PERSUASION WITH CORRELATION NEGLECT: A FULL MANIPULATION RESULT

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Online Appendix

A1. Proof of Lemma 2

A receiver with correlation neglect believes that the conditional joint distribution of the signals is the product of the conditional marginals:

$$q(s_1, s_2, ..., s_m \mid \omega) = \prod_{i=1}^{m} q_i(s_i \mid \omega).$$

Therefore upon observing realisation $s = (s_1, ..., s_m)$ which leads to posteriors $\boldsymbol{\mu} = (\mu_1, ..., \mu_m)$ with $\mu_i(\omega) = (p(\omega)q_i(s_i \mid \omega))/(\sum_\upsilon p(\upsilon)q_i(s_i \mid \upsilon))$, her posterior belief is:

$$\begin{aligned}
\frac{p(\omega)q(s_1,...,s_m|\omega)}{\sum_\upsilon p(\upsilon)q(s_1,...,s_m|\upsilon)} &= \frac{p(\omega)\prod_{i=1}^m q_i(s_i|\omega)}{\sum_\upsilon p(\upsilon)\prod_{i=1}^m q_i(s_i|\upsilon)} \\
&= \frac{(1/p(\omega)^{m-1})\prod_{i=1}^m p(\omega)q_i(s_i|\omega)/\sum_\nu p(\nu)q_i(s_i|\nu)}{\sum_\upsilon(1/p(\upsilon)^{m-1})\prod_{i=1}^m p(\upsilon)q_i(s_i|\upsilon)/\sum_\nu p(\nu)q_i(s_i|\nu)} \\
&= \frac{\prod_{i=1}^m \mu_i(\omega)\big/p(\omega)^{m-1}}{\sum_{\upsilon\in\Omega}\prod_{i=1}^m \mu_i(\upsilon)\big/p(\upsilon)^{m-1}}
\end{aligned}$$

Hence we can write,

$$\mu^{CN}(\boldsymbol{\mu})(\omega) = \frac{\prod_{i=1}^m \mu_i(\omega)\Big/ p(\omega)^{m-1}}{\sum_{\upsilon\in\Omega}\left(\prod_{i=1}^m \mu_i(\upsilon)\Big/ p(\upsilon)^{m-1}\right)}$$

□

A2. Additional proofs for Theorem 1

We prove that the joint conditional distribution described in (10) is well defined.

First, we show that $0 \le \gamma^\omega \le 1$. From the definition, γ^ω is trivially positive. Moreover, it cannot be that both $\alpha/z^\omega_\omega > 1$ and $\beta/(\max_{\upsilon\neq\omega}\{z^\omega_\upsilon\}) > 1$. If that were the case, then $z^\omega_\omega < \alpha$ and $\max_{\upsilon\neq\omega}\{z^\omega_\upsilon\} < \beta$. But then $\sum_\upsilon z^\omega_\upsilon \le z^\omega_\omega + (n-1)\max_{\upsilon\neq\omega}\{z^\omega_\upsilon\} < \alpha + (n-1)\beta = 1$, which contradicts that $\sum_\upsilon z^\omega_\upsilon = 1$. Therefore, $\gamma^\omega \le 1$.

Second we show that both $0 \le \lambda^\omega_\omega \le 1$ and $0 \le \lambda^\omega_\upsilon \le 1$. Again, from the definition it is trivial to see that these numbers are below 1. To see that they are positive note that, $\lambda^\omega_\omega = \alpha - \gamma^\omega z^\omega_\omega \ge \alpha - (\alpha/z^\omega_\omega)z^\omega_\omega = 0$ and $\lambda^\omega_\upsilon = \beta - \gamma^\omega z^\omega_\upsilon \ge \beta - (\beta/z^\omega_\upsilon)z^\omega_\upsilon = 0$.

Lastly, $\gamma^\omega + \sum_v \lambda_v^\omega = \gamma^\omega(1 - \sum_v z_v^\omega) + \alpha + (n-1)\beta = 1$, therefore $\tau(\cdot \mid \omega)$ is indeed a distribution.

We now address the case in which ρ^ω is not interior. Consider a sequence of interior posteriors $\{\rho_m^\omega\}_{m\in\mathbb{N}}$ converging to ρ^ω and we replicate the proof by replacing in equation (6) ρ^ω by ρ_m^ω. Hence with probability converging to one, the information structure generates vectors of proportions $z^\omega(m)$ that satisfy:

$$0 \leq \lim_{m\to\infty} |\mu^{CN}(z^\omega(m)) - \rho^\omega| \leq \lim_{m\to\infty} |\mu^{CN}(z^\omega(m)) - \rho_m^\omega| + \lim_{m\to\infty} |\rho_m^\omega - \rho^\omega| = 0.$$

□

A3. *Proof of Corollary 1*

Recall from Section II.B that for each $\omega \in \Omega$, we defined $v^\omega(\mu) = \max_{a\in A_\mu} v(a,\omega)$ and that $v^\omega(\cdot)$ is continuous for all $\mu \in \Delta(\Omega)$ but for a finite set of posteriors. If ρ^ω is such that v^ω is continuous at ρ^ω, then given Theorem 1, the expected utility of the sender conditional on state ω converges to $\bar{v}^\omega$.

Suppose now that ρ^ω is one of the finite points for which $A^\omega_{\rho^\omega}$ is not continuous. Then, by Assumption 1 there exists $a \in A^\omega_{\rho^\omega}$ and sequences $\{\rho_m^\omega\}_{m=1}^\infty$, $\{a_m\}_{m=1}^\infty$ with $\rho_m^\omega \neq \rho^\omega$ and $a_m \in A_{\rho_m^\omega}$, such that $\rho_m^\omega \to_{m\to\infty} \rho^\omega$ and $a_m \to_{m\to\infty} a$. Note that without loss of generality we can assume that for any m, v^ω is continuous at ρ_m^ω. Now, we use the proof of Theorem 1 in which at each m we replace ρ^ω in equation (6) by ρ_m^ω. Therefore, with probability converging to one, the information structure generates vectors of proportions $z^\omega(m)$ that satisfy:

$$|v(a_{\mu^{CN}(z^\omega(m))}, \omega) - \bar{v}^\omega| \leq |v(a_{\mu^{CN}(z^\omega(m))}, \omega) - v(a_m, \omega)| + |v(a_m, \omega) - \bar{v}^\omega| \longrightarrow_{m\to\infty} 0.$$

Therefore, the ex-ante expected utility of the sender converges to $\sum_{\omega\in\Omega} p(\omega)\bar{v}^\omega$. □

A4. *Proof of Proposition 1*

Suppose first that $\rho(\omega) \neq 0$ for all $\omega \in \Omega$. Let $\hat{\mu}_m \in \Delta(\Omega)$ be defined in the following way:

$$\hat{\mu}_m(\omega) = \frac{\rho(\omega)^{\frac{1}{m}} p(\omega)^{\frac{m-1}{m}}}{\sum_{v\in\Omega} \rho(v)^{\frac{1}{m}} p(v)^{\frac{m-1}{m}}},$$

which then implies that:

$$\mu_m^{FC}(\hat{\mu}_m) = \rho.$$

Note that since p is interior, we can always design a signal structure τ^m of fully positively correlated signals that have two vectors of posteriors in the support, $\hat{\boldsymbol{\mu}}_m = (\hat{\mu}_m, ..., \hat{\mu}_m)$ and $\boldsymbol{\mu}'_m = (\mu'_m, ..., \mu'_m)$, where μ'_m is at a fixed distance $\delta > 0$ from p. The weights are then pinned down by the Bayesian Plausibility constraint such that:

$$\tau^m(\hat{\boldsymbol{\mu}}_m)\hat{\mu}_m + (1 - \tau^m(\hat{\boldsymbol{\mu}}_m))\mu'_m = p.$$

Note that when $m \to \infty$, then $\hat{\mu}_m$ is arbitrarily close to p, as

$$\hat{\mu}_m(\omega) = \frac{p(\omega)^{1-\frac{1}{m}}}{\sum_{\upsilon\in\Omega}\left(\frac{\rho(\upsilon)}{\rho(\omega)}\right)^{\frac{1}{m}} p(\upsilon)^{1-\frac{1}{m}}} \to_{m\to\infty} p(\omega).$$

As a result, given the Bayesian Plausibility constraint, and maintaining μ'_m always at a fixed distance $\delta > 0$ away from p, $\tau^m(\hat{\boldsymbol{\mu}}_m) \to_{m\to\infty} 1$. This implies that for any $\epsilon > 0$,

$$\lim_{m\to\infty} \tau^m(\{\boldsymbol{\mu} = (\mu, ..., \mu) \text{ s.t. } \tau^m(\boldsymbol{\mu}) > 0 \mid |\mu_m^{FC}(\mu) - \rho| < \epsilon\}) = 1.$$

Lastly, consider the case in which $\rho(\omega) = 0$ for some $\omega \in \Omega$. Consider a sequence $\{\rho_m\}_{m=1}^{\infty}$ such that for any m, $\rho_m(\upsilon) > 0$ for any $\upsilon \in \Omega$ and $\lim_{m\to\infty}\rho_m = \rho$. Moreover, we can choose the sequence to satisfy for any $\upsilon, \omega \in \Omega$, $\lim_{m\to\infty}(\rho_m(\upsilon)/\rho_m(\omega))^{1/m} = 1$.

Let $\hat{\mu}_m \in \Delta(\Omega)$ be defined in the following way:

$$\hat{\mu}_m(\omega) = \frac{(\rho_m(\omega))^{\frac{1}{m}} p(\omega)^{\frac{m-1}{m}}}{\sum_{\upsilon\in\Omega}(\rho_m(\upsilon))^{\frac{1}{m}} p(\upsilon)^{\frac{m-1}{m}}},$$

which then implies that:

$$\mu_m^{FC}(\hat{\mu}_m) = \rho_m.$$

Note that since p is interior, we can always design a signal structure τ^m of fully positively correlated signals with two vectors of posteriors in the support, $\hat{\boldsymbol{\mu}}_m = (\hat{\mu}_m, ..., \hat{\mu}_m)$ and $\boldsymbol{\mu}'_m = (\mu'_m, ..., \mu'_m)$, where μ'_m is at a fixed distance $\delta > 0$ from p. The weights are then pinned down by the Bayesian Plausibility constraint such that:

$$\tau^m(\hat{\boldsymbol{\mu}}_m)\hat{\mu}_m + (1 - \tau^m(\hat{\boldsymbol{\mu}}_m))\mu'_m = p.$$

Note that when $m \to \infty$, then $\hat{\mu}_m$ is arbitrarily close to p, as

$$\hat{\mu}_m(\omega) = \frac{p(\omega)^{\frac{m-1}{m}}}{\sum_{\upsilon\in\Omega}\left(\frac{\rho_m(\upsilon)}{\rho_m(\omega)}\right)^{\frac{1}{m}} p(\upsilon)^{\frac{m-1}{m}}} \to_{m\to\infty} p(\omega).$$

As a result, given the Bayesian Plausibility constraint, and maintaining μ'_m always at a fixed distance $\delta > 0$ away from p, $\tau^m(\hat{\boldsymbol{\mu}}_m) \to_{m\to\infty} 1$. As $\lim_{m\to\infty}\rho_m = \rho$, this implies that for any $\epsilon > 0$,

$$\lim_{m\to\infty} \tau^m(\{\boldsymbol{\mu} = (\mu, ..., \mu) \text{ s.t. } \tau^m(\boldsymbol{\mu}) > 0 \mid |\mu_m^{FC}(\mu) - \rho| < \epsilon\}) = 1.$$

□