

Appendix
to
Estimating Risk Preferences in the Field

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1 Models of Risk Preferences: Further Details

In this section, we provide further details about the models of risk preferences that we describe in Section 3 of the review. We illustrate how the models work — and how their predictions differ — by describing their predictions in some examples. In addition, for some of the models, we also describe some additional issues that arise.

We use the following three examples.

Example 1. *What is a household’s willingness to pay for insurance with deductible d against the possibility of losing L with probability μ ? In other words, what is the z that makes the household indifferent between the lottery $(-z-d, \mu; -z, 1-\mu)$ and the lottery $(-L, \mu; 0, 1-\mu)$?*

Example 2. *What is a household’s certainty equivalent for the lottery $X \equiv (x_1, \mu, x_2, 1-\mu)$ with $x_1 < x_2$? In other words, what is the z that makes the household indifferent between the lottery X and the lottery $(z, 1)$?*

Example 3. *What is a household’s willingness to pay for an asset that pays out x_1 with probability μ_1 , x_2 with probability μ_2 , and x_3 with probability μ_3 ? In other words, what is the z that makes the household indifferent between the lottery $(0, 1)$ and the lottery $(-z + x_1, \mu_1; -z + x_2, \mu_2; -z + x_3, \mu_3)$?*

The conditions derived below for Example 1 are the conditions that appear in Table 2 of the review. The equations derived below for Example 2 — in particular, the utility equations for the generic binary lottery X — are those that appear in Section 4.4 of the review. Finally, Example 3 highlights some further details for RDEU and CPT.

1.1 Expected Utility

According to expected utility (EU) theory, given a choice set \mathbf{X} , a person will choose the option $X \in \mathbf{X}$ that maximizes

$$EU(X) \equiv \sum_{n=1}^N \mu_n u(w + x_n),$$

where u is a utility function that maps final wealth onto the real line.

In our three examples:

Example 1 (Under EU). *A household’s willingness to pay for insurance with deductible d against the possibility of losing L with probability μ is the z such that*

$$\mu u(w - z - d) + (1 - \mu)u(w - z) = \mu u(w - L) + (1 - \mu)u(w).$$

Example 2 (Under EU). *A household's certainty equivalent for the lottery $X \equiv (x_1, \mu, x_2, 1-\mu)$ with $x_1 < x_2$ is the z such that*

$$u(w+z) = \mu u(w+x_1) + (1-\mu)u(w+x_2).$$

Example 3 (Under EU). *A household's willingness to pay for an asset that pays out x_1 with probability μ_1 , x_2 with probability μ_2 , and x_3 with probability μ_3 is the z such that*

$$u(w) = \mu_1 u(w-z+x_1) + \mu_2 u(w-z+x_2) + \mu_3 u(w-z+x_3).$$

1.2 Rank-Dependent Expected Utility

Under rank-dependent expected utility (RDEU), we replace the EU equation with

$$V(X) \equiv \sum_{n=1}^N \omega_n u(w+x_n),$$

where ω_n is a decision weight associated with outcome x_n and may not be equal to a person's belief μ_n . When evaluating a lottery $X \equiv (x_1, \mu_1; x_2, \mu_2; \dots; x_N, \mu_N)$, if the outcomes are ordered such that $x_1 < x_2 < \dots < x_N$, then the weight on outcome n is

$$\omega_n = \begin{cases} \pi(\mu_1) & \text{for } n = 1 \\ \pi\left(\sum_{j=1}^n \mu_j\right) - \pi\left(\sum_{j=1}^{n-1} \mu_j\right) & \text{for } n \in \{2, \dots, N-1\} \\ 1 - \pi\left(\sum_{j=1}^{n-1} \mu_j\right) & \text{for } n = N \end{cases},$$

where π is a probability weighting function.

For our three examples, RDEU generates the following equations:

Example 1 (Under RDEU). *A household's willingness to pay for insurance with deductible d against the possibility of losing L with probability μ is the z such that*

$$\pi(\mu)u(w-d-z) + (1-\pi(\mu))u(w-z) = \pi(\mu)u(w-L) + (1-\pi(\mu))u(w).$$

Example 2 (Under RDEU). *A household's certainty equivalent for the lottery $X \equiv (x_1, \mu, x_2, 1-\mu)$ with $x_1 < x_2$ is the z such that*

$$u(w+z) = \pi(\mu)u(w+x_1) + (1-\pi(\mu))u(w+x_2).$$

Example 3 (Under RDEU). *A household's the willingness to pay for an asset that pays out*

x_1 with probability μ_1 , x_2 with probability μ_2 , and x_3 with probability μ_3 is the z such that

$$\begin{aligned} u(w) &= \pi(\mu_1)u(w - z + x_1) + (\pi(\mu_2 + \mu_1) - \pi(\mu_1))u(w - z + x_2) \\ &\quad + (1 - \pi(\mu_2 + \mu_1))u(w - z + x_3). \end{aligned}$$

Extrapolating from Example 2, under RDEU, when choosing among binary lotteries $X \equiv (x_1, \mu; x_2, 1 - \mu)$ with $x_1 < x_2$, a household chooses the lottery that maximizes

$$U(X) = \Omega(\mu)u(w + x_1) + (1 - \Omega(\mu))u(w + x_2),$$

where $\Omega(\mu) = \pi(\mu)$.

It's worth highlighting some implications of RDEU. Consider the implications of the Karmarkar (1978) probability weighting function with $\gamma = 0.50$ (as depicted in Figure 2 of the review). For binary lotteries, as in Examples 1 and 2, there will be overweighting of low probability events (events with $\mu < 1/2$) and underweighting of high probability events (events with $\mu > 1/2$). Hence, in Example 1, for instance, if the probability of a loss $\mu < 1/2$, then the weight $\pi(\mu)$ on the loss will be greater than the probability, and thus probability weighting generates a source of risk aversion. In contrast, if the probability of a loss $\mu > 1/2$, then the weight $\pi(\mu)$ on the loss will be less than the probability, and thus probability weighting generates a source of risk seeking.

For lotteries with more than two outcomes, such as in Example 3, an inverse-S-shaped probability weighting function (as discussed in the review) instead generates overweighting of extreme outcomes and underweighting of intermediate outcomes — and importantly two outcomes that are equally likely need not have the same decision weights. For instance, consider Example 3 when $\mu_1 = \mu_2 = \mu_3 = 1/3$. Given the Karmarkar function, the extreme outcomes of x_1 and x_3 will both be overweighted (i.e., $\pi(1/3) > 1/3$ and $1 - \pi(2/3) > 1/3$), while the intermediate outcome x_2 is underweighted (i.e., $\pi(2/3) - \pi(1/3) < 1/3$).

In the RDEU model outlined above, the probability weighting function is first applied to the worst outcome, and then successively applied to better and better outcomes — which we refer to as RDEU from the bottom. Some analyses do the reverse, so the probability weighting function is first applied to the best outcome, and then successively applied to worse and worse outcomes — which we refer to as RDEU from the top. Formally, if the outcomes are again ordered such that $x_1 < x_2 < \dots < x_N$, then, under RDEU from the top,

the weight on outcome n is

$$\omega_n = \begin{cases} \pi(\mu_N) & \text{for } n = N \\ \pi\left(\sum_{j=n}^N \mu_j\right) - \pi\left(\sum_{j=n+1}^N \mu_j\right) & \text{for } n \in \{2, \dots, N-1\} \\ 1 - \pi\left(\sum_{j=n+1}^N \mu_j\right) & \text{for } n = 1 \end{cases} .$$

In principle, the decision whether to apply RDEU from the bottom versus from the top might not matter. In particular, if we use π^T in RDEU from the top and π^B in RDEU from the bottom, the two forms yield identical predictions as long as these functions are symmetric, in the sense that $\pi^B(\mu) = 1 - \pi^T(1 - \mu)$ for all μ . Similarly, if one estimates an RDEU model with a nonparametric approach to π , it is not important which version one uses. However, if one uses a parametric functional form that does not satisfy the symmetry property, the direction along which RDEU is applied can yield different predictions. For instance, suppose we use the Prelec (1998) function with $\gamma = 0.61$ (as depicted in Figure 2 of the review) in Example 1 with $\mu = 0.40$. Under RDEU from the bottom, because $\pi(0.40) < 0.40$, we would underweight the loss event, whereas under RDEU from the top, because $1 - \pi(0.6) > 0.40$, we would overweight the loss event. In other words, applied from the bottom probability weighting would generate a source of risk seeking, whereas applied from the top it would generate a source of risk aversion.

In fact, both variants have been used in field applications. Typically, the version used depends on which outcome is more "focal" in a particular application. For insurance applications, where the loss event arguably is the focal event, researchers most often use RDEU from the bottom. For gambling applications, where the win event arguably is the focal event, researchers most often use RDEU from the top.

1.3 Cumulative Prospect Theory

Cumulative prospect theory (CPT) requires as an input a reference outcome s , and each outcome is coded as a gain or loss relative to this reference outcome. Consider a lottery $X \equiv (x_1, \mu_1; \dots; x_N, \mu_N)$ and a reference point s , and suppose $x_1 < \dots < x_{\bar{n}-1} \leq s < x_{\bar{n}} < \dots < x_N$. Under CPT, this lottery is evaluated as

$$V(X; r) \equiv \sum_{n=1}^N \omega_n v(x_n - s),$$

where the weight on outcome x_n is

$$\omega_n = \begin{cases} \pi^-(\mu_1) & \text{for } n = 1 \\ \pi^-\left(\sum_{j=1}^n \mu_j\right) - \pi^-\left(\sum_{j=1}^{n-1} \mu_j\right) & \text{for } n \in \{2, \dots, \bar{n} - 1\} \\ \pi^+\left(\sum_{j=n}^N \mu_j\right) - \pi^+\left(\sum_{j=n+1}^N \mu_j\right) & \text{for } n \in \{\bar{n}, \dots, N - 1\} \\ \pi^+(\mu_N) & \text{for } n = N \end{cases} .$$

In this formulation, π^- and π^+ are probability weighting functions applied to the loss and gain events, respectively. The value function v is assumed to have three key properties: (i) $v(0) = 0$ and it assigns positive value to gains and negative value to losses, (ii) it is concave over gains and convex over losses (often labelled "diminishing sensitivity"), and (iii) it is steeper in the loss domain than in the gain domain (often labelled "loss aversion").

For our three examples, CPT with a reference point $s = 0$ generates the following equations:

Example 1 (Under CPT). *A household's willingness to pay for insurance with deductible d against the possibility of losing L with probability μ is the z such that*

$$\pi^-(\mu)v(-d - z) + (1 - \pi^-(\mu))v(-z) = \pi^-(\mu)v(-L).$$

Example 2 (Under CPT). *A household's certainty equivalent for the lottery $X \equiv (x_1, \mu, x_2, 1 - \mu)$ with $x_1 < x_2$ is the z such that*

$$v(z) = \begin{cases} (1 - \pi^+(1 - \mu))v(x_1) + \pi^+(1 - \mu)v(x_2) & \text{if } s < x_1 \\ \pi^-(\mu)v(x_1) + \pi^+(1 - \mu)v(x_2) & \text{if } x_1 \leq s < x_2 \\ \pi^-(\mu)v(x_1) + (1 - \pi^-(\mu))v(x_2) & \text{if } s \geq x_2 \end{cases} .$$

Example 3 (Under CPT). *A household's willingness to pay for an asset that pays out x_1 with probability μ_1 , x_2 with probability μ_2 , and x_3 with probability μ_3 , where $x_1 < s < x_2 < x_3$ is the z such that¹*

$$\begin{aligned} v(0) &= \pi^-(\mu_1)v(-z + x_1) + (\pi^+(\mu_3 + \mu_2) - \pi^+(\mu_3))v(-z + x_2) \\ &\quad + \pi^+(\mu_3)v(-z + x_3). \end{aligned}$$

Extrapolating from Example 2, under CPT, when choosing among binary lotteries $X \equiv (x_1, \mu; x_2, 1 - \mu)$ with $x_1 < x_2$, we cannot reduce the model to one in which a household

¹Here we use π^+ to weigh the second event because we are assuming that z is such that $x_2 - z \geq 0$.

chooses the lottery that maximizes

$$U(X) = \Omega(\mu)u(w + x_1) + (1 - \Omega(\mu))u(w + x_2)$$

for some $\Omega(\mu)$. In particular the weights must also be a function of how the possible outcomes x_1 and x_2 compare to the reference point s , and moreover we must use the value function v in place of the utility function u , and the value function also takes as an input how x_1 and x_2 compare to the reference point s .

1.4 Expectations-Based Models

1.4.1 Kőszegi-Rabin Loss Aversion

Under the Kőszegi-Rabin (KR) loss aversion model (Kőszegi and Rabin 2006, 2007), the utility from choosing lottery $X \equiv (x_n, \mu_n)_{n=1}^N$ given a reference lottery $\tilde{X} \equiv (\tilde{x}_m, \tilde{\mu}_m)_{m=1}^M$ is

$$V(X|\tilde{X}) \equiv \sum_{n=1}^N \sum_{m=1}^M \mu_n \tilde{\mu}_m [u(w + x_n) + v(w + x_n | w + \tilde{x}_m)].$$

The function u represents standard "intrinsic" utility defined over final wealth, just as in EU. The function v represents "gain-loss" utility that results from experiencing gains or losses relative to the reference lottery. For the value function, KR use

$$v(y|\tilde{y}) = \begin{cases} \eta [u(y) - u(\tilde{y})] & \text{if } u(y) > u(\tilde{y}) \\ \eta\lambda [u(y) - u(\tilde{y})] & \text{if } u(y) \leq u(\tilde{y}) \end{cases}.$$

KR propose that the reference lottery equals recent expectations about outcomes — i.e., if a person expects to face lottery \tilde{X} , then her reference lottery becomes \tilde{X} . However, because situations vary in terms of when a person deliberates about and then commits to her choices, KR offer multiple solution concepts for the determination of the reference lottery. Here, we focus on two solution concepts that are perhaps most relevant for field data.

Definition 1 (KR-PPE). *Given a choice set \mathbf{X} , a lottery $X \in \mathbf{X}$ is a personal equilibrium if for all $X' \in \mathbf{X}$, $V(X|X) \geq V(X'|X)$, and it is a preferred personal equilibrium if there does not exist another $X' \in \mathbf{X}$ such that X' is a personal equilibrium and $V(X'|X') > V(X|X)$.*

Definition 2 (KR-CPE). *Given a choice set \mathbf{X} , a lottery $X \in \mathbf{X}$ is a choice-acclimating personal equilibrium if for all $X' \in \mathbf{X}$, $V(X|X) \geq V(X'|X')$.*

To date, the literature on estimating risk preferences using field data has focused exclusively on KR-CPE, and thus we focus on that solution concept here. Because the derivations

for KR-CPE are slightly more complicated than for the models above, it is useful to start with the utility equations.

For a certain lottery $X \equiv (x, 1)$, we have $V(X|X) = u(w + x)$.

For a binary lottery $X \equiv (x_1, \mu; x_2, 1 - \mu)$ with $x_1 < x_2$, we have

$$\begin{aligned}
V(X|X) &= \mu u(w + x_1) + (1 - \mu)u(w + x_2) \\
&\quad + \mu(1 - \mu)\eta [u(w + x_2) - u(w + x_1)] \\
&\quad + \mu(1 - \mu)\eta\lambda [u(w + x_1) - u(w + x_2)] \\
&= \mu [1 + \eta(\lambda - 1)(1 - \mu)] u(w + x_1) \\
&\quad + [1 - \mu [1 + \eta(\lambda - 1)(1 - \mu)]] u(w + x_2) \\
&= \mu [1 + \Lambda(1 - \mu)] u(w + x_1) \\
&\quad + [1 - \mu [1 + \Lambda(1 - \mu)]] u(w + x_2),
\end{aligned}$$

where $\Lambda \equiv \eta(\lambda - 1)$. Note two implications. First, under KR-CPE with binary lotteries, the parameters η and λ always appear in the lottery evaluation as the product Λ . In fact, this is true under KR-CPE for lotteries with any number of outcomes, and thus under KR-CPE only Λ can be identified. Second, under KR-CPE, when choosing among binary lotteries $X \equiv (x_1, \mu; x_2, 1 - \mu)$ with $x_1 < x_2$, a household chooses the lottery that maximizes

$$U(X) = \Omega(\mu)u(w + x_1) + (1 - \Omega(\mu))u(w + x_2),$$

where $\Omega(\mu) = \mu [1 + \Lambda(1 - \mu)]$.

With these equations in hand, it is straightforward to derive the implications of KR-CPE in Examples 1 and 2.

Example 1 (Under KR-CPE). *A household's willingness to pay for insurance with deductible d against the possibility of losing L with probability μ is the z such that*

$$\begin{aligned}
&\mu [1 + \Lambda(1 - \mu)] u(w - d - z) \\
+ [1 - \mu [1 + \Lambda(1 - \mu)]] u(w - z) &= \mu [1 + \Lambda(1 - \mu)] u(w - L) \\
&\quad + [1 - \mu [1 + \Lambda(1 - \mu)]] u(w).
\end{aligned}$$

Example 2 (Under KR-CPE). *A household's certainty equivalent for the lottery $X \equiv (x_1, \mu, x_2, 1 - \mu)$ with $x_1 < x_2$ is the z such that*

$$u(w + z) = \mu [1 + \Lambda(1 - \mu)] u(w + x_1) + [1 - \mu [1 + \Lambda(1 - \mu)]] u(w + x_2).$$

There is one further point with regard to KR-CPE: it sometimes can yield strange predictions. For instance, one might naturally think that, *ceteris paribus*, if the probability μ of a loss increases, the willingness to pay z for full insurance should also increase. However, if $\Lambda > 1$, one can show that for μ close enough to one, z declines with μ . The intuition for this result is that, under CPE, a person has a strong aversion to risk. Hence, because intermediate probabilities (close to $1/2$) involve more risk than probabilities close to one, the person is willing to pay more for full insurance (even though the expected loss is smaller). In fact, one can also show that, for any $\Lambda > 0$, under CPE a person can choose a dominated lottery. The intuition is much the same: the aversion to risk can be so strong that the person would rather choose a certain outcome over a risky lottery that dominates that certain outcome.

1.4.2 Disappointment Aversion

Under the Bell (1985) disappointment aversion model, a lottery $X \equiv (x_n, \mu_n)_{n=1}^N$ is evaluated as

$$V(X) = \sum_{n=1}^N \mu_n u(w + x_n) - \beta \sum_{n=1}^N \mu_n [I(u(w + x_n) < \bar{U}) (\bar{U} - u(w + x_n))],$$

where I is an indicator function and $\bar{U} \equiv \sum_{n=1}^N \mu_n u(w + x_n)$. The first term is the standard expected utility of lottery X . The second term reflects the expected disutility from disappointment that arises when the realized utility from an outcome is less than the standard expected utility of the lottery. The parameter β captures the magnitude of disappointment aversion, where the model reduces to expected utility for $\beta = 0$.²

Again, it is useful to start with the utility equations.

For a certain lottery $X \equiv (x, 1)$, we have $V(X) = u(w + x)$.

For a binary lottery $X \equiv (x_1, \mu; x_2, 1 - \mu)$ with $x_1 < x_2$, disappointment is experienced if and only if x_1 is realized, and thus we have

$$\begin{aligned} V(X) &= \mu u(w + x_1) + (1 - \mu)u(w + x_2) \\ &\quad - \beta \mu ([\mu u(w + x_1) + (1 - \mu)u(w + x_2)] - u(w + x_1)) \\ &= \mu [1 + \beta(1 - \mu)] u(w + x_1) \\ &\quad + [1 - \mu [1 + \beta(1 - \mu)]] u(w + x_2). \end{aligned}$$

²Bell (1985) further assumes that (i) $u(x) = x$ and (ii) a person might also experience utility from elation when the realized outcome is larger than the expected utility. Even with the latter, however, his model reduces to the model in the text where β represents the difference between the marginal disutility from disappointment and the marginal utility from elation. Loomes and Sugden (1986) also use this formulation, except they study nonlinear disappointment.

It follows that, under Bell-DA, when choosing among binary lotteries $X \equiv (x_1, \mu; x_2, 1 - \mu)$ with $x_1 < x_2$, a household chooses the lottery that maximizes

$$U(X) = \Omega(\mu)u(w + x_1) + (1 - \Omega(\mu))u(w + x_2),$$

where $\Omega(\mu) = \mu [1 + \beta(1 - \mu)]$.

In Examples 1 and 2, these equations imply:

Example 1 (Under Bel-DA). *A household's willingness to pay for insurance with deductible d against the possibility of losing L with probability μ is the z such that*

$$\begin{aligned} \mu [1 + \beta(1 - \mu)] u(w - d - z) \\ + [1 - \mu [1 + \beta(1 - \mu)]] u(w - z) &= \mu [1 + \beta(1 - \mu)] u(w - L) \\ &+ [1 - \mu [1 + \beta(1 - \mu)]] u(w). \end{aligned}$$

Example 2 (Under Bell-DA). *A household's certainty equivalent for the lottery $X \equiv (x_1, \mu, x_2, 1 - \mu)$ with $x_1 < x_2$ is the z such that*

$$u(w + z) = \mu [1 + \beta(1 - \mu)] u(w + x_1) + [1 - \mu [1 + \beta(1 - \mu)]] u(w + x_2).$$

Under the Gul (1991) disappointment aversion model, a lottery $X \equiv (x_n, \mu_n)_{n=1}^N$ is evaluated as $V(X) = \bar{V}$ such that

$$\bar{V} = \sum_{n=1}^N \mu_n u(w + x_n) - \beta \sum_{n=1}^N \mu_n [I(u(w + x_n) < \bar{V}) (\bar{V} - u(w + x_n))].$$

The z that solves $u(w + z) = \bar{V}$ is one's certainty equivalent for lottery X in this model. On the right-hand side, the first term is the standard expected utility of lottery X , while the second term reflects the expected disutility from disappointment that arises when the realized utility from an outcome is less than the utility from the certainty equivalent of the lottery. The parameter β captures the magnitude of disappointment aversion, where the model reduces to expected utility for $\beta = 0$.

Again, it is useful to start with the utility equations.

For a certain lottery $X \equiv (x, 1)$, clearly $V(X) = u(w + x)$.

For a binary lottery $X \equiv (x_1, \mu; x_2, 1 - \mu)$ with $x_1 < x_2$, disappointment is experienced if and only if x_1 is realized, and thus we have

$$\bar{V} = \mu u(w + x_1) + (1 - \mu)u(w + x_2) - \beta \mu (\bar{V} - u(x_1)).$$

Solving for \bar{V} and recalling that $V(X) = \bar{V}$ yields

$$V(X) = \frac{(1 + \beta)\mu}{1 + \beta\mu}u(w + x_1) + \left(1 - \frac{(1 + \beta)\mu}{1 + \beta\mu}\right)u(w + x_2).$$

It follows that, under Gul-DA, when choosing among binary lotteries $X \equiv (x_1, \mu; x_2, 1 - \mu)$ with $x_1 < x_2$, a household chooses the lottery that maximizes

$$U(X) = \Omega(\mu)u(w + x_1) + (1 - \Omega(\mu))u(w + x_2),$$

where $\Omega(\mu) = (1 + \beta)\mu / (1 + \beta\mu)$.

In Examples 1 and 2, these equations imply:

Example 1 (Under Gul-DA). *A household's willingness to pay for insurance with deductible d against the possibility of losing L with probability μ is the z such that*

$$\begin{aligned} & \frac{(1 + \beta)\mu}{1 + \beta\mu}u(w - d - z) \\ & + \left(1 - \frac{(1 + \beta)\mu}{1 + \beta\mu}\right)u(w - z) = \frac{(1 + \beta)\mu}{1 + \beta\mu}u(w - L) \\ & \qquad \qquad \qquad + \left(1 - \frac{(1 + \beta)\mu}{1 + \beta\mu}\right)u(w). \end{aligned}$$

Example 2 (Under Gul-DA). *A household's certainty equivalent for the lottery $X \equiv (x_1, \mu, x_2, 1 - \mu)$ with $x_1 < x_2$ is the z such that*

$$u(w + z) = \frac{(1 + \beta)\mu}{1 + \beta\mu}u(w + x_1) + \left(1 - \frac{(1 + \beta)\mu}{1 + \beta\mu}\right)u(w + x_2).$$

When applied to binary lotteries, Bell-DA is equivalent to KR-CPE, and Gul-DA, while having slightly different equations, also has much the same structure. One can show that the three models are more distinct when applied to lotteries with more than two outcomes.

1.5 Combining RDEU and KR-CPE

As mentioned in the review, sometimes one might want to consider a model that combines features from the different models discussed above. To illustrate how one might do so, in this section we develop a model that combines RDEU and KR-CPE. This combination is particularly interesting because — as mentioned in Section 4.4 of the review — one can show that, for lotteries with any number of outcomes, the combination of RDEU and KR-CPE reduces to an equivalent RDEU model using effective probability weighting $\Omega(\mu) =$

$\pi(\mu)(1 + \Lambda(1 - \pi(\mu)))$. In turn, this implies that it is never possible to separately identify the RDEU probability weighting function $\pi(\mu)$ and the KR-CPE loss aversion parameter Λ .³

Recall that, under KR, the utility from choosing lottery $X \equiv (x_n, \mu_n)_{n=1}^N$ given a reference lottery $\tilde{X} \equiv (\tilde{x}_m, \tilde{\mu}_m)_{m=1}^M$ is

$$V(X|\tilde{X}) \equiv \sum_{n=1}^N \sum_{m=1}^M \mu_n \tilde{\mu}_m [u(w + x_n) + v(w + x_n | w + \tilde{x}_m)].$$

When we add probability weighting as in RDEU, we assume the RDEU weight ω_n on outcome x_n given lottery X is used in place of the probability μ_n , and we further assume that the RDEU weight $\tilde{\omega}_m$ on outcome \tilde{x}_m given lottery \tilde{X} is used in place of the comparison weight $\tilde{\mu}_m$. The former is natural, as the RDEU model is framed as generating weights to replace probabilities. The latter is perhaps less natural, as $\tilde{\mu}_m$ is best thought of as a weight rather than a probability — i.e., it is the weight used when comparing a realized outcome to the possible reference outcome \tilde{x}_m . That said, just as KR argue that the natural weight to use when making this comparison is the probability of \tilde{x}_m , once we move to a person who is subject to probability weighting in how they react to probabilities, it seems natural to use the RDEU weight $\tilde{\omega}_m$.

Given these assumptions, we can rewrite $V(X|\tilde{X})$ as

$$V(X|\tilde{X}) \equiv \sum_{n=1}^N \sum_{m=1}^M \omega_n \tilde{\omega}_m [u(w + x_n) + v(w + x_n | w + \tilde{x}_m)],$$

where the ω_n 's are generated as in RDEU applied to lottery X , and the $\tilde{\omega}_m$'s are generated as in RDEU applied to lottery \tilde{X} . Given this $V(X|\tilde{X})$, the definition of CPE is exactly as in Definition 3 in the review — i.e., a household chooses the lottery X that maximizes $V(X|X)$.

Consider the evaluation of a generic discrete lottery $X \equiv (x_1, p_1; \dots; x_N, p_N)$ with $x_1 \leq \dots \leq x_N$.

³Masatlioglu and Raymond (2014) make a similar point using a decision-theoretic approach.

$$\begin{aligned}
V(X|X) &\equiv \omega_1 u(w + x_1) + \cdots + \omega_N u(w + x_N) \\
&+ \omega_1 \left[\sum_{n=2}^N [\omega_n \eta \lambda (u(w + x_1) - u(w + x_n))] \right] \\
&+ \omega_2 \left[\omega_1 \eta (u(w + x_2) - u(w + x_1)) + \sum_{n=3}^N [\omega_n \eta \lambda (u(w + x_2) - u(w + x_n))] \right] \\
&+ \omega_3 \left[\sum_{n=1}^2 [\omega_n \eta (u(w + x_3) - u(w + x_n))] + \sum_{n=4}^N [\omega_n \eta \lambda (u(w + x_3) - u(w + x_n))] \right] \\
&\vdots \\
&+ \omega_N \left[\sum_{n=1}^{N-1} [\omega_n \eta (u(w + x_N) - u(w + x_n))] \right],
\end{aligned}$$

where

$$\begin{aligned}
\omega_1 &= \pi(\mu_1) \\
\omega_2 &= \pi(\mu_1 + \mu_2) - \pi(\mu_1) \\
&\vdots \\
\omega_N &= 1 - \pi(\mu_1 + \cdots + \mu_{N-1}).
\end{aligned}$$

Defining $\Lambda \equiv \eta(\lambda - 1)$, we can rewrite this as

$$\begin{aligned}
V(X|X) &= u(w + x_1) \omega_1 \left[1 + \Lambda \sum_{n=2}^N \omega_n \right] \\
&+ u(w + x_2) \omega_2 \left[1 + \Lambda \sum_{n=3}^N \omega_n - \Lambda \omega_1 \right] \\
&+ u(w + x_3) \omega_3 \left[1 + \Lambda \sum_{n=4}^N \omega_n - \Lambda \sum_{n=1}^2 \omega_n \right] \\
&\vdots \\
&+ u(w + x_N) \omega_N \left[1 - \Lambda \sum_{n=1}^{N-1} \omega_n \right] \\
&= \sum_{n=1}^N u(w + x_n) \omega_n \left[1 + \Lambda \sum_{n'=n+1}^N \omega_{n'} - \Lambda \sum_{n'=1}^{n-1} \omega_{n'} \right].
\end{aligned}$$

Result. For any probability-weighting function $\pi(\mu)$, define $\tilde{\pi}(\mu, \Lambda)$ such that

$$\tilde{\pi}(\mu, \Lambda) (1 + \Lambda(1 - \tilde{\pi}(\mu, \Lambda))) = \pi(\mu).$$

Then any combination of loss aversion $\Lambda \geq 0$ and probability weighting $\tilde{\pi}(\mu, \Lambda)$ generates the same utility function $V(X|X)$.

Proof: Fix $\Lambda \geq 0$ and consider the weights under the combination Λ and $\tilde{\pi}(\mu, \Lambda)$. The weight on outcome x_1 is

$$\omega_1 \left[1 + \Lambda \sum_{n=2}^N \omega_n \right] = \tilde{\pi}(\mu_1, \Lambda) [1 + \Lambda(1 - \tilde{\pi}(\mu_1, \Lambda))] = \pi(\mu_1).$$

Defining $M_n \equiv \sum_{n'=1}^n \mu_{n'}$, the weight on outcome $x_n \in \{x_2, \dots, x_N\}$ is

$$\begin{aligned} & \omega_n \left[1 + \Lambda \sum_{n'=n+1}^N \omega_{n'} - \Lambda \sum_{n'=1}^{n-1} \omega_{n'} \right] \\ &= [\tilde{\pi}(M_n, \Lambda) - \tilde{\pi}(M_{n-1}, \Lambda)] [1 + \Lambda(1 - \tilde{\pi}(M_n, \Lambda)) - \Lambda \tilde{\pi}(M_{n-1}, \Lambda)] \\ &= \tilde{\pi}(M_n, \Lambda) [1 + \Lambda(1 - \tilde{\pi}(M_n, \Lambda))] - \tilde{\pi}(M_{n-1}, \Lambda) [1 + \Lambda(1 - \tilde{\pi}(M_{n-1}, \Lambda))] \\ &= \pi(M_n) - \pi(M_{n-1}). \end{aligned}$$

Hence, all weights are independent of Λ , and thus the utility function $V(X|X)$ is the same for all Λ and $\tilde{\pi}(\mu, \Lambda)$. ■

From this result, it follows that the combination of RDEU with probability weighting function $\pi(\mu)$ and KR-CPE with loss aversion Λ reduces to an equivalent RDEU model using effective probability weighting $\Omega(\mu) = \pi(\mu)(1 + \Lambda(1 - \pi(\mu)))$. It further follows that, in this model, one cannot separately identify $\pi(\mu)$ and Λ ; all one can do is estimate $\Omega(\mu)$.

2 Evidence on Moral Hazard and Adverse Selection

For many of the property insurance contexts, moral hazard appears to play a small role (for a recent review of the literature, see Cohen and Siegelman (2010)). Most studies that test for the presence of asymmetric information in auto insurance markets using cross-sectional data do not find evidence of a positive correlation between risk and coverage (Chiappori 1999; Chiappori and Salanié 2000; Chiappori, Jullien, Salanié, and Salanié 2006; Dionne, Gouriéroux,

and Vanasse 1999, 2001).⁴ These studies have been interpreted as casting doubt on the presence of moral hazard (Cohen 2005), at least in auto insurance. More recently, a handful of papers separately test for moral hazard in longitudinal auto insurance data using a dynamic approach pioneered by Abbring, Chiappori, and Pinquet (2003) and Abbring, Chiappori, Heckman, and Pinquet (2003). Abbring, Chiappori, and Pinquet (2003) find no evidence of moral hazard in French auto insurance data. Israel (2004) reports a small, but statistically significant moral hazard effect for drivers in Illinois. Ceccarini (2007), Abbring, Chiappori, and Zavadil (2008), Pinquet, Dionne, Vanasse, and Mathieu (2008), and Dionne, Michaud, and Dahchour (2007) present stronger evidence of moral hazard using auto insurance data from Italy, the Netherlands, Quebec, and France, respectively. Each of the foregoing papers, however, identifies a moral hazard effect with respect to either liability coverage or a composite coverage that confounds liability coverage with other coverages. None of them identifies or quantifies the separate moral hazard effect (if any) directly attributable to collision and comprehensive auto coverages.⁵

There is a significant body of work assessing the presence of asymmetric information in other markets, in particular the health and life insurance markets. Cardon and Hendel (2001) estimate a structural model of health insurance and health care choices using individual-level data. They find no evidence of informational asymmetries. Cawley and Philipson (1999) look for the presence of asymmetric information in term life insurance markets, using measures for both actual and self-reported risk. They too find no evidence of asymmetric information. More recent work has found increasing evidence for informational asymmetries. As in the prior two works, Finkelstein and Poterba (2004) find no informational asymmetries in the U.K. annuity market based only on annuity amounts or "insurance payout." However, they find informational asymmetries for other annuity characteristics — e.g., payout timing and whether an estate is guaranteed payment. Further, in long-term care insurance, Finkelstein and McGarry (2006) find evidence for multiple dimensions of private information, which can separately lead to moral hazard, adverse selection, or advantageous selection. Thus, focusing on the failure to reject a positive correlation between insurance coverage and risk occurrence ignores the fact that these selection forces may "cancel" each other out. The findings of Fang, Keane, and Silverman (2008) support the notion of advantageous selection

⁴We are aware of two exceptions: Puelz and Snow (1994) and Cohen (2005). Recent work criticizes the methodology of Puelz and Snow (1994) and suggests that their results are spurious (Chiappori 1999; Dionne, Gouriéroux, and Vanasse 1999). Cohen (2005) presents mixed evidence of asymmetric information. She finds that a positive coverage-accident correlation exists for new customers with more than three years of driving experience, but not for new customers with fewer than three years of driving experience. To our knowledge, there are no studies that test for asymmetric information in home insurance.

⁵These are the auto coverages that have been used to date, to estimate risk preferences with property insurance data

in Medigap insurance. Bajari, Dalton, Hong, and Khwaja (2014), using semi-parametric methods, report significant moral hazard/adverse selection in a recent (2002-2004) dataset from a large self-insured U.S. employer. He (2009) finds evidence of asymmetric information in life insurance markets in a sample of potential new buyers. Bundorf, Levin, and Mahoney (2012) calculate that asymmetric information between consumers and the risk-adjustment software used by health insurance providers, lowers annual welfare by \$35–\$100 per enrollee in data from small employers. Einav, Finkelstein, Ryan, Schrimpf, and Cullen (2013) find evidence of moral hazard in individuals’ choices of health insurance plans.

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