Online Appendix of "Bank Leverage Cycles"

Galo Nuño
Banco de España

Carlos Thomas
Banco de España

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A. Data appendix

Data on equity capital and total assets of the four leveraged financial subsectors we consider (US-chartered commercial banks, savings institutions, security brokers and dealers, and finance companies) are from the Z.1 files of the US Flow of Funds.[1] The series corresponding to savings institutions are the sum of OTS and FDIC reporters. Data on *levels* in the Z.1 files (denoted by 'FL' in the series identifier) suffer from discontinuities that are caused by changes in the definition of the series. The Flow of Funds accounts correct for such changes by constructing *discontinuities* series (denoted by 'FD').[2] In particular, for each series the *flow* (denoted by 'FU') is equal to the change in level outstanding less any discontinuity. That is: $\mathrm{FU}_t = \mathrm{FL}_t - \mathrm{FL}_{t-1} - \mathrm{FD}_t$. Therefore, the flow data are free from such discontinuities. In order to construct discontinuity-free level series, we take the value of the level in the first period of the sample and then accumulate the flows onwards.

For each subsector, the leverage ratio is the ratio between total assets and equity capital, both in dollars. In the tables and figures, 'assets' and 'equity' refer to real total assets and equity capital, i.e. deflated by the GDP Implicit Price Deflator.[3] The latter and real GDP are both from the Bureau of Economic

[1] Website: http://www.federalreserve.gov/datadownload/Choose.aspx?rel=Z1

[2] For instance, changes to regulatory report forms and/or accounting rules typically trigger 'FD' entries for the affected series.

[3] Real assets and equity, and their estimated trends, are displayed in Figure 1.

Analysis (BEA). Investment is 'Real Gross Private Domestic Investment', also from the BEA. Consumption is computed in a model-consistent way, i.e. as the difference between GDP and investment. Hours are 'Nonfarm Business Sector: Hours of All Persons', from the Bureau of Labor Statistics. All these series are readily available at the Federal Reserve Bank of St. Louis FRED database.[4]

Our proxy for bank asset prices is a weighted average of Barclays Capital US Mortgage Backed Securities (MBS) and Asset Backed Securities (ABS) indexes, both obtained from Datastream.[5] As documented by He and Krishnamurthy (2013), total outstanding MBS and ABS totaled \$8.9 and \$2.5 trillion respectively in 2007. Assuming that leveraged intermediaries held both asset types in the same proportion, we assign a weight of 3/4 for the MBS index, which closely corresponds to the observed weights (78-22%). We then rescale the resulting series by its sample mean (101.9), so that it fluctuates around 1 (the steady-state value of Q_t in the model).

In order to obtain an empirical proxy for aggregate log TFP, we use the quarterly change in the Business sector log TFP series (labelled 'dtfp') constructed by the Center for the Study of Income and Productivity (CSIP) at the Federal Reserve Bank of San Francisco.[6] We then accumulate the log changes to obtain the log level series.

Finally, in order to construct a proxy for island-specific volatility, we use the annual TFP series for all 6-digit NAICS manufacturing industries constructed by the National Bureau of Economic Research (NBER) and the US Census Bureau's Center for Economic Studies (CES), which run until 2009 (included).[7] We discard those industries that enter the sample in 1997 due to the change in industry classification from SIC to NAICS. We log and linearly detrend each industry TFP series, and then aggregate all 6-digit industries into 3-digit industries. We then obtain a time series for volatility by computing the cross-industry variance in TFP in each year. In order to construct a quarterly volatility series, we distribute the annual series into a quarterly one by adapting the methodology of Stock and Wat-

[4] Website: http://research.stlouisfed.org/fred2/

[5] Tickers: LHMNBCK and LHASSBK, respectively. The Barclays Capital ABS index starts in 1992:Q1. Thus, our composite index starts on that date.

[6] Website: http://www.frbsf.org/csip/tfp.php

[7] Website: http://www.nber.org/data/nbprod2005.html

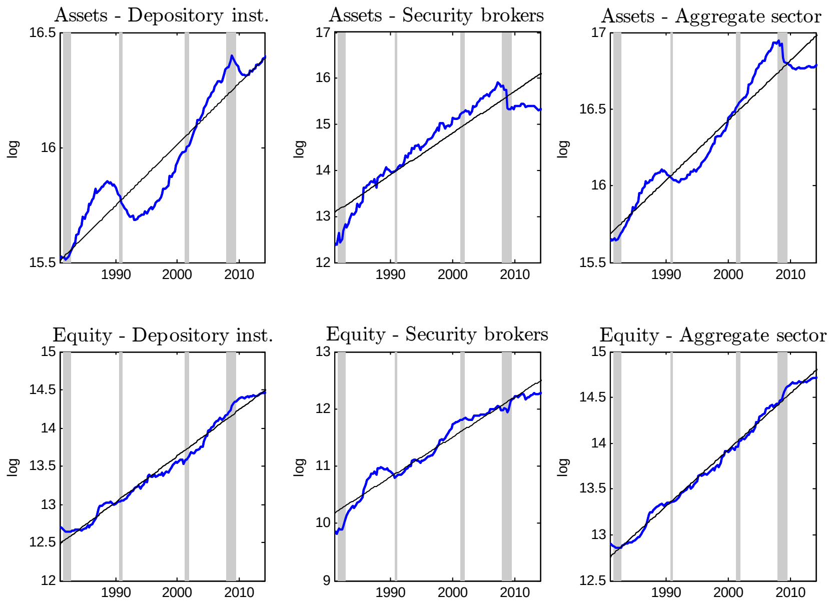

Figure 1: Real total assets and equity

Source: US Flow of Funds. See Data Appendix for details. The series have been logged. Shaded areas represent NBER-dated recessions.

son (2010). We use as a quarterly dispersion indicator the variance in industrial production across 3-digit NAICS manufacturing industries, using quarterly data from the G.17 (Industrial Production and Capacity Utilization) files of the US Federal Reserve.[8] The distribution is performed as follows. Let $h_t \equiv \sigma^2_{t-1}$ be the unobserved quarterly cross-industry variance in TFP.[9] The quarterly dispersion indicator, x_t, is assumed to be related to h_t by

$$x_t = \frac{1}{\vartheta} h_t - \frac{\vartheta_0}{\vartheta} - \frac{1}{\vartheta} \varpi_t \Leftrightarrow h_t = \vartheta x_t + \vartheta_0 + \varpi_t, \tag{1}$$

where ϖ_t follows

$$\varpi_t = \rho_{\varpi} \varpi_{t-1} + \sigma_{\varpi} \varepsilon_t^{\varpi}, \tag{2}$$

with $\varepsilon_t^{\varpi} \overset{iid}{\sim} N(0,1)$. Let $H_T \equiv \frac{1}{4} \sum_{q=0}^{3} h_{4T-q}$ denote the observed annual cross-industry variance in TFP in year T. Using (1) to substitute for h_{4T-q}, we obtain

$$H_T = \frac{\vartheta}{4} \sum_{q=0}^{3} x_{4T-q} + \vartheta_0 + \frac{1}{4} \sum_{q=0}^{3} \varpi_{4T-q}. \tag{3}$$

Let $X_T \equiv \frac{1}{4} \sum_{q=0}^{3} x_{4T-q}$ denote the annual average of the quarterly dispersion indicator. We have

$$cov\left(H_T, X_T\right) = \vartheta var\left(X_T\right) \Rightarrow \vartheta = \frac{cov\left(H_T, X_T\right)}{var\left(X_T\right)},$$

$$E\left(H_T\right) = \vartheta E\left(X_T\right) + \vartheta_0 \Rightarrow \vartheta_0 = E\left(H_T\right) - \vartheta E\left(X_T\right).$$

We use the sample moments of H_T and X_T to estimate ϑ and ϑ_0. We can then

[8] Website: http://www.federalreserve.gov/datadownload/Choose.aspx?rel=G17.

Analogously to our treatment of the NBER-CES data, we log and linearly detrend each industry series, and then compute the cross-industry variance in industrial production in each quarter. The 'Major Industry Groups' option of the G17 files contains data on 3-digit NAICS industries over our whole sample period, so unlike for the NBER-CES data there is no need to exclude industries or to aggregate into lower digits.

[9] As explained in section 2, σ^2_{t-1} is the variance of island-specific shocks at time t, only such variance is known one period in advance.

construct the following annual observable variable,

$$\begin{aligned}\tilde{H}_T &\equiv H_T - \vartheta X_T - \vartheta_0 \\ &= \frac{1}{4}\sum_{q=0}^{3} \varpi_{4T-q}, \end{aligned} \tag{4}$$

where the second equality follows from (3). The state equation (2) and the observation equation (4) constitute a state-space model. We estimate ρ_ϖ and σ_ϖ by maximum likelihood and employ the Kalman smoother to obtain the quarterly series for ϖ_t up until 2009:Q4 (the last quarter in the annual sample for $\tilde{H}_T$); as an estimate of ϖ_t for 2010:Q1-2014:Q2 we use its expectation conditional on ϖ_{4T}, $T = 2009$. We finally use (1) to construct a quarterly series for $h_t = \sigma_{t-1}^2$ for the whole sample period 1981Q1:2014Q2.

B. Proof of Lemma 1

In the text, we have defined $\pi_t(x) \equiv \int^x (x-\omega) dF_t(\omega)$. Using integration by parts, it is possible to show that

$$\pi_t(x) = \int^x F_t(\omega)\, d\omega.$$

We then have

$$\Delta\pi_t(x) \equiv \tilde{\pi}_t(x) - \pi_t(x) = \int^x \left(\tilde{F}_t(\omega) - F_t(\omega)\right) d\omega.$$

Notice first that $\Delta\pi_t(0) = 0$. We also have $\Delta\pi_t'(x) = \tilde{F}_t(x) - F_t(x)$. Thus, from Assumption 2, $\Delta\pi_t(x)$ is strictly increasing for $x \in (0, \omega_t^*)$, reaches a maximum at $x = \omega_t^*$, and then strictly decreases for $x > \omega_t^*$. Using integration by parts, it is also possible to show that

$$\begin{aligned}\lim_{x\to\infty} \Delta\pi_t(x) &= \int (1 - F_t(\omega))\, d\omega - \int \left(1 - \tilde{F}_t(\omega)\right) d\omega \\ &= \int \omega dF_t(\omega) - \int \omega d\tilde{F}_t(\omega) > 0, \end{aligned}$$

where the inequality follows from Assumption 1 in the main text. Therefore, for $x > \omega_t^*$ the function $\Delta\pi_t(x)$ decreases asymptotically towards $\mathbb{E}(\omega) - \mathbb{E}(\tilde{\omega})$. It follows that $\Delta\pi_t(x) > 0$ for all $x > 0$. It also follows that $\Delta\pi_t(x)$ cuts $\mathbb{E}(\omega) - \mathbb{E}(\tilde{\omega})$ precisely once and from below.

C. The bank's problem

We start by defining the ratio $\bar{b}_{t-1}^j \equiv \bar{B}_{t-1}^j/(Q_{t-1}A_{t-1}^j)$ and using the latter to substitute for $\bar{B}_{t-1}^j = \bar{b}_{t-1}^j Q_{t-1} A_{t-1}^j$. Given the choice of investment size A_t^j, the bank then chooses the ratio $\bar{b}_t^j$. With this transformation, and abusing somewhat the notation V_t and $\bar{V}_t$ in the main text, the bank's maximization problem can be expressed as

$$V_t\left(\omega, A_{t-1}^j, \bar{b}_{t-1}^j\right) = \max_{N_t^j} \left\{ \begin{array}{c} \left(\omega - \bar{b}_{t-1}^j/R_t^A\right) R_t^A Q_{t-1} A_{t-1}^j - N_t^j + \bar{V}_t\left(N_t^j\right) \\ +\mu_t^j \left[\left(\omega - \bar{b}_{t-1}^j/R_t^A\right) R_t^A Q_{t-1} A_{t-1}^j - N_t^j\right] \end{array} \right\}, \quad (5)$$

$$\bar{V}_t\left(N_t^j\right) = \max_{A_t^j, \bar{b}_t^j} \mathbb{E}_t \Lambda_{t,t+1} \int_{\bar{b}_t^j/R_{t+1}^A} \left[\theta V_{t+1}\left(\omega, A_t^j, \bar{b}_t^j\right) + (1-\theta)\left(\omega - \bar{b}_t^j/R_{t+1}^A\right) R_{t+1}^A Q_t A_t^j\right] dF_t(\omega)$$

subject to the participation constraint,

$$\mathbb{E}_t \Lambda_{t,t+1} R_{t+1}^A Q_t A_t^j \left\{ \int^{\bar{b}_t^j/R_{t+1}^A} \omega dF_t(\omega) + \frac{\bar{b}_t^j}{R_{t+1}^A} \left[1 - F_t\left(\frac{\bar{b}_t^j}{R_{t+1}^A}\right)\right] \right\} \geq Q_t A_t^j - N_t^j,$$

and the IC constraint

$$\begin{aligned} & \mathbb{E}_t \Lambda_{t,t+1} \int_{\bar{b}_t^j/R_{t+1}^A} \left\{ \theta V_{t+1}\left(\omega, A_t^j, \bar{b}_t^j\right) + (1-\theta) R_{t+1}^A Q_t A_t^j \left(\omega - \frac{\bar{b}_t^j}{R_{t+1}^A}\right) \right\} dF_t(\omega) \\ \geq\; & \mathbb{E}_t \Lambda_{t,t+1} \int_{\bar{b}_t^j/R_{t+1}^A} \left\{ \theta V_{t+1}\left(\omega, A_t^j, \bar{b}_t^j\right) + (1-\theta) R_{t+1}^A Q_t A_t^j \left(\omega - \frac{\bar{b}_t^j}{R_{t+1}^A}\right) \right\} d\tilde{F}_t(\omega). \end{aligned}$$

The first order condition with respect to N_t^j is given by

$$\mu_t^j = \bar{V}_t'\left(N_t^j\right) - 1.$$

We can now guess that $\bar{V}_t'(N_t^j) > 1$. Then $\mu_t^j > 0$ and the non-negativity constraint on dividends is binding, such that a continuing bank optimally decides to retain all earnings,

$$N_t^j = \left(\omega - \frac{\bar{b}_{t-1}^j}{R_t^A}\right) R_t^A Q_{t-1} A_{t-1}^j.$$

From (5), we then have $V_t(\omega, A_{t-1}^j, \bar{b}_{t-1}^j) = \bar{V}_t((\omega - \bar{b}_{t-1}^j/R_t^A) R_t^A Q_{t-1} A_{t-1}^j)$. Using the latter, we can express the Bellman equation for $\bar{V}_t(N_t^j)$ as

$$\begin{aligned}
\bar{V}_t(N_t^j) = \max_{A_t^j, \bar{b}_t^j} \{ & \mathbb{E}_t \Lambda_{t,t+1} \int_{\bar{b}_t^j / R_{t+1}^A} \left[\theta \bar{V}_{t+1} \left(\left(\omega - \frac{\bar{b}_t^j}{R_{t+1}^A} \right) R_{t+1}^A Q_t A_t^j \right) + (1-\theta) \left(\omega - \frac{\bar{b}_t^j}{R_{t+1}^A} \right) R_{t+1}^A Q_t A_t^j \right] dF_t(\omega) \\
& + \lambda_t^j \left\{ \mathbb{E}_t \Lambda_{t,t+1} R_{t+1}^A Q_t A_t^j \left[\int^{\bar{b}_t^j / R_{t+1}^A} \omega dF_t(\omega) + \frac{\bar{b}_t^j}{R_{t+1}^A} \left(1 - F_t \left(\frac{\bar{b}_t^j}{R_{t+1}^A} \right) \right) \right] - \left(Q_t A_t^j - N_t^j \right) \right\} \\
& + \kappa_t^j \mathbb{E}_t \Lambda_{t,t+1} \int_{\bar{b}_t^j / R_{t+1}^A} \left[\theta \bar{V}_{t+1} \left(\left(\omega - \frac{\bar{b}_t^j}{R_{t+1}^A} \right) R_{t+1}^A Q_t A_t^j \right) + (1-\theta) R_{t+1}^A Q_t A_t^j \left(\omega - \frac{\bar{b}_t^j}{R_{t+1}^A} \right) \right] dF_t(\omega) \\
& - \kappa_t^j \mathbb{E}_t \Lambda_{t,t+1} \int_{\bar{b}_t^j / R_{t+1}^A} \left[\theta \bar{V}_{t+1} \left(\left(\omega - \frac{\bar{b}_t^j}{R_{t+1}^A} \right) R_{t+1}^A Q_t A_t^j \right) + (1-\theta) R_{t+1}^A Q_t A_t^j \left(\omega - \frac{\bar{b}_t^j}{R_{t+1}^A} \right) \right] d\tilde{F}_t(\omega) \},
\end{aligned}$$

where λ_t^j and κ_t^j are the Lagrange multipliers associated to the participation and IC constraints, respectively. The first order conditions with respect to A_t^j and $\bar{b}_t^j$ are given by

$$\begin{aligned}
0 \;=\; & \mathbb{E}_t \Lambda_{t,t+1} R_{t+1}^A \int_{\bar{\omega}_{t+1}^j} \left[\theta \bar{V}_{t+1}' \left(N_{t+1}^j \right) + 1 - \theta \right] \left(\omega - \bar{\omega}_{t+1}^j \right) dF_t(\omega) \\
& + \lambda_t^j \left\{ \mathbb{E}_t \Lambda_{t,t+1} R_{t+1}^A \left[\int^{\bar{\omega}_{t+1}^j} \omega dF_t(\omega) + \bar{\omega}_{t+1}^j \left[1 - F_t \left(\bar{\omega}_{t+1}^j \right) \right] \right] - 1 \right\} \\
& + \kappa_t^j \mathbb{E}_t \Lambda_{t,t+1} R_{t+1}^A \int_{\bar{\omega}_{t+1}^j} \left\{ \theta \bar{V}_{t+1}' \left(N_{t+1}^j \right) + 1 - \theta \right\} \left(\omega - \bar{\omega}_{t+1}^j \right) dF_t(\omega) \\
& - \kappa_t^j \mathbb{E}_t \Lambda_{t,t+1} R_{t+1}^A \int_{\bar{\omega}_{t+1}^j} \left\{ \theta \bar{V}_{t+1}' \left(N_{t+1}^j \right) + 1 - \theta \right\} \left(\omega - \bar{\omega}_{t+1}^j \right) d\tilde{F}_t(\omega),
\end{aligned}$$

$$
\begin{aligned}
0 \;=\; & -\mathbb{E}_t\Lambda_{t,t+1}\int_{\bar{\omega}^j_{t+1}}\left[\theta\bar{V}'_{t+1}\left(N^j_{t+1}\right)+(1-\theta)\right]dF_t(\omega)-\mathbb{E}_t\Lambda_{t,t+1}\theta\frac{\bar{V}_{t+1}(0)}{R^A_{t+1}Q_tA^j_t}f_t\left(\bar{\omega}^j_{t+1}\right)\\
& +\lambda^j_t\mathbb{E}_t\Lambda_{t,t+1}\left[1-F_t\left(\bar{\omega}^j_{t+1}\right)\right]\\
& -\kappa^j_t\mathbb{E}_t\Lambda_{t,t+1}\int_{\bar{\omega}^j_{t+1}}\left\{\theta\bar{V}'_{t+1}\left(N^j_{t+1}\right)+(1-\theta)\right\}dF_t(\omega)-\kappa^j_t\mathbb{E}_t\Lambda_{t,t+1}\theta\frac{\bar{V}_{t+1}(0)}{R^A_{t+1}Q_tA^j_t}f_t\left(\bar{\omega}^j_{t+1}\right)\\
& +\kappa^j_t\mathbb{E}_t\Lambda_{t,t+1}\int_{\bar{\omega}^j_{t+1}}\left\{\theta\bar{V}'_{t+1}\left(N^j_{t+1}\right)+(1-\theta)\right\}d\tilde{F}_t(\omega)+\kappa^j_t\mathbb{E}_t\Lambda_{t,t+1}\theta\frac{\bar{V}_{t+1}(0)}{R^A_{t+1}Q_tA^j_t}\tilde{f}_t\left(\bar{\omega}^j_{t+1}\right),
\end{aligned}
$$

respectively, where we have used $\bar{b}^j_t/R^A_{t+1}=\bar{\omega}^j_{t+1}$. We also have the envelope condition

$$\bar{V}'_t\left(N^j_t\right)=\lambda^j_t.$$

At this point, we guess that in equilibrium $\bar{V}_t(N^j_t)=\lambda^j_tN^j_t$, and that the multipliers λ^j_t and κ^j_t are equalized across islands: $\lambda^j_t=\lambda_t$ and $\kappa^j_t=\kappa_t$ for all j. Using this, the IC constraint simplifies to

$$\mathbb{E}_t\Lambda_{t,t+1}R^A_{t+1}\left\{\theta\lambda_{t+1}+(1-\theta)\right\}\left[\int_{\bar{\omega}^j_{t+1}}\left(\omega-\bar{\omega}^j_{t+1}\right)dF_t(\omega)-\int_{\bar{\omega}^j_{t+1}}\left(\omega-\bar{\omega}^j_{t+1}\right)d\tilde{F}_t(\omega)\right]\geq 0. \tag{6}$$

The first order conditions then become

$$
\begin{aligned}
0 \;=\; & \mathbb{E}_t\Lambda_{t,t+1}R^A_{t+1}\left[\theta\lambda_{t+1}+1-\theta\right]\int_{\bar{\omega}^j_{t+1}}\left(\omega-\bar{\omega}^j_{t+1}\right)dF_t(\omega) \qquad (7)\\
& +\lambda_t\left\{\mathbb{E}_t\Lambda_{t,t+1}R^A_{t+1}\left[\int^{\bar{\omega}^j_{t+1}}\omega dF_t(\omega)+\bar{\omega}^j_{t+1}\left[1-F_t\left(\bar{\omega}^j_{t+1}\right)\right]\right]-1\right\},
\end{aligned}
$$

$$
\begin{aligned}
0 \;=\; & \lambda_t\mathbb{E}_t\Lambda_{t,t+1}\left[1-F_t\left(\bar{\omega}^j_{t+1}\right)\right]-\mathbb{E}_t\Lambda_{t,t+1}\left[\theta\lambda_{t+1}+1-\theta\right]\left[1-F_t\left(\bar{\omega}^j_{t+1}\right)\right]\\
& +\kappa_t\mathbb{E}_t\Lambda_{t,t+1}\left\{\theta\lambda_{t+1}+1-\theta\right\}\left[F_t\left(\bar{\omega}^j_{t+1}\right)-\tilde{F}_t\left(\bar{\omega}^j_{t+1}\right)\right], \qquad (8)
\end{aligned}
$$

where in (7) we have used the fact that κ^j_t times the left-hand side of (6) must be zero as required by the Kuhn-Tucker conditions, and in (8) we have used the fact that, according to our guess, $\bar{V}_{t+1}(0)=0$. Solving for the Lagrange multipliers, we obtain

$$\lambda_t = \frac{\mathbb{E}_t \Lambda_{t,t+1} R^A_{t+1} \left[\theta \lambda_{t+1} + 1 - \theta\right] \int_{\bar{\omega}^j_{t+1}} \left(\omega - \bar{\omega}^j_{t+1}\right) dF_t\left(\omega\right)}{1 - \mathbb{E}_t \Lambda_{t,t+1} R^A_{t+1} \left[\int^{\bar{\omega}^j_{t+1}} \omega dF_t\left(\omega\right) + \bar{\omega}^j_{t+1}\left[1 - F_t\left(\bar{\omega}^j_{t+1}\right)\right]\right]}, \tag{9}$$

$$\kappa_t = \frac{\lambda_t \mathbb{E}_t \Lambda_{t,t+1}\left[1 - F_t\left(\bar{\omega}^j_{t+1}\right)\right] - \mathbb{E}_t \Lambda_{t,t+1}\left[\theta\lambda_{t+1} + 1 - \theta\right]\left[1 - F_t\left(\bar{\omega}^j_{t+1}\right)\right]}{\mathbb{E}_t \Lambda_{t,t+1}\left\{\theta\lambda_{t+1} + 1 - \theta\right\}\left[\tilde{F}_t\left(\bar{\omega}^j_{t+1}\right) - F_t\left(\bar{\omega}^j_{t+1}\right)\right]}. \tag{10}$$

In the steady state, the Lagrange multipliers are

$$\lambda = \frac{\beta R^A \left(1-\theta\right) \int_{\bar{\omega}^j} \left(\omega - \bar{\omega}^j\right) dF\left(\omega\right)}{1 - \beta R^A + \left(1-\theta\right)\beta R^A \int_{\bar{\omega}^j}\left(\omega - \bar{\omega}^j\right) dF\left(\omega\right)},$$

$$\kappa = \frac{\left(\lambda - 1\right)\left(1-\theta\right)}{\theta\lambda + 1 - \theta} \frac{\left[1 - F\left(\bar{\omega}^j\right)\right]}{\tilde{F}\left(\bar{\omega}^j\right) - F\left(\bar{\omega}^j\right)},$$

where we have used $\int \left(\omega - \bar{\omega}^j\right) dF\left(\omega\right) = 1 - \bar{\omega}^j$. Provided the parameter values are such that

$$0 < \beta R^A - 1 < \left(1-\theta\right)\beta R^A \int_{\bar{\omega}^j} \left(\omega - \bar{\omega}^j\right) dF\left(\omega\right),$$

then $\lambda > 1$. Also, notice that for $\bar{\omega}^j \geq \omega^*$ we have $\Delta\pi\left(\bar{\omega}^j\right) > \mathbb{E}\left(\omega\right) - \mathbb{E}\left(\tilde{\omega}\right)$ and thus the IC constraint is violated in the steady state.[10] Therefore, in equilibrium it must be the case that $\bar{\omega}^j < \omega^*$. But from Assumption 2 in the main text this implies $\tilde{F}\left(\bar{\omega}^j\right) > F\left(\bar{\omega}^j\right)$ which, together with $\lambda > 1$, implies in turn $\kappa > 0$. That is, both the participation and IC constraints hold in the steady state.[11] Provided aggregate shocks are sufficiently small, we will also have $\lambda_t > 1$, $\kappa_t > 0$ and $\bar{\omega}^j_t < \omega^*_t$ along the cycle. But if $\lambda_t > 1$, then our guess that $\bar{V}'_t(N^j_t) > 1$ is verified. Also, given that $\bar{\omega}^j_{t+1} = \bar{b}^j_t / R_{t+1}$, the ratio $\bar{b}^j_t$ is then pinned down by the IC constraint (equation 6) holding with equality. Since we have guessed that the multiplier λ_t is equalized across islands, so are $\bar{b}^j_t = \bar{b}_t$ and $\bar{\omega}^j_{t+1} = \bar{\omega}_{t+1} = \bar{b}_t / R_{t+1}$. But if $\bar{\omega}_{t+1}$ is equalized, then from (9) and (10) our guess that λ_t and κ_t are symmetric across

[10] As shown in Appendix B, $\Delta\pi(x)$ increases initially, reaches a maximum at $x = \omega^*$ and then decreases asymptotically towards $\mathbb{E}\left(\omega\right) - \mathbb{E}\left(\tilde{\omega}\right)$. This implies that $\Delta\pi(x) > \mathbb{E}\left(\omega\right) - \mathbb{E}\left(\tilde{\omega}\right)$ for $x \geq \omega^*$.

[11] Our calibration in Table 2 implies $\lambda = 1.3688$ and $\kappa = 32.8067$.

islands is verified too.

The participation constraint (holding with equality) is given by

$$\mathbb{E}_t \Lambda_{t,t+1} R_{t+1}^A Q_t A_t^j \left\{ \int^{\bar{\omega}_{t+1}} \omega dF_t(\omega) + \bar{\omega}_{t+1} \left[1 - F_t(\bar{\omega}_{t+1})\right] \right\} = Q_t A_t^j - N_t^j.$$

Using the latter to solve for A_t^j, we obtain

$$Q_t A_t^j = \frac{1}{1 - \mathbb{E}_t \Lambda_{t,t+1} R_{t+1}^A \left\{ \bar{\omega}_{t+1} - \pi_{t+1}(\bar{\omega}_{t+1}) \right\}} N_t^j \equiv \phi_t N_t^j,$$

where we have also used the definition of the put option value, $\pi_t(\bar{\omega}_{t+1}) = \int^{\bar{\omega}_{t+1}} (\bar{\omega}_{t+1} - \omega) dF_t(\omega)$. Therefore, the leverage ratio $Q_t A_t^j / N_t^j = \phi_t$ is equalized across firms too. Finally, using $\bar{V}_{t+1}(N_{t+1}^j) = \lambda_{t+1} N_{t+1}^j$, $N_{t+1}^j = (\omega - \bar{\omega}_{t+1}) R_{t+1}^A Q_t A_t^j$ and $Q_t A_t^j = \phi_t N_t^j$, the value function $\bar{V}_t(N_t^j)$ can be expressed as

$$\bar{V}_t(N_t^j) = \phi_t N_t^j \mathbb{E}_t \Lambda_{t,t+1} R_{t+1}^A \left[\theta \lambda_{t+1} + 1 - \theta\right] \int_{\bar{\omega}_{t+1}} (\omega - \bar{\omega}_{t+1}) dF_t(\omega),$$

which is consistent with our guess that $\bar{V}_t(N_t^j) = \lambda_t N_t^j$ only if

$$\begin{aligned} \lambda_t &= \phi_t \mathbb{E}_t \Lambda_{t,t+1} R_{t+1}^A \left[\theta \lambda_{t+1} + 1 - \theta\right] \int_{\bar{\omega}_{t+1}} (\omega - \bar{\omega}_{t+1}) dF_t(\omega) \\ &= \frac{\mathbb{E}_t \Lambda_{t,t+1} R_{t+1}^A \left[\theta \lambda_{t+1} + 1 - \theta\right] \left\{1 - \bar{\omega}_{t+1} + \pi_t(\bar{\omega}_{t+1})\right\}}{1 - \mathbb{E}_t \Lambda_{t,t+1} R_{t+1}^A \left\{\bar{\omega}_{t+1} - \pi_t(\bar{\omega}_{t+1})\right\}}. \end{aligned}$$

But the latter corresponds exactly with (9) without j subscripts, once we use the definition of $\pi_t(\bar{\omega}_{t+1})$. Our guess is therefore verified.

D. Model summary and comparison to standard RBC model

Our model can be reduced to the following 12-equation system,

$$\frac{v'(L_t)}{u'(C_t)} = (1 - \alpha) \frac{Y_t}{L_t}, \tag{S1}$$

$$Y_t = Z_t L_t^{1-\alpha} \left(\xi_t K_t\right)^{\alpha}, \tag{S2}$$

$$K_{t+1} = \left[1 - S(I_t/I_{t-1})\right] I_t + (1-\delta)\, \xi_t K_t, \tag{S3}$$

$$Y_t = C_t + I_t \tag{S4}$$

$$1 = Q_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right] + \mathbb{E}_t\left[\Lambda_{t,t+1} Q_{t+1} S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right], \tag{S5}$$

$$R_t^A = \xi_t \frac{(1-\delta)\, Q_t + \alpha \frac{Y_t}{\xi_t K_t}}{Q_{t-1}}, \tag{S6}$$

$$1 = \beta \mathbb{E}_t \left\{ \frac{u'(C_{t+1})}{u'(C_t)} R_{t+1}^A \left[\bar{\omega}_{t+1} - \pi\left(\bar{\omega}_{t+1}; \sigma_t\right)\right] \frac{\phi_t}{\phi_t - 1} \right\}, \tag{S7}$$

$$Q_t K_{t+1} = \phi_t N_t, \tag{S8}$$

$$\bar{\omega}_t = \bar{b}_{t-1}/R_t^A, \tag{S9}$$

$$N_t = \theta R_t^A \left[1 - \bar{\omega}_t + \pi_{t-1}\left(\bar{\omega}_t\right)\right] Q_{t-1} K_t + \left\{1 - \theta\left[1 - F_{t-1}\left(\bar{\omega}_t\right)\right]\right\} \tau Q_t K_t, \tag{S10}$$

$$\lambda_t = \frac{\mathbb{E}_t \beta u'(C_{t+1}) R_{t+1}^A \left[\theta \lambda_{t+1} + 1 - \theta\right] \left\{1 - \bar{\omega}_{t+1} + \pi_t\left(\bar{\omega}_{t+1}\right)\right\}}{u'(C_t) - \mathbb{E}_t \beta u'(C_{t+1}) R_{t+1}^A \left\{\bar{\omega}_{t+1} - \pi_t\left(\bar{\omega}_{t+1}\right)\right\}}, \tag{S11}$$

$$1 - \int \omega d\tilde{F}_t(\omega) = \mathbb{E}_t \left\{ \frac{u'(C_{t+1}) R_{t+1}^A \left(\theta \lambda_{t+1} + 1 - \theta\right)}{\mathbb{E}_t u'(C_{t+1}) R_{t+1}^A \left(\theta \lambda_{t+1} + 1 - \theta\right)} \left[\tilde{\pi}_t\left(\frac{\bar{b}_t}{R_{t+1}^A}\right) - \pi_t\left(\frac{\bar{b}_t}{R_{t+1}^A}\right)\right] \right\}, \tag{S12}$$

which jointly determine the dynamics of 12 endogenous variables: C_t, L_t, K_t, I_t, Y_t, R_t^A, Q_t, N_t, $\bar{\omega}_t$, $\bar{b}_t$, λ_t, ϕ_t.

The standard RBC model is given by equations (S1) to (S6), *plus* the following investment Euler equation,

$$1 = \beta \mathbb{E}_t \left\{ \frac{u'(C_{t+1})}{u'(C_t)} R_{t+1}^A \right\}, \tag{S7'}$$

which jointly determine the path of 7 endogenous variables: C_t, L_t, K_t, I_t, Y_t, R_t^A Q_t.

References

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