

# Online Appendix to “Rational Inattention during an RCT”

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April 2025

This Online Appendix contains additional results for “Rational Inattention during an RCT.”

**An analytical solution for the posterior variances  $\Sigma_{0|0}$  and  $\Sigma_{0|0,s}$ .** If the inequality constraint (3) in the paper is non-binding in period  $t = 0$  before the survey, the first-order condition for the before-the-survey posterior variance  $\Sigma_{0|0}$  is given by equation (12) in the paper. It is a quadratic equation with the unique positive solution given by

$$\Sigma_{0|0} = \frac{-(\gamma\sigma_\varepsilon^2 - \mu\rho^2 + \beta\mu\rho^2) + \sqrt{(\gamma\sigma_\varepsilon^2 - \mu\rho^2 + \beta\mu\rho^2)^2 + 4\gamma\rho^2\mu\sigma_\varepsilon^2}}{2\gamma\rho^2}.$$

The same expression is also the solution for the steady-state posterior variance in stage 1 and in stage 3,  $\bar{\Sigma}_{t|t}$ .

If the inequality constraint (3) in the paper is non-binding in period  $t = 0$  during the survey and in period  $t = 1$ , the first-order condition for the after-the-survey posterior variance  $\Sigma_{0|0,s}$  is given by equation (13) in the paper. It is a quadratic equation with the unique positive solution given by

$$\Sigma_{0|0,s} = \frac{-(\gamma\sigma_\varepsilon^2 - \lambda\rho^2 + \beta\mu\rho^2) + \sqrt{(\gamma\sigma_\varepsilon^2 - \lambda\rho^2 + \beta\mu\rho^2)^2 + 4\gamma\rho^2\lambda\sigma_\varepsilon^2}}{2\gamma\rho^2}.$$

**Persistence of the treatment effect.** Rationally inattentive agents optimally decrease attention for some time after an information treatment, which reduces the persistence of the treatment effect compared with a model with exogenous imperfect information.

In the exogenous-imperfect-information model introduced in the paper: (i) outside the survey, in every period each agent updates their prior based on a realization of the signal “current state plus exogenous noise,” with noise  $\psi$  drawn from a normal distribution with mean zero and variance  $\sigma_\psi^2$ ,

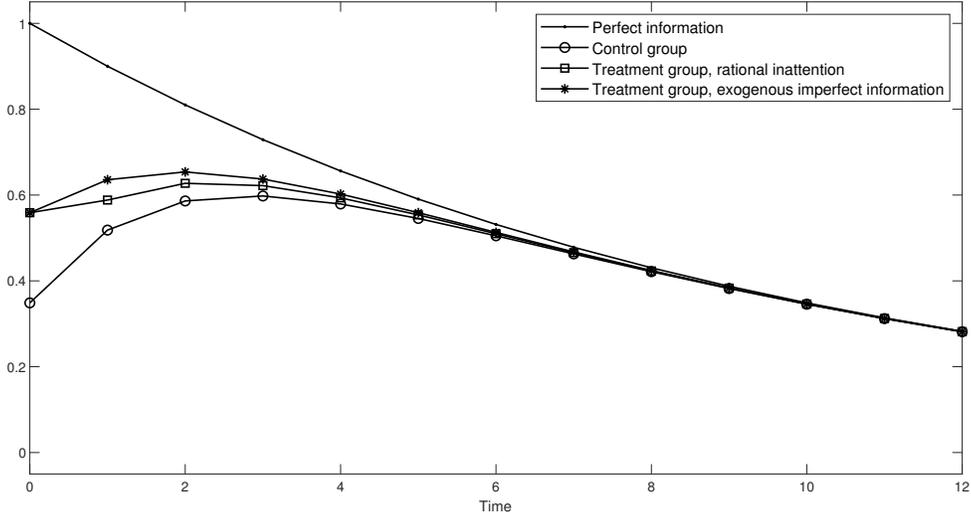
(ii) during the survey, each agent updates their prior based on a realization of the signal “current state plus exogenous noise,” with noise  $\eta$  drawn from a normal distribution with mean zero and variance  $\sigma_\eta^2$ .

Figure 1 compares the impulse response of the conditional expectation of the state,  $\pi_{t|t}$ ,  $t \geq 0$ , to an innovation in the state in period  $t = 0$  for three types of agents: (i) agents from the RI model who did not participate in the survey (“Control group”), (ii) agents from the RI model who participated in the survey (“Treatment group, rational inattention”), and (iii) agents from the exogenous-imperfect-information model who participated in the survey (“Treatment group, exogenous imperfect information”). The figure also reports the impulse response for hypothetical agents with perfect information. The parameter values are:  $\rho = 0.9$ ,  $\sigma_\varepsilon^2 = 1$ ,  $\gamma = 1$ ,  $\beta = 0.99$ ,  $\mu = 2/\ln(2)$ ,  $\lambda = 0.75\mu$ ,  $\sigma_\eta^2 = 2.8954$ ,  $\sigma_\psi^2 = 3.9529$ . With this parameterization, both treatment groups absorb the same amount of information during the survey. Furthermore, the treatment group from the exogenous-imperfect-information model absorbs the same amount of information in each period after the survey as does the control group. That is, agents in the RI model choose  $\sigma_\eta^2 = 2.8954$  during the survey and  $\sigma_\psi^2 = 3.9529$  in the steady state; in the steady state, agents update their belief as if in each period  $t$  they observed a realization of  $x_t + \psi$ , where  $\psi \sim i.i.d.N(0, \sigma_\psi^2)$  and  $\sigma_\psi^2 = 3.9529$ .

As can be seen in Figure 1, both treatment groups start out together in period zero, closer to the perfect-information benchmark than the control group. The initial treatment effect is the same for both treatment groups. In period one, the period after the survey, the treatment group from the RI model optimally chooses to pay less attention than in the steady state. The extra information from the survey pushes the prior variance below its steady-state value, making it optimal to decrease attention. This effect reduces the persistence of the treatment effect compared with the exogenous-imperfect-information model. In period one in Figure 1, the treated agents from the exogenous-imperfect-information model are about 70 percent further from the control group than the treated agents from the RI model, which is a non-trivial difference in the persistence of the treatment effect. In this AR(1) model, convergence to the steady state is immediate in the RI model, and therefore the aforementioned effect operates only in period one. Note that in the exogenous-imperfect-information model, the prior variance is likewise below its steady-state value after the survey, which also affects the Kalman gain in that model; furthermore, convergence to

the steady state is gradual.

Figure 1: **Impulse response of  $\pi_{t|t}$  to an innovation in  $\pi_0$**



**Forecast treatment.** During a survey respondents may be shown a forecast of a future value of the state, and they may be asked to report their forecast. Suppose that in stage 2 of the model agents are constrained to learn from a forecast of the state. The signal absorbed during the survey is a realization of  $S = \rho\pi_0 + \eta$ , with  $\rho > 0$  and  $\sigma_\eta^2$  chosen in the same stage-2 problem, where  $\rho\pi_0$  is the best forecast of next period's state at the time of the survey. The updating equation reads:

$$\pi_{1|0,s} = \pi_{1|0} + \frac{\Sigma_{0|0}}{\Sigma_{0|0} + \sigma_\eta^2/\rho^2} (S - \pi_{1|0})$$

where  $\pi_{1|0,s}$  is the posterior (after-the-survey) mean of period-1 state, and  $\pi_{1|0}$  is the prior (before-the-survey) mean. The treatment effect in the baseline model is  $G = \Sigma_{0|0} / (\Sigma_{0|0} + \sigma_\eta^2)$ . Here the treatment effect is  $\Sigma_{0|0} / (\Sigma_{0|0} + \sigma_\eta^2/\rho^2)$ , which is smaller for a given  $\sigma_\eta^2$  than in the baseline model. However, a rationally inattentive agent chooses a lower  $\sigma_\eta^2$  here than in the baseline model. In fact, the agent chooses a  $\sigma_\eta^2$  such that  $\sigma_\eta^2/\rho^2$  is equal to  $\sigma_\eta^2$  in the baseline. The reason is that during the survey the agent wants to pay the same amount of attention to the current value of the state as in the baseline. As a result, in equilibrium the treatment effect is equal to the treatment effect in the baseline. It does not matter if the provided information is the current realization ( $\pi_0$ ) or the forecast ( $\rho\pi_0$ ).

Outside the AR(1) case, we no longer expect the current-realization and the best-forecast treatments to yield exactly the same treatment effect. Being provided with the current realization or the best forecast are no longer equivalent options for a rationally inattentive agent, and both options are suboptimal (the optimal signal is on the state vector, see Maćkowiak et al., 2018). As an example, we solved numerically the model in the case when the state follows an AR(2) process,  $\pi_t = \rho_1\pi_{t-1} + \rho_2\pi_{t-2} + \varepsilon_t$ , assuming  $\rho_1 = 1.2$ ,  $\rho_2 = -0.3$  with the rest of the baseline parameterization unchanged. We used the code of Afrouzi and Yang (2021) because the stage-3 problem is a special case of the problem that they study. We obtained very similar treatment effects: 0.288 for the current-realization treatment, and 0.293 for the best-forecast treatment. Coibion et al. (2022) elicit households’ inflation expectations and provide different pieces of information regarding inflation during the survey. They find very similar treatment effects when the provided information is the most recent inflation rate or the FOMC’s inflation forecast (or the Federal Reserve’s inflation target). The inflation-forecast and the inflation-target treatments appear to result in slightly more persistent belief revisions.

**“The rate of change in my cost-of-living is different from the official inflation rate.”**

We study a version of the model where the treatment consists of showing respondents a noisy measure of the current value of the state, with noise beyond the respondents’ control. To fix ideas, think of the state as inflation. Suppose the agent cares about an inflation rate that is different from the official inflation rate, because the agent’s consumption basket differs from the consumption basket underlying the official inflation rate or because there is measurement error in the official inflation rate. During a survey experiment, the agent is treated with the official inflation rate.

In a model with perfect information, an agent knows all official statistics and consequently shows a treatment effect of zero, even if the official statistics are only a noisy measure of the state relevant for the agent’s optimal action. In a model with RI, an agent learns directly about the state (i.e., not about official statistics). During a survey, the rationally inattentive agent reports their conditional expectation of the official statistic; during an information treatment, the agent updates their conditional expectations of the state and of the official statistic. Let us now interpret  $\pi_t$  as the inflation rate that the agent cares about. Suppose that the official inflation rate is given by  $\pi_t + z_t$ , where  $z_t$  follows a Gaussian AR(1) process independent of  $x_t$ . During the survey agents are constrained to learn from the current official inflation rate and they decide how much attention

to pay to that variable. The signal absorbed during the survey is a realization of  $S = \pi_0 + z_0 + \eta$ , with  $\sigma_\eta^2$  chosen in the same stage-2 problem. The updating equation reads:

$$\pi_{0|0,s} = \pi_{0|0} + \frac{\Sigma_{0|0}}{\Sigma_{0|0} + \Sigma_z + \sigma_\eta^2} (S - \pi_{0|0})$$

where  $\Sigma_z$  is the unconditional variance of  $z_t$ .<sup>1</sup> The treatment effect,  $\Sigma_{0|0} / (\Sigma_{0|0} + \Sigma_z + \sigma_\eta^2)$ , is smaller than in the baseline model because the signal during the survey is suboptimal for the agent, implying less attention during the survey compared with the baseline. We solve this version of the model numerically using the code of Afrouzi and Yang (2021). We find qualitatively the same comparative statics of the treatment effect with respect to  $\gamma$  and  $\sigma_\varepsilon^2$  as in the baseline model. It does not matter for the comparative statics if the agent cares about an inflation rate that is different from the official inflation rate.

**Respondents are incentivized to pay attention during the survey.** The result in Proposition 1 in the paper is robust to giving agents additional monetary incentives during the experiment. Suppose that the first term in the loss function in stage 2 is not  $\gamma \Sigma_{0|0,s}$  but  $(\gamma + \alpha) \Sigma_{0|0,s}$ , where  $\alpha \geq 0$  captures additional monetary incentives that are provided during the experiment (e.g., agents are given a show-up compensation and some money is deducted from this show-up compensation when the action, such as the reported posterior, is incorrect). Agents who are called into the survey and receive the information treatment then solve the following RI problem at the end of period  $t = 0$  (during the survey):

$$\min_{\Sigma_{0|0,s}} \left\{ \left( (\gamma + \alpha) \Sigma_{0|0,s} + \lambda \ln \left( \frac{\Sigma_{0|0}}{\Sigma_{0|0,s}} \right) \right) + \beta V_1(\Sigma_{1|0}) \right\}$$

subject to

$$\Sigma_{0|0,s} \leq \Sigma_{0|0}$$

$$\Sigma_{1|0} = \rho^2 \Sigma_{0|0,s} + \sigma_\varepsilon^2$$

$$\Sigma_{0|0} \text{ given.}$$

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<sup>1</sup>In principle, it is the prior variance of  $z_t$  at the beginning of the survey, equal to the steady-state posterior variance from stage 1, but the agent had no incentive to pay attention to  $z_t$  in stage 1. Note also that the updating equation for the official inflation rate (as opposed to the inflation rate that the agent cares about) is very similar; the numerator reads  $\Sigma_{0|0} + \Sigma_z$  instead of  $\Sigma_{0|0}$ .

The model in the paper is the special case of  $\alpha = 0$ . The statements of the RI problems in stages 1 and 3 do not change, since the additional monetary incentives are only given during the experiment.

If the inequality constraint (3) in the paper is non-binding in period  $t = 0$  before the survey, the first-order condition for  $x \equiv \Sigma_{0|0}$  is again given by equation (16) in the paper (Proof of Proposition 1) and the expression for  $\left(\frac{\partial x}{\partial \gamma}/x\right)$  is again given by equation (18) in the paper (Proof of Proposition 1), since the additional monetary incentives during the experiment do not affect agents' choices before the experiment.

Next, if the inequality constraint  $\Sigma_{0|0,s} \leq \Sigma_{0|0}$  is non-binding during the survey and the inequality constraint (3) in the paper is non-binding in period  $t = 1$  after the survey, the first-order condition for  $y \equiv \Sigma_{0|0,s}$  reads

$$\gamma + \alpha - \lambda \frac{1}{y} + \beta \mu \frac{1}{y + \frac{\sigma_\varepsilon^2}{\rho^2}} = 0. \quad (1)$$

It follows from the implicit function theorem that

$$\frac{\partial y}{\partial \gamma} = -\frac{1}{\mu} \left[ \frac{\lambda}{\mu} \frac{1}{y} - \beta \frac{y}{\left(y + \frac{\sigma_\varepsilon^2}{\rho^2}\right)^2} \right]^{-1}. \quad (2)$$

Using equation (1) to substitute for  $\frac{\lambda}{\mu} \frac{1}{y}$  in equation (2) yields

$$\frac{\partial y}{\partial \gamma} = -\frac{1}{\mu} \left[ \frac{\gamma + \alpha}{\mu} + \beta \frac{\frac{\sigma_\varepsilon^2}{\rho^2}}{\left(y + \frac{\sigma_\varepsilon^2}{\rho^2}\right)^2} \right]^{-1}. \quad (3)$$

Equations (1)-(3) are the generalizations of equations (19)-(21) in the paper (Proof of Proposition 1) for the case of  $\alpha \geq 0$ .

Comparing the expression for  $\left(\frac{\partial y}{\partial \gamma}/y\right)$  given by equation (3) to the expression for  $\left(\frac{\partial x}{\partial \gamma}/x\right)$  given by equation (18) in the paper (Proof of Proposition 1) yields again that  $\left(\frac{\partial y}{\partial \gamma}/y\right)$  is strictly smaller than  $\left(\frac{\partial x}{\partial \gamma}/x\right)$  in absolute value. It follows from equation (15) in the paper (Proof of Proposition 1) that the treatment effect  $G$  is strictly decreasing in  $\gamma$ .

In sum, the presence of the additional monetary incentives during the experiment ( $\alpha > 0$ ) reduces  $y \equiv \Sigma_{0|0,s}$  because agents are paying more attention during the experiment. See equation (1). However, the additional monetary incentives during the experiment does not affect the result in Proposition 1: the treatment effect  $G$  is strictly decreasing in  $\gamma$ .

## References

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