Estimating Dynamic Games of Electoral Competition to Evaluate Term Limits in U.S. Gubernatorial Elections: Online Appendix

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I. States with Term Limits

Table 1. Term Limitations by State in 2013

State law	State
States with no term limits	CT, IA, ID, IL, MA, MN, ND, NH, NY, TX, UT, VT, WA, WI
States limiting governors to 1 term in office	VA
States limiting governors to 2 consecutive terms in office	AL(1968), AZ(1993), CO(1991), FL(1968), GA(1976), KS(1974), KY(1992), LA(1966),MD(1954), ME(1966), NC(1993), NE(1967), NJ, NM(1991), OH(1966), PA(1967), RI(1994), SC(1980), SD(1956), TN(1978), WV(1970)
States limiting governors to 8 out of 12 years in office	IN(1972), OR(1987)
States limiting governors to 2 lifetime terms in office	AR(1987), CA(1991), DE, MI(1993), MO(1966), MS(1986), NV(1971), OK(2010)
States limiting governors to 8 out of 16 years in office	MT(1993), WY(1993)

Note: The table summarizes the use of term limits in states in the U.S. Note: Source: the Book of the States.

The parenthesis shows the year of change if it was after 1950. We have considered 48 constitutional states. Note that a) NC adopted 2 lifetime term limit from 1977 to 1992; b) NM adopted 2 consecutive term limit prior to 1971 and adopted 1 term limit from 1971 to 1990; and c) OR adopted 2 consecutive term limit before 1987. OK adopts 2 consecutive term limit from 1966 to 2009.

II. Equilibrium Beliefs

This equilibrium can be supported by the following voters' beliefs:

- If $x_t < \underline{\rho}_R(a)$, then $P_R(\rho|x_t, a) = 0$ for all $\rho < x_t$ and $P_R(\rho|x_t, a) = 1$ for all $\rho \geq x_t$. (Left Extremists)
- If $x_t = \underline{s}_R(a)$ then $P_R(\rho|x_t, a) = F_R^{\rho}(\rho|\rho \in [\underline{\rho}_R(a), \underline{s}_R(a))$. (Left-leaning Moderates)
- If $x_t \in (\underline{s}_R(a), \bar{s}_R(a))$, then $P_R(\rho|x_t, a) = 0$ for all $\rho < x_t$ and $P_R(\rho|x_t, a) = 1$ for all $\rho \geq x_t$. (Centrists)
- If $x_t = \bar{s}_R(a)$ then $P_R(\rho|x_t, a) = F_R^{\rho}(\rho|\rho \in [\bar{s}_R(a), \bar{\rho}_R(a)])$. (Right-leaning Moderates)
- If $x_t > \bar{\rho}_R(a)$, then $P_R(\rho|x_t, a) = 0$ for all $\rho < x_t$ and $P_R(\rho|x_t, a) = 1$ for all $\rho \geq x_t$. (Right Extremists)
- If $x_t \in (\underline{\rho}_R(a), \underline{s}_R(a))$, then $P_R(\rho|x_t, a) = 0$ for all $\rho < \underline{\rho}_R(a)$ and $P_R(\rho|x_t, a) = 1$ for all $\rho \geq \underline{\rho}_R(a)$. (Beliefs when off-equilibrium deviations occur left center.)
- If $x_t \in (\bar{s}_R(a), \bar{\rho}_R(a))$, then $P_R(\rho|x_t, a) = 0$ for all $\rho < \bar{\rho}_R(a)$ and $P_R(\rho|x_t, a) = 1$ for all $\rho \geq \bar{\rho}_R(a)$. (Beliefs when off-equilibrium deviations occur right center.)

III. Proof of Proposition 1

In the sub-population described in Proposition 1 we have $x = \rho$. Substituting this equation into the measurement system, we obtain:

$$\begin{aligned} p_1 &= \rho + \epsilon_1 \\ p_2 &= \mu_{21}\,\rho + \epsilon_2 \\ p_3 &= \mu_{31}\,\rho + a + \epsilon_3 \\ p_4 &= \mu_{41}\,\rho + \mu_{42}\,a + \epsilon_4 \\ p_5 &= \mu_{51}\,\rho + \mu_{52}\,a + \epsilon_5 \end{aligned} \tag{1}$$

We assume that ρ, a, and ϵ_j's are mutually independent, the means of ρ and a are finite, and $E(\epsilon_j) = 0$ for all j. We further assume that ρ, a, and ϵ_j's satisfy the conditions of Fubini's theorem and have non-vanishing (a.e.) characteristic functions.

CHH (2003) show identification of the factor loadings. Here we just briefly summarize their argument. Let $j = 1, 2$ and $l = 1, 2, 3, 4, 5$ $(j \neq l)$, we then have:

$$cov(p_j, p_l) = \mu_{j1}\,\mu_{l1}\sigma_\rho^2 \tag{2}$$

In particular

$$\begin{aligned} cov(p_1, p_l) &= \mu_{l1}\sigma_\rho^2 \\ cov(p_2, p_l) &= \mu_{l1}\mu_{21}\sigma_\rho^2 \end{aligned} \tag{3}$$

Hence we have:

$$\mu_{21} = \frac{cov(p_2, p_3)}{cov(p_1, p_3)} = \frac{cov(p_2, p_4)}{cov(p_1, p_4)} \tag{4}$$

and $cov(p_1, p_2) = \mu_{21}\sigma_\rho^2$ identifies σ_ρ^2. $cov(p_1, p_3) = \mu_{31}\sigma_\rho^2$ identifies μ_{31}. $cov(p_1, p_4) = \mu_{41}\sigma_\rho^2$ identifies μ_{41}. $cov(p_1, p_5) = \mu_{51}\sigma_\rho^2$ identifies μ_{51}.

Proceeding to the next equations and note that for $j = 3, 4$ and $l = 3, 4, 5$ ($j \neq l$) we have:

$$cov(p_j, p_l) = \mu_{j1}\ \mu_{l1}\sigma_\rho^2 + \mu_{j2}\ \mu_{l2}\sigma_a^2 \tag{5}$$

In particular:

$$\begin{aligned} cov(p_3, p_5) - \mu_{31}\mu_{51}\ \sigma_\rho^2 &= \mu_{52}\ \sigma_a^2 \\ cov(p_4, p_5) - \mu_{41}\mu_{51}\ \sigma_\rho^2 &= \mu_{42}\mu_{52}\ \sigma_a^2 \end{aligned} \tag{6}$$

Hence we have:

$$\mu_{42} = \frac{cov(p_4, p_5) - \mu_{41}\mu_{51}\ \sigma_\rho^2}{cov(p_3, p_5) - \mu_{31}\mu_{51}\ \sigma_\rho^2} \tag{7}$$

and $cov(p_3, p_4) - \mu_{31}\ \mu_{41}\ \sigma_\rho^2 = \mu_{42}\ \sigma_a^2$ identifies σ_a^2. $cov(p_3, p_5) - \mu_{31}\mu_{51}\ \sigma_\rho^2\ = \mu_{52}\ \sigma_a^2$ identifies μ_{52}.

CHS (2010) show non-parametric identification of the underlying density functions. Again, we briefly summarize their argument. Define:

$$\bar{p}_1 = p_1 = \rho + \epsilon_1 = \rho + \bar{\epsilon}_1 \tag{8}$$

$$\bar{p}_2 = \frac{p_2}{\mu_{21}} = \rho + \frac{\epsilon_2}{\mu_{21}} = \rho + \bar{\epsilon}_2 \tag{9}$$

Kotlarski's Theorem then implies that the characteristic functions of ρ and $\bar{\epsilon}_i$ are given by:

$$\varphi_\rho(t) = \exp\left(\int^t \frac{\varphi_n^1(0, u)}{\varphi_n(0, u)} du\right) \tag{10}$$

$$\varphi_{\bar{\epsilon}_1}(t) = \frac{\varphi_n(t, 0)}{\varphi_\rho(t)} \tag{11}$$

$$\varphi_{\bar{\epsilon}_2}(t) = \frac{\varphi_n(0, t)}{\varphi_\rho(t)} \tag{12}$$

where φ_n is the joint characteristic function of $\bar{p}_j^1$ and $\bar{p}_j^2$ for the restricted sample, and $\varphi_n^1(0, u)$ denotes the derivative of this function with respect to its first argument. We can then use the standard inversion formula to estimate the densities based on the characteristic functions:

$$f_\rho(x) = \frac{1}{2\pi} \int_{-T}^{T} \exp(-itx)\varphi_\rho(t)dt \tag{13}$$

$$f_{\bar{\epsilon}_i}(x) = \frac{1}{2\pi} \int_{-T}^{T} \exp(-itx)\varphi_{\bar{\epsilon}_i}(t)dt \quad i = 1, 2 \tag{14}$$

where T is a smoothing parameter. Next define:

$$\bar{p}_3 = p_3 - \mu_{31} p_1 = a + \epsilon_3 + \mu_{31}\epsilon_1 = a + \bar{\epsilon}_3 \tag{15}$$

$$\bar{p}_4 = \frac{1}{\mu_{42}} p_4 - \frac{\mu_{41}}{\mu_{42}\mu_{21}} p_2 = a + \frac{1}{\mu_{42}}\epsilon_4 + \frac{\mu_{41}}{\mu_{42}\mu_{21}}\epsilon_2 = a + \bar{\epsilon}_4 \tag{16}$$

Applying Kotlarski's Theorem on the two transformed measurements above yields the distribution of a.

IV. Model without Term Limits

The key equations that define cut-off rules and ideological thresholds in the model without term limits are the following:

$$\frac{-|\bar{\rho}_R(a) - \bar{s}_R(a)| + \gamma\,(\psi\, + \lambda\, a)}{1-\gamma} = 0 \tag{17}$$

$$\frac{-|\underline{\rho}_R(a) - \underline{s}_R(a)| + \gamma\,(\psi\, + \lambda\, a)}{1-\gamma} = 0 \tag{18}$$

and

$$\frac{-|\underline{s}_R(a)| + \lambda a}{1-\beta} = V^D(0) \tag{19}$$

$$\frac{-|\bar{s}_R(a)| + \lambda a}{1-\beta} = V^D(0) \tag{20}$$

Similarly, we can derive election standards for Democratic incumbents denoted by $\underline{s}_D(a)$ and $\bar{s}_D(a)$, as well as cut-off points $\underline{\rho}_D(a)$ and $\bar{\rho}_D(a)$. Value functions satisfy:

$$\begin{aligned} V^D(\theta) &= \int_A \int_{-\infty}^{\underline{\rho}_D(a)} -|\rho-\theta| + \lambda a + \beta\, V^R(\theta)\; dF_D^{\rho}(\rho)\; dF_D^{a}(a) \\ &+ \int_A \int_{\underline{\rho}_D(a)}^{\underline{s}_D(a)} \frac{-|\underline{s}_D(a)-\theta| + \lambda a}{1-\beta}\; dF_D^{\rho}(\rho)\; dF_D^{a}(a) \\ &+ \int_A \int_{\underline{s}_D(a)}^{\bar{s}_D(a)} \frac{-|\rho-\theta| + \lambda a}{1-\beta}\; dF_D^{\rho}(\rho)\; dF_D^{a}(a) \\ &+ \int_A \int_{\bar{s}_D(a)}^{\bar{\rho}_D(a)} \frac{-|\bar{s}_D(a)-\theta| + \lambda a}{1-\beta}\; dF_D^{\rho}(\rho)\; dF_D^{a}(a) \\ &+ \int_A \int_{\bar{\rho}_D(a)}^{\infty} -|\rho-\theta| + \lambda a + \beta\, V^R(\theta)\; dF_D^{\rho}(\rho)\; dF_D^{a}(a) \end{aligned} \tag{21}$$

A similar equation holds for $V^R(\theta)$.

V. Additional Empirical Evidence

A. *Evidence Supporting Policy Moderation*

The key prediction of our dynamic game is that a subset of two-term governors will engage in policy moderation to win reelection. We have denoted these types as Moderates to distinguish them from Centrists, who are very close to the ideal point of the median voter and do not need to moderate. In this section we present evidence that supports this key prediction and shows that the data are broadly consistent with our model.

One way to measure policy moderation is to analyze the differences in the standard deviation of policies adopted in the first and second term restricting attention to a subsample of two-term governors. Broadly speaking, our model implies that the observed standard deviation of policies of successful incumbents should be larger in the second term than in the first term.

Table 2. A Policy Moderation Test

	std deviation 1st term	std deviation 2nd term	One sided Test p-value
expenditures	109.86	127.21	0.003
taxes	60.78	61.96	0.359

Note: The table reports the empirical results for the two outcomes studied that are primarily driven by ideology.

Source: Authors' calculations.

Table 2 reports the empirical results for the two outcomes studied that are primarily driven by ideology. We find that the standard deviation of first term policies is smaller than the standard deviation of second term policies for both outcome measures that are strongly correlated with ideology. Using conventional levels of significance, the difference is significantly different from zero for expenditures. We have also conducted the same analysis for each party. Our qualitative findings are similar once we condition on party membership.

Second, our model also suggests that the effect of policy moderation depends on which side of the median voter a governor is located. All moderates need to move towards the center to win reelection. A fiscally conservative moderate must adopt higher taxes and expenditures in the first term than in the second term to win reelection. A fiscally liberal moderate must adopt lower taxes and expenditures in the first term than in the second term. Note that this prediction holds for both parties. The degree of policy moderation is, however, party specific.

Table 3. Modified Besley-Case Regressions

Variables	expenditure		tax	
Ideology	liberal	conservative	liberal	conservative
Democratic incumbent	-37.50	10.93	-4.00	4.10
1st term	(13.79)	(14.47)	(5.80)	(7.84)
Republican incumbent	-17.97	60.15	-5.61	24.20
1st term	(15.79)	(14.00)	(6.09)	(8.23)
Governor's party	18.44	11.46	-10.79	14.48
is Democratic	(15.03)	(14.39)	(6.03)	(8.13)
Constant	77.27	-86.17	48.02	-56.38
	(11.31)	(10.00)	(4.36)	(5.88)

Note: The table reports the results of these modified Besley & Case regression exercises.

Source: Authors' calculations.

This insight then suggests a modified version of the Besley & Case regression. Again, we restrict the sample to successful two-term governors. We split the sample not only based on party affiliation, but also based on an indicator of ideology, which uses the second period tax or expenditure policies to classify governors as liberal or conservative. Table 3 reports the results of these modified Besley & Case regression exercises. We find that these results are very supportive of our modeling strategy. Conservatives adopt higher tax and spending policies in the first period while liberals do exactly the opposite regardless of their party affiliation. Our findings thus suggests that the ideology of the candidate may be more important than party membership in explaining outcomes. We, therefore, conclude that this evidence provides strong support for one of the key predictions of our model.

B. *Evidence Supporting Extremism of One-Term Governors*

In this section we provide additional evidence that supports the prediction of our model that there exists a class of extremists that do not engage in policy moderation during the first term. Table 4 compares one-term governors (extremists) with two-term governors (non-extremists). First, consider the policies that are largely a function of ideology: expenditures and taxes. Not surprisingly, we do not find large difference in the mean policies since extremists from both sides of the political spectrum tend to cancel each other out. More relevant is the fact that the standard deviation of tax and expenditure policies is larger for one-term governors than the standard deviation of first term policies of two-term governors. This indicates that one term governors tend to favor more extreme policies just as our model predicts.

Table 4. Comparison Between One- and Two-Term Governors

			Means		
	expenditure	tax	income growth	borrowing cost	workers comp
Two-term governor	5.774	5.450	.00098	.00275	-2.078
One-term governor	-2.296	-1.835	-.00309	.00071	3.921
Dif in Means Test	0.412	0.192	0.052	0.632	0.003
			Std deviations		
	expenditure	tax	income growth	borrowing cost	workers comp
Two-term governor	109.86	60.78	.0223	.0586	13.87
One-term governor	110.70	65.66	.0254	.0107	32.40
Dif in Variance Test:	0.45	0.11	0.02	1.00	0.00

Note: The table shows that one-term governors have significantly lower GDP growth rates and higher worker's compensation than two-term governors. We report p-values for the difference in means and variances test.

Source: Authors' calculations.

Next consider the policies that are primarily a function of ability: income growth, workers compensation, and borrowing costs. Table 4 shows that one-term governors have significantly lower GDP growth rates and higher worker's compensation than two-term governors. There measures are highly correlated with ability according to our estimates. We, therefore, find some strong evidence that failure of reelection is not just due to ideological extremism, but also due to lack of ability or valence.

We thus conclude that there is strong evidence that one-term governors are more extreme on the ideological scale and less competent than two-term governors. These findings are broadly consistent with the predictions of our model.

C. *One-Term Governors by Type*

In this section we consider the subsample that consists of one term governors that unsuccessfully ran for reelection and those who retired without seeking reelection. Table 5 summarizes the differences in mean policies among these one-term governors.

Table 5. Mean Policies of One-Term Governors by Type

	expenditure	tax	income growth	borrowing cost	workers comp
lost	5.14	-3.64	-.0034	.00004	2.78
retire	-19.18	2.27	-.0024	.00222	6.52
Dif in Means Test	0.16	0.57	0.81	0.19	0.46

Note: The table summarizes the differences in mean policies among these one-term governors.

We report p-values for the difference in means tests.

Source: Authors' calculations.

Overall, we find that the differences between the two subsamples are small. Expenditure is lower and borrowing costs is higher for governors that retired and did not seek reelection. If anything, governors that retired instead of running for reelection appear to be of lower ability and potentially more extreme than those who unsuccessfully ran for the reelection.

The sample of one-term governors that did not seek reelection is too small to determine whether these governors strategically decided to not seek reelection. A case-by-case analysis suggests that one-term governors do not run for reelection for variety of reasons (such as scandals, campaigns for senate seats, health problems). We did not uncover any systematic patterns that would indicate strategic retirement.

VI. Some Additional Comments on Identification of the Extended Model

In the baseline model, we have two sets of moments that primarily depend on the benefits of holding office, ψ. These are the unconditional probability of winning reelection and the ratio of the variances of expenditures and taxes. The baseline model is, therefore, over-identified. We have only one parameter to explain two very different sets of moments.

In the extended model, we add one more parameter, κ, to the model. The key insight is that the two parameters, κ and ψ, affect the predicted moments mentioned above in a different non-linear way. More specifically, the benefit of holding office, ψ, affects the ideological thresholds and hence the willingness of moderates to compromise. The extended versions of equations (4) and (5) are still given by

$$-|\bar{\rho}_R(a) - \bar{s}_R(a)| + \gamma\,(\psi\ + \lambda\, a) = 0 \tag{22}$$

$$-|\underline{\rho}_R(a) - \underline{s}_R(a)| + \gamma\,(\psi\ + \lambda\, a) = 0 \tag{23}$$

Note that both equations include a linear term in ψ.* As a consequence ψ directly affects the ideological thresholds. In contrast, κ directly affects the election standards. The extended version of the equations (10) and (11) are given by:

$$-E\left(|\rho|\ \middle|\rho \in [\underline{\rho}_R(a), \underline{s}_R(a)]\right) + \lambda a + \kappa + \beta V^o(0) = V^D(0) \tag{24}$$

$$-E\left(|\rho|\ \middle|\rho \in [\bar{s}_R(a), \bar{\rho}_R(a)]\right) + \lambda a + \kappa + \beta V^o(0) = V^D(0) \tag{25}$$

Note that both equations have linear term in κ. As a consequence κ strongly affects the election standards. The extended model thus gives us more flexibility to disentangle the effects of model parameters on election standards and ideological thresholds. This allows us to disentangle the benefits of holding office, ψ, from the tenure effect, κ.

Stacking again all moment conditions, gives us a system of non-linear equations in the structural parameters. These parameters are, therefore, identified if this system has a unique solution. Our model is too complicated to analytically verify this condition. For any finite sample, uniqueness of the solution can be numerically verified during estimation.

Note that this argument is consistent with our empirical findings. We find that the parameters are significantly different from zero. More importantly, the goodness of fit analysis reveals that the extended model improves primarily the fit of the two sets of moments discussed above.

VII. The Expected Discounted Costs

Let us assume for simplicity that, $\kappa_D = \kappa_R = \kappa$. Let C^D and C^R be expected discounted values of negative tenure effect from electing a Democratic candidate and Republican candidate respectively. With two term limit, C^D can be expressed as following:

$$\begin{aligned} C^D &= \int_A \int_{-\infty}^{\underline{\rho}_D(a)} \beta\, C^R\, dF_D^{\rho}(\rho)\, dF_D^{a}(a) \\ &+ \int_A \int_{\underline{\rho}_D(a)}^{\rho_D(a)} -\beta\kappa + \beta^2 \left(p_D C^D + (1-p_D) C^R\right)\ dF_D^{\rho}(\rho)\, dF_D^{a}(a) \\ &+ \int_A \int_{\bar{\rho}_D(a)}^{\infty} \beta\, C^R\, dF_D^{\rho}(\rho)\, dF_D^{a}(a) \\ &= Pr^D \left(\frac{-\beta\ \kappa}{1-\beta} + \beta^2 (p_D C^D + (1-p_D) C^R)\right) + (1 - Pr^D)\ \beta C^R \end{aligned} \tag{26}$$

*In addition, the value functions in equations are functions of λ, κ, and ψ. As a consequence the model is nonlinear in all three parameters.

where Pr^D is the probability of winning a reelection for Democratic party and p_D is the probability of winning an open election for Democratic party. C^R is similarly defined. We can solve the two equation and two unknown problem analytically.

Without term limits, C^D can be expressed as following:

$$
\begin{aligned}
C^D &= \int_A \int_{-\infty}^{\underline{\rho}_D(a)} \beta\, C^R\, dF_D^{\rho}(\rho)\, dF_D^{a}(a) \\
&+ \int_A \int_{\underline{\rho}_D(a)}^{\bar{\rho}_D(a)} \frac{-\beta\ \kappa}{1-\beta}\, dF_D^{\rho}(\rho)\, dF_D^{a}(a) \\
&+ \int_A \int_{\bar{\rho}_D(a)}^{\infty} \beta\, C^R\, dF_D^{\rho}(\rho)\, dF_D^{a}(a) \\
&= Pr^D\, \frac{-\beta\ \kappa}{1-\beta} + (1 - Pr^D)\, \beta C^R
\end{aligned}
\tag{27}
$$

where C^R is similarly defined. We can solve the two equation and two unknown problem analytically.

When $Pr^D = Pr^R = 1$, one can show that $C^D = C^R = \frac{-\beta\kappa}{1-\beta^2}$ with two term limit. Thus, the voters bear the cost of tenure effect every two periods. On the other hand, without term limit, $C^D = C^R = \frac{-\beta\kappa}{1-\beta}$ if $Pr^D = Pr^R = 1$. This implies that governors incur the cost of tenure effect every period except the first term.