Supplemental Appendix

Environmental Subsidies to Mitigate Net-Zero Transition Costs

Eric Jondeau Gregory Levieuge
Jean-Guillaume Sahuc Gauthier Vermandel

CONTENTS

A. The Environmental Goods and Services Sector

A.1. **Definition.** The Environmental Goods and Services Sector (EGSS) consists of a heterogeneous set of producers of goods and services aiming at the protection of the environment and the management of natural resources. Environmental goods and services are products manufactured or services rendered for the main purpose of: *(i)* preventing or minimising pollution, degradation or natural resources depletion; *(ii)* repairing damage to air, water, waste, noise, biodiversity and landscapes; *(iii)* reducing, eliminating, treating and managing pollution, degradation and natural resource depletion; *(iv)* carrying out other activities such as measurement and monitoring, control, research and development, education, training, information and communication related to environmental protection or resource management. The EGSS framework was developed by Eurostat and detailed in a practical handbook, whose first version dates from 1999 and the last from 2016 (https://ec.europa.eu/eurostat/web/products-manuals-and-guidelines/-/ks-ra-09-012). The EGSS was later incorporated into the System of Environmental-Economic Accounting (SEEA), an international statistical standard developed by the United Nations Statistical Division in collaboration with organizations such as the European Commission, International Monetary Fund (IMF), Organisation for Economic Co-operation and Development (OECD), and the World Bank (https://seea.un.org). Consequently, the EGSS serves as a national accounting framework comparable to the GDP by offering a comprehensive breakdown by activity within the environmental sector.

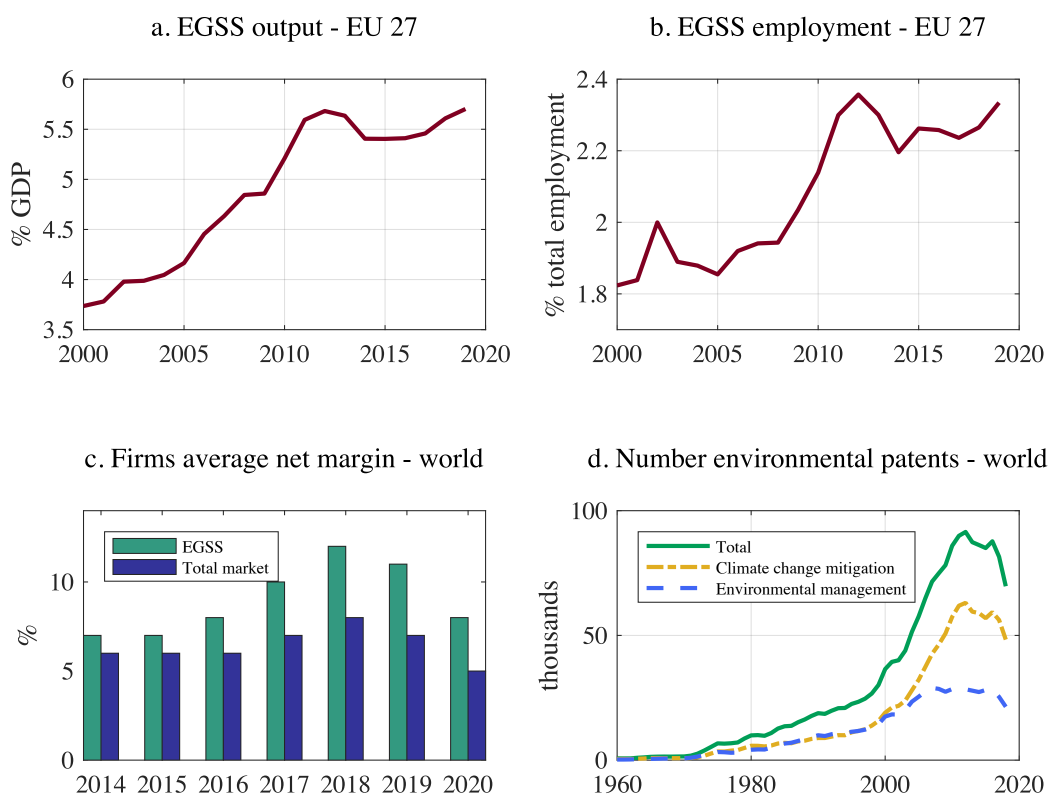
The EGSS includes both: *(i)* mitigative measures, i.e., technologies like carbon capture, air scrubbers, or emission filters that reduce pollution without altering the energy source; and *(ii)* preventive measures, i.e., systemic changes, including energy substitution (e.g., transitioning to renewable energy), to reduce emissions by eliminating fossil fuel dependence.[1] This means that energy substitution is included under the umbrella of abatement in a broad sense, as it achieves the same goal of reducing emissions, albeit through transformative rather than mitigative means. Some exemples are: *(i)* replacing coal-fired power plants with solar or wind farms as a direct emissions reduction measure; *(ii)* substituting gasoline vehicles with electric vehicles powered by renewable energy; and *(iii)* switching from fossil-fuel-based energy to green hydrogen in manufacturing and heavy industry. These efforts are part of the broader abatement goal of decarbonization, even though they involve a more transformative

[1]This distinction is well documented in the literature, notably by Hassler et al. (2021), who develop a framework where both energy-saving and energy-substitution emerge endogenously through directed technical change. Their findings highlight how innovation responds to resource scarcity, steering the economy toward cleaner energy sources.

approach. In summary, the EGSS, referred to as *abatement goods sector* in this paper for brevity, aims to reduce environmental harm, whether by abating pollution from current systems or by introducing cleaner technologies and processes.

A.2. **Statistics on EGSS.** The gross output of the EGSS is estimated at 5.5% and 1.9% of GDP in the European Union (Panel a of Figure SA.1) and the United States (Fixler et al., 2024), respectively. Figure SA.1 also indicates that this sector is more concentrated than the overall economy, with net margins significantly exceeding the average across all industries (Panel c). Furthermore, after decades of growth, the number of environment-related patents worldwide began to decline in 2012 (Panel d).

FIGURE SA.1. Key characteristics of the environmental goods and services sector



Note: Panel A reports the share of EGSS output in the total GDP. The EGSS output consists of the value of *(i)* products that become available for use outside the producer unit, *(ii)* any goods and services produced for own final use and *(iii)* goods that remain in the inventories at the end of the period in which they are produced. It is valued at basic prices (i.e., the prices receivable by the producer from the purchaser minus taxes and plus subsidies on products). Panel B reports the share of EGSS employment in the total employment. It is measured in full-time equivalent jobs engaged in the production of environmental goods and services. The full-time equivalent is defined as the total hours worked divided by the average annual working hours in a full-time job. Panel C reports the average net margin (i.e., the net income on total revenue) computed from a panel of 600 firms worldwide for EGSS (represented by "Green and Renewable Energy" and "Environmental and Waste Services") and 46,500 firms for the total market. Panel D displays the annual number of environment-related patents by category (Haščič and Migotto, 2015). The data cover all family sizes worldwide. Source: Eurostat, OECD, Bloomberg, Morningstar, Capital IQ, and Compustat.

B. A DISCUSSION ON ABATEMENT TIMING

This section discusses how duration can be incorporated into abatement, either within the current single-period framework (using net present value) or across multiple periods (through durable abatement goods).

B.1. **Definition.** In our economy, abatement goods are considered perishable, meaning they generate emissions reductions only for a single period upon deployment. This approach follows the Marginal Abatement Cost Curve (MACC) framework, which links each mitigation action to its associated cost and effectiveness. The MACC illustrates the relationship between abatement costs and effectiveness, plotting the marginal cost (typically in monetary terms per ton of CO2 abated) on the vertical axis and the cumulative emissions reduction potential on the horizontal axis. Each point on the curve represents a specific mitigation measure, such as energy efficiency improvements, fuel switching, or carbon capture technologies. A key feature of the MACC is its convexity, reflecting the increasing marginal cost of stricter emissions reductions. This principle is embedded in the DICE model, where the marginal abatement function takes the form:

$$\frac{\partial(\theta_{1,t}\mu_t^{\theta_2})}{\partial\mu_t} > 0, \tag{sa.1}$$

which is highly convex, consistent with empirical evidence on MAC curves. However, not all abatement goods are purely short-lived. Some, such as solar power plants or energy-efficient infrastructure, have a lifespan extending beyond a single period. Consequently, the calculation of the abatement curve must account for the specific duration and investment structure of each mitigation action.

B.2. **Reformulating the marginal abatement cost curve in terms of net present value.** The MACC typically represents the cost of reducing one unit of emissions (e.g., a ton of CO2) across different mitigation options. Rather than expressing it solely as a cost per ton, the MACC can be interpreted in terms of the Net Present Value (NPV) of each mitigation option. The NPV of a mitigation option considers the upfront investment costs, which include the initial capital required to implement the abatement measure. Additionally, ongoing operating and maintenance costs play a crucial role in determining the overall economic viability of the option. Some abatement measures generate savings over time, such as reduced energy consumption or fuel switching benefits, which should also be incorporated into the evaluation. We define the NPV of an abatement option a as:

$$\text{NPV}(a) = \sum_{t=0}^{\mathcal{T}} \frac{1}{(1+r)^t} \frac{\mathscr{C}_t(a)}{\mathscr{E}_t(a)} \tag{sa.2}$$

where r denotes the discount rate that adjusts future cash flows to their present value, and $\mathcal{T}$ represents the project's lifetime, defining the period over which costs and savings are evaluated. The term $\mathscr{C}_t(a)$ captures costs incurred at time t, including initial investment expenditures in $t = 0$, ongoing operational and maintenance costs throughout the project's lifespan, and dismantling costs at the end of the investment. Finally, $\mathscr{E}_t(a)$ represents the flow of emissions avoided (e.g. energy efficiency gains, fuel-switching benefits) at time t. The economic viability of a mitigation project is determined by comparing its NPV to the carbon tax τ. If $NPV < \tau$, the project is economically beneficial while reducing emissions. Conversely, if $NPV > \tau$, subsidies or financial incentives are required for adoption.

In the DICE framework, abatement costs are modeled as recurring expenses, resembling coupon payments on an annuity rather than a one-time lump-sum investment. Under this approach, abatement expenditures are spread over time rather than being incurred upfront, aligning with the valuation of a financial asset that generates periodic payments (Equation (sa.2)). If financial markets efficiently fund mitigation efforts, firms do not need to bear the entire upfront cost. Instead, they can smooth expenditures over the project's lifetime, effectively paying a periodic coupon-like cost. This interpretation suggests that the flow-based treatment of abatement in DICE reflects a scenario where firms continually finance emissions reduction rather than making an irreversible investment in long-term infrastructure.

B.3. **Abatement goods as durable goods.** Abatement can also be explicitly treated as a durable good. In this context, polluters cannot roll over the costs and benefits of abatement infrastructure through financial markets, as if their participation in financial markets were restricted. As a result, we modify Equation (4) from the main text as follows:

$$\Lambda_{i,t} = \left(\theta_{1,t} x_{i,t}^{\theta_2}\right) y_{i,t} / \delta_\mu^{\theta_2}, \tag{sa.3}$$

where

$$x_{i,t} = \mu_{i,t} - \left(1 - \delta_\mu\right) \mu_{i,t-1}, \tag{sa.4}$$

and δ_μ captures the lifetime of an investment in abating carbon emissions. Investing $\Lambda_{i,t}$ for firm i yields $x_{i,t}$ % additional reduction in emissions. Dividing Equation (sa.3) by $\delta_\mu^{\theta_2}$ allows the backstop price (obtained when $\mu_{i,t} = \mu_b = 1$) to remain the same for any value of δ_μ. We adopt a 8% annual rate for this parameter, which assumes a lifetime of 50 years for abatement infrastructures, and evaluate a net zero transition in our scenario.

FIGURE SA.2. Transition in the baseline scenario with abatement as one-period (blue) and durable (red) goods.

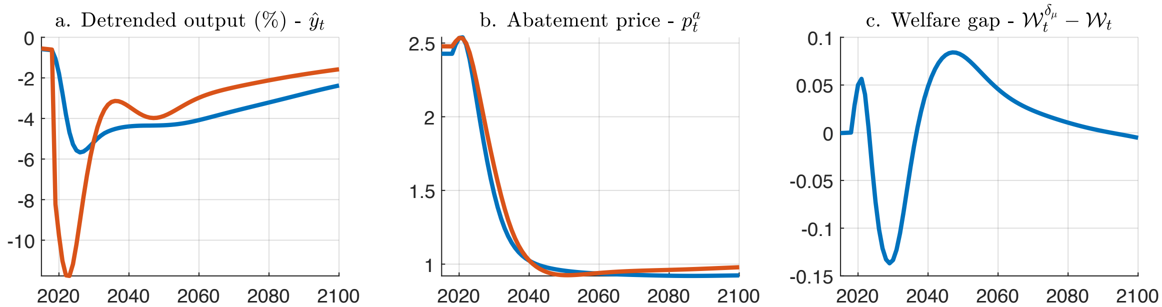


Figure (SA.2) illustrates the transition dynamics in the baseline scenario (without a subsidy policy) under two different assumptions about the lifetime of abatement goods. The blue curve represents the standard case, where abatement goods are non-durable and have a one-period lifespan with $\delta_\mu = 1$. In contrast, the red curve depicts a framework incorporating durable abatement goods, allowing investments to persist across multiple periods. The results indicate that under the durable abatement framework, the initial transition cost is significantly higher. Achieving net-zero emissions requires large upfront investments, without the flexibility to spread costs over time. As a result, the initial GDP loss reaches -11%, nearly twice as severe as in the non-durable abatement scenario. However, once these investments are made, the economic downturn is less pronounced because the abatement infrastructure remains operational, reducing future costs.

Should limited abatement be considered as a durable product? Our simulation, which projects an 11% GDP loss during the transition, significantly exceeds the 4-6% GDP loss estimated in current NGFS scenarios for Greening the Financial System (NGFS) and IPCC projections. This discrepancy suggests that modeling abatement as a durable investment may overestimate the short-term economic burden of transition policies.

Consequently, a one-period cost assumption appears more consistent with economic realities, where firms can *(i)* leverage financial instruments to distribute the transition costs efficiently over time, and *(ii)* smooth gains and losses associated with carbon pricing policies across the lifespan of newly deployed technologies and infrastructure.

C. Full model

C.1. **Climate block.** The climate block relies on a derived version of Nordhaus (1992, 2018) with minor changes to make it more consistent with the climate dynamics. The law of motion of the atmospheric loading of CO_2 (in gigatons of CO_2) is given by:

$$M_t = M_{1750} + (1 - \delta_M)(M_{t-1} - M_{1750}) + \xi_M E_t, \tag{sa.5}$$

where E_t denotes anthropogenic carbon emissions in t, $\delta_M \in [0, 1]$ represents the rate of transfer of atmospheric carbon to the deep ocean, and $\xi_M \geq 0$ is the atmospheric retention ratio.[2] The term $M_{t-1} - M_{1750}$ represents the excess carbon in the atmosphere net of its (natural) removal, with M_{1750} representing the stock of carbon in the preindustrial era, i.e., the steady-state level in the absence of anthropogenic emissions.

The heat received at the Earth's surface F_t (in watts per square meter, W/m^2) is the sum of the forcing caused by atmospheric CO_2 and the non-CO_2 forcing:

$$F_t = \eta \ \log_2 \left(\frac{M_t}{M_{1750}} \right) + F_{EX,t}, \tag{sa.6}$$

where η denotes the effect on temperature from doubling the stock of atmospheric CO_2. As in DICE models, non-CO_2 forcing $F_{EX,t}$ is an exogenous process:

$$F_{EX,t} = \min(F_{EX,t-1} + F_\Delta, F_{\max}), \tag{sa.7}$$

where the parameter F_Δ denotes the fixed increase in exogenous radiative forcing and $F_{\max}$ is a cap that is met by 2100.

The global mean temperature anomalies of the surface T_t and deep oceans T_t^* with respect to the preindustrial period are given by:

$$T_t = \phi_{11} T_{t-1} + \phi_{12} T_{t-1}^* + \xi_T F_t + \varepsilon_{T,t}, \tag{sa.8}$$

$$T_t^* = \phi_{21} T_{t-1} + \phi_{22} T_{t-1}^*, \tag{sa.9}$$

where $\xi_T \geq 0$ is the elasticity of the surface temperature to Earth's surface heat, while parameters ϕ_{11}, ϕ_{12}, ϕ_{21}, and ϕ_{22} capture either the persistence or interaction between the temperature of the surface and deep oceans. To disentangle transitory changes in temperature versus

[2]More advanced climate blocks have been developed to portray the link between temperature and carbon emissions more accurately. Although these refinements are crucial in assessing physical risks (Folini et al., 2024), they offer limited additional value in the context of transition risk and do not significantly alter the policy recommendations of our study. Furthermore, Dietz and Venmans, 2019 demonstrate that the transient climate response to carbon emissions is adequately approximated as long as δ_M in Equation (sa.5) is sufficiently small.

permanent drifts, we introduce an exogenous stochastic process, $\varepsilon_{T,t} = \rho_T \varepsilon_{T,t-1} + \eta_{T,t}$ with $\eta_{T,t} \sim \mathcal{N}(0, \sigma_T^2)$, which captures cyclical changes in temperature.

C.2. **Household sector.** The world economy is populated by a mass L_t of atomistic, identical, and infinitely lived households. This mass is time varying and captures the upward trend of the world population observed over the last sixty years. Formally, it is assumed that the world population asymptotically converges to a long-run level $L_\infty > 0$, such as $L_t = L_{t-1} \left(L_\infty / L_{t-1} \right)^{\ell_g}$, with $\ell_g \in [0,1]$ being the geometric rate of convergence to L_∞. This formulation for population growth dynamics fits perfectly well with the observed path of the world population from 1960 to the present. Each household indexed by $i \in [0, L_t]$ maximizes its sequence of present and future utility flows that depend positively on consumption $c_{i,t}$ and negatively on hours worked $h_{i,t}$:

$$\mathbb{E}_t \left\{ \sum_{\tau=0}^{\infty} \beta^\tau \left(\frac{c_{i,t+\tau}^{1-\sigma_c}}{1-\sigma_c} - \psi_t \frac{h_{i,t+\tau}^{1+\sigma_h}}{1+\sigma_h} \right) \right\}, \tag{sa.10}$$

subject to the sequence of real budget constraints

$$c_{i,t} + b_{i,t} \leq w_t h_{i,t} + \xi_{i,t} + d_{i,t} + r_{t-1} b_{i,t-1}, \tag{sa.11}$$

where $\mathbb{E}_t$ denotes the expectation conditional upon information available at t, $b_{i,t}$ is the one-period riskless government bond, w_t is the real wage, $\xi_{i,t}$ denotes lump-sum government transfers (or taxes if negative), $d_{i,t}$ is the dividend payments received from holding shares of firms in both the intermediate goods and abatement goods sectors, and r_t is the gross real interest rate. $\beta \in (0,1)$ is the subjective discount factor, $\sigma_c > 0$ is the inverse of the intertemporal elasticity of substitution in consumption, $\sigma_h > 0$ is the inverse of the Frisch labor supply elasticity, and $\psi_t > 0$ is a time-varying parameter that cancels out the effects of the productivity trend on labor supply. Such a feature is necessary to obtain a balanced growth path on hours.[3] Anticipating symmetry across households, the maximization problem gives *(i)* the aggregate labor supply equation $w_t c_t^{-\sigma_c} = \psi_t h_t^{\sigma_h}$ and *(ii)* the Euler equation $\mathbb{E}_t \beta_{t,t+1} r_t = 1$, where $\beta_{t,t+\tau} = \beta^\tau (c_{t+\tau}/c_t)^{-\sigma_c}$ is the stochastic discount factor. Note that carbon tax revenues reversed to households through social transfers do not lead to any change in consumption, as this policy does not materialize as a permanent increase in income.

[3]Note that ψ_t must grow proportionally to the flow of current consumption. Thus, if Z_t denotes the trend in per capita consumption, then $\psi_t = \psi_h Z_t^{1-\sigma_c}$, with ψ_h as a scaling parameter.

The optimal control problem faced by households is given by:

$$\max_{\{c_{i,t},h_{i,t},b_{i,t}\}} \mathbb{E}_t \left\{ \sum_{\tau=0}^{\infty} \beta^{\tau} \left(\frac{c_{i,t+\tau}^{1-\sigma_c}}{1-\sigma_c} - \psi_t \frac{h_{i,t+\tau}^{1+\sigma_h}}{1+\sigma_h} \right) \right.$$
$$\left. + \sum_{\tau=0}^{\infty} \beta^{\tau} \lambda_{t+\tau}^{c} \left(w_{t+\tau} h_{i,t+\tau} + \xi_{i,t+\tau} + d_{i,t+\tau} + r_{t-1+\tau} b_{i,t-1+\tau} - c_{i,t+\tau} - b_{i,t+\tau} \right) \right\}, \tag{sa.12}$$

where λ_t^c denotes the Lagrange multiplier associated to the constraint given by Equation (sa.11). When $\lambda_t^c > 0$, the budget constraint binds equality.

The first-order conditions are given by:

$$(c_{i,t}) : \lambda_t^c = c_{i,t}^{-\sigma_c}, \tag{sa.13}$$

$$(h_{i,t}) : w_t \lambda_t^c = \psi_t h_{i,t}^{\sigma_h}, \tag{sa.14}$$

$$(b_{i,t}) : \lambda_t^c = \beta \, \mathbb{E}_t \{ \lambda_{t+1}^c r_t \}. \tag{sa.15}$$

Substituting the Lagrange multiplier from the previous conditions provides:

$$w_t c_{i,t}^{-\sigma_c} = \psi_t h_{i,t}^{\sigma_h}, \tag{sa.16}$$

$$\mathbb{E}_t \beta_{t,t+1} = \beta \mathbb{E}_t (c_{t+1}/c_t)^{-\sigma_c} \tag{sa.17}$$

where $\mathbb{E}_t \beta_{t,t+1} = 1/r_t$ is the the stochastic discount factor between period t and $t+1$.

C.3. **Business sector.**

C.3.1. *Final good sector.* At every point in time t, a perfectly competitive sector produces a final good Y_t by combining a continuum of intermediate goods $y_{i,t}$, $i \in [0, L_t]$, according to the technology

$$Y_t = \left[L_t^{-1/\zeta} \int_0^{L_t} y_{i,t}^{\frac{\zeta-1}{\zeta}} \mathrm{d}i \right]^{\frac{\zeta}{\zeta-1}}. \tag{sa.18}$$

The number of intermediate good firms owned by households is equal to the size of the population L_t. $\zeta > 1$ measures the substitutability across differentiated intermediate goods. Final good firms take their output price, P_t, and their input prices, $P_{i,t}$, as given and beyond their control.

The final good sector must solve the following problem:

$$\max_{\{Y_t, y_{i,t}\}} P_t Y_t - \int_0^{L_t} P_{i,t} y_{i,t} \mathrm{d}i + \lambda_t^f \left[L_t^{\frac{-1}{\zeta}} \int_0^{L_t} y_{i,t}^{\frac{\zeta-1}{\zeta}} \mathrm{d}i - Y_t^{\frac{\zeta-1}{\zeta}} \right], \tag{sa.19}$$

where λ_t^f is the Lagrangian multiplier on the supply curve.

The first-order conditions are given by:

$$(Y_t) : 0 = P_t - \frac{\zeta - 1}{\zeta} \lambda_t^f Y_t^{\frac{-1}{\zeta}}, \tag{sa.20}$$

$$(y_{i,t}) : 0 = -P_{i,t} + \frac{\zeta - 1}{\zeta} \lambda_t^f L_t^{\frac{-1}{\zeta}} y_{i,t}^{\frac{-1}{\zeta}}. \tag{sa.21}$$

Profit maximization implies the demand curve:

$$y_{i,t} = \frac{1}{L_t} \left(\frac{P_{i,t}}{P_t} \right)^{-\zeta} Y_t, \tag{sa.22}$$

from which we deduce the relationship between the prices of the final good and intermediate goods

$$P_t \equiv \left[\frac{1}{L_t} \int_0^{L_t} P_{i,t}{}^{1-\zeta} \mathrm{d}i \right]^{\frac{1}{1-\zeta}}. \tag{sa.23}$$

C.3.2. *Intermediate good sector.* Intermediate good i is produced by a monopolistic firm using the following production function:

$$y_{i,t} = \Gamma_t h_{i,t}^I, \tag{sa.24}$$

where Γ_t is total factor productivity (TFP), which affects labor demand $h_{i,t}^I$.

TFP is determined by the following three components:

$$\Gamma_t = \Phi(T_t) Z_t \varepsilon_{Z,t}, \tag{sa.25}$$

where

$$\Phi(T_t) = 1/(1 + aT_t^2), \tag{sa.26}$$

$$\varepsilon_{Z,t} = (1 - \rho_Z) + \rho_Z \varepsilon_{Z,t-1} + \eta_{Z,t}, \text{ with } \eta_{Z,t} \sim N(0, \sigma_Z^2), \tag{sa.27}$$

$$\log Z_t = \log Z_{t-1} + (1 - \exp(-\delta_Z)) \left(\frac{\gamma_Z}{\delta_Z} - \log \left(\frac{Z_t}{Z_{t0}} \right) \right). \tag{sa.28}$$

Firm's emissions take the following form

$$e_{i,t} = \sigma_t (1 - \mu_{i,t}) y_{i,t} \varepsilon_{E,t}, \tag{sa.29}$$

where:

$$\log \sigma_t = \log \sigma_{t-1} + (1 - \exp(-\delta_\sigma)) \left(\frac{\gamma_\sigma}{\delta_\sigma} - \log \left(\frac{\sigma_t}{\sigma_{t0}} \right) \right),$$

$$\varepsilon_{E,t} = (1 - \rho_E) + \rho_E \varepsilon_{E,t-1} + \eta_{E,t}, \text{ with } \eta_{E,t} \sim N(0, \sigma_E^2)$$

Firms have access to a set of abatement actions. Following Nordhaus (2018), we assume that the cost of abatement equipment (in proportion to output) is given by:

$$\Lambda_{i,t} = \left(\theta_{1,t}\, \mu_{i,t}^{\theta_2}\right) y_{i,t}, \tag{sa.30}$$

where

$$\theta_{1,t} = \frac{p_b}{\theta_2}(1-\delta_{pb})^{t-t_0}\sigma_t.$$

Here, $p_b > 0$ is a parameter that determines the initial cost of abatement, $0 < \delta_{pb} < 1$ captures technological progress, $\theta_2 > 0$ represents the curvature of the abatement cost function.

The intermediate good firm i maximizes its one-period profits:

$$\max_{\{h_{i,t}^I, \mu_{i,t}\}} p_{i,t}y_{i,t} - w_t h_{i,t}^I - p_t^A \Lambda_{i,t} - \tau_t e_{i,t}, \tag{sa.31}$$

where $p_{i,t} = P_{i,t}/P_t$ is the relative price of intermediate goods, $p_t^A = P_t^A/P_t$ is the relative abatement price, τ_t is the carbon tax.

The problem can be rewritten as follow:

$$\max_{\{h_{i,t}^I, \mu_{i,t}\}} \left[p_{i,t}\Gamma_t - w_t - p_t^A\left(\theta_{1,t}\, \mu_{i,t}^{\theta_2}\right)\Gamma_t - \tau_t\sigma_t\left(1-\mu_{i,t}\right)\Gamma_t\varepsilon_{E,t}\right] h_{i,t}^I \tag{sa.32}$$

The first-order conditions read as follows:

$$(h_{i,t}^I) : p_{i,t} = \frac{w_t}{\Gamma_t} + p_t^A\left(\theta_{1,t}\, \mu_{i,t}^{\theta_2}\right) + \tau_t\sigma_t\left(1-\mu_{i,t}\right)\varepsilon_{E,t}, \tag{sa.33}$$

$$(\mu_{i,t}) : p_t^A\left(\theta_{1,t}\, \theta_2\mu_{i,t}^{\theta_2-1}\right) = \tau_t\sigma_t\varepsilon_{E,t}. \tag{sa.34}$$

Under imperfect competition, net profit is the distance between the total gains from selling and cost of production,

$$\max_{\{p_{i,t}\}} (p_{i,t} - mc_{i,t})y_{i,t} \tag{sa.35}$$

$$s.t.\;\; y_{i,t} = \frac{1}{L_t}\left(\frac{P_{i,t}}{P_t}\right)^{-\zeta} Y_t$$

with $mc_{i,t}$ denoting the firm's real marginal cost.

Maximizing this profit under the demand curve from final good firms and the production function provides the following pricing scheme:

$$\frac{mc_{i,t}}{p_{i,t}} = \frac{\zeta-1}{\zeta}. \tag{sa.36}$$

Combining (sa.33) and (sa.36) and assuming asymmetry lead to the following expression of the real wage offered by firms:

$$w_t = \Gamma_t \left[\frac{\zeta - 1}{\zeta} - p_t^A \left(\theta_{1,t}\, \mu_t^{\theta_2} \right) - \tau_t \sigma_t \left(1 - \mu_t \right) \varepsilon_{E,t} \right]. \tag{sa.37}$$

In addition, the optimal decision of abatement effort is given by:

$$\mu_t = \left(\frac{\tau_t \sigma_t \varepsilon_{E,t}}{\theta_2 \theta_{1,t} p_t^A} \right)^{1/(\theta_2 - 1)}. \tag{sa.38}$$

C.4. **Abatement goods sector.**

C.4.1. *Abatement goods packers.* Packers produce homogeneous abatement goods $y_{i,t}^A$, $i \in [0, L_t]$, by combining a continuum of varieties of abatement goods $y_{i,\omega,t}^A$, $\omega \in \Omega$, according to the technology:

$$y_{i,t}^A = \left[\int_{\omega \in \Omega} (y_{i,\omega,t}^A)^{\frac{\zeta_A - 1}{\zeta_A}} \mathrm{d}\omega \right]^{\frac{\zeta_A}{\zeta_A - 1}}, \tag{sa.39}$$

where $\zeta_A > 1$ measures the substitutability across varieties.

Packers take their output price, $P_{i,t}^A$, and their input prices, $P_{i,\omega,t}^A$, as given and beyond their control. These packers solve the following optimal control problem:

$$\max_{\left\{ y_{i,t}^A, y_{i,\omega,t}^A \right\}} P_{i,t}^A y_{i,t}^A - P_{i,\omega,t}^A y_{i,\omega,t}^A + \lambda_t^\omega \left[\int_{\omega \in \Omega} (y_{i,\omega,t}^A)^{\frac{\zeta_A - 1}{\zeta_A}} \mathrm{d}\omega - (y_{i,t}^A)^{\frac{\zeta_A - 1}{\zeta_A}} \right]. \tag{sa.40}$$

The first-order conditions are given by:

$$(y_{i,t}^A) : 0 = P_{i,t}^A - \frac{\zeta_A - 1}{\zeta_A} \lambda_t^\omega (y_{i,t}^A)^{\frac{-1}{\zeta_A}}, \tag{sa.41}$$

$$(y_{i,\omega,t}^A) : 0 = -P_{i,\omega,t}^A + \frac{\zeta_A - 1}{\zeta_A} \lambda_t^\omega (y_{i,t})^{\frac{-1}{\zeta_A}}. \tag{sa.42}$$

Profit maximization implies the optimal quantity of goods demanded by packer i for each variety of abatement ω,

$$y_{i,\omega,t}^A = \left(\frac{P_{i,\omega,t}^A}{P_{i,t}^A} \right)^{-\zeta_A} y_{i,t}^A, \tag{sa.43}$$

from which we deduce the relationship between the price of the homogeneous abatement good and the prices of abatement varieties $P_{i,t}^A = \left[\int_{\omega \in \Omega} \left(P_{i,\omega,t}^A \right)^{1 - \zeta_A} \mathrm{d}\omega \right]^{\frac{1}{1 - \zeta_A}}$.

C.4.2. *Intensive margin.* Each variety ω from already established firms, *incumbents* for short, is produced using labor, which is subject to TFP as follows:

$$y_{i,\omega,t}^A = \Gamma_t h_{i,\omega,t}^A, \tag{sa.44}$$

where $h^A_{i,\omega,t}$ is the labor demand from firm ω held by household i.

Real profits operating in the abatement goods market are given by:

$$\Pi^A_{i,\omega,t} = \frac{P^A_{i,\omega,t}}{P_t} y^A_{i,\omega,t} - w_t h^A_{i,\omega,t} \left(1 - s^A_t\right), \tag{sa.45}$$

where s^A_t is a subsidy rate to incumbents.

Combining Equations (sa.43) and (sa.44) in Equation (sa.45), the problem is as follows:

$$\max_{\{P^A_{i,\omega,t}\}} \left[\frac{P^A_{i,\omega,t}}{P_t} - \frac{w_t}{\Gamma_t}\left(1 - s^A_t\right)\right] \left(\frac{P^A_{i,\omega,t}}{P^A_{i,t}}\right)^{-\zeta_A} y^A_{i,t}. \tag{sa.46}$$

The first-order condition reads as:

$$(1 - \zeta_A) \frac{P^A_{i,\omega,t}}{P_t} = -\zeta_A \frac{w_t}{\Gamma_t}\left(1 - s^A_t\right).$$

Using $p^A_{i,\omega,t} = P^A_{i,\omega,t}/P_t$ and isolating the price, we find:

$$p^A_{i,\omega,t} = \frac{\zeta_A}{\zeta_A - 1}\left(1 - s^A_t\right)\frac{w_t}{\Gamma_t}. \tag{sa.47}$$

C.4.3. *Extensive margin.* While each household manages a continuum of abatement varieties Ω, only a subset of goods $\Omega_t \in \Omega$ is available at any given time t. We denote by $N_{i,t}$ the number of firms owned by household i in the abatement goods sector (a mass of Ω_t) and by $N^E_{i,t}$ the number of startups created by the household. As in Bilbiie et al. (2012), startups at time t only start producing at $t+1$, which features one period of time-to-build. This assumption is necessary to capture the empirically observed lag between entry and economic growth. The number of firms owned by household i in the abatement goods sector is given by the following law of motion:

$$N_{i,t} = (1 - \delta_A)\left[N_{i,t-1} + \varepsilon_{N,t-1}\left(1 - f_N\left(\frac{N^E_{i,t-1}}{N^E_{i,t-2}}\right)\right) N^E_{i,t-1}\right], \tag{sa.48}$$

where $\delta_A \in [0,1]$ is the probability that any firm incurs an exogenous exit-inducing shock. In addition to the exit shock, startups also face another exit probability $f_N\left(N^E_{i,t-1}/N^E_{i,t-2}\right) = 0.5\chi(N^E_{i,t-1}/N^E_{i,t-2} - 1)^2$. Finally firm entry is subject to an exogenous shock that follows an AR(1) process:

$$\varepsilon_{N,t} = (1 - \rho_N) + \rho_N \varepsilon_{N,t-1} + \eta_{N,t}, \text{ with } \eta_{N,t} \sim N(0, \sigma^2_N). \tag{sa.49}$$

Following Bilbiie et al. (2012), setting up a new firm requires labor services, such as

$$\Gamma_t h_{i,t}^E = X_{w,t} N_{i,t}^E, \tag{sa.50}$$

where $h_{i,t}^E$ is the number of hours worked necessary to establish a startup (subject to total factor productivity Γ_t), while $X_{w,t}$ is a sunk cost subject to trends $\theta_{1,t}$ and Z_t to ensure that the barrier to entry is trend-neutral, $X_{w,t} = \theta_{1,t} Z_t X_w$.

The balance sheet of the investor/household is given by:

$$(1-\delta_A)(\Pi_t^A + v_t)\left[x_{i,t-1} + \varepsilon_{N,t-1}\left(1 - f_N\left(\frac{N_{i,t-1}^E}{N_{i,t-2}^E}\right)\right) N_{i,t-1}^E\right] \\ = d_{i,t}^E + \left(1 - s_t^E\right) h_{i,t}^E w_t + X_{q,t} N_{i,t}^E + x_{i,t} v_t. \tag{sa.51}$$

The right-hand term of this equation represents the expenditure side of the household-investor, composed of the labor cost ($h_{i,t}^E w_t$) used in Equation (sa.50) for the creation of startups subject to subsidy policy $\left(1 - s_t^E\right)$. Shares purchase ($x_{i,t}$) valued at market price (v_t) and pay dividends equal to the next-period profits (Π_t^A). Firm entry is also subject to a barrier to entry, $X_{q,t} = X_q Z_t$, which also grows at the same rate as TFP and the cost of abatement to ensure that the entry barrier remains the same across time in relative terms. Investing in existing firms and startups provides profits denoted by $d_{i,t}^E$, which go to zero under perfect competition across investors.

An investor/household willing to establish a startup solves the following optimization problem:

$$\max_{\left\{h_{i,t}^E, N_{i,t}^E, x_{i,t}\right\}} \mathbb{E}_t \left\{\sum_{\tau=0}^{\infty} \beta_{t,t+\tau} d_{i,t+\tau}^E\right\}, \tag{sa.52}$$

which can be rewritten as follows:

$$\max_{\left\{N_{i,t}^E, x_{i,t}\right\}} \mathbb{E}_t \left\{\sum_{\tau=0}^{\infty} \beta_{t,t+\tau} \left[(1-\delta_A)(\Pi_{t+\tau}^A + v_{t+\tau})\left(x_{i,t-1+\tau} + (1 - f_{N,t-1})\, N_{i,t-1+\tau}^E\right) \right.\right. \\ \left.\left. - \left(N_{i,t+\tau}^E X_{i,t+\tau} + x_{i,t+\tau}\, v_{t+\tau}\right)\right]\right\},$$

where $f_{N,t-1} = f_N\left(N_{i,t-1}^E / N_{i,t-2}^E\right)$ and $X_{i,t} = X_t = \left[X_w(1 - s_t^E) w_t / \Gamma_t + X_q\right]$.

The first-order conditions solving the optimal control problem are:

$$(x_{i,t}) : v_t = \mathbb{E}_t \left\{ \beta_{t,t+1}(1-\delta_A)(\Pi^A_{t+1} + v_{t+1}) \right\}, \tag{sa.53}$$

$$\begin{aligned}(N^E_{i,t}) : X_{i,t} = \mathbb{E}_t &\left\{ \beta_{t,t+1}(1-\delta_A)(\Pi^A_{t+1} + v_{t+1}) \left(1 - \frac{\partial f_{N,t} N^E_{i,t}}{\partial N^E_{i,t}}\right) \right\} \\ &- \mathbb{E}_t \left\{ \beta_{t,t+2}(1-\delta_A)(\Pi^A_{t+2} + v_{t+2}) \frac{\partial f_{N,t+1}}{\partial N^E_{i,t}} N^E_{i,t+1} \right\}.\end{aligned} \tag{sa.54}$$

To rewrite the system in a state-space form, one needs to eliminate the $t+2$ terms in the first-order condition associated with $(N^E_{i,t})$ by exploiting forward recursion in the first-order condition associated with $(x_{i,t})$. To do so, consider first:

$$\lambda_t v_t = \beta \lambda_{t+1}(1-\delta_A)(\Pi^A_{t+1} + v_{t+1}).$$

Iterating forward:

$$\lambda_{t+1} v_{t+1} = \beta \lambda_{t+2}(1-\delta_A)(\Pi^A_{t+2} + v_{t+2}).$$

Therefore, the term in $t+2$ of the second first-order condition can be rewritten as:

$$\begin{aligned}\mathbb{E}_t \left\{ \beta_{t,t+2}(1-\delta_A)(\Pi^A_{t+2} + v_{t+2}) \frac{\partial f_{N,t+1}}{\partial N^E_{i,t}} N^E_{i,t+1} \right\} &= \mathbb{E}_t \left\{ \beta^2 \frac{\lambda_{t+2}}{\lambda_t} (1-\delta_A)(\Pi^A_{t+2} + v_{t+2}) \frac{\partial f_{N,t+1}}{\partial N^E_{i,t}} N^E_{i,t+1} \right\} \\ &= \mathbb{E}_t \left\{ \beta \frac{1}{\lambda_t} \beta \lambda_{t+2}(1-\delta_A)(\Pi^A_{t+2} + v_{t+2}) \frac{\partial f_{N,t+1}}{\partial N^E_{i,t}} N^E_{i,t+1} \right\} \\ &= \mathbb{E}_t \left\{ \beta \frac{1}{\lambda_t} \lambda_{t+1} v_{t+1} \frac{\partial f_{N,t+1}}{\partial N^E_{i,t}} N^E_{i,t+1} \right\} \\ &= \mathbb{E}_t \left\{ \beta_{t,t+1} v_{t+1} \frac{\partial f_{N,t+1}}{\partial N^E_{i,t}} N^E_{i,t+1} \right\}\end{aligned}$$

Combining these two first-order conditions allows us to get:

$$Z_t \theta_{1,t} \left[\left(1 - s^E_t\right) X_w N^E_{i,t} \frac{w_t}{\Gamma_t} + X^q \right] = v_t \left(1 - \frac{\partial f_{N,t} N^E_{i,t}}{\partial N^E_{i,t}}\right) - \mathbb{E}_t \left\{ \beta_{t,t+1} v_{t+1} \frac{\partial f_{N,t+1}}{\partial N^E_{i,t}} N^E_{i,t+1} \right\}, \tag{sa.55}$$

where:

$$\frac{\partial f_{N,t} N^E_{i,t}}{\partial N^E_{i,t}} = 0.5\chi \left(3 \left(\frac{N^E_{i,t}}{N^E_{i,t-1}} \right)^2 + 1 - 4 \frac{N^E_{i,t}}{N^E_{i,t-1}} \right),$$

$$\frac{\partial f_{N,t+1}}{\partial N^E_{i,t}} N^E_{i,t+1} = -\chi \left(\frac{N^E_{i,t+1}}{N^E_{i,t}} \right)^2 \left(\frac{N^E_{i,t+1}}{N^E_{i,t}} - 1 \right).$$

C.5. **Public sector and environmental policies.** The government issues bonds, collects the carbon tax from firms' emissions, repays the issued bonds with interest payments, makes some unproductive expenditures, pays (or collects) a lump-sum transfer (or tax) to (from) households, and may provide subsidies to the abatement goods sector. The budget constraint is:

$$B_t + \tau_t E_t = r_{t-1} B_{t-1} + G_t + \xi_t + (s_t^A w_t L_t h_t^A + s_t^E w_t N_t^E L_t h_t^E). \tag{sa.56}$$

Public spending is determined exogenously as $G_t = g_y Y_t \varepsilon_{G,t}$, where $g_y \in [0, 1]$ is the steady-state share of public spending to output and $\varepsilon_{G,t}$ is a government spending shock. This shock captures exogenous shifts in aggregate demand and follows $\varepsilon_{G,t} = (1 - \rho_G) + \rho_G \varepsilon_{G,t-1} + \eta_{G,t}$, with $\eta_{G,t} \sim \mathcal{N}(0, \sigma_G^2)$. The total lump-sum transfer to households and the total issued bonds read as $\xi_t = \int_0^{L_t} \xi_{i,t} \mathrm{d}i$ and $B_t = \int_0^{L_t} b_{i,t} \mathrm{d}i$, respectively.

In the following, we assume that public expenditures are financed by a combination of bond issues (or equivalently debt) and lump-sum taxes. In addition, carbon tax revenues can either (*i*) be returned to households via lump-sum transfers and used for debt repayment, or (*ii*) be spent on subsidies to the abatement goods sector.

C.6. **Market clearing and equilibrium conditions.** First, the annual flow of emissions is given by the total emissions from firms $E_t = \int_0^{L_t} e_{i,t} \mathrm{d}i$, while output is given by $Y_t = \int_0^{L_t} y_{i,t} \mathrm{d}i$. Note that because firms are symmetric, the abatement rate is the same across firms $\mu_{i,t} = \mu_t$. Therefore, the aggregate flow of emissions is expressed as follows:

$$E_t = \sigma_t (1 - \mu_t) Y_t \, \varepsilon_{E,t}. \tag{sa.57}$$

Resource constraints determining aggregate demand are obtained from the aggregation of household consumption $C_t = L_t c_t = \int_0^{L_t} c_{i,t} \mathrm{d}i$, government spending, and the barrier to entry costs paid in terms of the final good:

$$Y_t = C_t + G_t + N_t^E L_t \theta_{1,t} Z_t X_q. \tag{sa.58}$$

In addition, we define detrended output as the percentage deviation of output Y_t from productivity and population trends, as follows:

$$\hat{Y}_t = 100 \times \log \left(\frac{Y_t}{Z_t L_t} \right). \tag{sa.59}$$

This metric allows us to compare the dynamics of output more easily than directly focusing on the output level.[4]

The aggregate demand of abatement goods reads as follows:

$$N_t Y_t^A = \left(\frac{\tilde{P}_t^A}{P_t^A}\right)^{-\zeta_A} L_t \Lambda_t. \tag{sa.60}$$

In this expression, because households are symmetric, the relative price ratio remains unchanged at the aggregate level $\tilde{P}_{i,t}^A / P_{i,t}^A = \tilde{P}_t^A / P_t^A$. The aggregate production function reads as follows:

$$N_t Y_t^A = \Gamma_t H_t^A, \tag{sa.61}$$

where $H_t^A = L_t h_t^A = \int_0^{L_t} \int_{\omega \in \Omega} h_{i,\omega,t}^A \mathrm{d}\omega \mathrm{d}i$ corresponds to the total labor input demand from incumbents in this sector and Y_t^A is the intensive margin in the abatement goods sector.[5] The aggregate selling price, which takes into account the number of incumbents in the determination of the selling price, is:

$$P_t^A = \tilde{P}_t^A \, N_t^{\frac{1}{1-\zeta_A}}. \tag{sa.62}$$

The labor market is at equilibrium when the total supply of households $H_t = L_t h_t = \int_0^{L_t} h_{i,t} \mathrm{d}i$ is equal to the demand from firms producing intermediate goods $H_t^I = \int_0^{L_t} h_{i,t}^I \mathrm{d}i$, abatement good incumbents H_t^A, and startups $H_t^E = L_t h_t^E = \int_0^{L_t} h_{i,t}^E \mathrm{d}i$:

$$H_t = H_t^I + H_t^A + H_t^E, \tag{sa.63}$$

where the aggregate supply of the final good is given by $Y_t = \Gamma_t \, H_t^I$.

Finally, we compute the share of abatement goods in output as follows:

$$\Psi_t = p_t^A \int_0^{L_t} \left(\frac{\Lambda_{i,t}}{Y_{i,t}}\right) \mathrm{d}i = p_t^A \theta_{1,t} \mu_t^{\theta_2}. \tag{sa.64}$$

C.7. **Model's summary.** This section reports the first-order conditions for agents' optimization problems and other relationships that define the equilibrium of the model.

[4] We do not remove the trend associated with the increase in temperature because it is endogenous and, thus, this would make it impossible to compare different policies.

[5] Aggregate labor demands include the number of firms, as in Bilbiie et al. (2012).

C.7.1. *Exogenous trends.*

$$
\begin{aligned}
F_{EX,t} &= \min(F_{EX,t-1} + F_{\Delta}, F_{\max}) \\
\log Z_t &= \log Z_{t-1} + (1 - \exp(-\delta_Z)) \left(\frac{\gamma_Z}{\delta_Z} - \log\left(\frac{Z_t}{Z_{t0}}\right)\right) \\
\log \sigma_t &= \log \sigma_{t-1} + (1 - \exp(-\delta_\sigma)) \left(\frac{\gamma_\sigma}{\delta_\sigma} - \log\left(\frac{\sigma_t}{\sigma_{t0}}\right)\right) \\
\theta_{1,t} &= \frac{p_b}{\theta_2}(1 - \delta_{pb})^{t-t_0}\sigma_t \\
L_t &= L_{t-1}\left(\frac{L_T}{L_{t-1}}\right)^{\ell_g}
\end{aligned}
$$

C.7.2. *Exogenous shocks.*

$$
\begin{aligned}
\varepsilon_{T,t} &= \rho_T \varepsilon_{T,t-1} + \eta_{T,t} \\
\varepsilon_{Z,t} &= (1-\rho_Z) + \rho_Z \varepsilon_{Z,t-1} + \eta_{Z,t} \\
\varepsilon_{E,t} &= (1-\rho_E) + \rho_E \varepsilon_{E,t-1} + \eta_{E,t} \\
\varepsilon_{N,t} &= (1-\rho_N) + \rho_N \varepsilon_{N,t-1} + \eta_{N,t} \\
\varepsilon_{G,t} &= (1-\rho_G) + \rho_G \varepsilon_{G,t-1} + \eta_{G,t}
\end{aligned}
$$

C.7.3. *Climate block.*

$$M_t = M_{1750} + (1-\delta_M)(M_{t-1} - M_{1750}) + \xi_M E_t \tag{sa.65}$$

$$F_t = \eta \ \log_2\left(\frac{M_t}{M_{1750}}\right) + F_{EX,t} \tag{sa.66}$$

$$T_t = \phi_{11} T_{t-1} + \phi_{12} T^*_{t-1} + \xi_T F_t + \varepsilon_{T,t} \tag{sa.67}$$

$$T^*_t = \phi_{21} T_{t-1} + \phi_{22} T^*_{t-1} \tag{sa.68}$$

C.7.4. *Household sector.*

$$w_t c_t^{-\sigma_c} = \psi_h Z_t^{1-\sigma_c} h_t^{\sigma_h} \tag{sa.69}$$

$$\beta_{t,t+1} = \beta \left(\frac{c_{t+1}}{c_t}\right)^{-\sigma_c} \tag{sa.70}$$

C.7.5. *Business sector.*

$$\Gamma_t = \frac{Z_t \varepsilon_{Z,t}}{(1 + aT_t^2)} \tag{sa.71}$$

$$\frac{w_t}{\Gamma_t} = \frac{\zeta - 1}{\zeta} - p_t^A(\theta_{1,t}\ \mu_t^{\theta_2}) - \tau_t \sigma_t \left(1 - \mu_t\right) \varepsilon_{E,t}$$

$$E_t = \sigma_t \left(1 - \mu_t\right) Y_t \tag{sa.72}$$

$$Y_t = \Gamma_t H_t^I \tag{sa.73}$$

$$\mu_t = \left(\frac{\tau_t \sigma_t \varepsilon_{E,t}}{\theta_{1,t}\theta_2 p_t^A}\right)^{1/(\theta_2 - 1)} \tag{sa.74}$$

C.7.6. *Abatement goods sector.*

$$\tilde{p}_t^A = \frac{\zeta_A}{\zeta_A - 1}\left(1 - s_t^A\right)\frac{w_t}{\Gamma_t} \tag{sa.75}$$

$$N_t Y_t^A = \Gamma_t H_t^A \tag{sa.76}$$

$$\Gamma_t h_t^E = \theta_{1,t} Z_t X_w N_t^E \tag{sa.77}$$

$$Z_t \theta_{1,t}\left[(1 - s_t^E) X_w N_t^E w_t / \Gamma_t + X_q\right] = v_t \left(1 - \frac{\partial f_{N,t} N_t^E}{\partial N_t^E}\right) - \mathbb{E}_t\left\{\beta_{t,t+1} v_{t+1} \frac{\partial f_{N,t+1}}{\partial N_t^E} N_{t+1}^E\right\} \tag{sa.78}$$

$$N_t = (1 - \delta_A)\left[N_{t-1} + \varepsilon_{N,t-1}\left(1 - \frac{\chi}{2}\left(\frac{N_{t-1}^E}{N_{t-2}^E} - 1\right)^2\right) N_{t-1}^E\right] \tag{sa.79}$$

$$v_t = \mathbb{E}_t\left\{\beta_{t,t+1}(1 - \delta_A)(\Pi_{t+1}^A + v_{t+1})\right\} \tag{sa.80}$$

$$N_t L_t \Pi_t^A = \tilde{p}_t^A Y_t^A - w_t H_t^A \left(1 - s_t^A\right) \tag{sa.81}$$

C.7.7. *Equilibrium conditions.*

$$L_t h_t = H_t^I + H_t^A + L_t h_t^E \qquad N_t Y_t^A = \left(\frac{\tilde{p}_t^A}{p_t^A}\right)^{-\zeta_A} (\theta_{1,t}\ \mu_t^{\theta_2}) Y_t$$

$$p_t^A = \tilde{p}_t^A\ N_t^{\frac{1}{1-\zeta_A}} \qquad Y_t = L_t c_t + g_y Y_t \varepsilon_{G,t} + N_t^E Z_t L_t \theta_{1,t} X_q$$

Thus, our system is composed of 17 economic variables/equations, $\{c_t, h_t, \beta_{t,t+1}, \Gamma_t, w_t, \mu_t, E_t, Y_t, H_t^I, \tilde{p}_t^A, p_t^A, H_t^A, Y_t^A, N_t, N_t^E, h_t^E, v_t, \Pi_t^A\}$, four climate variables $\{M_t, F_t, T_t, T_t^*\}$, five deterministic trends $\{F_{EX,t}, Z_t, \sigma_t, \theta_{1,t}, L_t\}$ and five exogenous disturbances $\{\varepsilon_{T,t}, \varepsilon_{Z,t}, \varepsilon_{E,t}, \varepsilon_{N,t}, \varepsilon_{G,t}\}$.

D. DETRENDING THE MODEL

The estimation procedure requires a large number of resolutions for the model. Therefore, it is necessary to reduce the time dedicated to resolution as much as possible. To this end, we remove trends from macroeconomic variables, while letting climate-related variables in level. Removing trends reduces the magnitude of the residuals in the dynamic equations when using Newton's optimization routines. In particular, the extended-path method that we use (see Appendix E) requires fewer iterations to obtain residuals below the tolerance value when the model is detrended.

We first define the growth rates of labor productivity Z_t and the cost of abatement $\theta_{1,t}$ as follows:

$$g_{z,t} = \frac{Z_t}{Z_{t-1}} \qquad \text{and} \qquad g_{\theta,t} = \frac{\theta_{1,t}}{\theta_{1,t-1}}.$$

D.1. **Household sector.**

- **Detrended Euler equation:**

$$w_t \tilde{c}_t^{-\sigma_c} = \psi_h h_t^{\sigma_h}, \tag{sa.82}$$

with $\tilde{c}_t = c_t / Z_t$, $\tilde{w}_t = w_t / Z_t$.

- **Detrended stochastic discount factor:**

$$\tilde{\beta}_{t,t+1} = g_{z,t+1}^{-\sigma_c} \beta \left(\frac{\tilde{c}_{t+1}}{\tilde{c}_t} \right)^{-\sigma_c}, \tag{sa.83}$$

with $\tilde{\beta}_{t,t+1} = g_{z,t+1}^{-\sigma_c} \beta_{t,t+1}$.

D.2. **Business sector.**

- **Detrended TFP:**

$$\tilde{\Gamma}_t = \frac{\varepsilon_{Z,t}}{1 + a T_t^2}, \tag{sa.84}$$

with $\tilde{\Gamma}_t = \Gamma_t / Z_t$.

- **Detrended real wage:**

$$\frac{\tilde{w}_t}{\tilde{\Gamma}_t} = \frac{\zeta - 1}{\zeta} - \theta_{1,t} \left[p_t^A \, \mu_t^{\theta_2} + \tilde{\tau}_t \left(1 - \mu_t\right) \varepsilon_{E,t} \right], \tag{sa.85}$$

with $\tilde{\tau}_t = \tau_t \sigma_t / \theta_{1,t}$.

- **Emissions (as a function of detrended output):**

$$E_t = \sigma_t \left(1 - \mu_t\right) L_t Z_t \tilde{Y}_t, \tag{sa.86}$$

where $\tilde{Y}_t = Y_t/(L_t Z_t)$. Note that we do not detrend E_t as it is a direct input for climate-related variables.

- **Detrended production:**

$$\tilde{Y}_t = \tilde{\Gamma}_t \tilde{H}_t^I, \tag{sa.87}$$

where $\tilde{H}_t^I = \tilde{H}_t^I/L_t$.

- **Detrended abatement share:**

$$\mu_t = \left(\frac{\tau_t \sigma_t \varepsilon_{E,t}}{\theta_{1,t}\theta_2 p_t^A}\right)^{1/(\theta_2-1)} \tag{sa.88}$$

D.3. **Abatement goods sector.**

- **The price of each variety:**

$$\tilde{p}_t^A = \frac{\zeta_A}{\zeta_A - 1}\left(1 - s_t^A\right)\frac{\tilde{w}_t}{\tilde{\Gamma}_t} \tag{sa.89}$$

- **Detrended production in the abatement goods sector:**

$$N_t \tilde{Y}_t^A = \tilde{\Gamma}_t \tilde{H}_t^A, \tag{sa.90}$$

with $\tilde{Y}_t^A = Y_t^A/(\theta_{1,t} L_t Z_t)$ and $\tilde{H}_t^A = H_t^A/(\theta_{1,t} L_t)$.

- **Detrended labor in startup creation:**

$$\tilde{\Gamma}_t \tilde{h}_t^E = X_w N_t^E, \tag{sa.91}$$

with $\tilde{h}_t^E = h_t^E/\theta_{1,t}$.

- **Detrended free-entry condition:**

$$\left(1 - s_t^E\right) X_w N_t^E \frac{\tilde{w}_t}{\tilde{\Gamma}_t} + X_q = \tilde{v}_t\left(1 - \frac{\partial f_{N,t} N_t^E}{\partial N_t^E}\right) - \mathbb{E}_t\left\{g_{\theta,t+1} g_{z,t+1}^{1-\sigma_c} \tilde{v}_{t+1} \tilde{\beta}_{t,t+1} \frac{\partial f_{N,t+1}}{\partial N_t^E} N_{t+1}^E\right\}, \tag{sa.92}$$

with $v_t = \tilde{v}_t/(Z_t \theta_{1,t})$.

- **Law of motion of firms:**

$$N_t = (1 - \delta_A)\left[N_{t-1} + \varepsilon_{N,t-1}\left(1 - \frac{\chi}{2}\left(\frac{N_{t-1}^E}{N_{t-2}^E} - 1\right)^2\right) N_{t-1}^E\right] \tag{sa.93}$$

- **Detrended firm value:**

$$\tilde{v}_t = (1 - \delta_A)\mathbb{E}_t\left\{g_{\theta,t+1} g_{Z,t+1}^{1-\sigma_c} \tilde{\beta}_{t,t+1}(\tilde{\Pi}_{t+1}^A + \tilde{v}_{t+1})\right\}, \tag{sa.94}$$

with $\tilde{\Pi}_t^A = \Pi_t^A/(Z_t \theta_{1,t})$.

- **Detrended profit per firm:**

$$\tilde{\Pi}_t^A = \frac{\tilde{p}_t^A \tilde{Y}_t^A - \tilde{w}_t \tilde{H}_t^A \left(1 - s_t^A\right)}{N_t} \tag{sa.95}$$

D.4. **Equilibrium conditions.**

- **Detrended equilibrium on the labor market:**

$$h_t = \tilde{H}_t^I + \theta_{1,t} \left(\tilde{H}_t^A + \tilde{h}_t^E\right) \tag{sa.96}$$

Note that the fraction of the abatement goods sector is not constant, even in the detrended version of the model.

- **Detrended equilibrium on the abatement goods market:**

$$N_t \tilde{Y}_t^A = \left(\frac{\tilde{p}_t^A}{p_t^A}\right)^{-\zeta_A} (\theta_{1,t}\, \mu_t^{\theta_2}) Y_t \tag{sa.97}$$

- **Relative abatement price:**

$$p_t^A = \tilde{p}_t^A \,(N_t)^{\frac{1}{1-\zeta_A}} \tag{sa.98}$$

- **Detrended resource constraint:**

$$\tilde{Y}_t = \tilde{c}_t + g_y \tilde{Y}_t \varepsilon_{G,t} + N_t^E \theta_{1,t} X_q \tag{sa.99}$$

E. Estimation method

E.1. **Solution method.** Our simulations are based on the extended path method initially proposed by Fair and Taylor (1983). Adjemian and Juillard (2014) describe recent work that considers future uncertainty. In this study, we use the deterministic version of the extended path, which assumes certainty equivalence but offers a computationally inexpensive method to estimate the model.

Let us consider the solution of a set of nonlinear deterministic equations:

$$\mathbb{E}_t\{f_\Theta\left(y_{t-1}, y_t, y_{t+1}, \epsilon_t\right)\} = 0, \tag{sa.100}$$

where y_t is a $N \times 1$ vector of endogenous variables in time period t, ϵ_t is a $M \times 1$ vector of exogenous variables, f_Θ is a set of N nonlinear equations based on a vector of parameters θ.

The perfect foresight algorithm. Let y_0 and y_T denote the initial and terminal states, respectively. Let also Y denote the matrix of endogenous variables and X denote the matrix of exogenous variables:

$$Y_{1:T-1} = \begin{pmatrix} y_1 \\ y_2 \\ ... \\ y_{T-1} \end{pmatrix} \quad \text{and} \quad X_{1:T-1} = \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \\ ... \\ \epsilon_{T-1} \end{pmatrix}.$$

The solution system (sa.100) can be represented by a set of N nonlinear equations over $T-1$ time period. Stacking these equations over all periods produces a set of $N \times (T-1)$ equations:

$$R\left(Y_{1:T-1}, X_{1:T-1}, y_{t-1}, y_T\right) = \begin{pmatrix} f_\Theta\left(y_0, y_1, y_2, \epsilon_1\right) & = 0 \\ f_\Theta\left(y_1, y_2, y_3, \epsilon_2\right) & = 0 \\ ... & \\ f_\Theta\left(y_{T-3}, y_{T-2}, y_{T-1}, \epsilon_{T-2}\right) & = 0 \\ f_\Theta\left(y_{T-2}, y_{T-1}, y_T, \epsilon_{T-1}\right) & = 0 \end{pmatrix}. \tag{sa.101}$$

A perfect foresight simulation simply solves:

$$Y^*_{1:T} = \arg \min_{\{Y_{1:T-1}\}} |R\left(Y_{1:T-1}, X_{1:T-1}, y_0, y_T\right)|, \tag{sa.102}$$

with residual matrix $R\left(Y^*_{1:T-1}, X_{1:T-1}, y_0, y_T\right)$ reaching some tolerance threshold.

In this paper, we use the relaxation algorithm developed by Laffargue (1990), Boucekkine (1995), and Juillard et al. (1996).

The extended path algorithm. The extended path approach is simply a perfect foresight solution that is consistent with rational expectations, i.e., $\mathbb{E}_t\{\epsilon_{t+s}\} = 0$ for $s > 0$. Considering the system given by Equation (sa.101) under rational expectations, to obtain the first simulation period y_1^*, the corresponding stacked equations are as follows:

$$R(Y_{1:T-1}, \epsilon_1, y_0, y_T) = \begin{pmatrix} \mathbb{E}_t\{f_\theta(y_0, y_1, y_2, \epsilon_1)\} & = 0 \\ \mathbb{E}_t\{f_\theta(y_1, y_2, y_3, 0)\} & = 0 \\ \dots & \\ \mathbb{E}_t\{f_\theta(y_{T-3}, y_{T-2}, y_{T-1}, 0)\} & = 0 \\ \mathbb{E}_t\{f_\theta(y_{T-2}, y_{T-1}, y_T, 0)\} & = 0 \end{pmatrix}.$$

Using Equation (sa.102), we find an initial value for y_1 consistent with both contemporaneous surprises ϵ_1 and rational expectations $\mathbb{E}_t\{\epsilon_{t+s}\} = 0$. Note also that the path of the expected variables $\mathbb{E}_t\{y_{t+s}\}$, $s = \{1, T\}$, is also updated.

The extended path solves recursively:

$$Y_{t:T-1}^* = \arg\min_{\{Y_{t:T-1}\}} \mathbb{E}_t\left\{R\left(Y_{t:T-1}, \epsilon_t, Y_{t-1}^*, y_T\right)\right\} \text{ for } t = \{1, T-1\}, \tag{sa.103}$$

with sequences of surprises ϵ_t, assuming that $\mathbb{E}_t\{\epsilon_{t+s}\} = 0$, for $s > 0$, and $Y_{t-1}^* = y_0$, for $t = 1$.

E.2. **Inversion filter.** The inversion filter from Fair and Taylor (1983) solves Equation (sa.103) by interverting the structural shocks with a subset of observable endogenous variables. In the context of the extended path, the endogenous variables are unknown and computed given a set of exogenous disturbances. Consider an inference based on a sample $\mathcal{Y}$ of size $T^* \times N^*$, where T^* is the number of periods and N^* is the number of observable variables. Let ω denote a selection matrix that selects observable variables within the endogenous variable vector (ωy_t) and z_t denote unobserved variables.

The new set of unknown variables that must be numerically computed for each period is given by:

$$W_t = \begin{pmatrix} w_t \\ y_{t+1} \\ \dots \\ y_{T-1} \end{pmatrix}, \text{ with } w_t = \begin{pmatrix} \epsilon_t \\ z_t \end{pmatrix}.$$

In this expression, the vector w_t stacks both current shocks and unobserved variables.

The new stacked residuals in t reads as:

$$F\left(W_{t:T^*},\mathcal{Y}_t,\hat{y}_{t-1},y_T\right)=\begin{pmatrix} \mathbb{E}_t\{f_\Theta\left(\hat{y}_{t-1},y_t,y_{t+1},\epsilon_t\right)\} & =0 & \text{with } \mathcal{Y}_t=\omega y_t \\ \mathbb{E}_t\{f_\Theta\left(y_1,y_2,y_3,0\right)\} & =0 & \\ \dots & & \\ \mathbb{E}_t\{f_\Theta\left(y_{T-3},y_{T-2},y_{T-1},0\right)\} & =0 & \\ \mathbb{E}_t\{f_\Theta\left(y_{T-2},y_{T-1},y_T,0\right)\} & =0 & \end{pmatrix},$$

where $\hat{y}_t$ denotes the smoothed endogenous variables. Shocks are now unknown, but are inferred through the constraint $\mathcal{Y}_t=\omega y_t$. As in the case of linearized models in Cuba-Borda et al. (2019) and Kollmann (2017), the number of shocks must equal the number of observable variables to have the same number of variables in both w_t and y_t, otherwise the system is indeterminate.

The inversion filter for extended path solves recursively the following optimization scheme:

$$W^*_{t:T-1}=\arg\min_{\{W_{t:T-1}\}}\mathbb{E}_t\{F\left(W_{t:T-1},\mathcal{Y}_t,\hat{y}_{t-1},y_T\right)\}\quad\text{for}\quad t=\{1,T^*\},\tag{sa.104}$$

with $T^*\leq T-1$. Smoothed shocks $\hat{\epsilon}_t$ and $\hat{y}_t$ are obtained recursively from Equation (sa.104).

E.3. **Likelihood function.** We use the inversion filter to extract the sequence of N^* shocks and T^* periods. When the structural innovations are drawn from a multivariate normal distribution with covariance matrix Σ, the log-likelihood is given by:

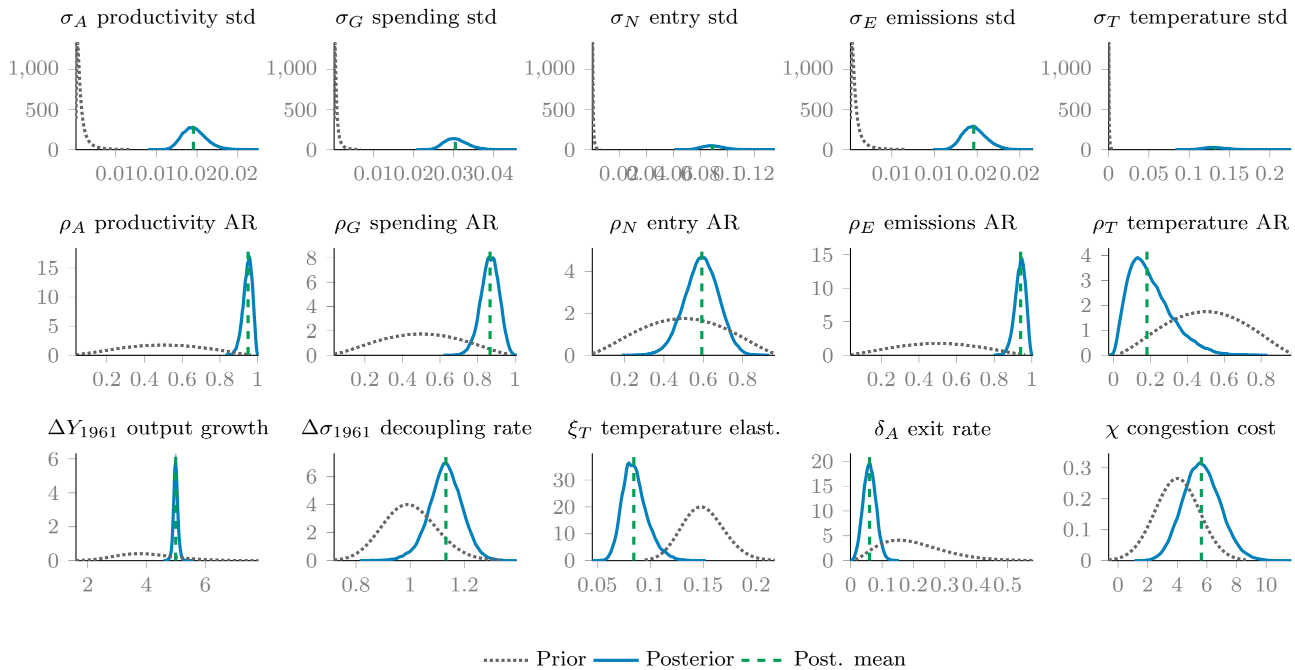
$$\mathcal{L}_t=-\frac{T^*N^*}{2}\log\left(2\pi\right)-\frac{T^*}{2}\log(\det\left(\Sigma\right))-\frac{1}{2}\textstyle\sum_{t=1}^{T^*}\hat{\epsilon}_t\Sigma^{-1}\hat{\epsilon}_t'+\frac{1}{2}\sum_{t=1}^{T^*}\log\left(\left|\det(\mathcal{J}_t)\right|\right),\tag{sa.105}$$

where $\mathcal{J}_t$ is the Jacobian matrix of the transformation of observable variables in innovations $\hat{\epsilon}_t$. Thus, we obtain the Jacobian of the endogenous variables as $-B_t^{-1}D_t$, where $B_t=\frac{\partial f_\Theta(.)}{\partial y_t}$ and $D_t=\frac{\partial f_\Theta(.)}{\partial \epsilon_t}$. We next use the selection matrix ω to select observable variables, such that:

$$\mathcal{J}_t=-\omega B_t^{-1}D_t.\tag{sa.106}$$

Although the Jacobian is not time-varying and can be computed directly from the policy function in linearized models, it is state-dependent and must be calculated for each period over the sample in nonlinear models.

FIGURE SA.3. Prior and posterior distributions



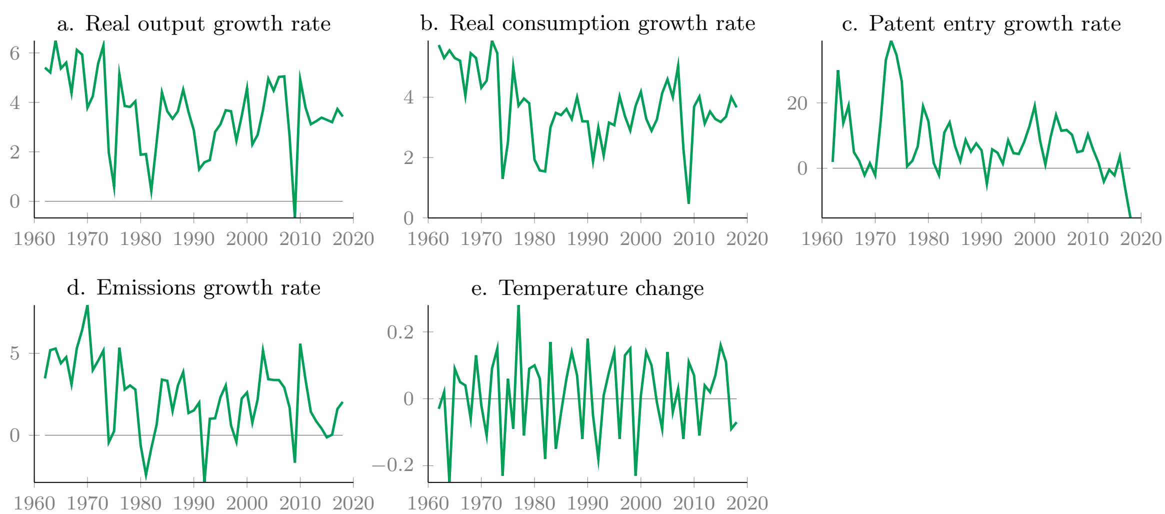
F. Data

Our sample is based on the following dataset:

- **GDP, PPP (constant 2017 international \$):** International Comparison Program, World Bank | World Development Indicators database, World Bank | Eurostat-OECD PPP Programme, "NY.GDP.MKTP.PP.KD", (denoted $\mathbf{Y}_t^{\$}$).
- **Households and NPISHs Final consumption expenditure (constant 2017 international \$):** World Bank national accounts data, and OECD National Accounts data files, "NE.CON.PRVT.PP.KD ", (denoted $\mathbf{C}_t^{\$}$).

  PPP real GDP and private consumption are backcasted based on the growth rates of non-PPP real GDP and private consumption (NY.GDP.MKTP.KD, NNE.CON.PRVT.CD, and OECDNAEXKP02IXOBSAQ) over 1962-1990.
- **Annual CO_2 emissions from fossil fuels, by world region (gigatonnes):** Global Carbon Project. "Our World in Data", (denoted $\mathbf{E}_t$).
- **Global Land and Ocean Temperature Anomaly:** NASA, degrees Celsius with base period of 1880–2022. "data.giss.nasa", (denoted $\mathbf{T}_t$).
- **Patent Environmental Related Technologies:** OECD Environment Directorate. "OECD Stats", (denoted $\mathbf{N}_t^E$).

Figure SA.4. Observable variables



The observable variable matrix is given by:

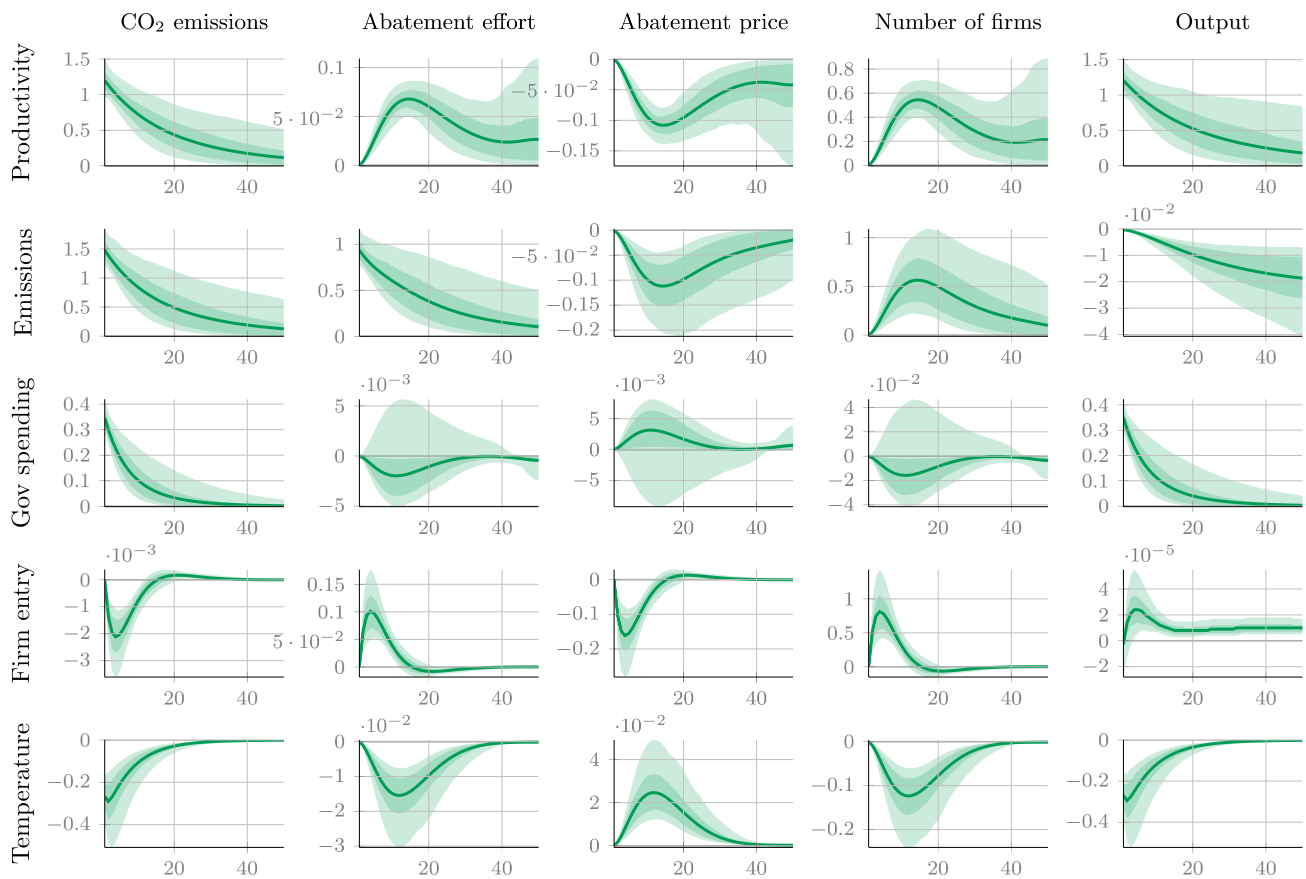
$$\begin{bmatrix} \text{Real output growth rate} \\ \text{Real consumption growth rate} \\ \text{CO}_2 \text{ emissions growth rate} \\ \text{Temperature anomaly change} \\ \text{Patent growth rate} \end{bmatrix} = \begin{bmatrix} \Delta \log(\mathbf{Y}_t^{\$}) \\ \Delta \log(\mathbf{C}_t^{\$}) \\ \Delta \log(\mathbf{E}_t) \\ \Delta \mathbf{T}_t \\ \Delta \log(\mathbf{N}_t^{E}) \end{bmatrix}. \tag{sa.107}$$

G. Model evaluation

This appendix examines the dynamic properties of the model by analyzing *(i)* the impulse response functions of several variables of interest to various shocks, *(ii)* the second moments of the observable variables, and *(iii)* the social cost of carbon. These analyses are useful in evaluating how economic shocks propagate through the system and in assessing whether the model accurately captures the statistical properties of macroeconomic and climate-related data and replicates a social cost of carbon trajectory consistent with existing literature.

G.1. **Impulse response functions.** Figure SA.5 shows the economy's response to a one-standard-deviation in five shocks – productivity, CO_2 emissions, government spending, firm entry, and temperature – in rows 1 to 5, respectively. Overall, these responses are broadly consistent with standard business cycle theory. First, a positive productivity shock (see the first row) increases aggregate output, which in turn raises CO_2 emissions. Hence, firms step up their abatement efforts to reduce their carbon tax burden. This effort stimulates the abatement goods sector, leading to a growing number of firms and a drop in abatement goods prices. Then, all variables gradually return to their initial (2019) levels as the highly persistent productivity shock dissipates. As shown in the second row, an exogenous increase in CO_2 emissions immediately raises abatement efforts, as firms seek to reduce their carbon tax burden. This additional effort in turn encourages new entrants to the abatement goods sector and drives down the abatement price as the sector expands. Meanwhile, the damaging effect of emissions on TFP, along with the burden of the carbon tax and the cost of abatement efforts, exerts downward pressure on output, which declines by about 0.02% in the short run relative to its initial level. The third row shows that, as a demand shock, an exogenous increase in government spending stimulates final good production – thereby raising CO_2 emissions – at the expense of abatement goods. Consequently, abatement effort and the number of firms decline relative to their initial levels, while the abatement price rises. The fourth row shows that an exogenous shock increasing firm entry intensifies competition, driving down the abatement price and thus encouraging abatement efforts, in line with Equation (sa.38). Aggregate output also benefits from higher revenues in the abatement sector, without causing any additional CO_2 emissions. Finally, the responses to an exogenous and temporary temperature rise –shown in the bottom row– provide insight into the economic effects of a climate-related shock. By intensifying the damage to firm productivity, this shock depresses output (by almost 0.3%), which in turn reduces CO_2 emissions. Consequently, abatement efforts and the number of new firms in the abatement goods sector both decline. Reduced competition in the abatement market then drives the abatement price higher.

FIGURE SA.5. Generalized impulse response functions



Note: This figure displays the generalized impulse response functions (GIRFs) of several variables to five one-standard-deviation shocks: productivity, CO_2 emissions, government spending, firm entry, and temperature, in rows 1 to 5, respectively. Generalized IRFs are well-suited for analyzing shock responses in models with nonlinearities, evolving trends, and policy-dependent dynamics (Koop et al., 1996). They are computed using 2019 as the initial state value for all variables and are expressed in percentage deviations from their 2019 values. Each GIRF is then averaged across 500 exogenous draws. Time units on the abscissa correspond to years after shock.

G.2. **Moments.** Table SA.1 provides the empirical second moments of our five observable variables and the 95% confidence interval obtained with our baseline model and an alternative specification without firm entry in the abatement goods sector (i.e, with perfect competition). The latter corresponds to what is found in DICE-2016R2, for instance. The estimation of the alternative specification includes the same observable variables except for patent growth (and no shock $\eta_{N,t}$). Consequently, the likelihood or standard information criteria cannot be employed to discriminate between the models. Thus, we rely on a comparison of the second moments. We find that both models accurately replicate the empirical moments, although

TABLE SA.1. Empirical and model-implied moments

	DATA	Baseline model [5%;95%]	No firm entry specification [5%;95%]
Standard deviations			
Output growth	1.50	[1.21;1.64]	[1.20;1.66]
Consumption growth	1.18	[1.18;1.60]	[1.21;1.64]
Emission growth	2.24	[1.67;2.39]	[1.70;2.43]
Temperature change	0.12	[0.11;0.16]	[0.11;0.17]
Patent growth	10.01	[7.62;13.15]	–
Autocorrelation			
Output growth	0.43	[-0.05;0.43]	[-0.08;0.45]
Consumption growth	0.51	[-0.05;0.43]	[-0.05;0.45]
Emission growth	0.50	[-0.18;0.34]	[-0.20;0.33]
Temperature change	-0.32	[-0.16;0.34]	[-0.19;0.35]
Patent growth	0.63	[0.26;0.73]	–

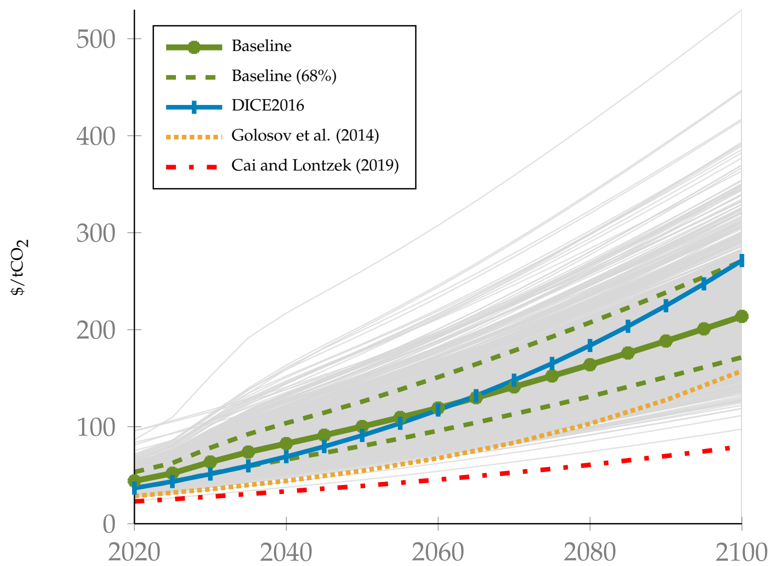
Note: Model-implied moments are computed across 1,000 random artificial series, each with the same size as the data sample (57). The "baseline model" corresponds to our macro-climate model with firm entry, while the "no firm entry specification" is an alternative version of our framework without firm entry.

they yield less persistence than in the data. Importantly, our baseline model reproduces the standard deviation and the autocorrelation of patent growth fairly well.

G.3. **The social cost of carbon.** Finally, we compute the model-implied social cost of carbon (SCC), which captures the net present value in dollars of the marginal damage that would result from emitting one additional ton of carbon dioxide into the atmosphere. This optimal price of carbon helps decision makers to set the right price of carbon, and serves as a metric to compare models. Figure SA.6 depicts the SCC associated with our baseline specification (green plain line) with its 68% confidence interval (green dashed lines), as well as the estimates from DICE2016 (blue line), Golosov et al. (2014) (orange dotted line), and Cai and Lontzek (2019) (red dashed line).

All previous social cost of carbon (SCC) estimates based on IAMs (e.g., DICE2016, Cai and Lontzek, 2019, and Golosov et al. (2014)) correspond to at least one of the Metropolis-Hastings draws (grey lines) generated by our model. Notably, the SCC from DICE2016 falls within the 68% confidence interval of our baseline model. Differences in the magnitude and trajectory of SCC estimates are primarily driven by the climate module. For example, DICE2016 and Cai and Lontzek (2019) assume greater persistence in the carbon cycle, leading to SCC growing relatively more as atmospheric carbon accumulates. In contrast, Cai and Lontzek (2019) reports a substantially lower SCC, likely due to the role of uncertainty, which exerts a downward effect.

FIGURE SA.6. The social cost of carbon

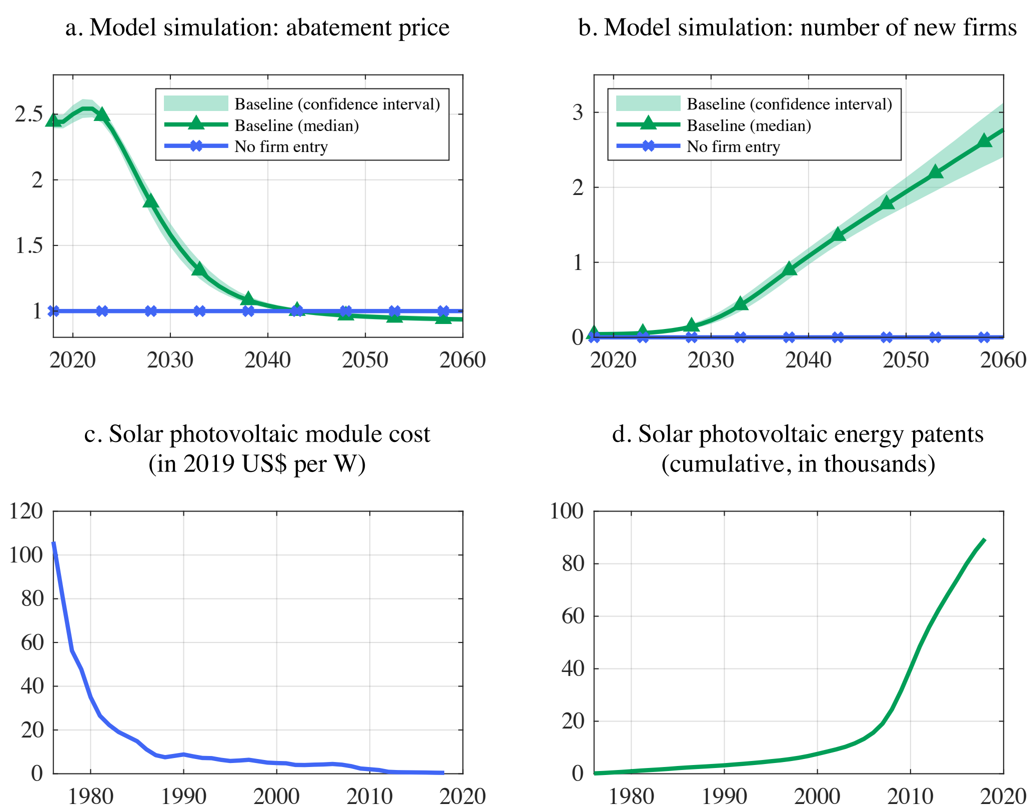


Note: The figure displays the social cost of carbon (SCC) on the vertical axis. The baseline model, where the carbon tax equals the SCC, is represented by a solid green line. Uncertainty interval, computed from 2,000 simulations using alternative parameter draws from the Metropolis-Hastings algorithm, are depicted by solid gray lines. Alternative SCC are also presented: (i) DICE2016 (solid blue with vertical markers), (ii) Golosov et al. (2014) (orange dotted line), and (iii) Cai and Lontzek, 2019 with a benchmark calibration assuming 1.5% total factor productivity growth (red dashed line).

H. Firm entry mechanism and the solar photovoltaic experience

The dynamics depicted by our macro-climate model with endogenous producer entry in the abatement goods sector are in line with the concomitant increase in the number of new environmentally related patents and the drop in their prices observed over the last forty years. As an illustration, the two bottom plots (*c* and *d*) of Figure SA.7 show that the substantial decrease in the cost of solar photovoltaic modules from the late 1970s to the present is associated with an impressive increase in the cumulative number of patents in this sector. The evolution of the solar photovoltaic sector was partly driven by government subsidies in several countries.

FIGURE SA.7. Model-implied dynamics of the abatement goods sector and historical evidence



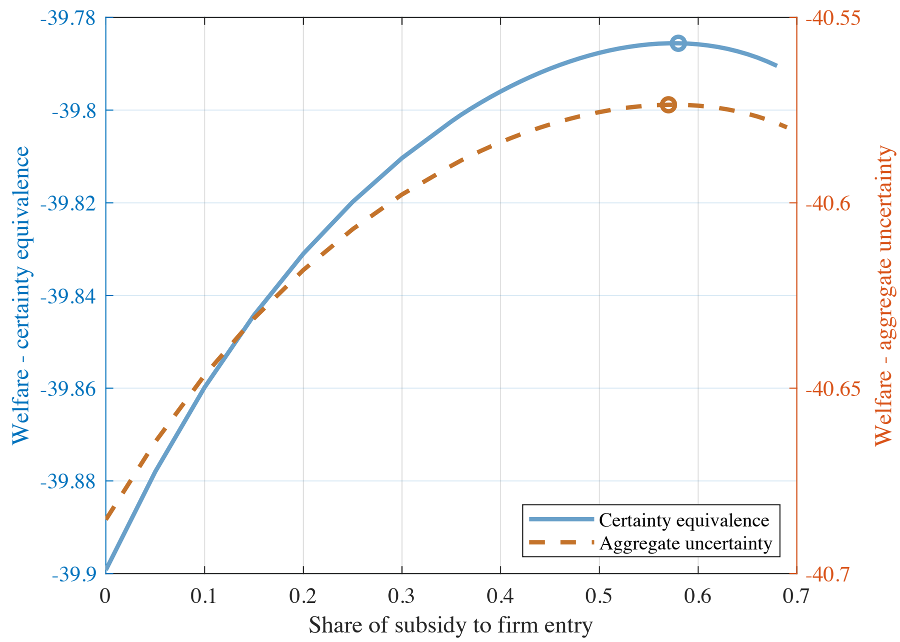
Note: Panels *a* and *b* display the temporal evolution of the abatement price and cumulative number of new firms under the assumption of a temperature increase below +2°C relative to preindustrial levels. "Baseline" corresponds to the macro-climate model with imperfect competition in the abatement goods sector (endogenous firm entry), and "no firm entry" corresponds to an alternative version with perfect competition in the abatement goods sector. The number of new firms corresponds to the number of additional startups per household. The light green area denotes both the parametric and stochastic uncertainties. The light green area represents the uncertainty interval, computed from 500 random draws using the Metropolis-Hastings sampler.

I. THE EFFECTS OF UNCERTAINTY ON THE OPTIMAL SUBSIDY RATE

I.1. **The role of aggregate uncertainty.** To address concerns regarding the perfect foresight assumption, we extend our framework to incorporate aggregate uncertainty following the stochastic extended version of Adjemian and Juillard (2014). Specifically, we introduce expectation corrections derived from second-order Taylor expansions, which allow us to approximate the effects of uncertainty while maintaining numerical tractability.

We then recompute our quantitative analysis under this framework and compare the results with the baseline case, which assumes certainty equivalence. Figure SA.8 presents the optimal environmental subsidy shares under both settings.

FIGURE SA.8. Social welfare for various subsidy rates to startups with aggregate uncertainty



Note: Social welfare is defined as the infinite discounted sum of future utilities. It is evaluated in 2019, when the carbon tax policy is announced, and reflects the expected future path of utilities under the net-zero transition.

As expected, welfare declines significantly under aggregate uncertainty (right axis), aligning with Robert Lucas's classic result on the welfare costs of fluctuations. However, the optimal subsidy share allocated to startups remains nearly unchanged between the two simulations. Under aggregate uncertainty, the initial subsidy share is 57%, compared to 58% under certainty equivalence. This marginal difference suggests that our main policy conclusions remain robust to the inclusion of aggregate uncertainty. One notable change is that welfare under uncertainty follows a flatter trajectory with respect to the subsidy policy. This reflects the dampened sensitivity of economic agents to policy adjustments when aggregate uncertainty is present.

The limited impact of uncertainty on the optimal subsidy share naturally arises from the functional form of the utility function. Our framework, consistent with IAMs such as DICE (Nordhaus, 2018) and models in Golosov et al. (2014), does not introduce mechanisms such as precautionary savings or state-dependent risk aversion, which could amplify the role of uncertainty in policy decisions.

Capturing higher-order effects of uncertainty would require risk-sensitive preferences that induce stronger responses to aggregate shocks. Such extensions have been explored in the literature (e.g., Cai and Lontzek 2019, Van den Bremer and Van der Ploeg 2021) and would provide a promising avenue for future research in an estimated framework such as ours. However, within the standard utility function of IAM (and many macro-models), our exercice here confirms that the optimal subsidy share is largely unaffected by the size of elements in the covariance matrix of stochastic variables.

I.2. **Sensitivity to calibrated parameters associated with the abatement sector.** We propose a sensitivity analysis by examining three key parameters in the abatement sector that are likely to influence our results. These parameters are the elasticity of substitution in abatement technologies (ζ_A), the backstop price for abatement (p_b), and the productivity scaling factor in the abatement sector (X_w).

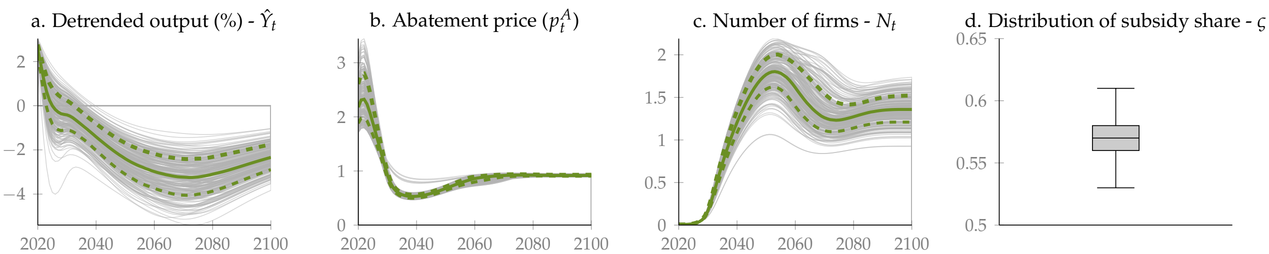
To assess their impact, we draw 2,000 random realizations from a multivariate Gaussian distribution defined as follows:

$$\begin{bmatrix} \zeta_A \\ p_b \\ X_w \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} 6 \\ 716.7 \\ 1 \end{bmatrix}, \begin{bmatrix} \sigma^2_{\zeta_A} & 0 & 0 \\ 0 & \sigma^2_{p_b} & 0 \\ 0 & 0 & \sigma^2_{X_w} \end{bmatrix} \right),$$

where the mean corresponds to the calibrated model parameters, while $\sigma^2_{\zeta_A}$, $\sigma^2_{p_b}$, and $\sigma^2_{X_w}$ represent the respective variances, which define the range of parameter values explored to assess uncertainty in the optimal subsidy policy.

The elasticity of substitution ζ_A typically lies between 4 (Bilbiie et al., 2012) and 11 (Smets and Wouters, 2007). To ensure that 99% of the parameter values fall within this range, we set $\sigma_{\zeta_A} = 1$. The backstop price of abatement p_b is taken from the DICE literature. Since the DICE2016 model's estimate has doubled in DICE2023, we define $\sigma_{p_b} = 716.7/3$, ensuring 99.9% of plausible values are captured within three standard deviations. Finally, according to Bilbiie et al. (2012), a plausible range of the productivity scaling factor X_w is 0.75 and 1.25. To ensure 99.9% of the distribution is covered, we set $\sigma_{X_w} = 0.25/3$

FIGURE SA.9. The effects of uncertainty on calibrated parameters associated with the abatement sector



Note: The gray lines correspond to the 2000 draws using the Metropolis-Hasting sampler and the dashed lines indicate the 68% uncertainty interval.

Figure SA.9 illustrates the time paths of detrended output (Panel a), the abatement price (Panel b), the number of firms (Panel c), and a boxplot of the optimal subsidy share allocated to startups (Panel d). The results confirm the robustness of our main findings, even when accounting for the uncertainty in these parameters. Specifically:

(1) detrended output and the number of firms remain largely unaffected by variations in ζ_A, p_b, and X_w;
(2) the abatement price exhibits some variation, but it only remains persistently high when ζ_A approaches its upper bound (equal to 9). In such cases, lower markups weaken the competition effect of the subsidy policy, through firm entry is still effectively promoted;
(3) the optimal subsidy share allocated to startups (ς) remains concentrated between 55% and 60%, with limited dispersion in the boxplot.

These findings reinforce the robustness of our conclusions, particularly regarding the optimal subsidy policy, and validate the reliability of our approach.

J. The model with physical capital

This section details the new equations that appear in the macro-climate model when physical capital is introduced.

The production function of intermediate firms is now given by:

$$y_{i,t} = \Gamma_t \left(h_{i,t}^I\right)^\alpha \left(k_{i,t}^I\right)^{1-\alpha}, \tag{sa.108}$$

where α corresponds to the labor share.

The profit maximization program becomes:

$$\max_{\{h_{i,t}^I, k_{i,t}^I, \mu_{i,t}\}} \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \beta_{t,t+s} \left[p_{i,t+s} y_{i,t+s} - w_{t+s} h_{i,t+s}^I - p_{t+s}^A \right.\right.$$
$$\left.\left. - \tau_{t+s} e_{i,t+s} + (1-\delta_k) k_{i,t+s}^I - k_{i,t+1+s}^I \right] \right\}, \tag{sa.109}$$

where δ_k is the depreciation rate of the physical capital. Note that the optimal control problem is now dynamic, as capital implies an intertemporal trade-off between cutting profits (as a result of investment) and boosting future output.

The modified equations solving the optimization problem are listed below.

First the marginal value of output, or the Lagrangian multiplier on the production function, is given by:

$$\phi_t = \frac{\zeta - 1}{\zeta} - p_t^A \theta_{1,t} \mu_{i,t}^{\theta_2} - \varrho_t \sigma_t \left(1 - \mu_{i,t}\right) \varepsilon_{E,t}. \tag{sa.110}$$

Second, the optimal labor demand is given by:

$$w_t = \alpha \frac{y_{i,t}}{h_{i,t}^I} \phi_t. \tag{sa.111}$$

Third, the Euler equation on physical capital reads as:

$$\mathbb{E}_t \left\{ \beta_{t,t+1} \left[(1-\delta_K) + (1-\alpha) \phi_{t+1} \frac{y_{i,t+1}}{k_{i,t}^I} \right] \right\} = 1. \tag{sa.112}$$

Finally, the production sector adresses capital goods. Investment appears in the resource constraint as follows:

$$Y_t = C_t + G_t + N_t^E L_t \theta_{1,t} Z_t X_q + K_{t+1}^I - (1-\delta_K) K_t^I. \tag{sa.113}$$

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