ONLINE APPENDIX for: Bailouts and the Preservation of Competition

The Case of the Federal Timber Contract Payment Modification Act

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A Multiple Equilibria and Equilibrium Selection

Even when we restrict attention to type-symmetric equilibria, a game with more than one bidder type may have multiple equilibria where different types of firm have different thresholds. For example, in our empirical setting, some parameters would support both equilibria where the mills have a lower entry threshold ($S'^{*}_{\text{mill}} < S'^{*}_{\text{logger}}$), and equilibria where loggers have a lower threshold ($S'^{*}_{\text{mill}} > S'^{*}_{\text{logger}}$).

This is illustrated in the first panel of Figure 1, which shows the reaction functions for the entry thresholds of both types of firm, when there are two firms of each type, $\sigma_V = 0.05, K = 4, \alpha = 0.1$ ($\sigma_\varepsilon = 0.0167$) and $\mu_1 = \mu_2 = 5$, so that the types are actually identical.[1] The reserve price R is set to 20. There are three equilibria (intersections of the reaction functions), one of which has the types using identical entry thresholds (45° line is dotted), and the others involving one of the types having the lower threshold (and so being more likely to enter). The fact that there are at most three equilibria follows from the inverse-S shapes of the reaction functions.

The second panel in Figure 1 shows the reaction functions when we set $\mu_1 = 5.025$ and $\mu_2 = 5$, holding the remaining parameters fixed. This change causes the reaction function of type 1 firms to shift down (for a given S'_2 they wish to enter for a lower signal) and the reaction function of the type 2 firms to shift outwards (for a given S'_1, type 2 firms are less

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[1] In this diagram the reaction function represents what would be the symmetric equilibrium best response between the two firms of a particular type when both firms of the other type use a particular S'.

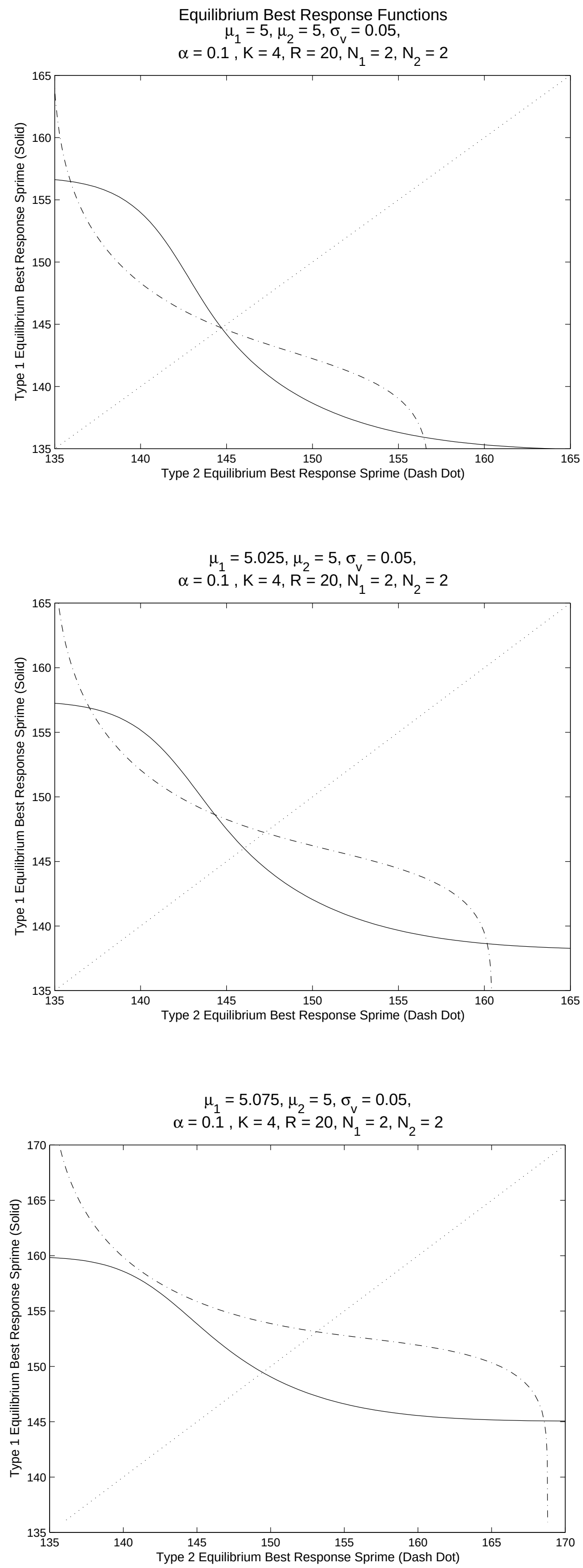

Figure 1: Reaction functions for symmetric and asymmetric bidders. In the top panel, the types are identical so that $\mu_1 = \mu_2 = 5$ and there are two firms of each type, $\sigma_V = 0.05, K = 4, \alpha = 0.1$ ($\sigma_\varepsilon = 0.0167$). In the next two panels firms are asymmetric in means only and the solid (dash dot) lines correspond to the type with the higher (lower) mean. In the middle (bottom) panel $\mu_1 = 5.025$ and $\mu_2 = 5$ ($\mu_1 = 5.075$ and $\mu_2 = 5$) and the remaining parameters are held fixed. The $45°$ line is dotted.

willing to enter). There are still three equilibria, but because of these changes in the reaction functions, there is only one equilibrium where the stronger type 1 firms have the lower entry threshold so that they are certainly more likely to enter. When the difference between μ_1 and μ_2 is increased, there is only one equilibrium and it has this form, as illustrated in the third panel of Figure 1.

The result that with two types of bidders there is a unique equilibrium with $S_1'^* < S_2'^*$ when $\mu_1 \geq \mu_2$ and σ_V, σ_ε and K are the same across types holds generally if the reaction functions have only one inflection point.[2] Under these assumptions it is also generally true that the game has a unique equilibrium, in which it will be the case that $S_1'^* < S_2'^*$, if $\mu_1 - \mu_2$ is large enough.

The empirical literature on estimating discrete choice games provides several approaches for estimating games with multiple equilibria including assuming that a particular equilibrium is played, estimating a statistical equilibrium selection rule that allows for different equilibria to be played in the data (Sweeting (2009) and Bajari, Hong and Ryan (2010)) and partial identification tecnhiques that may only give bounds on the parameters (e.g. Ciliberto and Tamer (2009) and Beresteanu, Molchanov and Molinari (2009)). In this paper we assume that the parameters σ_V, σ_ε and K are the same across types and that, if there are multiple equilibria, the equilibrium played will be the unique one where $S_1'^* < S_2'^*$. We view our focus on this type of equilibrium as very reasonable, given that it is clear in our data that mills (our type 1) tend to have significantly higher average values than loggers (our type 2), so that it is almost certain that only one equilibrium will exist (a presumption that we verify based on our parameter estimates).

B Monte Carlos

This Appendix describes a set of Monte Carlo exercises where we investigate the performance of our Simulated Maximum Likelihood (SML) estimator, which uses Importance Sampling, to approximate the likelihood of the observed outcome for a particular auction (Ackerberg (2009)). This evidence is important because SML estimators may perform poorly when the number of simulation draws is too small. We also study the performance of our estimator under alternative definitions of the likelihood, which make different assumptions about the data available to the researcher.

[2] In general, the exact shape of the reaction functions depends on the distributional assumptions made for the distributions of values and signal noise. Under our distributional assumptions, we have verified that the reaction functions have no more than one inflection point based on more than 40,000 auctions involving different draws of the parameters and different numbers of firms of each type.

Simulated Data

To generate data for the Monte Carlos, we allow the number of {mill, logger} potential entrants to take on values {3,3}, {5,5}, {8,8}, {6,2} and {2,6} with equal probability. For each auction a, there is one observed auction covariate x_a, which is drawn from a Uniform [0,1] distribution, and the vector X_a is equal to $[1\ x_a]$. We assume

$$\text{Location Parameter of Logger Value Distribution: } \mu_{a,\text{logger}} \sim N(X_a\beta_1, \omega^2_{\mu,\text{logger}})$$
$$\text{Difference in Mill/Logger Location Parameters: } \mu_{a,\text{mill}} - \mu_{a,\text{logger}} \sim TRN(X_a\beta_3, \omega^2_{\mu,\text{diff}}, 0, \infty)$$
$$\text{Scale Parameter of Mill and Logger Value Distributions: } \sigma_{Va} \sim TRN(X_a\beta_2, \omega^2_{\sigma_V}, 0.01, \infty)$$
$$\alpha\text{: } \alpha_a \sim TRN(X_a\beta_4, \omega^2_{\alpha}, 0, 1)$$
$$\text{Entry Costs: } K_a \sim TRN(X_a\beta_5, \omega^2_{K}, 0, \infty)$$

where $TRN(\mu, \sigma^2, a, b)$ is a truncated normal distribution with parameters μ and σ^2, and upper and lower truncation points a and b. The true values of the parameters are $\beta_1 = [2.8; 1.5]$, $\beta_2 = [0.3; 0.2]$, $\beta_3 = [0.5; -0.1]$, $\beta_4 = [0.5; 0]$, $\beta_5 = [4; 4]$, $\omega_{\mu,\text{logger}} = 0.2$, $\omega_{\sigma_V} = 0.3$, $\omega_{\mu,\text{diff}} = 0.2$, $\omega_\alpha = 0.2$ and $\omega_K = 2$. The reserve price can take on values of 10, 30 or 50. We allow for R to be correlated with x, as one would expect if the seller sets a higher reserve price when he believes the tract has higher value. Specifically, for each auction, we take a draw u_a from a uniform $[0, 1]$ distribution and set

$$R_a = 10 \text{ if } \frac{x_a + u_a}{2} < 0.33$$
$$R_a = 30 \text{ if } 0.33 \leq \frac{x_a + u_a}{2} \leq 0.66$$
$$R_a = 50 \text{ otherwise.}$$

For each auction we find the unique equilibrium that satisfies the constraint that $S'^{*}_{\text{mill}} < S'^{*}_{\text{logger}}$, and generate data using the equilibrium strategies assuming that the auction operates as a second price sealed-bid auction, or, equivalently, an English button auction. The exercises described below all use the same 100 data sets of 1,000 auctions each.

Having constructed the data we estimate the parameters in three different Monte Carlo exercises, which differ in the importance sampling density used to draw the simulated parameters.

B.1 Monte Carlo Exercise 1: Importance Sampling Density is the True Distribution of the Parameters

In the first exercise we make the (generally infeasible) assumption that the researcher knows the true distribution of each of the parameters, which depends on the value of x_a for a particular auction. The number of simulation draws per auction is set equal to 250, and different draws are used for each auction. We compute the results for four different definitions of the likelihood (the same simulation draws are used in each case) that make different assumptions about the information available to the researcher, which will vary with the exact format of the auction (open-outcry vs. sealed-bid) and with the information that the seller collects about entry decisions. The alternative assumptions are:

1. the researcher observes the values (as bids) and identities of all firms that pay the entry cost and have values above the reserve, and he observes the entry decision of each potential entrant;

2. the researcher observes the values (as bids) and identities of all firms that pay the entry cost and have values above the reserve, and he knows that these firms entered, but for other firms he does not know whether they paid the entry cost and found that their values were less than R, or they did not pay the entry cost;

3. the researcher observes the value and identity of the firm with the second highest value as the final price, the identity of the winning bidder (e.g. whether it is a mill or logger), the identity of all entering firms with values above the reserve price and he observes the entry decision of each potential entrant;

4. the researcher observes the value and identity of the firm with the second highest value as the final price, the identity of the winning bidder (e.g. whether it is a mill or logger), the identity of all entering firms with values above the reserve price, but for other firms he does not know whether they paid the entry cost and found that their values were less than R, or they did not pay the entry cost. This informational assumption forms the basis of the likelihood function shown in Equation (7).

Table 1 shows the mean value of each parameter and its standard deviation across the simulated datasets for each definition of the likelihood. With the true distribution as the importance sampling density and $S = 250$, all of the parameters are recovered accurately, including the standard deviation parameters. Several of the parameters appear to be recovered less precisely when less information is available to the researcher (likelihood definition 4), but the differences are never large.

			Likelihood Definition			
Parameter	Variable	True Value	1	2	3	4
Logger	Constant	2.8	2.7793	2.7830	2.7918	2.7873
Location Parameter			(0.1094)	(0.1224)	(0.0893)	(0.0776)
	x_a	1.5	1.4954	1.4962	1.4945	1.4950
			(0.0999)	(0.1070)	(0.1211)	(0.1247)
	Std. Dev.	0.2	0.1904	0.1930	0.1849	0.1848
			(0.0182)	(0.0204)	(0.0320)	(0.0312)
Difference in Mill and Logger	Constant	0.3	0.3128	0.3025	0.3195	0.3139
Location Parameters			(0.0633)	(0.1477)	(0.1037)	(0.0551)
	x_a	0.2	0.1815	0.1897	0.1981	0.1925
			(0.0909)	(0.1045)	(0.1042)	(0.0942)
	Std. Dev.	0.2	0.1873	0.1888	0.1872	0.1820
			(0.0190)	(0.0278)	(0.0286)	(0.0277)
Value Distribution	Constant	0.5	0.5153	0.5190	0.5205	0.5307
Scale Parameter			(0.1311)	(0.1221)	(0.1003)	(0.0534)
	x_a	-0.1	-0.1007	-0.1022	-0.0895	-0.0795
			(0.1099)	(0.1065)	(0.0892)	(0.0759)
	Std. Dev.	0.3	0.2797	0.2749	0.2804	0.2771
			(0.0267)	(0.0268)	(0.0321)	(0.0280)
α (Degree of selection)	Constant	0.5	0.4979	0.4561	0.4809	0.5100
			(0.1358)	(0.2151)	(0.1716)	(0.0815)
	x_a	0.0	-0.0145	-0.0403	-0.0167	0.0068
			(0.1266)	(0.2003)	(0.1649)	(0.1337)
	Std. Dev.	0.2	0.1886	0.1853	0.1869	0.1891
			(0.0199)	(0.0300)	(0.0299)	(0.0407)
Entry Cost	Constant	4.0	4.0269	4.0115	4.0604	4.0743
			(0.5182)	(0.5386)	(0.7512)	(0.8108)
	K	4.0	4.2420	4.2858	4.4381	4.5171
			(0.8683)	(0.8868)	(1.3409)	(1.4110)
	Std. Dev.	2.0	1.9076	1.9194	1.9117	1.8934
			(0.2855)	(0.3024)	(0.3733)	(0.3971)

Table 1: True Importance Sampling Density Monte Carlo. The table shows the mean and standard deviation (in parentheses) for each of the parameters estimates across the 100 repetitions based on the four different definitions of the likelihood when we use the true joint distribution of the parameters as the importance sampling density, with $S = 250$ draws. See paper for descriptions of the different likelihood definitions.

B.2 Monte Carlo Exercise 2: Importance Sampling Density is a Uniform Distribution

When the true distributions are unknown, it is necessary to choose importance sampling densities that provide good coverage of the possible parameter space. In this exercise we draw parameters from independent uniform distributions where $\mu_{a,\text{logger}} \sim U[2,6]$, $\sigma_{Va} \sim U[0.01, 2.01]$, $\mu_{a,\text{mill}} - \mu_{a,\text{logger}} \sim U[0, 1.5]$, $\alpha_a \sim U[0,1]$, $K_a \sim U[0, 20]$. In this case we set the number of simulation draws per auction equal to 1,000 to try to compensate for the fact that a relatively small proportion of the simulated draws are likely to be close to the parameters that really generate the data (in our empirical work we use 2,500 simulated draws per auction so that we get even better coverage). We use the four alternative definitions of the likelihood that we used for the first exercise.

Table 2 shows the mean value of each parameter and its standard deviation across the simulated datasets for each definition of the likelihood. The parameters which determine the means of each distribution are recovered accurately, but four out of the five standard deviation parameters are biased upwards. As in the first exercise, the alternative likelihood definitions appear to have only small effects on the precision of the estimates.

B.3 Monte Carlo Exercise 3: Two Step Estimation

As some of the parameter estimates appear to be biased using a uniform importance sampling density, the estimator we use in the paper uses the estimates based on a uniform importance sampling density to form new importance sampling densities that are used in a repetition of the estimation procedure. As long as the first step estimates are not too biased, this two step procedure should give accurate results, provided that the number of simulation draws is large enough.

To confirm that this is the case, we apply this two step procedure using likelihood definition 4 estimates from exercise 2 for each of the 100 datasets to form an importance sampling density from which we take $S = 250$ simulation draws for each auction (when we apply our estimator to the real data we use $S = 500$). We focus on likelihood definition 4 as it is the basis of our preferred estimates in the paper.

Table 3 shows the mean and standard deviation of the estimates for each of the parameters. We see that both the mean and standard deviation parameters are recovered accurately, although the estimated standard deviation of entry costs is recovered slightly less accurately than when we used the infeasible estimator in exercise 1. Overall, we regard these Monte Carlo results as providing strong support for our estimation procedure, especially as we use more than twice as many simulation draws when we apply our estimator to the actual data.

Parameter	Variable	True Value	Likelihood Definition 1	2	3	4
Logger	Constant	2.8	2.7051	2.7102	2.6895	2.7031
Location Parameter			(0.0969)	(0.0998)	(0.1324)	(0.1333)
	x_a	1.5	1.3921	1.3807	1.3331	1.2946
			(0.1766)	(0.1810)	(0.2090)	(0.2245)
	Std. Dev.	0.2	0.2536	0.2478	0.2379	0.2312
			(0.0163)	(0.0178)	(0.0223)	(0.0238)
Difference in Mill and Logger	Constant	0.3	0.3286	0.3255	0.3532	0.3407
Location Parameters			(0.0806)	(0.0848)	(0.0970)	(0.1007)
	x_a	0.2	0.3073	0.3148	0.3536	0.3819
			(0.1465)	(0.1505)	(0.1595)	(0.1713)
	Std. Dev.	0.2	0.2487	0.2913	0.2452	0.2418
			(0.0171)	(0.0232)	(0.0204)	(0.0194)
Value Distribution	Constant	0.5	0.5727	0.5694	0.5969	0.5921
Scale Parameter			(0.0812)	(0.0423)	(0.0719)	(0.0753)
	x_a	-0.1	-0.0729	-0.0607	-0.0476	-0.0176
			(0.0762)	(0.0760)	(0.1227)	(0.1289)
	Std. Dev.	0.3	0.2895	0.2913	0.3163	0.3174
			(0.0201)	(0.0232)	(0.0326)	(0.0351)
α (Degree of selection)	Constant	0.5	0.4678.	0.4811	0.4671	0.5034
			(0.0842)	(0.1112)	(0.1052)	(0.1380)
	x_a	0.0	-0.1070	-0.1164	-0.1394	-0.1849
			(0.1526)	(0.1878)	(0.1595)	(0.2112)
	Std. Dev.	0.2	0.2590	0.3088	0.2537	0.3077
			(0.0234)	(0.0385)	(0.0311)	(0.0600)
Entry Cost	Constant	4.0	4.3744	4.0931	4.7719	4.6044
			(0.7186)	(0.7834)	(1.0344)	(1.0318)
	K	4.0	3.5605	3.8494	3.0215	3.1088
			(1.5964)	(1.5916)	(1.9387)	(1.9433)
	Std. Dev.	2.0	3.5409	3.6859	3.3704	3.5099
			(0.3358)	(0.3446)	(0.3623)	(0.4164)

Table 2: Uniform Importance Sampling Density Monte Carlo. The table shows the mean and standard deviation (in parentheses) for each of the parameters estimates across the 100 repetitions based on the four different definitions of the likelihood when we use a uniform importance sampling density, with $S = 1,000$ draws. See paper for descriptions of the different likelihood definitions.

Parameter	Variable	True Value	Definition 4
Logger	Constant	2.8	2.7313
Location Parameter			(0.1389)
	x_a	1.5	1.3720
			(0.2138)
	Std. Dev.	0.2	0.1722
			(0.0349)
Difference in Mill and Logger	Constant	0.3	0.3308
Location Parameters			(0.0976)
	x_a	0.2	0.3138
			(0.1490)
	Std. Dev.	0.2	0.2039
			(0.0257)
Value Distribution	Constant	0.5	0.5741
Scale Parameter			(0.0639)
	x_a	-0.1	-0.0380
			(0.1078)
	Std. Dev.	0.3	0.2706
			(0.0292)
α (Degree of selection)	Constant	0.5	0.4725
			(0.1321)
	x_a	0.0	-0.0902
			(0.2193)
	Std. Dev.	0.2	0.2064
			(0.0590)
Entry Cost	Constant	4.0	4.2557
			(0.9945)
	K	4.0	3.5161
			(2.0808)
	Std. Dev.	2.0	2.5403
			(0.4681)

Table 3: Two Step Estimator Monte Carlo. The table shows the mean and standard deviation (in parentheses) for each of the parameters estimates across the 100 repetitions based on the fourth of the different definitions of the likelihood when we use the true joint distribution of the parameters as the importance sampling density, with $S = 250$ draws. See paper for the likelihood definition.

C Counterfactual with Strengthened Surviving Mills

In this appendix we present results of our counterfactual simulations when we increase the value distribution of the mills that we assume survive due to what would be an increased concentration of mills, as described at the end of Section VI. We focus on the case when only the mills that bought out at \$10/mbf shut down, which corresponds to the third row in Table 5 of the main body of the paper. Table 4 gives the results (the first row of this table is identical to the third row of Table 5 in the main body of the paper). The last three rows correspond to different assumptions about how the value distribution of surviving mills increases after those that faced insolvency shut down. For each row, we increase $\mu_{a,\text{mill}}$ in each auction, not only those in which there is a potential entrant who we assume exits, so that at the current value of σ_{Va}, the mean of the surviving mills value distribution in an auction a is increased by $X\%$, where $X = 2$, 5 and 10%.

	Non-strategic Reserve		Optimal Reserve	
	Δ Revenues	% Change	Δ Revenues	% Change
Baseline	\$40.83 m	10.46	\$33.19 m	8.50
	(\$2.87 m)	(0.35)	(\$2.60 m)	(0.36)
Surviving Mill Mean Values ↑ by:				
2 percent	\$35.46 m	9.08	\$27.59 m	7.07
	(\$2.57 m)	(0.35)	(\$2.27 m)	(0.34)
5 percent	\$27.22 m	6.97	\$18.86 m	4.83
	(\$2.15 m)	(0.35)	(\$1.78 m)	(0.31)
10 percent	\$13.21 m	3.38	\$4.20 m	1.08
	(\$1.60 m)	(0.38)	(\$1.09 m)	(0.29)

Table 4: The format of the table is identical to Table 5 in the main body of the paper and all cases here correspond to the assumption that only insolvent mills shut down. The first row is identical to the third row in Table 5 in the main body of the paper, and the next three rows assume that the average mill value in each auction increases by 2, 5 and 10 percent, respectively.

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