Supplemental Appendix for “On the Programmability and Uniformity of Digital Currencies”

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Online Appendix A. An example where buyers play a signaling game when tokens are imperfectly recognizable

Below, we consider a signaling game and show that for low α and high σ_H, a separating equilibrium exists: H-buyers hold M unprogrammed tokens and consume too much while L buyers hold $1/(1-\alpha) < M$ programmed tokens and consume efficiently.

Suppose on the North island, a fraction π of buyers learn in advance that, if they trade in the North, the sellers will be uninformed. When an uninformed seller is offered m tokens in a match, the seller's belief is that all the m tokens offered are NOT programmed whenever $m \geq M$, otherwise the belief is that they are all programmed. Suppose $u(q) = \log q$. We conjecture that, in equilibrium, $p_0 = 1$ and $p_1 = \sigma_L(1-\alpha)$. That is, buyers of different types separate by choosing different portfolio. L-buyers hold less than M $\mathfrak{p}_1$-tokens and H buyers hold more than M $\mathfrak{p}_0$-tokens. We will verify that these are the equilibrium prices.

Define

$$\mathbf{1}_{\boldsymbol{M}} = 1 \text{ iff } m_0 + m_1 \geq M.$$

The problem of a type i buyer is

$$\max_{m_{i0}, m_{i1}} \sigma_i u(q^{\pi}_{N,i}) + (1-\sigma_i) u_i(q_{S,i}) - p_0 m_0 - p_1 m_1$$

where

$$\begin{aligned}
q^{\pi}_{N,i} &= (m_{i0}+m_{i1})(1-\alpha)+\mathbf{1}_{\boldsymbol{M}}(m_{i0}+m_{i1})\alpha,\\
q_{S,i} &= m_{i0},\\
u_L(q_S) &= \varepsilon q_S,\\
u_H(q_S) &= u(q_S).
\end{aligned}$$

First, consider L-buyers: Suppose $\mathbf{1}_{\boldsymbol{M}}=0$, then:

$$\max_{m_{L0},m_{L1}} \sigma_L u((m_{L0}+m_{L1})(1-\alpha))+(1-\sigma_L)\varepsilon m_{L0}-m_{L0}-\sigma_L(1-\alpha)m_{L1}+\lambda_M\left[M-(m_{L0}+m_{L1})\right]$$

The FOCs are

$$\begin{aligned}
m_{L0}: \qquad & \sigma_L\frac{1}{(m_{L0}+m_{L1})}+(1-\sigma_L)\varepsilon-1-\lambda_M &\le& \ 0,\\
m_{L1}: \qquad & \sigma_L\frac{1}{(m_{L0}+m_{L1})}-\sigma_L(1-\alpha)-\lambda_M &\le& \ 0.
\end{aligned}$$

with ε small enough, $m_{L0}=0$. Then either $m_{L0}=1/(1-\alpha)<M$ or $m_{L0}=M$.

- Hence, the solution is

$$\begin{aligned}
m_{L1} &= \min\left\{\frac{1}{1-\alpha},M\right\},\\
m_{L0} &= 0.
\end{aligned}$$

Suppose $\mathbf{1}_{\boldsymbol{M}}=1$, then:

$$\max_{m_{L0}+m_{L1}\ge M} \sigma_L u((m_{L0}+m_{L1}))+(1-\sigma_L)\varepsilon m_{L0}-m_{L0}-\sigma_L(1-\alpha)m_{L1}$$

Hence, the solution is

$$\begin{aligned}
m_{L1} &= \max\left\{\frac{1}{1-\alpha},M\right\},\\
m_{L0} &= 0.
\end{aligned}$$

Then the buyer chooses between (i) revealing the true token type

$$m_{L1}=\min\left\{\frac{1}{1-\alpha},M\right\},$$

with a payoff

$$\sigma_L \log\left((1-\alpha)\min\left\{\frac{1}{1-\alpha}, M\right\}\right) - \sigma_L(1-\alpha)\min\left\{\frac{1}{1-\alpha}, M\right\},$$

or (ii) pretends to hold programmed tokens by choosing

$$m_{L1} = m_{H0} \geq M,$$

with a payoff

$$\sigma_L\{\log m_{H0} - (1-\alpha)m_{H0}\}.$$

Option (i) is better if

$$\log\left((1-\alpha)\min\left\{\frac{1}{1-\alpha}, M\right\}\right) - \sigma_L(1-\alpha)\min\left\{\frac{1}{1-\alpha}, M\right\} > \log m_{H0} - (1-\alpha)m_{H0}. \tag{1}$$

Second, consider H-buyers, suppose $\mathbf{1}_{\boldsymbol{M}} = 0$, then:

$$\max_{m_{H0}+m_{H1}<M} \sigma_H u((m_{H0}+m_{H1})(1-\alpha)) + (1-\sigma_H)u(m_{H0}) - m_{H0} - \sigma_L(1-\alpha)m_{H1} + \lambda_M\left[M - (m_{H0}+m_{H1})\right]$$

The FOCs are

$$\begin{aligned} m_{H0} :& \sigma_H \frac{1}{m_{H0}+m_{H1}} + (1-\sigma_H)\frac{1}{m_{H0}} - 1 - \lambda_M = 0, \\ m_{H1} :& \sigma_H \frac{1}{m_{H0}+m_{H1}} - \sigma_L(1-\alpha) - \lambda_M \leq 0. \end{aligned}$$

If $m_{H1} > 0$ and $\lambda_M = 0$, then we have

$$\frac{\sigma_H}{\sigma_L(1-\alpha)} = m_{H0} + m_{H1} < M,$$

and

$$m_{H0} = \frac{1-\sigma_H}{1-\sigma_L(1-\alpha)}.$$

Otherwise, $m_{H1} = 0$ and $\lambda_M = 0$, then

$$m_{H0} = 1 < M,$$

which is consistent with $m_{H1} = 0$ iff $\sigma_H < \sigma_L(1-\alpha)$.

If $\lambda_M > 0$ then $m_{H0} + m_{H1} = M$ and if $m_{H1} > 0$ then

$$\begin{aligned}
\sigma_H \frac{1}{m_{H0} + m_{H1}} + (1 - \sigma_H)\frac{1}{m_{H0}} - 1 - \left(\sigma_H \frac{1}{m_{H0} + m_{H1}} - \sigma_L(1 - \alpha)\right) &= 0, \\
(1 - \sigma_H)\frac{1}{m_{H0}} - 1 + \sigma_L(1 - \alpha) &= 0,
\end{aligned}$$

so that

$$m_{H0} = \frac{(1 - \sigma_H)}{1 - \sigma_L(1 - \alpha)}.$$

If $m_{H1} = 0$ then $m_{H0} = M$ which is consistent with $m_{H1} = 0$ iff

$$\begin{aligned}
\sigma_H \frac{1}{M} - \sigma_L(1 - \alpha) - \lambda_M &\leq 0, \\
\sigma_H \frac{1}{M} - \sigma_L(1 - \alpha) - \left[\sigma_H \frac{1}{M} + (1 - \sigma_H)\frac{1}{M} - 1\right] &\leq 0, \\
-\sigma_L(1 - \alpha) - (1 - \sigma_H)\frac{1}{M} + 1 &\leq 0,
\end{aligned}$$

or

$$M < \frac{(1 - \sigma_H)}{1 - \sigma_L(1 - \alpha)}.$$

Notice that all of this is consistent with the seller's beliefs that all payment below M is done with programmed money. Suppose $\mathbf{1_M} = 1$, then:

$$\max_{m_{H0}+m_{H1}>M} \sigma_H u((m_{H0}+m_{H1}))+(1-\sigma_H)u(m_{H0})-m_{H0}-\sigma_L(1-\alpha)m_{H1}+\lambda_M\left[m_{H0} + m_{H1} - M\right]$$

FOC:

$$\begin{aligned}
m_{H0} :& \sigma_H \frac{1}{m_{H0} + m_{H1}} + (1 - \sigma_H)\frac{1}{m_{H0}} - 1 + \lambda_M = 0, \\
m_{H1} :& \sigma_H \frac{1}{m_{H0} + m_{H1}} - \sigma_L(1 - \alpha) + \lambda_M \leq 0.
\end{aligned}$$

If $m_{H1} > 0$, then we have

$$m_{H0} + m_{H1} = \frac{\sigma_H}{\sigma_L(1 - \alpha)} > M,$$

or

$$m_{H0} + m_{H1} = M > \frac{\sigma_H}{\sigma_L(1 - \alpha)},$$

and

$$m_{H0} = \frac{1-\sigma_H}{1-\sigma_L(1-\alpha)}.$$

Otherwise, $m_{H1} = 0$ and (with $M > 1$):

$$m_{H0} = M > 1,$$

which is consistent with $m_{H1} = 0$ iff

$$\begin{aligned} \sigma_H \frac{1}{m_{H0}} - \sigma_L(1-\alpha) &\leq 0 \\ \frac{\sigma_H}{\sigma_L(1-\alpha)} &\leq M. \end{aligned}$$

Overall, if

$$M > \max\left\{1, \frac{\sigma_H}{\sigma_L(1-\alpha)}\right\} \tag{2}$$

then, conditional on $\mathbf{1_M} = 1$, it is optimal to choose

$$m_{H1} = 0, m_{H0} = M,$$

with a payoff:

$$\sigma_H \log M - M,$$

and conditional on $\mathbf{1_M} = 0$, if $\frac{\sigma_H}{\sigma_L(1-\alpha)} < 1$,

$$m_{H1} = 0, m_{H0} = 1,$$

with a payoff:

$$\sigma_H \log(1-\alpha) - 1.$$

Hence, H-buyers choose $m_{H0} = M$ to reveal the true type of tokens iff

$$\sigma_H \log M - M \geq \sigma_H \log(1-\alpha) - 1.$$

Under this condition, $m_{H0} = M$ and going back to the choice of L-buyers, option (i) is better if

$$\log((1-\alpha)\min\{\frac{1}{1-\alpha}, M\}) - \sigma_L(1-\alpha)\min\{\frac{1}{1-\alpha}, M\} > \log m_{H0} - (1-\alpha)m_{H0}, \tag{3}$$

$$\log((1-\alpha)\min\{\frac{1}{1-\alpha}, M\}) - \sigma_L(1-\alpha)\min\{\frac{1}{1-\alpha}, M\} > \log M - (1-\alpha)M. \tag{4}$$

We require $M > 1/(1-\alpha)$ so that L-buyers choose $m_{L1} = 1/(1-\alpha)$ and H-buyers choose $m_{H0} = M$. So this signaling equilibrium exists iff $\sigma_H < \sigma_L(1-\alpha)$ and

$$\begin{aligned} M &> \max\left\{1, \frac{\sigma_H}{\sigma_L(1-\alpha)}\right\} = 1, \\ \sigma_H \log M - M &\geq \sigma_H \log(1-\alpha) - 1, \\ -\sigma_L &> \log M - (1-\alpha)M. \end{aligned}$$

Online Appendix B. An example where sellers have trade surplus

Suppose the utility function of sellers in the final period is

$$u_s(q_{S,s}) = (1+\gamma)\min\{1, q_{S,s}\}.$$

Then the first-best allocation requires that the consumption of sellers in the final period to be $q_{S,s} = 1$. Then the marginal value of a token to a L-buyer is

$$\sigma_L u'(q_{N,L})[1-\alpha+\alpha(1-\mathfrak{p})(1+\gamma))] + (1-\sigma_L)\varepsilon(1-\mathfrak{p}),$$

and the marginal value to a H-buyer is

$$\sigma_H u'(q_{N,H})[1-\alpha+\alpha(1-\mathfrak{p})(1+\gamma))] + (1-\sigma_H)u'(q_{S,H}).$$

When $\mathcal{M}_H = \{0\}$, $\mathcal{M}_L = \{1\}$, the equilibrium conditions for L-, H- buyers and the banker are given by:

$$\begin{aligned}
\phi_1 &= \beta\sigma_L u'(q_{N,L})(1-\alpha) \\
\phi_1 &= \beta\sigma_L(1-\alpha) \\
\phi_0 &= \beta\sigma_H u'(q_{N,H})(1+\alpha\gamma) + \beta(1-\sigma_H)u'(q_{S,H}) \\
\phi_0 &= \beta
\end{aligned}$$

Assume $u(q) = \log(q)$. Then, the first two conditions imply that

$$q_{N,L} = 1 = m_L(1-\alpha).$$

The last two conditions above imply that

$$\begin{aligned}
m_H &= 1 \\
q_{N,H} &= 1+\alpha\gamma > 1 = q_{S,H}
\end{aligned}$$

H-buyers have no incentive to hold $\mathfrak{p}_1$ if

$$\phi_1 > \beta\sigma_H u'(q_H)(1-\alpha)$$

or

$$\sigma_L > \frac{\sigma_H}{1+\alpha\gamma},$$

which is satisfied. Finally, L-buyers have not incentives to hold $\mathfrak{p}_0$ if

$$\phi_0 > \beta\sigma_L u'(q_{N,L})(1+\alpha\gamma) + \beta(1-\sigma_L)\varepsilon$$

or

$$1 > \frac{\alpha\gamma}{1-\sigma_L} + \varepsilon.$$

So, for γ not too big, this is an equilibrium. However, the allocation is

$$\begin{aligned} q_{N,L} &= 1, q_{S,L} = 0 \\ q_{N,H} &= 1 + \alpha\gamma > 1, q_{S,H} = 1, \\ q_{S,s} &= \frac{f_H \sigma_H}{f_H \sigma_H + f_L \sigma_L} < 1. \end{aligned}$$

Hence, the sellers under-consume in the South while H-buyers over-consume in the North.