Online Appendix for "Information Choice in Auctions"

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B Calculations for Example 4

Pure CV equilibrium. Consider a pure CV equilibrium where bidders learn $X_i^{1,0} \sim \mathcal{N}(0, 1 + \frac{1}{2})$. By standard Gaussian updating, $X_2^{1,0} | X_1^{1,0} = x \sim \mathcal{N}(\frac{2x}{3}, \frac{1}{3} + \frac{1}{2})$, and

$$V_1 | X_1^{1,0} = x, X_2^{1,0} = y \sim \mathcal{N}(\frac{2(x+y)}{5}, \frac{1}{5} + \frac{1}{3}).$$

Then, for the log-additive valuation specification,

$$\mathbb{E}[V_1 | X_1^{1,0} = x, X_2^{1,0} = y] = exp\left(\frac{2(x+y)}{5} + \frac{1}{2}(\frac{1}{5} + \frac{1}{3})\right).$$

Plugging this into the standard expression for the bid and expected utility of a bidder (see Milgrom and Weber (1982)) yields an expected utility of approximately 0.3636 in the SPA, and 0.5426 in the FPA.

Without loss, let bidder 1 deviate to $X_1^{1,1}$, resolving all uncertainty about $S + T_1$: $\mathbb{E}[V_1 | X_1^{1,1} = x, X_2^{1,0} = y] = exp(x)$. With the deviation, bidder 1's marginal distribution is $X_1^{1,1} \sim \mathcal{N}(0, 1 + \frac{1}{3})$, and $X_2^{1,0} | X_1^{1,1} = x \sim \mathcal{N}(\frac{3x}{4}, \frac{1}{4} + \frac{1}{2})$. For the SPA, bidder 1's optimal bid with $X_1^{1,1} = x$ is the bid of type $X_1^{1,0} = \frac{5}{4}x - \frac{1}{3}$ in the candidate equilibrium. Calculating expected utility again for these distributions and optimal bid strategy shows that bidder 1 can achieve at most a deviation payoff of approximately 0.8626. This establishes the existence of a pure CV equilibrium in the SPA since the costs of learning

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$X_1^{1,1}$ instead of $X_1^{1,0}$ are $c = 0.5$, but the deviation payoff is only 0.4990 above the candidate equilibrium payoff.

For the FPA, consider a deviation strategy where a bidder with signal realization $X_1^{1,1} = x$ bids as if he observed a signal $X_1^{1,0} = \frac{5}{4}x - \frac{1}{3}$ (mimicking the same types as in the SPA deviation strategy). This possibly suboptimal deviation strategy yields an expected payoff of 1.0623: an additional payoff of approximately 0.5197 above the candidate equilibrium, and worth incurring the learning costs of 0.5. Hence, there exists no pure CV equilibrium in the FPA.

Perfectly revealing equilibrium. Next, I show that learning $X_i^{1,1} \sim \mathcal{N}(0, 1 + \frac{1}{3})$ is an equilibrium in the FPA, but not in the SPA. By standard Gaussian updating, it holds that $X_2^{1,1}|X_1^{1,1} = x \sim \mathcal{N}(\frac{3x}{4}, \frac{1}{4} + \frac{1}{3})$. Moreover, $\mathbb{E}[V_1|X_1^{1,1} = x, X_2^{1,1} = y] = exp(x)$. Plugging this into the standard expressions for the expected utility yields an expected payoff of 0.8451 for the SPA, and 1.0494 for the FPA.

Let bidder 1 deviate to learning $X_1^{1,0}$. Then, $X_2^{1,1}|X_1^{1,0} = x \sim \mathcal{N}(\frac{2x}{3}, \frac{1}{3} + \frac{1}{3})$ and

$$\mathbb{E}[V_1|X_1^{1,0} = x, X_2^{1,1} = y] = exp\left(\frac{2x + 3y}{6} + \frac{1}{2}\left(\frac{1}{6} + \frac{1}{3}\right)\right).$$

In the SPA, following this deviation and learning $X_1^{1,0} = x$, bidder 1's optimal bid is the same as that of $X_1^{1,1} = \frac{1}{6}(3 + 4x)$ in the candidate equilibrium. The expected utility achievable with the deviation is approximately 0.3826. In sum, the deviation saves $c = 0.5$ learning costs while decreasing the expected payoff by only approximately 0.4624. Therefore, no equilibrium exists in the SPA where both bidders learn $X_1^{1,1}$.

In the FPA, the optimal bid following the deviation can be calculated numerically. Bidder 1's expected payoff when deviating and bidding optimally is approximately 0.5204. The loss from the deviation of approximately 0.5289 is strictly more than the gain from saving the learning costs $c = 0.5$; learning $X_1^{1,0}$ is not a profitable deviation. Hence, in the FPA, there exists an equilibrium where both bidders learn $X_1^{1,1}$.

C Robustness and limitations of Proposition 2

In this section, I analyze the robustness of the insight of Proposition 2 that the SPA induces learning only about the private component. To set intuition and for tractability, I use a simplified model with a discrete information choice: learning only about the common component, or the private one. This provides a reasonable benchmark in many contexts where information is discrete. For example, drilling an exploratory well only halfway to the required depth, or performing only the first half of a laboratory procedure does not provide any useful information.

Consider $N = 2$ bidders with additive valuation $V_i = S + T_i$. Bidders can learn costlessly about the common component or their private one, but not both. If bidder i learns about the common component and if its realization is $S = s$, then she observes a random variable X_i^S with full support normalized on $[0, 1]$ and density $f^S(.|s)$. If bidder i learns about her private component and if its realization is $T_i = t$, then she observes X_i^T with full support normalized on $[0, 1]$ and density $f^T(.|t)$.

The *information choice* of bidder i is a probability $\sigma_i \in [0, 1]$ of learning X_i^S. With the remaining probability $1 - \sigma_i$, bidder i learns X_i^T. Again, the model nests a pure CV framework if $\sigma_1 = \sigma_2 = 1$, and an IPV framework if $\sigma_1 = \sigma_2 = 0$. Signals satisfy the following assumption:

Assumption 5.

(i) $X_1^T \perp\!\!\!\perp X_2^T$ *and* $X_i^S \perp\!\!\!\perp X_j^T$ *for* $i, j \in \{1, 2\}$*;*

(ii) $X_1^S \perp\!\!\!\perp X_2^S \mid S$*;*

(iii) for all $\ell' > \ell$ *and* $L \in \{S, T\}$, $\frac{f^L(x|\ell')}{f^L(x|\ell)}$ *is strictly increasing in* $x \in [0, 1]$.

Next, I derive all symmetric, monotonic Bayesian Nash equilibria $\{\sigma, \beta_S, \beta_T\}$ in which bidders have the same information choice σ, and use the same increasing bidding function: β_S if learning their common component signal X_i^S, and β_T if learning X_i^T.

C.1 Analysis: mixed learning strategies

The following main result shows that the SPA is the *ex-ante efficient* auction format.

Proposition 6. *Let* $\mathbb{E}[V_i|X_i^S] \overset{d}{=} \mathbb{E}[V_i|X_i^T]$ *for* $i = 1, 2$. *In any symmetric equilibrium of the SPA,* $\sigma^* = 0$. *There exists an equilibrium with* $\sigma^* = 0$ *and* $\beta_T^*(X_i^T) = \mathbb{E}[V_i|X_i^T = x]$.

Proof. The existence of an IPV equilibrium follows from Assumption 2. If bidder 2 learns X_2^T, then there is no winner's curse for bidder 1. If it is optimal for bidder 1 to bid b following $X_1^S = x$, then bidding b is also optimal when $X_1^T = x$. This is because both signals lead to the same expectation of V_i, and both are independent of the other bidder's bid. Both signals X_1^T and X_1^S are distributed identically, so a bidder is indifferent between learning either since they both lead to the same strategic problem.

Next, I show that no symmetric equilibrium with $\sigma > 0$ can exist. By contradiction, let there exist a candidate equilibrium $\{\sigma^c > 0, \beta_S^c, \beta_T^c\}$. Then, bidder 1 has a profitable deviation: learn X_1^T ($\sigma_1 = 0$) and bid $\beta_T(x) = \beta_S^c(x)$. This deviation yields a strictly higher payoff than learning X_1^S and bidding β_S^c, as I show next.

Lemma 7 (Equal winning probability)**.** *Let $\{\sigma^c > 0, \beta_S^c, \beta_T^c\}$ be a candidate equilibrium. Bidder 1 wins with the same probability if (i) she plays the candidate equilibrium and learns X_1^S, or (ii) she plays the above deviation.*

Proof. With probability σ^c, bidder 2 learns X_2^S. Then, winning probability in (i) the candidate equilibrium after learning X_1^S and (ii) in the deviation strategy is 1/2. This is because X_1^S and X_1^T are distributed identically, and both lead to the same bid distribution. Hence, both bidders have the same marginal bid distribution and win when they have the highest signal. Hence,

$$\Pr(X_1^T \geq X_2^S) = \int_0^1 f^S(x)\left[1 - F^T(x)\right] dx = 1 - \int_0^1 f^S(x)F^S(x)dx = \frac{1}{2}. \tag{1}$$

With probability $1 - \sigma^c$, bidder 2 learns X_2^T. As X_2^T is independent of both X_1^S and X_1^T, and X_1^T and X_1^S are distributed identically (Assumption 2), winning probability in (i) and (ii) also coincides,

$$\Pr\left[\beta_T(X_1^T) \geq \beta_T^c(X_2^T)\right] = \Pr\left[\beta_T(X_1^S) \geq \beta_T^c(X_2^T)\right] = \Pr\left[\beta_S^c(X_1^S) \geq \beta_T^c(X_2^T)\right]. \quad \square$$

Lemma 8 (Lower expected payment.)**.** *Let $\{\sigma^c > 0, \beta_S^c, \beta_T^c\}$ be a candidate equilibrium. Bidder 1's expected payment is strictly lower in the deviation than in the candidate equilibrium when learning X_1^S.*

Proof. With probability $1 - \sigma^c$, bidder 2 learns X_2^T and bids $\beta_T^c(X_2^T)$, which bidder 1 pays when winning. In both the deviation and in the candidate equilibrium when learning X_1^S, bidder 1 has the same marginal bidding function. Bids are independent in both cases. Hence, bidder 1's expected payment is the same.

With probability σ^c, bidder 2 learns X_2^S. In the candidate equilibrium after learning X_1^S, bidder 1 wins if $X_1^S \geq X_2^S$. Hence, the expected payment of bidder 1 when winning after learning X_1^S is $\int_0^1 \beta_S^c(x)dG_{(2)}(x)$, where $G_{(2)}(x)$ is the second-order statistic distribution of two identically distributed signals X_1^S and X_2^S as derived in (3).

In the deviation strategy, bidder 1 wins if $X_1^T \geq X_2^S$. Both random variables X_1^S and X_1^T are i.i.d. Hence, when winning bidder 1 wins, she pays the expected bid of the second-order statistic, $\int_0^1 \beta_S^c(x)d\hat{G}_{(2)}(x)$, where $\hat{G}_{(2)} = 2F^S(x) - F^S(x)^2$ is the distribution of the second-order statistic of two i.i.d. variables with distribution $F^S(.)$.

Lemma 2 in Appendix A.1 establishes that $G_{(2)}$ first-order stochastically dominates $\hat{G}_{(2)}$ (in the notation of the lemma, $X_1 = X_1^S$, $\hat{X}_1 = X_1^T$, and $X_2 = X_2^S$). Thus, because $G_{(2)}(x) < \hat{G}_{(2)}(x)$ for every x for which $F(x) \notin \{0, 1\}$ and as β_S^c is increasing, it follows that $\int_0^1 \beta_S^c(x) d\hat{G}_{(2)}(x) < \int_0^1 \beta_S^c(x) dG_{(2)}(x)$. If bidder 2 learns X_2^S, then bidder 1's expected payment conditional on winning is strictly lower in the deviation strategy than in the candidate equilibrium after learning X_1^S. Finally, by Lemma 7, overall winning probability is $\frac{1}{2}$ in the candidate equilibrium and the deviation if bidder 2 learns X_2^S. Hence, the unconditional expected payment is also strictly lower in the deviation. □

Lemma 9 (Equal expected value.)**.** *Let $\{\sigma^c > 0, \beta_S^c, \beta_T^c\}$ be a candidate equilibrium. In the candidate equilibrium and the deviation strategy, bidder 1's expected value when winning, $\mathbb{E}[V_1|\text{bidder 1 wins}]$, is identical.*

Proof. With probability $1 - \sigma^c$, bidder 2 learns X_2^T. Bidder 1 with signal $X_1^S = x$ in the candidate equilibrium wins with the same probability as with $X_1^T = x$ in the deviation. There is no winner's curse for bidder 1. By Assumption, $\mathbb{E}[V_1|X_1^S = x] = \mathbb{E}[V_1|X_1^T = x]$. Hence, the value of V_1 conditional on winning is the same due to the same marginal distribution of X_1^S and X_1^T.

With probability σ^c, bidder 2 learns X_2^S. Then, in the candidate equilibrium, the expected object value for bidder 1 who learns X_1^S and wins is $\mathbb{E}[S] + \mathbb{E}[T_1]$, as bidders are symmetric and win with probability $\frac{1}{2}$ at every $s \in S$.

In the deviation strategy, signals are independent and X_1^T is informative only about T_1, while X_2^S is informative only about S. As X_i^T and X_i^S are distributed i.i.d., bidder 1's expected value of the private component when winning can be written as

$$\begin{aligned}\mathbb{E}\left[T_1|\beta_T(X_1^T) \geq \beta_S^c(X_2^S)\right] &= \int_0^1 \mathbb{E}\left[T_1|X_1^T = x\right] f^T(x) F^S(x)\, dx \\ &= \int_0^1 \mathbb{E}\left[S|X_1^S = x\right] f^S(x) F^S(x)\, dx + \mathbb{E}[T_1] - \mathbb{E}[S], \qquad (2)\end{aligned}$$

where the last equality followed because $f^T(.) = f^S(.)$ and

$$\begin{aligned}&\mathbb{E}\left[V_1 = S + T_1|X_1^S = x\right] = \mathbb{E}\left[V_1 = S + T_1|X_1^T = x\right] \\ \Leftrightarrow \quad &\mathbb{E}\left[S|X_1^S = x\right] + \mathbb{E}[T_1] = \mathbb{E}\left[T_1|X_1^T = x\right] + \mathbb{E}[S].\end{aligned}$$

Note the similarity of (2) to bidder 2's common component value when winning:

$$\mathbb{E}\left[S|\beta_S^c(X_2^S) > \beta_T(X_1^T)\right] = \int_0^1 \mathbb{E}\left[S|X_2^S = x\right] f^S(x) F^T(x)\, dx. \tag{3}$$

The object is always sold, the common component surplus to be divided between the bidders is $\mathbb{E}[S]$, and each bidder wins with probability $\frac{1}{2}$ (Lemma 7) if bidder 2 learns X_2^S. Hence,

$$\mathbb{E}[S] = \frac{1}{2}\mathbb{E}[S|\underbrace{\beta_S^c(X_2^S) \le \beta_T(X_1^T)}_{\text{bidder 1 wins}}] + \frac{1}{2}\mathbb{E}[S|\underbrace{\beta_S^c(X_2^S) > \beta_T(X_1^T)}_{\text{bidder 2 wins}}].$$

This and Equations (2) and (3) pin down bidder 1's winning valuation when deviating,

$$\begin{aligned}
&\mathbb{E}\left[S + T_1|\beta_T(X_1^T) \ge \beta_S^c(X_2^S)\right] \\
=&2\mathbb{E}[S] - \mathbb{E}\left[S|\beta_S^c(X_2^S) > \beta_T(X_1^T)\right] + \mathbb{E}\left[T_1|\beta_S^c(X_2^S \le \beta_T(X_1^T)\right] \\
=&2\mathbb{E}[S] + \mathbb{E}[T_1] - \mathbb{E}[S] = \mathbb{E}[S] + \mathbb{E}[T_1],
\end{aligned}$$

which is the same expected value conditional on winning as in the candidate equilibrium when both bidders learn X_i^S. □

Together, Lemmas 7, 8 and 9 establish that bidder 1 has a strictly profitable deviation in any candidate equilibrium with $\sigma^c > 0$. When bidder 2 learns X_2^S (a positive probability event), then the deviation performs strictly better (lower expected payment and equal value of the object) than learning X_1^S in the candidate equilibrium. If bidder 2 learns X_2^T, the deviation strategy yields the candidate equilibrium payoff. □

C.2 Learning about opponent's private component and FPA

Learning about the opponent's private component. In the baseline model, bidders can learn only about their payoff-relevant components, but not the private component of the other bidder. How relevant is this assumption for the uniqueness of the IPV equilibrium in the SPA? Allowing bidders to learn about the other bidder's private component imposes further restrictions on the IPV equilibrium since it constitutes an additional deviation. In what follows, I show that the IPV outcome remains an equilibrium in the SPA even if bidders can learn about every component in the model.

As before, bidder $i \neq j$ chooses between X_i^T and X_i^S. In addition, each bidder can learn a signal about the private component of the other bidder, Y_i^T, which is informative only about $T_{j \neq i}$. I do not impose an accuracy ranking about V_j among Y_i^T and X_j^T, and Y_1^T and Y_2^T can have different distributions. The following result shows that the privacy-preserving IPV outcome remains an equilibrium in the SPA.

Proposition 7. *Let bidders choose one signal in $\{X_i^S, X_i^T, Y_i^T\}$ where $\mathbb{E}[V_i|X_i^S] \overset{d}{=} \mathbb{E}[V_i|X_i^T]$. Then, there exists an IPV equilibrium in the SPA in which bidders learn only X_i^T.*

Proof. The existence of an IPV equilibrium for a fixed information choice $\{X_1^T, X_2^T\}$ follows trivially. By Proposition 6, bidders do not have a profitable deviation involving X_i^S. Learning Y_i^T cannot be part of a strictly profitable deviation because bidder i's best-response bid with any payoff-irrelevant signal realization $Y_i^T = y$ is constant: $\beta(Y_i^T = y) = \mathbb{E}[V_i|Y_i^T = y] = \mathbb{E}[V_i]$. But bidder i can obtain the same payoff by learning X_i^T, and bidding $\beta(X_i^T) = \mathbb{E}[V_i]$ for any realization. Hence, her payoff after learning X_i^T and bidding optimally (as in the candidate equilibrium) is weakly higher. □

Comparison to an FPA. In addition to allowing bidders to learn about the private components of their opponent, I relax two further assumptions next: signals can be costly, and no informativeness ranking among signals is required. While an absolute prediction as in Proposition 6 (i.e., bidders learn only X_i^T in equilibrium) is no longer possible, I derive a relative prediction between the SPA and the FPA. The next result shows that not every auction format has the same efficiency properties of the SPA; the proof follows below after Example 5.

Proposition 8. *Let bidders choose among $\{X_i^T, X_i^S, Y_i^T\}$ with corresponding costs $\{c^T, c^S, \tilde{c}^T\}$. If there exists an IPV equilibrium in the FPA, then there exists an IPV equilibrium in the SPA. The reverse does not hold.*

The proof relies on revenue equivalence. In an IPV equilibrium, bidders' expected utility coincides in the FPA and the SPA. In addition, I show that the deviation payoff after learning X_i^S also coincides in the FPA and the SPA. This means that if learning about the common component is not a strictly profitable deviation in the FPA, then the same is true for the SPA. As in Proposition 7, learning Y_i^T cannot be a strictly profitable deviation in the SPA since it results in a constant best-response bid.

Why does the existence of an IPV equilibrium in the SPA not imply that there also exists an equilibrium in the FPA? Let bidder 2 learn only about T_2. In the FPA, anticipating bidder 2's bid can be useful for bidder 1 who does not want to "leave money on the table" by placing an unnecessarily high winning bid. Learning Y_1^T yields a better estimate of bidder 2's bid. However, learning Y_1^T also comes at the opportunity cost of not learning about V_1. In an FPA, bidders trade off these effects of exploiting higher correlation versus learning more about one's own valuation. Example 5 shows that the former effect can dominate and destroy the existence of an IPV equilibrium in the FPA.

Example 5. *Let $S, T_1, T_2 \sim \mathcal{U}[0,1]$ and $V_i = S + T_i$, and let signals be perfectly revealing: $X_1^S = X_2^S = S$ and $X_i^T = Y_{j\neq i}^T = T_i$. Since both signals yield the same distribution of the expected value of V_i, it follows that learning X_i^T and bidding $\mathbb{E}[V_i|X_i^T = T_i]$ constitutes an IPV equilibrium in the SPA.*

In contrast, there exists no IPV equilibrium in the FPA. Consider a candidate equilibrium in which bidders bid the standard IPV bidding function $\beta(X_i^T = x) = \mathbb{E}\left[V_j|\mathbb{E}[V_j|X_j^T] \leq \mathbb{E}[V_i|X_i^T = x]\right] = \frac{1+x}{2}$. Bidder i's expected payoff with $X_i^T = x$ is $(\frac{1}{2} + x - \frac{1+x}{2})x = \frac{x^2}{2}$. Overall, her expected payoff in this candidate equilibrium is $\frac{1}{6}$.

However, the following deviation yields a strictly higher payoff: learn $Y_1^T = T_2$ and slightly outbid the opponent by bidding $\frac{1+T_2}{2} + \epsilon$ for some small $\epsilon > 0$. This strategy always wins the object at its prior expected value of 1. For $\epsilon \to 0$, bidder 1 pays the expected bid of the opponent, which is $\frac{3}{4}$. Hence, for ϵ sufficiently small, bidder 1's payoff from this deviation approaches $1 - \frac{3}{4} = \frac{1}{4}$, making this a strictly profitable deviation.

Proof of Proposition 8. Let there exist an IPV equilibrium in the FPA. Without loss, consider bidder 1. By standard IPV arguments for the FPA, bidder 2 bids the symmetric equilibrium bidding function

$$\beta^I(X_2^T) = \mathbb{E}\left[\mathbb{E}[V_1|X_1^T]|\mathbb{E}[V_1|X_1^T] \leq \mathbb{E}[V_2|X_2^T]\right].$$

By the same argument, bidder 1's best-response bid when deviating to X_1^S is

$$\tilde{\beta}^I(X_1^S) = \mathbb{E}\left[\mathbb{E}[V_2|X_2^T]|\mathbb{E}[V_2|X_2^T] \leq \mathbb{E}[V_1|X_1^S]\right].$$

Now consider an IPV candidate equilibrium in the SPA with bidding function $\beta^{II}(X_i^T) = \mathbb{E}[V_i|X_i^T]$. By the same logic as in Proposition 7, bidders do not have a strictly profitable deviation to learn Y_i^T, because it is payoff irrelevant and leads to a constant best-response bid. It remains to be shown that learning X_i^S cannot be part of a strictly profitable deviation. Let bidder 1 deviate and learn X_1^S. Then, $\tilde{\beta}^{II}(X_1^S) = \mathbb{E}[V_1|X_1^S]$ is a weakly dominant strategy. Note that the optimal expected payoff for each realization $X_1^S = x$ is the same in the SPA and the FPA because

1. winning probability is identical,[1]

$$\Pr\left[\tilde{\beta}^I(X_1^S = x) \geq \beta^I(X_2^T)\right] = \Pr\left[\mathbb{E}[V_1|X_1^S = x] \geq \mathbb{E}[V_2|X_2^T]]\right]$$
$$= \Pr\left[\tilde{\beta}^{II}(X_1^S = x) \geq \beta^{II}(X_2^T)\right]$$

2. expected value if winning depends only on $X_1^S = x$, i.e., $E[V_1|X_1^S = x, 1 \text{ wins}] = E[V_1|X_1^S = x]$
3. expected payment when winning is identical,

$$\mathbb{E}[\beta^{II}(X_2^T)|\beta^{II}(X_2^T) \leq \tilde{\beta}^{II}(X_1^S = x)] = \mathbb{E}\left[\mathbb{E}[V_2|X_2^T]|\mathbb{E}[V_2|X_2^T] \leq \mathbb{E}[V_1|X_1^S = x]\right]$$
$$= \tilde{\beta}^I(X_1^S = x).$$

Hence, the deviation payoff after learning X_1^S and bidding optimally coincides in the FPA and the SPA. By revenue equivalence, bidder 1 obtains the same payoff in the IPV (candidate) equilibrium of the FPA and the SPA. By assumption, deviating to X_1^S is not a strictly profitable deviation in the FPA. As shown above, this deviation has the same payoff in the SPA, and is thus also not a strictly profitable deviation.

Finally, I show that the existence of an IPV equilibrium in the SPA does not imply the existence of an IPV equilibrium in the FPA. Example 5 provides a counterexample which relies on perfectly revealing signals, and thus does not satisfy Assumption 5. Consider the following variation on this example with noisy signals satisfying the requirements of the model, $X_i^S \sim \mathcal{N}(S, \xi^2)$ and $X_i^T \sim \mathcal{N}(T_i, \xi^2)$. For any $\epsilon > 0$, there exists a ξ^2 sufficiently small such that after bidder 1 learns Y_1^T, then she knows with probability approaching one that bidder 2's value falls within the ϵ-interval of $E[V_2|Y_1^T]$. Then, bidder 1 is almost sure of bidder 2's bid and can outbid her by bidding $\mathbb{E}[V_2|Y_1^T] + \epsilon$. Thus, as $\xi^2 \to 0$, bidder 1's deviation is strictly profitable as it approaches the strictly profitable full information benchmark. □

C.3 Many bidders

Next, I show that an IPV equilibrium exists in the SPA for any number of bidders:

[1] For $X_1^S = x$ and $X_1^T = y$ such that $\mathbb{E}[V_1|X_1^S = x] = \mathbb{E}[V_1|X_1^T = y]$, bidder 1 places the same bid in the deviation and the candidate equilibrium. By symmetry, $\mathbb{E}[V_1|X_1^T = y] = \mathbb{E}[V_2|X_2^T = y]$. Thus, bidder 1 with $X_1^S = x$ wins if $X_2^T \leq y$, or equivalently, if $\mathbb{E}[V_2|X_2^T] \leq \mathbb{E}[V_2|X_1^T = y] = \mathbb{E}[V_1|X_1^S = x]$.

Proposition 9 (IPV equilibrium exists.)**.** *Let $N \geq 2$ bidders choose between X_i^T or X_i^S with $\mathbb{E}[V_i|X_i^T] \stackrel{d}{=} \mathbb{E}[V_i|X_i^S]$. Then, there exists an IPV equilibrium in which bidders learn X_i^T with probability one and bid $\beta(X_i^T) = \mathbb{E}[V_i|X_i^T]$.*

Proof. Without loss, consider bidder 1. If all other bidders learn $X_{i\neq 1}^T$, then both X_1^S and X_1^T yield the same distribution of the expected value which is independent from the information of all other bidders. Hence, learning X_1^S and bidding optimally results in the same payoff as learning X_1^T and bidding optimally, and thus cannot lead to a strictly profitable deviation. After learning X_i^T, it is well known that bidding the expected value is a weakly dominant strategy in the SPA. □

Is the IPV equilibrium also the unique symmetric equilibrium? The answer to this is more involved with more than two bidders. If $N \geq 3$ and a bidder uses the same class of deviation strategies as described previously, then she might be strictly more likely to win when deviating. As a consequence, the deviating bidder's expected payment might be strictly higher than in the candidate equilibrium. However, under additional assumptions, previous results can be extended to any number of bidders. For example, for binary states, Proposition 10 shows that a pure CV framework cannot be an equilibrium outcome in the SPA.[2]

Proposition 10. *Let $S, T_i \in \{a, b\}$ uniformly with $a < b$, and $\mathbb{E}[V_i|X_i^T] \stackrel{d}{=} \mathbb{E}[V_i|X_i^S]$. For any $N \geq 2$, there exists no pure CV equilibrium in the SPA.*

Proof. The result for $N = 2$ follows by Proposition 6. Let $N \geq 3$. By contradiction, let bidders $1, ..., N$ learn X_i^S and use the same increasing bidding function $\beta_S(X_i^S)$. I show that bidder 1 has a strictly profitable deviation: learn X_1^T and bid $\hat{\beta}(X_1^T) = \beta_S(X_1^S)$. Let $Y_{(1)} = \max\{X_2^S, ..., X_N^T\}$ be the highest common-component signal of all other bidders. Let $G_{(1)}(x|s) := \Pr(Y_{(1)} \leq x|S = s) = F^S(y|S = s)^{N-1}$ be the distribution of $Y_{(1)}$, and $g_{(1)}$ the corresponding density.

Because $\mathbb{E}[V_i|X_i^T] \stackrel{d}{=} \mathbb{E}[V_i|X_i^S]$, define $f(x|a) := f^S(x|a) = f^T(x|a)$ and $f(x|b) := f^S(x|b) = f^T(x|b)$. Denote the corresponding distributions $F(x|a)$ and $F(x|b)$. Let $S = T_1 = a$. In the candidate equilibrium and the deviation, bidder 1 has the same marginal bid distribution, so the deviation has no effect on the payoff. Similarly, if $S = T_1 = b$, the above deviation has no effect on the payoff. Let $V_1 = a + b$, so that $(S = a, T_1 = b)$ and $(S = b, T_1 = a)$ are equally likely. The expected payoff in the

[2] This result can be extended further to rule out equilibria involving mixing between X_i^S and X_i^T.

candidate equilibrium is

$$\int_0^1 [a+b-\beta_S(x)] \left[\frac{1}{2}g_{(1)}(x|b)[1-F(x|b)]dx + \frac{1}{2}g_{(1)}(x|a)[1-F(x|a)]\right] dx. \tag{4}$$

The expected payoff in the deviation is

$$\int_0^1 [a+b-\beta_S(x)] \left[\frac{1}{2}g_{(1)}(x|b)[1-F(x|a)] + \frac{1}{2}g_{(1)}(x|a)[1-F(x|b)]\right] dx. \tag{5}$$

The net payoff from the deviation, (5) minus (4), can be written as

$$\frac{1}{2}\int_0^1 [\beta_S(x)-a-b] \left(g_{(1)}(x|a)-g_{(1)}(x|b)\right)(F(x|b)-F(x|a))\, dx. \tag{6}$$

Let $\alpha(x) \coloneqq \left(g_{(1)}(x|a)-g_{(1)}(x|b)\right)(F(x|b)-F(x|a))$. It captures the difference in winning probability between the candidate equilibrium and the deviation for bidder 1. Next, I show that by deviating, bidder 1 is more likely to win.

Lemma 10. $\int_0^1 \alpha(x)dx < 0$.

Proof. Using integration by parts, we can write

$$\begin{aligned}\int_0^1 \alpha(x)dx &= -\int_0^1 (f(x|b)-f(x|a))\left(F(x|a)^{N-1}-F(x|b)^{N-1}\right)dx \\ &= +\frac{2}{N} - \left[\frac{1}{N}+\frac{N-1}{N}\right]\int_0^1 \left(f(x|b)F(x|a)^{N-1}+f(x|a)F(x|b)^{N-1}\right)dx \end{aligned} \tag{7}$$

Integrating both terms of the middle $\frac{1}{N}$-term by parts yields

$$\begin{aligned}&\frac{1}{N}\int_0^1 \left(f(x|b)F(x|a)^{N-1}+f(x|a)F(x|b)^{N-1}\right)dx \\ &= \frac{2}{N} - \frac{N-1}{N}\int_0^1 f(x|b)F(x|b)^{N-2}F(x|a)+f(x|a)F(x|a)^{N-2}F(x|b)dx.\end{aligned}$$

Plugging this into (7) for the $\frac{1}{N}$-term, the expression simplifies to

$$-\frac{N-1}{N}\int_0^1 \left(F(x|a)^{N-2}-F(x|b)^{N-2}\right)(f(x|b)F(x|a)-f(x|a)F(x|b))\,dx$$

Due to $N \geq 3$ and the strong MLRP, for all interior x, $F(x|a)^{N-2}-F(x|b)^{N-2} > 0$ and $\frac{f(x|b)}{F(x|b)} > \frac{f(x|a)}{F(x|a)}$ (reverse-hazard-rate dominance). Hence, $\int_0^1 \alpha(x)dx < 0$. □

Lemma 11. *There exists* $\underline{x}, \overline{x}$ *with* $\underline{x} \leq \overline{x}$ *such that*

1. $\alpha(x)$ *crosses zero exactly once from below at some* $\overline{x} \leq \hat{x}$ *where* $f^S(\hat{x}|b) = f^S(\hat{x}|a)$.
2. $\int_{\underline{x}}^1 \alpha(x)dx = 0$.

Proof. By Proposition 2 and 4 in Milgrom and Weber (1982), $Y_{(1)}$ and S are affiliated (due to conditional independence of all X_i^S given S). Hence, $\frac{g_{(1)}(x|b)}{g_{(1)}(x|a)}$ is nondecreasing in x. MLRP implies first-order stochastic dominance, $F^S(x|b) \leq F^S(x|a)$. This establishes the single-crossing property of $\alpha(x)$. As $\int_0^1 \alpha(x)dx \leq 0$ by Lemma 10, and $\alpha(x)$ crosses zero once, there exists $\underline{x} \leq \overline{x}$ (the crossing point) as in the Lemma.

Since X_i^S satisfies MLRP, and $\int_0^1 f^S(x|s)dx = 1$, there exists a 'neutral' signal $\hat{x}$ such that $f(\hat{x}|1) = f(\hat{x}|0)$. At this signal $\hat{x}$, $\alpha(\hat{x}) \geq 0$ because $F(\hat{x}|b) \leq F(\hat{x}|a)$ and

$$\begin{aligned} g_{(1)}(\hat{x}|a) - g_{(1)}(\hat{x}|b) =& (N-1)\left[f(\hat{x}|b)F(\hat{x}|b)^{N-2} - f(\hat{x}|a)F(\hat{x}|a)^{N-2}\right] \\ \leq& (N-1)F(\hat{x}|a))^{N-2}\left[f(\hat{x}|b) - f(\hat{x}|a)\right] = 0. \end{aligned}$$

Hence, $\hat{x} \geq \overline{x}$: $\alpha(x)$ crosses zero at some realization below $\hat{x}$. □

Let $\gamma(x) := \beta_S(x) - a - b$, which is nondecreasing in x. Using this, (6) can be written as two sums,

$$\frac{1}{2}\int_0^{\underline{x}} \gamma(x)\alpha(x)dx + \frac{1}{2}\int_{\underline{x}}^1 \gamma(x)\alpha(x)dx.$$

The first summand is strictly positive. This is because by Lemma 11, $\alpha(x) < 0$ as $\underline{x} \leq \overline{x}$ where the crossing at zero from below occurs. In addition, the bidding function in an SPA is $\beta_S(x) = \mathbb{E}[V_i|X_i^S = x, Y_{(1)} = x]$ and increasing in x by Milgrom and Weber (1982). Thus, at any $x \leq \underline{x}$, $\beta_S(x) \leq \beta_S(\overline{x}) \leq \beta_S(\hat{x}) = \mathbb{E}[V_i|X_i^S = \hat{x}, Y_{(1)} = \hat{x}] \leq \mathbb{E}[V_i] = a + b$. Thus, for any $x \leq \underline{x}$, $\gamma(x)\alpha(x) \geq 0$, and for a positive mass of x, $\gamma(x)\alpha(x) > 0$.

Next, I use Lemma 4 again. As I established in Lemma 11, all assumptions of Lemma 4 are satisfied by $\alpha(.)$ and $\gamma(.)$ for $X_i^S \geq \underline{x}$. Hence, the second summand is nonnegative. This establishes that the deviation payoff is positive for $N \geq 3$. □

C.4 Asymmetric equilibria

How robust is the insight that the SPA is ex-ante efficient? I show that the SPA might lose its ex-ante efficiency property if players coordinate on asymmetric equilibria.

For brevity, I focus on the case where the common component is binary, $S \in \{\underline{s}, \overline{s}\}$, and I construct asymmetric equilibria in which one bidder learns about the common component, and the other bidder learns about her private component.

Proposition 11. *Let $S \in \{\underline{s}, \overline{s}\}$ and $\mathbb{E}[V_i|X_i^S] \stackrel{d}{=} \mathbb{E}[V_i|X_i^T]$. There exists an asymmetric equilibrium in the SPA where bidder 1 learns X_1^S, bidder 2 learns X_2^T and wins if $\mathbb{E}[T_2|X_2^T] > \mathbb{E}[T_1]$.*

Proof. Let bidder 1 learn X_1^S and bid $\beta_1(x_1) = \mathbb{E}[V_1|X_1^S = x_1] = \mathbb{E}[T_1] + \mathbb{E}[S|X_1^S = x_1]$. Let bidder 2 learn X_2^T and bid as follows: bid 0 if $\mathbb{E}[T_2|X_2^T = x_2] < \mathbb{E}[T_1]$ and bid $\mathbb{E}[T_1] + \sup_x \mathbb{E}[S|X_1^S = x]$ otherwise such that she wins with probability one. Next, I show that this constitutes an equilibrium.

Given bidder 1's information choice, her bidding strategy $\beta_1(x)$ is a weakly dominant strategy since bidder 2's information and hence, bid is payoff-irrelevant. If bidder 1 deviates and learns X_1^T instead, she faces the same distribution of her expected object value (because $\mathbb{E}[V_1|X_1^S] \stackrel{d}{=} \mathbb{E}[V_1|X_1^T]$) and the same payoff as when learning X_1^S. Hence, bidder 1 has no profitable deviation.

Next, consider bidder 2's best response after learning a private signal realization $X_2^T = x_2$. If bidder 1 has a signal realization $X_1^S = y$ and follows the above strategy, then bidder 2's expected payoff when winning is

$$\mathbb{E}[S|X_1^S = y] + \mathbb{E}[T_2|X_2^T = x] - \mathbb{E}[S|X_1^S = y] - \mathbb{E}[T_1],$$

where the first two summands capture bidder 2's expected value and the last two correspond to bidder 1's bid. Bidder 1's signal realization y is irrelevant for the expected payoff of bidder 2, and only bidder 2's signal realization determines whether winning is profitable: bidder 2's best response bid is to win if and only if $\mathbb{E}[T_2|X_2^T = x] - \mathbb{E}[T_1] \geq 0$, which corresponds to the strategy described in the candidate equilibrium above.

It is left to show that bidder 2 cannot be better off learning X_2^S. By contradiction, let bidder learn X_2^S and be strictly better off than with X_2^T. Bidder 2's expected payoff from winning with a signal $X_2^S = x$ when bidder 1 learns $X_1^S = y$ is

$$\mathbb{E}[S|X_1^S = y, X_2^S = x] + \mathbb{E}[T_2] - \mathbb{E}[S|X_1^S = y] - \mathbb{E}[T_1].$$

By assumption, $\mathbb{E}[T_1] = \mathbb{E}[T_2]$. Let $h(y;x) := \mathbb{E}[S|X_1^S = y, X_2^S = x] - \mathbb{E}[S|X_1^S = y]$. For

$S \in \{\underline{s}, \overline{s}\}$ with $\mu_S := \Pr(S = \overline{s}) \in (0,1)$, this expression simplifies to

$$\begin{aligned} h(y;x) &= (\overline{s} - \underline{s}) \Big(\Pr(S = \overline{s} | X_1^S = y, X_2^S = x) - \Pr(S = \overline{s} | X_1^S = y) \Big) \\ &= (\overline{s} - \underline{s}) \left(\frac{1}{1 + \frac{f^S(x|\underline{s})}{f^S(x|\overline{s})} \frac{f^S(y|\underline{s})}{f^S(y|\overline{s})} \frac{1-\mu_s}{\mu_s}} - \frac{1}{1 + \frac{f^S(y|\underline{s})}{f^S(y|\overline{s})} \frac{1-\mu_s}{\mu_s}} \right) \end{aligned}$$

where the last equality followed because $\Pr(S = \overline{s} | X_1^S = y) = \frac{f^S(y|\overline{s})\mu_s}{f^S(y|\overline{s})\mu_s + f^S(y|\underline{s})(1-\mu_s)}$ and with $\Pr(S = \overline{s} | X_1^S = y, X_2^S = x)$ derived accordingly. Crucially, the expression $\frac{f^S(x|\underline{s})}{f^S(x|\overline{s})}$ determines the sign of $h(y;x)$ irrespective of y. E.g., if $\frac{f^S(x|\underline{s})}{f^S(x|\overline{s})} > 1$ then bidder 2's signal is more indicative of a low common component, and winning means overpaying for the object for any X_1^S. By assumption, the ratio $\frac{f^S(x|\underline{s})}{f^S(x|\overline{s})}$ is strictly decreasing in x. Hence, there exists $\hat{x}$ such that the best response bid is to bid zero (i.e., lose) if $X_2^S < \hat{x}$, and to bid to win with probability one if $X_2^S \geq \hat{x}$. I refer to the deviation strategy in this paragraph as (DS) below.

Now, I show that the following alternative deviation strategy (AS) is weakly better for bidder 2 than (DS): learn X_2^T instead of X_2^S, and bid as in (DS), $\beta_2(X_2^T = x) = \beta_2(X_2^S = x)$. The probability of losing and winning is equal in (DS) and (AS), since $F^S(\hat{x}) = F^T(\hat{x})$. Conditional on winning, the value of the object to bidder 2 also coincides in (DS) and (AS) because bidder 2 wins against any signal realization of X_1^S (thus, the event of winning bears no additional information about bidder 2's object value) and $\mathbb{E}[V_2 | X_2^S \geq \hat{x}] = \mathbb{E}[V_2 | X_2^T \geq \hat{x}]$ (due to $\mathbb{E}[V_2 | X_2^T] \stackrel{d}{=} \mathbb{E}[V_2 | X_2^T]$). Finally, the expected payment in (AS) is weakly lower than in (DS): since X_1^S and X_2^S are affiliated and $\beta_1(.)$ is an increasing function, by Proposition 5 in Milgrom and Weber (1982), $\mathbb{E}[\beta_1(X_1^S) | X_2^S \geq a]$ is nondecreasing in a, and hence,

$$\mathbb{E}[\beta_1(X_1^S) | X_2^S \geq \hat{x}] \geq \mathbb{E}[\beta_1(X_1^S)] = \mathbb{E}[\beta_1(X_1^S) | X_2^T \geq \hat{x}],$$

where the last equality followed because X_1^S and X_2^T are independent. Finally, note that the payoff with (AS) is weakly lower than the payoff with the candidate equilibrium for which the optimal bidding strategy when learning X_2^T was established above. But this yields a contradiction to (DS) being a strictly profitable deviation. □

The resulting outcome in such an asymmetric equilibrium is ex-post efficient given the bidders' information choice, since the bidder with the highest expected private component gets the object. However, with this learning choice the resulting outcome cannot be ex-ante efficient because bidder 1 learns X_1^S — a signal which bears no information about the efficient allocation — in lieu of learning X_1^T. Finally, depending on the parameters, bidders can obtain a higher payoff in the symmetric IPV equilibrium of Proposition 6 or the asymmetric equilibrium of Proposition 11, leaving the question of equilibrium selection open for further research.[3]

One might fear that for a larger number of bidders, there always exist similar ex-ante inefficient asymmetric equilibria in which several bidders learn about the common component at the expense of allocative efficiency. This, however, is not the case. If more than one bidder learns about the common component in an asymmetric equilibrium, then their signals are interdependent and they might have a strictly profitable deviation to "decrease" this interdependence via learning X_i^T. For the assumptions of Online Appendix C.3, the proof of Proposition 10 can be adapted to show that in any asymmetric pure strategy equilibrium with $N \geq 2$ bidders there can be at most one single bidder learning about the common component.

C.5 Two-dimensional signals

If bidders are initially partially informed about their private component, and then choose whether to acquire an additional signal about the common or private component at the information acquisition stage, will the IPV setting arise again endogenously in equilibrium? I show that the IPV equilibrium in the SPA is robust to two-dimensional signals: under an assumption which makes a bidder indifferent between X_i^S and X_i^T in a posted price setting (similar to Assumption 2), there exists an equilibrium in the SPA in which both bidders acquire X_i^T in addition to their initial private-component signal, and no bidder learns about the common component.

Bidders observe a private-component signal Y_i with support $\mathcal{Y}$, drawn independently for each bidder. In addition, each bidder chooses between learning X_i^S or X_i^T at no cost. The following assumption — in parallel to Assumption 2 in the baseline model — guarantees that a bidder is indifferent between learning X_i^S and X_i^T in a posted price setting since learning either results in the same distribution of the expected value.

Assumption 6. *For every $y \in \mathcal{Y}$, $\mathbb{E}[V_i|X_i^S, Y_i = y] \overset{d}{=} \mathbb{E}[V_i|X_i^T, Y_i = y]$.*

This assumption is satisfied, for example, if each private component is the sum of

[3] For example, let $T_1, T_2, S \in \{0, 1\}$, $\Pr(S = 1) = \Pr(T_i = 1) = m$, $f^S(x|0) = f^T(x|0) = 2 - 2x$ and $f^S(x|1) = f^T(x|1) = 2x$. Then, for $m = 0.5$ (for $m = 0.01$) bidder 1 is strictly better off (strictly worse off) in a symmetric equilibrium than in the asymmetric equilibrium of Proposition 11.

two independently drawn private components, $T_i = T_{ia} + T_{ib}$, and Y_i is informative only about T_{ia} while X_i^T is informative only about T_{ib}.

Proposition 12 (IPV equilibrium exists)**.** *Let Assumption 6 hold. Then, there exists an equilibrium in the SPA where bidders learn X_i^T and bid $\beta_i(X_i^T, Y_i) = \mathbb{E}[V_i|X_i^T, Y_i]$.*

Proof. The bidding function in the proposition is a weakly dominant strategy following the mutual information choice (X_1^T, X_2^T). Moreover, bidder i cannot be better off learning X_i^S instead of X_i^T. This follows by the same argument as in Proposition 6: by Assumption 6, for every realization of Y_i, bidder 1 faces the same expected value distribution with either information choice. In addition, this expected value is independent of the signal and bid of the other bidder irrespective of her information choice. Hence, when deviating to X_i^S, bidder i obtains at most the same payoff as with X_i^T. □

Next, I show that no equilibrium where both bidders learn about the common component in addition to their initial private-component signal exists under an additional assumption. Even without information choice, pinning down the equilibrium in the SPA is often not possible for two-dimensional private information with one signal about the private component and one about the common component. There are known instances in which an equilibrium does not exist (Jackson, 2009).[4] I avoid this intractability of how a bidder scores several signals into one single bid via a conditions on the expected value, such that in equilibrium each bid is placed only by a unique signal combination.

Proposition 13. *Let Assumption 6 hold and $\mathcal{Y} = \{0, 1, ..., K\}$ be a finite set. Let $\inf_x \mathbb{E}[V_i|X_i^S = x, X_{j\neq i}^S = x, Y_i = k] \geq \sup_x \mathbb{E}[V_i|X_i^T = x, X_{j\neq i}^S = x, Y_i = k-1]$ for all $k \in \mathcal{Y}$. Then, in the SPA, there exists no symmetric pure strategy equilibrium in weakly undominated strategies where both bidders learn X_i^S.*

Proof. By contradiction, consider a pure strategy monotonic candidate equilibrium in which both bidders learn X_i^S. I show that bidder 1 has a strictly profitable deviation. In any candidate equilibrium where bidders learn about the common component, bidder 1 with $Y_1 = k$ wins with probability one against bidder 2 with $Y_2 < k$, and loses with probability one against bidder 2 with $Y_2 > k$. This is because by the premise, for any

[4]See Pesendorfer and Swinkels (2000) for a discussion of two-dimensional private information. They show existence of an epsilon equilibrium instead of an equilibrium. For an alternative approach, see Goeree and Offerman (2003) who make the two-dimensional signal framework tractable by assuming that signals about the common component are independent. This assumption, while elegant in their framework, would fully shut down the channel of bidders seeking lower correlation in the SPA.

k and any realization x, bidding below $\inf_x \mathbb{E}[V_1|X_1^S = x, X_2^S = x, Y_1 = k]$ or above $\sup_x \mathbb{E}[V_1|X_1^S = x, X_2^S = x, Y_1 = k]$ is weakly dominated.[5]

Let $\beta(X_i^S, Y_i)$ be the bidding function in the candidate equilibrium which is strictly increasing in both arguments. Bidder 1 is strictly better off learning X_1^T and using the same bidding function as in the candidate equilibrium, $\hat{\beta}_1(X_1^T, Y_1) = \beta(X_1^S, Y_1)$. For any $Y_1 = k$, bidder 1's deviation only affects her payoff if $Y_2 = k$. Then, conditional on $Y_1 = k$, by the same argument as the proof of Proposition 6, the above deviation yields a strictly higher payoff since it yields the same winning probability, the same expected value of the object conditional on winning, but a strictly lower expected payment. □

D Different accuracy

Is the SPA still an ex-ante efficient auction format if the available signals lead to different distributions of the expected object value? Next, I show how Proposition 6 can be extended beyond Assumption 2. This involves finding an auxiliary private-component experiment $\tilde{X}_i^T$ which (i) is less accurate about T_i than the available private-component experiment X_i^T, and (ii) leads to the same distribution of the expected object value as the available common-component signal X_i^S. If such an experiment $\tilde{X}_i^T$ exists, then the SPA remains the ex-ante efficient auction format as Proposition 15 shows.

D.1 Preliminaries

Recall the definition of Accuracy in Definition 1. Let an agent face the following decision problem: she chooses an action $a \in \mathcal{A}$ that, jointly with an unknown state Z, determines the payoff $u(a; z) : \mathcal{Z} \times \mathcal{A} \to \mathbb{R}$. Her payoff satisfies a standard single-crossing property:

Definition 2. *$u(a; z)$ satisfies a single-crossing property (SCP) in $(a; z)$ if, for $a' > a$ and $z' > z$, $u(a', z) - u(a; z) > 0 \quad \Rightarrow \quad u(a', z') - u(a; z') \geq 0$.*

The agent does not observe Z, but instead has access to a signal in $X_i \in \{X^a, X^b\}$, based on which she chooses her optimal action. When the concept of accuracy is applied to signals that satisfy MLRP with respect to Z, and payoffs satisfy the single-crossing

[5] This effectively reduces the two-dimensional signal model to tractable disjoint intervals of a one-dimensional framework in which only bidders with the same realization of Y_i are competing for the object. For a fixed information choice (X_1^S, X_2^S), it is straightforward to show that bidding $\beta(X_i^S = x, Y_i = k) = \mathbb{E}[V_i|X_i^S = x, X_{j\neq i}^S = x, Y_i = k]$ constitutes an equilibrium.

property, then higher accuracy corresponds to a higher payoff.[6]

Proposition 14 (Persico, 2000; Jewitt, 2007)**.** *Let X^a and X^b both satisfy the MLRP with respect to Z. Then, $X^a \succ_Z X^b$ if and only if the expected payoff with X^a is higher than with X^b for any payoff $u(a; z)$ which satisfies the SCP in $(a; z)$.*

D.2 Analysis for different accuracy

Next, I find a sufficient condition when a function $u_1 : \mathbb{R}^+ \times \mathcal{T} \to \mathbb{R}$ satisfies SCP in $(b; t_1)$, which will be used in the proof of Proposition 15.

Definition 3. *A function $y : \mathbb{R}^+ \times \mathcal{T} \to \mathbb{R}$ is supermodular (spm) if for every $t_1' > t_1$ and every $b' > b$, it holds that $y(b'; t_1') + y(b; t_1) \geq y(b; t_1') + y(b'; t_1)$.*

The following is a well-known result (see, e.g., Athey (2001)).

Lemma 12. *If $u_1 : \mathbb{R}^+ \times \mathcal{T} \to \mathbb{R}$ is spm, then it satisfies SCP in $(b; t_1)$.*

The following is sufficient for u_1 to be spm.[7]

Lemma 13. *Let $u_1(b; t_1) = f(t_1, b)g(b)$. If (i) $g(b)$ is nondecreasing in b and nonnegative, and (ii) $f(t_1, b)$ is nondecreasing in t_1 and spm, then $u_1(b; t_1)$ is spm.*

Proof. For any $b' > b$ and $t_1' > t_1$,

$$
\begin{aligned}
& f(t_1', b') - f(t_1, b') \geq f(t_1', b) - f(t_1, b) \\
\Rightarrow \quad & g(b') \left[f(t_1', b') - f(t_1, b') \right] \geq g(b) \left[f(t_1', b) - f(t_1, b) \right] \\
\Rightarrow \quad & f(t_1', b')g(b') + f(t_1, b)g(b) \geq f(t_1', b)g(b) + f(t_1, b')g(b').
\end{aligned}
$$

□

Hence, to show that a function $u_1(b; t_1) = f(t_1, b)g(b)$ satisfies SCP in $(b; t_1)$, it is sufficient to establish Properties (i) and (ii) in Lemma 13.

The main result on different accuracy follows:

Proposition 15. *Let two bidders choose between X_i^T and X_i^S. If there exists a private-component signal $\tilde{X}_i^T$ satisfying Assumption 5 such that $X_i^T \succ_{T_i} \tilde{X}_i^T$ and $\mathbb{E}[V_i|\tilde{X}_i^T] \stackrel{d}{=} \mathbb{E}[V_i|X_i^S]$, then bidders learn X_i^T in any symmetric equilibrium in the SPA.*

[6] Lehmann (1988) has shown this for a condition similar to the SCP: payoffs which are monotone in the sense of Karlin and Rubin (1956). See Jewitt (2007) for a discussion of assumptions on payoffs such that a version of Proposition 14 applies, and for a general proof for the SCP.

[7] A similar observation has been made in Athey (2001) in the proof of Proposition 7.

Proof. By contradiction, assume there exists a symmetric candidate equilibrium with $\sigma > 0$ and bidding functions β_S and β_T. In what follows, I show that bidder 1 has a strictly profitable deviation. Denote bidder 1's expected utility after learning $X_1 \in \{X_1^T, \tilde{X}_1^T\}$ and bidding optimally as $EU_1^*[X_1]$.

Claim 1. $EU_1^*[X_1^T] \geq EU_1^*[\tilde{X}_1^T]$.

Proof. Let $T_1 = t_1$. Let $\Pr[1 \text{ wins}|b]$ be the probability that bidder 1 wins with a bid b when bidder 2 plays the candidate equilibrium. When winning with b, denote bidder 1's object value as $\mathbb{E}[S + t_1|b, 1 \text{ wins}] \coloneqq t_1 + \mathbb{E}[S|b, 1 \text{ wins}]$, and her expected payment as $\mathbb{E}[\text{pay}|b, 1 \text{ wins}]$. Then, bidder 1's expected utility when she bids b is

$$u_1(t_1, b) = (t_1 + \mathbb{E}[S|b, 1 \text{ wins}] - \mathbb{E}[\text{pay}|b, 1 \text{ wins}]) \Pr[1 \text{ wins}|b]. \tag{8}$$

The above expression does not depend on whether bidder 1 learns X_1^T or $\tilde{X}_1^T$ and its realization. This is because with both signals, bidders' information is independent and hence, winning probability and inference about S only depends on the placed bid b.

Next, I show that $u_1(t_1, b)$ satisfies SCP in $(b; t_1)$, a prerequisite of Proposition 14. To establish this, by Lemma 13, it is sufficient to show that (i) the function $f(t_1, b) \coloneqq t_1 + \mathbb{E}[S|b, 1 \text{ wins}] - \mathbb{E}[\text{pay}|b, 1 \text{ wins}]$ is spm and nondecreasing in t_1, and (ii) $g(b) \coloneqq \Pr[1 \text{ wins}|b]$ is nondecreasing and nonnegative. $\Pr[1 \text{ wins}|b]$ is nonnegative and nondecreasing as a higher bid is weakly more likely to win in an SPA. Hence, (ii) is satisfied. The function $f(t_1, b)$ is nondecreasing in t_1, as only the first term t_1 depends on t_1 and is nondecreasing by assumption. Finally, none of the additive terms in f depend on both t_1 and b; it is straightforward that $f(.,.)$ is spm. Hence, (i) is satisfied. This establishes that $u_1(t_1, b)$ is SCP in $(b; t_1)$. Thus, Proposition 14 can be applied: bidder 1 weakly prefers X_1^T to $\tilde{X}_1^T$ because $X_1^T \succ_{T_1} \tilde{X}_1^T$. □

Denote bidder 1's expected utility in the candidate equilibrium when learning X_1^S as $EU_1^c[X_1^S]$.

Claim 2. $EU_1^*[\tilde{X}_1^T] > EU_1^c[X_1^S]$.

Proof. This is an immediate corollary of Proposition 6. Bidder 1 is strictly better off learning $\tilde{X}_1^T$ instead of X_1^S and using the same bidding function as in the candidate equilibrium. If bidder 2 learns X_2^S (which occurs with probability $\sigma > 0$), then this deviation yields a strictly lower expected payoff with no downside; if bidder 2 learns

X_2^T, then (due to independence of private signals and because $\mathbb{E}[V_1|\tilde{X}_1^T] \overset{d}{=} \mathbb{E}[V_1|X_1^S]$), bidder 1's deviation yields the same expected payoff as the candidate equilibrium. $\square$

Hence, bidder 1 has a strictly profitable deviation: she is better off learning X_1^T than $\tilde{X}_1^T$, and strictly better off learning $\tilde{X}_1^T$ instead of X_1^S (which she learns with positive probably in the candidate equilibrium). So, $\sigma > 0$ cannot arise in equilibrium. $\square$

Note that the auxiliary experiment $\tilde{X}_i^T$ is compared to the private-component signal in terms of its accuracy about the private component, not in terms of its accuracy about the total value V_i. Many experiments about the private component can be ranked by the order $\succ_{T_i}$, and (in contrast to $\succ_{V_i}$) this ranking depends neither on the distribution of S nor on how the common and private component enter the value. Assuming $X_i^T \succ_{V_i} X_i^S$ is problematic in a multi-component setup: It might substantially restrict set of experiments, and this set depends on the distribution of the components. And even if this set is nonempty, we cannot apply Proposition 14 to conclude that bidders prefer to learn X_i^T over X_i^S. This is because a bidder's signal is informative about whether the value is high due to a high common or a high private component, so the signal realization remains payoff relevant even when V_i is known.

On the other hand, the existence of an IPV equilibrium in the SPA breaks down if (i) the common-component signal is more accurate about the value than the private-component signal, $X_i^S \succ_{V_i} X_i^T$, or (ii) if there exists $\tilde{X}_i^S$ such that $\mathbb{E}[V_i|X_i^T] \overset{d}{=} \mathbb{E}[V_i|\tilde{X}_i^S]$ and $X_i^S \succ_S \tilde{X}_i^S$. In this case, if bidder i learns about her private component, neither her signal nor her bid is relevant for the other bidder j who then prefers a higher accuracy signal X_i^S about her value or component to a lower accuracy signal X_i^T.

References

Athey, Susan (2001): Single crossing properties and the existence of pure strategy equilibria in games of incomplete information. *Econometrica*, 69:861–889.

Goeree, Jacob K. and Theo Offerman (2003): Competitive bidding in auctions with private and common values. *The Economic Journal*, 113(489):598–613.

Jackson, Matthew O. (2009): Non-existence of equilibrium in Vickrey, second-price, and English auctions. *Review of Economic Design*, 13(1-2):137–145.

Jewitt, Ian (2007): Information Order in Decision and Agency Problems. Working paper.

Karlin, Samuel and Herman Rubin (1956): The theory of decision procedures for distributions with monotone likelihood ratio. *The Annals of Mathematical Statistics*, pp. 272–299.

Lehmann, Erich L. (1988): Comparing location experiments. *The Annals of Statistics*, 16:521–533.

Milgrom, Paul R. and Robert J. Weber (1982): A theory of auctions and competitive bidding. *Econometrica*, 50:1089–1122.

Persico, Nicola (2000): Information acquisition in auctions. *Econometrica*, 68:135–148.

Pesendorfer, Wolfgang and Jeroen M. Swinkels (2000): Efficiency and information aggregation in auctions. *American Economic Review*, 90:499–525.