Online Appendix

The optimal quantity of CBDC in a bank-based economy

Lorenzo Burlon* Manuel A. Muñoz† Frank Smets‡

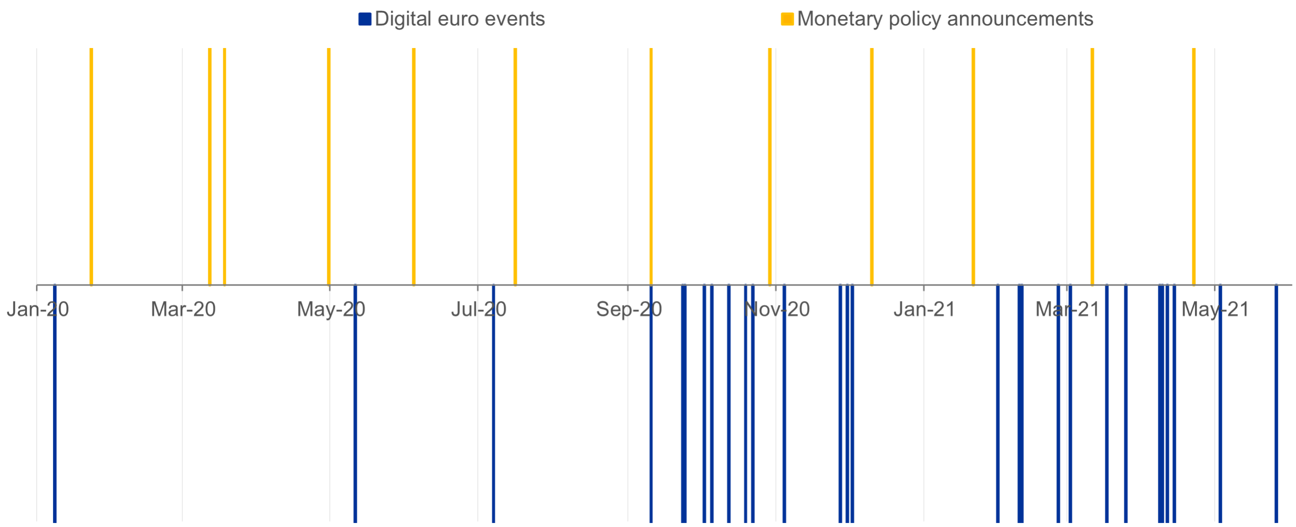
*European Central Bank. E-mail: lorenzo.burlon@ecb.europa.eu
†European Central Bank. E-mail: manuel.munoz@ecb.europa.eu
‡European Central Bank, Ghent University and CEPR. E-mail: frank.smets@ecb.europa.eu.

A Additional Empirical Evidence

Table A.1: List of digital euro events

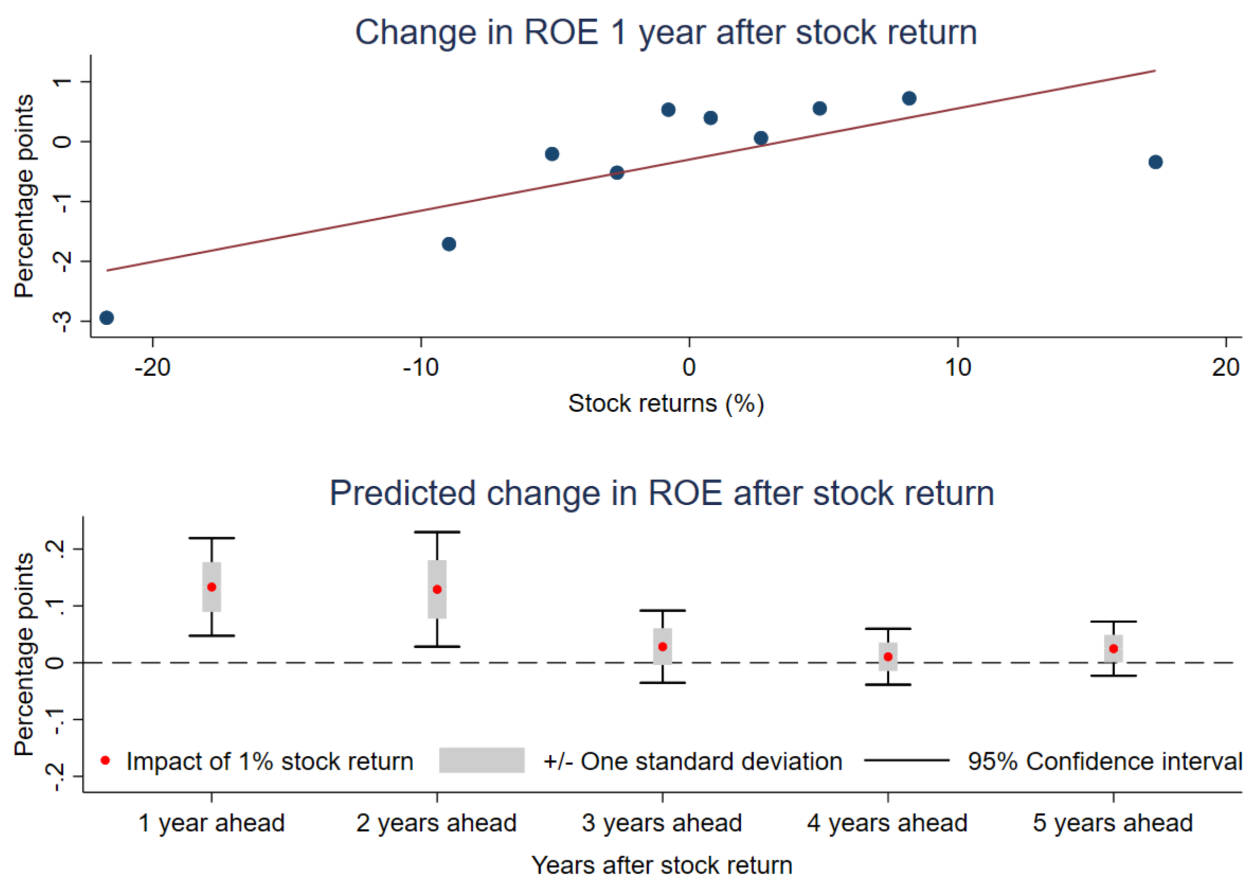
Date	Event	Subject	Type of event
08-Jan-20	INTERVIEW	Christine Lagarde: Interview with "Challenges" magazine	Neutral
11-May-20	SPEECH	Yves Mersch: An ECB digital currency – a flight of fancy?	Detailing D€
07-Jul-20	SPEECH	Fabio Panetta: Unleashing the euro's untapped potential at global level	Neutral
10-Sep-20	SPEECH	Christine Lagarde: Payments in a digital world	Neutral
23-Sep-20	INTERVIEW	Yves Mersch: Interview with Bloomberg	Neutral
24-Sep-20	INTERVIEW	Philip R. Lane: Q&A on Twitter	Neutral
02-Oct-20	THE ECB BLOG	Fabio Panetta: We must be prepared to issue a digital euro	Fostering D€
02-Oct-20	PRESS RELEASE	ECB intensifies its work on a digital euro	Fostering D€
05-Oct-20	VOXEU COLUMN	Fabio Panetta & Ulrich Bindseil: CBDC remuneration in a world with low or negative nominal interest rates	Detailing D€
12-Oct-20	SPEECH	Fabio Panetta: A digital euro for the digital era	Fostering D€
19-Oct-20	INTERVIEW	Christine Lagarde: Interview with Le Monde	Neutral
22-Oct-20	SPEECH	Fabio Panetta: On the edge of a new frontier: European payments in the digital age	Neutral
04-Nov-20	SPEECH	Fabio Panetta: The two sides of the (stable)coin	Neutral
27-Nov-20	SPEECH	Fabio Panetta: From the payments revolution to the reinvention of money	Fostering D€
30-Nov-20	INTERVIEW	Christine Lagarde: The future of money – innovating while retaining trust	Fostering D€
02-Dec-20	THE ECB BLOG	Fabio Panetta: Money in the digital era	Neutral
31-Jan-21	INTERVIEW	Isabel Schnabel: Interview with Deutschlandfunk	Neutral
09-Feb-21	INTERVIEW	Fabio Panetta: Interview with Der Spiegel	Detailing D€
10-Feb-21	SPEECH	Fabio Panetta: Evolution or revolution? The impact of a digital euro on the financial system	Detailing D€
25-Feb-21	INTERVIEW	Isabel Schnabel: Interview with LETA	Detailing D€
02-Mar-21	INTERVIEW	Luis de Guindos: Interview with Público	Neutral
17-Mar-21	INTERVIEW	Frank Elderson: Q&A on Twitter	Neutral
25-Mar-21	THE ECB BLOG	Fabio Panetta: Digital central bank money for Europeans – getting ready for the future	Neutral
08-Apr-21	SPEECH	Christine Lagarde: IMFC Statement	Neutral
09-Apr-21	INTERVIEW	Isabel Schnabel: Interview with Der Spiegel	Detailing D€
11-Apr-21	INTERVIEW	Fabio Panetta: Interview with El País	Detailing D€
14-Apr-21	PRESS RELEASE	ECB publishes the results of the public consultation on a digital euro	Neutral
14-Apr-21	SPEECH	Fabio Panetta: A digital euro to meet the expectations of Europeans	Neutral
03-May-21	INTERVIEW	Luis de Guindos: Interview with La Repubblica	Neutral
26-May-21	INTERVIEW	Fabio Panetta: Interview with Nikkei	Detailing D€

Figure A.1: Sequence of digital euro events and monetary policy announcements



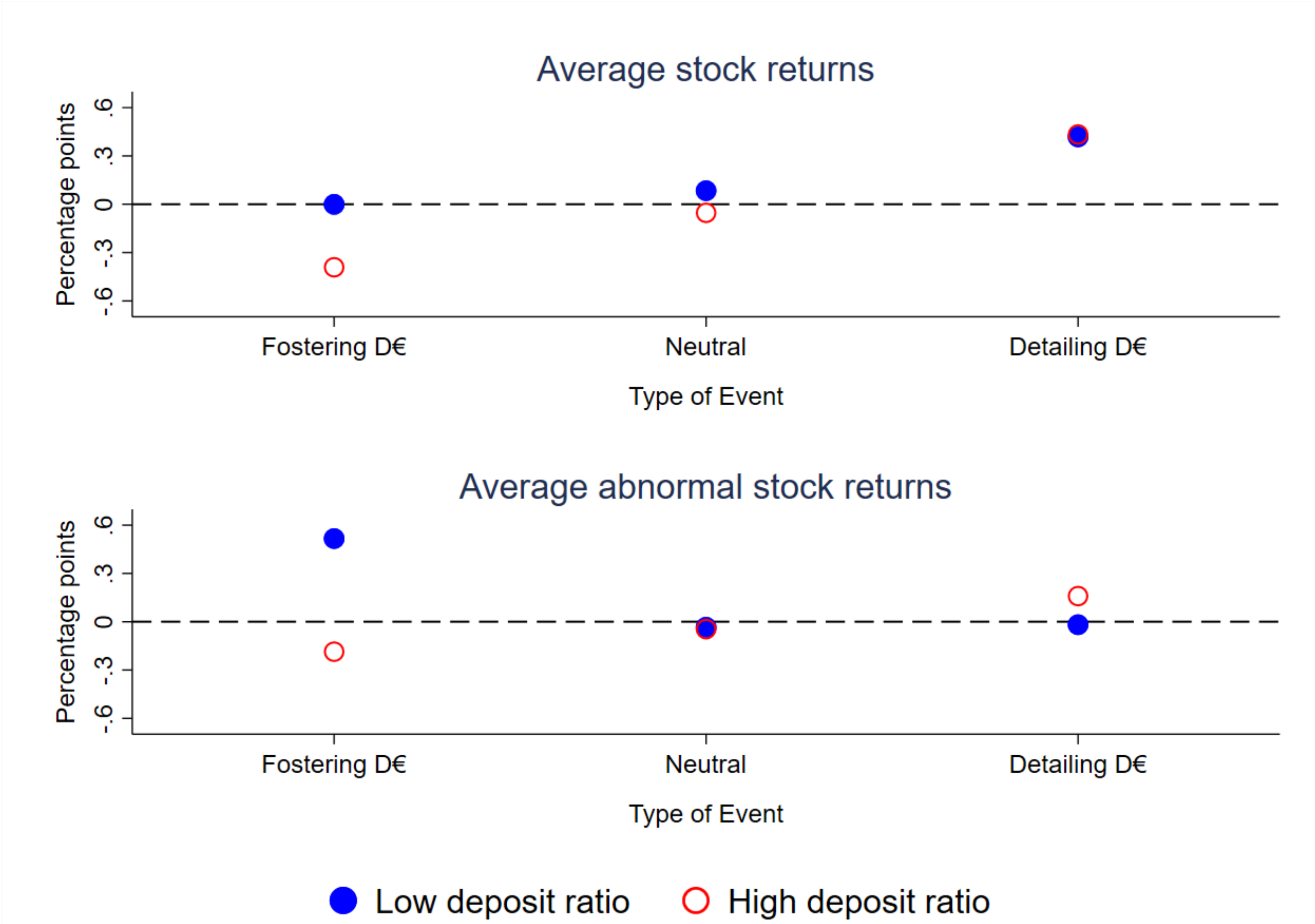
Notes: Blue lines indicate days corresponding to digital euro events as reported in Table A.1. Yellow lines indicate days corresponding to monetary policy announcements by the ECB.

Figure A.2: Historical correlation between bank stock market returns and subsequent bank profits



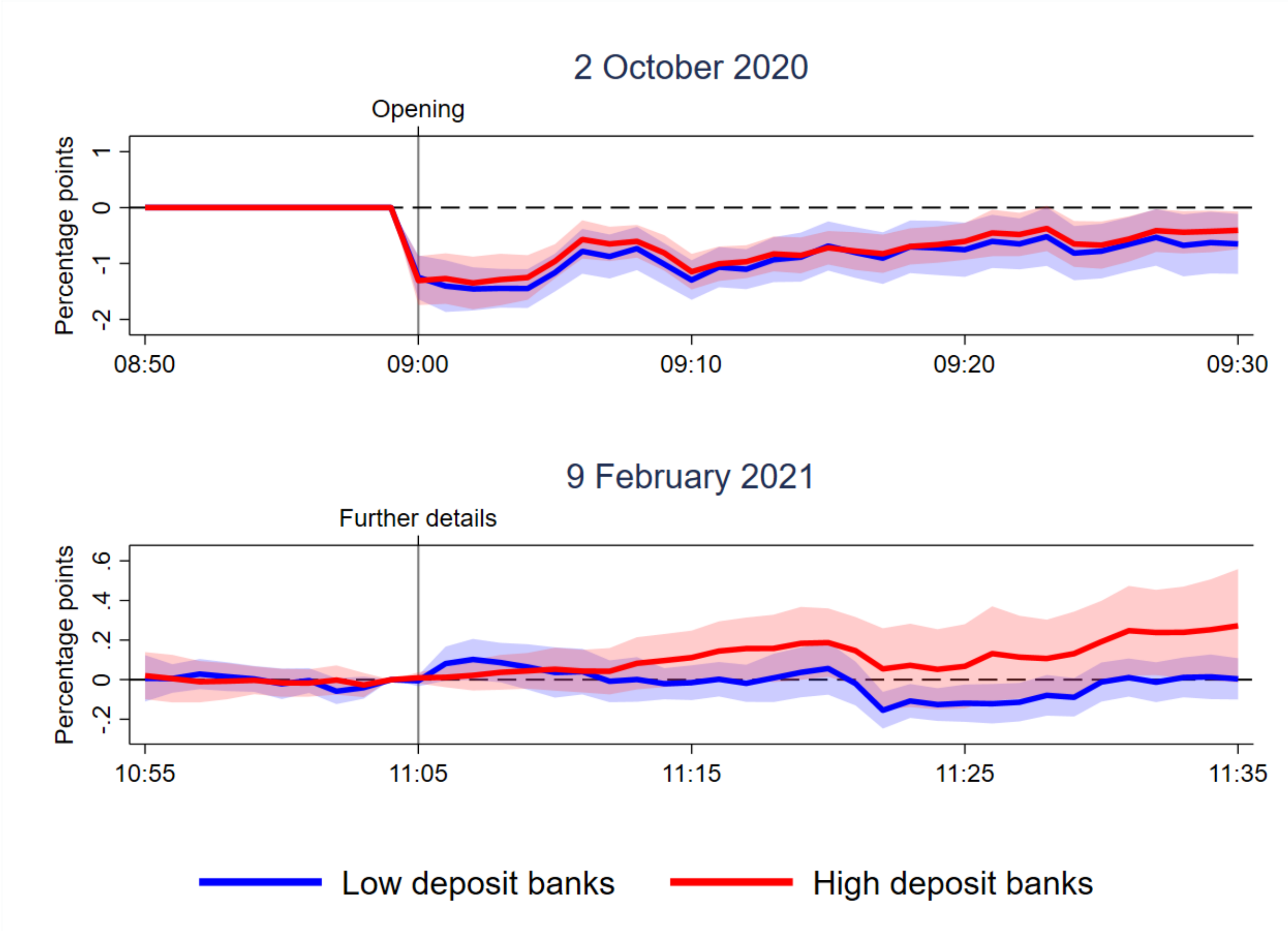
Notes: In the top panel, each dot represents a decile of the distribution of monthly bank stock returns in the euro area. The sample is an unbalanced panel of 323 euro area banks between June 2007 and September 2022. The vertical axis measures the average change in bank ROE one year after a change in a bank's stock price for each deciles of monthly bank stock returns in the euro area. The horizontal axis measures the average bank stock return within each decile. The line represents the linear prediction across the dots. In the bottom panel, the red dots report the estimation coefficients β^H of the regressions $\Delta \text{ROE}_{b,t,t+H} = \alpha_b^H + \alpha_{c,t}^H + \beta^H \text{Stock Return}_{b,t} + \gamma^H \Delta \text{ROE}_{b,t-1,t} + \Gamma^H X_{b,t} + \varepsilon_{b,t}^H$, for $H = \{1 \text{ year ahead}, \ldots, 5 \text{ years ahead}\}$. The sample is the same as the one used in the top panel. Each observation is a euro area bank b in a month t. $\Delta \text{ROE}_{b,t,t+H}$ is the change (in percentage points) in bank b's ROE between month t and month $t+H$. $\text{Stock Return}_{b,t}$ is the change (in percentages) of bank b's stock price between month $t-1$ and month t. $\text{ROE}_{b,t-1,t}$ is the change (in percentage points) in bank b's ROE between month $t-1$ and month t. $X_{b,t}$ are time-varying bank controls that include the log of bank assets, the deposit ratio, the securities and excess liquidity holdings as shares of assets, the CET1 ratio, the CDS spread, and the level of ROE. Bank and country-month fixed effects are represented by α_b^H and $\alpha_{c,t}^H$, respectively. The grey areas indicate the range of estimates spanned by plus or minus one standard deviation around the central estimate, the black dashes indicate the 95% confidence interval, with errors clustered at the bank level.

Figure A.3: Stock market returns of euro area banks by type of digital euro event and by reliance on deposit funding



Notes: The three columns of the two panels represent the three types of digital euro events detailed in Table A.1. In the top panel, we compute the average daily stock return of banks with a high (red circles) and low (blue dots) reliance on deposit funding across events belonging to each of the three categories. In the bottom panel, we do the same with the average abnormal daily returns estimated in Model (1).

Figure A.4: Minute-by-minute stock market returns of euro area banks around the key early digital euro events



Notes: The two panels report the minute-by-minute evolution of stock prices of euro area banks 10 minutes before and 30 minutes after two key digital euro events hit the markets. The events are the publication of the report on the digital euro on 2 October 2020 (top panel) and the publication of ECB Board member Panetta's interview on 9 February 2021 (bottom panel). For the event of 2 October 2020, the first Bloomberg News flash ("*ECB SAYS IT IS INTENSIFYING ITS WORK ON DIGITAL EURO") was at 8:00am (CET) and the second flash ("ECB Takes Major Step Toward Introducing a Digital Euro") was at 08:12am (CET). Stock prices were flat until market opening at 9:00am (CET). Hence, the event window goes from 08:50am to 09:30am. For the event of 9 February 2021, the Bloomberg News flash ("ECB's Panetta Floats 3,000-Euro Limit on Digital Cash: Spiegel") was at 11:04am (CET). Hence, the event window goes from 10:55am to 11:35am. The solid blue and red lines report the coefficients $\beta^E_{m,\text{Low}}$ and $\beta^E_{m,\text{High}}$ of the cross-sectional regressions $\Delta\text{Stock price}^E_{b,m} = \beta^E_{m,\text{Low}}\mathbb{1}_b(\text{Low deposit ratio}) + \beta^E_{m,\text{High}}\mathbb{1}_b(\text{High deposit ratio}) + \varepsilon^E_{b,m}$, for the two events $E = \{2\text{ October }2020, 9\text{ February }2021\}$, where the observation is a bank b. $\Delta\text{Stock price}^E_{b,m}$ is the percentage change in stock price between the minute before the key event and minute m, with m spanning 10 minutes before and 30 minutes after key event E (for the 2 October 2020, given that we start observing prices only after market opening, we consider as baseline level of stock prices the close-of-business level from the day before). $\mathbb{1}_b(\text{Low deposit ratio})$ and $\mathbb{1}_b(\text{High deposit ratio})$ are dummies indicating banks with low (below mean) and high (above mean) deposit ratios, respectively. The blue and red areas report the 95% confidence intervals computed with robust standard errors. The sample consists of 33 euro area banks with available intradaily stock market returns around the two events considered.

B Data and Sources

This appendix presents the full data set employed to calibrate the model in Section 3.2 of the paper.

Gross Domestic Product: Gross domestic product at market prices, Euro area 19 (fixed composition), Domestic (home or reference area), Total economy, Euro, Current prices, Non transformed data, Calendar and seasonally adjusted data. Source: ESA2010 National accounts, Main aggregates, Eurostat.

GDP Deflator: Gross domestic product at market prices, Euro area 19 (fixed composition), Domestic (home or reference area), Total economy, Index, Deflator (index), Non transformed data, Calendar and seasonally adjusted data. Source: ESA2010 National accounts, Main aggregates, Eurostat.

Private Consumption: Private final consumption, Individual consumption expenditure, Euro area 19 (fixed composition), World (all entities, including reference area, including IO), Households and non profit institutions serving households (NPISH), Euro, Current prices, Non transformed data, Calendar and seasonally adjusted data. Source: ESA2010 National accounts, Main aggregates, Eurostat.

Public Consumption: Government final consumption, Final consumption expenditure, Euro area 19 (fixed composition), World (all entities, including reference area, including IO), General government, Euro, Current prices, Non transformed data, Calendar and seasonally adjusted data. Source: ESA2010 National accounts, Main aggregates, Eurostat.

Gross fixed capital formation: Gross fixed capital formation, Euro area 19 (fixed composition), World (all entities, including reference area, including IO), Total economy, Fixed assets by type of asset (gross), Euro, Current prices, Non transformed data, Calendar and seasonally adjusted data.

Bank Deposits (Counterpart: MFIs): Deposit liabilities vis-a-vis euro area MFI reported by MFI excluding ESCB in the euro area (stock), Euro area (changing composition), Outstanding amounts at the end of the period (stocks), MFIs excluding ESCB reporting sector, Deposit liabilities, Total maturity, Euro, Euro area (changing composition) counterpart, Monetary financial institutions (MFIs) sector, denominated in Euro, data Neither seasonally nor working day adjusted. Source: MFI Balance Sheet Items (BSI Statistics), European Central Bank.

Bank Deposits (Counterpart: Non-MFIs): Deposit liabilities vis-a-vis euro area non-MFI reported by MFI excluding ESCB in the euro area (stock), Euro area (changing composition), Outstanding amounts at the end of the period (stocks), MFIs excluding ESCB reporting sector - Deposit liabilities, Total maturity, Euro - Euro area (changing composition) counterpart, Non-MFIs sector, denominated in Euro, data Neither seasonally nor working day adjusted. Source: MFI Balance Sheet Items (BSI Statistics), European Central Bank.

Bank Capital and Reserves: Capital and reserves reported by MFI excluding ESCB in the euro area (stock), Euro area (changing composition), Outstanding amounts at the end of the period (stocks), MFIs excluding ESCB reporting sector - Capital and reserves, All currencies combined, World not allocated (geographically) counterpart, Unspecified counterpart sector sector, denominated in Euro, data Neither seasonally nor working day adjusted. Source: MFI Balance Sheet Items (BSI Statistics), European Central Bank.

Bank Loans to Households: Loans vis-a-vis euro area households reported by MFI excluding ESCB in the euro area (stock), Euro area (changing composition), Outstanding amounts at the end of the period (stocks), MFIs excluding ESCB reporting sector, Loans, Total maturity, All currencies combined, Euro area (changing composition) counterpart, Households and non-profit institutions serving households (S.14 and S.15) sector, denominated in Euro, data Neither seasonally nor working day adjusted. Source: MFI Balance Sheet Items (BSI Statistics), European Central Bank.

Bank Loans to NFCs: Loans vis-a-vis euro area NFC reported by MFI excluding ESCB in the euro area (stock), Euro area (changing composition), Outstanding amounts at the end of the period (stocks), MFIs excluding ESCB reporting sector - Loans, Total maturity, All currencies combined - Euro area (changing composition) counterpart, Non-Financial corporations (S.11) sector, denominated in Euro, data Neither seasonally nor working day adjusted. Source: MFI Balance Sheet Items (BSI Statistics), European Central Bank.

Bank Holdings of Government Debt: Holdings of debt securities issued by euro area General Government reported by MFI excluding ESCB in the euro area (stock), Euro area (changing composition), Outstanding amounts at the end of the period (stocks), MFIs excluding ESCB reporting sector, Debt securities held, Total maturity, All currencies combined, Euro area (changing composition) counterpart, General Government sector, denominated in Euro, data Neither seasonally nor working day adjusted. Source: MFI Balance Sheet Items (BSI Statistics), European Central Bank.

Reserves: Liabilities to euro area credit institutions related to MPOs denominated in euro - Eurosystem, Euro area (changing composition), Eurosystem reporting sector, Liabilities to euro area credit institutions related to MPOs denominated in euro, Euro, Euro area (changing composition) counterpart. Source: Internal Liquidity Management (ILM Statistics), European Central Bank.

Banknotes (Cash): Banknotes in circulation - Eurosystem, Euro area (changing composition), Eurosystem reporting sector, Banknotes in circulation, Euro, World not allocated (geographically) counterpart. Source: MFI Balance Sheet Items (BSI Statistics), European Central Bank.

Deposit Interest Rate: Bank interest rates, overnight deposits from households - euro area, Euro area (changing composition), Annualised agreed rate (AAR) / Narrowly defined effective rate (NDER), Credit and other institutions (MFI except MMFs and central banks) reporting sector, Overnight deposits, Total original maturity, New business coverage, Households and non-profit institutions serving households (S.14 and S.15) sector, denominated in Euro. Source: MFI Interest Rate Statistics (MIR Statistics), European Central Bank.

NFC Loans Interest Rate: Bank interest rates, loans to corporations with an original maturity of up to one year (outstanding amounts) - euro area, Euro area (changing composition), Annualised agreed rate (AAR) / Narrowly defined effective rate (NDER), Credit and other institutions (MFI except MMFs and central banks) reporting sector, Loans, Up to 1 year original maturity, Outstanding amount business coverage, Non-Financial corporations (S.11) sector, denominated in Euro. Source: MFI Interest Rate Statistics (MIR Statistics), European Central Bank.

Deposit Facility Rate: ECB Deposit facility, date of changes (raw data), Level. Euro area (changing composition), Key interest rate, ECB Deposit facility, date of changes (raw data), Level, Euro, provided by ECB. Source: Financial market data (MF Statistics), European Central Bank.

Lending Facility Rate: ECB Marginal lending facility - date of changes (raw data) - Level. Euro area (changing composition), Key interest rate, ECB Marginal lending facility, date of changes (raw data), Level, Euro, provided by ECB. Source: Financial market data (MF Statistics), European Central Bank.

C Equations of the Model

This section presents the full set of equilibrium equations of the DSGE model under the baseline scenario.

C.1 Patient Households

Patient households seek to maximize their objective function subject to the following budget constraint:

$$\begin{aligned} c_{p,t} + q_t(h_{p,t} - h_{p,t-1}) + m_t + f(m_t) + cbdc_t + d_t + b_{p,t} + \omega_T T_t \\ = \frac{m_{t-1}}{\pi_t} + R_{cbdc,t-1}\frac{cbdc_{t-1}}{\pi_t} + R_{d,t-1}\frac{d_{t-1}}{\pi_t} + R_{g,t-1}\frac{b_{p,t-1}}{\pi_t} + w_t n_{p,t} + \Omega_t. \end{aligned} \quad \text{(C.1)}$$

Their choice variables are $c_{p,t}$, $h_{p,t}$, d_t, m_t, $cbdc_t$, $b_{p,t}$ and $n_{p,t}$. The optimality conditions of the problem read

$$\lambda_t^p = \left[c_{p,t} - \frac{n_{p,t}^{1+\phi}}{(1+\phi)}\right]^{-\sigma_h}, \quad \text{(C.2)}$$

$$q_t \lambda_t^p = \frac{j_{p,t}}{h_{p,t}} + \beta_p E_t\left(q_{t+1}\lambda_{t+1}^p\right), \quad \text{(C.3)}$$

$$\lambda_t^p = \beta_p E_t\left(\lambda_{t+1}^p R_{d,t}/\pi_{t+1}\right) + \frac{\chi_{z,t}}{z_t}\omega_d\left(\frac{z_t}{d_t}\right)^{1/\eta_{z,t}}, \quad \text{(C.4)}$$

$$\lambda_t^p = \beta_p E_t\left(\lambda_{t+1}^p R_{cbdc,t}/\pi_{t+1}\right) + \frac{\chi_{z,t}}{z_t}\left(\frac{z_t}{cbdc_t}\right)^{1/\eta_{z,t}}, \quad \text{(C.5)}$$

$$\lambda_t^p(1+f_m) = \beta_p E_t\left(\lambda_{t+1}^p/\pi_{t+1}\right) + \frac{\chi_{z,t}}{z_t}\left(\frac{z_t}{m_t}\right)^{1/\eta_{z,t}}, \quad \text{(C.6)}$$

$$\lambda_t^p = \beta_p E_t\left(\lambda_{t+1}^p R_{g,t}/\pi_{t+1}\right), \quad \text{(C.7)}$$

$$w_t = n_{p,t}^{\phi}, \quad \text{(C.8)}$$

where λ_t^p is the Lagrange multiplier on the budget constraint of the representative patient household.

C.2 Impatient Households

The representative impatient household chooses the trajectories of consumption $c_{i,t}$, property housing $h_{i,t}$, hours worked $n_{i,t}$, and demand for loans $l_{i,t}$ that maximizes its objective function subject to a budget constraint and a borrowing limit:

$$c_{i,t} + q_t\left(h_{i,t} - h_{i,t-1}\right) + R_{i,t-1}\frac{l_{i,t-1}}{\pi_t} + (1-\omega_T)T_t = l_{i,t} + w_t n_{i,t}, \quad \text{(C.9)}$$

$$l_{i,t} \leq m_{H,t} E_t\left(\frac{q_{t+1}}{R_{i,t}}h_{i,t}\pi_{t+1}\right). \quad \text{(C.10)}$$

The resulting optimality conditions are

$$\lambda_t^i = \left[c_{i,t} - \frac{n_{i,t}^{1+\phi}}{(1+\phi)} \right]^{-\sigma_h}, \tag{C.11}$$

$$\lambda_t^i \left[q_t - E_t \left(m_{H,t} \frac{q_{t+1}}{R_{i,t}} \pi_{t+1} \right) \right] = \frac{j_{i,t}}{h_{i,t}} + \beta_i E_t \left[q_{t+1} \lambda_{t+1}^i \left(1 - m_{H,t}\right) \right], \tag{C.12}$$

$$w_t = n_{i,t}^{\phi}, \tag{C.13}$$

where λ_t^i is the Lagrange multiplier on the budget constraint of the representative impatient household.

C.3 Banks

Banks maximize their objective function subject to a balance sheet identity, a cash flow restriction, a capital adequacy constraint, a liquidity (reserves) requirement and a central banks' collateral requirement

$$L_{i,t} + L_{e,t} + b_{b,t} + \widetilde{R}_{b,t} = e_t + D_t + f_t, \tag{C.14}$$

$$\begin{aligned} &\Omega_{b,t} + e_t - (1-\delta^e)\frac{e_{t-1}}{\pi_t} \\ &= \frac{\left(r_{i,t-1} L_{i,t-1} + r_{e,t} L_{e,t-1} + r_{g,t-1} b_{b,t-1} + r_{\widetilde{R},t-1} \widetilde{R}_{b,t-1} - r_{d,t-1} D_{t-1} - r_{f,t-1} f_{t-1} \right)}{\pi_t}, \end{aligned} \tag{C.15}$$

$$D_t + f_t \leq \gamma_i L_{i,t} + \gamma_e L_{e,t} + \gamma_b b_{b,t} + \gamma_{\widetilde{R}} \widetilde{R}_{b,t}, \tag{C.16}$$

$$\theta_{R,t} D_t \leq \widetilde{R}_{b,t}, \tag{C.17}$$

$$f_t \leq \theta_{b,t} E_t \left(\frac{b_{b,t}}{R_{f,t}} \pi_{t+1} \right). \tag{C.18}$$

The law of motion for bank equity reads

$$e_t = J_{b,t} - \ \Omega_{b,t} + \ (1-\delta^e) e_{t-1}/\pi_t. \tag{C.19}$$

Their choice variables are $\Omega_{b,t}$, $L_{i,t}$, $L_{e,t}$, $b_{b,t}$, $\widetilde{R}_{b,t}$, D_t and f_t.The resulting optimality conditions read

$$\frac{1}{\Omega_{b,t}^{\frac{1}{\sigma}}} + \mu_{e,t}\gamma_e = E_t \left[\Lambda_{t,t+1} \frac{\left(r_{e,t+1} + 1 - \delta^e\right)/\pi_{t+1}}{\Omega_{b,t+1}^{\frac{1}{\sigma}}} \right], \tag{C.20}$$

$$\frac{1}{\Omega_{b,t}^{\frac{1}{\sigma}}} + \mu_{e,t}\gamma_i = E_t \left[\Lambda_{t,t+1} \frac{\left(r_{i,t} + 1 - \delta^e\right)/\pi_{t+1}}{\Omega_{b,t+1}^{\frac{1}{\sigma}}} \right], \tag{C.21}$$

$$\frac{1}{\Omega_{b,t}^{\frac{1}{\sigma}}} + \mu_{\widetilde{R},t} + \mu_{e,t} = E_t \left[\Lambda_{t,t+1} \frac{\left(r_{\widetilde{R},t} + 1 - \delta^e\right)/\pi_{t+1}}{\Omega_{b,t+1}^{\frac{1}{\sigma}}} \right], \tag{C.22}$$

$$\frac{1}{\Omega_{b,t}^{\frac{1}{\sigma}}}+\mu_{f,t}\theta_{f,t}E_t\left(\frac{\pi_{t+1}}{R_{f,t}}\right)+\mu_e=E_t\left[\Lambda_{t,t+1}\frac{\left(r_{g,t}+1-\delta^e\right)/\pi_{t+1}}{\Omega_{b,t+1}^{\frac{1}{\sigma}}}\right], \tag{C.23}$$

$$\frac{1}{\Omega_{b,t}^{\frac{1}{\sigma}}}+\mu_{e,t}+\mu_{\widetilde{R},t}\theta_{R,t}=E_t\left[\Lambda_{t,t+1}\frac{\left(r_{d,t}+1-\delta^e\right)/\pi_{t+1}}{\Omega_{b,t+1}^{\frac{1}{\sigma}}}\right], \tag{C.24}$$

$$\frac{1}{\Omega_{b,t}^{\frac{1}{\sigma}}}+\mu_{e,t}+\mu_{f,t}=E_t\left[\Lambda_{t,t+1}\frac{\left(r_{f,t}+1-\delta^e\right)/\pi_{t+1}}{\Omega_{b,t+1}^{\frac{1}{\sigma}}}\right], \tag{C.25}$$

where $\mu_{e,t}$, $\mu_{\widetilde{R},t}$, and $\mu_{f,t}$ are the multipliers on the capital adequacy constraint, the reserve requirement, and the central bank's collateral constraint, respectively.

C.4 Entrepreneurial Managers

Entrepreneurs seek to maximize their objective function subject to a budget constraint and the corresponding borrowing limit:

$$\Omega_{e,t}+R_{e,t}\frac{l_{e,t-1}}{\pi_t}+q_{k,t}\left[k_{e,t}-(1-\delta_t^k)k_{e,t-1}\right]+q_t(h_{e,t}-h_{e,t-1})=r_{h,t}h_{e,t-1}+r_{k,t}u_tk_{e,t-1}+l_{e,t}+J_{er,t}, \tag{C.26}$$

$$l_{e,t}\leq m_{K,t}E_t\left[\frac{q_{k,t+1}}{R_{e,t+1}}(1-\delta_{t+1}^k)k_{e,t}\pi_{t+1}\right], \tag{C.27}$$

where

$$\delta_t^k\left(u_t\right)=\delta_0^k+\delta_1^k(u_t-1)+\frac{\delta_2^k}{2}\left(u_t-1\right)^2. \tag{C.28}$$

Their choice variables are $\Omega_{e,t}$, $l_{e,t}$, $k_{e,t}$, $h_{e,t}$ and u_t. The following optimality condition can be derived from the first order conditions of the problem

$$\Omega_{e,t}^{-\frac{1}{\sigma}}q_t=\Lambda_{t,t+1}E_t\left[\Omega_{e,t+1}{}^{-\frac{1}{\sigma}}\left(q_{t+1}+r_{h,t+1}\right)\right], \tag{C.29}$$

$$\begin{aligned}\Omega_{e,t}^{-\frac{1}{\sigma}}&\left\{q_{k,t}-m_{K,.t}E_t\left[\frac{q_{k,t+1}}{R_{e,t+1}}(1-\delta_{t+1}^k)\pi_{t+1}\right]\right\}\\&=\Lambda_{t,t+1}E_t\left\{\Omega_{e,t+1}{}^{-\frac{1}{\sigma}}\left[q_{k,t+1}(1-\delta_t^k)(1-m_{K,t})+u_{t+1}r_{k,t+1}\right]\right\},\end{aligned} \tag{C.30}$$

$$\delta_1^k+\delta_2^k\left(u_t-1\right)=r_{k,t}. \tag{C.31}$$

C.5 Entrepreneurial Retailers

There is a continuum of entrepreneurial retailers (also referred to as intermediate non-housing good producers). Each intermediate good producer j operates the following Cobb-Douglas production function:

$$Y_t(j)=A_t\left[u_t(j)k_{e,t-1}(j)\right]^{\alpha}h_{e,t-1}(j)^{\nu}N_t(j)^{(1-\alpha-\nu)}, \tag{C.32}$$

Intermediate good producers solve a two-stage problem. In the first stage, they choose the trajectories of $k_{e,t-1}(j)$, $h_{e,t-1}(j)$ and $N_t(j)$ that minimize total real costs, $r_{k,t}k_{e,t-1}(j) + r_{h,t}h_{e,t-1}(j) + w_t N_t(j)$:

$$\frac{w_t}{r_{k,t}} = \frac{(1-\alpha-\nu)}{\alpha}\frac{u_t k_{e,t-1}}{N_t}, \tag{C.33}$$

$$\frac{r_{h,t}}{r_{k,t}} = \frac{\nu}{\alpha}\frac{u_t k_{e,t-1}}{h_{e,t-1}}, \tag{C.34}$$

$$mc_t = \frac{(w_t)^{(1-\alpha-\nu)}(r_{k,t})^{\alpha}(r_{h,t})^{\nu}}{A_t(1-\alpha-\nu)^{(1-\alpha-\nu)}\alpha^{\alpha}\nu^{\nu}}. \tag{C.35}$$

The firms that can change prices in period t set them to satisfy:

$$g_t^1 = \lambda_t^p mc_t Y_t + \beta_p \theta E_t \left(\frac{\pi_t^{\chi_\pi}}{\pi_{t+1}}\right)^{-\varepsilon} g_{t+1}^1, \tag{C.36}$$

$$g_t^2 = \lambda_t^p \pi_t^* Y_t + \beta_p \theta E_t \left(\frac{\pi_t^{\chi_\pi}}{\pi_{t+1}}\right)^{1-\varepsilon}\left(\frac{\pi_t^*}{\pi_{t+1}^*}\right) g_{t+1}^2, \tag{C.37}$$

$$\varepsilon g_t^1 = (\varepsilon - 1)\, g_t^2. \tag{C.38}$$

The price level and price dispersion υ_t , respectively, evolve according to:

$$1 = \theta\left(\frac{\pi_{t-1}^{\chi_\pi}}{\pi_t}\right)^{1-\varepsilon} + (1-\theta)\,\pi_t^{*1-\varepsilon}, \tag{C.39}$$

and

$$\upsilon_t = \theta\left(\frac{\pi_{t-1}^{\chi_\pi}}{\pi_t}\right)^{-\varepsilon}\upsilon_{t-1} + (1-\theta)\,\pi_t^{*-\varepsilon}. \tag{C.40}$$

Profits from each intermediate good producer j are transferred to entrepreneurial managers:

$$J_{er,t}(j) = Y_t(j) - \left[r_{k,t}k_{e,t-1}(j) + r_{h,t}h_{e,t-1}(j) + w_t N_t(j)\right]. \tag{C.41}$$

C.6 Capital and Final Goods Producers

The representative final goods producer maximizes $P_t Y_t - \int_0^1 P_t(j)\, Y_t(j)\, dj$ with respect to the demand for the intermediate good, $Y_t(j)$. The homogeneous final good is produced by means of a Dixit-Stiglitz technology, $Y_t = \left[\int_0^1 Y_t(j)^{(\varepsilon-1)/\varepsilon} dj\right]^{\varepsilon/(\varepsilon-1)}$, where $\varepsilon > 1$ is the elasticity of substitution across intermediate goods. Profit maximization yields demand functions for intermediate good j: $Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} Y_t$, $\forall j$.

Capital-good-producing firms seek to maximize their objective function with respect to net investment in physical capital, I_t. The resulting optimal condition is

$$1 = q_{k,t}\left[1 - \frac{\psi_I}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 - \psi_I\left(\frac{I_t}{I_{t-1}} - 1\right)\frac{I_t}{I_{t-1}}\right] + E_t\left[\Lambda_{t,t+1} q_{k,t+1}\psi_I\left(\frac{I_{t+1}}{I_t} - 1\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right]. \tag{C.42}$$

The law of motion for physical capital reads

$$K_t = (1-\delta_t^k)K_{t-1} + I_t \left[1 - \frac{\psi_I}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right]. \quad \text{(C.43)}$$

C.7 Government

Tax revenues are collected from households in a lump-sum fashion and determined according to a fiscal rule

$$T_t = \phi_p b_{p,t-1} + \phi_b b_{b,t-1}. \quad \text{(C.44)}$$

Government spending is assumed to be equal to a constant fraction of steady state real output

$$G_t = \varrho Y^{ss}. \quad \text{(C.45)}$$

Supply of short-term government bonds is endogenously determined by the following intertemporal budget constraint

$$R_{g,t-1}\frac{B_{g,t-1}}{\pi_t} + G_t = T_t + B_{g,t} + \Omega_{cb,t}. \quad \text{(C.46)}$$

C.8 Central Bank

The central bank sets the lending facility rate $r_{f,t}$ according to a Taylor-type policy rule:

$$r_{f,t} = \rho_r r_{f,t-1} + (1-\rho_r)\left(r_f^{ss} + \alpha_\pi \tilde{\pi}_t + \alpha_Y \tilde{y}_t\right) + e_{r_{f,t}}. \quad \text{(C.47)}$$

A constant corridor of width $\alpha > 0$ is assumed to be maintained between the lending facility rate and the deposit facility rate,

$$r_{\widetilde{R},t} = r_{f,t} - \alpha. \quad \text{(C.48)}$$

According to the balance sheet of the central bank:

$$F_t = \widetilde{R}_t + M_t + CBDC_t. \quad \text{(C.49)}$$

Central bank's profits evolve as

$$\Omega_{cb,t} = \widetilde{R}_t + M_t + CBDC_t + R_{f,t-1}\frac{F_{t-1}}{\pi_t} - R_{\widetilde{R},t-1}\frac{\widetilde{R}_{t-1}}{\pi_t} - \frac{M_{t-1}}{\pi_t} - R_{cbdc,t-1}\frac{cbdc_{t-1}}{\pi_t} - F_t. \quad \text{(C.50)}$$

In the baseline scenario, CBDC supply is set according to the following policy rule:

$$CBDC_t = \phi_Y Y^{ss}. \quad \text{(C.51)}$$

C.9 Aggregation and Market Clearing

Market clearing is implied by the Walras' law, by aggregating all the budget constraints. The aggregate resource constraint of the economy represents the equilibrium condition for the final goods market:

$$Y_t = C_t + q_{k,t} I_t + G_t + \delta^e e_{t-1} + f(m_t). \tag{C.52}$$

Similarly, in equilibrium labor demand equals total labor supply,

$$N_t = n_{p,t} + n_{i,t}. \tag{C.53}$$

The stock of physical capital produced by capital goods producers must equal the demand for this good coming from households

$$K_t = k_{e,t}. \tag{C.54}$$

The stock of real estate must equal the demand coming from households and entrepreneurs

$$\overline{H} = h_{p,t} + h_{i,t} + h_{e,t}. \tag{C.55}$$

Similarly, in equilibrium demand for loans of households and entrepreneurs equals bank credit supply

$$l_{i,t} + l_{e,t} = L_t. \tag{C.56}$$

In equilibrium, the supply of government bonds equals the demand for this asset coming from patient households and banks

$$b_{p,t} + b_{b,t} = B_{g,t}. \tag{C.57}$$

Bank's reserves are a liability of the central bank

$$\widetilde{R}_{b,t} = \widetilde{R}_t. \tag{C.58}$$

CBDC issued by the central bank equals demand for that means of payment

$$CBDC_t = cbdc_t. \tag{C.59}$$

Cash issued by the central bank equals demand for that monetary instrument

$$M_t = m_t. \tag{C.60}$$

The stock of bank deposits held by households must be equal to banks' deposit funding

$$d_t = D_t. \tag{C.61}$$

In equilibrium, banks' demand for central bank funding equals central bank's supply of funding to banks

$$f_t = F_t. \tag{C.62}$$

C.10 Shocks

The following zero-mean, AR(1) shocks are present in the baseline calibration model: $\varepsilon_{h,t}$, $\varepsilon_{z,t}$, $\varepsilon_{\eta,t}$, $\varepsilon_{mh,t}$, $\varepsilon_{mk,t}$, A_t, $\varepsilon_{\theta_R,t}$, ${}_{\theta_b,t}$. These shocks follow the processes given by:

$$\log \varepsilon_{h,t} = \rho_h \log \varepsilon_{h,t-1} + e_{h,t},\ \ e_{h,t} \sim N(0, \sigma_h), \tag{C.63}$$

$$\log \varepsilon_{z,t} = \rho_z \log \varepsilon_{z,t-1} + e_{z,t},\ \ e_{z,t} \sim N(0, \sigma_z), \tag{C.64}$$

$$\log \varepsilon_{\eta,t} = \rho_z \log \varepsilon_{\eta,t-1} + e_{\eta,t},\ \ e_{\eta,t} \sim N(0, \sigma_\eta), \tag{C.65}$$

$$\log \varepsilon_{mh,t} = \rho_{mh} \log \varepsilon_{mh,t-1} + e_{mh,t},\ \ e_{mh,t} \sim N(0, \sigma_{mh}), \tag{C.66}$$

$$\log \varepsilon_{mk,t} = \rho_{mk} \log \varepsilon_{mk,t-1} + e_{mk,t},\ \ e_{mk,t} \sim N(0, \sigma_{mk}), \tag{C.67}$$

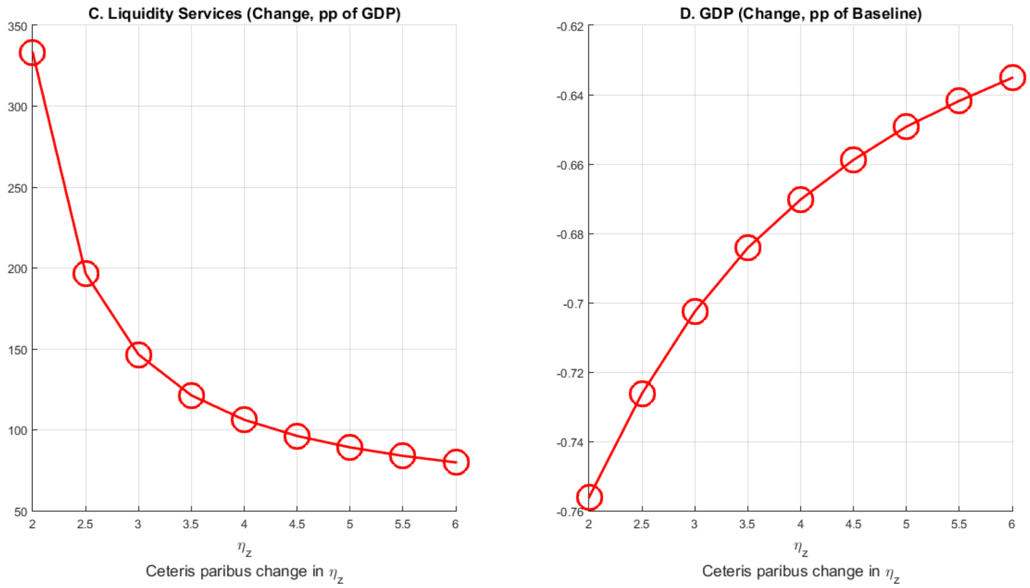
$$\log A_t = \rho_A \log A_{t-1} + e_{A,t},\ \ e_{A,t} \sim N(0, \sigma_A), \tag{C.68}$$

$$\log \varepsilon_{\theta_R,t} = \rho_{\theta_R} \log \varepsilon_{\theta_R,t-1} + e_{\theta_R,t},\ \ e_{\theta_R,t} \sim N(0, \sigma_{\theta_R}), \tag{C.69}$$

$$\log \varepsilon_{\theta_b,t} = \rho_{\theta_b} \log \varepsilon_{\theta_b,t-1} + e_{\theta_b,t},\ \ e_{\theta_b,t} \sim N(0, \sigma_{\theta_b}). \tag{C.70}$$

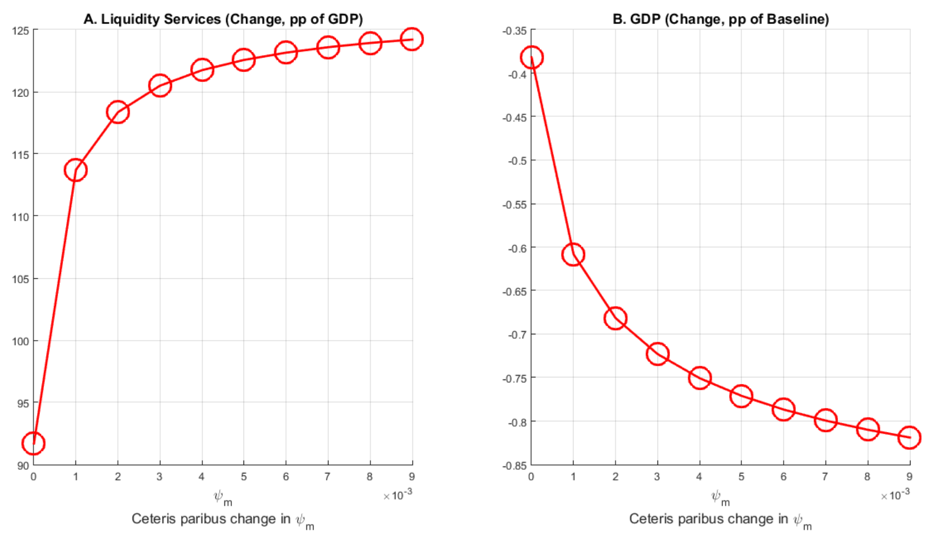
D Additional Quantitative Assessment

Figure D.1: Liquidity services expansion: η_z



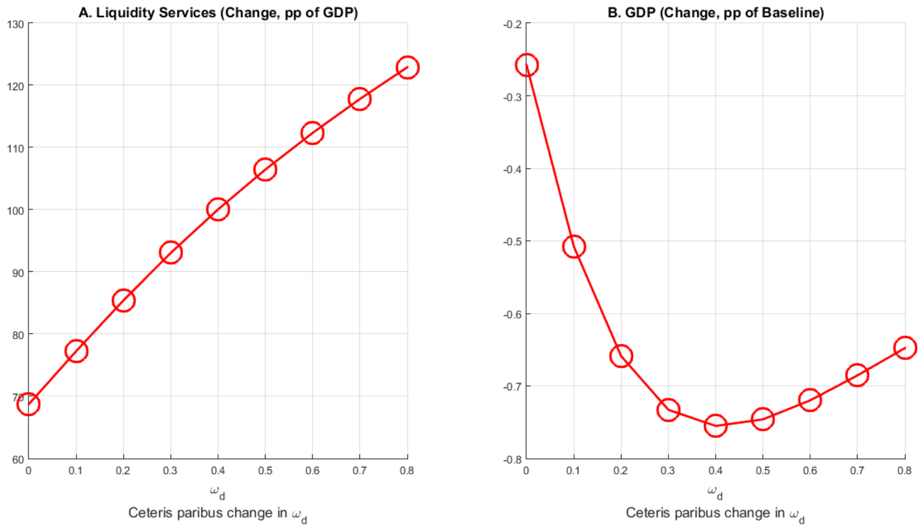
Notes: CBDC-induced changes in the steady state level of liquidity services (expressed in percentage points of GDP) and GDP (expressed in percentage points of its baseline level) - associated with the scenario under which CBDC supply in equilibrium is equal to 64.4% of quarterly GDP (i.e., zero CBDC interest rate) - for different values of the elasticity of substitution across monetary instruments, η_z.

Figure D.2: Liquidity services expansion: ψ_m



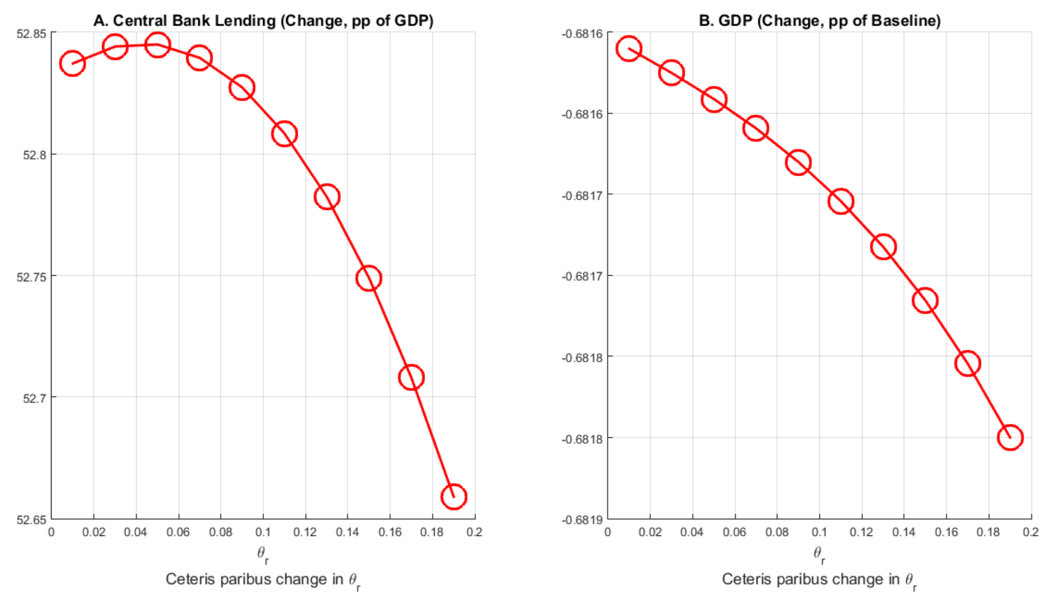
Notes: CBDC-induced changes in the steady state level of liquidity services (expressed in percentage points of GDP) and GDP (expressed in percentage points of its baseline level) - associated with the scenario under which CBDC supply in equilibrium is equal to 64.4% of quarterly GDP (i.e., zero CBDC interest rate) - for different values of the cash storage cost parameter, ψ_m.

Figure D.3: Liquidity services expansion: ω_d



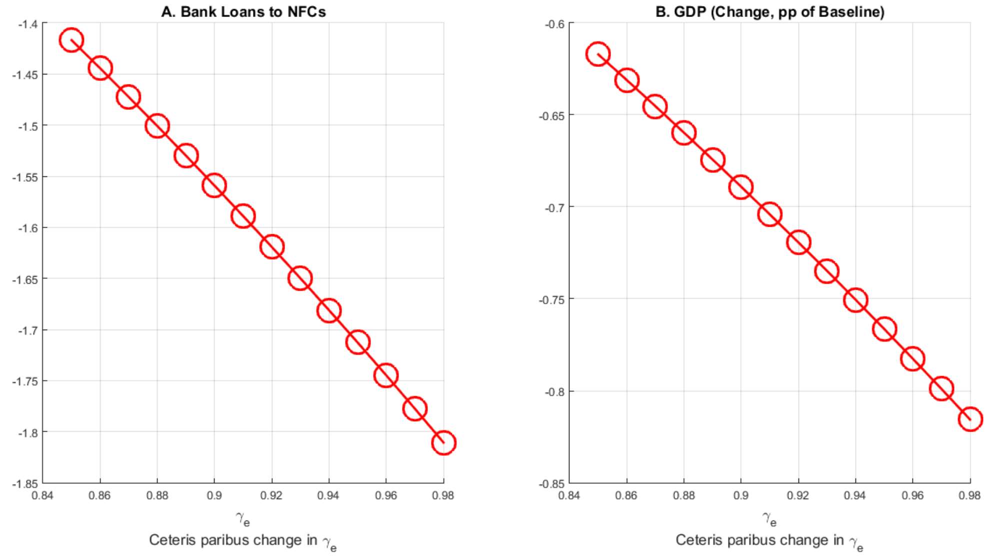
Notes: CBDC-induced changes in the steady state level of liquidity services (expressed in percentage points of GDP) and GDP (expressed in percentage points of its baseline level) -associated with the scenario under which CBDC supply in equilibrium is equal to 64.4% of quarterly GDP (i.e., zero CBDC interest rate) - for different values of the weight parameter of bank deposits in liquidity services, ω_d.

Figure D.4: Central bank balance sheet expansion: θ_r



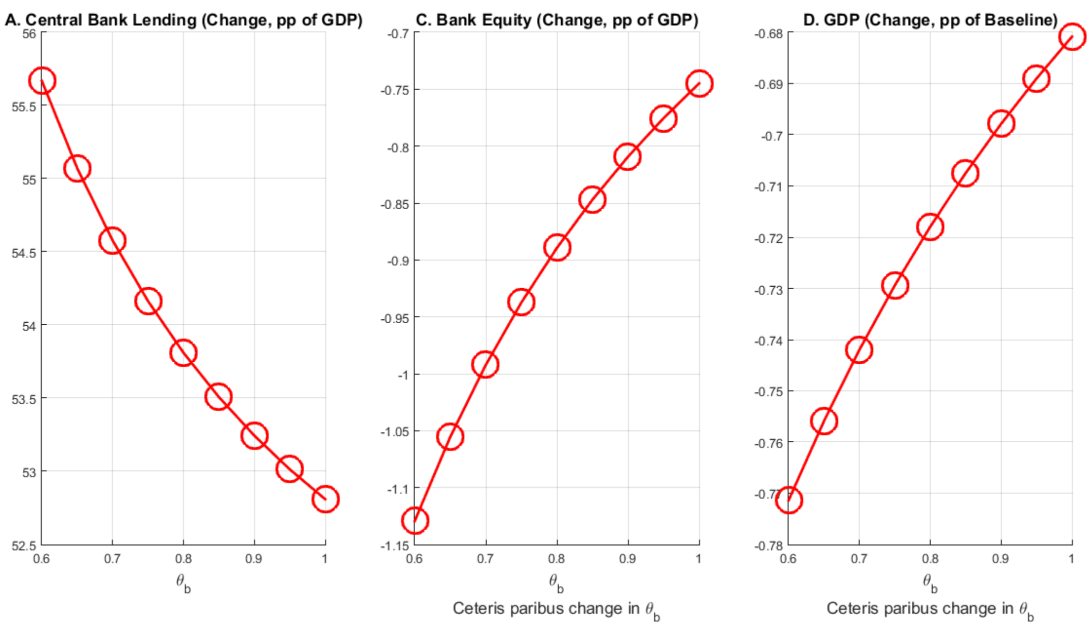
Notes: CBDC-induced changes in the steady state level of central bank lending (expressed in percentage points of GDP) and GDP (expressed in percentage points of its baseline level) - associated with the scenario under which CBDC supply in equilibrium is equal to 64.4% of quarterly GDP (i.e., zero CBDC interest rate) - for different values of the reserve requirements parameter, θ_r.

Figure D.5: Bank disintermediation: γ_e



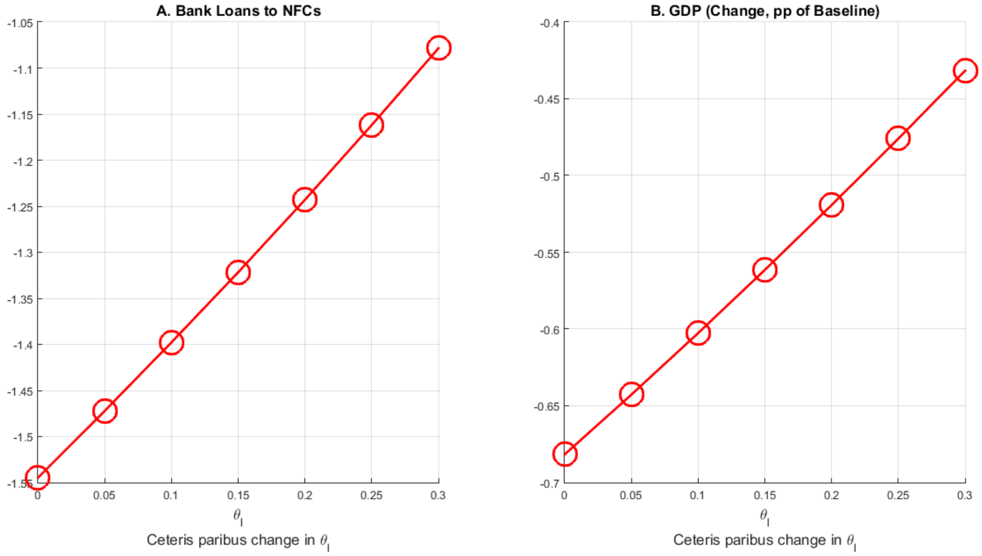
Notes: CBDC-induced changes in the steady state level of bank loans to non-financial corporations (expressed in percentage points of GDP) and GDP (expressed in percentage points of its baseline level) associated with the scenario under which CBDC supply in equilibrium is equal to 64.4% of quarterly GDP (i.e., zero CBDC interest rate) - for different values of sectoral (NFCs) capital requirements parameter γ_e.

Figure D.6: Bank disintermediation: θ_b



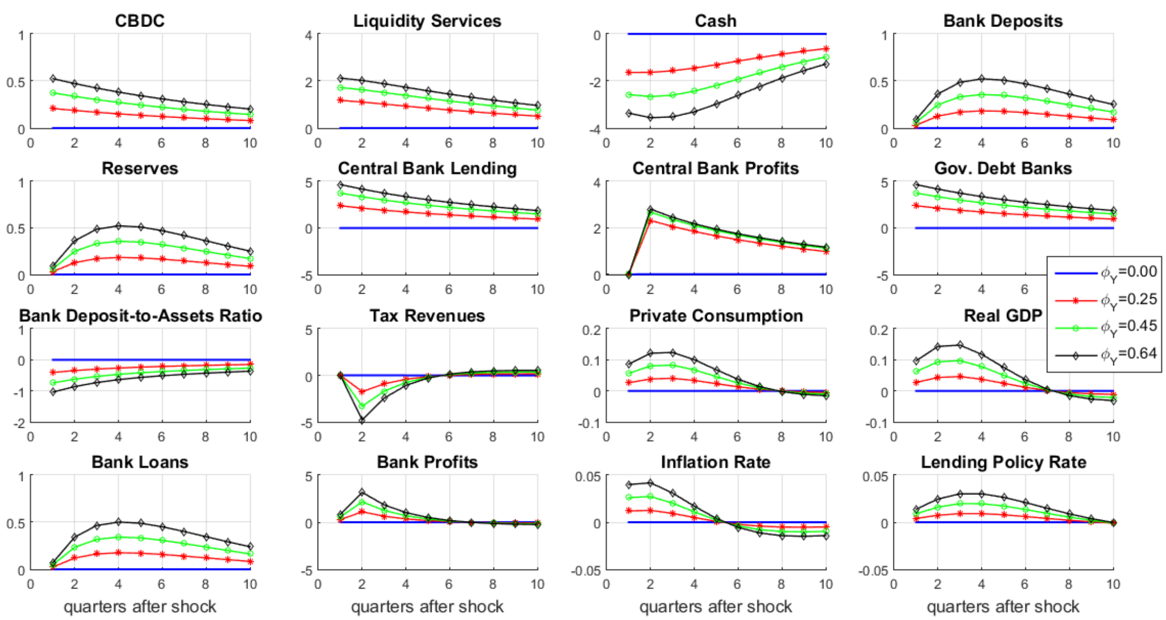
Notes: CBDC-induced changes in the steady state level of central bank lending, bank profits and equity (expressed in percentage points of GDP) as well as GDP (expressed in percentage points of its baseline level) - associated with the scenario under which CBDC supply in equilibrium is equal to 64.4% of quarterly GDP (i.e., zero CBDC interest rate) - for different values of central bank collateral requirement parameter θ_b.

Figure D.7: Bank disintermediation: θ_l



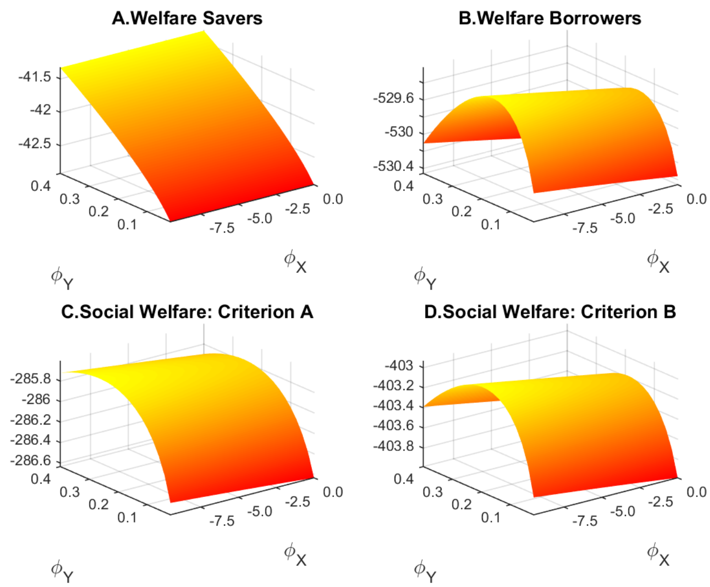
Notes: CBDC-induced changes in the steady state level of bank loans to non-financial corporations (expressed in percentage points of GDP) and GDP (expressed in percentage points of its baseline level) associated with the scenario under which CBDC supply in equilibrium is equal to 64.4% of quarterly GDP (i.e., zero CBDC interest rate) - for different values of the collateral requirement on NFC loans parameter, θ_l.

Figure D.8: Transmission and cyclical effects. Impulse-responses to a positive CBDC supply shock



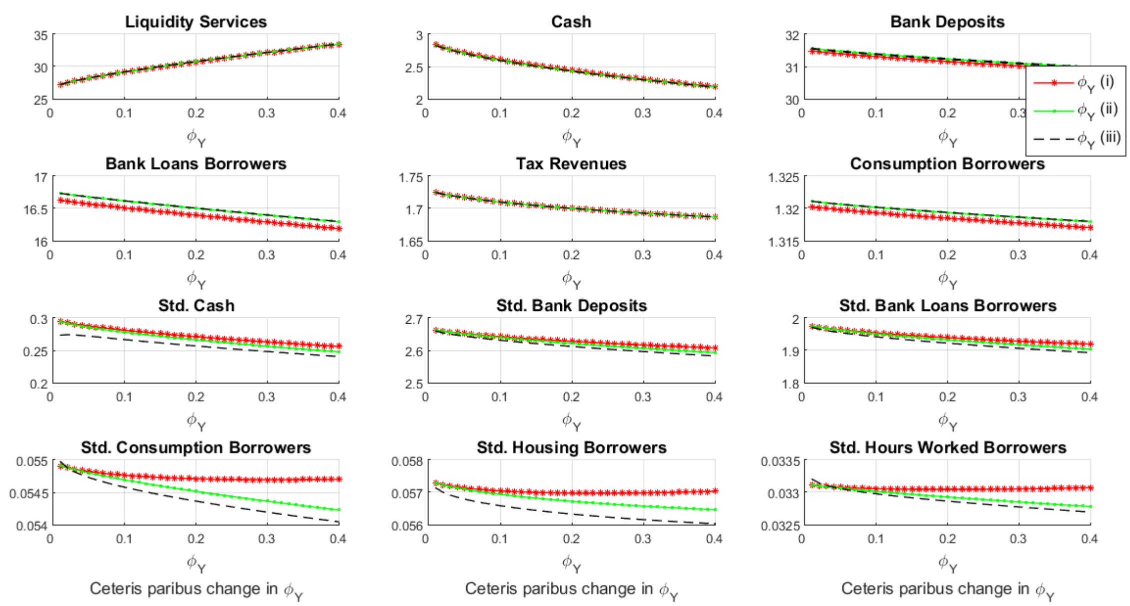
Notes: Variables are expressed in percentage deviations from the steady state with the exceptions of CBDC, the inflation rate and the lending policy rate, which are shown as absolute deviations from the steady state. These two rates have been annualized and are expressed in percentage points. The solid line refers to the baseline (no CBDC) scenario. The starred, dotted, and diamond lines make reference to alternative scenarios under which CBDC supply in equilibrium is equal to 25%, 45% and 64% of quarterly real GDP, respectively.

Figure D.9: Welfare effects of CBDC quantity rule (iii) (welfare effects of ceteris paribus changes in $\phi_Y - \phi_X$)



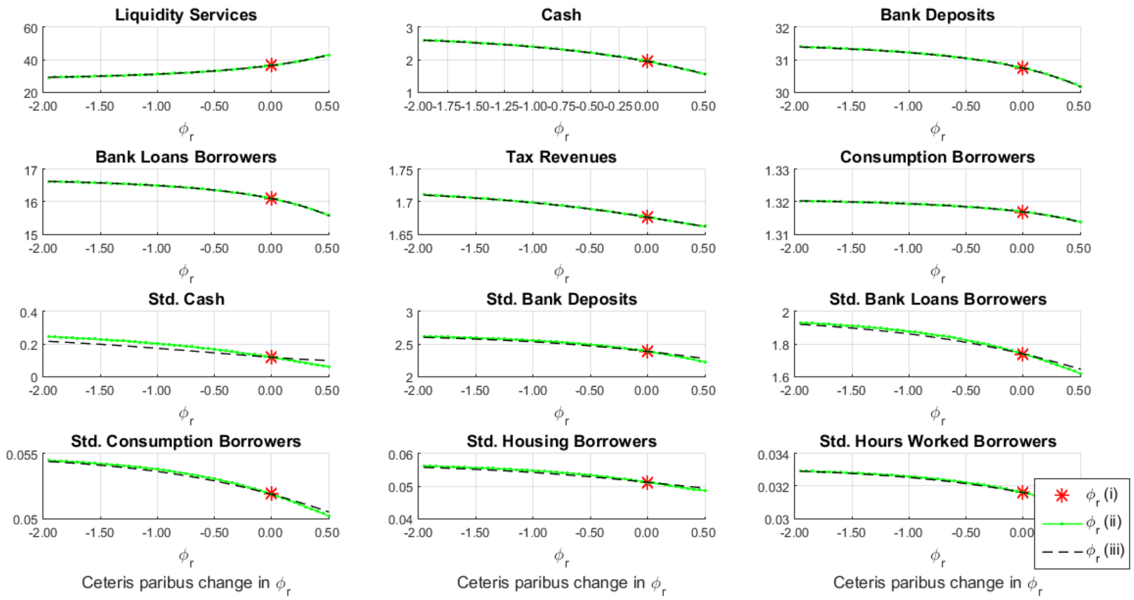
Notes: Second-order approximation to the unconditional welfare of savers and borrowers as well as to the unconditional social welfare under welfare criteria "A" and "B" as a function of CBDC policy parameters ϕ_Y and ϕ_X under CBDC quantity rule (iii).

Figure D.10: Mean and volatility effects of CBDC quantity rules (welfare effects of ceteris paribus changes in ϕ_Y)



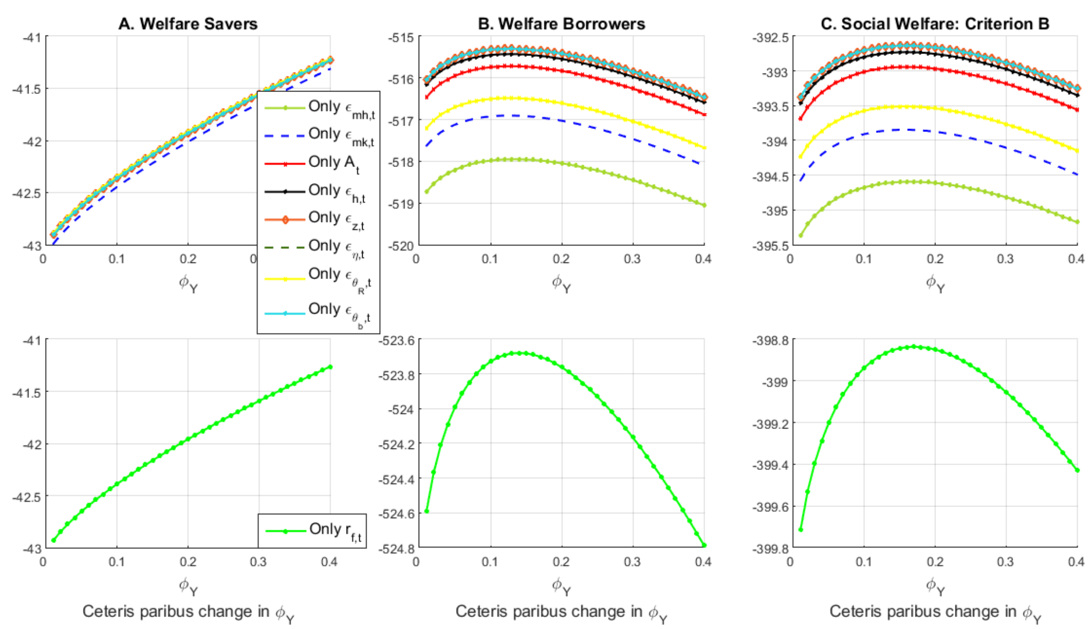
Notes: Second-order approximation to the stochastic mean and standard deviation of key selected aggregates as a function of CBDC policy parameter ϕ_Y. The starred line, the dotted line, and the dashed line relate to CBDC quantity rules (i), (ii) and (iii), respectively.

Figure D.11: Mean and volatility effects of CBDC interest rate rules (welfare effects of ceteris paribus changes in ϕ_r)



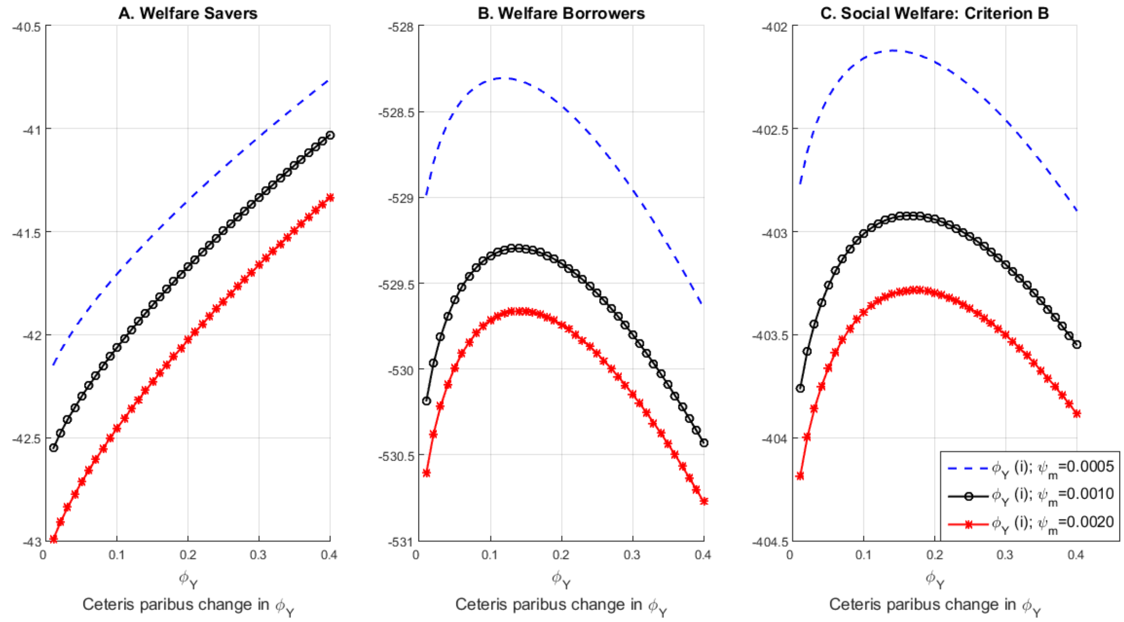
Notes: Second-order approximation to the stochastic mean and standard deviation of key selected aggregates as a function of CBDC policy parameter ϕ_r. The star, the dotted line, and the dashed line relate to CBDC interest rate rules (i), (ii) and (iii), respectively.

Figure D.12: Welfare effects of CBDC quantity rules by types of shocks (shutting down shocks)



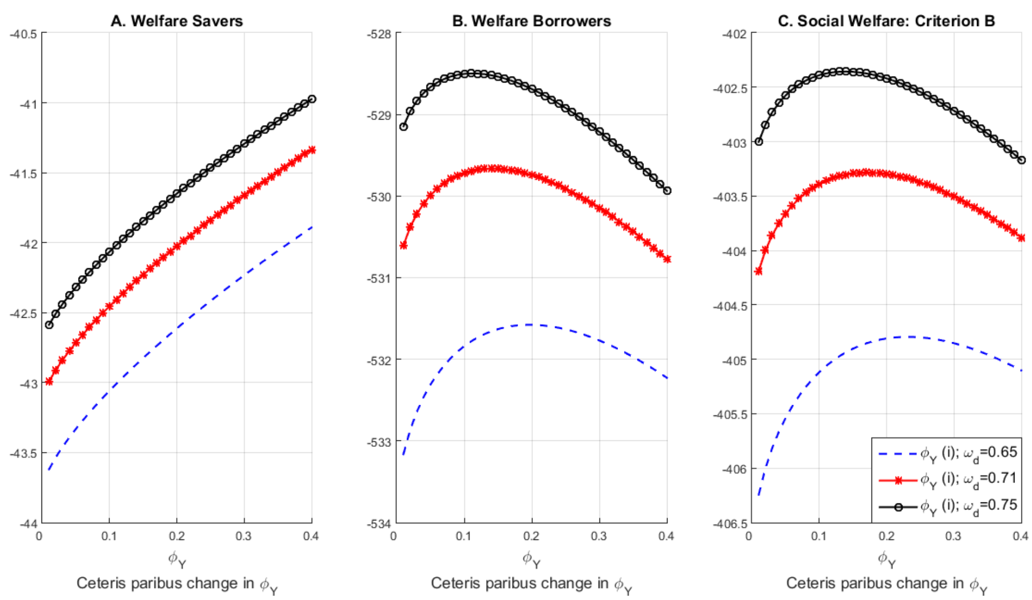
Notes: Second-order approximation to the unconditional welfare of savers and borrowers as well as to the unconditional social welfare under welfare criterion "B" as a function of CBDC policy parameter ϕ_Y under quantity rule of type (i). Each of the 9 lines informs about the welfare effects of ceteris paribus changes in ϕ_Y when only one of the nine types of shocks that are considered under the baseline calibration hits this model economy.

Figure D.13: Robustness Checks: ψ_m (welfare effects of ceteris paribus changes in ϕ_Y)



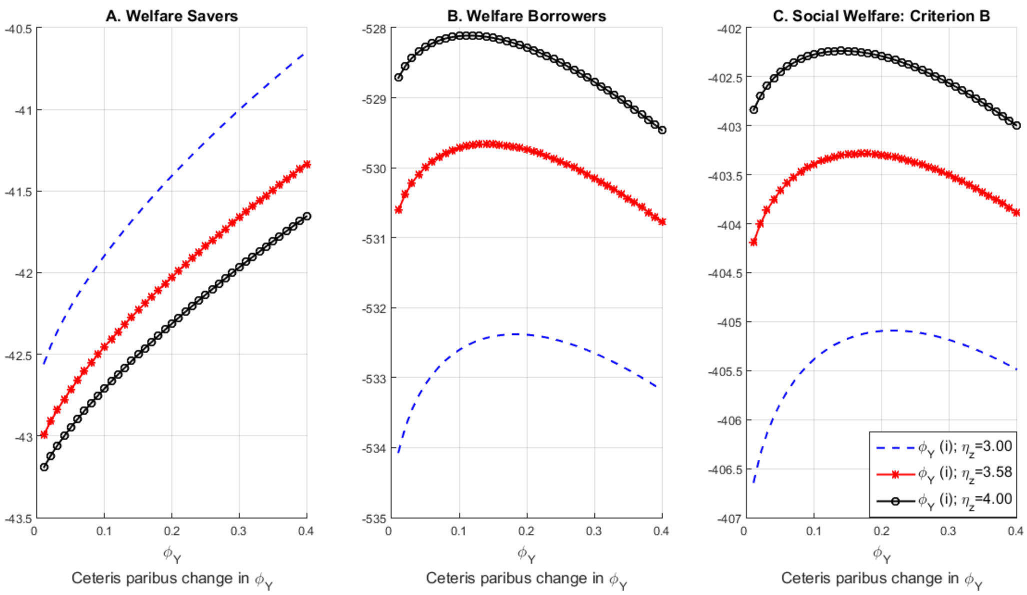
Notes: Second-order approximation to the unconditional welfare of savers and borrowers as well as to the unconditional social welfare under welfare criterion "B" for CBDC quantity rule (i) as a function of policy parameter ϕ_Y, for alternative values of the cash storage cost parameter, ψ_m. The starred line refers to the baseline calibration whereas the dotted and dashed lines relate to alternative parameterization scenarios.

Figure D.14: Robustness Checks: ω_d (welfare effects of ceteris paribus changes in ϕ_Y)



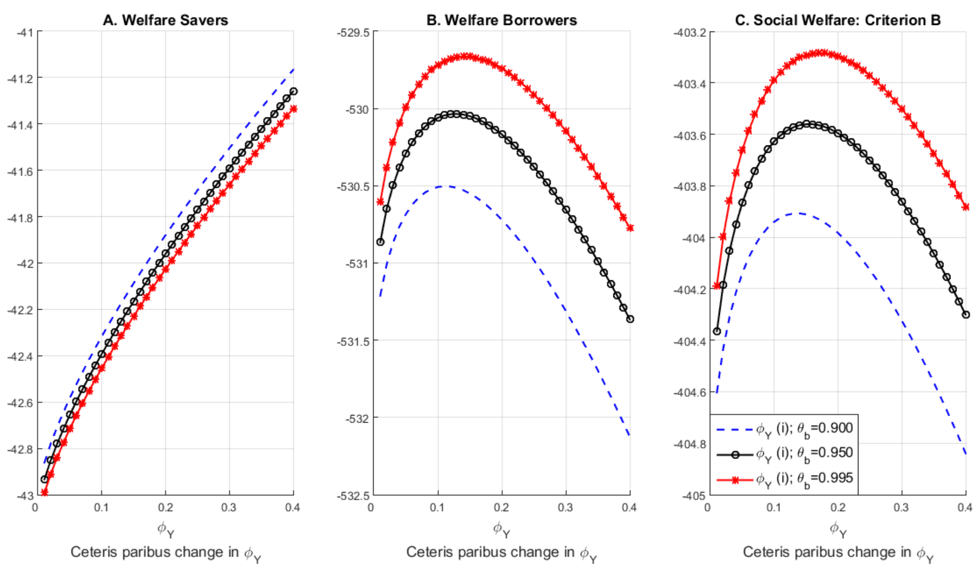
Notes: Second-order approximation to the unconditional welfare of savers and borrowers as well as to the unconditional social welfare under welfare criterion "B" for CBDC quantity rule (i) as a function of policy parameter ϕ_Y, for alternative values of the deposits preference parameter ω_d. The starred line refers to the baseline calibration whereas the dotted and dashed lines relate to alternative parameterization scenarios.

Figure D.15: Robustness Checks: η_z (welfare effects of ceteris paribus changes in ϕ_Y)



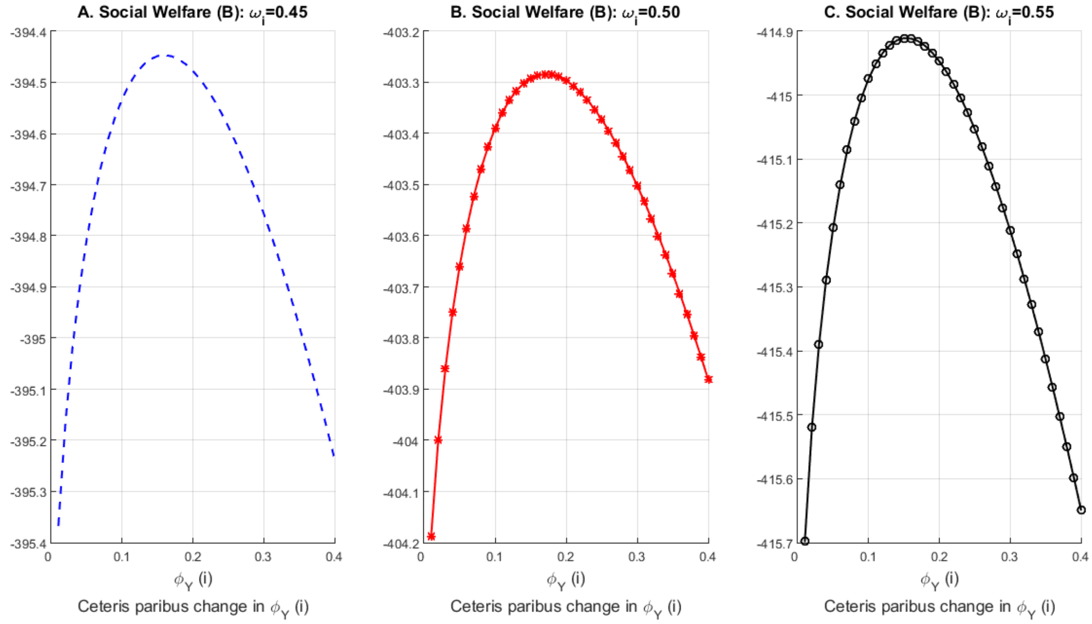
Notes: Second-order approximation to the unconditional welfare of savers and borrowers as well as to the unconditional social welfare under welfare criterion "B" for CBDC quantity rule (i) as a function of policy parameter ϕ_Y, for alternative values of the elasticity of substitution across forms of money, η_z. The starred line refers to the baseline calibration whereas the dotted and dashed lines relate to alternative parameterization scenarios.

Figure D.16: Robustness Checks: θ_b (welfare effects of ceteris paribus changes in ϕ_Y)



Notes: Second-order approximation to the unconditional welfare of savers and borrowers as well as to the unconditional social welfare under welfare criterion "B" for CBDC quantity rule (i) as a function of policy parameter ϕ_Y, for alternative values of the central bank's collateral requirement parameter for government bonds, θ_b. The starred line refers to the baseline calibration whereas the dotted and dashed lines relate to alternative parameterization scenarios.

Figure D.17: Robustness Checks: ω_i (welfare effects of ceteris paribus changes in ϕ_Y)



Notes: Second-order approximation to the unconditional social welfare under welfare criterion "B" for CBDC quantity rule (i) as a function of policy parameter ϕ_Y, for alternative values of borrowers' weight in the economy, ω_i. The starred line refers to the baseline calibration whereas the dotted and dashed lines relate to alternative parameterization scenarios.