ONLINE APPENDIX

for

Eliciting time preferences when income and consumption vary: Theory, validation & application to job search

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A Online Appendix: Theory - Main Model Extensions and Illustrations

A.1 Illustration with real effort elicitation

Assume individuals value money and leisure. In the simplest setting their utility function per period can be represented as $E\left[u_i(c_{i,t}) + v_i(h_{i,t})\right]$, where $u_i(\cdot)$ and $y_{i,t}$ are as in the main body, and $v_i(\cdot)$ is an increasing and convex utility-of-leisure function and $h_{i,t}$ is the leisure in the given period. If individuals can choose between r_τ units of time spent on a real effort task in period τ, and r_t units of real effort to spent on a task in period $t > \tau$ (and they might get some additional payment in the first period as a show-up fee to make them willing to take either one of these), their level of indifference is now given by

$$Ev_i(h_{i,\tau} - r_\tau) + \gamma_{i,\tau,t} Ev_i(y_t) = Ev_i(h_{i,\tau}) + \gamma_{i,\tau,t} Ev_i(h_{i,t} - r^*_{i,t}). \tag{1'}$$

In this simple example we obtain the analogous equation to (1), only that we are now working with leisure time rather than income. So we can again derive the analogue to (2)

$$\gamma_{i,\tau,t} = \frac{Ev_i(h_{i,\tau} - r_\tau) - Ev_i(y_{i,\tau})}{E\ v_i(h_{i,t} - r^*_{i,t}) - Ev_i(y_{i,t})} \approx \frac{r_\tau}{r^*_{i,t}} \frac{Ev_i'(h_{i,\tau})}{Ev_i'(h_{i,t})}, \tag{2'}$$

where the third expression uses the Taylor approximation $v_i(h_{i,t} - r) \approx v_i(h_{i,t}) - v_i'(h_{i,t})r$.

Now a similar example as that in the main body goes as follows: Consider two individuals A and B with identical utility-of-leisure functions $v_i(h) = \log(h)$ and identical discount factor, who differ only in their expectations about finding a job and associated changes in leisure time: They are both unemployed in the early period with full leisure time $h_{A,1} = h_{B,1} = 1$. In the late period individual A believes to have found a job that cuts her leisure to $h_{A,2} = 1/4$, while individual B believes that she will stay unemployed with $h_{B,2} = 1$. Assume real effort r_1 is sufficiently small that the approximation in (2') is valid and we get

$$\begin{aligned} & \gamma_{A,1,2} = \frac{r_\tau}{r^*_{A,t}} \frac{Ev_i'(1)}{Ev_i'(1/4)} = \frac{r_\tau}{4r^*_{A,t}} \\ = \quad & \gamma_{B,1,2} = \frac{r_\tau}{r^*_{B,t}} \frac{Ev_i'(1)}{Ev_i'(1)} = \frac{r_\tau}{r^*_{B,t}}, \end{aligned}$$

where the two lines equal because of the assumption of identical discount factor across individuals. So $\frac{r_\tau}{r^*_{A,t}} = 4\frac{r_\tau}{r^*_{B,t}}$, and if one neglects the change in expected leisure time person A appears as if she has a higher discount factor than person B. Person A also finds a job more quickly. If one just took $r_\tau / r^*_{i,t}$ as a measure of patience, the more patient person here finds the job more quickly. But truly they both hold the same discount factor and person A only does effort early because she anticipate a reduced leisure endowment in the second period.

A.2 Illustration of direct method elicitation

The direct method of Attema et al. (2016) asks questions of the kind: Do you want to get 10 Euro in each of the first X weeks, or would you rather get 10 Euro in each week after X until some maximum week. Let the maximum week for simplicity be week ten. Clearly if

$X = 9$ most people would like to be paid early since they would be paid nine periods while they will only be paid in one period if they take the late payments. Similarly, if $X = 1$ most people would presumable take the late payments as it will lead to nine payments and despite discounting this will for most be preferred to a single early payment. Varying X elicits the point where the individual is indifferent between early and late payment. The discount factor can be recovered from versions of such questions when background consumption does not change. People with identical discount factors have to answer the questions identically.

But now consider two people that have identical discount factors and preferences but different job finding expectations. Person A expects to find a job in week 6. Person B does not expect to find a job within the ten weeks. Both get low unemployment benefits during unemployment and consume them. Upon finding a job person A gets a higher wage w.

Consider first individual B: If she does not discount at all, she would choose indifference point X=5, as she would like to have the maximum number of periods payout. If she has mild time preferences, she might choose X=4 because early payments are valued more. Now consider individual A: Assume w is so high that the marginal utility $u'(w)$ is close to zero. So she essentially does not value additional money in periods after period 5. That means she would choose X=2.5 if she does not otherwise discount, or with some mild discounting she would choose X=2. So even with this method, it would look as if A is much more impatient.

A.3 Large Stakes in Multiple Price Lists Method

In this section we explore the role of large stakes under the multiple price lists (MPL) method discussed in Section 3.

For MPL, we discussed that the discount factor is related to rewards via equation 2, and equals the usual empirical counterpart $r_\tau / r^*_{i,t}$ only if background consumption is constant and marginal utility is approximately constant between background consumption with and without rewards. If reward r_τ becomes large, also the reward $r^*_{i,t}$ becomes large. For given distributions of background consumption $y_{i,\tau}$ and $y_{i,t}$ we therefore obtain the approximation when r_τ becomes large that:

$$\gamma_{i,\tau,t} = \frac{Eu_i(y_{i,\tau} + r_\tau) - Eu_i(y_{i,\tau})}{Eu_i(y_{i,t} + r^*_{i,t}) - Eu_i(y_{i,t})} \approx \frac{u_i(r_\tau)}{u_i(r^*_{i,t})}. \tag{1}$$

This means that $u_i(r_\tau)/u_i(r^*_{i,t})$ approximately identifies the discount factor, independently of background consumption. Since $\gamma_{i,\tau,t} \approx \frac{u_i(r_\tau)}{u_i(r_\tau(r^*_{i,t}/r_\tau))}$, also the probability ratio $r^*_{i,t}/r_\tau$ is approximately tied down independently of background consumption. Just that now the discount factor can no longer directly be equated to this probability ratio, but is identified by the ratio of utilities $u_i(r_\tau)/u_i(r^*_{i,t})$, and thus requires knowledge of the individual's utility function. So moving from small to large stakes changes more then just taking out background consumption: going to large stakes also invalidates the second approximation in equation (2) that rendered measurement at low stakes (in the absence of varying background assumption) independent of details of each individual's utility function.

Alternative methods like the Convex Time Budget method of Andreoni and Sprenger (2012) are designed to allow for curvature and to back out individual utility parameters. They did not deal with changing background consumption if each individual can have his/her own utility function and background consumption path. Allowing for large rewards in their setting is likely to combine the benefits of eliciting the utility function and abstracting from background consumption.

A.4 Setup with Savings after Lottery Win

Here we outline the setting where individuals can save after their lottery win. Recall that individual i's continuation value of consumption from period t onwards, evaluated with the type preferences of the decision-maker at time $t_0 = 0$, is given by utility

$$E \sum_{s=t}^{T} \gamma_{i,t,s}(t_0) u_i(c_{i,t}) \tag{2}$$

where the total life span T could be infinite. (Note that a decision-maker at time t_1 might have a different continuation value because she applies discount factors $\gamma_{i,t,z}(t_1)$.) Individuals are hand-to-mouth unless they win the lottery ($c_{i,t} = y_{i,t}$). But in case of a lottery win they can freely save or borrow at interest rate ι as long as their wealth W (i.e., the net present value of past savings plus current and future income) stays weekly positive. So starting a period with W allows consumption

$$c \in [0, W] \tag{3}$$

and next period wealth

$$W' = (1 + \iota)(W - c). \tag{4}$$

In particular, this means that individuals can use proceeds from lottery wins over many periods. For ease of exposition also assume that the income stream is deterministic, though possibly heterogeneous across individuals and time.

Here we focus only *exponential* discounters or *quasi-hyperbolic* discounters as defined in the setup of the basic model. We assume $\delta_i(1 - \iota) \leq 1$ so exponential discounters dis-save after a lottery win. In the quasi-hyperbolic case, we follow the literature and distinguish between *naive* individuals who believe that they will behave as exponential discounters in the future, and *sophisticated* individuals who understand that in their future "selves" will also have more interest in immediate consumption. Both exponential discounters and naive quasi-hyperbolic individuals believe that any sequence of savings choices that is optimal is temporary today will also be optimal for their future selves, so their savings problem after a lottery win is simply an optimization problem: choose sequence $c_t, c_{t+1}, ...$ to maximize (2) subject to constraints (3) and (4). Sophisticates on the other hand understand that future selves discount the future different from themselves and will take different actions from the ones that the individual would like to commit to today. It plays a game with its future selves, as, e.g., in Laibson (1996).

For finitely-lived individuals the savings problem has a unique solution. In the case of sophisticates it is found by backward induction: under constant relative risk aversion an individual with $T - t$ remaining periods of life consumes a fraction λ_{T-t} of her wealth, and this fraction is increasing in remaining lifetime (Laibson (1996)). Since individuals might benefit from a lottery win for a long time, it will be useful to consider T large, and we use the limit at $T \to \infty$ to capture infinitely-lived individuals. This has no restriction for exponential discounters and naives. For sophisticates this constitutes a particular selection among all possible markov equilibria in infinite settings. It implies that individuals consume fraction λ_∞ of wealth, and this fraction increases in an individual's present-bias all else equal (see equation (9) in Laibson (1996)). There exist other markov equilibria with different constant savings rate in these infinite games, and our results apply as long as they inherit the same comparative statics:

Assumption: Infinitely-lived sophisticated quasi-hyperbolic discounters have Bernoulli *utility function with constant absolute risk aversion $\rho \geq 1$, play an equilibrium markov strategy in the savings game where consumption is a constant fraction λ of wealth, and λ is increasing in impatience (β) all else equal.*

A.5 Proof of Proposition 3

Consider an infinitely-lived person i who wins the lottery reward R at time τ and has no other wealth. Consider her sequence of consumption choices $C_0, C_1, C_2, ..$ going forward, i.e., in periods τ, $\tau+1$, $\tau+2$ etc. Now consider the same individual who wins the lottery reward R at time $t > \tau$ and has no other wealth. If $\tau > 0$, it is obvious that this person will choose exactly the same consumption sequence going forward: She will choose $C_0, C_1, C_2, ..$in periods t, $t{+}1$, $t{+}2$ etc. The reason is that the environment going forward is exactly identical. That also implies that their continuation utilities are identical, so that $U_{i,\tau}(R) = U_{i,t}(R)$. This immediately establishes relationship (12) as a direct consequence of (11). This also holds for exponential discounters when $\tau = 0$ by the same logic.

It does not hold for quasi-hyperbolic discounters at $\tau = 0$. Consider first a sophisticate. At any point in time, this person is aware that her future selves are as present-biased as she is currently. Given our assumption on Markov equilibria (for which the limit of the game of finitely-lived players is a special case) the consumption sequence from time τ onwards rests exactly the same as the consumption sequence from time t onwards. So that step from the previous paragraph remains. But the discount factors that are applied differ when the sum starts at zero compared to a future date:

$$U_{i,0}(R) = u(C_0) + \beta \sum_{s \geq 1} \delta^s u_i(C_s) < u(C_0) + \sum_{s \geq 1} \delta^s u_i(C_s) \leq U_{i,t}(R) \tag{5}$$

where in fact the last inequality holds with equality. By (11) it holds that $\varepsilon_\tau / \varepsilon^*_{i,t} \approx \gamma_{i,t,\tau} U_{i,t}(R)/U_{i,0}(R)$, so the probability ratio overstates the true discount factor as stated in the proposition.

To make the same statement for naive quasi-hyperbolics we will use a related but slightly more sophisticated argument: let $C_0, C_1, C_2, ...$ be the optimal consumption sequence of a naive quasi-hyperbolic after lottery with wealth R at time zero. Then $U_{i,0}(R)$ can be constructed with the same equality as in (5). Also the strict inequality in (5) still holds. But now the weak inequality in fact holds strictly: from time t onwards the person could use the same consumption choices, but in fact he might even find a better sequence of consumption choices moving forward. Note that naives believe that their future selves will carry out their optimal decisions, so this logic applies. Again we conclude that the probability ratio understates the true discount factor as stated in the proposition.

We are left to show that nevertheless the probability ratio ranks individuals correctly. Consider two naive quasi-hyperbolic discounters i and j who only differ in respect to their present-bias parameter $\beta_i > \beta_j$ (i.e., they only differ in $\gamma_{i,0,t} > \gamma_{j,0,t}$ for all t). We have to show that $\beta_i \delta^t U_{i,0}(R)/U_{i,t}(R)$ is larger than $\beta_j \delta^t U_{j,0}(R)/U_{j,t}(R)$, i.e., that the probability ratio as in (11) is higher for person i than for person j. Here we omit the person index on the long-run discount factor because it is identical among them.

Clearly $U_{i,t}(R) = U_{j,t}(R)$ because in the future ($t > 0$) they expect both to discount exponentially with identical long-run discount factor. So we have to show that $U_{i,0}(R)/\beta_i$ is smaller than $U_{j,0}(R)/\beta_j$. Analogous to the previous arguments, consider a sequence of consumption choices $C_0, C_1, C_2, ...$ that is optimal for individual i at $\tau = 0$. Clearly:

$\frac{U_{i,0}(R)}{\beta_i} = \frac{u(C_0)}{\beta_i} + \sum_{s>0} \delta^s u(C_s) < \frac{u(C_0)}{\beta_j} + \sum_{s>0} \delta^s u(C_s) \le \frac{U_{j,0}}{\beta_j}$, where the weak inequality arises because individual j might choose an even better sequence. This establishes the result for naives.

For sophisticates, we cannot use the same argument as they play a game rather than face an optimally chosen sequence, so in particular the last inequality of the previous argument is not obvious. So here we exploit brute-force the closed-form expression $\gamma_{i,0,t} U_{0,\tau}(R)/U_{i,t}(R) = \beta_i \delta^t \left[1 - (1-\beta_i)\delta(1-\iota)^{1-\rho}(1-\lambda_i)^{(1-\rho)}\right]^{-1}$ when individuals save at rate λ_i, (see Laibson (1996), equation (29) for $U_{i,0}$, and for $U_{i,t}$ use the same equation but omit the present-bias). We will simply take comparative statics with respect to β_i directly, and indirectly through the change in λ_i. Clearly the first is positive. For the second, since λ_i is increasing in β_i, we have to show that $(1-\lambda_i)^{1-\rho}$ is increasing in λ_i, or equivalently that $\rho \ge 1$. This completes the proof of Proposition 3.

A.6 Proof of Proposition 3

We want to show the following: Consider infinitely-lived naive quasi-hyperbolic discounters with discount factors $\gamma_{i,\tau,t} = \delta_i^{t-\tau}$ if $\tau > 0$ and $\gamma_{i,0,t} = \beta_i \delta_i^t$ otherwise, who has a time-varying income stream $y_{i,t}$. She can save at person-specific interest rate τ_i^L in any period after winning our lottery and at rate τ_i otherwise. Fix any distance d, and fix a different infinitely lived naive quasi-hyperbolic discounter j. For R sufficiently large, there exists an open ball of winning probability around zero ε_τ such that for $\tau > 0$: $\gamma_{k,\tau,t} - \varepsilon_\tau/\varepsilon^*_{k,t} < d$ for each individual $k \in i, j$. Moreover, if both individuals are otherwise identical except for their short-run discount factors $\gamma_{i,0,t}$ and $\gamma_{j,0,t}$ and their usual interest rates τ_i and τ_j, then the probability ratio ranks correctly also relative to their short-run discount factor (i.e., if $\gamma_{i,0,t} < \gamma_{j,0,t}$ then $\varepsilon_\tau/\varepsilon^*_{i,t} < \varepsilon_\tau/\varepsilon^*_{j,t}$).

To show this, write agent $k \in \{i, j\}$'s problem in period $t > 0$ with net present value W in the absence of our lottery as:

$$\begin{aligned} U_{k,+}(W,\iota) = \quad & \max_c u_k(c) + \delta_k U_{k,+}(W') \\ \text{s.t.} \quad & W' = (1+\iota)(W-c) \\ & W' \ge 0. \end{aligned}$$

Note that it is independent of the exact time period t. In case $t = 0$ it is

$$\begin{aligned} U_{k,0}(W,\iota) = \quad & \max_c u_k(c) + \beta_k \delta_k U_{k,+}(W') \\ \text{s.t.} \quad & W' = (1+\iota)(W-c) \\ & W' \ge 0. \end{aligned}$$

Let $W_k = \sum_s y_{k,s}/(1+\iota_k)^s$ be the net present value of person k's income stream when discounted at rate ι_i. From this initial net present value, denote by $C_{k,0}, C_{k,1}, C_{k,2}, \ldots$ the sequence of consumption choices that maximize this recursive program. Standard arguments for such a simple recursive problem establish that $U_{k,+}(\cdot)$ and $U_{k,0}(\cdot)$ are strictly increasing, concave and differentiable. For ease of exposition write with slight abuse of notation $U_{k,t}(\cdot) := U_{k,+}(\cdot)$ when $t > 0$, even though the only variation in the continuation utility arises relative to time zero.

The ex-ante problem at time zero with a lottery that additionally pays R with probability ϵ at time $t > 0$ is then

$$\begin{aligned}
\max_{c_0, c_1, \ldots, c_{t-1}} \quad & u_k(c_0) + \beta_i \sum_{s=1}^{t} \delta_k^s u_k(c_s) + \beta_i \delta_k^t [(1-\epsilon) U_{k,t}(W', \iota_k) + \epsilon U_{k,t}(W' + R, \iota_k^L)] \\
\text{s.t.} \quad & W' = W_k (1 + \iota_k)^t - \sum_{s=0}^{t} c_s (1 + \iota_k)^{t-s} \\
& W' \geq 0,
\end{aligned}$$

Call the value of this program $\mathcal{U}_k(W_k, R, \epsilon, t)$. Clearly $C_{k,0}, C_{k,1}, ..., C_{k,t-1}$ defined above are maximizers of this program when $\epsilon = 0$, and we can write $W'_{k,t} = W_k(1+\iota_k)^t - \sum_{s=0}^{t} C_{k,s}(1+\iota_k)^{t-s}$ for the net present value from period t onward given this consumption path. If the lottery is already at time time zero we have simply $\mathcal{U}_k(W_k, R, \epsilon, 0) = (1-\epsilon)U_{k,0}(W_k, \iota_k) + \epsilon U_{k,0}(W_k + R, \iota_k^L)$.

The envelope theorem (e.g., Theorem 7, **?**)[1] establishes:

$$\left. \frac{d\mathcal{U}_k(W_k, R, \epsilon, t)}{d\epsilon} \right|_{\epsilon=0} = U_{k,t}(W'_{k,t} + R, \iota_k^L) - U_{k,t}(W'_{k,t}, \iota_k).$$

That is, the (right-)derivative with respect to the winning probability can be computed as if the actual choices of consumption are unchanged. Therefore, we can write as first order Taylor approximation

$$\mathcal{U}_k(W_k, R, \epsilon, t) = \mathcal{U}_k(W_k, R, 0, t) + \epsilon[U_{k,t}(W'_{k,t} + R, \iota_k^L) - U_{k,t}(W'_{k,t}, \iota_k)] + O(\epsilon^2),$$

where $O(\epsilon^2)$ is the Bachmann–Landau notation for a function that vanishes at least at quadratic order when epsilon tends to zero, and $W'_{k,t}$ is the continuation net present value under the original consumption plan as defined above.

Recall that our elicitation method fixes an early winning probability ϵ_τ at time τ and elicits the winning probability $\epsilon^*_{k,t}$ at time t that makes person k indifferent, i.e., such that $U_k(W_k, R, \epsilon_\tau, \tau) = U_k(W_k, R, \epsilon^*_{k,t}, t)$. By the previous argument, for ϵ_τ close to zero this implies that $\epsilon^* k, t$ has to be close to zero, and by the previous approximation this indifference be written as

$$\epsilon_\tau [U_{k,\tau}(W'_{k,\tau} + R, \iota_k^L) - U_{k,\tau}(W'_{k,\tau}, \iota_k)] \approx \epsilon^*_{k,t}[U_{k,t}(W'_{k,t} + R, \iota_k^L) - U_{k,t}(W'_{k,t}, \iota_k)].$$

This means that we can use the indifference condition approximately as if the person had a fixed consumption stream $C_{k,0}, C_{k,1}, ...$ in the absence of a lottery win when the winning probability is small. All remaining arguments proceed along the lines of the proof for Proposition 3. This concludes the proof of Proposition 4.

A.7 Theory for Implementation via Lottery Tickets

Consider the implementation via lottery tickets discussed in Section 4.3. Lottery tickets that pay out a reward R with probability $\hat{\epsilon}$. When we vary R, assume that the expected

[1] This particular envelope theorem is designed to accommodate parameters at the boundary of the domain; in our case: evaluation of the derivative at $\epsilon = 0$.

value stays constant at some low value κ, so that $\kappa = \hat{\epsilon}R$.

Consider at most a finite number N of lottery tickets, and assume that the consumer is indifferent between n_τ lottery tickets at the early time τ, or $n^*_{i,t}$ lottery tickets at a later time t.

Let $\mathcal{P}(m|n,\hat{\epsilon}) = \frac{n!}{m!(n-m)!}\hat{\epsilon}^k(1-\hat{\epsilon})^{n-k}$ denote the Binomial probability of winning m times the reward R for someone who receives n lottery tickets.

The period-τ utility of n_τ lottery tickets is given by

$$\begin{aligned} & (1-\sum_{m=1}^{n_\tau}\mathcal{P}(m|n_\tau,\hat{\epsilon}))Eu_i(y_{i,\tau}) + n_\tau\hat{\epsilon}(1-\hat{\epsilon})^{n_\tau-1}Eu_i(y_{i,\tau}+R) \\ + & \sum_{m=2}^{n_\tau}\mathcal{P}(m|n_\tau,\hat{\epsilon})Eu_i(y_{i,\tau}+mR) \end{aligned}$$

which lies strictly between the lower bound

$$(1-n_\tau\hat{\epsilon}(1-\hat{\epsilon})^{n_\tau-1})Eu_i(y_{i,\tau}) + n_\tau\hat{\epsilon}(1-\hat{\epsilon})^{n_\tau-1}Eu_i(y_{i,\tau}+R)$$

and the upper bound

$$(1-n_\tau\hat{\epsilon}(1-\hat{\epsilon})^{n_\tau-1})Eu_i(y_{i,\tau}) + n_\tau\hat{\epsilon}(1-\hat{\epsilon})^{n_\tau-1}Eu_i(y_{i,\tau}+R) + Eu_i(y_{i,\tau}+n_\tau R)\sum_{m=2}^{n_\tau}\mathcal{P}(m|n_\tau,\hat{\epsilon}),$$

where the upper bound is itself smaller than

$$\begin{aligned} & (1-n_\tau\hat{\epsilon}(1-\hat{\epsilon})^{n_\tau-1})Eu_i(y_{i,\tau}) + n_\tau\hat{\epsilon}(1-\hat{\epsilon})^{n_\tau-1}Eu_i(y_{i,\tau}+R) \\ + & Kn_\tau R\sum_{m=2}^{n_\tau}\mathcal{P}(m|n_\tau,\hat{\epsilon}) \end{aligned}$$

for some $K > u'(R)$.

We will show that the term $Kn_\tau R\sum_{m=2}^{n_\tau}\mathcal{P}(m|n_\tau,\kappa/R)$ vanishes for large R:

$$\begin{aligned} & Kn_\tau R\sum_{m=2}^{n_\tau}\mathcal{P}(m|n_\tau,\hat{\epsilon}) \\ = & Kn_\tau R(1-(1-\hat{\epsilon})^{n_\tau} - n_\tau\hat{\epsilon}(1-\hat{\epsilon})^{n_\tau-1}) \\ = & Kn_\tau\frac{1-(1-\kappa/R)^{n_\tau} - n_\tau\kappa/R(1-\kappa/R)^{n_\tau-1}}{1/R} \end{aligned}$$

where we used the relationsship $\kappa = \hat{\epsilon}R$ to replace $\hat{\epsilon}$. Both numerator and denominator go to zero for large R, so after applying L'Hopital's rule we get equally

$$Kn_\tau\frac{n_\tau(n_\tau-1)\kappa^2/R^3(1-\kappa/R)^{n_\tau-2}}{1/R^2} \to 0.$$

So for large R the upper and lower bounds approximately coincide. Replacing the utility by its lower bound, and doing the same for time τ, indifference along the same lines leading to

(4) now means:

$$\begin{aligned}
& (1-n_\tau\hat\epsilon(1-\hat\epsilon)^{n_\tau-1})Eu_i(y_{i,\tau}) + n_\tau\hat\epsilon(1-\hat\epsilon)^{n_\tau-1}Eu_i(y_{i,\tau}+R) + \gamma_{i,\tau,t}Eu_i(y_{i,t}) \\
\approx\quad & Eu_i(y_{i,\tau}) + \gamma_{i,\tau,t}\left[(1-n^*_{i,t}\hat\epsilon(1-\hat\epsilon)^{n^*_{i,t}-1})Eu_i(y_{i,t}) + n^*_{i,t}\hat\epsilon(1-\hat\epsilon)^{n^*_{i,t}-1}Eu_i(y_{i,t}+R)\right]
\end{aligned}$$

which can be rearranged to

$$\begin{aligned}
\gamma_{i,\tau,t} \quad &\approx\quad \frac{n_\tau(1-\hat\epsilon)^{n_\tau-1}}{n^*_{i,t}(1-\hat\epsilon)^{n^*_{i,t}-1}}\frac{Eu_i(y_{i,\tau}+R)-Eu_i(y_{i,\tau})}{Eu_i(y_{i,t}+R)-Eu_i(y_{i,t})} \\
&\approx\quad \underbrace{n_\tau/n^*_{i,t}}_{probability\ ratio} \qquad \text{for large } R \text{ or } Eu_i(y_{i,t}) \approx Eu_i(y_{i,\tau}), \qquad (6)
\end{aligned}$$

where the last approximation follows exactly the same steps as that for (4) with the addition that $\hat\epsilon \to 0$ and therefore the terms involving it drops out.

A.8 Probability Weighting

Consider an individual who obtains $y_{i,t}$ in period t, unless she wins the lottery in which case she obtains $y_{i,t}+R$. We abstract from uncertainty in $y_{i,t}$ for illustrative purposes. Such uncertainty would need to be taken into account with appropriate probability weights, which does not alter the final result but significantly increases notational complexity. Let w be a mapping from [0,1] onto [0,1] representing the probability weighting function, and we adopt rank-dependence. Indifference between the early and late lottery now requires

$$\begin{aligned}
& [(1-w(\varepsilon_\tau))u_i(y_{i,\tau}) + w(\varepsilon_\tau)u_i(y_{i,\tau}+R)] + \gamma_{i,\tau,t}u_i(y_{i,t}) \\
= \quad & u_i(y_{i,\tau}) + \gamma_{i,\tau,t}[(1-w(\varepsilon^*_{i,t}))u_i(y_{i,t}) + w(\varepsilon^*_{i,t})u_i(y_{i,t}+R)],
\end{aligned}$$

where winning the lottery is always the most attractive outcome and is weighted by the probability weight. For the less attractive outcome rank-dependence means that it is assessed with $1-w(p)$. This reduces to

$$\begin{aligned}
\gamma_{i,\tau,t} \quad &= \quad \frac{w(\varepsilon_\tau)}{w(\varepsilon^*_{i,t})}\frac{u_i(y_{i,\tau}+R)-u_i(y_{i,\tau})}{u_i(y_{i,t}+R)-u_i(y_{i,t})}. \\
&\approx \quad \frac{w(\varepsilon_\tau)}{w(\varepsilon^*_{i,t})} \quad \text{for large } R. \qquad (7)
\end{aligned}$$

The steps to show the approximation are identical to those used in (4). We assume that $\varepsilon_\tau R = K$ for some fixed K, and take limits as R becomes large.

Let $r_i = \varepsilon_\tau/\varepsilon^*_{i,t} \in (0,1)$ be the limit of individual's choices as R becomes large. We can then derive the following approximation around r_i of unity:

$$\begin{aligned}\ln\left(\frac{w(\varepsilon_\tau)}{w(\varepsilon^*_{i,t})}\right)=\ln\left(\frac{w(\varepsilon^*_{i,t}r_i)}{w(\varepsilon^*_{i,t})}\right) &= \ln w(\varepsilon^*_{i,t}r_i)-\ln w(\varepsilon^*_{i,t}) \\ &\approx \ln w(\varepsilon^*_{i,t})+\frac{w'(\varepsilon^*_{i,t})}{w(\varepsilon^*_{i,t})}(\varepsilon^*_{i,t}r_i-\varepsilon^*_{i,t})-\ln w(\varepsilon^*_{i,t}) \\ &\approx \phi(r_i-1)\end{aligned}$$

where the first approximation is simply a Taylor expansion and the second reflects that $w'(\varepsilon^*_{i,t})\varepsilon^*_{i,t}/w(\varepsilon^*_{i,t})$ captures the elasticity of the weighting function around zero, which we denote by ϕ. Therefore (7) can now be written as

$$\begin{aligned}\gamma_{i,\tau,t} &\approx e^{\phi(r_i-1)} \\ &\approx 1+\phi(r_i-1)\end{aligned}$$

where the second approximation reflects that standard approximation of exponential functions around an argument of zero.

B Online Appendix: Calibration exercise

In the paper we provide a simple calibration example with log-utility, income levels based on our job seeker survey and no savings. Here we show that our qualitative findings are robust to various variations of the calibration exercise.

B.1 Baseline calibration: no saving

In Figure B.1 we reproduce our baseline calibration in panels (a) and (b) (the same as Figure 1 in the paper). Panel (a) depicts a job seeker with low (current) income of 668 per month ($\tau = 0$), and future (expected) income of 1440 per month ($t = 2$). The annual true discount factor is 0.96 and the experimental reward (for MPL) is 10. In this scenario the MLL estimate converges towards the true value for large enough headline prizes. Panel (b) shows qualitatively similar results for the case where income levels are taken from the respondents in the validation experiment on Prolific. The income level is substantially higher than that of the job seekers, and the expected income fluctuation is smaller in relative terms. As a result, the bias in the MPL preference estimate is smaller, but still substantial and MLL still produces accurate results for a lottery prize exceeding 100,000. In panel (c) we find that our results remain when using square-root utility instead of log utility. Panel (d) shows results for CRRA utility (a = 2). Since utility is bounded in this case, we indeed find that the MLL estimate only partly converges towards to the true value. Panel (e) reproduces the baseline result from panel (a), but implements increasing experimental rewards for the MPL estimates (see appendix A.3). In line with the theory, even with very large experimental rewards MPL does not recover the true discount factor.

B.2 Saving after lottery win

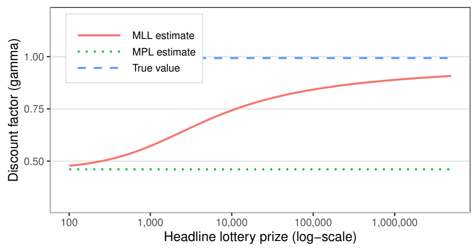
For simplicity, these results consider an individual that cannot save. In particular when considering a large lottery prize, this may be unrealistic. As an extension we therefore consider the case where individuals can save only if they win the headline lottery prize. Saving is allowed for an infinite number of periods. For simplicity we do not include savings in the non-winning scenario, and as a result the benchmark MPL estimate is not affected.

Deriving optimal consumption dynamic requires additional parameters. We set the annual interest rate at 3%. Furthermore we consider a quasi-hyperbolic discounter and show results for the naïve case (with robustness for the sophisticated case). In the baseline calibration we set present-bias β to 0.97 and the long-run discount factor δ to 0.99751 per period (0.97 annually). This reflects a scenario where individuals have constant savings after winning the lottery, because $\delta(1 + \iota) = 1$. All other parameters are equal to those in the case without saving.

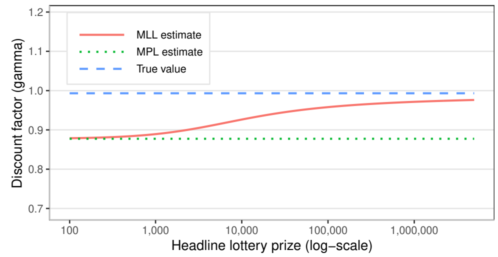
We show results in Figure B.2. The y-axis now depicts the discount factor $\gamma = \beta\delta^{t-\tau}$, with $\tau = 0, t = 2$ in this example. For a naïve decision-maker, the results are qualitatively similar to the no-savings case, as our theory predicts (see section 4.1). The ability to save does not alter the convergence of the MLL estimates towards the true discount factor. For headline prizes exceeding 1,000, MLL outperforms MPL estimates and for prizes above 10,000 MLL estimates are very close to the true value. For very high prizes the MLL estimate exceeds the true value, as proven in Proposition 3, since rewards are consumed not only in the present but also in the future. As a result, MLL converges to a value above $\beta\delta^{t-\tau}$ closer to $\delta^{t-\tau}$. Where in this interval the discount factor converges depends on the

Figure B.1: Calibration of time preference estimates without saving. Job seeker income levels (current income is 668 per month, future (expected) income is 1440 per month). The annual true discount factor is 0.96 and the experimental reward (for MPL) is 10.

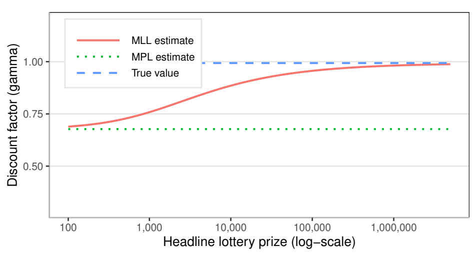
(a) Log utility



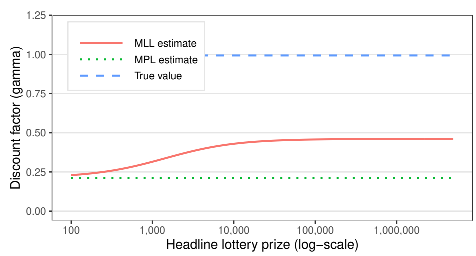
(b) Income levels from Prolific validation experiment: low (current) income = 3567, high (future) income = 4048



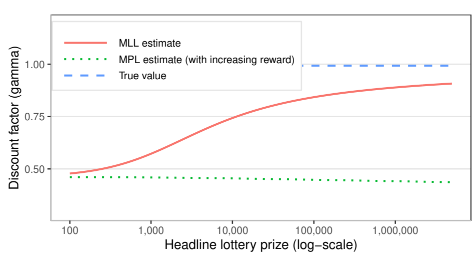
(c) Square-root utility



(d) CRRA (a=2) utility

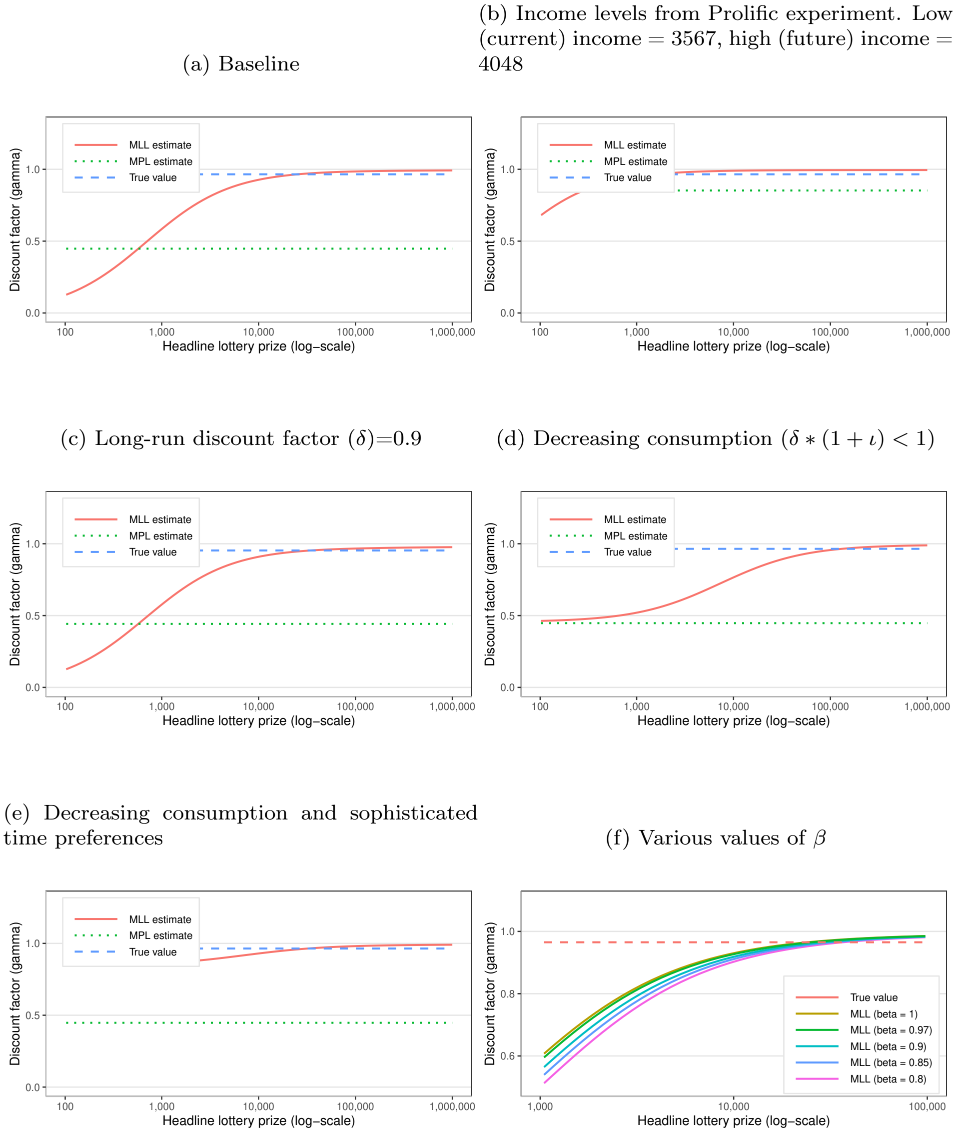


(e) Log-utility with increasing rewards for MPL



number of periods N during which individuals can save (higher N tends to load more into the future and we use the extreme case of infinite N here) and the level of the discount factors (low discount factors imply more immediate consumption and more loadings on the immediate period). In any case, the difference with the MPL estimate remains striking as the MPL estimate is far from the true discount factor. Alternative calibrations such as Prolific income levels (panel b), stronger discounting (panel c) or a dis-saving scenario (panel d) lead to qualitatively similar conclusions. In panel e we show estimates for the case where the individual is sophisticated in the sense that they foresee their time-inconsistent behavior. Again, MLL performs well compared to MPL, especially for larger headline prizes. Finally, we compare MLL estimates for varying levels of present bias (β) in panel f. We find that our method is able to correctly distinguish (rank) the various scenarios, although this ability fades when the headline prize grows too large and savings are loaded into the future so that the measurement overshoots towards the long-run discount factor. This is due to the large δ and the large number of saving periods ($N = \infty$) that underlie this calibration, and would change if individuals only consider savings for a limited number of periods, in analogue of the finite N underlying the equal-split assumption in Proposition 2.

Figure B.2: Calibration of preference estimates with saving after a lottery win. Baseline uses job seeker income levels, log utility, $\beta = 0.97, \delta = 0.99751$ (0.97 annual) and $\iota = 0.0025$ (3% annual)



(a) Baseline

(b) Income levels from Prolific experiment. Low (current) income = 3567, high (future) income = 4048

(c) Long-run discount factor (δ)=0.9

(d) Decreasing consumption ($\delta * (1 + \iota) < 1$)

(e) Decreasing consumption and sophisticated time preferences

(f) Various values of β

C Online Appendix: Additional Material on the Validation Experiment

C.1 Preference estimation

Below we describe how we estimate the preference parameters β and δ from the responses in the validation experiment.

The premise underlying the CTB approach is that individuals have "beta-delta" preferences, where each individual i is characterized by tuple (β_i, δ_i). The second entry δ denotes the long-run discount factor and the first entry β the present bias. An individual who considers an early period t_1 and a late period t_2 discounts the time between them at factor $\delta_i^{t_2-t_1}$ if the early period is in the future, but discounts it with factor $\beta_i\delta_i^{t_2-t_1}$ if the early time period includes the present. Andreoni and Sprenger (2012) propose a regression model to generate a point estimate $(\hat{\beta}_i, \hat{\delta}_i)$ for each individual i (equation (6) in their paper). We apply their method to our CTB data for individuals in treatments (2), (3) and (4) to generate these point estimates for each individual i, and call them $(\hat{\beta}_{i,CTB}, \hat{\delta}_{i,CTB})$. For treatment (1) we apply their method separately to the answers elicited before prompting $(\hat{\beta}_{i,CTB-NP}, \hat{\delta}_{i,CTB-NP})$ and to the answers elicited after prompting $(\hat{\beta}_{i,CTB-P}, \hat{\delta}_{i,CTB-P})$.[2]

For the MLL data we also construct estimates of (β_i, δ_i). To do this, consider participant i who chooses between a 0.5 chance to get a lottery ticket at the early time t_1 or a higher probability of obtaining a lottery ticket at a later time t_2. In our questions, t_1 can be zero or two months, and t_2 can be two, four or six months. Define $P_i(t_1, t_2)$ as the highest late probability such that the individual chooses the early option whenever the late probability is weakly lower. Define $Q_i(t_1, t_2)$ as the lowest late probability such that the individual chooses the late option whenever the late probability is weakly higher. Set $P_i(t_1, t_2) = 0.5$ if the individual never chooses the early ticket. We set $Q_i(t_1, t_2) = 1.2$ if the individual always chooses the early ticket.

Define the ratio of the early winning probability and the midpoint between P and Q as $D_i(t_1, t_2) = \frac{0.5}{P_i(t_1,t_2)+0.5(Q_i(t_1,t_2)-P_i(t_1,t_2))}$. This is our empirical counterpart of the discount factor between the early and late period for individual i. We then define as the present bias:

$$\beta_{i,MLL} = 0.5\Big[\frac{D_i(0,2)}{D_i(2,4)} + \frac{D_i(0,4)}{D_i(2,6)}\Big] \tag{8}$$

Each ratio defines how much the individual additional discounts when the early answer is in the present compared to when both answers are in the future. The first ratio does this when answer periods are two months apart. The second ratio when periods are four months apart. We average both ratios.

We define the long-run (monthly) discount factor as:

$$\delta_{i,MLL} = 0.5\big[D_i(2,4)^{1/2} + D_i(2,6)^{1/4}\big] \tag{9}$$

It uses only the discounting between periods that do not involve the present period. For two-month discounting it takes the square root to get a monthly discount factor. For four-month discounting it takes the forth root to get the monthly discount factor. It then takes

[2] Following Andreoni and Sprenger (2012) we use Tobit regression if a respondent picked at least two interior choices. If not, we used OLS. If all tokens were always placed on the late option, we set $\beta = \delta = 1$ and if all tokens were always placed on the early option we set $\beta = 1$, $\delta = 0.5$.

the average.

C.2 Additional tables and figures

Table C.1: Question parameters

Convex budget set (CTB) questions					Lottery ticket (MLL) questions				
τ early period	$t-\tau$ delay	a_τ early rate	a_t late rate	$1+r$	τ early period	$t-\tau$ delay	a_τ early prob. of ticket	a_t late prob. of ticket	$1+r$
Within 3 days	2 months	0.1	0.11	1.1	Within 3 days	2 months	50%	55%	1.1
Within 3 days	2 months	0.1	0.12	1.2	Within 3 days	2 months	50%	60%	1.2
Within 3 days	2 months	0.1	0.15	1.5	Within 3 days	2 months	50%	75%	1.5
Within 3 days	2 months	0.1	0.18	1.8	Within 3 days	2 months	50%	90%	1.8
Within 3 days	2 months	0.1	0.2	2	Within 3 days	2 months	50%	100%	2
Within 3 days	4 months	0.1	0.11	1.1	Within 3 days	4 months	50%	55%	1.1
Within 3 days	4 months	0.1	0.12	1.2	Within 3 days	4 months	50%	60%	1.2
Within 3 days	4 months	0.1	0.15	1.5	Within 3 days	4 months	50%	75%	1.5
Within 3 days	4 months	0.1	0.18	1.8	Within 3 days	4 months	50%	90%	1.8
Within 3 days	4 months	0.1	0.2	2	Within 3 days	4 months	50%	100%	2
2 months	2 months	0.1	0.11	1.1	2 months	2 months	50%	55%	1.1
2 months	2 months	0.1	0.12	1.2	2 months	2 months	50%	60%	1.2
2 months	2 months	0.1	0.15	1.5	2 months	2 months	50%	75%	1.5
2 months	2 months	0.1	0.18	1.8	2 months	2 months	50%	90%	1.8
2 months	2 months	0.1	0.2	2	2 months	2 months	50%	100%	2
2 months	4 months	0.1	0.11	1.1	2 months	4 months	50%	55%	1.1
2 months	4 months	0.1	0.12	1.2	2 months	4 months	50%	60%	1.2
2 months	4 months	0.1	0.15	1.5	2 months	4 months	50%	75%	1.5
2 months	4 months	0.1	0.18	1.8	2 months	4 months	50%	90%	1.8
2 months	4 months	0.1	0.2	2	2 months	4 months	50%	100%	2

Each row is one question, and specifies τ (the early period), $t-\tau$ (delay, the difference between the early and late period in months), a_τ (token exchange rate early period for CTB, probability of receiving an early period lottery ticket for MLL), a_t (same for late period) and $1+r$ (implied exchange rate).

Table C.2: Balance table

Variable	(1) Treatment 1 Mean/(SE)	(2) Treatment 2 Mean/(SE)	(3) Treatment 3 Mean/(SE)	(4) Treatment 4 Mean/(SE)	(1)-(2) P-value	(1)-(3) P-value	(1)-(4) Pairwise t-test P-value	(2)-(3) P-value	(2)-(4) P-value	(3)-(4) P-value
Female	0.49 (0.06)	0.47 (0.06)	0.48 (0.06)	0.48 (0.06)	0.81	0.86	0.88	0.94	0.93	0.98
Age	35.22 (1.52)	35.04 (1.49)	35.52 (1.52)	35.32 (1.35)	0.93	0.89	0.96	0.82	0.89	0.92
Expenditures (pre-prompting)	2986.69 (462.63)	3347.43 (900.40)	4020.41 (980.37)	7685.38 (2014.37)	0.72	0.33	0.03**	0.61	0.05*	0.11
Number of observations	77	76	73	79	153	150	156	149	155	152

Columns (1)-(4) shows means for the four treatment groups. 7 respondents are excluded because they did not report their age.

Figure C.1: Expenditure estimates before and after prompting

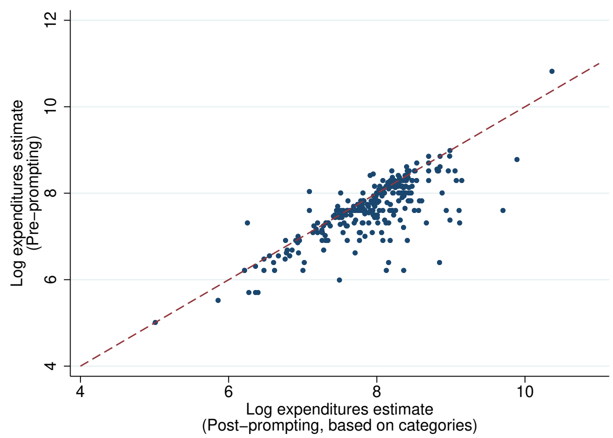


Figure C.2: Mean expenditure estimates

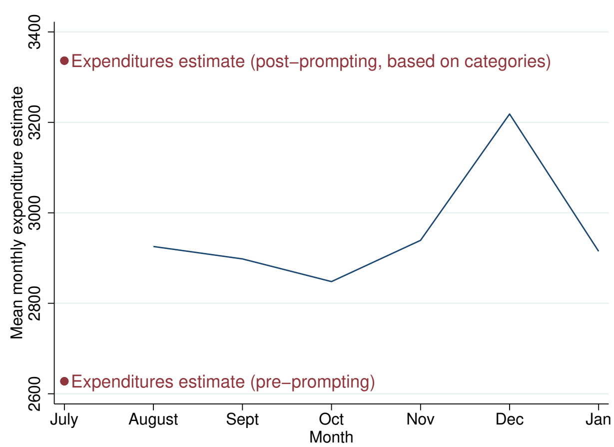


Table C.3: First-stage tests (hypotheses 0.1 and 0.2): alternative approach to deal with outliers

	Pre-prompting	Post-prompting	P-value one-sided t-test
Hyp. 0.1 (between)			
Expenditures, mean	2,584.61	3,200.24	0.012
Standard error mean	113.38	248.67	
Observations	141	139	
Hyp. 0.2 (within)			
Expenditures, mean	2,584.61	3,070.09	0.000
Standard error mean	113.38	130.93	
Observations	141	141	

Excluding respondents for whom the ratio of the pre-prompting response to the post-prompting response was below 1/3 or above 3.

Table C.4: First-stage tests (hypotheses 0.1 and 0.2): alternative approach to deal with outliers

	Pre-prompting	Post-prompting	P-value one-sided t-test
Hyp. 0.1 (between)			
Expenditures, mean	3,175.67	3,445.35	0.313
Standard error mean	490.81	250.16	
Observations	157	155	
Hyp. 0.2 (within)			
Expenditures, mean	2,542.74	3,141.29	0.000
Standard error mean	100.15	117.49	
Observations	137	137	

The highest and lowest 5% of expenditure responses were trimmed.

Table C.5: Summary statistics of preference estimates

	Mean	St.error	Min	Max	Obs
delta (MLL), no prompting	0.899	0.00618	0.748	0.982	151
delta (MLL), prompting	0.908	0.00576	0.748	0.982	152
delta (CTB), no prompting	0.860	0.0146	0.500	1.500	157
delta (CTB), prompting	0.903	0.0147	0.500	1.500	155
beta (MLL), no prompting	0.987	0.0120	0.500	1.500	148
beta (MLL), prompting	0.980	0.0107	0.500	1.339	148
beta (CTB), no prompting	1.005	0.0197	0.500	1.500	157
beta (CTB), prompting	0.997	0.0194	0.500	1.500	155

Only the first responses are included for treatment groups 1 and 2. Estimates are censored to the 0.5 - 1.5 (as we do in all baseline analysis).

Figure C.3: Distribution of preference estimates (with δ the monthly discount factor and β the present-bias factor). The box plot shows the lower adjacent value, the first quartile, the median, the third quartile, the upper adjacent value and outside values. Estimates are censored to the 0.5 - 1.5 (as we do in all baseline analysis).

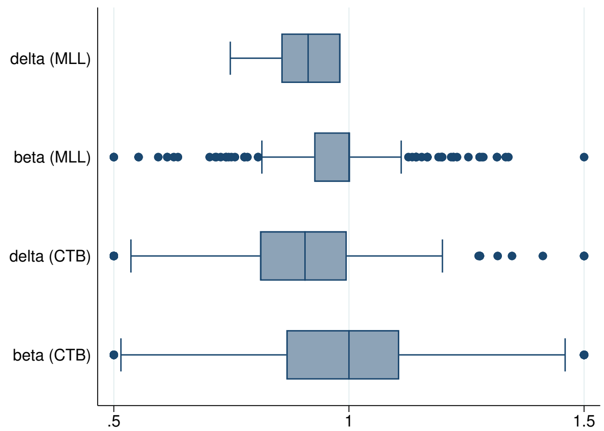


Table C.6: Core hypotheses: parameter estimates censored to 0.3-2.0

	No prompting	Prompting	P-value one-sided t-test
Hyp. 1.1 (between)			
Beta (CTB estimates), mean	1.020	1.014	0.436
Standard error mean	0.025	0.025	
Observations	157	155	
Hyp. 1.2 (between)			
Beta (MLL estimates), mean	0.987	0.980	0.326
Standard error mean	0.012	0.011	
Observations	148	148	
Hyp. 2.1 (within)			
Beta (CTB estimates), mean	1.055	1.013	0.195
Standard error mean	0.036	0.036	
Observations	79	79	
Hyp. 2.2 (within)			
Beta (MLL estimates), mean	0.982	0.978	0.426
Standard error mean	0.021	0.021	
Observations	71	71	

All t-tests are one-sided. $\hat{\beta}$ in all cases censored to values in the range 0.3-2.0 (replacing more extreme values with these limits).

Table C.7: Core hypotheses: parameter estimates censored to 0.6-1.4

	No prompting	Prompting	P-value one-sided t-test
Hyp. 1.1 (between)			
Beta (CTB estimates), mean	1.003	0.996	0.396
Standard error mean	0.018	0.017	
Observations	157	155	
Hyp. 1.2 (between)			
Beta (MLL estimates), mean	0.988	0.981	0.319
Standard error mean	0.011	0.010	
Observations	148	148	
Hyp. 2.1 (within)			
Beta (CTB estimates), mean	1.034	0.973	0.024
Standard error mean	0.025	0.022	
Observations	79	79	
Hyp. 2.2 (within)			
Beta (MLL estimates), mean	0.982	0.980	0.453
Standard error mean	0.019	0.019	
Observations	71	71	

All t-tests are one-sided. $\hat{\beta}$ in all cases censored to values in the range 0.6-1.4 (replacing more extreme values with these limits).

Table C.8: Secondary hypotheses: regressions with MLL estimates as dependent variables

	(1) Delta (MLL)	(2) Delta (MLL)	(3) Beta (MLL)	(4) Beta (MLL)
Delta (CTB)	0.204***	0.107***		
	(0.030)	(0.030)		
Beta (CTB)			0.153***	0.114**
			(0.048)	(0.044)
Constant	0.724***	0.811***	0.835***	0.867***
	(0.026)	(0.028)	(0.049)	(0.045)
Sample	No prompting (Treatment 1,2)	Prompting (Treatment 3,4)	No prompting (Treatment 1,2)	Prompting (Treatment 3,4)
Observations	151	152	148	148

Standard errors in parentheses
All parameters censored to (.5 - 1.5).
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table C.9: Secondary hypotheses: regressions with CTB estimates as dependent variables

	(1) Delta (CTB)	(2) Delta (CTB)	(3) Beta (CTB)	(4) Beta (CTB)
Delta (MLL)	1.163***	0.723***		
	(0.171)	(0.204)		
Beta (MLL)			0.428***	0.389**
			(0.134)	(0.149)
Constant	-0.185	0.246	0.572***	0.612***
	(0.154)	(0.185)	(0.134)	(0.148)
Sample	No prompting (Treatment 1,2)	Prompting (Treatment 3,4)	No prompting (Treatment 1,2)	Prompting (Treatment 3,4)
Observations	151	152	148	148

Standard errors in parentheses

All parameters censored to (.5 - 1.5).

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table C.10: Exploratory analysis

	(1) Beta (MLL, pre-prompting)	(2) Beta (CTB, pre-prompting)
Beta (MLL, post-prompting)	0.566***	
	(0.098)	
Beta (CTB, post-prompting)		0.197
		(0.125)
Constant	0.428***	0.846***
	(0.097)	(0.125)
Sample	Treatment group 2	Treatment group 1
Observations	71	79

Standard errors in parentheses

All parameters censored to (.5 - 1.5).

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

The difference between the coefficients in column (1) and (2) is statistically significant with p-value 0.037.

D Online Appendix: Computation of Preference Parameters Job Seekers

We calculated time preference parameters in analogous way to our validation experiment.

Time preferences were elicited using the MLL method. Job seekers were asked three sets of questions involving a choice between receiving 5 lottery tickets at an early date vs a higher number (6, 7, 8, 9 or 10) at a later point. The dates involved in each of these choices were as follows:

1. Today vs in a week (first set)
2. Today vs in 4 weeks (second set)
3. 8 weeks vs 12 weeks (third set)

We use the choice data to construct estimates of (β_i, δ_i). To do this, consider participant i who chooses between reciving 5 lottery tickets at the early time t_1 or a higher number of tickets at a later time t_2.

Define $M_i(t_1, t_2)$ as the number of tickets corresponding to the 'last early choice', and set to 5 if no early choice was taken. The individual always chooses the late option whenever the late choice involves a strictly higher number of tickets. That is, the individual always chooses the late choice when the late option involves at least $Q_i(t_1, t_2) = M_i(t_1, t_2) + 1$ lottery tickets (as long as $M_i(t_1, t_2) + 1 \leq 10$ so that at least the highest number of late lottery tickets induces a late choice). Similarly, define $N_i(t_1, t_2)$ as the number of tickets corresponding to the 'first late choice', and set it to 11 if no late choice was taken. The individual chooses the early option whenever the late number of tickets is strictly smaller. That is, the individual always chooses early when the late choice involves weakly less than $P_i(t_1, t_2) = N_i(t_1, t_2) - 1$ lottery tickets (as long as $N_i(t_1, t_2) - 1 \geq 6$ so that the lowest number of lottery tickets induces an early choice). If an individual is consistent and switches only once, $P_i(t_1, t_2) = Q_i(t_1, t_2) - 1$. But if the individual switches multiple times, this is not necessarily the case.

If an individual picks the early option even at the maximum number of tickets for the late lottery (i.e., $M_i(t_1, t_2) + 1 \leq 10$), we do not know the level above which this person would choose the late option consistently. In analogy to our validation experiment we set it 20% above the highest late ticket number, so that $Q_i(t_1, t_2) = 12$. Conversely, if an individual picks the early option even at the lowest number of late tickets, we do not know how low that number would have to be for them to consistently choose the early option. We set this lower bound to $P_i(t_1, t_2) = 5$.

We then compute the mid-point between $P_i(t_1, t_2)$ and $Q_i(t_1, t_2)$. The discount factor between the early and later period corresponds to $D_i(t_1, t_2) = \frac{5}{0.5(P_i(t_1,t_2) + Q_i(t_1,t_2))}$.

We then define as the present bias:

$$\beta_{i,MLL} = \frac{D_i(0,4)}{D_i(8,12)} \tag{10}$$

Note that we only use the data for the second and third set of choices (today vs in 4 weeks, and 4 weeks vs 8 weeks).

Figure D.1: Distribution of preference estimates for job seekers (with δ the monthly discount factor and β the present-bias factor). The box plot shows the lower adjacent value, the first quartile, the median, the third quartile, the upper adjacent value and outside values.