Online Appendix to Bailouts, Time Inconsistency, and Optimal Regulation:

A Macroeconomic View[*]

V. V. Chari

University of Minnesota
and Federal Reserve Bank of Minneapolis

Patrick J. Kehoe

University of Minnesota,
University College London,
and Federal Reserve Bank of Minneapolis

ABSTRACT

This Appendix contains proofs of results not contained in the paper.
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4. Appendix

Here we provide the proofs for some of our propositions and lemmas.

A. Proving Proposition 2

We begin with a lemma that is helpful in proving Proposition 2.

Lemma A1. There exist cutoffs $\varepsilon_s^*(U_s)$ such that the optimal contract has this form: continue if $\varepsilon > \varepsilon_s^*(U_s)$ and declare bankruptcy otherwise.

Proof. Suppose by way of contradiction that a contract that is immune to renegotiation has this form: there is a nonbankruptcy region $N = (\varepsilon_1, \varepsilon_2)$ and a bankruptcy region to the right of it, namely, $M = (\varepsilon_2, \varepsilon_3)$, where $\varepsilon_1 < \varepsilon_2 < \varepsilon_3$.

We first develop a simple inequality that will be useful in our argument. Note that since M is part of the bankruptcy region from (7) and $R < 1$, it follows that

$$(49) \quad d_s(\varepsilon) < d_s \text{ for all } \varepsilon \in M.$$

Now consider an alternative (continuation) contract (holding fixed k and p), denoted by $\{\hat{c}_s(\varepsilon), \hat{d}_s(\varepsilon), \hat{B}_s(\varepsilon)\}$. In terms of bankruptcy, this contract is the same as the original allocation except that it turns M from a bankruptcy region to a nonbankruptcy region. In terms of payments to the financial intermediary, it reduces the payments everywhere except the region M by a constant amount a and raises payments in region M so as to give the financial intermediary the same expected payments as in the original contract. Finally, the manager's new consumption is defined residually from the resource constraint. Of course, since this manager is paying the same expected amount to the investor but reaps the benefit $(1 - R)A_s\varepsilon$ for all $\varepsilon \in M$, this manager's expected utility increases.

More formally, define the bankruptcy function ϕ by $\phi_s(\varepsilon) = 1$ if $\varepsilon \in N_s$ and $\phi_s(\varepsilon) = 0$ if $\varepsilon \in B_s$. Now let $\hat{\phi}_s(\varepsilon) = 1$ for $\varepsilon \in M$ and coincide with $\phi_s(\varepsilon)$ for all other realizations of the idiosyncratic shocks. Let $\hat{d}_s(\varepsilon) = d_s - a$ for $\varepsilon \in M$, and for other ε, let $\hat{d}_s(\varepsilon) = d_s(\varepsilon) - a$, where the constant a is chosen so that the payment to the financial intermediary is the same as in the original contract:

$$(50) \quad \bar{d}_s = \int_{N_s} d_s \, dH(\varepsilon) + \int_{B_s} d_s(\varepsilon) \, dH(\varepsilon) =$$

$$\int_{N_s} (d_s - a)\, dH(\varepsilon) + \int_M (d_s - a)\, dH(\varepsilon) + \int_{B_s/M} [d_s(\varepsilon) - a]\, dH(\varepsilon).$$

Subtracting the left side from the right side of the second equality in (50) gives $a = \int_M [d_s - d_s(\varepsilon)]\, dH(\varepsilon)$, which we know from (49) is strictly positive. The expected consumption of the managers in the original contract is given by

$$\bar{c}_s = \int_{N_s} [A_s \varepsilon - d_s]\, dH(\varepsilon) + \int_{B_s} [R A_s \varepsilon - d_s(\varepsilon)]\, dH(\varepsilon),$$

and in the alternative contract their expected consumption is given by

$$(51) \quad \int_{N_s} [A_s \varepsilon - d_s + a]\, dH(\varepsilon) + \int_M [A_s \varepsilon - d_s + a]\, dH(\varepsilon) + \int_{B_s/M} [R A_s \varepsilon - d_s(\varepsilon) + a]\, dH(\varepsilon),$$

which we know, from (50), equals $\bar{c}_s + \int_M (1 - R) A_s \varepsilon\, dH(\varepsilon)$.

Under this alternative contract, the consumption of the managers satisfies the non-negativity constraint. To see this, note that in all states but those in M, we have simply added a positive number a to the managers' consumption. To argue that consumption is positive for states in M, we note that under our contradiction hypothesis, the set N is to the left of M. Since the consumption of the managers in the alternative contract $A_s \varepsilon - d_s + a$ satisfies nonnegativity for any $\varepsilon \in N$, this same expression clearly satisfies nonnegativity for the region M, which has larger idiosyncratic shocks.

This alternative contract is clearly incentive compatible. For all states besides those in M, we have subtracted off a constant from the repayments of the managers so that the incentive constraints are automatically satisfied. We have switched M to a nonbankruptcy region, and the only incentive constraint that applies in this region is that the repayments are constant, which is satisfied by construction. Thus, we have established a contradiction. Q.E.D.

We now characterize the payments in the optimal contract. Because payments by firms in say, state L, have no effect on feasible payments in the other state, say state H, it follows that we can analyze the contract in each health state separately, holding fixed the payments in the other state.

We let ε_s^* be shorthand for $\varepsilon_s^*(U_s)$. Any contract that is immune to renegotiation

must maximize, say, the payoffs to the manager subject to the constraint that the financial intermediary receives at least d_s. Furthermore, Lemma A1 implies that any contract that is immune to renegotiation must be of the form $c_s(\varepsilon) = A_s\varepsilon - d_s$ for $\varepsilon \geq \varepsilon_s^*$. Nonnegativity then implies that

(52) $d_s \leq A_s\varepsilon_s^*$ for $\varepsilon > \varepsilon_s^*$.

Incentive compatibility requires that

(53) $c_s(\varepsilon) = RA_s\varepsilon - d_s(\varepsilon) \geq A_s\varepsilon - d_s$ for $\varepsilon \leq \varepsilon_s^*$,

and nonnegativity requires that

(54) $d_s(\varepsilon) \leq RA_s\varepsilon$ for $\varepsilon \leq \varepsilon_s^*$.

Therefore, any contract that is immune to renegotiation must solve

$$\max_{\varepsilon_s^*, d_s(\varepsilon), d_s} \int_{\underline{\varepsilon}}^{\varepsilon_s^*} [RA_s\varepsilon - d_s(\varepsilon)] \, dH(\varepsilon) + \int_{\varepsilon^*}^{\bar{\varepsilon}} (A_s\varepsilon - d_s) \, dH(\varepsilon)$$

subject to (52), (53), (54), and

(55) $\displaystyle\int_{\underline{\varepsilon}}^{\varepsilon_s^*} d_s(\varepsilon) dH(\varepsilon) + d_s[1 - H(\varepsilon_s^*)] \geq \bar{d}_s.$

The solution to this problem depends on the size of the payments $\bar{d}_s$ owed to the financial intermediary. If these payments are low enough, then there is no default, and managers pay a constant amount less than $A_s\underline{\varepsilon}$, whereas if these payments are higher, then there is default and payments are as we said. Finally, if $\bar{d}_s$ is too large, then this problem does not have a solution because there is a maximal amount of expected payments $\bar{d}_s$ that can be raised by any contract that satisfies the constraints on this problem.

We now turn to the proof of Proposition 2.

Proof. It is immediate that a debt-equity contract is immune to renegotiation. We

now show that if a contract is immune to renegotiation, it must be a debt-equity contract. Consider the case that $\bar{d}_s > A_s \underline{\varepsilon}$. Clearly, to generate these payments to the financial intermediary, some bankruptcy is required, so that $\varepsilon_s^* > \underline{\varepsilon}$. We now show that the payments in the nonbankruptcy region $d_s = A_s \varepsilon_s^*$. The argument is by contradiction. Since $d_s \leq A_s \varepsilon_s^*$, we need only show that $d_s < A_s \varepsilon_s^*$ leads to a contradiction.

To do so, we construct an alternative contract that satisfies (52)–(55) and raises the payoffs to the manager. This alternative contract has a bankruptcy region $[\underline{\varepsilon}, \hat{\varepsilon}]$, where $A_s \hat{\varepsilon} = d_s$, so that $\hat{\varepsilon} < \varepsilon^*$. In this contract, set $\hat{d}_s(\varepsilon) = d_s(\varepsilon) - a$, where a is constructed so that it satisfies (55) with equality. Hence, a satisfies

$$(56) \quad \bar{d}_s = \int_{\underline{\varepsilon}}^{\hat{\varepsilon}} d_s(\varepsilon) \, dH(\varepsilon) + \int_{\hat{\varepsilon}}^{\varepsilon^*} d_s(\varepsilon) \, dH(\varepsilon) + \int_{\varepsilon^*}^{\bar{\varepsilon}} d_s \, dH(\varepsilon) =$$

$$\int_{\underline{\varepsilon}}^{\hat{\varepsilon}} [d_s(\varepsilon) - a] \, dH(\varepsilon) + \int_{\hat{\varepsilon}}^{\varepsilon^*} (d_s - a) \, dH(\varepsilon) + \int_{\varepsilon^*}^{\bar{\varepsilon}} (d_s - a) \, dH(\varepsilon).$$

Hence, $a = \int_{\hat{\varepsilon}}^{\varepsilon^*} [d_s - d_s(\varepsilon)] \, dH(\varepsilon)$, which (53) indicates is strictly positive. This alternative contract also satisfies (52)–(54) because we have simply reduced d_s and $d_s(\varepsilon)$ by a.

We now show that in the alternative contract, the expected consumption of managers is higher than in the original contract. The consumption of the managers in the original contract is given by

$$\bar{c}_s = \int_{\underline{\varepsilon}}^{\hat{\varepsilon}} [R A_s \varepsilon - d_s(\varepsilon)] \, dH(\varepsilon) + \int_{\hat{\varepsilon}}^{\varepsilon^*} [R A_s \varepsilon - d_s(\varepsilon)] \, dH(\varepsilon) + \int_{\varepsilon^*}^{\bar{\varepsilon}} (A_s \varepsilon - d_s) \, dH(\varepsilon),$$

and in the alternative contract it is given by

$$(57) \quad \int_{\underline{\varepsilon}}^{\hat{\varepsilon}} [R A_s \varepsilon - d_s(\varepsilon) + a] \, dH(\varepsilon) + \int_{\hat{\varepsilon}}^{\varepsilon^*} (A_s \varepsilon - d_s + a) \, dH(\varepsilon) + \int_{\varepsilon^*}^{\bar{\varepsilon}} (A_s \varepsilon - d_s + a) \, dH(\varepsilon),$$

which, using (56), equals $\bar{c}_s + \int_{\hat{\varepsilon}}^{\varepsilon^*} (1 - R) A_s \varepsilon \, dH(\varepsilon)$. Since $R < 1$, the managers' expected payoff is strictly higher. Hence, we have proved the desired result for the case that $\bar{d}_s > A_s \underline{\varepsilon}$.

Next, consider the case that $\bar{d}_s \leq A_s \underline{\varepsilon}$. Clearly, it is feasible to have no bankruptcy and repay the financial intermediary $\bar{d}_s$. Since bankruptcy simply wastes resources, it is optimal to set $\varepsilon_s^* = \underline{\varepsilon}$, and from (52), $\bar{d}_s \leq A_s \underline{\varepsilon}$.

4

We now show that the managers' and the investors' consumption has the desired form in the bankruptcy region. Suppose, by way of contradiction, that $\int_{\underline{\varepsilon}}^{\varepsilon^*} c_s(\varepsilon) dH(\varepsilon) > 0$. Consider an alternative contract that leaves the bankruptcy set as well as the expected consumption of managers and investors unchanged. This contract reduces the managers' consumption in the bankruptcy set to zero and raises the managers' consumption in the nonbankruptcy interval by an amount that leaves overall expected consumption the same. Since the bankruptcy region is unchanged, this alternative contract gives the same expected payoffs to the investors as the original contract but has the property that $A_s \varepsilon_s^* > \bar{d}_s$. From the first step, however, we know that any such contract is strictly dominated by the optimal contract. This gives us a contradiction. $Q.E.D.$

B. Proving Lemma 1

Proof. We assume throughout that the bankruptcy cutoff is interior. First we substitute out for c_H using the manager's incentive constraint and drop c_H as a choice variable so that the problem becomes

(58) $\quad \max_{k, \varepsilon^*, p} \left[c_L(\varepsilon^*) + p v'(p) - v(p) \right] g(k)$

subject to

(59) $\quad \left[p(A_H - v'(p)) + (1 - p) y_L(\varepsilon^*) - c_L(\varepsilon^*) \right] g(k) \geq k.$

The Lagrangian for this problem is

$\quad \left[c_L(\varepsilon^*) + p v'(p) - v(p) \right] g(k) +$

$\quad (1 + \hat{\lambda}) \left\{ \left[p(A_H - v'(p)) + (1 - p) y_L(\varepsilon^*) - c_L(\varepsilon^*) \right] g(k) - k \right\},$

where $1 + \hat{\lambda}$ denotes the multiplier on (59), which can be rewritten as

(60) $\quad \left[p A_H + (1 - p) y_L(\varepsilon^*) - v(p) \right] g(k) +$

$$\hat{\lambda} \left\{ [p(A_H - v'(p)) + (1-p)y_L(\varepsilon^*) - c_L(\varepsilon^*)] \, g(k) - k \right\}.$$

The first-order condition for ε^* to this modified problem is

$$-\hat{\lambda}c'_L = -(1+\hat{\lambda})(1-p)y'_L.$$

Since c'_L and y'_L are both negative and $1+\hat{\lambda}$ is nonnegative, it follows that $\hat{\lambda} > 0$. Thus, (60) is the Lagrangian of the contract in the desired form. $Q.E.D.$

C. Proving Proposition 3

Proof. Clearly, the bailout authority's objective function (26) is maximized by setting $\varepsilon_b = \underline{\varepsilon}$ so that no bankruptcies occur. For any cutoff ε_R of the representative firm, this outcome is achieved by making an offer $d_b = d(\varepsilon_R)$ and setting a new bankruptcy cutoff $\varepsilon_b = \underline{\varepsilon}$. Firms will accept such an offer since the financial intermediary and managers are made better off by doing so. Thus, in any equilibrium, the outcomes differ from the efficient outcomes and are therefore inefficient.

To show that $\varepsilon_R > \underline{\varepsilon}$ in any equilibrium, suppose by way of contradiction that $\varepsilon_R = \underline{\varepsilon}$ so that $\tau_b = 0$. Then we will show that the voluntary acceptance constraint (32) is violated at $\underline{\varepsilon}$. To see this result, note that it is optimal for an individual firm to deviate to the efficient contract x_{CE}, which has $\varepsilon_{CE} > \underline{\varepsilon}$. Since the associated debt payments $d(\varepsilon_{CE}) > d(\underline{\varepsilon})$, the financial intermediary will reject the bailout authority's offer and the efficient contract will be implemented, contradicting that $\varepsilon_R = \underline{\varepsilon}$. Since $\varepsilon_R > \underline{\varepsilon}$, taxes are positive in any equilibrium.

In order to show that if (32) holds as a strict inequality, we have a continuum of equilibria, note that both the left and right sides of (32) are continuous functions of ε_R and that the inequality is violated at $\varepsilon_R = \underline{\varepsilon}$. Thus, there is some value of ε_R at which (32) holds with equality. Let $\varepsilon_{\min}$ denote the largest value of ε_R such that (32) holds with equality. Then any $\varepsilon_R \in [\varepsilon_{\min}, \varepsilon_{\max}]$ is part of an equilibrium. Note that if a firm deviates to a lower bankruptcy cutoff than ε_R, this firm will accept the bailout with payments $d_b = d(\varepsilon_R)$ to the financial intermediary and bankruptcy cutoff $\underline{\varepsilon}$ and therefore will receive the same payoff as under the representative contract. Thus, no such deviation is profitable. Clearly, it is not optimal for any firm to deviate to a higher bankruptcy cutoff. Since no deviations are

profitable, any $\varepsilon_R \in [\varepsilon_{\min}, \varepsilon_{\max}]$ is part of an equilibrium. *Q.E.D.*

D. Proving Proposition 4

Proof. Suppose first that the outcomes (x_t, π_t) are those of a bailout equilibrium. Since the contracting problem is static, these outcomes must solve the one-period contracting problem. Clearly, in any equilibrium the government budget constraint is satisfied. Next, we show that under our assumption on the severity of trigger strategies (38), they must satisfy the sustainability constraint. To see why, suppose by way of contradiction that in equilibrium these outcomes violate the sustainability constraint (39). Then the authority, by setting the bankruptcy set to be empty in the current period, obtains current payoffs equal to the first term on the right side of (39), and under (38), its continuation payoff is at least as large as the last term. Thus, outcomes that violate the sustainability constraint contradict optimality by the bailout authority.

Suppose, next, that a set of candidate equilibrium outcomes $(\hat{x}_t, \hat{\pi}_t)$ with associated history $\hat{H}_t$ satisfy $(i), (ii)$, and (iii) of Proposition 4. We will construct revert-to-static strategies that support these outcomes as an equilibrium. For private agents, these strategies specify that if the history $H_t = \hat{H}_t$, then the contract x_t equals the desired one $\hat{x}_t$; otherwise, the contract x_t equals the full bailout contract x_b. For the bailout authority, these strategies specify that if $H_t = \hat{H}_t$, then the policies equal the desired ones $\hat{\pi}_t$; otherwise, they equal the full bailout policy of purchasing all the debt in the distressed state and eliminating all the bankruptcies.

Now consider the bailout authority. If there has been no deviation from these specified outcomes in or before period t, in that $H_t = \hat{H}_t$, then the payoffs associated with choosing the desired policy $\hat{\pi}_t$ are given by the left side of the sustainability constraint. The payoffs associated with any deviation are smaller than the right side of the sustainability constraint because the first term on the right side represents the best one-shot deviation. The inequality in (39) guarantees that the desired policies are indeed optimal. If there has been a deviation in or before t, so that $H_t \neq \hat{H}_t$, then the continuation payoffs of the bailout authority are independent of the current policy. Hence, the bailout authority's optimal choice is the statically optimal full bailout policy.

Clearly, the private agent's strategies are optimal by construction. $Q.E.D.$

E. Proving Proposition 5

Proof. To prove the first part of the proposition, suppose that by way of contradiction that $\beta < \bar{\beta}$ but no bailouts occur in that $\varepsilon_{bt} = \varepsilon_{Rt}$ and $\tau_t = 0$. That taxes are zero implies $d_{bt} = d(\varepsilon_{bt}) = d(\varepsilon_{Rt})$. By definition of $\bar{\beta}$, the surplus in any period t is strictly less than the surplus under commitment, so that the commitment outcomes are not sustainable. Thus $\varepsilon_{bt} \neq \varepsilon_{CE}$. We will show since $\tau_t = 0$, it is optimal for the firm to choose the commitment outcome anticipating that it will reject the bailout offer. To see this result, suppose first that $\varepsilon_{bt} < \varepsilon_{CE}$ so that $d_{bt} = d(\varepsilon_{bt}) < d(\varepsilon_{CE})$. If the firm chooses the commitment outcome the value of the surplus will be strictly higher than in the purported equilibrium and the financial intermediary will reject the bailout offer so that the commitment outcome will be implemented. Suppose next that $\varepsilon_{bt} > \varepsilon_{CE}$. Then the firm can choose the commitment outcome, raise the surplus, and the manager will reject the bailout offer.

To show that the outcome is sustainably inefficient, note that if τ_t is positive the (k, p) decisions are distorted for the same reasons they are in the one-period model. $Q.E.D.$

F. Proving Proposition 7

Proof of first part. We will show that the best orderly resolution outcome has zero taxes. Since taxes are strictly positive in the best bailout outcome, the two outcomes differ. Since the bailout outcome is feasible under orderly resolution, the orderly resolution outcome yields strictly higher surplus.

To show that the best orderly resolution outcome has zero taxes, consider a modified version of the orderly resolution problem in which we hold all future allocations fixed and only vary the current allocations. This modification implies that we keep the continuation value V_{t+1} fixed while we vary the period t allocations. Clearly, if for any V_{t+1} the best allocations at t have zero taxes, then so does the allocations in all periods. Here, by way of contradiction, we allow for positive bailouts in that $\varepsilon_b < \varepsilon_O$ so that the implemented bankruptcy cutoff is ε_b. The modified problem can be written as

$$\max_{k, p, \varepsilon_O, \varepsilon_b} U(p, \varepsilon_b) g(k) + \omega - k$$

8

subject to

(61) $\quad f(p, \varepsilon_b)g(k) \geq k,$

(62) $\quad \left[(1-p)(1-R)A_L \int_{\underline{\varepsilon}}^{\varepsilon_b} \varepsilon dH(\varepsilon) \right] g(k) \leq \beta(V_{t+1} - V_{FB}),$

(63) $\quad [U(p, \varepsilon_b)] g'(k) - 1 = -\dfrac{\tilde{U}_p(p, \varepsilon_b, d(\varepsilon_O))}{\tilde{f}_p(p, \varepsilon_b, d(\varepsilon_O))} [(f(p, \varepsilon_b) g'(k) - 1],$

where $\varepsilon_b \leq \varepsilon_O$. Here we have substituted out the government budget constraint into (27) and (29) to obtain (61) and (63).

Since (61) holds with equality, $g(k) = k^\alpha$ implies that $f = k^{1-\alpha}$ and $fg' = \alpha$ so that (63) can be written as

(64) $\quad \dfrac{U(p, \varepsilon_b)}{f(p, \varepsilon_b)} \alpha - 1 = -(1-\alpha) \dfrac{\tilde{U}_p(p, \varepsilon_b, d(\varepsilon_O))}{\tilde{f}_p(p, \varepsilon_b, d(\varepsilon_O))},$

which implicitly defines the effort function $p(\varepsilon_b, \varepsilon_O)$, which, importantly, does not depend on k. We later show in Lemma A2 that under our assumptions on $v(p)$, the effort function is decreasing in ε_O. From the implementability constraint (61) written as $f(p(\varepsilon_b, \varepsilon_O), \varepsilon_b) = k^{1-\alpha}$, it follows that if, as we show later, f_p is negative, then reducing ε_O reduces k by reducing effort.

Now suppose by way of contradiction that it is optimal to have $\varepsilon_O > \varepsilon_b$, so that there are taxes in equilibrium. Consider reducing ε_O. This reduction increases effort and, since effort is below the full information level, raises surplus. This variation relaxes the sustainability constraint because effort rises and k falls, thus making it possible to raise welfare and thereby establishing the contradiction.

To establish that $f_p < 0$, note that the first-order condition with respect to ε_O is

$$\left\{ U_p + \lambda f_p + \mu \left[(1-R)A_L \int_{\underline{\varepsilon}}^{\varepsilon_b} \varepsilon dH(\varepsilon) \right] g(k) \right\} \frac{\partial p(\varepsilon_b, \varepsilon_O)}{\partial \varepsilon_O} = 0,$$

where λ and μ are the multipliers on the implementability and sustainability constraints, which are both positive. Since effort is below the full information level, U_p is positive. Clearly,

9

the last term in brackets is positive. It follows that f_p is negative. $Q.E.D.$

We turn now to our lemma.

Lemma A2. Under our sufficient conditions (42), the partial derivative $\partial p(\varepsilon_b, \varepsilon_O)/\partial \varepsilon_O$ is negative.

Proof. The effort function $p(\varepsilon_b, \varepsilon_O)$ is implicitly defined by (64). Let $M(p, \varepsilon_b) = U(p, \varepsilon_b)/f(p, \varepsilon_b)$ and $N(p, \varepsilon_b, \varepsilon_O) = \tilde{U}_p(p, \varepsilon_b, \varepsilon_O)/\tilde{f}_p(p, \varepsilon_b, \varepsilon_O)$. To show that the relevant partial derivative of effort is negative, take the total derivative of (64) to get

$$(65) \quad [\alpha M_p + (1 - \alpha) N_p] \frac{\partial p(\varepsilon_b, \varepsilon_O)}{\partial \varepsilon_O} = -(1 - \alpha) \frac{\partial N}{\partial \varepsilon_O}.$$

Next, we will show that M_p, N_p, and $\partial N/\partial \varepsilon_O$ are all positive, to obtain the desired result. To show that M_p is positive, we rewrite M as

$$M(p, \varepsilon_b) = 1 + \frac{p v'(p)}{f(p, \varepsilon_b)}$$

so that M_p has the same sign as

$$f [v'(p) + p v''(p)] - p v'(p) f_p,$$

which under (42) has the same sign as

$$[f (1 + a) - p f_p] v'(p),$$

which is positive because $f - p f_p = y_L(\varepsilon_b) - c_L(\varepsilon_b) + p^2 v''(p) > 0$ since $y_L(\varepsilon_b) > c_L(\varepsilon_b)$. To see that N_p and $\partial N/\partial \varepsilon_O$ are positive, note that

$$N(p, \varepsilon_b, \varepsilon_O) = 1 + \frac{p v''(p)}{\tilde{f}_p(p, \varepsilon_b, \varepsilon_O)},$$

where $\tilde{f}_p = A_H - c_L - v'(p) - p v''(p) - d(\varepsilon_O)$. Next, note that since $\tilde{f}_p$ is clearly decreasing in ε_O, it follows that $\partial N/\partial \varepsilon_O$ is positive. Since $a \geq 1$, $p v''(p)$ is increasing in p and since $\tilde{f}_p$ is decreasing in p, it follows that N_p is positive. It thus follows from (65) that $\partial p(\varepsilon_b, \varepsilon_O)/\partial \varepsilon_O$.

Q.E.D.

Proof of second part. We first show a preliminary result that we use in proving the main result: if $0 < \beta < \bar{\beta}$, the sustainably efficient outcome has bankruptcies. To see this result, suppose by way of contradiction that the sustainably efficient outcome has no bankruptcies. Then there is no static gain to canceling bankruptcies, so the first terms on the left and right sides of the sustainability constraint (39) are the same. The continuation payoffs are strictly greater than the full bailout continuation payoffs, however, so that the sustainability constraint holds as a strict inequality. This is a contradiction since in any sustainably efficient outcome below commitment, the sustainability constraint binds.

We now show that the orderly resolution outcome is sustainably inefficient. Consider the outcomes of an orderly resolution equilibrium denoted $(k_O, \varepsilon_O, p_O)$ in some particular period t. Consider the alternative allocations that alter period t outcomes, but let future outcomes coincide with those of the given orderly resolution equilibrium. These alternative allocations at time t maximize surplus subject to the implementability constraint and the sustainability constraint except that here the continuation surplus V in the sustainability constraint is the surplus associated with the given orderly resolution equilibrium. Since the original outcomes satisfy the sustainability constraint, it is clear that $(k_O, \varepsilon_O, p_O)$ is feasible for the alternative maximization problem. We have dropped the combined (k, p) first order condition (43) at time t, so it is clear that surplus in the alternative allocation is weakly higher than in the orderly resolution equilibrium. Clearly, since the sustainability constraint binds and the sustainably efficient outcome has bankruptcies, the first-order conditions for this alternative allocation with respect to ε^* and k will not satisfy the first-order conditions with respect ε^* and k in an orderly resolution equilibrium. Since the sustainably efficient outcome yields even higher welfare than the alternative allocations, it follows that the orderly resolution outcome is sustainably inefficient. *Q.E.D.*

G. Setup and Proof of Proposition 8

Setup: We begin by deriving the objective function in (45) and the implementability constraint with taxes and transfers (46), both reproduced here:

(66) $U(p, \varepsilon^*)g(k) + \omega + T - (1 + \theta)k,$

(67) $f(p, \varepsilon^*)g(k) + T \geq (1 + \theta)k.$

To derive these let $\tilde{c}_H g(k)$ and $\tilde{c}_L(\varepsilon^*)g(k)$ denote the expected payments from the firm to the manager so that the expected consumption of the manager in the two states, inclusive of the transfer T, is given by

(68) $c_H g(k) = \tilde{c}_H g(k) + T$ and $c_L(\varepsilon^*)g(k) = \tilde{c}_L(\varepsilon^*)g(k) + T.$

The objective function (66) is immediate: the sum of manager and the investor's utilities is increased by the lump sum transfer T and reduced by the tax θk. The implementability constraint is given by

(69) $[p(A_H - \tilde{c}_H) + (1 - p)(y_L(\varepsilon^*) - \tilde{c}_L(\varepsilon^*))]g(k) \geq (1 + \theta)k.$

Substituting for $\tilde{c}_H$ and $\tilde{c}_L(\varepsilon^*)$ from (68) into (69) yields (67). Clearly, since the manager receives the same lump sum payment T in both the healthy and distressed states, the manager's effort incentive constraint is unaffected.

Proof. The basic idea of the proof is to use the solution to the sustainable efficiency problem to construct the multipliers for the regulatory problem and the tax rate θ and show that they are positive. To do so, consider the first-order conditions to the sustainable efficiency problem for (k, ε^*, p) in the current period, holding future allocations fixed:

(70) $U_p + \lambda f_p - \mu L_p = 0$

(71) $U_\varepsilon + \lambda f_\varepsilon - \mu L_\varepsilon = 0$

(72) $U g'(k) - 1 + \lambda [f g'(k) - 1] - \mu L g'(k) = 0,$

where

$$L(p, \varepsilon^*)g(k) = [U(p, \underline{\varepsilon}) - U(p, \varepsilon^*)] g(k) = (1-R)(1-p)g(k) \int_{\underline{\varepsilon}}^{\varepsilon^*} \varepsilon dH(\varepsilon)$$

so that the sustainability constraint can be written as

$$L(p, \varepsilon^*)g(k) \leq \beta(V_{SE} - V_{FB}),$$

λ and μ are the multipliers on the implementability and sustainability constraints, and V_{SE} denotes the continuation value associated with the sustainably efficient outcome. Writing the constraint (44) as $d(\varepsilon^*) \leq vk$, the first-order conditions to (45) evaluated at the sustainably efficient outcomes satisfy

(73) $\quad U_p + \hat{\lambda} f_p = 0$

(74) $\quad U_\varepsilon + \hat{\lambda} f_\varepsilon - \hat{\mu} \dfrac{d'(\varepsilon^*)}{g(k)} = 0$

(75) $\quad Ug'(k) - (1+\theta) + \hat{\lambda} [fg'(k) - (1+\theta)] + \hat{\mu}v = 0$

for some positive multipliers $\hat{\lambda}$ and $\hat{\mu}$ on (46) and (44). Note that (73) implies that $\hat{\lambda} > 0$ since $U_p > 0$ and $f_p < 0$. Equating the first-order conditions in the two problems yields

(76) $\quad \hat{\lambda} = \lambda - \mu L_p / f_p$

(77) $\quad \hat{\mu} \dfrac{d'(\varepsilon^*)}{g(k)} = (\hat{\lambda} - \lambda) f_\varepsilon + \mu L_\varepsilon$

(78) $\quad \theta(1 + \hat{\lambda}) = (\lambda - \hat{\lambda})(1 - \alpha) + \mu Lg'(k) + \hat{\mu}v$

where we have used that when the constraint (46) holds with equality and $g(k) = k^\alpha$, then $fg' = \alpha$.

Next we show that, under our sufficient conditions, the constructed multiplier $\hat{\mu}$ and the tax rate θ are both positive. First note from the definition of L that $L_p < 0$, $L_\varepsilon > 0$, and $Lg'(k) > 0$. Since $f_p < 0$ and $L_p < 0$, from (76) it follows that $\hat{\lambda} < \lambda$. Note, next, once we

show that $\hat{\mu} > 0$ it follows from (78) that $\theta > 0$.

To show that $\hat{\mu} > 0$, we substitute for $\hat{\lambda}$ from (73) and solve for λ and μ from (71) and (70) to obtain

$$\hat{\mu}\frac{d'(\varepsilon^*)}{g(k)} = -\frac{U_p}{f_p}f_\varepsilon + U_\varepsilon.$$

Substituting for U_p, f_p, U_ε, and f_ε, simplifying, and noting that $d' > 0$, gives that $\hat{\mu}$ is positive if

$$(A_H - y_L(\varepsilon^*) - v'(p))\,(1 - H(\varepsilon^*)) - (1 - p)v''(p)(1 - R)\varepsilon^* h(\varepsilon^*) > 0.$$

Since $\varepsilon^* \leq \varepsilon_{\max}$, we know that $H(\varepsilon^*)$ is uniformly bounded away from 1 as A_H is increased. All the other terms are also uniformly bounded. Thus, for A_H sufficiently large, this inequality holds. $Q.E.D.$

H. Allowing for the Bailout Authority to Observe Individual Actions

In the body of the paper, in the history of decisions we recorded only the history of past policies and not the history of past private actions. We did so to capture the idea that private agents are competitive (or anonymous) in the sense that no individual private agent perceives that the government or other private agents will change their decisions in response to changes in that agent's actions. In Section 5 of Chari and Kehoe (1990) we proved that the symmetric perfect Bayesian equilibria of an anonymous game coincided with the sustainable equilibria in an environment in which we recorded only the history of past policies. In a similar vein, here we show how informational assumptions in a game generate similar results for the environment in this paper.

We start by arguing that allowing for the history to record the actions of each individual agent, without making other assumptions on information and costs of observing individual actions, allows the government to induce private agents to take whatever action it desires by effectively threatening each private agent with severe consequences if that particular agent deviates from the desired action.

The most trivial example that illustrates this point is a static one in which a continuum

of homogeneous consumers are taxed on their labor income to provide for a given amount of government spending. Each consumer solves

$$\max u(c, l) \text{ subject to } c \leq (1 - \tau(l))l.$$

The government budget constraint is $g = \tau(l)l$, and the resource constraint is $c + g = l$. Here the tax rate is individualized in that a given consumer's tax rate depends on that consumer's labor supply. Clearly, by setting

$$\tau(l) = \left\{ \begin{array}{c} 1 \text{ if } l \neq l^* \\ \tau \text{ otherwise} \end{array} \right\}$$

with appropriate assumptions on u, the government can engineer essentially any desired feasible labor supply l^* that it wants.

Note that once we allow such threats, the government can indirectly control actions that have no obvious connection to the instrument. Suppose, for example, that we modify private agents' utility to be $u(c, l) + w(a)$ where a is some other action, say, attention to personal health. Then the government can pick a desired outcome (l^*, a^*) and implement it with a tax system $\tau(l, a)$ that equals a low number if the desired policies are followed and 1 otherwise. Clearly, such a setup gives the government an enormous amount of control over its citizens.

With these issues in mind, we now turn to our dynamic model with a bailout authority. We show that simply allowing the government to observe individual actions, the government has so much power to control private agents that it can achieve any sustainable outcome. We then add reasonable informational assumptions and show that our results go through essentially unchanged.

We make two minor modifications to our dynamic model. We assume that debt purchases are absolute rather than scaled and that the tax rate is on the absolute receipts of investors rather than on their scaled receipts. These changes lead to small differences in the first-order conditions but have no effect on the inefficiency of bailout equilibria, the inefficiency of orderly resolution equilibria, or the efficiency of the regulatory equilibrium. As

we elaborate on later, these modifications make the policies easier to interpret given the information structure.

We now index firms by i to help make clear how policies are individualized. Suppose that the bailout policy of a given firm i can depend on that firm's contract x_i but the tax rate τ is common to all firms. Specifically, the bailout is a function of an individual firm's size and bankruptcy cutoff: an unscaled debt purchase offer $D_b(x_i)$, a renegotiated debt level indexed by $\varepsilon_b(x_i)$, and a common tax rate τ (now on unscaled receipts by investors).

We start by showing that if all agents can observe the actions of every private agent and the government in all periods, then any desired sustainable outcome can be supported by trigger-like policies by the government. Here we focus on supporting the sustainably efficient outcome, but the logic applies to any sustainable outcome.

Consider the following trigger-like strategies. We construct the strategies to ensure both private and government optimality. To construct the government strategies, let x^s denote the sustainably efficient contract and suppose the individualized policy is: if $x_i = x^s$, then the government does not intervene on this particular contract, whereas if $x_i \neq x^s$, then the government purchases the debt at, say, its face value $A_L \varepsilon_i^*$ and sets $\varepsilon_b(x_i) = \underline{\varepsilon}$. Along the equilibrium path and off the path with single (measure zero) deviations, the tax rate is $\tau = 0$.

Under sufficient conditions on $v(p)$, it is optimal for firms to choose x^s. The reason is that each firm understands that if it deviates, the consumption of the manager conditional on being in the distressed state will be high, but this deviation lowers the incentives for the manager to provide effort enough so that ex ante surplus is lower (here is where we need a condition on $v(p)$). Here the government has encoded a sufficiently dire threat into its individualized policy so that it can induce the agents to take whatever actions it wants.

To make this policy optimal for the government, we assume that if the government deviates from such a policy, then the continuation equilibrium is the full bailout equilibrium. These policies support the sustainably efficient outcome in much the same way that in the labor tax example the policies supported the lump-sum tax outcome.

We can make reasonable assumptions on what is observed by the government and what is recorded in the history to rule out such extreme outcomes and restore our results.

The assumption on what is observed by the government at t about period t actions is the following. If the government does not pay a fixed cost η, it obtains no information about individual choices, whereas if it does pay, it sees the entire vector (x_{it}) of actions for all agents in the current period.

The assumption on what is recorded in the past history is just the past history of average policies $\left\{ \int \pi_{is-1} di \right\}_{s=0}^{t}$ where π_{is-1} is the vector of government interventions. Note well that the history does *not* record whether the previous government paid the fixed cost.

Under these assumptions we have the following.

Lemma A3. The best bailout equilibrium in the model with individualized policies coincides with the best bailout equilibrium in the model with uniform policies.

Proof. We first show that in any equilibrium, the government will not pay the fixed cost. To see why, suppose by way of contradiction that the government pays the fixed cost in some period t. Recall that the government chooses to pay the fixed cost after the private agents have chosen their contracts. Moreover, in any equilibrium along the equilibrium path, all private agents will choose the same contract.
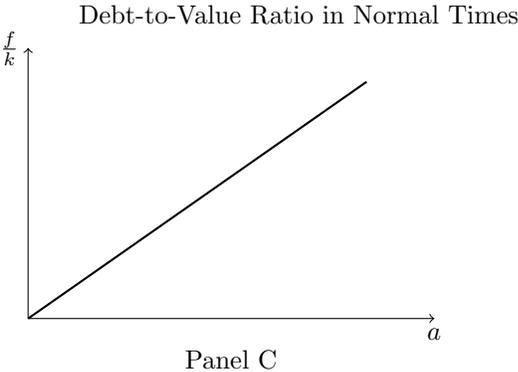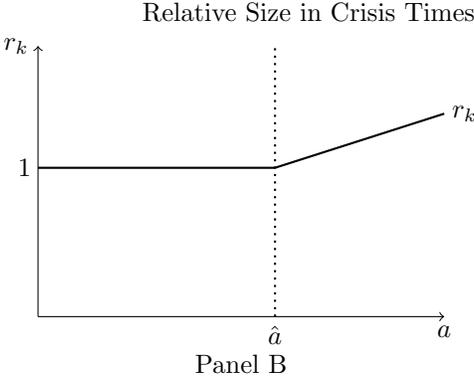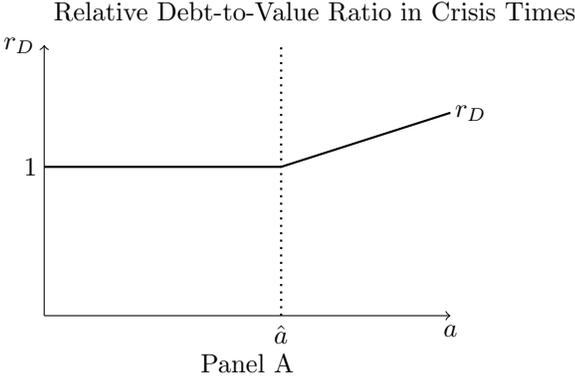
Suppose the government in period t deviates by not paying the fixed cost and offers a uniform policy that implies the same average outcomes as under the purported equilibrium. In the current period, the government gains by not paying the fixed cost, and, by construction, this deviation does not set off a trigger so that the government's future payoffs are unaffected. Hence, this deviation is profitable and we have a contradiction.

Next, because the government does not incur the fixed cost, we can show it is without loss of generality to restrict attention to uniform policies (that are not individualized). To see this result, consider an equilibrium in which the government offers an individualized policy without incurring the fixed cost, in that the government asks agents to voluntarily disclose their contracts. Clearly, in this equilibrium, all agents will pick the most desirable reported contract so that the payoffs are identical to those if the government offered a uniform policy (nonindividualized) that coincides with the individualized policy outcomes at the most desirable reports. *Q.E.D.*

We briefly elaborate on why we modified the model so that here debt purchases are absolute rather than scaled and that the tax rate is on the absolute receipts of investors

rather than on their scaled receipts. When the government does not pay the fixed cost, it does not see any individual firm's k, and thus one would need an elaborate story for how the government actually collects taxes and makes debt purchases that are scaled by $g(k)$. When policies are levied on unscaled variables, we do not need such an elaborate story.

Figure 3: Regulatory Policy Across Industries

Relative Debt-to-Value Ratio in Crisis Times

Panel A

Relative Size in Crisis Times

Panel B

Debt-to-Value Ratio in Normal Times

Panel C

Note: Here r_D is the debt-to-value ratio in the best bailout equilibrium divided by the debt-to-value ratio in the sustainably efficient outcome while r_k is the corresponding ratio of sizes. The debt-to-value ratio $f/k = A_L \varepsilon^* g(k)/k$.