(e), b^N_\omega(e)\}$. We already know from the first part of the proof that $b^\sigma_\omega(e)$ and $V^\sigma_\omega(e)$ are not decreasing in e. To show that $b^\sigma_\omega(e)$ is weakly convex in e, it is sufficient to show that the operator $\mathcal{T}$ maps weakly convex functions into weakly convex functions. Let $g \in B$ be a weakly convex function. We first show that the operator $\mathcal{K}$ maps g to a weakly convex function. Let $x^\sigma(e,g) := \arg\max_{x \in [0,1]} Q^\sigma_{g,n}(e,x)$ and $\overline{Q}^\sigma_{g,n}(e) := \max_{x \in [0,1]} Q^\sigma_{g,n}(e,x)$. Using (B6), the first-order condition implies that $R'_{g,n}(x^\sigma(e,g)) + nf(x^\sigma(e,g))F(x^\sigma(e,g))^{n-1}g(e) = 0$. Hence $\overline{Q}'_{g,n}(e) = F(x^\sigma(e,g))^n g'(e)$, which is increasing in e because $x^\sigma(e,g)$ is increasing in e and $g(e)$ is weakly convex. Hence, $\overline{Q}_{g,n}$ is weakly convex. From Eq. (B7) we have that $\mathcal{K}^\sigma g(e) = \max\{g^\sigma(e), \overline{Q}^\sigma_{g,n}(e)\}$, which is weakly convex because it is the maximum of two weakly convex functions. Let $g \in \mathcal{B}$ be weakly convex. The operator (B4) can then be rewritten as:

$$(\mathcal{T}_{\omega_t} g)(e) := \rho^\sigma_{\omega_t}(e) + \delta \sum_{\omega' \in \Omega} p_{\omega_t}(\omega') \left(\pi^\sigma \mathcal{K}^D g_{\omega'}(e)[g] + (1-\pi^\sigma)\mathcal{K}^N g_{\omega'}(e)[g]\right), \tag{B13}$$

which is the sum of weakly convex functions in e and therefore weakly convex itself.

Next, we show that the equilibrium function b is continuous in κ. The proof of continuity in the other parameters follows the same logic and is omitted. Observe first that the mapping $\mathcal{T}$ is continuous in κ. Namely, let $|\kappa - \kappa'| < \varepsilon$ and let $\mathcal{T}_\kappa$ and $\mathcal{T}_{\kappa'}$ be the $\mathcal{T}$ in (B4) for recovery probability values κ and κ', respectively. Observe that $0 \leq V^\sigma_\omega(e) < \max_\omega \rho_\omega/(1-\delta)$ because an owner can, first, always secure a non-negative payoff from auctioning with a zero reserve price, and, second, cannot value the artwork at more than the value of holding the asset forever and enjoying the maximum possible emotional dividend. Let $M := \frac{\max_\omega \rho_\omega}{1-\delta}$. Now take any state/status-contingent bidding function b. From (B4) we have that $|(\mathcal{T}^\sigma_\kappa)b^\sigma(e) - (\mathcal{T}^\sigma_{\kappa'})b^\sigma(e)| < \varepsilon M$. Thus, for any state/status-contingent function $b \in B$, we have that:

$$||\mathcal{T}_\kappa[b] - \mathcal{T}_{\kappa'}[b]|| \leq \varepsilon M.$$

Let b_κ and $b_{\kappa'}$ be the fixed point of $\mathcal{T}_\kappa$ and $\mathcal{T}_{\kappa'}$, respectively. We want to show that $\lim_{\kappa' \to k} b_{\kappa'} = b_\kappa$. If not, then there is a $Z > 0$ such that, for any $\varepsilon > 0$, there exist κ, κ' such that $|\kappa - \kappa'| < \varepsilon$ and $||b_{\kappa'} - b_\kappa|| \geq Z$. Now, note that because $\mathcal{T}$ is a contraction there is a $\mu < 1$ such that for any state/status-contingent function $b \in B$, one has that $||\mathcal{T}_\kappa[b] - \mathcal{T}_\kappa[b']|| \leq \mu||b - b'||$. Thus:

$$||\mathcal{T}_\kappa[b_{\kappa'}] - \mathcal{T}_\kappa[b_\kappa]|| = ||\mathcal{T}_\kappa[b_{\kappa'}] - b_\kappa|| \leq \mu||b_{\kappa'} - b_\kappa||.$$

Also $||\mathcal{T}_\kappa[b_{\kappa'}] - b_{\kappa'}|| = ||\mathcal{T}_\kappa[b_{\kappa'}] - \mathcal{T}_{\kappa'}[b_{\kappa'}]|| < \varepsilon M$. Applying the triangular inequality we have:

$$||b_{\kappa'} - b_\kappa|| \leq ||\mathcal{T}_\kappa[b_{\kappa'}] - b_\kappa|| + ||\mathcal{T}_\kappa[b_{\kappa'}] - b_{\kappa'}|| \leq \mu||b_{\kappa'} - b_\kappa|| + \varepsilon M,$$

which implies that $||b_{\kappa'} - b_\kappa|| \leq \frac{\varepsilon}{1-\mu} M$. But if one takes $0 < \varepsilon < Z(1-\mu)/M$, then $||b_{\kappa'} - b_\kappa|| < Z$. Hence a contradiction. ■

Proof of Lemma 1

Let us consider the equilibrium behavior of an owner of type $e \in [0, 1]$ in status σ when the state is ω. The owner needs to make two decisions: whether to consign the artwork to auction or not, and the reserve price level when auctioning. Let $R_\omega(x)$ be the net expected revenue when auctioning in state ω with a reserve price of $b_\omega(x)$, i.e., using (B5), $R_\omega(x) := R_{b_\omega, n_\omega}(x)$. Let $Q^\sigma_\omega(e, x)$ be the owner's expected continuation payoff in state ω when auctioning with a reserve price of $b^N_\omega(x)$:

$$Q^\sigma_\omega(e, x) := R_\omega(x) + F(x)^{n_\omega} b^\sigma_\omega(e).$$

Because the owner's expected payoff from holding the artwork is $b^\sigma_\omega(e)$, he will prefer consigning the artwork if and only if there is an $x \in [0, 1]$ such that $Q^\sigma_\omega(e, x) > b^\sigma_\omega(e)$, or equivalently:

$$Q_\omega(x) := \frac{R_\omega(x)}{1 - F(x)^{n_\omega}} \geq b^\sigma_\omega(e). \tag{B14}$$

Let $\hat{Q}_\omega := \max_{x \in [0,1]} Q_\omega(x)$. Note that $\hat{Q}_\omega \leq (1-\gamma)b^N_\omega(1)$. Let e^σ_ω be as defined in (7). Now consider an owner of type $e \geq e^\sigma_\omega$. Because $b^\sigma_\omega(e)$ is increasing, it must be that $b^\sigma_\omega(e) \geq b^\sigma_\omega(e^\sigma_\omega) = \hat{Q}_\omega \geq Q_\omega(x)$ for any x, implying that there exists no x satisfying inequality (B14), and hence the owner will not sell. Let us now consider $e < e^\sigma_\omega$. It is sufficient to show that there exists an x satisfying inequality (B14), which is true for $x = e^\sigma_\omega$, because $b^\sigma_\omega(e) \leq b^\sigma_\omega(e^\sigma_\omega) \leq \hat{Q}_\omega$. Thus, an owner prefers auctioning the artwork if and only if $e \leq e^\sigma_\omega$.

(i) The first inequality follows from the fact that $\hat{Q}_\omega$ cannot exceed the maximum possible net revenues from reselling $(1-\gamma)b_\omega^N(1) < b_\omega^N(1)$. The second inequality follows from the fact that $b_\omega^D(e) \leq b_\omega^N(e)$, where the inequality is strict for $e > 0$.

(ii) Suppose that for some status $\sigma \in \{D, N\}$, one has that $e_\omega^\sigma < 0$ for all ω. This implies that no owner with status σ would ever sell. Consider a type-0 individual. No matter his status, he derives a per-period dividend of at most 0. Therefore, if he never sells, he would value the artwork at 0 or less. However, by auctioning with a reserve price of 0, he can secure $R_\omega(0) > 0$. Thus he must sell in some state and some distress status. Clearly, if he sells in state ω when his status is N, then he will also sell when in distress in state ω. It cannot be optimal to never sell when not in distress, as the owner can guarantee a higher payoff by selling in some state before having to pay the distress cost c.

(iii) No matter an individual's e, for $\kappa = 0$ recovery is impossible, so once hit by a liquidity shock an owner must sell in some state, otherwise his continuation payoff would not exceed $c \leq 0$, while he can guarantee $R_\omega(0) > 0$ by auctioning. The result then follows from the continuity of b with respect to κ. ■

Proof of Proposition 2

The consignment strategy directly follows from Lemma 1. Hence let us consider the choice of the optimal reserve price. In state ω, an owner of type $e < e_\omega^\sigma$ and status σ will set a reserve price equal to $r_\omega^\sigma(e) = b_\omega^N(x_\omega^\sigma(e))$, where $x_\omega^\sigma(e)$ maximizes $Q_\omega^\sigma(e, x)$. Without loss of generality, we can set $x_\omega^\sigma(e) \in [0, 1]$. Differentiating with respect to x and then e, we get:

$$\begin{aligned}\frac{\partial Q_\omega^\sigma(e,x)}{\partial x} &= n_\omega F(x)^{n_\omega - 1}\left(b_\omega^{N\prime}(x)(1-F(x))(1-\gamma) - f(x)(b_\omega^N(x)(1-\gamma+\gamma_{BI}) - b_\omega^\sigma(e))\right) \\ &\quad -\gamma_{BI} b_\omega^{N\prime}(x) F(x)^{n_\omega}, \qquad \text{(B15)} \\ \frac{\partial^2 Q_\omega^\sigma(e,x)}{\partial e \partial x} &= n_\omega F(x)^{n_\omega - 1} f(x) b_\omega^{\sigma\prime}(e) > 0.\end{aligned}$$

Because for $e < 0$, $f(e) = 0$, the first expression implies that $x_\omega^\sigma(e) \in [0, 1]$. The second expression implies that $Q_\omega^\sigma(e, x)$ is a quasi-concave function in x. From this we deduce that $x_\omega^\sigma(e) \in [0, 1]$. Furthermore, whenever $x_\omega^\sigma(e) \in (0, 1)$, it must be that $\partial Q_\omega^\sigma(e, x)/\partial x|_{x=x_\omega^\sigma(e)} = 0$. Because the right-hand side of (B15) is continuously increasing in e and larger for $\sigma = N$ than for $\sigma = D$, we have that $x_\omega^\sigma(e)$—and hence the optimal reserve price—is continuously increasing in e and larger for $\sigma = N$ than for $\sigma = D$. ■

Proof of Proposition 3

The proof follows directly from the argument following the Proposition. ■

Proof of Corollary A1

The proof follows directly from the fact that in any given state ω the bidding function is strictly increasing in the bidder type e. ■

Proof of Corollary A2

For any given state ω and distress status σ, the reserve type x is weakly increasing in the owner type e. Because d and κ do not depend on the owner's type, an increase in the owner's type decreases the chance of a successful auction, and hence increases the expected holding period. ■

Proof of Corollary A3

Consider a stationary economy with $\kappa = 0$, in which an individual of type e buys at time 0. Let $O_N(e, \tau + 1)$ denote the probability for this individual of being a non-distressed at time $\tau + 1$. It is equal to $O_N(e, \tau)$, times the probability of not selling at τ, times the probability of not being hit by a liquidity shock at the beginning of $\tau + 1$. Similarly let $O_D(e, \tau + 1)$ denote the buyer's probability of being a distressed owner at $\tau + 1$. It equals the probability of being a distressed owner at τ, times the probability of not selling at τ, plus the probability of being a non-distressed owner at τ, times the probability of not selling at τ, times the probability of a liquidity shock at the start of $\tau + 1$. We thus have:

$$\begin{cases} O_N(e, \tau + 1) = O_N(e, \tau)F(x^N(e))^n(1 - d) \\ O_D(e, \tau + 1) = O_D(e, \tau)F(x^D(e))^n + O_N(e, \tau)F(x^N(e))^n d, \end{cases} \tag{B16}$$

Considering that $O_D(e, 1) = d$ and $O_N(e, 1) = (1 - d)$ and solving the difference equation, we get:

$$\begin{cases} O_N(e, \tau) = (1 - d)^\tau F(x^N(e))^{n(\tau-1)} \\ O_D(e, \tau) = d\frac{F(x^D(e))^{n\tau} - (1-d)^\tau F(x^N(e))^{n\tau}}{F(x^D(e))^n - (1-d)F(x^N(e))^n}. \end{cases} \tag{B17}$$

The unconditional probability that a type-e buyer sells after τ periods then depends on the probability of being a non-distressed owner at τ and that of being a distressed owner at τ, and on the probabilities of finding a buyer in each case. Formally:

$$\pi(e, \tau) := O_N(e, \tau)(1 - F(x^N(e))^n) + O_D(e, \tau)(1 - F(x^D(e))^n). \tag{B18}$$

Replacing Eq. (B17) in $\pi(e, \tau)$, one has the probability that a type-e buyer sells after τ periods. The expression for $T(e)$ is given by $\sum_{\tau \geq 1} \tau\pi(e, \tau)$. Note that because $\kappa = 0$, one has that $x^D(e) = x^D(0)$ for all e. Hence, $T(e)$ is increasing in $x^N(e)$, which is increasing in e for $e < e^N$ and maximum for $e \geq e^N$. ■

Proof of Corollary A4

Consider a seller of type e and status σ when the state is ω. The average hammer price is the $s_\omega^\sigma(e)$ given by equation (A2), and thus it is an increasing function of the seller's reserve type $x_\omega^\sigma(e)$. Then the statement about the average hammer price follows immediately from the fact that a non-distressed seller's reserve price is increasing in the seller's type and larger than the reserve price of a distressed seller of the same type (Proposition 2). The statement about dispersion follows from Proposition 2 and from the fact that the selling price is continuously distributed on the interval $[b_\omega^N(x_\omega^\sigma(e)), b_\omega^N(1)]$. ■

Proof of Corollary A5

(i) Let $\pi_D(e,\tau)$ denote the probability of selling in distress conditional on selling after τ periods, which is equal to:

$$\pi_D(e,\tau) = \frac{O_D(e,\tau)(1-F(x^D(e))^n)}{O_D(e,\tau)(1-F(x^D(e))^n) + O_N(e,\tau)(1-F(x^N(e))^n)}.$$

where $O_N(.)$ and $O_D(.)$ are given by expression (B17). Because a non-distressed owner of type $e > e^N$ does not sell, one has that $(1-F(x^N(e))^n) = 0$ and $\pi_D(\tau, e) = 1$. Consider an owner of type $e \leq e^N$. We want to show that $\pi_D(e,\tau) < \pi_D(e,\tau+1)$, which is equivalent to:

$$\frac{O_D(e,\tau+1)}{O_D(e,\tau)} > \frac{O_N(e,\tau+1)}{O_N(e,\tau)}.$$

Because of Eq. (B16), we can rewrite this inequality as:

$$dF(x^N(e))^n O_N(e,\tau) > ((1-d)F(x^N(e))^n - F(x^D(e))^n)O_D(e,\tau). \tag{B19}$$

We prove this by induction. Note that $O_N(e,1) = 1-d$ and $O_D(e,1) = d$, implying that the inequality is satisfied for $\tau = 1$. Now suppose that Eq. (B19) is satisfied for τ, then we can verify that it is satisfied for $\tau+1$, which would mean that $dF(x^N(e))^n O_N(e,\tau+1) > (1-d)F(x^N(e))^n - F(x^D(e))^n O_D(e,\tau+1)$. Indeed, replacing in this inequality $O_N(e,\tau+1)$ and $O_D(e,\tau+1)$ by the expressions given in (B16) and simplifying, we get back to (B19).

The statement regarding $S(e,\tau)$ follows from $S(e,\tau) = s^D(e)\pi_D(e,\tau) + (1-\pi_D(e,\tau))s^N(e)$, with $s^N(e) > s^D(e)$ for $e \leq e^N$, and from the fact that $\pi_D(e,\tau)$ is increasing in τ for $e < e^n$ and equals 1 for $e \geq e^N$.

(ii) The expression for $S(e)$ results from $\sum_{\tau\geq 1} S(e,\tau)\pi(e,\tau)$. Observe that for $e > e^N$ we have that $x^N(e) = 1$, implying that $S(e) = s^D(e)$, which is smaller than $s^N(e')$ for any $e' < e^N$. To prove quasi-concavity, observe that $S(e)$ can be rewritten as follows:

$$S(e) = \frac{ds^D(e) + \frac{1-d}{1-\gamma}(R(x^N(e)) + \gamma_{BI} b(x^N(e))F(x^N(e))^n)}{1-(1-d)F(x^N(e))^n},$$

where $R(x)$ is given by Eq. (B1). Recall that for $e \in (0, e^N)$ we have that $x^N(e)$ maximizes $R(x) + F(x)^n b^N(e)$. From the envelope theorem, we then have that:

$$\partial R(x^N(e))/\partial e = -b^N(e)x'(e)nf(x^N(e))F(x(e))^{n-1}.$$

Differentiating $S(e)$ and considering this expression for $\partial R(x(e))/\partial e$, we have that $S'(e) > 0$ if and only if:

$$b^N(e) < (1-\gamma)S(e) + \gamma_{BI}\left(b^N(x^N(e)) + \frac{b'(x^N(e))F(x^N(e))}{nf(x^N(e))}\right).$$

Observe that $S'(0) > 0$, because an individual who does not enjoy the artwork will never bid more than the present value of the net expected selling price. This is strictly less than $\delta(1-\gamma)S(0)$, which is smaller than the r.h.s. ■

Proof of Corollary A6

We prove the corollary for $\kappa = 0$. The result follows from the continuity of b with respect to κ. Take $e \geq e^N_\omega$ and $e' < e^N_\omega$. Note that the average purchase price is greater for a type-e owner than for a type-e' owner (Corollary A1). Let us consider the average selling price. If a type-e owner sells when the state is ω, it must be that he sells in distress and hence he sets a reserve type $x^D_\omega(e)$. For $k = 0$, one has that $x^D_\omega(e) = x^D_\omega(0) \leq x^N_\omega(e')$ and $s^D_\omega(e) = s^D_\omega(0) \leq s^N_\omega(e')$. Consider now a type-$e'$ owner. If he sells when the state is ω, he could either be in distress or not. Let $\theta \in (0,1)$ be the probability he sells in distress, then on average his selling price is $s^D_\omega(e) + (1-\theta)s^N_\omega(e') = s^D_\omega(0) + (1-\theta)s^N_\omega(e') > s^D_\omega(0) > s^D_\omega(e)$. ■

Proof of Corollary 1

We prove the corollary for a stationary economy with $\kappa = 0$. The result follows by continuity of b with respect to κ. It is sufficient to note that purchase prices and average holding periods are higher for a type $e > e^N$ than for a type $e \leq e^N$, and that the expected selling price is higher for a type $e \leq e^N$ than for a type $e > e^N$. ■

Proof of Corollary 2

Observe that the volatility of returns in an economy with business cycles is not lower that in a stationary economy. Hence it is sufficient to prove the statement for a stationary economy. Because $\gamma_{BI} > 0$, it must be that optimal reserve types $x^\sigma(e) < 1$ for all status σ and types e. Thus $\overline{x} := \max_{\sigma,e} x^\sigma(e) < 1$ and $M := \text{Var}[\max\{b^N(\overline{x}), b^N(\tilde{e}^{(2)})\}] > 0$. Consider a type-$e$ individual who bought at time t for some price p_t and let $t' > t$ be the time at which he sells and $p_{t'}$ the selling price. Then for any e and for any t', the selling price is distributed in a support that includes the interval $[b^N(\overline{x}), b^N(1)]$ and so $\text{Var}[\tilde{p}_{t'}] \geq M$. Thus, the variance of return is bounded away from 0 independently of the length of the holding period. ■

Proof of Corollary 3

Note that in a stationary economy non-distressed sellers set reserve prices guaranteeing them a positive net return. Because the transaction cost γ is typically substantial, sales from non-distressed sellers generate positive and relatively large gross returns. Such non-distressed sellers are flippers and investors. Distressed sales are associated with the lowest reserve prices if κ is small. Moreover, distressed sellers also include collectors, who paid a high price and hence are most likely to realize a strictly negative gross return. ■

Proof of Corollary 4

We prove the corollary for a stationary economy with $\kappa = 0$. The result follows by continuity of b with respect to κ. Note that, if $\kappa = 0$, all distressed owners set the same reserve price. Owners of type $e > e^N$ only auction when in distress; for them there is no relation between the time since purchase and the probability of a buy-in. Owners of type $e < e^N$ auction in every period. The probability that such an owner in distress is increasing in the time since purchase. Given that the reserve type of a distressed owner is lower than the reserve type of a non-distressed owner, a buy-in is less likely when the owner is in distress. ■

Proof of Proposition 4

We prove the results for $\varepsilon = 0$, which means that $p_1 - p_2 = \gamma_{BI} = \kappa = 0$. The results extend to the case of $\varepsilon > 0$ but small by continuity of the function b with respect to the parameters of the model (Proposition 1).

(i) Let $p_\omega = \Pr(\omega_{t+1} = 1 | \omega_t = \omega)$. Let us consider the mapping $\mathcal{T} : B \times \Omega \to B \times \Omega$. Let $g \in B$ be such that $g_1(e) < g_2(e)$, and let $p_1 \geq p_2$ and $n_2 \geq n_1$. Then $\mathcal{T}_1 g(e) < \mathcal{T}_2 g(e)$, implying that under Assumption 1 $b_2^\sigma(e) > b_1^\sigma(e)$. The argument is analogous to the one used in Proposition 1 to prove the convexity of $b_\omega^\sigma(e)$ and is omitted. Because $b_2^N(e) > b_1^N(e)$ and $n_2 \geq n_1$, both an owner's expected revenues in case of a sale and the owner's valuation of holding are larger in expansion than in recession, and as a consequence $V_2^\sigma(e) > V_1^\sigma(e)$. Observe that for $\kappa = 0$ one has that for any $e \in [0, 1]$ and any ω, $V_\omega^D(e) = V_\omega^D(0)$. Therefore:

$$b_\omega^N(e) = \rho_\omega e + \delta \left(p_\omega (d V_1^D(0) + (1-d) V_1^N(e)) + (1 - p_\omega)(d V_2^D(0) + (1-d) V_2^N(e)) \right).$$

Hence $b_2^N(e) \geq b_1^N(e) + (\rho_2 - \rho_1) e$, because $V_2 > V_1$ and $p_1 \geq p_2$.

(ii) Define $A := \frac{\rho_1 + \delta(1-p)(1-d)(\rho_2 - \rho_1)}{1 - \delta(1-d)} > 0$ and $B := \frac{p V_1^D(0) + (1-p) V_2^D(0)}{1 - \delta(1-d)} \delta d$. We then have the following lemma.

Lemma B3 *Under Assumptions 1.(i)–1.(iv) with $\varepsilon = 0$, we have:*
(i) For $e \geq e_2^N$, $b_1^N(e) = Ae + B$.
(ii) $e_2^N = 1 - \gamma \frac{A + B + \rho_2 - \rho_1}{A + \rho_2 - \rho_1} < 1 - \gamma$.

(iii) $1-\gamma\frac{A+B}{A+B-b_1(0)} < e_1^N < 1-\gamma\frac{A+B}{A} < e_2^N$.

Proof. Note first that for $p_1 = p_2$ and $e \geq 0$, we have that:

$$b_2^N(e) = b_1^N(e) + (\rho_2 - \rho_1)e. \tag{B20}$$

(i) A non-distressed individual of type $e \geq \max\{e_1^N, e_2^N\}$ does not sell, which implies that $V_\omega^N(e) = b_\omega^N(e)$. Hence:

$$b_2^N(e) = \rho_2 e + \delta(p(dV_1^D(0) + (1-d)b_1^N(e)) + (1-p)(dV_2^D(0) + (1-d)b_2^N(e)).$$

Taken together with (B20), this means that $b_1(e) = Ae + B$.

(ii) Recall that e_2^N solves $b_2(e_2^N) = \hat{Q}_2$. For $\gamma_{BI} = 0$, one has that:

$$\begin{aligned}\hat{Q}_\omega &= \max_{x\in[0,1]} \frac{R_\omega(x)}{1-F^{n_\omega}(x)} \\ &= \max_{x\in[0,1]}(1-\gamma)E[\max\{b_\omega^N(e^{(2)}), b_\omega^N(x)\}|e^{(1)} \geq x] \\ &= (1-\gamma)b_\omega^N(1),\end{aligned}$$

and so using (B20):

$$\begin{aligned}\hat{Q}_2 &= (1-\gamma)b_2^N(1) \\ &= (1-\gamma)(b_1^N(1) + \rho_2 - \rho_1).\end{aligned}$$

Now, using (B20) and (i), one has that $b_2^N(e_2^N) = b_1^N(e_2^N) + (\rho_2-\rho_1)e_2^N = Ae_2^N + B + (\rho_2-\rho_1)e_2^N$ and that $\hat{Q}_2 = (1-\gamma)(A+B+\rho_2-\rho_1)$. The expression for e_2^N comes from the solution of $Ae_2^N + B + (\rho_2-\rho_1)e_2^N = (1-\gamma)(A+B+\rho_2-\rho_1)$.

(iii) Observe that because $b_1^N(e)$ is convex, it satisfies:

$$b_1^N(0) + (b_1^N(1) - b_1^N(0))e \geq b_1^N(e) \geq Ae + B$$

for all $e \in [0,1]$. The threshold e_1^N solves $b_1^N(e) = (1-\gamma)b_1^N(1)$ and hence it must be included between the solutions of the two equations $b_1^N(0)+(b_1^N(1)-b_1^N(0))e = (1-\gamma)b_1^N(1)$ and $Ae+B = (1-\gamma)b_1^N(1)$. The result follows from $b_1^N(1) = A+B$. ■

(iii) Let us show that, under Assumptions 1.(i)–1.(v), one has that $r_2^N(e) > r_1^N(e)$. Let $Z_\omega(r) := F((b_\omega^N)^{-1}(r))$ and $z_\omega := Z'_\omega$. Then the first-order condition for the choice of the optimal reserve price by a non-distressed owner of type e in state ω is equivalent to:

$$(1-\gamma)(1-Z_\omega(r)) - z_\omega(r)((1-\gamma)r - b_\omega^N(e)) = 0,$$

or equivalently:

$$\frac{1-Z_\omega(r)}{z_\omega(r)} - \frac{(1-\gamma)r - b_\omega^N(e)}{1-\gamma} = 0.$$

Because this expression is increasing in $b_\omega^N(e)$, to show that $r_2^N(e) > r_1^N(e)$, it is sufficient to show that $\frac{1-Z_2(r)}{z_2(r)}$ is increasing in $\Delta := \rho_2 - \rho_1$. Let $y = b_2^{N^{-1}}(r)$, i.e., $b_1^N(y) + \Delta y = r$. Then:

$$\frac{\partial \left[\frac{1-Z_2(r)}{z_2(r)}\right]}{\partial \Delta} = \frac{-\frac{\partial y}{\partial r}\frac{\partial y}{\partial \Delta}\left(f(y)^2 + (1-F(y)f'(y)) - (1-F(y))f(y)\frac{\partial^2 y}{\partial r \partial \Delta}\right.}{z(r)^2} > 0, \tag{B21}$$

where the inequality follows from $\frac{\partial y}{\partial r} = 1/(b_1'(y) + \Delta) > 0$, $\frac{\partial y}{\partial \Delta} = -y/(b_1'(y) + \Delta) < 0$, $\frac{\partial^2 y}{\partial r \partial \Delta} = -1/(b_1'(y) + \Delta)^2 < 0$ and $f(y)^2 + (1-F(y))f'(y) > 0$ because of Assumption 1.(iv).

Let us now show that, under Assumptions 1.(i)–1.(iii), one has that $x_2^N(e) < x_1^N(e)$. Note that expression (B15) provides the first-order condition for $x_\omega^2(e)$. For $\gamma_{BI} = 0$, this is equivalent to:

$$b^{N\prime}{}_\omega(x)(1-\gamma)(1-F(x)) - f(x)(b_\omega^N(x)(1-\gamma) - b_\omega^N(e))\Big|_{x=x_\omega^N(e)} = 0. \tag{B22}$$

To show that $x_2^N(e) < x_1^N(e)$, it is sufficient to prove that:

$$b^{N\prime}{}_2(x)(1-\gamma)(1-F(x)) - f(x)(b_2^N(x)(1-\gamma) - b_2^N(e))\Big|_{x=x_1^N(e)} < 0.$$

Using (B22) for $\omega = 1$, and the fact that for $x > 0$ one has that $b_2^N(x) = b_1^N(x) + (\rho_2 - \rho_1)x$, the previous inequality is equivalent to:

$$(\rho_2 - \rho_1)(1-\gamma)(1-F(x_1(e))) - f(x_1(e))((1-\gamma)(\rho_2-\rho_1)x_1(e) - b_2^N(e) + b_1^N(e)) < 0. \tag{B23}$$

Observe that $p_1 = p_2$ implies $b_1^N(0) = b_2^N(0)$. Thus the l.h.s. of (B23) computed for $e = 0$ equals:

$$(\rho_2 - \rho_1)((1-\gamma)(1-F(x_1^N(e))) - f(x_1^N(e))(1-\gamma)x_1^N(e)),$$

which is strictly smaller than:

$$(\rho_2 - \rho_1)((1-\gamma)(1-F(x_1^N(e))) - f(x_1^N(e))((1-\gamma)x_1^N(e) - e)),$$

which is equal to the l.h.s. of (B23) computed for $e \in [0,1]$, because in this case $b_2^N(e) = b_1^N(e) + (\rho_2 - \rho_1)e$. Hence it is sufficient to show that the latter expression is negative. Using again (B22) for $\omega = 1$, this is equivalent to showing that:

$$(1-\gamma)(b_1^N(x_1^N(e)) - b_1^{N\prime}(x_1^N(e))x_1^N(e)) + b_1^{N\prime}(x_1^N(e))e - b_1^N(e) < 0. \tag{B24}$$

Note that $b_1^N(x_1^N(e)) - b^{N\prime}{}_1(x_1^N(e))x_1^N(e) \geq b_1^N(x_1^N(e)) - Ax_1^N(e) > b_1^N(x_1^N(e)) - Ax_1^N(e) - B \geq 0$, where the first and last inequality follow from the fact that $b_1^N(e)$ is weakly convex and that $b_1^N(e) = Ae + B$ for $e > e_2^N$, whereas the second inequality follows from the fact that $B > 0$. Thus, the l.h.s. of (B24) is strictly decreasing in γ, and so it is sufficient to show that it is not positive for $\gamma = 0$. We thus need that $b_1^N(x_1^N(e)) - b_1^N(e) \geq b_1'^N(x_1^N(e))(x_1^N(e) - e)$, which is true because b_1 is weakly convex and $x_1^N(e) > e$. ■

Proof of Corollary 5

The result follows immediately from Proposition 4 and the discussion in the text. ■

Proof of Corollary 6

Consider the distribution of time periods since purchase for owners who consign their artworks for the first time. For initial consignments in recessions, this distribution is composed of dispersed time periods for distressed owners and the shortest possible time periods for flippers. The distribution in expansions is obtained by adding the consignments of non-distressed investors. Because such non-distressed investors try to sell at the earliest expansion, their time period since purchase cannot exceed the time elapsed since the previous expansion and so it is bounded from above. ■

Proof of Corollary 7

Sales in recessions after a short holding period result either from distressed owners or from non-distressed flippers, whereas sales after an intermediate holding period mostly result from distressed sellers. Thus, when looking at sales in recessions, the average returns for short holding periods are above those for intermediate holding periods. In expansions, sales after short holding periods come from distressed owners, non-distressed flippers, and non-distressed investors. Intermediate holding periods are realized mostly by distressed owners and non-distressed investors. Because investors' returns are on average between those of distressed owners and those of flippers, the difference between the average returns for short holding periods and the average returns for intermediate holding period should be lower for sales in expansions than for sales in recessions. ■

Proof of Corollary 8

Let us focus on consignments of artworks at some time t by owners who have held for a relatively long period. It is likely that all these consignments come from distressed owners who, for κ small, all choose a reserve type close to $x^D_{\omega_t}(0)$. Let μ^L_t denote the fraction of buy-ins and η^L_t the fraction of transactions occurring at the reserve price among these auctions. Observe that:

$$\begin{aligned}
\mu^L_t &\simeq F(x^D_{\omega_t}(0))^{n_{\omega_t}}, \\
\eta^L_t &\simeq n_{\omega_t} F(x^D_{\omega_t}(0))^{n_{\omega_t}-1}(1 - F(x^D_{\omega_t}(0))), \\
I^L_t &\simeq \frac{A_t + B_t}{1 - \mu^L_t}, \\
M_t &\simeq A_t + C_t,
\end{aligned}$$

where

$$\begin{aligned} A_t &= \int_{x^D_{\omega_t}(0)}^{1} n_{\omega_t}(n_{\omega_t}-1)f(z)F(z)^{n_{\omega_t}-2}(1-F(z))b_{\omega_t}(z)dz, \\ B_t &= b_t(x^D(0))\eta^L_t, \\ C_t &= E[b_{\omega_t}(\tilde{e}^{(2)}_t)|\tilde{e}^{(2)}_t < x^D_{\omega_t}(0)](\eta^L_t + \mu^L_t). \end{aligned}$$

Therefore:

$$M_t \simeq I^L_t(1-\mu^L_t) + C_t - B_t.$$

One can expect B_t and C_t to be comparable, and hence $M_t \simeq I^L_t(1-\mu^L_t)$. ■

Proof of Corollary 9

The first statement follows directly from the discussion preceding the corollary. The second statement follows from the fact that if in state ω the buy-in rate is very low, almost all transactions are observed. Because some might occur at the reserve price, the bias $I_\omega - M_\omega$ remains positive but will be small. For a very high buy-in rate, in most of the auctions the highest bid is below the reserve price, and hence the few observed transactions occur at prices substantially larger than the average second-highest bid in the economy. Thus the bias $I_\omega - M_\omega$ is positive and large. We can use the argument of Corollary 8 for a more formal proof. For any given $\omega \in \{1, 2\}$ we can approximate M_ω by $(1-\mu^L_\omega)I^L_\omega$. Thus if μ^L_1 is substantially larger than μ^L_2, then $I^L_2/I^L_1 < M_2/M_1$. ■

Proof of Corollary 10

The result follows directly from the negative correlation between holding periods and returns and the discussion in the text.

Appendix C Stationary Owner Type Distribution

The aim of this Appendix is to show that a stationary owner type distribution exists. Consider a stationary economy for a single artwork for which emotional dividends are i.i.d. according to a c.d.f. F. Assume that $\kappa = 0$. Observe that because $x^D(e) = x^D(0)$, and eventually all owners are hit by a liquidity shock, only buyers of type $e \geq x^D(0)$ can purchase in the long run. Let Γ be the set of distribution functions over the interval $[x^D(0), 1]$. If a stationary owner type distribution G^* exists, it belongs to Γ. Thus, without loss of generality we can focus on $G_t \in \Gamma$ in what follows. Let $\mathcal{P} : [x^D(0), 1] \to \Gamma$ be the transition function of the Markov process describing how the owner type changes between t and $t+1$. Formally, for any measurable subset A of $[x^D(0), 1]$, let $\mathcal{P}(e, A)$ denote the probability that at the end of time $t+1$ the owner of the artwork is an individual of type $e' \in A$, conditionally on the the fact that at the end of time t the owner is of type $e \in [x^D(0), 1]$. For a distribution $G \in \Gamma$, let T be the adjoint operator associated with $\mathcal{P}$:

$$[TG](A) = \int \mathcal{P}(z, A) G(dz).$$

We interpret $[TG](A)$ as the probability that time-$t+1$ owner's type lies in the set A, if the time-t owner's type is drawn from a probability distribution G. A distribution G^* is an invariant measure of T if $[TG^*] = G^*$.

Definition C1 *The economy has a stationary equilibrium if the Markov equilibrium adjoint operator T has an invariant measure G^* and any $G_0 \in \Gamma$ converges to G^*.*

Consider now a market of many artworks, for which bidder types are i.i.d. according to the same c.d.f. F. If the economy has a stationary equilibrium, then one can interpret G^* as the long-term cross-sectional distribution of owners' types for these artworks.

Proposition C1 *If $\kappa = 0$, then a stationary economy has a unique stationary equilibrium.*

Proof. We use some properties of strong convergence of Markov processes that can be found in chapters 8 and 11 of Stokey and Lucas (1989) (henceforth SL). In particular, we will use the definition of "Condition M" (page 348 of SL) and "Theorem 11.12" (page 350 of SL). Let $\mathcal{S}$ be a measurable space, let Γ be the space of probability measures on $\mathcal{S}$ with the total variation norm, let $\mathcal{P}$ be a transition function on $\mathcal{S}$, let T be the adjoint operator associated with $\mathcal{P}$, let $\mathcal{P}^m$ denote the m-th iteration of the transition function $\mathcal{P}$, and let A^c denote the complement of A in $\mathcal{S}$ for a set $A \in \mathcal{S}$.

Condition M *There exists $\varepsilon > 0$ and an integer $m \geq 1$ such that for any measurable set $A \in \mathcal{S}$, either $\mathcal{P}^m(e, A) \geq \varepsilon$, all $e \in \mathcal{S}$, or $\mathcal{P}^m(e, A^c) \geq \varepsilon$, all $e \in \mathcal{S}$.*

Theorem 11.12 *If $\mathcal{P}$ satisfies Condition M for some $m \geq 1$ and $\varepsilon > 0$, then there exists a unique $G^* \in \Gamma$ such that:*

$$||T^{mk} G_0 - G^*|| \leq (1 - \varepsilon) ||G_0 - G^*||,$$

all $G_0 \in \Gamma$, $k = 1, 2, \ldots$

So if $\mathcal{P}$ satisfies Condition M, then the mapping T is a contraction, and hence has a unique invariant measure to which converge all initial distributions of owner types $G_0 \in \Gamma$.

Let us apply Theorem 11.12 to our economy, where $\mathcal{S} = [x^D(0), 1]$. To prove the proposition, it is sufficient to show that the $\mathcal{P}$ defined by our Markov equilibrium satisfies Condition M. Observe that because in every period t the owner is in distress with at least probability d, the probability that the next period owner's type is in a set $A \subseteq \mathcal{S}$ satisfies:

$$\mathcal{P}(e, A) \geq d \int_A nf(e')F(e')^{n-1}de' > d\alpha U(A)$$

where $\alpha := \min_{e \in \mathcal{S}} nf(e)F(e)^{n-1} > 0$, and $U(A)$ is the measure of the set A according to the uniform distribution on $[x^D(0), 1]$. Note that for any A, either $U(A) \geq 1/2$ or $U(A^c) \geq 1/2$. Then either $\mathcal{P}(e, A) \geq d\alpha U(A) \geq d\alpha/2$ or $\mathcal{P}(e, A^c) \geq d\alpha U(A^c) \geq d\alpha/2$, for all e. Thus condition M is satisfied for $\varepsilon = d\alpha/2$ and $m = 1$. ■

Appendix D Long-Lived Agents with Known Preferences

In this Appendix we present a stylized model in which owners are long-lived and know each other well. The purpose is to show that there exists an equilibrium that is intuitive, and in which holding periods and returns are negatively correlated, just like in our baseline model.

Consider an economy where two long-lived individuals, 1 and 2, meet repeatedly over time at auctions for a given artwork. As in our main model, ownership of the artwork in any given period t provides individual i with an emotional dividend e_i if the individual is not in distress, and of c if he is in distress. In every period t, a non-distressed individual is hit by a liquidity shock with exogenous probability d, whereas a distressed individual recovers with probability κ. With an abuse of notation we let σ now indicate the distress status profile for both individuals. The set of status profiles is $\Sigma := \{NN, DN, ND, DD\}$. (For example, $\sigma_t = DN$ indicates that at time t individual 1 is in distress and individual 2 is not.) Let $e_i(\sigma)$ equal e_i if individual i is not in distress and c if he is in distress. We assume full information: in every period t, individuals know each other's emotional dividend and distress status. Without loss of generality, we can assume that $c \leq 0$, and $e_1 \leq e_2$. In any t, we will call the individual owning the asset the "owner" and the other individual the "bidder". In this simple setting, the state of the economy at any time t is a triplet indicating who is the current owner of the artwork, the status of individual 1, and the status of individual 2.

Because this is a relatively standard repeated-game setting, the Folk theorem applies and for δ close enough to 1 there is a continuum of equilibria. Rather than characterizing these equilibria, we focus on Markov equilibria that hold no matter the discount factor $\delta > 0$, in which in any period t consignment, reserve price, and bidding strategies only depend on the state of the economy.

Observe first that because there is complete information about each individual's taste and status, individuals know each other's equilibrium value function of becoming an owner. Thus, conditional on auctioning the artwork, the owner will set a reserve price equal to the other individual's valuation. This has three implications. First, the value function of being a bidder is nil, because a bidder purchases at a price that equals his valuation as an owner. Second, buy-ins do not occur in equilibrium, and hence γ_{BI} is irrelevant. Third, a Markov equilibrium can be represented by a partition of Σ into three sets $\{\Sigma 0, \Sigma 1, \Sigma 2\}$. When $\sigma_t \in \Sigma 0$, then the owner does not sell no matter whether he is individual 1 or 2. When $\sigma_t \in \Sigma 1$ the artwork is auctioned only if the owner is individual 1, whereas when $\sigma_t \in \Sigma 2$ the artwork is auctioned only if the owner is individual 2. The equilibrium also specifies the transaction price $p(\sigma)$ for all $\sigma \notin \Sigma 0$.

For any $\sigma \in \Sigma$, let $V_i(\sigma)$ and $B_i(\sigma)$ denote the equilibrium value function for individual i when he is the owner or the bidder, respectively. Denote by $E\left[V_i(\tilde{\sigma})|\sigma\right]$ and $E\left[B_i(\tilde{\sigma})|\sigma\right]$ individual i's expected owner and bidder value functions in $t+1$ conditional on the status profile in t being σ. We then have:

Lemma D1 *If $\{\Sigma 0, \Sigma 1, \Sigma 2\}$ is a Markov equilibrium, then we have the following results:*
(i) If $\sigma \in \Sigma i$, then the reserve price and the transaction price are both equal to $V_{-i}(\sigma)$.
(ii) $B_i(\sigma) = 0$ for all $i = 1, 2$ and all $\sigma \in \Sigma$.

(iii) A status $\sigma \in \Sigma i$, $i = 1, 2$, if and only if:

$$\begin{aligned} V_{-i}(\sigma) &= e_{-i}(\sigma) + \delta E\left[V_{-i}(\tilde{\sigma})|\sigma\right], && \text{(D1)} \\ V_i(\sigma) &= (1-\gamma)V_{-i}(\sigma) > 0, && \text{(D2)} \\ e_i(\sigma) + \delta E\left[V_i(\tilde{\sigma})|\sigma\right] &\leq (1-\gamma)V_{-i}(\sigma). && \text{(D3)} \end{aligned}$$

(iv) A status $\sigma \in \Sigma 0$ if and only if for all $i = 1, 2$:

$$e_i(\sigma) + \delta E\left[V_i(\tilde{\sigma})|\sigma\right] \geq (1-\gamma)V_{-i}(\sigma), or \tag{D4}$$

$$V_i(\sigma) < 0. \tag{D5}$$

Proof. Without loss of generality, let σ_t be the status profile at time t, and the owner be individual i. Let $r_i(\sigma_t)$ denote the owner's reserve price if he chooses to sell.

Decision to buy. Because there is no competition among bidders in this simple economy, if individual $-i$ chooses to bid when the artwork is for sale he will acquire it for the reserve price $r_i(\sigma_t)$. Alternatively, he could wait for the next period. Thus, faced with the opportunity to purchase the artwork for $r_i(\sigma_t)$, individual $-i$ would bid only if:

$$e_{-i}(\sigma_t) - r_i(\sigma_t) + \delta E[V_{-i}(\tilde{\sigma}_{t+1})|\sigma_t] \geq \delta E[B_{-i}(\tilde{\sigma}_{t+1})|\sigma_t]. \tag{D6}$$

In other words, individual $-i$ purchases if the emotional dividend net of the purchase price plus the discounted expected continuation payoff of being the owner in the next period exceeds the discounted expected continuation payoff of being the bidder in the next period.

Reserve price. Knowing bidder $-i$'s optimal decision to bid, conditional on choosing to sell the artwork, owner i's optimal reserve price in state ω_t equals:

$$r_i(\sigma_t) = e_{-i}(\sigma_t) + \delta E[V_{-i}(\tilde{s}_{t+1}) - B_{-i}(\tilde{s}_{t+1})|\sigma_t]. \tag{D7}$$

This makes sure that the auction is successful and that the proceeds are maximized.

Decision to sell. Consider now the decision to sell for owner i. In state σ_t, the owner will sell only if:

$$(1-\gamma)r_i(\sigma_t) + \delta E[B_i(\tilde{\sigma}_{t+1})|\sigma_t] \geq e_i(\sigma_t) + \delta E[V_i(\tilde{\sigma}_{t+1})|\sigma_t]. \tag{D8}$$

This means that owner i will sell only if the auction proceeds net of fees plus the discounted expected continuation payoff of being a bidder in the next period exceed the current emotional dividend plus the discounted expected continuation payoff of being an owner in the next period. From Eqs. (D7) and (D8) we have that $\sigma_t \in \Sigma i$ (i.e., owner i sells in status profile σ_t) if and only if:

$$(1-\gamma)e_{-i}(\sigma_t) - e_i(\sigma_t) \geq \delta E[V_i(\tilde{\sigma}_{t+1}) - B_i(\tilde{\sigma}_{t+1}) - (1-\gamma)(V_{-i}(\tilde{\sigma}_{t+1}) - B_{-i}(\tilde{\sigma}_{t+1})|\sigma_t]. \tag{D9}$$

We thus have that for $\sigma_t \notin \Sigma i$ (i.e., when owner i does not sell) owner and bidder equilibrium continuation payoffs are:

$$V_i(\sigma_t) = e_i(\sigma_t) + \delta E[V_i(\tilde{\sigma}_{t+1})|\sigma_t], \quad \text{(D10)}$$
$$B_{-i}(\sigma_t) = \delta E[B_{-i}(\tilde{\sigma}_{t+1})|\sigma_t]. \quad \text{(D11)}$$

If $\sigma_t \in \Sigma i$ (i.e., when owner i sells), then we have:

$$V_i(\sigma_t) = (1-\gamma)(e_{-i}(\sigma_t) + \delta E[V_{-i}(\tilde{\sigma}_{t+1}) - B_{-i}(\tilde{\sigma}_{t+1})|\sigma_t]) + \delta E[B_i(\tilde{\sigma}_{t+1})|\sigma_t] \quad \text{(D12)}$$
$$B_{-i}(\sigma_t) = e_{-i}(\sigma_t) + \delta E[V_{-i}(\tilde{\sigma}_{t+1})|\sigma_t] - r_i(\sigma_t) = \delta E[B_{-i}(\tilde{\sigma}_{t+1})|\sigma_t] \quad \text{(D13)}$$

Observe that because an auctioning owner sets a reserve price such that the bidder is indifferent between buying and not, we have that there is no positive utility from being a bidder: $B_i(\sigma) = 0$ for each individual i and status profile σ, which is result (ii). Replacing the bidder's continuation payoffs by 0, we obtain: result (i) from Eqs. (D7) and (D10); Eqs. (D1) and (D2) from equalities (D10) and (D12), respectively; conditions (D3) and (D4) from inequality (D9). Finally, to see that no trade is possible when both individuals have negative continuation values (condition (D5)), it is sufficient to focus on the case where $V_i(\sigma) < V_{-i}(\sigma) < 0$. In this case, the owner of the artwork would have to—and be happy to—sell for a negative price. But this is impossible in an auction. ■

The specific Markov equilibrium of this economy will depend on the value of the exogenous parameters e_1, e_2, c, d, k, γ and δ. In what follows, we study the equilibrium for parameter values (or value ranges) $e_1 = 0.25$, $0.25 \leq e_2 < 1$, $-1 < c < 0$, $0 < d, k < 0.25$, $\gamma = 0.15$, and $\delta = 1/1.1$. To guide our analysis, we start from some reasonable claims for which we then verify that they are satisfied in equilibrium. First, we claim that a non-distressed owner will not sell to a bidder in distress. Second, because $e_1 \leq e_2$ implies that a non-distressed owner 2 does not value the artwork less than a non-distressed owner 1 and because there are transaction costs, we claim that in status profile NN owner 2 does not sell.

We can identify four scenarios as candidates for a Markov equilibrium that satisfies these claims. Let us start with a symmetric setting where e_2 is equal to e_1. It is then reasonable to focus on a symmetric Markov equilibrium where a trade occurs only if the owner is in distress and the bidder is not. Formally:

Scenario 1 (symmetric): $\{\Sigma 0, \Sigma 1, \Sigma 2\} = \{\{NN, DD\}, \{DN\}, \{ND\}\}$.

For e_2 sufficiently larger than e_1 it become sensible for owner 1 to sell to bidder 2 whenever the latter is not in distress. Moreover, a distressed owner 1 may sell to a distressed bidder 2. Thus, we have the following two scenarios:

Scenario 2: $\{\Sigma 0, \Sigma 1, \Sigma 2\} = \{\{DD\}, \{NN, DN\}, \{ND\}\}$.

Scenario 3: $\{\Sigma 0, \Sigma 1, \Sigma 2\} = \{\{\emptyset\}, \{NN, DN, DD\}, \{ND\}\}$.

Finally if the distress cost c is close to 0, the probability of recovery from distress k is large and e_2 is substantially larger than e_1, then it is not optimal for owner 2 to sell even when in distress. Furthermore, in this case it becomes optimal for owner 1 not to sell when the status profile is DD. He prefers to pay the small distress cost and to wait for the recovery of individual 2, so as to sell at a higher price. In this case we have:

Scenario 4: $\{\Sigma 0, \Sigma 1, \Sigma 2\} = \{\{ND, DD\}, \{NN, DN\}, \{\emptyset\}\}$.

To each one of the above scenarios corresponds a system of eight equations where the unknowns are the owners' value functions in the four possible status profiles. After solving for the value functions, one needs to check whether they satisfy the inequalities in Lemma D1.

We illustrate this method with Scenario 2. Suppose that $\{\{DD\}, \{NN, DN\}, \{ND\}\}$ forms a Markov equilibrium and apply the conditions of Lemma D1. Consider, for example, status profile $\sigma = NN$. Because $NN \in \Sigma 1$, if the owner is individual 1, he will sell to individual 2 for a reserve price of $V_2(NN)$ and will receive net proceeds of $V_1(NN) = (1-\gamma)V_2(NN)$. This must exceed the value of keeping the artwork, i.e., enjoying the emotional dividend and being the owner in the next period, which equals $e_1 + \delta E[V_1(\tilde{s})|NN]$, or the first line of system (D14). Similarly, for owner 2, $V_2(NN) = e_2 + \delta E[V_2(\tilde{\sigma})|NN]$. This must exceed $(1-\gamma)V_1(NN)$, which gives us the second line of system (D14). Applying an analogous approach to the other status profiles, we get the following system of equalities and inequalities:

$$\begin{cases} V_1(NN) = & (1-\gamma)V_2(NN) & \geq & e_1 + \delta E[V_1(\tilde{\sigma})|NN] \\ V_2(NN) = & e_2 + \delta E[V_2(\tilde{\sigma})|NN] & \geq & (1-\gamma)V_1(NN) \\ V_1(ND) = & e_1 + \delta E[V_1(\tilde{\sigma})|ND] & \geq & (1-\gamma)V_2(ND) \\ V_2(ND) = & (1-\gamma)V_2(ND) & \geq & c + \delta E[V_2(\tilde{\sigma})|ND] \\ V_1(DN) = & (1-\gamma)V_2(DN) & \geq & c + \delta E[V_1(\tilde{\sigma})|DN] \\ V_2(DN) = & e_2 + \delta E[V_2(\tilde{\sigma})|DN] & \geq & (1-\gamma)V_1(DN) \\ V_1(DD) = & c + \delta E[V_1(\tilde{\sigma})|DD] & \geq & (1-\gamma)V_2(DD) \\ V_2(DD) = & c + \delta E[V_2(\tilde{\sigma})|DD] & \geq & (1-\gamma)V_1(DD), \end{cases} \tag{D14}$$

where $E[V_i(\tilde{\sigma})|\sigma]$ is obtained using the following transition probability matrix from $\sigma_t \in \Sigma$ to $\sigma_{t+1} \in \Sigma$.

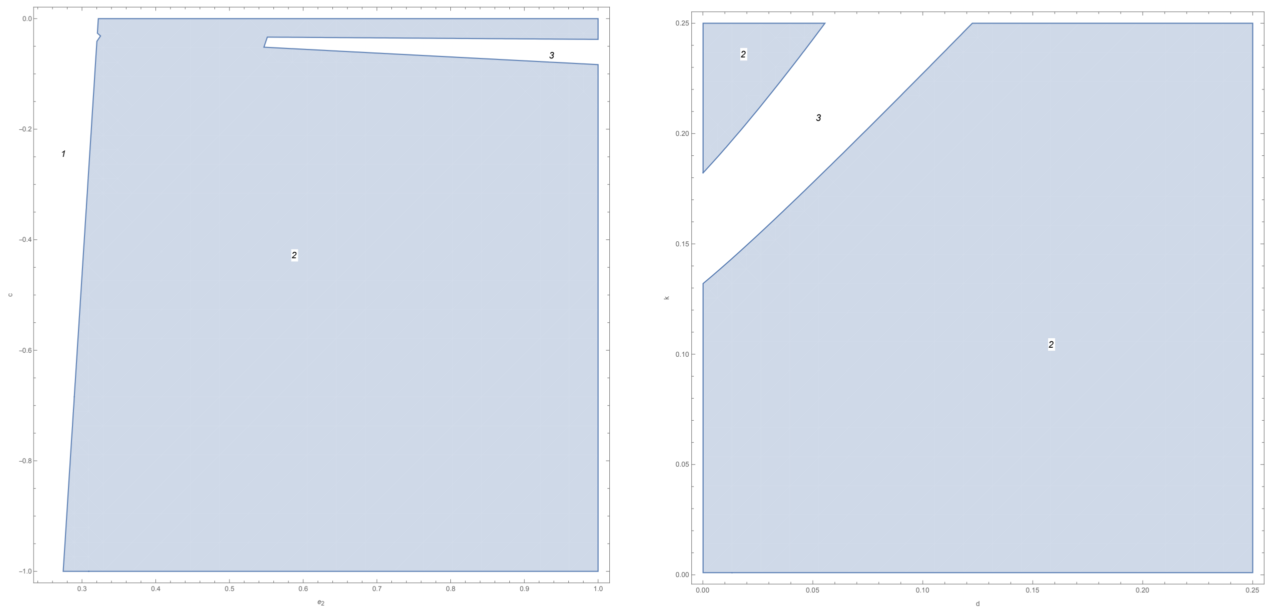
$$\begin{pmatrix} (1-d)^2 & d(1-d) & d(1-d) & d^2 \\ (1-d)\kappa & (1-d)(1-\kappa) & dk & d(1-\kappa) \\ (1-d)\kappa & d\kappa & (1-d)(1-\kappa) & d(1-\kappa) \\ \kappa^2 & \kappa(1-\kappa) & \kappa(1-\kappa) & (1-\kappa)^2 \end{pmatrix}$$

One can first solve the system of eight equalities to obtain the values for $V_i(\sigma)$.[1] Then, one needs to check that these values satisfy the eight inequalities in the system.

The figure below illustrates the regions of parameter values for which scenarios 1, 2, and 3 emerge as a Markov equilibrium. Scenario 4 only emerges for c sufficiently small and κ sufficiently large. The left panel is obtained by setting $d = 0.05$, $\kappa = 0.01$, $\delta = 1/1.1$, $\gamma = 0.15$, $e_1 = 0.25$ and varying $(e_2, c) \in [e_1, 1] \times [-1, 0]$. Each region is labeled with numbers that correspond to the scenario that emerges as a Markov equilibrium for parameters (c, e_2) in that region. As expected, Scenario 1 occurs only if e_2 is sufficiently close to $e_1 = 0.25$, whereas Scenario 4 does not emerge for this range of the parameters. For the other values of e_2 and c in the considered range, Scenarios 2 and 3 emerge as Markov equilibria. In the right panel, we set $e_1 = 0.25$, $e_2 = 0.5$, $c = -0.1$, $\delta = 1/1.1$, $\gamma = 0.15$, and vary $(d, k) \in [0, 0.25] \times [0, 0.25]$. The asymmetry in emotional dividends

[1] While this system has a closed-form solution, we spare the reader its long algebraic expression, as it would not add much to the understanding of the equilibrium.

renders scenario 1 incompatible with equilibrium, and neither in this case Scenario 4 emerges. For most other values of d and k in the considered range, Scenario 2 emerges. Also Scenario 3 can be an equilibrium for some specific values.



(a) (b)

Figure D1: The shaded area is the region of parameters for which the equilibrium corresponding to scenario 2 emerges. Panel (a): Markov equilibria for different values of e_2 and c, for $e_1 = 0.25$, $d = 0.05$, $\kappa = 0.01$, $\delta = 1/1.1$, and $\gamma = 0.15$. Panel (b): Markov equilibria for different values of d and κ, for $e_1 = 0.25$, $e_2 = 0.5$, $c = -0.1$, $\delta = 1/1.1$, and $\gamma = 0.15$.

Scenario 4 is clearly the less interesting for our analysis. Individual 2 never sells, so no repeated sale shall be observed in this scenario. This scenario only emerges if the probability k of recovery is sufficiently large or the probability of default is low. Scenario 3, which emerges as an equilibrium for a relatively small set of parameter values, involves distressed investor 1 selling to distressed investor 2. Whereas a transaction of this type is in principle possible, it would be unlikely to occur if one extends the analysis to more than two bidders. In this case the probability of all individuals simultaneously being in distress is small, and a distressed owner would prefer to sell to a non-distressed buyer. Scenario 1 is an equilibrium only if individuals have sufficiently similar tastes. Thus, the most sensible scenario is Scenario 2, which is also the one that emerges for the widest range of parameter values.

We can now show that, within the framework of this simple model, two fundamental predictions of the baseline model in our paper survive the introduction of long-lived bidders who repeatedly interact with each other:

Lemma D2 *(i) In all four scenarios, the average holding period is weakly larger for the individual with the largest emotional dividend.*

(ii) In Scenarios 1 and 2, the average return is smaller for the individual with the largest emotional dividend.

Proof. Recall that individual 2's emotional dividend is larger than individual 1's.

(i) In Scenario 1, individual 1 buys in status profile ND and sells in status profile DN, while no trade occurs otherwise. Because the transition probability matrix is symmetric, the average holding period is the same for owner 1 and 2. In Scenario 2, individual 1 buys in status ND but sells in status profiles DN and NN. Therefore, individual 2 is expected to hold the artwork for a longer period than individual 1. The difference in average holding periods is even stronger in Scenario 3: individual 1 buys in status ND but sells in status profiles DN, NN, and DD. In Scenario 4, individual 1 sells in status profiles NN and DN, and hence has a finite expected holding period. Individual 2 never sells, so his holding period is infinitely long.

(ii) Consider that the return of individual 1 is the opposite of the return of individual 2. It is sufficient to show that the average return for individual 1 is positive. In Scenario 1, individual 1's total return is $p(DN)/p(ND) - 1 = V_2(DN)/V_1(ND) - 1 \geq 0$, because the equilibrium behavior is symmetric but $e_2 \geq e_1$. In Scenario 2, there are two possible returns. If individual 1 sells in state NN, then the holding period return is $p(NN)/p(ND) - 1 > p(DN)/p(ND) - 1$, which is the return when the sale is in state DN. Considering that $p(DN)/p(ND) = V_2(DN)/V_1(ND)$, it is sufficient to show that $V_2(DN) - V_1(ND) > 0$. Considering that $V_2(DN) = e_2 + \delta E[V_2|DN]$ and $V_1(DN) = e_1 + \delta E[V_2|DN]$, we have that:

$$V_2(DN) - V_1(ND) = \frac{e_2 - e_1 + \delta(\gamma k(1-d)V_2(NN) + (1-k)d(V_2(DD) - V_1(DD))}{1 - \delta((1-k)(1-d) - (1-\gamma)dk)} > 0.$$

■

Appendix E A Calibration Exercise

Given that our model is stylized, it is useful to examine whether it can (simultaneously) account for the magnitudes of the price and volume changes that we see over business cycles in the data.[1] We therefore conduct a simple calibration exercise using a market consisting of 1,500 hypothetical artworks. We let a period equal a calendar year, since we see virtually zero resale attempts within twelve months of the purchase in the data. We set p_1 and p_2 equal to 0.50 and 0.25, respectively. These numbers reflect the transition probabilities in the time series of macroeconomic cycles from 1900 to 2015.[2] Motivated by available evidence on auction house commissions, we set our transaction cost parameters $\gamma = 15\%$ and $\gamma_{BI} = 2\%$.[3] No direct evidence exists on how many bidders enter the auction for any individual work. However, anecdotal reports from auction rooms suggest that turnout is highly procyclical, and that there are often only a handful of (potential) bidders for each individual item. We set the number of bidders equal to five in recessions and let this number double in expansions, i.e., $n_1 = 5$ and $n_2 = 10$. Furthermore, we normalize ρ_2 to 1, and calibrate ρ_2 to 0.75 to match the substantial destruction of private wealth that is typical of recessions.[4] We choose $d = 5\%$ to reflect that the types of liquidity shocks relevant in the art market—death, debt, divorce—are infrequent but not too rare. Finally, we set $\kappa = 0$ and $c = -0.1$, and use a discount rate of 10%.[5]

We calibrate the two parameters that control the beta distribution of types to match a level of both consignment and transaction volume that is one third lower in recessions,[6] which is roughly the average total drop over the first two years of each of the recessions in our sample period. Such a (consignment) volume decrease is matched by a beta distribution with parameters $\alpha = 0.4$ and $\mu = 7$. With a heavily positively skewed distribution of types—which reflect variation in both tastes and wealth—the model can thus account for the sizeable shifts in auction volume that we observe in the data.

The next question is then what is the associated predicted change in price levels—an endogenous moment not targeted by our procedure? We find that average transaction prices are about 15%

[1] Even more fundamentally, note that the predictions of our model on the cyclicality of prices and volume (Proposition 4) were derived under a number of assumptions on the (relative) values of certain parameters (Assumption 1). It is therefore useful to check whether a model calibrated to realistic values for these parameters produces the same results as the ones in Corollary 5.

[2] We classify a year as recessionary if it includes three or more months that fall into an NBER recession period. Of course, unlike in our model, we observe secular economic growth over time, and also variation in growth rates *between* different recessions or expansions. We will here focus on what happens on average when the economy transitions from an expansion into a recession (or vice versa).

[3] Sotheby's (2017) reported reported "auction commission margins", i.e., total commission revenues as a percentage of sales, of between 14.3% and 17.1% for the years 2012–2016. The fraction of the reserve price that potential sellers have to pay to the auction house in case of a buy-in varies across auctions and consignors, but is never more than a few percentage points.

[4] Based on data from the Panel Study of Income Dynamics, Pfeffer, Danziger, and Schoeni (2013) report a decrease in net worth of 27% between 2007 and 2011 for households at the 95th percentile of the net worth distribution.

[5] The results are robust to the introduction of a positive but small κ, and also to small changes in the other parameters. We can, however, not match the cyclicality in the market if we set the discount rate close to zero.

[6] We let our artworks trade over 116 years, using the actual history of macroeconomic states over the period 1900–2015. Artworks are randomly allocated to types at the start of 1900. We measure changes in volume and prices over the period 1976–2015 in our calibration.

lower in recessions than in expansions. This is about half the average drop that we observe at the start of recessions in the data. Taken altogether, the calibration clarifies that our model—despite the simplifying assumptions—can generate economically significant procyclicality in consignment volume, transaction volume, and prices.[7]

[7] The average holding period (computed over purchase and sale sample periods matching our data set) equals 10 years in our calibrated model, which is close to what we observe empirically. The average buy-in rate is higher in our calibration than in the data, which suggests that there may be an implicit cost of a failure to meet the reserve price that is outside our model (e.g., Beggs and Graddy (2008)).

Appendix F Additional Empirical Results on Business Cycles

Cyclicality in distribution of time periods since purchase. One implication of our model is that, when computing the periods since purchase for consignments in expansions, we should see a higher fraction of short to intermediate period lengths, because we also have investors trying to sell then (Corollary 5). Do we see such a pattern in our data? Figure F1 shows the empirical distribution of time intervals since purchase for resale attempts during the 2008–2009 recession and compares it to the distribution for auctions in 2007 and 2010–2015. Our analysis is complicated by the relatively small number of observations in the recession subsample, and by the fact that in any year the observed distribution of periods since purchase will be a complex function of the whole history of market conditions (and data coverage). Yet, the findings are suggestive of state-dependent behavior on the part of art owners such as modeled in this paper.

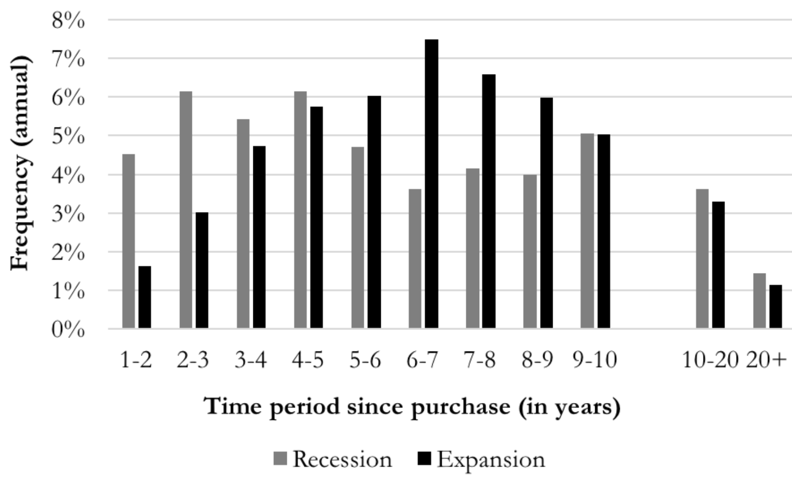


Figure F1: This figure shows the distribution of time periods since purchase in our database of resale attempts. We split the sample by the state of the economy in the year of the resale attempt. For holding periods of 10–20 and 20+ years, we compute the average frequency for one-year period lengths.

Cyclicality in relation between holding periods and returns. Finally, our model predicted that the negative relation between realized holding periods and returns should be less strong—at least over short to intermediate holding periods—for resales in expansions (Corollary 7). Table F1 repeats regression model (2) of Table 1 relating performance to realized holding periods separately on the subsamples of resales in recession and in expansion. The recession model shows a coefficient on the shortest holding period category that is large—and much larger than that on the next holding period variables. (The coefficient on the dummy for holding periods of 1–3 years has a p-value of 0.13. Note that the number of observations is small in this subsample.) By contrast, in the expansion subsample, we see coefficients of similar magnitudes up to holding periods of 7 years, after which (market-adjusted) returns start to decline.

Dependent variable	Total (market-adjusted) return	
Model	(1)	(2)
Resale state	Recession	Expansion
1–3 years since purchase	0.196	0.135
	0.129	0.088
3–5 years since purchase	0.068	0.154**
	0.126	0.066
5–7 years since purchase	0.032	0.143**
	0.140	0.061
7–10 years since purchase	-0.029	0.103*
	0.122	0.057
10–20 years since purchase	0.079	0.066
	0.098	0.050
20+ years since purchase	[left out]	[left out]
N	540	1,757

Table F1: This table shows the results of a set of ordinary least squares regressions relating total market-adjusted returns on successful artwork resales (as measured by the residuals of a repeat-sales regression) to the holding period in years. We split the sample by the state of the economy in the year of resale. Both specifications include a constant. *, **, and *** indicate statistical significance at the 0.10, 0.05, and 0.01 level, respectively.

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