Leverage and Beliefs: Personal Experience and Risk Taking in Margin Lending

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Online Appendix

Appendix A: Sample contract – original and English translation (SAA 10,602, F. 1309)

Heden den 2e November 1772 compareerde voor mij Daniel van den Brink Openbaar Notaris binnen Amsterdam de heer Raphael de Abraham Mendes da Costa, voor en in de naam van zijn Compagnie luidende Abraham de Raphael Mendes da Costa & Co, Kooplieden binnen deeze stadt

Today, November 2, 1772, appeared before me, Daniel van den Brink, Public Notary in the City of Amsterdam, Mr. Raphael de Abraham Mendes da Costa, for and in the name of his company called Abraham de Raphael Mendes da Costa & Co, merchants in this town (hereafter: “the party present”).

en bekende bij deeze wel en deugdelijk schuldig te wezen aan de Heer Ananias Willink, meede Coopman alhier de somma van 24.000 guldens bankgeld spruytende uyt hoofden en ter saake van sodanige somma als de selve den 22e Oktober laatstleden aan syn comp[arants] voorn[oemde] Compagnie heeft afgeschreven, […] en welke somma van f. 24.000 Bankgeld hij Comparant in de naam van zijn voorn[oemde] compagnie aanneemt

And declared to be indebted to Mr. Ananias Willink, also merchant in this city for the sum of 24,000 guilders banco, originating from and relating to a withdrawal of such sum on October 22 last in favor of the present party’s said company, and the present party accepting that sum of 24,000 guilders banco in the name of said company.

en belooft aan voorn[oemde] Heer Ananias Willink of zijn Co[mpagnies] rechthebbende kosten schadeloos alhier weeder te zullen restitueren en voldoen binnen de tijdt van ses maanden te reekenen van den 6 Oktober deeses jaars met den Interest van dien tegens vier percent ’t jaar en bij prolongatie gelijke interest

And promises to said Mr. Ananias Willink, or his company’s legal representative, to return this sum (including any costs incurred), within the time of six months, counting from October 6 this year, with the interest of 4% annual, and in case of prolongation the same interest.

en zulks tot de volle en effectueele betaalinge toe tog de interessen te betaalen ieder 6 maanden des zo zal bij opeischinge of aflossinge den een den ander ses weeken voor de vervaltijd waarschouwent

And [promises] to pay the full and effective payment of the interest every six months
In case that the contract is not prolonged he will be notified 6 weeks in advance.

tot nakominge deezes verbind hij comparant zijn en zijn gemelde Compagnons persoon en goederen als na rechten en specialijk

To honor this agreement, the present party pledges his own body and goods and especially 1500 Pounds Sterling capital in *sodanige vijftienhonderd ponden sterling capitaal actien in de d'Oost Indische Compagnie van Engeland als tot London voor reekening van zijn comparants gemelde compagnie als pand ter minnen op de naam en reekening van gemelde H[eer] Ananias Willink zijn getransporteerd […]*

the stocks of the English East India Company, which have been transferred in London from the account of the present party's company to the account of said Mr. Ananias Willink as collateral.
[…]

en zulks meede een somma van f. 1500 indien deselve actien mogten komen te daalen op 180% en zo vervolgens van 10 tot 10 % om bij aflossinge en voldoeninge van gemelde capitaale somma gerescontreerd en geluiqideerd te werden, zullende de interessen van zodaanige restitutie kon te resteeren van dien dag af dat dezelve restitutie geschied is

And he also [promises] to transfer an amount of 1500 guilders banco if the price of said stock were to fall below 180% and similarly with every additional fall of 10%. Interest payments associated with these sums of money will be calculated until the moment the money is effectively transferred.

en hy comparant belooft meede in de naam van zyn gemelde Compagnie te zullen goed doen de provisie en onkosten die by 't transporteren van dezelve Actien aan zijn compagnie zullen komen te vallen welk transport by aflossinge zal met ten geschieden door de correspondenten van zijn comparants gemelde Compagnie.

And he, the party present, promises in the name of his said Company to pay for the fees and other costs associated with transferring the stock to his Company the moment the loan is repaid, which will be arranged by the correspondents of the present party's said company

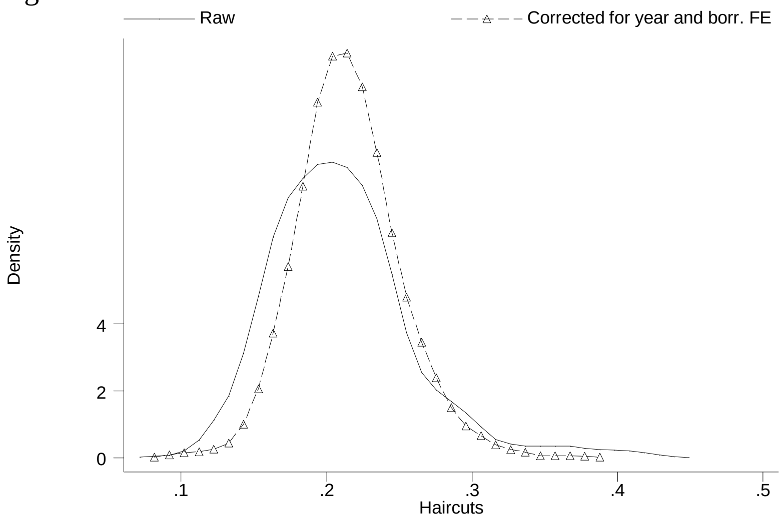
Voorts verklaarde hy Comparant dezelve Heer Anianas Willink specialijk te authoriseeren en consititueeren ommeindien zijn comparants gemelde compagnie in gebreken mogt komen te blijven de voorsz[egde] capitaale somma van f. 24000 bankgeld en interessen promptelijk te betaalen en voldoen ofte […] en meede zo wanneer bij vermindering der waarde van voornoemde Actien zijn comparants gemelde Compagnie op de eerste aanzegginge 't surplus niet kwam te voldoen dezelve actien door een makelaar alhier ofte tot London te mogen verkopen omme daar uit te vinden 't geene syn Ed[eles] uit kragte deezes zal zijn Competeerende 't geene hy Comparant in de naam van zyn voornoemde Compagnie belooft voor goed vast en van waarde te houden en zoo wanneer dezelve minder mogten renderen zoo belooft hij comparant 't mindere aan zijn Ed[elste] zullen opleggen en voldoen waar tegens gemelde Heer Ananias Willink als meerdere

Furthermore, the present party declares that, in case the present party's company defaults on the obligation to repay said sum of 24,000 guilders banco and associated interest payments in a timely fashion, or when he fails (due to the fall in value of said stocks) to provide additional surplus after a first instigation, he authorizes Mr. Ananias Willink especially to have the said stock sold through an official broker, either here or in London, and to retrieve from the proceeds the amount of money he is entitled according to this agreement with the present party's company.
In case the sale yields less than the full amount, the present party promises to make up the difference. In case it yields more, Mr. Ananias Willink will remit the resulting surplus.
The party present declares that he has received a counter-deed in reference to said stock.

aan zijn comparants gemelde Compagnie zal goed doen en hij Comparant bekende van syn Ed[ele] wegens voorsz[egde] actien een renvers[aal] te hebben ontvangen	
Actum Amsterdam, 2 November 1772	Signed in Amsterdam, November 2, 1772

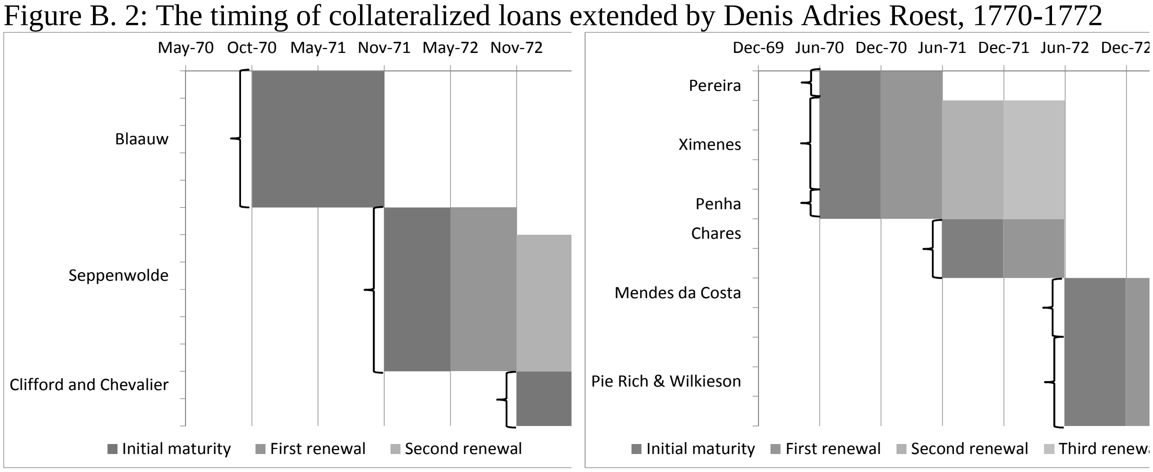
Appendix B: Additional figures and tables

Figure B. 1: Kernel densities haircuts before Christmas 1772



Raw vs corrected for year dummies and borrower fixed effects

Figure B. 2: The timing of collateralized loans extended by Denis Adries Roest, 1770-1772



Panel A: November (May) cycle　　Panel B: June (December) cycle

This figure illustrates the importance of timing in determining matches between lenders and borrowers with the example of lender Denis Adrien Roest. Loan contracts were signed for 6 (or 12) months and were often silently renewed with another 6 (or 12) months. Roest extended his loans either in the beginning of May/November or June/December. When loans were repaid after a multiple of 6 months, funds became available for new borrowers. The vertical axis indicates borrowing by different borrowers; the width of the bars indicates the size of the collateral behind a loan (in face value). The horizontal axis plots time and indicates when loans were originally extended and renewed.

Figure B. 3: Lender and borrower network – 1770-75

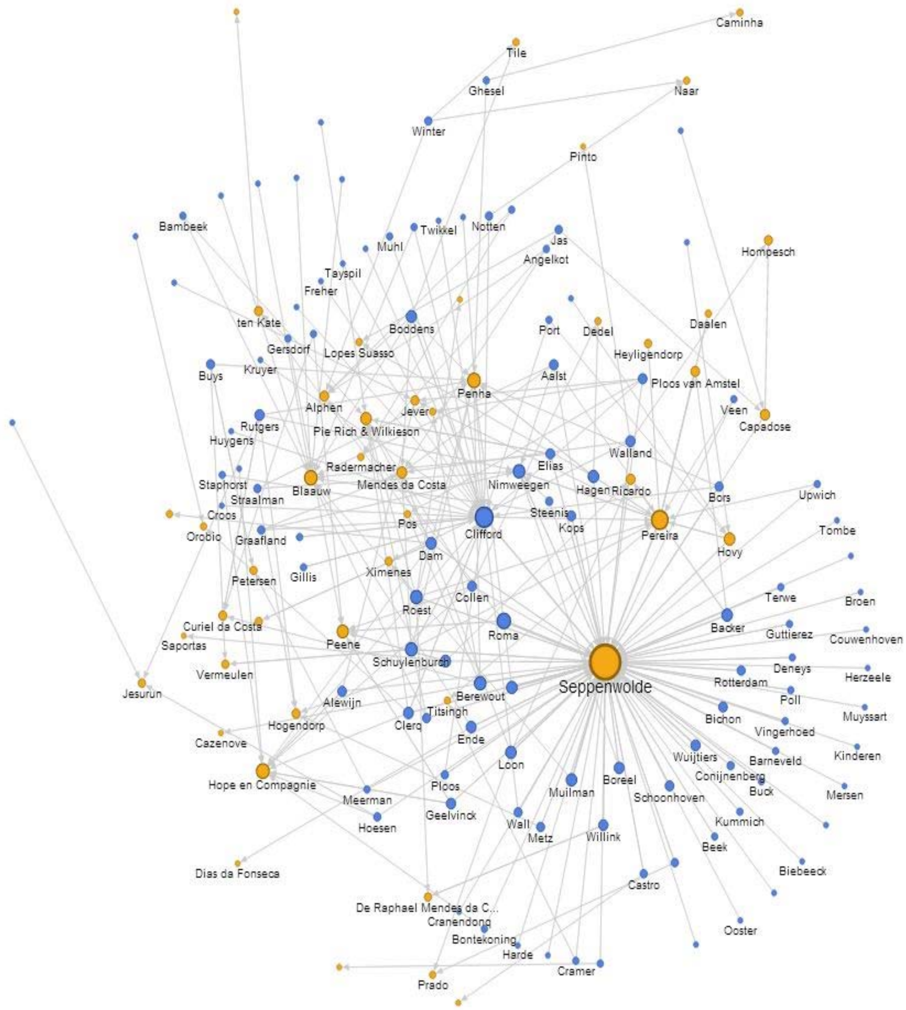
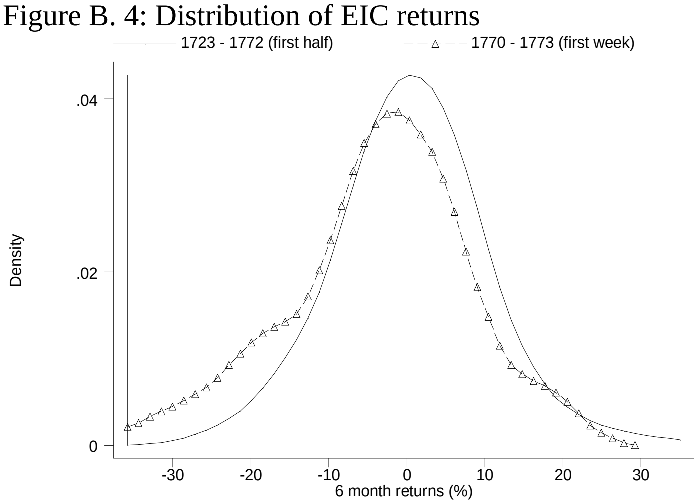
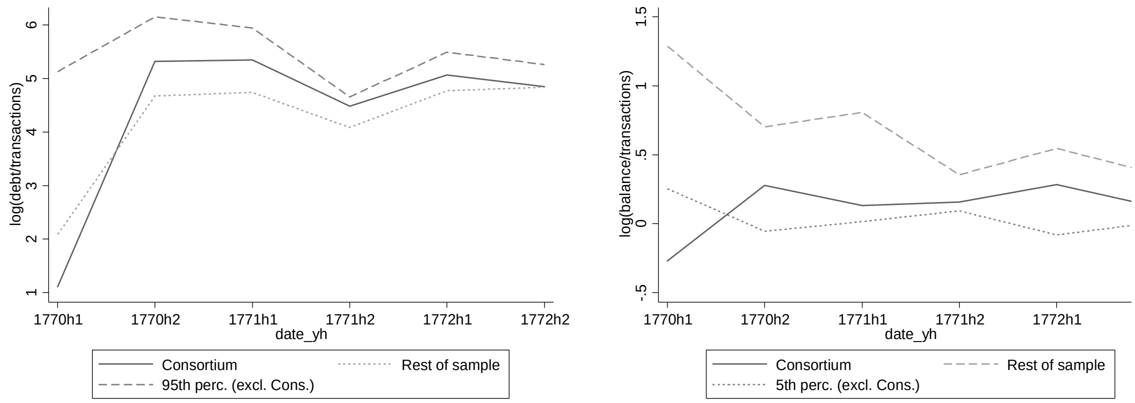


Figure B. 4: Distribution of EIC returns



Returns calculated over 6 month periods (overlapping). Vertical line indicates the 6 month return over the second half of 1772.

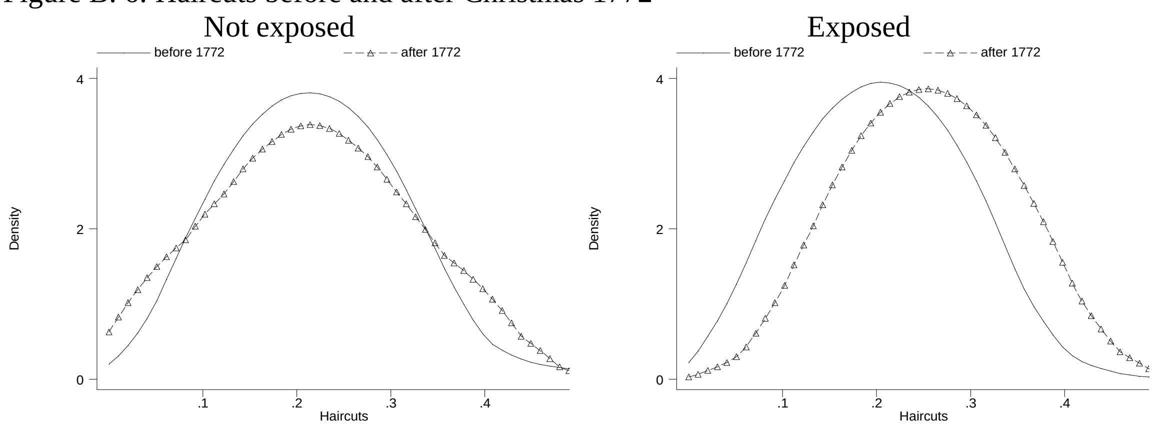
Figure B.5: Debt and cash positions Consortium before Christmas 1772



Panel A: $\log(debt_{i,t}\ /\ transaction_{i,t})$ Panel B: $\log(balance_{i,t}\ /\ transactions_{i,t})$

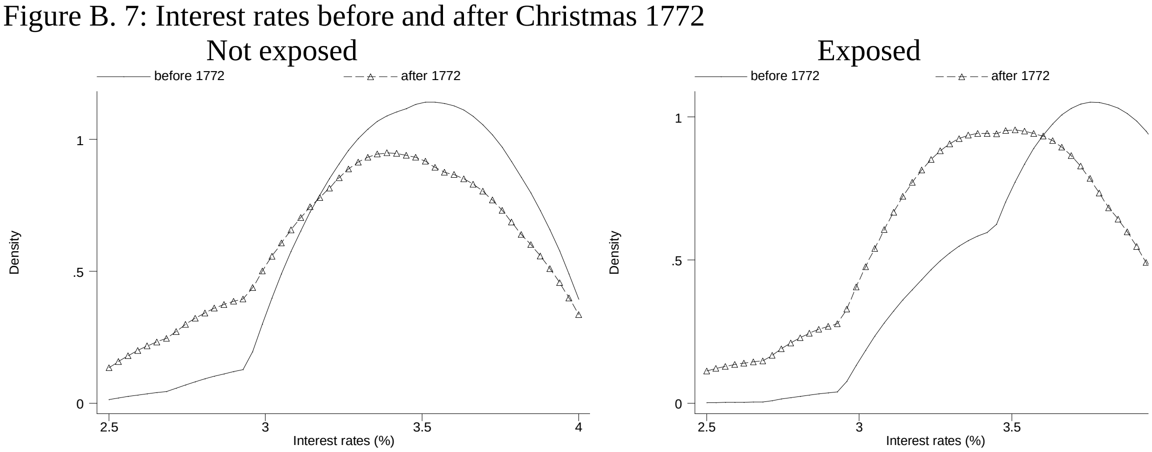
These two figures calculate half-yearly averages of log debt and cash positions for the consortium compared to the mean and 95th/5th percentile of the sample.

Figure B. 6: Haircuts before and after Christmas 1772



Haircuts before and after Christmas 1772, differentiated by exposed and non-exposed lenders

Figure B. 7: Interest rates before and after Christmas 1772



Interest rates before and after Christmas 1772, differentiated by exposed and non-exposed lenders

Figure B.8: Haircuts as a function of debt, before and after Christmas 1772

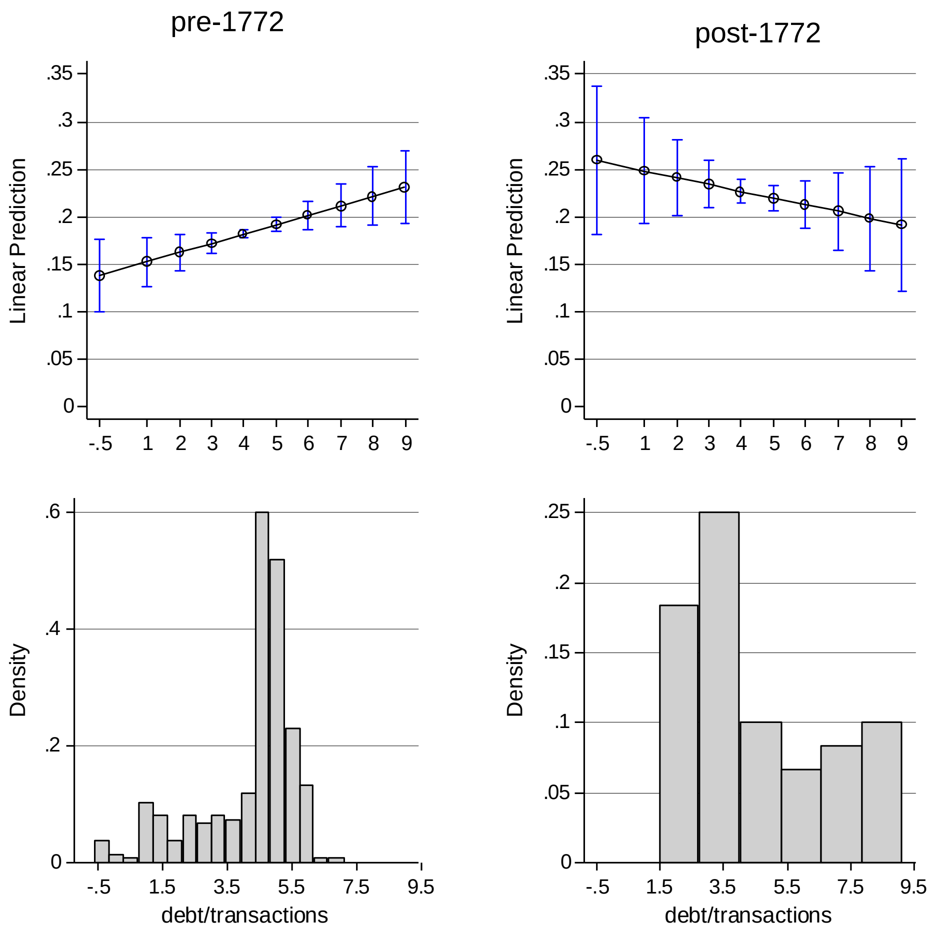


Table B. 1: Descriptive statistics, EIC stock returns over 6 month periods (overlapping)

Sample	Prior to distress	Distress period	Full
	1723-1772*	1770-73**	1723-1794
Mean	0.0051	-0.034	0.0028
Median	0.0068	-0.019	0.0053
σ	0.089	0.108	0.089
Skewness	0.248	-0.49	-0.07
Maximum loss	-0.256	-0.358	-0.358
% of observations with loss>0.2	0.011	0.075	0.022

* first half ** first week of 1773

Table B.2: Haircuts and time varying borrower risks (dependent variable: haircuts)

	(1)	(2)	(3)	(4)	(5)	(6)
		Full sample			Pre-1773	
$\log(debt_{i,t} / transactions_{i,t})$	0.011***		0.009**	0.011**		0.011***
	(0.004)		(0.004)	(0.004)		(0.004)
$\log(balance_{i,t} / transactions_{i,t})$		-0.022***	-0.018**		-0.009	-0.016
		(0.007)	(0.007)		(0.013)	(0.012)
Non-EIC	-0.048***	-0.051***	-0.049***	-0.050***	-0.053***	-0.051***
	(0.008)	(0.008)	(0.008)	(0.008)	(0.008)	(0.008)
Lender type dummies	Y	Y	Y	Y	Y	Y
Borrower fixed effects	Y	Y	Y	Y	Y	Y
Adj. R^2	0.565	0.560	0.573	0.541	0.522	0.542
N	317	317	317	272	272	272

This table presents estimates of the impact of two borower risk measures on haircuts, for the sample as a whole (cols 1-3) and for the period before the Seppenwolde default (cols 4-6). $debt_{i,t}$: : total margin loan position borrower i at time t. $balance_{i,t}$ ($transactions_{i,t}$): average daily balance (transaction volume) of borrower i in the Amsterdam Bank of Exchange during the 52 weeks prior to time i. All estimates include borrower fixed effects and lender type dummies. Robust standard errors (clustered at the lender level) are presented in parentheses, * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table B. 3: Lender attrition

	(1)	(2)	(3)	(4)
		Probit, 1 = remains in sample		
Exposed	0.075	0.079	0.142	0.123
	(0.054)	(0.052)	(0.087)	(0.096)
Total lending before 1773 (£000)		6.230	5.488	5.894
		(3.612)*	(3.615)	(3.613)
Fraction total lending to consortium			-0.086	-0.064
			(0.099)	(0.113)
Lender type dummies	N	N	N	Y
N	177	174	174	149
Pseudo-R^2	0.012	0.041	0.045	0.062

Estimates of a probit model predicting whether lenders will remain in the sample. The table presents marginal effects, e.g. in Col (1) a lender is 7.5% more likely to stay in the sample if it was exposed to the consortium. Robust standard errors are presented in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table B. 4: Total lending

	Including Van Seppenwolde			Excluding Van Seppenwolde		
	(1)	(2)	(3)	(4)	(5)	(6)
	Pooled OLS	Pooled OLS	FE	Pooled OLS	Pooled OLS	FE
Exposed	1.911	2.144		1.433	1.783	
	(0.935)**	(1.166)*		(0.923)	(1.212)	
Post 1772	0.326	0.175	-0.115	0.742	0.764	0.135
	(0.487)	(0.610)	(1.700)	(0.402)*	(0.517)	(2.234)
Exposed * Post 1772	-1.186	-1.465	-3.487	-0.749	-1.232	-1.634
	(0.913)	(1.163)	(3.279)	(0.988)	(1.081)	(2.406)
non-EIC	3.337	3.499	2.727	2.243	1.832	1.762
	(1.346)**	(1.700)**	(3.609)	(0.895)**	(0.954)*	(3.240)
Constant	2.190	3.050	3.643	1.884	4.147	2.948
	(0.462)***	(1.405)**	(0.992)***	(0.335)***	(1.518)***	(0.763)***
Year dummies	Y	Y	Y	Y	Y	Y
Lender type dummies	N	Y		N	Y	
Lender FE	N	N	Y	N	N	Y
N	202	175	202	128	113	128
N (if balanced)			50			30
R^2	0.040	0.080	0.880	0.050	0.150	0.955
# lenders	177	152	177	113	99	113

Regression estimates for total lending at the lender level on the collateral of all English securities. Total lending is calculated before and after Christmas 1772; in £000s of face value of collateral. Exposed lenders are those who were forced to liquidate collateral after the events of Christmas 1772. The interaction between the post-1772 and the exposed dummies captures the diff-in-diff effect. Lender type dummies are as in Table 3. Lender fixed effects refer to fixed effects on the family level. Robust standard errors (clustered at the lender level) are reported in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table B. 5: Haircuts and time since event (dependent variable: haircuts)

	(1)	(2)	(3)	(4)	(5)
	OLS	OLS	FE	FE	FE
Exposed	-0.005	-0.003		-0.000	
	(0.005)	(0.005)		(0.006)	
Exposed * Post 1772	0.097	0.086	0.086	0.054	0.095
	(0.030)***	(0.033)***	(0.046)*	(0.030)*	(0.047)**
Time since event	-0.001	0.023	0.008	-0.065	-0.005
	(0.058)	(0.059)	(0.066)	(0.058)	(0.070)
Exposed * time since event	-0.051	-0.041	-0.051	-0.031	-0.048
	(0.044)	(0.044)	(0.045)	(0.042)	(0.048)
non-EIC	-0.058	-0.055	-0.047	-0.053	-0.048
	(0.006)***	(0.007)***	(0.012)***	(0.008)***	(0.015)***
Constant	0.218	0.244	0.243	0.212	0.205
	(0.007)***	(0.018)***	(0.026)***	(0.012)***	(0.036)***
Year dummies	Y	Y	Y	Y	Y
Lender FE	N	N	Y	N	Y
Borrower FE	N	N	N	Y	Y
Lender type dummies	N	Y		Y	
Borrower type dummies	N	Y	Y		
N	418	387	418	387	418
N (if balanced panel)			166	77	33
R^2	0.342	0.444	0.637	0.664	0.802
# groups (lenders)	177	152	177	152	177
# groups (borrowers)	72	70	72	70	72

Regression estimates for all English securities. Observations refer to new contracts and are weighted by the face value of the collateral. Haircuts are calculated as the fraction of the collateral value that is not financed with a loan. Exposed lenders are those who were forced to liquidate collateral after the events of Christmas 1772. The interaction between the *Exposed* and the *Post 1772* dummies captures the diff-in-diff effect. *Time since event* is measured in years. The interaction between the *Exposed* and *Time since event* dummies captures the reversion of the treatment effect. For example, in Column 3 the immediate treatment effect on haircuts is .08 and decreases by .04 every year. Lender and borrower type dummies are as in Table 3. Lender and borrower fixed effects are at the family/firm level. Robust standard errors (clustered at the lender level) are reported in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table B. 6: Extensive and intensive margin (dependent variable: haircuts)

	(1) Pooled OLS	(2) Pooled OLS	(3) Pooled OLS
Exposed	-0.003	-0.003	-0.003
	(0.005)	(0.004)	(0.006)
Exposed * Post 1772	0.066	0.052	0.077
	(0.023)***	(0.028)*	(0.039)**
non-EIC	-0.056	-0.056	-0.056
	(0.006)***	(0.007)***	(0.006)***
Absolute position with consortium (£ 000s)		-0.000	
		(0.000)	
Absolute position with consortium * Post 1772		0.002	
		(0.003)	
Relative position with consortium (fraction)			-0.001
			(0.011)
Relative position with consortium * Post 1772			-0.026
			(0.038)
Constant	0.245	0.247	0.246
	(0.017)***	(0.016)***	(0.017)***
Year dummies	Y	Y	Y
Lender type dummies	Y	Y	Y
Borrower type dummies	Y	Y	Y
N	387	387	384
R^2	0.440	0.443	0.442

Regression estimates for all English securities. Observations refer to new contracts and are weighted by the face value of the collateral. Haircuts are calculated as the fraction of the collateral value that is not financed with a loan. Exposed lenders are those who were forced to liquidate collateral after the events of Christmas 1772. The interaction between the *Exposed* and the *Post 1772* dummies captures the **extensive** margin of adjustment. The *absolute position with the consortium* measures the total amount of the collateral the consortium had pledged with a specific lender around Christmas 1772 (in (£ 000s face value). The *relative position with the consortium* divides this measure by the total amount of collateral that was pledged with a specific lender before Christmas 1772. The interactions with the post-1772 dummy capture the **intensive** margin of adjustment. We do not measure this with a triple interaction because the position with the consortium for non-exposed lenders is always 0. Standard errors for the absolute and relative position measures are 5.26 and 0.39 respectively. Robust standard errors (clustered at the lender level) are reported in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table B. 7: Haircuts, excluding January 1773

	(1) Pooled OLS	(2) Pooled OLS	(3) FE	(4) FE	(5) FE
Exposed	-0.005	-0.002		-0.001	
	(0.005)	(0.005)		(0.006)	
Exposed * Post 1772	0.068	0.058	0.050	0.039	0.062
	(0.022)***	(0.023)**	(0.035)	(0.024)	(0.036)*
non-EIC	-0.059	-0.055	-0.047	-0.053	-0.047
	(0.006)***	(0.007)***	(0.012)***	(0.008)***	(0.015)***
Constant	0.218	0.246	0.245	0.210	0.190
	(0.006)***	(0.016)***	(0.024)***	(0.012)***	(0.037)***
Year dummies	Yes	Yes	Yes	Yes	Yes
Lender FE	No	No	Yes	No	Yes
Borrower FE	No	No	No	Yes	Yes
Lender observables	No	Yes		Yes	
Borrower observables	No	Yes	Yes		
N	412	381	412	381	412
N (if balanced panel)			160	73	32
R^2	0.302	0.422	0.625	0.636	0.788
# lenders	177	152	177	152	177
# borrowers	69	67	69	67	69

Regression estimates for all English securities. Observations refer to new contracts and are weighted by the face value of the collateral. Observations for January 1773 are excluded. Haircuts are calculated as the fraction of the collateral value that is not financed with a loan. Exposed lenders are those who were forced to liquidate collateral after the events of Christmas 1772. The interaction between the *Exposed* and the *Post 1772* dummies captures the diff-in-diff effect. Lender and borrower observables are as in Table 3. Lender and borrower fixed effects are at the family/firm level. Robust standard errors (clustered at the lender level) are reported in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table B. 8: Bertrand et al. analysis on haircuts – collapsing data into pre- and post-1772

	(1)	(2)	(3)	(4)
Exposed	-0.020		-0.005	
	(0.010)*		(0.006)	
Exposed * Post 1772	0.080	0.052	0.085	0.080
	(0.025)***	(0.023)**	(0.016)***	(0.019)***
Post 1772	-0.027	0.007	-0.025	-0.012
	(0.020)	(0.019)	(0.014)*	(0.017)
Non-EIC	-0.042	-0.037	-0.057	-0.039
	(0.013)***	(0.022)*	(0.008)***	(0.017)**
Lender type dummies	Y		Y	
Lender fixed effects	N	Y	N	Y
Weighted	N	N	Y	Y
N	175	202	175	202
R^2	0.157	0.848	0.439	0.845

In this table, we collapse our data into two periods only: pre- and post-1772, as suggested by Bertrand, Duflo, and Mullainathan (2004). That means that we have at most two observations per lender. In cols 1 and 2 we assign each lender-period observation the same weight. In cols 3 and 4 we weight observations by the total lending activity of lender in that period (measured by the total face value of accepted collateral). Standard errors are reported in parentheses * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Appendix C: Concentration of lending

To test if random matching of lenders and borrowers can explain the nature of lending in our sample, we calculate the Herfindahl index for every lender during the pre-crisis period:

$$H_i = \sum_j s_{i,j}^2$$

where $s_{i,j}$ is the share of lending by lender i to an individual borrower j. If lenders repeatedly lent to the same borrower, to the exclusion of other investors, we would expect a high Herfindahl index. The left panel of Figure C. 1 presents the actual distribution of these Herfindahl indices for all lenders in ours sample. Many lenders only entered into a single transaction; these are highlighted for the observations where the Herfindahl index equals 1.[43] The distribution is discontinuous, with zero weight between 0.68 and 1. This is the result of the way a Herfindahl index is constructed and the fact that most lenders only do a few transactions.

To compare the actual distribution with one arising by chance, we randomly pick a lender from our set of actual lenders. We determine how many new loan contracts he or she entered into before Christmas 1772, and then randomly draw a corresponding number of counterparties (taking into account that some borrowers are more active than others). Finally, we calculate the resulting Herfindahl index, and repeat the exercise 10,000 times. As the figure demonstrates, the two distributions are nearly identical. Both the Pearson Chi2 and the log likelihood test for the equality of distributions fail to reject.[44]

We use the Herfindahl indices to test whether the (possible) destruction of existing credit networks after the Seppenwolde bankruptcy might explain our empirical findings. The idea is that lenders who lost their network would have been forced to lend to new borrowers. Since these individuals were relatively unknown, they would have initially charged higher haircuts. We start from the assumption that lenders that are heavily invested in a particular client relationship will have more concentrated portfolios. We then estimate the following equation

$$Haircut_{i,t} = \beta_1 Exposed_i + \beta_2 Exposed_i * Post1772_t + \beta_3 Herfin_i + \beta_4 Herfin_i * Post1772_t + \beta_5 nonEIC + \bar{\varepsilon}_{it} + \zeta_{i,t}$$

where $\bar{\varepsilon}_{it}$ includes time effects and both borrower and lender characteristics. $\zeta_{i,t}$ is a random error. β_4 captures whether lenders increased haircuts more if they engaged in more

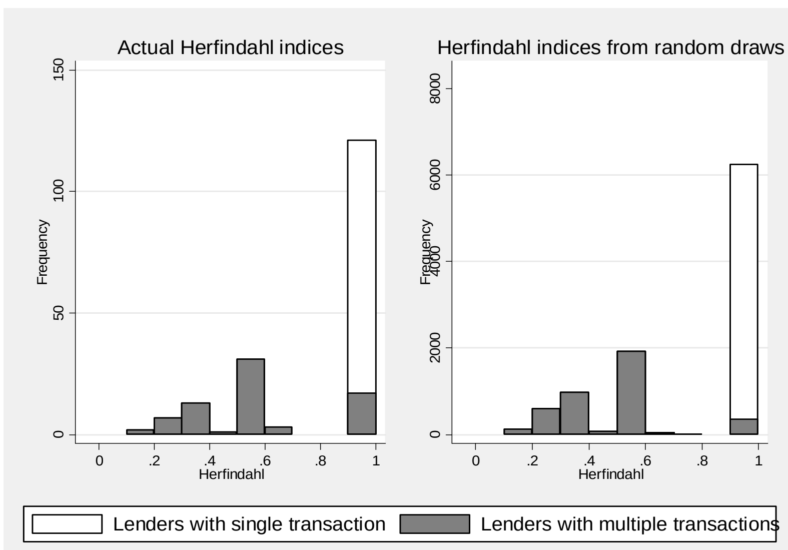
[43] The y-axes are aligned to reflect equal fractions. Grey bars reflect lenders who entered into at least 2 transactions. The white bars indicated lenders who only lent out once.

[44] P-values 0.43 and 0.505.

relationship lending before Christmas 1772 (a higher Herfindahl index). Table C.1 (Col 1) shows that this is not the case; if anything a higher degree of concentration before Christmas 1772 (more relationship lending) leads to lower haircuts. This effect is not statistically significant though.

In Col 2 we include a triple interaction effect between the Herfindahl index, the post-1772 dummy and the exposed dummy. This captures whether exposed lenders who had a more concentrated lending portfolio changed haircuts more aggressively after Christmas 1772. The idea is that exposed lenders with a relatively concentrated loan portfolio would have faced a larger disruption of their network. The triple interaction effect is insignificant and negative, suggesting that, if anything, exposed lenders with a more concentrated loan portfolio charged lower haircuts after the Seppenwolde default.

Figure C. 1: Herfindahl indices – actual vs simulated



For each lender we calculate the Herfindahl index of its lending before Christmas 1772. In addition, we construct a random distribution of Herfindahl indices. We randomly pick a lender from our set of lenders; we determine how many (x) new loan contracts it entered into before Christmas 1772; we randomly draw x counterparties (taking into account that some borrowers are more active than others); and we calculate the resulting Herfindahl index. We do this 10,000 times. The y-axes are aligned to reflect equal fractions. Grey bars reflect lenders who entered into at least 2 transactions. The white bars indicated lenders who only lent out once.

Tests on the equality of the distributions:

	Test statistic	pvalue
Pearson's Chi2	83.8	0.435
Log likelihood ratio	37.9	0.505
Obs. (Unique values)	178 (84)	

Table C. 1: Haircuts and concentration lending before Christmas 1772

	(1)	(2)
Exposed	-0.002	0.022
	(0.005)	(0.012)*
Exposed * Post 1772	0.056	0.075
	(0.028)**	(0.056)
non-EIC	-0.056	-0.055
	(0.006)***	(0.006)***
Herfindahl (pre-event)	0.006	0.028
	(0.008)	(0.014)**
Herfindahl (pre-event) * Post 1772	-0.030	-0.011
	(0.036)	(0.070)
Herfindahl (pre-event) * Exposed		-0.037
		(0.017)**
Herfindahl (pre-event) * Exposed * Post 1772		-0.022
		(0.088)
Constant	0.244	0.228
	(0.017)***	(0.018)***
Year dummies	Y	Y
Lender & borrower type dummies	Y	Y
N	384	384
R^2	0.443	0.452
# lenders	149	149

Pooled OLS estimates for all English securities. Observations refer to new contracts and are weighted by the face value of the collateral. Haircuts are calculated as the fraction of the collateral value that is not financed with a loan. Exposed lenders are those who were forced to liquidate collateral after the events of Christmas 1772. The interaction between the *Exposed* and the *Post 1772* dummies captures the diff-in-diff effect. The Herfindahl index (0-1) measures the concentration of a lender's portfolio before Christmas 1772. The double interaction between Herfindahl and *Post 1772* captures whether *all* lenders with higher degrees of concentration charged higher haircuts after Christmas 1772. The triple interaction between Herfindahl, the *Exposed* and *Post 1772* captures whether *exposed* lenders with a higher degree of concentration adjusted haircuts more. Lender and borrower type dummies are as in Table 3. Robust standard errors (clustered at the lender level) are reported in parentheses.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Appendix D: Direct exposure to EIC price movements

It is possible that individuals who lent to the consortium overall had strong exposure to EIC stock through other portfolio holdings. Then, changes in haircuts could reflect managing this risk, rather than the shock of the default.

To investigate this issue we estimate the following equation

$$Haircut_{i,t} = \beta_1 Exposed_i + \beta_2 Exposed_i * EICprice_t + \beta_3 Exposed_i * Post1772_t + \beta_4 EICprice_t + \overline{\varepsilon}_{it} + \zeta_{i,t}$$

where $\overline{\varepsilon}_{it}$ includes time effects and both borrower and lender characteristics. $\zeta_{i,t}$ is a random error. This equation tests whether exposed lenders in general charge higher haircuts when EIC prices are lower. Results are presented in Table 12.

Col 1 includes the interaction between the exposed dummy and the EIC stock price. The economic size of the coefficient is small and statistically insignificant. The average EIC price during 1770-1772 was 212%; in 1773-1775, it was 155%. The price decline corresponds an increase in haircuts by 1.9% (0.57*0.033). This is less than a third of the impact of the interaction effect with the post-1772 dummy (Table 6, Col 2). Col 2 includes both interaction effects to perform a horserace: what has more explanatory power, the post-1772 dummy or changes in the price of EIC stock? The estimates show that the interaction effect with the post-1772 dummy is much stronger; it increased haircuts by 6.8%. The coefficient on the interaction between exposed and the EIC price is now wrongly signed. Overall, these results show that EIC stock prices have no additional predicative power above and beyond the post-event dummy.

Table D.1: EIC factor (dependent variable: haircuts)

	(1) Pooled OLS	(2) Pooled OLS
Exposed	0.004	-0.010
	(0.007)	(0.008)
Exposed * EIC price	-0.033	0.047
	(0.030)	(0.038)
EIC price	0.049	-0.015
	(0.029)*	(0.035)
Exposed * Post 1772		0.068
		(0.035)*
Constant	0.245	0.252
	(0.022)***	(0.023)***
Year dummies	Y	Y
Lender type dummies	Y	Y
N	288	288
R^2	0.320	0.332
# lenders	127	127

Pooled OLS regression estimates for EIC stock only. Observations refer to new contracts and are weighted by the face value of the collateral. Haircuts are calculated as the fraction of the collateral value that is not financed with a loan. Exposed lenders are those who were forced to liquidate collateral after the events of Christmas 1772. EIC prices are in fractions of the face value. Average price before Christmas 1772 2.12, after Christmas 1772 1.55. The estimates in Col 1 indicate that such a price fall causes haircuts demanded by exposed to increase by 0.019 (0.57*0.033). The interaction between the *Exposed* and the *Post 1772* dummies in Col 2 captures the benchmark diff-in-diff effect. Lender type dummies are as in Table 3. Robust standard errors (clustered at the lender level) are reported in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Appendix E: Further robustness checks

Disaggregation of haircut components

The change in the haircut we document can be disaggregated into two parts – the difference between the price at which a contract is signed and the pre-agreed level when a margin call is triggered, and the difference between the trigger level and the value of the loan. In Table E. 1, we analyse the shift in the haircut for its two components separately.

In Panel A, we examine the difference between market price and the trigger level for a margin call. The lenders who were exposed to the default increased the trigger level substantially, by 4-5 percent – very close to the change in the overall collateral requirements. In Panel B, we analyze the distance to loss, the difference between the margin trigger and the value of the loan. Here, there are only relatively small and mostly insignificant effects – lenders adjusted the risk profile of their lending by demanding margin earlier, and keeping the value of the loan overall lower relative to the market value at the time of signing.

Table E. 1: Disaggregation of haircut components

	(1) Pooled OLS	(2) Pooled OLS	(3) FE	(4) FE	(5) FE
Panel (A): Distance to margin call					
Exposed	-0.009	-0.005		-0.006	
	(0.007)	(0.006)		(0.006)	
Exposed * Post 1772	0.063	0.042	0.039	0.028	0.046
	(0.023)***	(0.024)*	(0.036)	(0.028)	(0.043)
non-EIC	-0.036	-0.029	-0.029	-0.027	-0.031
	(0.006)***	(0.006)***	(0.009)***	(0.007)***	(0.011)***
Constant	0.131	0.167	0.167	0.116	0.069
	(0.006)***	(0.014)***	(0.020)***	(0.009)***	(0.026)***
R^2	0.130	0.294	0.589	0.521	0.760
Panel (B): distance to loss					
Exposed	0.004	0.002		0.007	
	(0.006)	(0.005)		(0.006)	
Exposed * Post 1772	0.012	0.022	0.025	0.010	0.027
	(0.012)	(0.012)*	(0.018)	(0.015)	(0.018)
non-EIC	-0.024	-0.027	-0.020	-0.027	-0.019
	(0.007)***	(0.006)***	(0.008)**	(0.007)***	(0.010)*
Constant	0.087	0.081	0.088	0.095	0.098
	(0.005)***	(0.017)***	(0.021)***	(0.008)***	(0.026)***
R^2	0.307	0.395	0.637	0.615	0.786
Year dummies	Yes	Yes	Yes	Yes	Yes
Lender FE	No	No	Yes	No	Yes
Borrower FE	No	No	No	Yes	Yes
Lender type dummies	No	Yes		Yes	
Borrower type dummies	No	Yes	Yes		
N	405	374	405	374	405
N (if balanced panel)			154	76	33
# lenders	176	151	176	151	176
# borrowers	67	65	67	65	67

Regression estimates for all English securities. Haircut = distance to margin call + distance to loss. Observations refer to new contracts and are weighted by the face value of the collateral. Exposed lenders are those who were forced to liquidate collateral after the events of Christmas 1772. The interaction between the *Exposed* and the *Post 1772* dummies captures the diff-in-diff effect. Lender and borrower type dummies are as in Table 3. Lender and borrower fixed effects are at the family/firm level. Robust standard errors (clustered at the lender level) are reported in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

East India Stock only

In the baseline results, we use lending against all assets in our database – East India stock, 3% annuities, and Bank of England stock. While we control for compositional change, it is interesting to examine how much of a shift we can find by focusing on EIC stock exclusively (the asset against which the Seppenwolde syndicate predominantly borrowed).

In Table E. 2, Panel A, we show that lending requirements in EIC stock changes in very much the same fashion as in the universe of all assets. In the pooled estimation (Col 2), the coefficient suggests a rise in collateral requirements by 6.8 percent. The fixed effect estimates look very similar to the benchmark numbers in Table 6. However, estimates become (borderline) insignificant. This is because with fixed effects, the effective number of observations that can be used to identify the interaction effect is constrained to those that are in the sample before and after 1772. In addition, we lose observations by constraining the sample to EIC transactions.

In Panel B, we analyze lending against non-EIC assets only. Due to the limited number of observations, the fixed effect specifications cannot be estimated. The pooled OLS estimates are very similar to those for loan contracts collateralized with EIC stock. For example, the estimate of the interaction effect in Col 2 is 6.6% (versus 6.8% in Panel A). Overall, there is no reason to think that the estimated effects in our baseline specification only reflect changes in haircuts in one type of asset.

Table E. 2: Haircuts – different types of collateral

	(1) Pooled OLS	(2) Pooled OLS	(3) FE	(4) FE	(5) FE
Panel A: EIC only					
Exposed	-0.002	0.000		-0.001	
	(0.008)	(0.008)		(0.008)	
Exposed * Post 1772	0.072	0.068	0.074	0.034	0.056
	(0.025)***	(0.028)**	(0.045)	(0.026)	(0.044)
Constant	0.222	0.240	0.228	0.210	0.180
	(0.006)***	(0.016)***	(0.027)***	(0.014)***	(0.046)***
N	314	288	314	288	314
N (if balanced panel)			134	65	29
# lenders	176	151	176	151	176
# borrowers	67	65	67	65	67
R^2	0.132	0.296	0.561	0.601	0.787
Panel B: BoE, SSC and 3% Annuities					
Exposed	-0.007	-0.005			
	(0.008)	(0.007)			
Exposed * Post 1772	0.102	0.066			
	(0.016)***	(0.027)**			
Constant	0.158	0.226			
	(0.007)***	(0.019)***			
N	104	99			
# lenders	70	64			
# borrowers	27	26			
R^2	0.072	0.284			
Year dummies	Yes	Yes	Yes	Yes	Yes
Lender FE	No	No	Yes	No	Yes
Borrower FE	No	No	No	Yes	Yes
Lender type dummies	No	Yes		Yes	
Borrower type dummies	No	Yes	Yes		

Regression estimates for EIC and BoE, SSC and 3% Annuities separately. Observations refer to new contracts and are weighted by the face value of the collateral. Haircuts are calculated as the fraction of the collateral value that is not financed with a loan. Exposed lenders are those who were forced to liquidate collateral after the events of Christmas 1772. The interaction between the *Exposed* and the *Post 1772* dummies captures the diff-in-diff effect. Lender and borrower type dummies are as in Table 3. Lender and borrower fixed effects are at the family/firm level. Due to a limited number of observations the fixed effects models cannot be estimated for the non-EIC securities. Robust standard errors (clustered at the lender level) are reported in parentheses.* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Outliers

It is possible that a few, extreme values for the haircuts influence our results. A standard way to deal with this issue is to winsorize the data. We winsorize the top and bottom 5 percent of observations, and re-estimate (see Table E.3). The results are largely unchanged. Coefficients are significant throughout, and are statistically indistinguishable from those in the baseline specification. For completeness we do the same for interest rates and re-estimate our benchmark results. Again, results are virtually unchanged.

Table E. 3: Haircuts – Winsorized dependent variable

	(1) Pooled OLS	(2) Pooled OLS	(3) FE	(4) FE	(5) FE
Exposed	-0.005 (0.005)	-0.003 (0.004)		-0.001 (0.006)	
Exposed * Post 1772	0.072 (0.020)***	0.064 (0.022)***	0.060 (0.032)*	0.040 (0.022)*	0.059 (0.031)*
Non-EIC	-0.057 (0.006)***	-0.054 (0.006)***	-0.047 (0.011)***	-0.051 (0.008)***	-0.045 (0.014)***
Constant	0.219 (0.006)***	0.240 (0.014)***	0.236 (0.022)***	0.214 (0.011)***	0.199 (0.033)***
R^2	0.365	0.466	0.630	0.638	0.785
Year dummies	Y	Y	Y	Y	Y
Lender FE	N	N	Y	N	Y
Borrower FE	N	N	N	Y	Y
Lender type dummies	N	Y		Y	
Borrower type dummies	N	Y	Y		
N	418	387	418	387	418
N (if balanced panel)			166	77	33
# lenders	177	152	177	152	177
# borrowers	72	70	72	70	72

Regression estimates for all English securities. Observations refer to new contracts. Haircuts are calculated as the fraction of the collateral value that is not financed with a loan. Exposed lenders are those who were forced to liquidate collateral after the events of Christmas 1772. The interaction between the *Exposed* and the *Post 1772* dummies captures the diff-in-diff effect. Observations are weighted by the face value of the collateral; the top and bottom 5% of the haircut distribution are Winsorized. Lender and borrower type dummies are as in Table 3. Lender and borrower fixed effects are at the family/firm level. Robust standard errors (clustered at the lender level) are reported in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table E. 4: Interest rates – Winsorized dependent variable

	(1) Pooled OLS	(2) Pooled OLS	(3) FE	(4) FE	(5) FE
Exposed	0.064 (0.035)*	0.042 (0.033)		0.060 (0.041)	
Exposed * Post 1772	-0.019 (0.077)	-0.008 (0.077)	-0.061 (0.087)	0.026 (0.080)	0.028 (0.159)
non-EIC	-0.072 (0.036)**	-0.087 (0.033)**	-0.077 (0.046)*	-0.104 (0.049)**	-0.081 (0.052)
Constant	3.534 (0.033)***	3.617 (0.092)***	3.628 (0.093)***	3.583 (0.070)***	3.658 (0.156)***
R^2	0.464	0.515	0.733	0.659	0.824
Year dummies	Y	Y	Y	Y	Y
Lender FE	N	N	Y	N	Y
Borrower FE	N	N	N	Y	Y
Lender type dummies	N	Y		Y	
Borrower type dummies	N	Y	Y		
N	418	386	418	386	418
N (if balanced panel)			166	77	33
# lenders	177	152	177	152	177
# borrowers	72	70	72	70	72

Regression estimates for all English securities. Exposed lenders are those who were forced to liquidate collateral after the events of Christmas 1772. The interaction between the *Exposed* and the *Post 1772* dummies captures the diff-in-diff effect. Observations are weighted by face value of the collateral; the top and bottom 5% of the distribution are Winsorized. Robust standard errors (clustered at the lender level) are reported in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

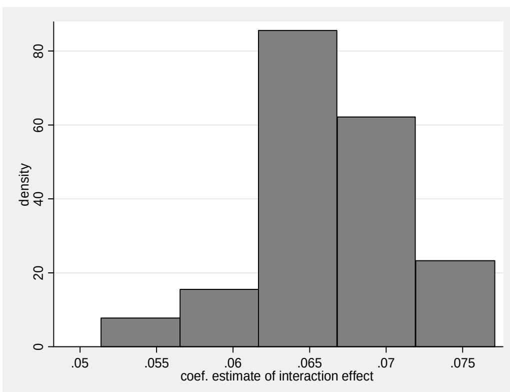
Extreme observations

The final step is to examine the sensitivity of our results to the influence of a single lender or borrower. To this end, we re-estimate the baseline specification (Table 6, Col 2), dropping one lender or borrower at a time. Figure E. 1 Panels A-D shows the distribution of coefficients (first row) and t-statistics (second row). The range of estimated coefficients is small, with results ranging from 5.5 to 7.5 percent. The t-statistics never falls below 2. This shows that our results are not driven by a single lender or borrower.

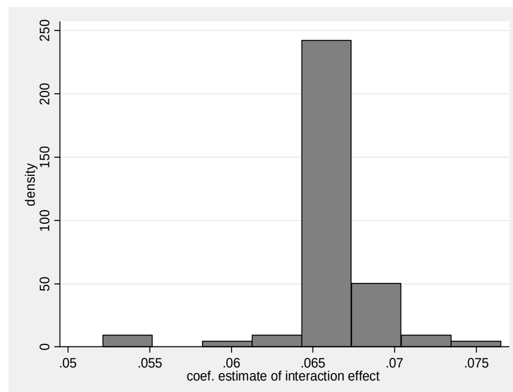
Figure E. 1: Outlier analysis, dropping one lender (borrower) at a time

Dropping one lender at a time | Dropping one borrower at a time

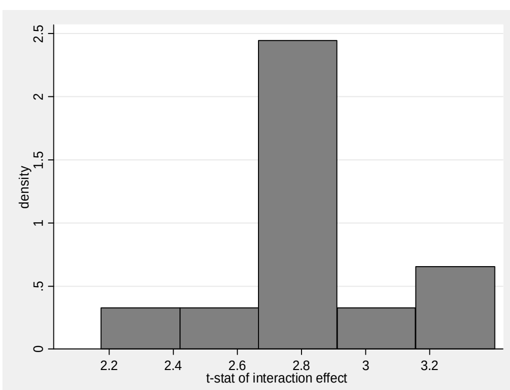
(A) Coefficient estimate



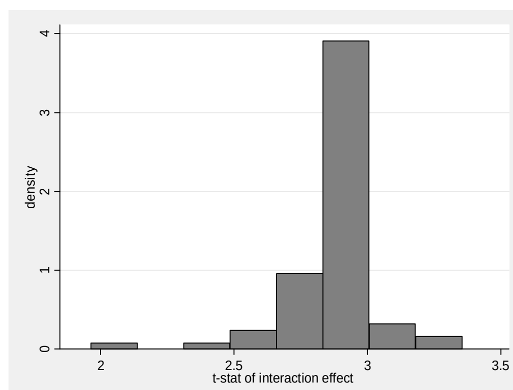
(B) Coefficient estimate



(C) t-statistic



(D) t-statistic



Coefficients on the interaction effect and t-statistics are generated dropping one lender (or borrower) at a time. All estimates include lender and borrower observables.

Appendix F: Primary Sources

GAR: Gemeentearchief Rotterdam (City Archives Rotterdam); NA: Nationaal Archief (Dutch National Archives); OSA: Oud Archief van de Stad Rotterdam (Old Archives City of Rotterdam); SAA: Stadsarchief Amsterdam (City Archives Amsterdam)

SAA (library), 'Stukken betreffende den boedel van Clifford en Zoonen', 1773-1779

SAA, Notariële protocollen Daniel van den Brink, 5075: 10,593 - 10,613 (various notary protocols)

SAA, Tex den Bondt aanvulling 1 en 2, 30269: 347 ('Staat en inventaris van de boedel van Johannes van Seppenwolde')

SAA, Archief van de Stads Beleenkamer, 5043: 1 ('Notulen van de vergaderingen van het 'Fonds tot maintien van het publiek crediet' (1773)')

NA, Archief van de familie Van der Staal van Piershil, 3.20.54: 381, 386, 396 (various correspondence)

OSA, 1.01: 3710 ('Stukken betreffende de kasgeldlening groot fl. 300.000 door de stad aan J. en H. van Seppenwolde, kooplieden te Amsterdam')

GAR, Archief van de Maatschappij van Assurantie, Belening, etc., 199: 5, 40, 354 (various accounts and letters)

GAR, Archief van Kuyls Fundatie, 90: 52, 56 (various letters)

De Koopman, Vol. IV (1772-1773) (Dutch periodical)

Appendix G: Full model solution[1]

The appendix is structured as follows. In Section G.1 we first describe the setup of the model and the underlying assumptions, including the specific loan contract and matching frictions that we consider. Next, in Section G.2, we analyze the Nash bargaining problem that borrowers and lenders solve when they meet. To do so, we first derive each agent's value function from obtaining a loan. We prove Proposition 3 in the main text that describes under what conditions borrowers will always accept a loan from a relatively pessimistic agent. In Section G.3 we then derive the equilibrium price as a function of outcomes in the loan market. We provide the conditions under which the general equilibrium exists and is unique. In Section G.4 we prove Proposition 1 in the main text that states that the optimal contract is always risk-free. We also show that Proposition 2 in the main text follows logically from the preceding results. In Section G.5, we analyse the local comparative statics of the model in closed-form and we prove Lemmas 4 – 7. Finally, in Section G.6 we provide global results through numerical analysis.

G.1 Setup and Assumptions

We model the market for collateralized lending as a matching market with frictions. Time is continuous and there is an infinite horizon. We focus on a steady state solution.

Apart from a risk free storage technology (with an instantaneous interest rate of zero), there is a single risky asset that is in unit supply. Following Geanakoplos (2003), the asset has a binomial payout where the good and the bad state occur with probability $1/2$. The timing of the payout is uncertain and follows a Poisson process with intensity $\pi > 0$.

There are three type of agents in the market $i \in \{1, 2, 3\}$ with masses N_i who are all risk neutral and have a subjective discount rate of zero. Each agent is infinitesimally small. Crucially, the agents have different beliefs about the asset payout. They all agree that in the good state of the world the asset will pay $\overline{r}$, they disagree about the payoff in the bad state of the world: $\underline{r}_1 < \underline{r}_2 < \underline{r}_3$. Expected payouts are given by v_i. All agents have a cash endowment of c_i. For simplicity, we assume that only type 3 agents will ever be cash constrained.

We focus on the case where $v_2 < p < v_3$. After the derivation of the equilibrium, we provide sufficient restrictions on the model's primitives to arrive at this case. In this scenario, only type 3 agents are willing to buy the asset. They will try to borrow from type 1 and type 2 agents to

[1]We are indebted to Dmitry Orlov and Victor Westrupp for their assistance in the development of this appendix.

increase their asset holdings above and beyond what they can buy with their own capital c_3. In what follows, we will initially derive the general case where $N_1 \neq N_2$. However, to generate closed-form solutions, we will restrict the analysis to the special case where $N_1 = N_2$.

G.1.1 Contract Space and Loan Contracts

We consider the following (restricted) contract space. First, there are shorting restrictions and agents can only buy and hold the asset. Second, agents can contract loans from each other to increase their asset holdings. Loans are collateralized with the asset and are non-recourse. (In equilibrium this assumption turns out to be irrelevant since a borrower will invest all of her wealth in the asset.)

A loan contract stipulates the size of the loan per unit of the asset l_j and an interest rate ρ_j for $j \in \{1, 2\}$. In our stylized setting, loan contracts do not have a fixed maturity: a contract ends if (1) the asset pays out, or if (2) the contract breaks down with an exogenous Poisson intensity λ. This greatly facilitates the computation of the steady state equilibrium. Interest payments are made lump sum at the end of the contract and are independent of the realized contract length. This may seem unrealistic, but notice that agents' subjective discount rates are zero. Interest rates therefore only reflect the underlying risk of the collateral or the transfer of surplus from borrower to lender, both of which are independent of the length of the contract.

We assume that if the contract ends before the asset pays out, the interest payment is zero. The borrower simply uses the proceeds from selling the collateral to repay the principal of the loan. We need to make this assumption to ensure the existence of a steady state solution, but there is a clear economic intuition behind it. Again, interest rates only capture risk or the transfer of surplus. In steady state, a loan contract ending before the asset pays out is always risk free: the price at which the asset was bought and the price at which the collateral is sold to repay the loan has to be the same by the definition of a steady state. Furthermore, if a loan contract ends before the asset pays out, there is no surplus generated yet that can be shared.

If the loan contract ends upon the asset payout, there are two possible scenarios. In the good state of the world, the asset payout is always sufficient to settle both principal and interest. In the bad state of the world, this may not be the case: the borrower may be forced to default. In that case, a lender can seize the asset's payout without any additional costs. It will charge a risk premium in the good state of the world to be compensated for this default risk. Crucially, borrowers and lenders can disagree about whether there will be default in the bad state of the world or not. If a borrower is more optimistic, he might believe that the return in the bad state of world will be sufficient to repay principal and interest, while the lender believes this is not the case.

G.1.2 Population Dynamics

We closely follow Duffie, Garleanu and Pedersen (2005) in setting up the population dynamics. Define $M_j(t)$ as the total mass of matches between type 3 agents (borrowers) and type $j \in \{1, 2\}$ agents (lenders) at time t. Let $U_j(t) = N_j - M_j(t)$ be the unmatched mass of type j lenders and $U_3(t) = N_3 - M_1(t) - M_2(t)$ the unmatched mass of borrowers. N is the total mass of agents in the economy. Lower case variables refer to proportions relative to N and are, by definition, bounded in between 0 and 1.

For simplicity, we assume that only borrowers actively search for lenders. They search with intensity μ and they cannot ex ante distinguish between type-1 and type-2 lenders or other type-3 agents. Under the exact law of large numbers for random independent matches, a borrower is matched with a lender of type-j at a total rate of $\mu u_3(t) u_j(t)$. At the same time, a fraction of existing contracts breaks down with intensity λ. The rates of change for $u_3(t)$ and $m_j(t)$ for $j \in \{1, 2\}$ are therefore given by

$$du_3(t) = -\mu u_3(t) [u_1(t) + u_2(t)] + \lambda [m_1(t) + m_2(t)] \tag{G.1}$$

$$dm_j(t) = \mu u_3(t) u_j(t) - \lambda m_j(t) \tag{G.2}$$

with the following restrictions

$$m_j(t) + u_j(t) = n_j \tag{G.3}$$

$$u_3(t) = n_3 - m_1(t) - m_2(t) \tag{G.4}$$

$$n_1 + n_2 + n_3 = 1 \tag{G.5}$$

Proposition G.1 *There exists a unique steady-state solution to equations (G.1)-(G.5).*

Proof. Conjecture that a steady-state exists. Start by substituting (G.3)-(G.5) in (G.1) and setting the right hand side of (G.1) to 0. This gives the following quadratic equation:

$$G(u_3) \equiv u_3^2 \mu + u_3(\mu + \lambda - 2\mu n_3) - \lambda n_3 = 0.$$

Now notice that $G(0) < 0$ and $G(1) > 0$, which implies that one of the roots of $G(.)$ must be negative while the other must lie in $(0, 1)$. Let u_3 be the positive root for the quadratic problem above. To show that $u_3 < n_3$, all we need to prove is that $G(n_3) > 0$. This follows directly from the fact that $G(n_3) = n_3\mu(1 - n_3) > 0$.

After setting the right hand side of (G.2) to zero, using (G.3), and realizing that $m_j = \frac{u_3\mu}{u_3\mu+\lambda}n_j$ and u_3 is bounded in $(0, 1)$, it is immediate that the equilibrium values for m_1 and m_2 are also bounded in $(0, 1)$. When we analyze the special case where $N_1 = N_2$, it is straightforward to see that $u_1 = u_2$ and $m_1 = m_2$. ■

G.2 The Loan Decision

G.2.1 Value Functions

We first define the borrowers' steady state value functions. We define q_j^* as the number of assets a borrower can buy in equilibrium when he is matched to a type j lender, and $\Pi_{3,j}^*$ as the expected profit per unit of the asset from a type-j loan for $j \in \{1, 2\}$. V_j is the value function from signing a loan contract with a type-j lender. q_0^* and $\Pi_{3,0}^*$ are defined as quantities and expected profits when a borrower is not matched to a lender; V_0 is the corresponding value function.

We first conjecture that a steady state exists in which the price p, loan quantities l_j^* and interest rates ρ_j^* are constant and in which $V_j \geq V_0$ (full matching). In this case, the value functions are given by the following:

$$V_0 = \frac{\pi}{\pi+\mu}q_0^*\Pi_0^* + \frac{\mu}{\pi+\mu}[u_1V_1 + u_2V_2 + (1-u_1-u_2)V_0] \tag{G.6}$$

$$V_1 = \frac{\pi}{\pi+\lambda}q_1^*\Pi_1^* + \frac{\lambda}{\pi+\lambda}V_0 \tag{G.7}$$

$$V_2 = \frac{\pi}{\pi+\lambda}q_2^*\Pi_2^* + \frac{\lambda}{\pi+\lambda}V_0 \tag{G.8}$$

where

$$\Pi_0^* = v_3 - p \tag{G.9}$$

and

$$\Pi_j^* = \frac{1}{2}\left[\overline{r} - (1+\rho_j)l_j^* - \frac{c_3}{q_j^*}\right] + \frac{1}{2}\max\left[\underline{r}_3 - (1+\rho_j)l_j^* - \frac{c_3}{q_j^*}, -\frac{c_3}{q_j^*}\right],$$

which, realizing that $-c_3/q_j^* = l_j^* - p$, can be rewritten as

$$\Pi_j^* = \frac{1}{2}(\overline{r} - \rho_j l_j^*) + \frac{1}{2}\left\{l_j^* + \max\left[r_3 - (1+\rho_j)l_j^*, 0\right]\right\} - p. \tag{G.10}$$

The total amount of assets q_j^* that a borrower can purchase depends on the amount of capital c_3 he has and loan size l_j^*:

$$q_j^*(p - l_j^*) \leq c_3.$$

Defining haircuts as

$$h_j^* = \frac{p - l_j^*}{p}, \tag{G.11}$$

this constraint can be rewritten more intuitively as

$$q_j^* \times p \times h_j^* \leq c_3:$$

the borrower's capital should cover the total value of the assets purchased times the haircut. As long as Π_j^*, the borrower's expected profit per unit of the asset, is strictly positive, the borrower will want buy as much of the asset as possible and the constraint will be binding, in which case

$$q_j^* = \frac{c_3}{p - l_j^*}$$

for $j \in \{1, 2\}$, and

$$q_0^* = \frac{c_3}{p}.$$

Solving equations (G.6)-(G.8) in terms of quantities and expected profits, we arrive at

$$\begin{aligned}
V_0 &= \frac{(\pi+\lambda)^2 q_0^*\Pi_0^* + (\pi+\lambda)\mu u_1 q_1^*\Pi_1^* + (\pi+\lambda)\mu u_2 q_2^*\Pi_2^*}{(\pi+\lambda)[\pi+\lambda+\mu(u_1+u_2)]} && \text{(G.12)}\\
V_1 &= \frac{(\pi+\lambda)\lambda q_0^*\Pi_0^* + [(\pi+\lambda)(\pi+\mu u_1)+\pi\mu u_2]q_1^*\Pi_1^* + \lambda\mu u_2 q_2^*\Pi_2^*}{(\pi+\lambda)[\pi+\lambda+\mu(u_1+u_2)]} && \text{(G.13)}\\
V_2 &= \frac{(\pi+\lambda)\lambda q_0^*\Pi_0^* + \lambda\mu u_1 q_1^*\Pi_1^* + [(\pi+\lambda)(\pi+\mu u_2)+\pi\mu u_1]q_2^*\Pi_2^*}{(\pi+\lambda)[\pi+\lambda+\mu(u_1+u_2)]}. && \text{(G.14)}
\end{aligned}$$

Intuitively, each value function is a weighted average of all the possible payoffs. Under the exact law of large numbers, these weights can interpreted as the probabilities that a certain payoff Π_j^* will be realized.

We then consider the lenders' value functions, where L_0^j is the value of not having a contract for a type-j lender and L^j is the value of being in a loan contract. In steady state, the value functions are given by:

$$\begin{aligned}
L_0^j &= \frac{\mu}{\pi+\mu}(u_j L^j + (1-u_j)L_0^j) && \text{(G.15)}\\
L^j &= \frac{\pi}{\pi+\lambda} q_j^* \left\{ \frac{1}{2}\rho_j^* l_j^* + \frac{1}{2}[\min\{r_j, l_j^*(1+\rho_j^*)\} - l_j^*] \right\} + \frac{\lambda}{\pi+\lambda} L_0^j, && \text{(G.16)}
\end{aligned}$$

solving the system of equations in terms of interest rates and loan sizes, we arrive at the following results:

$$L_0^j = \frac{\mu u_j}{\pi + \lambda + \mu u_j} q_j^* \left\{ \frac{1}{2}\rho_j^* l_j^* + \frac{1}{2}[\min\{r_j, l_j^*(1+\rho_j^*)\} - l_j^*] \right\} \tag{G.17}$$

$$L_j^j = \frac{\pi + \mu u_j}{\pi + \lambda + \mu u_j} q_j^* \left\{ \frac{1}{2}\rho_j^* l_j^* + \frac{1}{2}[\min\{r_j, l_j^*(1+\rho_j^*)\} - l_j^*] \right\}. \tag{G.18}$$

G.2.2 Nash Bargaining Solution For The Optimal Contract

When a borrower and lender meet they negotiate over the terms of the contract. We assume that they Nash bargain over the total surplus of the contract, where borrowers have bargaining power $\theta \in [0,1]$ that is determined outside the model. Before we solve the bargaining problem, note the following:

- We conjecture that the optimal debt contract is always risk free from both the perspective of the borrower and lender. We prove this after the derivation of the equilibrium. This means that $l_j(1+\rho_j) \leq \underline{r}_j$.

- As long as $\left(l_j^*, \rho_j^*\right) \geq 0$, a lender will always be at least indifferent between signing a loan contract or not. We assume that a lender will always go ahead with the loan contract if the borrower strictly prefers to sign the contract.

- Borrowers do not always strictly prefer to go ahead with a loan. Specifically, they might decide to turn down a type-1 contract. In what follows, we focus on a "full matching" equilibrium where $V_j \geq V_0$ for $j = 1, 2$ and a borrower will always accept a type-1 loan. After the derivation of the equilibrium, we will calculate the constraints on the model's primitives that are sufficient to guarantee this.

- There are cases (specifically when lenders have all the bargaining power) that a borrower is indifferent between signing a loan contract or not. As long as the lender strictly prefers to go through with the loan, we assume that the borrower will go ahead and sign the contract.

The Nash bargaining problem can be formulated as

$$\max_{l_j,\rho_j} (V_j - V_0)^{\theta} (L_j^j - L_0^j)^{1-\theta} \tag{G.19}$$

such that

$$
\begin{aligned}
l_j &\geq 0 \\
\rho_j &\geq 0 \\
l_j(1+\rho_j) &\leq \underline{r}_j
\end{aligned}
$$

We define $S_j^B = V_j - V_0$ as the borrower's surplus associated with a type-j loan and $S_j^L = L^j - L_0^j$ as the lender's surplus. We can rewrite each agent's surplus as follows:

$$
\begin{aligned}
S_j^B &= (\gamma + \nu u_{-j}) q_j \Pi_j - \gamma q_0 \Pi_0 - \nu u_{-j} q_{-j} \Pi_{-j} \\
S_j^L &= \vartheta_j q_j \left\{ \frac{1}{2}\rho_j l_j + \frac{1}{2}[\min\{r_j, l_j(1+\rho_j)\} - l_j] \right\}
\end{aligned}
$$

where

$$
\gamma = \frac{\pi}{\pi + \lambda + \mu(u_1 + u_2)}, \tag{G.20}
$$

$$
\delta = \frac{\pi \mu}{(\pi + \lambda)(\pi + \lambda + \mu(u_1 + u_2))}, \text{ and} \tag{G.21}
$$

$$
\vartheta_j = \frac{\pi}{\pi + \lambda + \mu u_j}. \tag{G.22}
$$

As long as the optimal contract is risk-free, we can use the expressions from (G.9)-(G.10) to rewrite the surpluses as:

$$
S_j^B = (\gamma + \delta u_{-j}) \frac{c_3}{p - l_j}(v_3 - p - \rho_j l_j) - \gamma \frac{c_3}{p}(v_3 - p) - \delta u_{-j} \frac{c_3}{p - l_{-j}}(v_3 - p - \rho_{-j} l_{-j}) \tag{G.23}
$$

$$
S_j^L = \vartheta_j \frac{c_3}{p - l_j} \rho_j l_j \tag{G.24}
$$

The Lagrangian is given by:

$$
\mathcal{L}(l_j, \rho_j; \eta, l_{-j}^*, \rho_{-j}^*) = (S_j^B)^\theta (S_j^L)^{1-\theta} + \eta_1 l_j + \eta_2 \rho_j + \eta_3(\underline{r}_j - l_j(1+\rho_j))
$$

and the Kuhn-Tucker necessary conditions for the constrained maximization problem are the following:

$$
\theta (S_j^B)^{\theta-1} (S_j^L)^{1-\theta} \frac{\partial S_j^B}{\partial l_j} + (1-\theta)(S_j^B)^\theta (S_j^L)^{-\theta} \frac{\partial S_j^L}{\partial l_j} + \eta_1 - \eta_3(1+\rho_j) = 0 \tag{G.25}
$$

$$
\theta (S_j^B)^{\theta-1} (S_j^L)^{1-\theta} \frac{\partial S_j^B}{\partial \rho_j} + (1-\theta)(S_j^B)^\theta (S_j^L)^{-\theta} \frac{\partial S_j^L}{\partial \rho_j} + \eta_2 - \eta_3 l_j = 0 \tag{G.26}
$$

$$
(l_j, \rho_j, \underline{r}_j - l_j(1+\rho_j)) \geq 0 \tag{G.27}
$$

$$
(\eta_1 l_j, \eta_2 \rho_j, \eta_3(\underline{r}_j - l_j(1+\rho_j))) = 0. \tag{G.28}
$$

We solve this problem in multiple steps:

1. Multiply (G.25) by l_j and (G.26) by ρ_j, thus eliminating the restrictions related to η_1 and η_2. We can thus rewrite (G.25) and (G.26) as the following:

$$(S_j^B)^{\theta-1}(S_j^L)^{-\theta}\left[\theta S_j^L l_j \frac{\partial S_j^B}{\partial l_j} + (1-\theta) S_j^B l_j \frac{\partial S_j^L}{\partial l_j}\right] = \eta_3 l_j(1+\rho_j) \quad \text{(G.29)}$$

$$(S_j^B)^{\theta-1}(S_j^L)^{-\theta}\left[\theta S_j^L \rho_j \frac{\partial S_j^B}{\partial \rho_j} + (1-\theta) S_j^B \rho_j \frac{\partial S_j^L}{\partial \rho_j}\right] = \eta_3 \rho_j l_j \quad \text{(G.30)}$$

2. Suppose that the last constraint is binding such that $\underline{r}_j = l_j(1+\rho_j)$. Isolate η_3 in (G.30) and substitute in (G.29) to obtain the following:

$$\theta S_j^L l_j \frac{\partial S_j^B}{\partial l_j} + (1-\theta) S_j^B l_j \frac{\partial S_j^L}{\partial l_j} = \theta S_j^L (1+\rho_j) \frac{\partial S_j^B}{\partial \rho_j} + (1-\theta) S_j^B (1+\rho_j) \frac{\partial S_j^L}{\partial \rho_j}.$$

This leads to the following solution for the optimal value of l_j:

$$l_j(\theta, l_{-j}) = \beta(\theta, l_{-j})\underline{r}_j + (1-\beta(\theta, l_{-j}))\left[\frac{\underline{r}_j - (v_3 - p) + \frac{\Gamma_{-j}}{\kappa_{-j}} p)}{1 + \frac{\Gamma_{-j}}{\kappa_{-j}}}\right]$$

where:

$$\beta(\theta, l_{-j}) = \frac{\theta(v_3 - \underline{r}_j)}{\theta(v_3 - \underline{r}_j) + (1-\theta)(p - \underline{r}_j)\left[1 + \frac{\Gamma_{-j}}{\kappa_{-j}}\right]} \in [0,1]$$

and

$$\begin{aligned}
\kappa_{-j} &= \gamma + \delta u_{-j} \\
\Gamma_{-j} &= \gamma \frac{v_3 - p}{p} + \delta u_{-j} \frac{v_3 - p - \rho_{-j} l_{-j}}{p - l_{-j}}
\end{aligned}$$

Note that the optimal loan contract $l_j(\theta, l_{-j})$ is the best response function, given the equilibrium contract with a type $-j$ lender. We have therefore arrived at a Nash equilibrium that is characterized by the following system of equations:

$$l_1 = \beta(\theta, l_2)\underline{r}_1 + (1-\beta(\theta, l_2))\left[\frac{\frac{\Gamma_2}{\kappa_2} p - (p - \underline{r}_1)(v_3 - p - \underline{r}_1)}{p\left(1 + \frac{\Gamma_2}{\kappa_2}\right) - \underline{r}_1}\right] \quad \text{(G.31)}$$

$$l_2 = \beta(\theta, l_1)\underline{r}_2 + (1-\beta(\theta, l_1))\left[\frac{\frac{\Gamma_1}{\kappa_1} p - (p - \underline{r}_2)(v_3 - p - \underline{r}_2)}{p\left(1 + \frac{\Gamma_1}{\kappa_1}\right) - \underline{r}_2}\right], \quad \text{(G.32)}$$

which yields the following solution:

$$l_j^*(\theta, p) = \frac{\underline{r}_j\left[\phi_\theta + \frac{\theta}{1-\theta}\left(\frac{v_3 - \underline{r}_j}{p - \underline{r}_j}\right)\right] + (v_3 - p)\left[a_{-j} + (1-\theta)a_j(1-a_{-j}) - \phi_\theta + (1-a_{-j})\theta \frac{p}{p - \underline{r}_{-j}}\right]}{\left[\phi_\theta + \frac{\theta}{1-\theta}\left(\frac{v_3 - \underline{r}_j}{p - \underline{r}_j}\right)\right] + \frac{(v_3 - p)}{p}\left[a_{-j} + (1-\theta)a_j(1-a_{-j}) + (1-a_{-j})\theta \frac{p}{p - \underline{r}_{-j}}\right]}, \quad \text{(G.33)}$$

where:

$$a_j = \frac{\gamma}{\kappa_j} = \frac{\gamma}{\gamma + \delta u_{-j}} = \frac{\pi + \lambda}{\pi + \lambda + \mu u_j}, \text{ and} \tag{G.34}$$

$$\phi_\theta = 1 - (1-\theta)(1-a_1)(1-a_2) \tag{G.35}$$

$$= 1 - (1-\theta)\frac{\mu^2 u_1 u_2}{(\pi + \lambda + \mu u_1)(\pi + \lambda + \mu u_2)}. \tag{G.36}$$

Note that in the special case where $N_1 = N_2$, we get that $u_1 = u_2$ and $a_1 = a_2 = a$.

If the constraint $l_j(1+\rho_j) \leq \underline{r}_j$ is not binding (such that $\eta_3 = 0$), it can be shown that no equilibrium exists. Borrowers and lenders can always find a better allocation by adjusting l_j and ρ_j up to the point that the constraint binds. Intuitively, type-3 agents will borrow the maximum amount at which the loan contract is still risk-free.

3. Next, we explore the boundary cases where $\theta = \{0, 1\}$. For these two cases, we can solve the maximization problem in the same way as before. The solutions to $l_j^*(\theta, p)$ are identical to the limiting cases when we take $\theta \to 1$ or $\theta \to 0$ in equation (G.33), proving that the solution is continuous in θ. The solutions are given by $l_j \to \underline{r}_j$ when $\theta \to 1$ (full bargaining power to the borrower) and $l_j \to \underline{r}_j \frac{p}{v_3}$ when $\theta \to 0$ (full bargaining power to the lender). Note that in the case where $\theta = 0$,

$$\begin{aligned} h_j^*(\theta, p) &= h_j^*(\theta) \\ &= \frac{v_3 - \underline{r}_j}{v_3} \end{aligned} \tag{G.37}$$

and the equilibrium haircut does not depend on the price.

4. Finally, we calculate the equilibrium value for $\rho_j^*(\theta, p)$. Notice that the constraint $l_j(1 + \rho_j) \leq \underline{r}_j$ is binding. This means that $\rho_j^*(\theta, p)$ is given by:

$$\rho_j^*(\theta, p) = \frac{\underline{r}_j - l_j^*(\theta, p)}{l_j^*(\theta, p)}. \tag{G.38}$$

Is the solution $l_j^*(\theta, p)$ feasible? The solution is bounded between the interval $\left[\underline{r}_j \frac{p}{v_3}, \underline{r}_j\right]$, which is positive if $\underline{r}_j > 0$. Since $l_j(1 + \rho_j) \leq \underline{r}_j$, this implies that the interest rate interval for $\rho_j^*(\theta, p)$ is also bounded on the nonnegative side. We must now check if both $S_j^B \geq 0$ and $S_j^L \geq 0$. Since $\rho_j l_j \geq 0$ in equilibrium for any value of θ, it follows that $S_j^L \geq 0$.

G.2.3 *Proof of Proposition 3*

As indicated before, it will not always be the case that $S_j^B \geq 0$. Specifically, it might not be optimal for a borrower to accept a loan from a type-1 lender. We can calculate the restrictions on the model's primitives that will guarantee that a borrower will always accept this loan.

Proof. of Proposition 3 ($V_1 \geq V_0$).

The first step of the proof is provided by the following Lemma:

Lemma G.2 *If* $S_j^B\left(l_j^*(1,p),\rho_j^*(1,p)\right) > 0$*, then* $S_j^B\left(l_j^*(\theta,p),\rho_j^*(\theta,p)\right) \geq 0 \;\forall\theta\in[0,1]$

Proof. The case for $\theta = 0$ is trivial since the S_j^B equals 0 if the lender has all the bargaining power. Define $S(\theta)$ as short-hand for $S(l_j^*(\theta,p),\rho_j^*(\theta,p))$. We prove the Lemma by contradiction. Suppose $\exists\hat{\theta}\in[0,1]$ such that $S_j^B(\hat{\theta}) < 0$. Since $S_j^B(\theta)$ is continuous in θ, $\exists\tilde{\theta}\in(\hat{\theta},1)$ such that $S^B(\tilde{\theta}) = 0$. This implies that $S_j^B(\hat{\theta})^{\hat{\theta}}S_j^L(\hat{\theta})^{1-\hat{\theta}} \geq S_j^B(\tilde{\theta})^{\hat{\theta}}S_j^L(\tilde{\theta})^{1-\hat{\theta}} = 0 \Rightarrow S_j^L(\hat{\theta}) = 0$. If the lender's surplus is equal to 0, this is only possible if $l_j^*(\hat{\theta},p)\rho_j^*(\hat{\theta},p) = 0$. Since the loan contract is risk free, this implies that $l_j^*(\hat{\theta},p) = \underline{r}_j$, resulting in $S_j^B(\hat{\theta}) = S_j^B(1) > 0$, a contradiction. ■

The intuition for this Lemma is straightforward. The reason a borrower would not want to accept a type-1 loan would be that he is better off waiting for a type-2 lender. When $\theta = 1$, the borrower has full bargaining power and will capture all the surplus from the transaction. When $\theta < 1$, he will have to give some of that surplus to the lender. This means that if it is optimal to accept a type-1 loan when $\theta = 1$, it also has to be optimal when $\theta < 1$.

As a result, we only have to verify the case when $\theta = 1$, which is relatively simple since $(l_j^*(1,p),\rho_j^*(1,p)) = (\underline{r}_j, 0)$. Setting $\theta = 1$ and substituting the expressions for Π_j^* from (G.9)-(G.10) into expression (G.23) for S_1^B, the surplus from a type-1 loan, we find that the borrower's surplus from this contract is strictly positive when $\theta = 1$ if and only if:

$$1 > a_2\underbrace{\frac{p-\underline{r}_1}{p}}_{<1} + (1-a_2)\underbrace{\frac{p-\underline{r}_1}{p-\underline{r}_2}}_{>1}$$

where a_2 is given by (G.34). Isolating $\underline{r}_2$, and focusing on the special case where $a_1 = a_2$, we arrive at inequality (2) in the main text. We can also isolate $\underline{r}_1$ to obtain the following inequality:

$$\underline{r}_1 > \frac{p(1-a_2)\underline{r}_2}{p-a_2\underline{r}_2}$$

The RHS is strictly decreasing in p. Therefore, if we substitute v_2 for p, we can derive the following general condition that holds for all $v_2 < p < v_3$.

$$\underline{r}_1 > \underbrace{\left(\frac{v_2 - a_2 v_2}{v_2 - a_2 \underline{r}_2}\right)}_{<1} \underline{r}_2$$

The inequality is intuitive. It states that the beliefs of type 1 and 2 lenders should not be too far apart. The maximum distance depends on a_2. If a_2 is close to 1, matching frictions are quite severe – either μ lies close to zero or u_2, the fraction of free type-2 lenders in the population, is close to 0 – and the inequality holds for any $\underline{r}_2$. If a_2 is close to 0 and matching frictions are negligible, the inequality will never hold. ■

G.3 General Equilibrium and Proof of Existence.

G.3.1 Market Clearing and Equilibrium Prices

The market clearing condition requires that total demand equals the (unit) supply of the asset:

$$\begin{aligned} \frac{1}{N} &= q_0^*(n_3 - m_1 - m_2) + q_1^* m_1 + q_2^* m_2 \\ &= \frac{c_3}{p} u_3 + \frac{c_3}{p - l_1^*(\theta, p)} m1 + \frac{c_3}{p - l_2^*(\theta, p)} m_2 \end{aligned} \tag{G.39}$$

It is not feasible to find a closed form solution for all values of θ, but we can easily calculate the solution for the case when $\theta = 0$:

$$p^*(\theta = 0) = N c_3 \left(u_3 + m_1 \frac{v_3}{v_3 - \underline{r}_1} + m_2 \frac{v_3}{v_3 - \underline{r}_2} \right) \tag{G.40}$$

G.3.2 Proof of existence

Starting with the closed form solution for p^* when $\theta = 0$, we can prove the existence of a unique equilibrium for each possible value of θ, restricting the set of parameters such that $v_2 < p^*(0) < v_3$. The proof proceeds as follows.

1. We first calculate the restrictions on the parameter space such that $\partial p / \partial \theta \geq 0 \; \forall (\theta, p)$. This is an intuitive condition that implies that if borrowers have more bargaining power, they will have more funding at their disposal, and this will lead to a higher price for the asset.

2. We then apply a version of Picard's Existence and Uniqueness Theorem for Ordinary Differential Equations (ODEs) to prove existence and uniqueness.

Lemma G.3 *As long as $v_3 > p$, $v_2 > v_3 - \underline{r}_1$ will guarantee that $\partial p/\partial\theta \geq 0\ \forall(\theta, p)$.*

Proof. Applying the implicit function theorem to (G.39), the derivative is equal to the following expression:

$$\frac{\partial p}{\partial \theta} = -\frac{\left(\frac{m_1}{h_1(\theta,p)^2}\frac{\partial h_1}{\partial \theta} + \frac{m_2}{h_2(\theta,p)^2}\frac{\partial h_2}{\partial \theta}\right)}{\frac{1}{Nc_3} + \frac{m_1}{h_1(\theta,p)^2}\frac{\partial h_1}{\partial p} + \frac{m_2}{h_2(\theta,p)^2}\frac{\partial h_2}{\partial p}} \tag{G.41}$$

where $h_j(\theta, p) \equiv \left[p - l_j^*(\theta, p)\right]/p$ is the equilibrium haircut, given by:

$$h_j(\theta,p) = \frac{\left(v_3 - \underline{r}_j\right)\left(\phi_\theta + \frac{\theta}{1-\theta}\right)}{p\phi_\theta + (v_3 - \underline{r}_j)\frac{\theta}{1-\theta}\left(\frac{p}{p-\underline{r}_j}\right) + (v_3 - p)\left[a_{-j} + (1 - a_{-j})\left[\theta\frac{p}{p-\underline{r}_{-j}} + (1-\theta)a_j\right]\right]}. \tag{G.42}$$

Expression (G.42) shows that the sign of $\partial p/\partial\theta$ directly depends on the signs of $\partial h_j/\partial\theta$ and $\partial h_j/\partial p$.

The first order derivative of h_j with respect to p is equal to:

$$\frac{\partial h_j}{\partial p} = \frac{\left(v_3 - \underline{r}_j\right)\left(\phi_\theta + \frac{\theta}{1-\theta}\right)\left[\frac{\theta}{1-\theta}\frac{(v_3-\underline{r}_j)\underline{r}_j}{(p-\underline{r}_j)^2} + (1-a_{-j})\theta\frac{(v_3-\underline{r}_{-j})\underline{r}_{-j}}{(p-\underline{r}_{-j})^2}\right]}{\left\{p\phi_\theta + (v_3 - \underline{r}_j)\frac{\theta}{1-\theta}\left(\frac{p}{p-\underline{r}_j}\right) + (v_3 - p)\left[a_{-j} + (1 - a_{-j})\left[\theta\frac{p}{p-\underline{r}_{-j}} + (1-\theta)a_j\right]\right]\right\}^2} \tag{G.43}$$

where a_j and ϕ_θ are given by (G.34)-(G.35). This derivative is always positive for $\forall(\theta, p)$. We proceed by listing the conditions under which the derivative of h with respect to θ is positive. Given the complexity of the solution for $\partial h_1/\partial\theta$, we list the conditions for the function to be strictly decreasing in θ based on expression (G.42) for $h(\theta, p)$ itself. If the denominator grows at a rate higher than the numerator with respect to θ, then it means that the function decreases. Given that both ϕ_θ and $\frac{\theta}{1-\theta}$ are strictly increasing in θ, $p > v_3 - \underline{r}_1$ and $v_3 - p > 0$ are necessary and sufficient conditions for $\partial h_1/\partial\theta > 0$.

Having $v_3 - p$ is a necessary restriction for borrowers to be willing purchase the asset, which we assume is always the case. Condition $p > v_3 - \underline{r}_1$ is less straightforward. Since there is no closed form solution for the equilibrium price p, we must define a lower bound that will hold for any feasible value of p. Given that $p > v_2$, assuming that $v_2 > v_3 - \underline{r}_1$ ensures that the price is strictly increasing in θ, no matter the equilibrium p, as long as $v_2 < p < v_3$. ■

Proposition G.4 *As long as $v_2 < p^*(0) < v_3$ and $v_2 > v_3 - \underline{r}_1$, there exists an unique solution to the problem defined by (G.19).*

Proof. We use a variant of Picard's Existence and Uniqueness Theorem for ODEs to prove this proposition.[2] The initial condition for the ODE is provided by $p^*(0) \equiv p_0$, defined by (G.40).

[2]See Adkins and Davidson (2012) pp. 87 Theorem 5 for one of the many references in the literature.

We approximate the differential with the first order derivative of the conjectured equilibrium price function $dp/d\theta \approx \partial p/\partial\theta$. Let the RHS of (G.41) be defined as $F(\theta, p)$, then we can rewrite (G.41) as

$$p' = F(\theta, p). \tag{G.44}$$

If $F(\theta, p)$ is continuous in both θ and p and continuously differentiable with respect to p on the rectangle $\mathcal{R} = \{(\theta, p) : 0 \leq \theta \leq 1, a \leq p \leq b\}$, then if $(0, p_0) \in \mathcal{R}$, there exists an unique solution to (G.44) with $p(0) = p_0$.

First, we must define $\mathcal{R}$ in our specific setup. Under the restriction listed in Lemma G.3, the equilibrium price is increasing in terms of θ. It is therefore natural to define $\{a, b\} = \{p_0, p_1\}$ where $p_1 \equiv p^*(1)$. We already defined p_0 – clearly $(0, p_0) \in \mathcal{R}$. Now, we must characterize p_1. Given $l_j^*(\theta = 1, p) = \underline{r}_j$, p_1 is defined by the following equation:

$$p_1 = Nc_3 \left(u_3 + m_1 \frac{p_1}{p_1 - \underline{r}_1} + m_2 \frac{p_1}{p_1 - \underline{r}_2} \right). \tag{G.45}$$

where p_1 is a positive real root of the underlying cubic equation. By Descartes' rule of signs, the solution can have either one or three positive real roots. Algebraic manipulation reveals that there is only one real root, which therefore has to be positive. Let $b = p_1$ be this root.

Next we ensure that $p_1 < v_3$. We start from equation (G.45). Notice that the LHS is strictly increasing in p_1 while the RHS is strictly decreasing in p_1. Therefore, if v_3 is such that:

$$v_3 > p_0 = Nc_3 \left(u_3 + m_1 \frac{v_3}{v_3 - \underline{r}_1} + m_2 \frac{v_3}{v_3 - \underline{r}_2} \right)$$

then it must be that $p_1 < v_3$. Note that this restriction is an upper bound and might be too restrictive.

Finally, we need to make sure that $F(.)$ is continuous and continuously differentiable on p over $\mathcal{R}$. Analyzing equations (G.41)-(G.43), first notice that the image of $h_j(.)$ for $(\theta, p) \in \mathcal{R}$ is bounded. The solution $p(\theta)$ is an increasing function of θ, and therefore, if $p_1 < v_3$ and $p_0 > v_2$, it follows that $v_2 < p < v_3$ for any $p \in \mathcal{R}$. This condition ensures that all the functions inside (G.41) are continuous except potentially when $\theta = 1$. Now we already know that $h_j(1, p) = p/\left(p - \underline{r}_j\right) < \infty$ for any $p \in \mathcal{R}$, which implies that $\partial h_j/\partial p|_{\theta=1} < \infty$ and $\partial h_j/\partial\theta|_{\theta=1} < \infty$, ensuring that $F(.)$ is continuous in $\mathcal{R}$. In order to show that $F(.)$ is continuously differentiable with respect to p, one can easily define an open neighborhood U such that $\mathcal{R} \subset U$ but none of the prior restrictions are violated, and show that partial derivatives of $F(.)$ exist for $(\theta, p) \in U$, thus proving that $F \in C^1$. The partial derivatives will depend on $\frac{\partial^2 h_j}{\partial\theta p}$, $\frac{\partial^2 h_j}{\partial\theta^2}$ and $\frac{\partial^2 h_j}{\partial p^2}$ which again can be shown to be continuous in U , including the case when $\theta = 1$. ■

G.3.3 Aggregating Restrictions on Parameters

In the preceding analysis we have listed a number of restrictions to ensure that a unique, full matching equilibrium exists where $v_2 < p < v_3$. We now summarize these restrictions:

1. The proof of Proposition 3 states that $\underline{r}_1 > \frac{v_2 - a_2 v_2}{v_2 - a_2 \underline{r}_2} \underline{r}_2$ for $V_1 > V_0$ (full matching equilibrium). Define
$$g_2 \equiv \frac{v_2 - a_2 v_2}{v_2 - a_2 \underline{r}_2}. \tag{G.46}$$
2. We know that for the price to be increasing in θ (a necessary condition for uniqueness and existence of the equilibrium), $v_2 > v_3 - \underline{r}_1$, or $2\underline{r}_1 + \underline{r}_2 > \underline{r}_3$.
3. In addition, we need that $v_2 < p_0 < v_3$, where $p_0 = Nc_3 \left(u_3 + m_1 \frac{v_3}{v_3 - \underline{r}_1} + m_2 \frac{v_3}{v_3 - \underline{r}_2} \right)$.

Grouping these three restrictions, the model's primitives must satisfy the following:

$$g_2 \underline{r}_2 \quad < \quad \underline{r}_1 < \underline{r}_2 \tag{G.47}$$
$$\underline{r}_2 \quad < \quad \underline{r}_3 < (1 + 2g_2)\underline{r}_2 \tag{G.48}$$
$$\underline{r}_2 \quad < \quad 2p_0 - \overline{r} < \underline{r}_3 \tag{G.49}$$

Intuitively, the first two restrictions state that beliefs cannot lie too far apart. If beliefs diverge by too much, the equilibrium price will fall below v_2, or borrowers will start to decline type-1 loan contracts. The third restriction effectively defines an upper and lower bound for what c_3 can be. If c_3 is too small, the price will dip below v_2, if it is too large, the price will exceed v_3.

G.4 Proofs of Propositions 1 and 2

Next, we prove Proposition 1 in the main text that states that the risk-free contract is the optimal contract, under the set of parameter restrictions defined before. Borrowers and lenders have different beliefs about $\underline{r}$. This means that their expectations about whether the borrower will default in the bad state of the world might also differ.

Proof. of Proposition 1 (the equilibrium loan contract is always risk free: $l_j(1 + \rho_j) = \underline{r}_j$).

We consider two deviations from the risk-free contract, one in which both borrowers and lenders belief that default in the bad state of the world will occur, one in which only the (more pessimistic) lenders think the borrower will default.

1. Default according to both borrowers and lenders: $\underline{r}_3 < l_j(1+\rho_j) \leq \overline{r}$.

   This case is ruled out because there are no gains from trade. We prove this by contradiction. Suppose there exists a contract with gains from trade. From (G.10) and (G.15) we can rewrite borrowers' and type-j lenders' profit functions (per unit of the asset) as

$$\begin{aligned} \frac{1}{2}\left[\overline{r} - p - \rho_j l_j\right] + \frac{1}{2}\left[l_j - p\right] &\geq 0 \\ \frac{1}{2}\left[\rho_j l_j\right] + \frac{1}{2}\left[\underline{r}_j - l_j\right] &\geq 0 \end{aligned}$$

   respectively. Combining the two inequalities implies that

$$\underline{r}_j \geq l_j(1-\rho_j) \geq 2p - \overline{r} \Rightarrow \underline{r}_j \geq 2p - \overline{r} \Rightarrow v_j \geq p.$$

   This is a contradiction since $p > v_j$.

2. Default according to lenders; full repayment according to borrowers: $\underline{r}_j < l_j(1+\rho_j) \leq \underline{r}_3$.

   In this case, the lender's surplus is given by

$$S_j^L = \vartheta_j \frac{c_3}{p - l_j}\frac{1}{2}(\rho_j l_j - l_j + \underline{r}_j).$$

   This is the average of the contract's payoff in the good and bad state of the world. Since the borrower does not expect to default, his surplus function is unchanged and given by (G.23).

   Define

$$x_j^B(\theta) = \frac{v_3 - p - \rho_j^*(\theta,p)l_j^*(\theta,p)}{p - l_j^*(\theta,p)} \text{ and} \tag{G.50}$$

$$x_j^L(\theta) = \frac{\rho_j^*(\theta,p)l_j^*(\theta,p)}{p - l_j^*(\theta,p)} \tag{G.51}$$

   where $l_j^*()$ and $\rho_j^*()$ are the optimal risk-free contracts. $x_j^B(\theta)$ can be interpreted as a borrower's profit from the risk-free contract with a type-j lender, per unit of his own capital, c_3. x_j^j is the lender's profit. Any risky contract $(\rho_j, l_j) > (\rho_j^*, l_j^*)$ that attempts to maximize total surplus for any $\theta \in [0,1]$ must satisfy the following conditions:

$$\frac{c_3}{p - l_j}(v_3 - p - \rho_j l_j) \geq c_3 x_j^B(\theta) \tag{G.52}$$

$$\frac{c_3}{p - l_j}\frac{1}{2}(\rho_j l_j + \underline{r}_j - l_j) \geq c_3 x_j^L(\theta) \tag{G.53}$$

$$\underline{r}_j < l_j(1+\rho_j) < \underline{r}_3. \tag{G.54}$$

We concentrate on conditions (G.52) and (G.53), from which we can derive the following condition for $\rho_j l_j$:

$$v_3 - p - x_j^B(\theta)(p - l_j) \geq \rho_j l_j \geq 2x_j^L(\theta)(p - l_j) + l_j - \underline{r}_j$$

This interval is only non-empty if

$$v_3 - p - x_j^B(\theta)(p - l_j) \geq 2x_j^L(\theta)(p - l_j) + l_j - \underline{r}_j.$$

After substituting for (G.50) and (G.51), we can rewrite the inequality as:

$$\underbrace{(v_3 + r_j - 2p)}_{<0}\underbrace{\left(\frac{l_j - l_j^*}{p - l_j^*}\right)}_{>0} \geq 0,$$

which is a contradiction. The first LHS term is strictly negative since $v_3 + \underline{r}_j < 2v_2 < 2p$. To verify this, consider $j = 2$ and notice that $2v_2 = \overline{r} + \underline{r}_2 > v_3 + \underline{r}_2$. The second term on the LHS is always strictly positive since any risky contract must feature a loan size that is strictly larger than the risk-free choice. This implies that a risky contract with $l_j > l_j^*$ (and $\rho_j > \rho_j^*$) makes borrowers worse off compared to the risk-free contract.

In sum, following the intuition from Geanakoplos (2003), a risky loan is never optimal. ■

The proof of Proposition 2 in the main text follows logically from these results:

Proof. of Proposition 2 (as long as $\underline{r}_1 < \underline{r}_2 \Rightarrow h_1 > h_2$).

The inequality $h_1 > h_2$ follows directly from (G.11) and the fact that the optimal contract is risk free and $l_1^* < l_2^*$. ■

G.5 Comparative Statics: proofs of Lemmas 4 – 7

Next, we analyse the comparative statics of the model and, in particular, provide proofs for Lemmas 4 – 7. Starting point is a situation where beliefs of type 1 and 2 lenders are identical, i.e. $\underline{r}_1 = \underline{r}_2$. We then analyse the impact of a shock to $\underline{r}_1$, keeping $\underline{r}_2$ constant. In order to guarantee closed form solutions, we simplify the model in two dimensions. First, we calculate all derivatives at the point where $\underline{r}_1 = \underline{r}_2 = \underline{r}$. The comparative static results are therefore local and only valid for small changes in $\underline{r}_1$. In the next section, we present global results through numerical analysis. Second, we restrict the analysis to the special case where $N_1 = N_2$ and $a_1 = a_2 = a$.

Proof. of Lemma 4: $\left.\frac{\delta h_1}{\delta \underline{r}_1} - \frac{\delta h_2}{\delta \underline{r}_1}\right|_{\underline{r}_1 = \underline{r}_2} < 0$ for $\forall(\theta, p) \in [0,1] \times (v_2, v_3)$

First, we calculate the differences in first order derivatives from expression (G.11):

$$\left.\frac{\delta h_1}{\delta \underline{r}_1} - \frac{\delta h_2}{\delta \underline{r}_1}\right|_{\underline{r}_1=\underline{r}_2} = -\frac{1}{p}\left(\frac{\delta l_1^*}{\delta \underline{r}_1} - \frac{\delta l_2^*}{\delta \underline{r}_1}\right) - \frac{1}{p}\frac{\delta p}{\delta \underline{r}_1}\left(\frac{\delta l_1^*}{\delta p} - \frac{\delta l_2^*}{\delta p}\right) + \frac{\delta p}{\delta \underline{r}_1}\left(\frac{l_1^* - l_2^*}{p^2}\right)\Bigg|_{\underline{r}_1=\underline{r}_2} \tag{G.55}$$

The third term on the RHS will be equal to zero as $l_1^* = l_2^* = l^*$. The same is true for the second term. To see this, start from expression (G.33) and notice that l_1^* and l_2^* depend on p in exactly the same way. As a result, the derivatives of l_1^* and l_2^* with respect to p will be identical when $\underline{r}_1 = \underline{r}_2$.

The first term on the RHS is different from zero. We first calculate the derivatives of l_j^* with respect to $\underline{r}_1$:

$$\begin{aligned}\frac{\delta l_1^*}{\delta \underline{r}_1} &= \frac{1}{\mathcal{D}(l_1^*)}\left\{\phi_\theta + \frac{\theta}{1-\theta}\left[\frac{v_3 - \underline{r}_1}{p - \underline{r}_1} + \frac{v_3 - p}{(p-\underline{r}_1)^2}(\underline{r}_1 - l_1^*)\right]\right\} \\ \frac{\delta l_2^*}{\delta \underline{r}_1} &= \frac{1}{\mathcal{D}(l_2^*)}\frac{v_3 - p}{(p-\underline{r}_2)^2}(1-a)\,\theta(p - l_2^*)\end{aligned}$$

where $\mathcal{D}(l_j^*)$ is the denominator in expression (G.33) and ϕ_θ is given by (G.35). As long as $\underline{r}_1 = \underline{r}_2$, $\mathcal{D}(l_1^*) = \mathcal{D}(l_2^*) = \mathcal{D}(l^*)$. We then calculate the difference:

$$\left.\frac{\delta l_1^*}{\delta \underline{r}_1} - \frac{\delta l_2^*}{\delta \underline{r}_1}\right|_{\underline{r}_1=\underline{r}_2} = \frac{1}{\mathcal{D}(l^*)}\left\{\phi_\theta + \frac{\theta}{1-\theta}\frac{(v_3 - \underline{r})(p - \underline{r}) + (v_3 - p)\left[(\underline{r} - l^*) - (1-a)(1-\theta)(p - l^*)\right]}{(p-\underline{r})^2}\right\}. \tag{G.56}$$

If $\theta = 0$, the difference is always positive since, from expression (G.35), $\phi_\theta > 0$ for $\forall\theta \in [0,1]$. The second term on the RHS will be smallest when $(1-a)(1-\theta)$ is close to one, its maximum value. Since

$$(v_3 - \underline{r})(p - \underline{r}) + (v_3 - p)(\underline{r} - l^*) - (v_3 - p)(p - l^*) = (p - \underline{r})^2 > 0, \tag{G.57}$$

the RHS will always be positive. ■

Proof. of Lemma 5: $\left.\frac{\delta h_1}{\delta c_3} - \frac{\delta h_2}{\delta c_3}\right|_{\underline{r}_1=\underline{r}_2} = 0$ for $\forall(\theta, p) \in [0,1] \times (v_2, v_3)$

This proof follows from the fact that

$$\left.\frac{\delta h_1}{\delta c_3} - \frac{\delta h_2}{\delta c_3}\right|_{\underline{r}_1=\underline{r}_2} = -\frac{1}{p}\frac{\delta p}{\delta c_3}\left(\frac{\delta l_1^*}{\delta p} - \frac{\delta l_2^*}{\delta p}\right) - \frac{\delta p}{\delta c_3}\left(\frac{l_1^* - l_2^*}{p^2}\right). \tag{G.58}$$

As long as $\underline{r}_1 = \underline{r}_2$, the second term on the RHS will be equal to zero. So will the first term: since l_1^* and l_2^* depend on p in exactly the same way, the derivatives of l_1^* and l_2^* with respect to p will be identical. ■

Proof. of Lemma 6: $\left.\frac{\delta \rho_1}{\delta \underline{r}_1} - \frac{\delta \rho_2}{\delta \underline{r}_1}\right|_{\underline{r}_1=\underline{r}_2} \lesseqgtr 0$

In equilibrium, loans are risk free and the total loan payment equals the return in the bad state of the world (from the point of the lender), that is $l_j(1+\rho_j) = \underline{r}_j$. Therefore

$$\begin{aligned}\frac{\delta\rho_1}{\delta\underline{r}_1} &= \frac{1}{(l_1^*)^2}\left(l_1^* - \underline{r}_1\frac{\delta l_1^*}{\delta\underline{r}_1}\right)\\ \frac{\delta\rho_2}{\delta\underline{r}_1} &= -\frac{1}{(l_1^*)^2}\underline{r}_2\frac{\delta l_2^*}{\delta\underline{r}_1}\,.\end{aligned}$$

Taking the difference and imposing that $\underline{r}_1 = \underline{r}_2$ yields

$$\left.\frac{\delta\rho_1}{\delta\underline{r}_1} - \frac{\delta\rho_2}{\delta\underline{r}_1}\right|_{\underline{r}_1=\underline{r}_2} = \frac{1}{(l^*)^2}\left[l^* - \underline{r}\left.\left(\frac{\delta l_1^*}{\delta\underline{r}_1} - \frac{\delta l_2^*}{\delta\underline{r}_1}\right)\right|_{\underline{r}_1=\underline{r}_2}\right].$$

Using expressions (G.33) and (G.56) for l^* and $\left.\frac{\delta l_1^*}{\delta\underline{r}_1} - \frac{\delta l_2^*}{\delta\underline{r}_1}\right|_{\underline{r}_1=\underline{r}_2}$ to simplify, we arrive at

$$\left.\frac{\delta\rho_1}{\delta\underline{r}_1} - \frac{\delta\rho_2}{\delta\underline{r}_1}\right|_{\underline{r}_1=\underline{r}_2} = \underbrace{\frac{1}{(l^*)^2}\frac{1}{\mathcal{D}(l^*)}\frac{\theta}{1-\theta}\frac{v_3-p}{(p-\underline{r})^2}}_{>0}\left[(1-a)(1-\theta)\left[\underline{r}(p-\underline{r}) + p - l^*\right] - (\underline{r} - l^*)\right]. \tag{G.59}$$

While the first term is always weakly positive, the sign of the second term is ambiguous and depends on the exact values of θ and a. Note that if $\theta = 0$, the difference in derivatives is zero. In this case, lenders extract all surplus, such that $\rho_j^*(0,p) = \frac{v_3-p}{v_3}$, which does not depend on $\underline{r}_1$. If $\theta = 1$, interest rates will be zero, as borrowers have all bargaining power, and the difference in derivatives is not defined. ■

Proof. of Lemma 7: $\left|\frac{\partial h_1}{\partial\underline{r}_1} - \frac{\partial h_2}{\partial\underline{r}_1}\right|_{\underline{r}_1=\underline{r}_2}\frac{1}{h} > \left|\frac{\partial\rho_1}{\partial\underline{r}_1} - \frac{\partial\rho_2}{\partial\underline{r}_1}\right|_{\underline{r}_1=\underline{r}_2}\frac{1}{\rho}$

Let

$$\begin{aligned}\Delta\varepsilon_{h,\underline{r}_1} &\equiv \left[\frac{\partial h_1}{\partial\underline{r}_1} - \frac{\partial h_2}{\partial\underline{r}_1}\right]_{\underline{r}_1=\underline{r}_2}\frac{1}{h}\\ \Delta\varepsilon_{\rho,\underline{r}_1} &\equiv \left[\frac{\partial\rho_1}{\partial\underline{r}_1} - \frac{\partial\rho_2}{\partial\underline{r}_1}\right]_{\underline{r}_1=\underline{r}_2}\frac{1}{\rho},\end{aligned}$$

where h and ρ are the initial haircut and interest rate given by $\frac{p-l^*}{p}$ and $\frac{\underline{r}-l^*}{l^*}$, respectively. $\Delta\varepsilon_{h,\underline{r}_1}$ and $\Delta\varepsilon_{\rho,\underline{r}_1}$ are the differences in semi-elasticities of type 1 and 2 haircuts and interest rates with respect to a change in $\underline{r}_1$. The semi-elasticities measure the percentage change in haircuts or interest rates in response to a unit change in $\underline{r}_1$. For example, if $\underline{r}_1$ falls by x, $\Delta\varepsilon_{h,\underline{r}_1}$ captures the percentage difference in responses between h_1 and h_2.

Using expressions (G.55) and (G.56), we arrive at the following absolute value of $\Delta\varepsilon_{h,\underline{r}_1}$:

$$\left|\Delta\varepsilon_{h,\underline{r}_1}\right| = \frac{1}{p-l^*}\frac{1}{\mathcal{D}(l^*)}\left[\phi_\theta + \frac{\theta}{1-\theta}\frac{(v_3-\underline{r})(p-\underline{r}) + (v_3-p)[(\underline{r}-l^*) - (1-a)(1-\theta)(p-l^*)]}{(p-\underline{r})^2}\right] \tag{G.60}$$

We use l^*'s functional form from (G.33) to simplify this expression. We first rewrite (G.33), noting that we are considering the case where $a_1 = a_2 = a$, and plugging in for ϕ_θ from (G.35):

$$\begin{aligned} l^*(p,\theta)\big|_{\underline{r}_1=\underline{r}_2} &= \frac{1}{\mathcal{D}(l^*)}\left[\underline{r}\left(\phi_\theta + \frac{\theta}{1-\theta}\frac{v_3-\underline{r}}{p-\underline{r}}\right) + (v_3-p)\frac{\theta(1-a)\underline{r}}{p-\underline{r}}\right] \\ &= \frac{\underline{r}}{\mathcal{D}(l^*)}\left[\phi_\theta + \frac{\theta}{1-\theta}\frac{(v_3-\underline{r})(p-\underline{r}) + (v_3-p)(1-a)(1-\theta)(p-\underline{r})}{(p-\underline{r})^2}\right] \end{aligned} \quad \text{(G.61)}$$

Combining expressions (G.60) and (G.61), we arrive at

$$\left|\Delta\varepsilon_{h,\underline{r}_1}\right| = \frac{l^*}{(p-l^*)\,\underline{r}} + \frac{1}{p-l^*}\frac{1}{\mathcal{D}(l^*)}\frac{\theta}{1-\theta}\frac{(v_3-p)}{(p-\underline{r})^2}\underbrace{[(\underline{r}-l^*) - (1-a)(1-\theta)(2p-l^*-\underline{r})]}_{>0} \quad \text{(G.62)}$$

From (G.57) we can see that $\Delta\varepsilon_{h,\underline{r}_1}$ is always positive.

Using (G.59), it is straightforward to derive the following expression for $\Delta\varepsilon_{\rho,\underline{r}_1}$:

$$\Delta\varepsilon_{\rho,\underline{r}_1} = \frac{1}{(\underline{r}-l^*)}\frac{1}{l^*}\frac{1}{\mathcal{D}(l^*)}\frac{\theta}{1-\theta}\frac{(v_3-p)}{(p-\underline{r})^2}\underbrace{[(1-a)(1-\theta)[2p-\underline{r}-l^*] - (\underline{r}-l^*)]}_{\lesseqgtr 0}. \quad \text{(G.63)}$$

Following the discussion below expression (G.59), the sign of $\Delta\varepsilon_{\rho,r_1}$ is ambiguous and depends on the exact parameter values.

To calculate the difference between $|\Delta\varepsilon_{h,r_1}|$ and $|\Delta\varepsilon_{\rho,r_1}|$, we need to consider two cases:

1. $[(1-a)(1-\theta)[2p-\underline{r}-l^*] - (\underline{r}-l^*)] < 0$ and $|\Delta\varepsilon_{\rho,r_1}| = -\Delta\varepsilon_{\rho,r_1}$

   In this case we have that:

$$\begin{aligned} |\Delta\varepsilon_{h,r_1}| - |\Delta\varepsilon_{\rho,r_1}| &= \underbrace{\frac{l^*}{(p-l^*)\,\underline{r}}}_{>0} + \frac{1}{\mathcal{D}(l^*)}\frac{\theta}{1-\theta}\frac{(v_3-p)}{(p-\underline{r})^2}\underbrace{[(\underline{r}-l^*) - (1-a)(1-\theta)(2p-l^*-\underline{r})]}_{>0} \\ &\quad \times \underbrace{\left[\frac{1}{p-l^*} - \frac{1}{\underline{r}-l^*}\frac{\underline{r}}{l^*}\right]}_{<0} \\ &= \frac{\phi_\theta}{\mathcal{D}(l^*)(p-l^*)} + \frac{1}{\mathcal{D}(l^*)(p-l^*)}\frac{\theta}{1-\theta}\frac{(v_3-p)}{(p-\underline{r})^2} \times \Phi \end{aligned}$$

   with

$$\begin{aligned} \Phi &= \underbrace{(v_3-\underline{r})(p-\underline{r})\left[\frac{1}{v_3-p} + (1-a)(1-\theta)\right]}_{>0} \\ &\quad \underbrace{+[r-l^* - (1-a)(1-\theta)(2p-\underline{r}-l^*)]\,\frac{1}{\underline{r}-l^*}}_{>0}\underbrace{\left[\underline{r}-l^* - (p-l^*)\frac{\underline{r}}{l^*}\right]}_{<0}. \end{aligned}$$

It can be shown that $\Phi > 0$:

$$\begin{aligned}
\Phi >_{I} \quad & (v_3-\underline{r})(p-\underline{r})\left[\frac{1}{v_3-p}+(1-a)(1-\theta)\right]+[1-(1-a)(1-\theta)]\left[\underline{r}-l^*-(p-l^*)\frac{r}{l^*}\right] \\
\geq_{II} \quad & (v_3-\underline{r})\frac{(p-\underline{r})}{(v_3-p)}+\left[\underline{r}-l^*-(p-l^*)\frac{r}{l^*}\right] \\
>_{III} \quad & (v_3-\underline{r})\frac{(p-\underline{r})}{(v_3-p)}-(v_3-\underline{r}) \\
= \quad & (v_3-\underline{r})\left(\frac{2p-\underline{r}-v_3}{v_3-p}\right) >_{IV} 0,
\end{aligned}$$

where $(>_{(I)})$ comes from replacing $(2p-\underline{r}-l^*)$ with $(\underline{r}-l^*)$. Since $p>\underline{r}$ and

$$r-l^*-(1-a)(1-\theta)(2p-\underline{r}-l^*)>0,$$

replacing p by $\underline{r}$ increases a positive coefficient multiplying a negative term, thus strictly decreasing the expression. $(>_{(II)})$ comes from setting $(1-a)(1-\theta)=0$, its minimal value. $(>_{(III)})$ comes from setting l^* and v_3 at their maximum attainable values ($l^*(\theta=1)=\underline{r}$ and $p=v_3$) in the negative term. Finally, $(>_{(IV)})$ comes from $p>v_2$ and $2v_2-\underline{r}-v_3=\frac{1}{2}(\overline{r}-\underline{r}_3)>0$.

2. $[(1-a)(1-\theta)[2p-\underline{r}-l^*]-(\underline{r}-l^*)]>0$ and $|\Delta\varepsilon_{\rho,r_1}|=\Delta\varepsilon_{\rho,r_1}$

   For this case, we were unable to find a closed-form proof. Extensive numerical analysis (available upon request) indicated, however, that $|\Delta\varepsilon_{h,r_1}|>|\Delta\varepsilon_{\rho,r_1}|$ for all feasible parameter values.

In sum, the Lemma holds for both cases. ■

G.6 Numerical simulations and global results

In the previous section, we derived closed form solutions for the relevant comparative statics when $\underline{r}_1=\underline{r}_2$. These results indicate how haircuts and interest rates change in response to relatively small changes in $\underline{r}_1$. It is possible that responses look different when we consider larger shocks. In this section, we analyze this numerically. In general, results are consistent with the previous section.

Starting point for the numerical simulations is $\underline{r}_1=\underline{r}_2$. We then reduce $\underline{r}_1$, keeping $\underline{r}_2$ constant, and trace out the impact on haircuts and interest rates. As discussed in Section G.3.3, to guarantee the existence of a (unique equilibrium), we can only let $\underline{r}_1$ decrease to $g_2\underline{r}_2$, where g_2 is given by expression (G.46). For this exercise, we normalize the model in two dimensions.

First, we impose that the average valuation of the asset by type 2 and 3 agents equals one, that is

$$\frac{v_2 + v_3}{2} = 1. \tag{G.64}$$

This is a pure normalization. Second, we impose that in the scenario where lenders have all bargaining power ($\theta = 0$) and $\underline{r}_1 = \underline{r}_2$, the price also equals unity: $p(\theta = 0, \underline{r}_1 = \underline{r}_2) = 1$. A price lower than one would mean than a shock to $\underline{r}_1$ would sooner drive the price below v_2, at which point the equilibrium breaks down. A price higher than one would imply that allocating more bargaining power to borrowers ($\theta \to 1$) would sooner drive prices above v_3, also destroying the equilibrium. Imposing that $p(\theta = 0, \underline{r}_1 = \underline{r}_2) = 1$ trades off these two potential (numerical) problems. Keeping all else equal, equation (G.40) pins down the amount of optimist capital c_3.

For our simulations, we fix the haircut a lender charges when $\theta = 0$ and $\underline{r}_1 = \underline{r}_2 = \underline{r}$ at $h(\theta = 0, \underline{r}_1 = \underline{r}_2) \equiv h_0$. Together with expression (G.64), this imposes a number of restrictions on $\underline{r}$ and $\overline{r}$.

1. When $\theta = 0$, the haircut is given by expression (G.37):

$$h_0 = \frac{v_3 - \underline{r}}{v_3}.$$

   Combining this with expression (G.64), and noting that $\underline{r}_2 = \underline{r}$, implies that

$$\underline{r} = (1 - h_0)\,\frac{4 - \overline{r}}{3 - h_0}. \tag{G.65}$$

2. The condition that $v_3 < \overline{r}$ implies a lower bound on the values that $\overline{r}$ can take. Plugging in for (G.64) and (G.65), we arrive at:

$$\overline{r} > \frac{4}{4 - h_0}.$$

3. Finally, we need that $\underline{r}_3 > \underline{r}$. This implies an upper bound on the values that $\overline{r}$ can take. To see this, plug in for (G.64) and (G.65) to arrive at:

$$\overline{r} < 1 + h_0.$$

Points (2.) and (3.) define an interval for possible values of $\overline{r}$. For simplicity, we use the midpoint of this interval in our simulations (results are generally robust to using other feasible values). Given h_0, this choice for $\overline{r}$ pins down $\underline{r}$ through expression (G.65).

Apart from these normalizations, we need to fix the other parameters of the model. Most important is h_0. Changing the other parameter values does not have important quantitative implications. We cannot use any value of h_0 in $(0,1)$: there is an upper bound. As discussed in Section G.3.3, differences of beliefs in the model cannot be too extreme, specifically,

$$\underline{r}_2 > \frac{\underline{r}_3}{1+g_2}$$

Using (G.64), and noting that $\underline{r}_2 = \underline{r}$, implies that:

$$\underline{r} < \frac{2-\overline{r}}{1+g_2}.$$

Plugging in for (G.65) yields

$$h_0 < \frac{2(\overline{r}-1)+g_2(4-\overline{r})}{2+g_2(4-\overline{r})} \equiv \overline{h}_0.$$

In our simulations we set h_0 equal to this upper bound. Qualitatively, results are similar for other $h_0 \in (0, \overline{h}_0]$, but using $\overline{h}_0$ is most conservative. Lemma 7 states that the difference (in absolute value) in semi-elasticities between type 1 and 2 loans with respect to changes in $\underline{r}_1$ is larger for haircuts than it is for interest rates. Unreported simulation results indicate that this difference is smallest when $h_0 = \overline{h}_0$.

Table G.1 gives an overview of the different normalizations and parameter choices and Figure G.1 presents the simulation results. On the x-axis we display $\underline{r}_1 \in [g_2\underline{r}_2, \underline{r}_2]$ Panels A and C show how haircuts charged by type 1 and type 2 lenders change as we move $\underline{r}_1$ away from $\underline{r}_2$. We consider three scenarios where $\theta \in \{0.1, 0.5, 0.9\}$. As expected, in response to a decrease in $\underline{r}_1$, type 1 haircuts increase. At the same time, market wide leverage falls and the equilibrium price p drops. This causes type 2 haircuts to decrease. Panel E presents the difference between the two type of haircuts, normalized by their initial level where $\underline{r}_1 = \underline{r}_2 = \underline{r}$ and $h_1 = h_2 = h$:

$$\frac{h_1(\underline{r}_1) - h_2(\underline{r}_1)}{h(\underline{r})}. \tag{G.66}$$

This expression shows how much the difference in haircuts goes up in response to a given decrease in $\underline{r}_1$ *relative* to the initial level of haircuts. In the main text, we calculate the difference in semi-elasticities at the point where $\underline{r}_1 = \underline{r}_2$. Expression (G.66) is the global equivalent. Consistent with Lemma 4, expression (G.66) is always positive.

Panels B and D repeat the exercise for interest rates. For both type 1 and 2 loans, interest rates increase as $\underline{r}_1$ falls. The interest rate is defined as surplus payments divided by the loan amount. Loans extended by type 1 agents become smaller when $\underline{r}_1$ falls ("size effect") and generate a smaller overall surplus which leads to a reduction in surplus payments to the lenders ("surplus effect"). The net impact on the interest rate is ambiguous. In the simulations, the size effect dominates and interest rates increase. Loans extended by type 2 agents tend to increase due to a fall in the equilibrium price and generate a higher overall surplus. Again, the net impact on the interest rate is unclear. In the simulations, the surplus effect always dominates and interest rates go up. Panel F shows the difference in interest rates, normalized by their initial level where $\rho_1 = \rho_2 = \rho$. In the simulations, type 2 interest rates always increase more than type 1 interest rates, but differences are small: Panels B and D are difficult to distinguish from each other. This discussion echoes the results from Lemma 6 that establishes that the sign of the difference is ambiguous.

Panel G looks at the differences between Panels E and F: are relative changes in haircuts larger than those for interest rates? Consistent with Lemma 7 in the main text, the impact on haircuts dominates for feasible changes in $\underline{r}_1$:

$$\left| \frac{h_1(\underline{r}_1) - h_2(\underline{r}_1)}{h(\underline{r})} \right| > \left| \frac{\rho_1(\underline{r}_1) - \rho_2(\underline{r}_1)}{\rho(\underline{r})} \right|$$

Unreported simulation results indicate that when θ approaches unity and the change in $\underline{r}_1$ is large, there are cases where the response in interest rates is larger than for haircuts. However, these are instances where interest rates are low to begin with, implying that we divide the difference between ρ_1 and ρ_2 by a number close to zero. We interpret this result as an artefact of the model, and our definition of semi-elasticties, rather than a substantive economic result. In any case, for moderate changes in $\underline{r}$, Lemma 7 is always valid.

References

Adkins, William, and Mark G. Davidson (2012). *Ordinary Differential Equations.* New York: Springer.

Duffie, Darrell, Nicolae Gârleanu, and Lasse Heje Pedersen (2005). "Over-the-Counter Markets". *Econometrica*, 73(6), 1815-1847.

Figure G.1: Simulation results

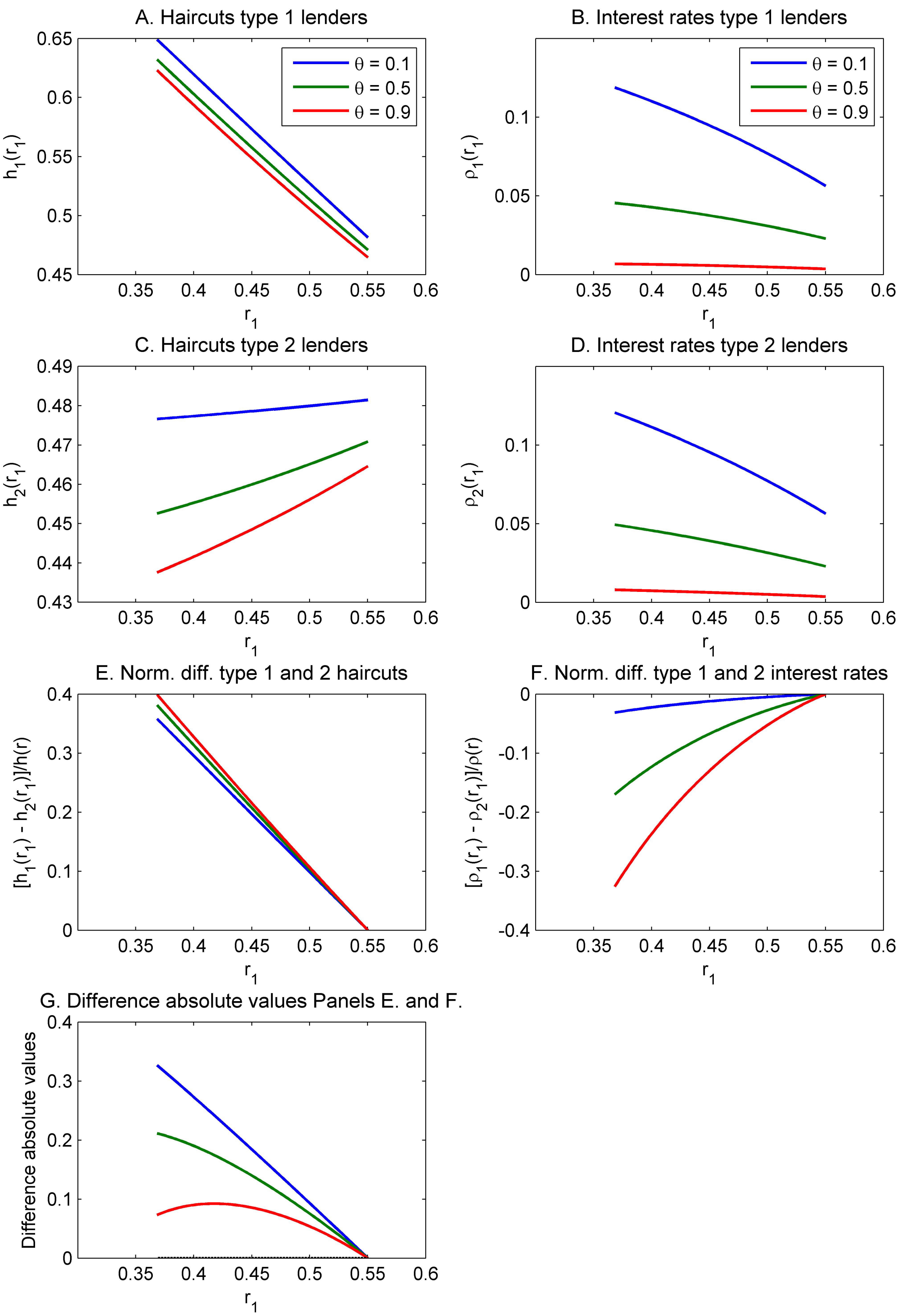


Table G.1: Parameter values numerical analysis

Parameter:	π	μ	λ	n_1	n_2	n_3
Values:	1	1	1	$1/3$	$1/3$	$1/3$

Parameter	c_3	$\underline{r}_2$	$\underline{r}_3$	$\overline{r}$	h_0
Values:	2.181	0.550	0.826	1.311	0.485
Normalization(s):	$p(\theta = 0, \underline{r}_1 = \underline{r}_2) = 1$	$v_2 + v_3 = 2$ $h_0 = \frac{v_3 - \underline{r}_j}{v_3}$ $\overline{r} = \frac{2}{4-h_0} + \frac{1+h_0}{2}$			$h_0 = \frac{2(\overline{r}-1)+g_2(4-\overline{r})}{2+g_2(4-\overline{r})}$