

## APPENDIX

### *Aggregate Demand*

A central equation of the aggregate demand block is given by an aggregate goods market clearing condition, i.e. a condition equating output to the sum of the components of aggregate demand. In log-linear form we can write it as:

$$y_t = \gamma_c c_t + \gamma_i i_t + d_t \tag{AD1}$$

where  $y$  is output,  $c$  is consumption,  $i$  is investment, and  $d_t$  captures the combined effect of other demand components (including government purchases and external demand). For simplicity we take those components as exogenous. The four variables are expressed in log deviations from a steady state.  $\gamma_c$  and  $\gamma_i$  represent the steady state shares of consumption and investment in output, respectively.

The evolution of aggregate consumption is described by the difference equation

$$c_t = \sigma (r_t - E_t \pi_{t+1}) + E_t c_{t+1} + \varepsilon_{c,t} \tag{AD2}$$

where  $r_t$  is the nominal interest rate,  $\pi_{t+1}$  denotes the rate of price inflation between  $t$  and  $t + 1$  and  $\varepsilon_{c,t}$  is an exogenous preference shock.

Let  $q$  denote the logarithm of the cost, expressed in terms of current consumption, of increasing next period's stock of productive capital by one unit (or, equivalently, the shadow value of installed capital at the end of the period, also known as Tobin's Q). In the presence of convex adjustment

costs, aggregate investment, expressed as a ratio to the capital stock  $k_t$ , is a function of (log) Tobin's  $q$  and an exogenous disturbance  $\varepsilon_{i,t}$ .

$$i_t - k_t = \eta q_t + \varepsilon_{i,t} \quad (\text{AD3})$$

Tobin's  $q$  is in turn determined by the difference equation

$$q_t = (1 - \beta(1 - \delta)) E_t(y_{t+1} - k_{t+1} - \mu_{p,t+1}) - (r_t - E_t\pi_{t+1}) + \beta E_t q_{t+1} \quad (\text{AD4})$$

where  $\mu_p$  denotes the (log) price markup. Thus, we see that  $q$  depends positively on the expected returns to investment, which in equilibrium is given by the expected marginal product of capital adjusted by the markup (or, equivalently, the equilibrium rental cost)  $E_t(y_{t+1} - k_{t+1} - \mu_{p,t+1})$ , and negatively on its opportunity cost, given by the expected real interest rate,  $r_t - E_t\pi_{t+1}$ .

### *Aggregate Supply*

The aggregate supply block of the prototype new monetary model has several elements. Next we describe the log-linear relationships that emerge a first-order approximation to the true equilibrium conditions in a neighborhood of the steady state.

First, a production function of the form

$$y_t = a_t + \alpha k_t + (1 - \alpha) n_t \quad (\text{AS1})$$

where  $k$  and  $n$  respectively denote (log) capital and (log) hours, and  $a$  represents the (log) of total factor productivity

It is assumed that each period a fraction  $\theta$  of firms cannot adjust their price. Thus, the evolution of the aggregate price level  $p$  is described by

$$p_t = \theta p_{t-1} + (1 - \theta) p_t^* \quad (\text{AS2})$$

where  $p^*$  is the price set by firms adjusting their price in the current period.

Value maximization by firms resetting their price implies that the latter is given by

$$p_t^* = (1 - \beta\theta) (w_t - (y_t - n_t)) + \beta\theta E_t p_{t+1}^* \quad (\text{AS3})$$

where  $w_t$  denotes the (log) nominal wage, and  $(w_t - (y_t - n_t))$  is the (log) nominal marginal cost. Notice that, when solved forward, equation (3) implies that firms choose a price to be equal to a discounted sum of current and expected future nominal marginal costs.

Finally, we define the price and wage markups as follows:

$$\mu_{p,t} = (y_t - n_t) - (w_t - p_t) \quad (\text{AS4})$$

$$\begin{aligned} \mu_{w,t} &= (w_t - p_t) - mrs_t & (\text{AS5}) \\ &= (w_t - p_t) - (\varphi n_t + \sigma c_t) \end{aligned}$$

Second, a capital accumulation equation

$$k_{t+1} = (1 - \delta) k_t + \delta i_t \quad (\text{AS6})$$

## 4.1 Equilibrium

Let  $\tilde{x}_t = x_t - x_t^n$  and assume  $k_t = 0$ .

Aggregate Demand

$$\tilde{c}_t = -\sigma (r_t - E_t \pi_{t+1} - rr_t^n) + E_t \tilde{c}_{t+1}$$

$$\tilde{i}_t = \eta \tilde{q}_t$$

implying

$$\tilde{y}_t = -\gamma_c \tilde{r}r_t^l + \gamma_i \eta \tilde{q}_t$$

where

$$\tilde{r}r_t^l = E_t \sum_{i=0}^{\infty} (r_{t+i} - \pi_{t+1+i} - rr_{t+i}^n)$$

$$\tilde{q}_t = (1 - \beta(1 - \delta)) E_t (\tilde{y}_{t+1} - \mu_{p,t+1}) - (r_t - E_t \pi_{t+1} - rr_t^n) + \beta E_t \tilde{q}_{t+1}$$

Aggregate Supply:

$$\pi_t = \beta E_t \pi_{t+1} - \lambda \mu_{p,t}$$

where

$$\begin{aligned} \mu_{p,t} &= -\left(\frac{\alpha + \varphi}{1 - \alpha} + \sigma\right) \tilde{y}_t - \sigma(\tilde{c}_t - \tilde{y}_t) - \mu_{w,t} \\ &= -\left(\frac{\alpha + \varphi}{1 - \alpha} + \frac{\sigma}{\gamma_c}\right) \tilde{y}_t + \frac{\gamma_i}{\gamma_c} \sigma \eta \tilde{q}_t - \mu_{w,t} \end{aligned}$$

Monetary Policy

$$r_t = rr_t^n + \phi_\pi \pi_t + \phi_y \tilde{y}_t$$

$$\mu_{p,t} = -\left(\frac{\alpha + \varphi}{1 - \alpha} + \frac{\sigma}{\gamma_c}\right) \tilde{y}_t + \frac{\gamma_i}{\gamma_c} \sigma \eta \tilde{q}_t - \mu_{w,t} \quad (7)$$