

Supplemental Appendix – Procyclical Fiscal Policy and Asset Market Incompleteness¹

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Appendix A: Static model (for online publication)

7.1 Financial autarky case

Consider a static model with a stochastic endowment and consumption tax. Output (endowment) can take two different values with probability p and $1 - p$:

$$y = \begin{cases} y_H = \bar{y} + \gamma, & \text{with probability } p, \\ y_L = \bar{y} - \gamma, & \text{with probability } 1 - p. \end{cases} \quad (33)$$

For simplicity, p is assumed to be equal to $1/2$. It can be easily checked that

$$\begin{aligned} E(y) &= \bar{y}, \\ V(y) &= \gamma^2. \end{aligned}$$

Preferences are given by

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$$U(c_i, g_i) = \begin{cases} E_{i=H,L} \left[\alpha \frac{c_i^{1-\frac{1}{\sigma_c}} - 1}{1-\frac{1}{\sigma_c}} + (1-\alpha) \frac{g_i^{1-\frac{1}{\sigma_g}} - 1}{1-\frac{1}{\sigma_g}} \right], & \sigma_c \neq 1 \text{ and } \sigma_g \neq 1, \\ E_{i=H,L} \left[\alpha \ln(c_i) + (1-\alpha) \frac{g_i^{1-\frac{1}{\sigma_g}} - 1}{1-\frac{1}{\sigma_g}} \right], & \sigma_c = 1 \text{ and } \sigma_g \neq 1, \\ E_{i=H,L} \left[\alpha \frac{c_i^{1-\frac{1}{\sigma_c}} - 1}{1-\frac{1}{\sigma_c}} + (1-\alpha) \ln(g_i) \right], & \sigma_c \neq 1 \text{ and } \sigma_g = 1. \end{cases} \quad (34)$$

We can solve this problem as a social planner because the consumption tax does not distort intertemporally (i.e., the model is static) or intratemporally (i.e., there is no labor/leisure choice). The economy's resource constraints take the form

$$y_i = c_i + g_i, \quad i = L, H. \quad (35)$$

The planner's choice variables are $\{c_H, g_H, c_L, g_L\}$. The Lagrangian is given by

$$\begin{aligned} &= p \left[\alpha \frac{c_H^{1-\frac{1}{\sigma_c}} - 1}{1-\frac{1}{\sigma_c}} + (1-\alpha) \frac{g_H^{1-\frac{1}{\sigma_g}} - 1}{1-\frac{1}{\sigma_g}} \right] + (1-p) \left[\alpha \frac{c_L^{1-\frac{1}{\sigma_c}} - 1}{1-\frac{1}{\sigma_c}} + (1-\alpha) \frac{g_L^{1-\frac{1}{\sigma_g}} - 1}{1-\frac{1}{\sigma_g}} \right] \\ &\quad + \lambda_H (y_H - c_H - g_H) + \lambda_L (y_L - c_L - g_L). \end{aligned}$$

Once we have solved the planner's problem, the government constraint in each state of nature can be used to find out the corresponding tax rates:

$$g_i = \tau_i c_i, \quad i = L, H. \quad (36)$$

The first-order conditions for $\{c_H, g_H, c_L, g_L\}$ are given by, respectively,

$$\alpha c_H^{-\frac{1}{\sigma_c}} = \lambda_H, \quad (37)$$

$$(1-\alpha) g_H^{-\frac{1}{\sigma_g}} = \lambda_H, \quad (38)$$

$$\alpha c_L^{-\frac{1}{\sigma_c}} = \lambda_L, \quad (39)$$

$$(1-\alpha) g_L^{-\frac{1}{\sigma_g}} = \lambda_L. \quad (40)$$

These first-order conditions imply that the marginal utilities of private and public consumption are equalized in each state of nature (i.e., $U_{cH} = U_{gH}$ and $U_{cL} = U_{gL}$) but not across states of nature (because there is no full insurance).

Specifically, combining first-order conditions (37) and (38), we obtain:

$$c_H = g_H^{\frac{\sigma_c}{\sigma_g}} \left(\frac{\alpha}{1-\alpha} \right)^{\sigma_c}. \quad (41)$$

By the same token, from (39) and (40):

$$c_L = g_L^{\frac{\sigma_c}{\sigma_g}} \left(\frac{\alpha}{1-\alpha} \right)^{\sigma_c}.^2 \quad (42)$$

Combining (35), (41), and (42), it follows that

$$\begin{aligned} y_H &= g_H^{\frac{\sigma_c}{\sigma_g}} \left(\frac{\alpha}{1-\alpha} \right)^{\sigma_c} + g_H, \\ y_L &= g_L^{\frac{\sigma_c}{\sigma_g}} \left(\frac{\alpha}{1-\alpha} \right)^{\sigma_c} + g_L. \end{aligned}$$

Define

$$\begin{aligned} \phi(g_i) &\equiv g_i^{\frac{\sigma_c}{\sigma_g}} \left(\frac{\alpha}{1-\alpha} \right)^{\sigma_c} + g_i, \quad i = L, H, \\ \phi'(g_i) &= 1 + \left(\frac{\alpha}{1-\alpha} \right)^{\sigma_c} \frac{\sigma_c}{\sigma_g} g_i^{\frac{\sigma_c}{\sigma_g}-1} > 0. \quad i = L, H. \end{aligned}$$

Then,

$$\begin{aligned} y_i &= \phi(g_i), \quad i = L, H, \\ g_i &= \Gamma(y_i) > 0, \quad i = L, H, \\ \Gamma'(y_i) &> 0, \quad i = L, H, \end{aligned} \quad (43)$$

²To show that the relative size of σ_c and σ_g captures preferences for c/g , divide equation (41) by equation (42) and rewrite this equation as $\log(c_H/c_L) = (\sigma_c/\sigma_g) \log(g_H/g_L)$. Then, $\sigma_c = \sigma_g$ implies $(c_H/g_H) = (c_L/g_L)$; $\sigma_c > \sigma_g$ implies $(c_H/g_H) > (c_L/g_L)$, and $\sigma_c < \sigma_g$ implies $(c_H/g_H) < (c_L/g_L)$.

where $\Gamma \equiv \phi^{-1}$ and

$$\frac{dg_i}{dy_i} = \frac{1}{\phi'} > 0, \quad i = L, H.$$

Define the cyclicity of government spending as

$$\theta_g \equiv \log\left(\frac{g_H}{g_L}\right) = \log(g_H) - \log(g_L).$$

Using (43), this expression can be rewritten as

$$\theta_g = \log(\Gamma(y_H)) - \log(\Gamma(y_L)) > 0,$$

because $\Gamma'(y_i) > 0$ and $y_H > y_L$, which shows that government spending is procyclical regardless of the values of σ_c and σ_g (Proposition 1 in the text).

Taking into account the binomial distribution, given by (1), we now show that θ_g is increasing in γ (Proposition 3 in the text):

$$\frac{d\theta_g}{d\gamma} = \frac{1}{\Gamma(y_H)}\Gamma'(y_H) + \frac{1}{\Gamma(y_L)}\Gamma'(y_L) > 0.$$

To obtain a reduced form for τ_i , combine (36) with (41) and (42) to obtain

$$\tau_i = \left(\frac{1-\alpha}{\alpha}\right)^{\sigma_c} g_i^{\frac{1-\sigma_c}{\sigma_g}}, \quad i = L, H. \quad (44)$$

Now define the cyclicity of the tax rate as

$$\theta_\tau \equiv \log\left(\frac{\tau_H}{\tau_L}\right)$$

which, using (44), can be rewritten as

$$\theta_\tau = \log\left(\frac{g_H^{\frac{1-\sigma_c}{\sigma_g}}}{g_L^{\frac{1-\sigma_c}{\sigma_g}}}\right) = \left(1 - \frac{\sigma_c}{\sigma_g}\right) \log\left(\frac{g_H}{g_L}\right) = \left(1 - \frac{\sigma_c}{\sigma_g}\right) \theta_g \geq 0. \quad (45)$$

As stated in Proposition 2, it follows that

$$\theta_\tau = \begin{cases} + & \text{(countercyclical), } \sigma_c < \sigma_g, \\ 0 & \text{(acyclical), } \sigma_c = \sigma_g, \\ - & \text{(procyclical), } \sigma_c > \sigma_g. \end{cases}$$

What happens in response to a mean-preserving spread in output? Differentiating (45) with respect to τ ,

$$\frac{d\theta_\tau}{d\gamma} = \left(1 - \frac{\sigma_c}{\sigma_g}\right) \frac{d\theta_g}{d\gamma},$$

where $(d\theta_g/d\gamma) > 0$. Hence,

$$\frac{d\theta_\tau}{d\gamma} = \begin{cases} + & \sigma_c < \sigma_g, \\ 0 & \sigma_c = \sigma_g, \\ - & \sigma_c > \sigma_g. \end{cases}$$

As stated in Proposition 3 in the text, if θ_τ is countercyclical (i.e., positive), it will become more countercyclical. If it is zero, it remains zero of course. If it is negative (procyclical), it becomes more negative (i.e., more procyclical). In other words, a mean-preserving spread always amplifies the cyclicity of tax rates.

7.2 Complete markets

While the model is static, we assume that households have access to contingent claims that can insure them against the outcomes in each state of nature (i.e., high and low output). The state-contingent bonds are intra-period; that is, they are purchased at the beginning of the period (i.e., before the shock materializes), and the households receive the pay-off at the end of the period (i.e., after the shock takes place).

As in the case of financial autarky, we can solve the planner's problem (with the planner having access to complete markets abroad) since, in the absence of any distortions, the government will be able to implement the first-best policy.

The planner chooses $\{c_H, g_H, c_L, g_L\}$ to maximize

$$pU(c_H, g_H) + (1 - p)U(c_L, g_L),$$

subject to

$$q_H y_H + q_L y_L = q_H(c_H + g_H) + q_L(c_L + g_L),$$

where $U(c_i, g_i)$ is given by (34).

The Lagrangian is given by

$$\begin{aligned} &= pU(c_H, g_H) + (1-p)U(c_L, g_L) \\ &\quad + \lambda [q_H y_H + q_L y_L - q_H(c_H + g_H) - q_L(c_L + g_L)]. \end{aligned}$$

The first-order conditions are given by

$$pU_{c_H}(c_H, g_H) = \lambda q_H, \quad (46)$$

$$pU_{g_H}(c_H, g_H) = \lambda q_H, \quad (47)$$

$$(1-p)U_{c_L}(c_L, g_L) = \lambda q_L, \quad (48)$$

$$(1-p)U_{g_L}(c_L, g_L) = \lambda q_L. \quad (49)$$

Notice that, as in the financial autarky case, the marginal utilities of private and public consumption are equalized in each state of the world; that is, $U_{c_H} = U_{g_H}$ and $U_{c_L} = U_{g_L}$.

Combining first-order conditions across states of the world (i.e., (46) and (47) on the one hand, and (48) and (49) on the other), we obtain

$$\begin{aligned} \frac{pU_{c_H}(c_H, g_H)}{q_H} &= \frac{(1-p)U_{c_L}(c_L, g_L)}{q_L}, \\ \frac{pU_{g_H}(c_H, g_H)}{q_H} &= \frac{(1-p)U_{g_L}(c_L, g_L)}{q_L}. \end{aligned}$$

Assuming actuarially fair insurance (i.e., $q_H/q_L = p/(1-p)$), we can rewrite these optimality conditions as

$$U_{c_H}(c_H, g_H) = U_{c_L}(c_L, g_L), \quad (50)$$

$$U_{g_H}(c_H, g_H) = U_{g_L}(c_L, g_L). \quad (51)$$

Marginal utilities of private and public consumption are equalized across states of nature (implying full risk sharing). Since the utility function, given by (34), is separable, conditions (50) and (51) imply, respectively, that $c_H = c_L$ and $g_H = g_L$. The latter implies that government spending is acyclical (Proposition 1). In other words, under complete markets, government spending is acyclical regardless of the relation between σ_c and σ_g . Further, given (36), $c_H = c_L$ and $g_H = g_L$ imply that $\tau_H = \tau_L$. Hence, tax policy is also acyclical for any values of σ_c and σ_g (Proposition 2). In either case (spending or tax rates), fiscal policy is acyclical regardless of the variance of the distribution (Proposition 3).

7.3 Income tax

Suppose that we have an income tax in the form of an endowment tax (to keep it symmetrical with the consumption tax case). In other words, the consumer's budget constraints are given by:

$$c_i = (1 - \xi_i)y_i, \quad i = L, H,$$

where ξ_i is the endowment tax.

Other than this, the model is exactly the same as in the case analyzed above. As in the consumption tax case, this endowment tax is non-distortionary. We can thus solve the planner's problem as we did for the consumption tax case. All results for both financial autarky and complete markets go through since they do not depend on what is the government's source of tax income as long as it is non-distortionary.

Given the planner's optimal choices (which will be the same as in the consumption tax case), the income tax will follow from the government budget constraint:

$$g_i = (1 - \xi_i)y_i, \quad i = L, H.$$

We conclude that Propositions 1-3 would also hold for the case of an income (i.e., endowment) tax.

8 Appendix B: DSGE model (for online publication)

This appendix formally sets up and solves the four variations of the main DSGE model used in the text. First, we consider a small open economy operating under financial autarky (i.e., no borrowing from/lending to the rest of the world). Second, we deal with a standard small open economy that has access to a world risk-free bond (i.e., incomplete asset markets). Third, we consider a small open economy that is operating under complete markets. Finally, we present a variation of the incomplete asset markets case with a labor tax.

8.1 Financial autarky model

8.1.1 Households' problem

The household's problem is given by:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, g_t, l_t),$$

subject to

$$\begin{aligned} d_t &= (1 + r_{t-1}) d_{t-1} - y_t + (1 + \tau_t) c_t + \Phi(d_t) - \Pi_t, \\ y_t &= A_t l_t, \\ \ln(A_t/\bar{A}) &= \rho_A \ln(A_{t-1}/\bar{A}) + \varepsilon_t^A, \quad \varepsilon_t^A \sim NIID(0, \sigma_A^2), \end{aligned} \tag{52}$$

and a no-Ponzi condition.

We assume households face convex portfolio transactions costs, $\Phi(d_t)$. These administrative services (for either assets or liabilities) are provided by a government agency at zero cost. Profits (Π) are transferred to households in a lump-sum way so as to get rid of any wealth effects associated with these portfolio adjustment/transaction costs.

Notice also that here the portfolio costs are internalized by both the household and the Ramsey planner. In the SOE case (see below), only the Ramsey planner internalizes the upward sloping supply of funds.

8.1.2 Lagrangian

The Lagrangian is given by

$$\mathcal{L} = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U(c_t, g_t, l_t) + \beta^t \lambda_t [d_t - (1 + r_{t-1}) d_{t-1} + A_t l_t - (1 + \tau_t) c_t - \Phi(d_t) + \Pi_t] \right\}.$$

The first-order conditions are given by

$$\begin{aligned} [c_t] &: \beta^t U_c(c_t, g_t, l_t) - \beta^t \lambda_t (1 + \tau_t) = 0, \\ [l_t] &: \beta^t U_l(c_t, g_t, l_t) + \beta^t \lambda_t A_t = 0, \\ [d_t] &: \beta^t \lambda_t [1 - \Phi'(d_t)] - (1 + r_t) \beta^{t+1} E_t \lambda_{t+1} = 0. \end{aligned}$$

Simplifying the first-order conditions, we obtain:

$$\begin{aligned} U_c(c_t, g_t, l_t) &= \lambda_t (1 + \tau_t), \\ U_l(c_t, g_t, l_t) &= -\lambda_t A_t, \\ \lambda_t [1 - \Phi'(d_t)] &= (1 + r_t) \beta E_t \lambda_{t+1}. \end{aligned}$$

8.1.3 Government's flow budget constraint

The government's flow budget constraint is given by

$$d_t^g = (1 + r_{t-1}) d_{t-1}^g - \tau_t c_t + g_t - \Phi(d_t) + \Pi_t. \quad (53)$$

8.1.4 Aggregate constraints

Financial autarky implies:

$$d_t + d_t^g = 0, \quad \forall t.$$

Combining the household's and government's flow budget constraints, given by (52) and (53), respectively, with the restriction above (in t and $t - 1$) yields:

$$c_t + g_t = y_t.$$

8.1.5 Implementability conditions

The implementability conditions follow from the first-order conditions and are given by:

$$\begin{aligned} A_t &= -\frac{U_l(c_t, g_t, l_t)}{U_c(c_t, g_t, l_t)}(1 + \tau_t) = \Gamma_t \equiv \Gamma(c_t, g_t, l_t, \tau_t), \\ \frac{U_c(c_t, g_t, l_t)}{1 + \tau_t} &= \lambda_t \equiv \lambda(c_t, g_t, l_t, \tau_t) = \frac{1 + r_t}{1 - \Phi(d_t)} \beta E_t \lambda(c_{t+1}, g_{t+1}, l_{t+1}, \tau_{t+1}). \end{aligned}$$

8.1.6 Ramsey problem

The Lagrangian for the Ramsey problem is given by

$$= E_0 \left\{ \begin{array}{l} \sum_{t=0}^{\infty} \beta^t U(c_t, g_t, l_t) \\ + \beta^t \mu_{1,t} (\Gamma_t - A_t) \\ + \beta^t \mu_{2,t} \left[\lambda_t - \frac{1+r_t}{1-\Phi(d_t)} \beta E_t \lambda_{t+1} \right] \\ + \beta^t \mu_{3,t} [d_t - (1 + r_{t-1}) d_{t-1} + A_t l_t - (1 + \tau_t) c_t] \\ + \beta^t \mu_{4,t} (A_t l_t - c_t - g_t) \end{array} \right\}.$$

The first-order conditions are given by

$$\begin{aligned}
[c_t] &: U_{c_t} + \mu_{1,t}\Gamma_{c_t} + \mu_{2,t}\lambda_{c_t} - \mu_{2,t-1}\frac{1+r_{t-1}}{1-\Phi'(d_{t-1})}\lambda_{c_t} - \mu_{3,t}(1+\tau_t) - \mu_{4,t} = 0, \\
[l_t] &: U_{l_t} + \mu_{1,t}\Gamma_{l_t} + \mu_{2,t}\lambda_{l_t} - \mu_{2,t-1}\frac{1+r_{t-1}}{1-\Phi'(d_{t-1})}\lambda_{l_t} + \mu_{3,t}A_t + \mu_{4,t}A_t = 0, \\
[g_t] &: U_{g_t} + \mu_{1,t}\Gamma_{g_t} + \mu_{2,t}\lambda_{g_t} - \mu_{2,t-1}\frac{1+r_{t-1}}{1-\Phi'(d_{t-1})}\lambda_{g_t} - \mu_{4,t} = 0, \\
[\tau_t] &: \mu_{1,t}\Gamma_{\tau_t} + \mu_{2,t}\lambda_{\tau_t} - \mu_{2,t-1}\frac{1+r_{t-1}}{1-\Phi'(d_{t-1})}\lambda_{\tau_t} - \mu_{3,t}c_t = 0, \\
[d_t] &: \mu_{2,t}\left[-\frac{1+r_t}{[1-\Phi'(d_t)]^2}\Phi''(d_t)\beta E_t\lambda_{t+1}\right] + \mu_{3,t} - \beta(1+r_t)E_t\mu_{3,t+1} = 0, \\
[r_t] &: -\frac{1}{1-\Phi'(d_t)}\mu_{2,t}\beta E_t\lambda_{t+1} - \beta d_t E_t\mu_{3,t+1} = 0, \\
[\mu_{1,t}] &: \Gamma_t - A_t = 0, \\
[\mu_{2,t}] &: \lambda_t - \frac{1+r_t}{1-\Phi'(d_t)}\beta E_t\lambda_{t+1} = 0, \\
[\mu_{3,t}] &: d_t - (1+r_{t-1})d_{t-1} + A_t l_t - (1+\tau_t)c_t = 0, \\
[\mu_{4,t}] &: A_t l_t - c_t - g_t = 0.
\end{aligned}$$

8.1.7 Parameterization

The portfolio transactions costs take the following quadratic form (see, for example, Schmitt-Grohe and M. Uribe, 2003):

$$\begin{aligned}
\Phi(d_t) &= \frac{\phi}{2}(d_t - \bar{d})^2, \\
\Phi'(d_t) &= \phi(d_t - \bar{d}), \quad \Phi''(d_t) = \phi.
\end{aligned}$$

where ϕ and \bar{d} are parameters.

$$\begin{aligned}
U(c_t, g_t, l_t) &= \frac{c_t^{1-1/\sigma_c} - 1}{1-1/\sigma_c} + \frac{g_t^{1-1/\sigma_g} - 1}{1-1/\sigma_g} + \ln(1-l_t), \\
U_{c_t} &= c_t^{-1/\sigma_c}; \quad U_{g_t} = g_t^{-1/\sigma_g}; \quad U_{l_t} = -\frac{1}{1-l_t}.
\end{aligned}$$

The derivatives of Γ_t and λ_t are given by, respectively,

$$\begin{aligned}\Gamma_t &= -\frac{U_l(c_t, g_t, l_t)}{U_c(c_t, g_t, l_t)} (1 + \tau_t) = \frac{1 + \tau_t}{(1 - l_t)c_t^{-1/\sigma_c}}, \\ \Gamma_{c_t} &= \frac{1 + \tau_t}{\sigma_c(1 - l_t)c_t^{1-1/\sigma_c}}; \quad \Gamma_{l_t} = \frac{1 + \tau_t}{(1 - l_t)^2 c_t^{-1/\sigma_c}}; \quad \Gamma_{g_t} = 0; \quad \Gamma_{\tau_t} = \frac{1}{(1 - l_t)c_t^{-1/\sigma_c}}. \\ \lambda_t &= \frac{U_c(c_t, g_t, l_t)}{1 + \tau_t} = \frac{c_t^{-1/\sigma_c}}{1 + \tau_t}, \\ \lambda_{c_t} &= -\frac{c_t^{-1/\sigma_c - 1}}{\sigma_c(1 + \tau_t)}; \quad \lambda_{l_t} = 0; \quad \lambda_{g_t} = 0; \quad \lambda_{\tau_t} = -\frac{c_t^{-1/\sigma_c}}{(1 + \tau_t)^2}.\end{aligned}$$

8.1.8 Dynamic system

We have a system of 12 endogenous and 1 exogenous variables:

$$\{c_t, \tau_t, l_t, g_t, d_t, r_t, y_t, \mu_{1,t}, \mu_{2,t}, \mu_{3,t}, \mu_{4,t}, \lambda_t\} \text{ and } \{A_t\}, \text{ respectively,}$$

that are determined by 13 equations:

$$\begin{aligned} & U_{c_t} + \mu_{1,t}\Gamma_{c_t} + \mu_{2,t}\lambda_{c_t} - \mu_{2,t-1}\frac{1 + r_{t-1}}{1 - \Phi'(d_{t-1})}\lambda_{c_t} \\ & \quad - \mu_{3,t}(1 + \tau_t) - \mu_{4,t} = 0, \\ [1] \quad c_t^{-1/\sigma_c} + \mu_{1,t}\frac{1 + \tau_t}{\sigma_c(1 - l_t)c_t^{1-1/\sigma_c}} - \mu_{2,t}\frac{c_t^{-1/\sigma_c - 1}}{\sigma_c(1 + \tau_t)} + \mu_{2,t-1}\frac{1 + r_{t-1}}{1 - \phi(d_{t-1} - \bar{d})}\frac{c_t^{-1/\sigma_c - 1}}{\sigma_c(1 + \tau_t)} \\ & \quad - \mu_{3,t}(1 + \tau_t) - \mu_{4,t} = 0, \end{aligned}$$

$$\begin{aligned} & U_{l_t} + \mu_{1,t}\Gamma_{l_t} + \mu_{2,t}\lambda_{l_t} - \mu_{2,t-1}\frac{1 + r_{t-1}}{1 - \Phi'(d_{t-1})}\lambda_{l_t} + \mu_{3,t}A_t + \mu_{4,t}A_t = 0, \\ [2] \quad & -\frac{1}{1 - l_t} + \mu_{1,t}\frac{1 + \tau_t}{(1 - l_t)^2 c_t^{-1/\sigma_c}} + \mu_{3,t}A_t + \mu_{4,t}A_t = 0, \end{aligned}$$

$$U_{g_t} + \mu_{1,t}\Gamma_{g_t} + \mu_{2,t}\lambda_{g_t} - \mu_{2,t-1}\frac{1+r_{t-1}}{1-\Phi'(d_{t-1})}\lambda_{g_t} - \mu_{4,t} = 0,$$

$$[3] \quad g_t^{-1/\sigma_g} - \mu_{4,t} = 0,$$

$$\mu_{1,t}\Gamma_{\tau_t} + \mu_{2,t}\lambda_{\tau_t} - \mu_{2,t-1}\frac{1+r_{t-1}}{1-\Phi'(d_{t-1})}\lambda_{\tau_t} - \mu_{3,t}c_t = 0,$$

$$[4] \quad \mu_{1,t}\frac{1}{(1-l_t)c_t^{-1/\sigma_c}} - \mu_{2,t}\frac{c_t^{-1/\sigma_c}}{(1+\tau_t)^2} + \mu_{2,t-1}\frac{1+r_{t-1}}{[1-\phi(d_{t-1}-\bar{d})]}\frac{c_t^{-1/\sigma_c}}{(1+\tau_t)^2} - \mu_{3,t}c_t = 0,$$

$$\mu_{2,t}\left[-\frac{1+r_t}{[1-\Phi'(d_t)]^2}\Phi''(d_t)\beta E_t\lambda_{t+1}\right] + \mu_{3,t} - \beta(1+r_t)E_t\mu_{3,t+1} = 0,$$

$$[5] \quad \mu_{2,t}\left[-\frac{1+r_t}{[1-\phi(d_t-\bar{d})]^2}\phi\beta E_t\lambda\right] + \mu_{3,t} - \beta(1+r_t)E_t\mu_{3,t+1} = 0,$$

$$-\frac{1}{1-\Phi'(d_t)}\mu_{2,t}\beta E_t\lambda_{t+1} - \beta d_t E_t\mu_{3,t+1} = 0,$$

$$[6] \quad -\frac{1}{1-\phi(d_t-\bar{d})}\mu_{2,t}\beta E_t\lambda_{t+1} - \beta d_t E_t\mu_{3,t+1} = 0,$$

$$\Gamma_t - A_t = 0,$$

$$[7] \quad \frac{1+\tau_t}{(1-l_t)c_t^{-1/\sigma_c}} - A_t = 0,$$

$$\lambda_t - \frac{1+r_t}{1-\Phi'(d_t)}\beta E_t\lambda_{t+1} = 0,$$

$$[8] \quad \lambda_t - \frac{1+r_t}{1-\phi(d_t-\bar{d})}\beta E_t\lambda_{t+1} = 0,$$

$$[9] \quad d_t - (1+r_{t-1})d_{t-1} + A_t l_t - (1+\tau_t)c_t = 0,$$

$$[10] \quad A_t l_t - c_t - g_t = 0,$$

$$[11] \quad y_t = A_t l_t,$$

$$[12] \quad \lambda_t = \frac{c_t^{-1/\sigma_c}}{1 + \tau_t},$$

$$[13] \quad \ln(A_t/\bar{A}) = \rho_A \ln(A_{t-1}/\bar{A}) + \varepsilon_t^A, \quad \varepsilon_t^A \sim NIID(0; \sigma_A^2),$$

with associated 8 parameters

$$\{\sigma_c, \sigma_g, \beta, \rho_A, \sigma_A^2, \bar{A}, \phi, \bar{d}\}.$$

8.1.9 Steady state

The steady state is a system of 13 equations with 21 unknowns, comprised of 13 steady-state variables given by

$$\{c, \tau, l, g, d, r, y, \lambda, \mu_1, \mu_2, \mu_3, \mu_4, A\},$$

and 8 parameters $\{\sigma_c, \sigma_g, \beta, \rho_A, \sigma_A^2, \bar{A}, \phi, \bar{d}\}$:

$$c_t^{-1/\sigma_c} + \mu_{1,t} \frac{1 + \tau_t}{\sigma_c(1 - l_t)c_t^{1-1/\sigma_c}} - \mu_{2,t} \frac{c_t^{-1/\sigma_c-1}}{\sigma_c(1 + \tau_t)} + \mu_{2,t-1} \frac{1 + r_{t-1}}{1 - \phi(d_{t-1} - \bar{d})} \frac{c_t^{-1/\sigma_c-1}}{\sigma_c(1 + \tau_t)} - \mu_{3,t}(1 + \tau_t) - \mu_{4,t} = 0,$$

$$[1] \quad c^{-1/\sigma_c} + \mu_1 \frac{1 + \tau}{\sigma_c(1 - l)c^{1-1/\sigma_c}} + \mu_2[r] \frac{c^{-1/\sigma_c-1}}{\sigma_c(1 + \tau)} - \mu_3(1 + \tau) - \mu_4 = 0,$$

$$-\frac{1}{1 - l_t} + \mu_{1,t} \frac{1 + \tau_t}{(1 - l_t)^2 c_t^{-1/\sigma_c}} + \mu_{3,t} A_t + \mu_{4,t} A_t = 0,$$

$$[2] \quad -\frac{1}{1 - l} + \mu_1 \frac{1 + \tau}{(1 - l)^2 c^{-1/\sigma_c}} + \mu_3 A + \mu_4 A = 0,$$

$$g_t^{-1/\sigma_g} - \mu_{4,t} = 0,$$

$$[3] \quad g^{-1/\sigma_g} - \mu_4 = 0,$$

$$\mu_{1,t} \frac{1}{(1-l_t)c_t^{-1/\sigma_c}} - \mu_{2,t} \frac{c_t^{-1/\sigma_c}}{(1+\tau_t)^2} + \mu_{2,t-1} \frac{1+r_{t-1}}{[1-\phi(d_{t-1}-\bar{d})]} \frac{c_t^{-1/\sigma_c}}{(1+\tau_t)^2} - \mu_{3,t}c_t = 0,$$

$$[4] \quad \mu_1 \frac{1}{(1-l)c^{-1/\sigma_c}} + \mu_2[r] \frac{c^{-1/\sigma_c}}{(1+\tau)^2} - \mu_3c = 0,$$

$$\mu_{2,t} \left[-\frac{1+r_t}{[1-\phi(d_t-\bar{d})]^2} \phi\beta E_t\lambda_{t+1} \right] + \mu_{3,t} - \beta(1+r_t) E_t\mu_{3,t+1} = 0,$$

$$[5] \quad \mu_2 \left[-\frac{1+r}{[1-\phi(d-\bar{d})]^2} \phi\beta\lambda \right] + \mu_3 - \beta(1+r)\mu_3 = 0,$$

$$-\frac{1}{1-\phi(d_t-\bar{d})} \mu_{2,t}\beta E_t\lambda_{t+1} - \beta d_t E_t\mu_{3,t+1} = 0,$$

$$[6] \quad -\frac{1}{1-\phi(d-\bar{d})} \mu_2\lambda - d\mu_3 = 0,$$

$$\frac{1+\tau_t}{(1-l_t)c_t^{-1/\sigma_c}} - A_t = 0,$$

$$[7] \quad \frac{1+\tau}{(1-l)c^{-1/\sigma_c}} - A = 0,$$

$$\lambda_t - \frac{1+r_t}{1-\phi(d_t-\bar{d})} \beta E_t\lambda_{t+1} = 0,$$

$$[8] \quad 1 - \frac{1+r}{1-\phi(d-\bar{d})} \beta = 0,$$

$$[9] \quad d - (1+r)d + Al - (1+\tau)c = 0,$$

$$[10] \quad Al - c - g = 0,$$

$$[11] \quad y = Al,$$

$$[12] \quad \frac{c^{-1/\sigma_c}}{1+\tau} = \lambda,$$

$$[13] \quad A = \bar{A}.$$

After calibrating some of these parameters (see text for details), we solve this system numerically.

8.2 Small open economy with incomplete asset markets

8.2.1 Households' problem

The household's problem is given by:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, g_t, l_t),$$

subject to

$$\begin{aligned} d_t &= (1 + r_{t-1}) d_{t-1} - y_t + (1 + \tau_t) c_t, \\ y_t &= A_t l_t, \\ r_t &= r_t^* + p(\tilde{d}_t), \\ \ln(A_t/\bar{A}) &= \rho_A \ln(A_{t-1}/\bar{A}) + \varepsilon_t^A, \quad \varepsilon_t^A \sim NIID(0, \sigma_A^2), \\ \ln(R_t^*/\bar{R}^*) &= \rho_{R^*} \ln(R_{t-1}^*/\bar{R}^*) + \varepsilon_t^{R^*}, \quad \varepsilon_t^{R^*} \sim NIID(0, \sigma_{R^*}^2). \end{aligned}$$

and a no-Ponzi condition.

Note that, in equilibrium, $\tilde{d}_t = d_t$. This is not internalized by the household, but is internalized by the Ramsey planner (see below).

8.2.2 Lagrangian

The Lagrangian is given by

$$= E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U(c_t, g_t, l_t) + \beta^t \lambda_t [d_t - (1 + r_{t-1}) d_{t-1} + A_t l_t - (1 + \tau_t) c_t] \right\}.$$

The first-order conditions are given by

$$\begin{aligned}
[c_t] &: \beta^t U_c(c_t, g_t, l_t) - \beta^t \lambda_t (1 + \tau_t) = 0, \\
[l_t] &: \beta^t U_l(c_t, g_t, l_t) + \beta^t \lambda_t A_t = 0, \\
[d_t] &: \beta^t \lambda_t - (1 + r_t) \beta^{t+1} E_t \lambda_{t+1} = 0.
\end{aligned}$$

The first-order conditions reduce to

$$\begin{aligned}
U_c(c_t, g_t, l_t) &= \lambda_t (1 + \tau_t), \\
U_l(c_t, g_t, l_t) &= -\lambda_t A_t, \\
\lambda_t &= (1 + r_t) \beta E_t \lambda_{t+1}.
\end{aligned}$$

8.2.3 Government's flow budget constraint

The government's flow budget constraint is given by

$$\begin{aligned}
d_t^g &= (1 + r_{t-1}^g) d_{t-1}^g - \tau_t c_t + g_t, \\
r_t^g &= r_t^* + p(d_t^g).
\end{aligned}$$

8.2.4 Implementability conditions

The implementability conditions are given by

$$\begin{aligned}
A_t &= -\frac{U_l(c_t, g_t, l_t)}{U_c(c_t, g_t, l_t)} (1 + \tau_t) = \Gamma_t \equiv \Gamma(c_t, g_t, l_t, \tau_t), \\
\frac{U_c(c_t, g_t, l_t)}{1 + \tau_t} &= \lambda_t \equiv \lambda(c_t, g_t, l_t, \tau_t) = (1 + r_t) \beta E_t \lambda(c_{t+1}, g_{t+1}, l_{t+1}, \tau_{t+1}).
\end{aligned}$$

8.2.5 Ramsey problem

The Ramsey problem takes the form:

$$= E_0 \left\{ \begin{array}{l} \sum_{t=0}^{\infty} \beta^t U(c_t, g_t, l_t) \\ + \beta^t \mu_{1,t} (\Gamma_t - A_t) \\ + \beta^t \mu_{2,t} [\lambda_t - (1 + r_t^* + p(d_t)) \beta E_t \lambda_{t+1}] \\ + \beta^t \mu_{3,t} [d_t - (1 + r_{t-1}^* + p(d_{t-1})) d_{t-1} + A_t l_t - (1 + \tau_t) c_t] \\ + \beta^t \mu_{4,t} [d_t^g - (1 + r_{t-1}^* + p(d_{t-1}^g)) d_{t-1}^g + \tau_t c_t - g_t] \end{array} \right\}.$$

The first-order conditions are given by

$$\begin{aligned} [c_t] &: U_{c_t} + \mu_{1,t} \Gamma_{c_t} + \mu_{2,t} \lambda_{c_t} - \mu_{2,t-1} (1 + r_{t-1}^* + p(d_{t-1})) \lambda_{c_t} - \mu_{3,t} (1 + \tau_t) + \mu_{4,t} \tau_t = 0, \\ [l_t] &: U_{l_t} + \mu_{1,t} \Gamma_{l_t} + \mu_{2,t} \lambda_{l_t} - \mu_{2,t-1} (1 + r_{t-1}^* + p(d_{t-1})) \lambda_{l_t} + \mu_{3,t} A_t = 0, \\ [g_t] &: U_{g_t} + \mu_{1,t} \Gamma_{g_t} + \mu_{2,t} \lambda_{g_t} - \mu_{2,t-1} (1 + r_{t-1}^* + p(d_{t-1})) \lambda_{g_t} - \mu_{4,t} = 0, \\ [\tau_t] &: \mu_{1,t} \Gamma_{\tau_t} + \mu_{2,t} \lambda_{\tau_t} - \mu_{2,t-1} (1 + r_{t-1}^* + p(d_{t-1})) \lambda_{\tau_t} - \mu_{3,t} c_t + \mu_{4,t} c_t = 0, \\ [d_t] &: -\mu_{2,t} p'(d_t) \beta E_t \lambda_{t+1} + \mu_{3,t} - \beta (p'(d_t) d_t + 1 + r_t^* + p(d_t)) E_t \mu_{3,t+1} = 0, \\ [d_t^g] &: \mu_{4,t} - \beta (p'(d_t^g) d_t^g + 1 + r_t^* + p(d_t^g)) E_t \mu_{4,t+1} = 0, \\ [\mu_{1,t}] &: \Gamma_t - A_t = 0, \\ [\mu_{2,t}] &: \lambda_t - (1 + r_t^* + p(d_t)) \beta E_t \lambda_{t+1} = 0, \\ [\mu_{3,t}] &: d_t - (1 + r_{t-1}^* + p(d_{t-1})) d_{t-1} + A_t l_t - (1 + \tau_t) c_t = 0, \\ [\mu_{4,t}] &: d_t^g - (1 + r_{t-1}^* + p(d_{t-1}^g)) d_{t-1}^g + \tau_t c_t - g_t = 0. \end{aligned}$$

8.2.6 Parameterization

Preferences are given by:

$$\begin{aligned} U(c_t, g_t, l_t) &= \frac{c_t^{1-1/\sigma_c} - 1}{1 - 1/\sigma_c} + \frac{g_t^{1-1/\sigma_g} - 1}{1 - 1/\sigma_g} + \ln(1 - l_t), \\ U_{c_t} &= c_t^{-1/\sigma_c}; \quad U_{g_t} = g_t^{-1/\sigma_g}; \quad U_{l_t} = -\frac{1}{1 - l_t}. \end{aligned}$$

The derivatives of Γ_t and λ_t are given by, respectively,

$$\begin{aligned}\Gamma_t &= -\frac{U_l(c_t, g_t, l_t)}{U_c(c_t, g_t, l_t)} (1 + \tau_t) = \frac{1 + \tau_t}{(1 - l_t)c_t^{-1/\sigma_c}}, \\ \Gamma_{c_t} &= \frac{1 + \tau_t}{\sigma_c(1 - l_t)c_t^{1-1/\sigma_c}}; \quad \Gamma_{l_t} = \frac{1 + \tau_t}{(1 - l_t)^2 c_t^{-1/\sigma_c}}; \quad \Gamma_{g_t} = 0; \quad \Gamma_{\tau_t} = \frac{1}{(1 - l_t)c_t^{-1/\sigma_c}}; \\ \lambda_t &= \frac{U_c(c_t, g_t, l_t)}{1 + \tau_t} = \frac{c_t^{-1/\sigma_c}}{1 + \tau_t}, \\ \lambda_{c_t} &= -\frac{c_t^{-1/\sigma_c - 1}}{\sigma_c(1 + \tau_t)}; \quad \lambda_{l_t} = 0; \quad \lambda_{g_t} = 0; \quad \lambda_{\tau_t} = -\frac{c_t^{-1/\sigma_c}}{(1 + \tau_t)^2}.\end{aligned}$$

Following Uribe and Schmitt-Grohé (2003), the debt-elastic specification for the household is given by:

$$\begin{aligned}p(d_t) &= \psi^c [\exp(d_t - \bar{d}) - 1], \\ p'(d_t) &= \psi^c \exp(d_t - \bar{d}),\end{aligned}$$

and for the government:

$$\begin{aligned}p(d_t^g) &= \psi^g [\exp(d_t^g - \bar{d}^g) - 1], \\ p'(d_t^g) &= \psi^g \exp(d_t^g - \bar{d}^g).\end{aligned}$$

8.2.7 Dynamic system

The dynamic system consists of 13 endogenous and 2 exogenous variables, respectively:

$$\{c_t, \tau_t, l_t, g_t, d_t, d_t^g, r_t, y_t, \mu_{1,t}, \mu_{2,t}, \mu_{3,t}, \mu_{4,t}, \lambda_t\} \text{ and } \{A_t, r_t^*\},$$

that are determined by 15 equations:

$$\begin{aligned}
U_{c_t} + \mu_{1,t}\Gamma_{c_t} + [\mu_{2,t} - \mu_{2,t-1} (1 + r_{t-1}^* + p(d_{t-1}))] \lambda_{c_t} - \mu_{3,t} (1 + \tau_t) + \mu_{4,t}\tau_t &= 0, \\
[1] \quad c_t^{-1/\sigma_c} + \mu_{1,t} \frac{1 + \tau_t}{\sigma_c(1 - l_t)c_t^{1-1/\sigma_c}} & \\
- [\mu_{2,t} - \mu_{2,t-1} (1 + r_{t-1}^* + \psi^c [\exp(d_{t-1} - \bar{d}) - 1])] \frac{c_t^{-1/\sigma_c-1}}{\sigma_c(1 + \tau_t)} & \\
- \mu_{3,t} (1 + \tau_t) + \mu_{4,t}\tau_t &= 0,
\end{aligned}$$

$$\begin{aligned}
U_{l_t} + \mu_{1,t}\Gamma_{l_t} + [\mu_{2,t} - \mu_{2,t-1} (1 + r_{t-1}^* + p(d_{t-1}))] \lambda_{l_t} + \mu_{3,t}A_t &= 0, \\
[2] \quad \frac{-1}{1 - l_t} + \mu_{1,t} \frac{1 + \tau_t}{(1 - l_t)^2 c_t^{-1/\sigma_c}} + \mu_{3,t}A_t &= 0,
\end{aligned}$$

$$\begin{aligned}
U_{g_t} + \mu_{1,t}\Gamma_{g_t} + [\mu_{2,t} - \mu_{2,t-1} (1 + r_{t-1}^* + p(d_{t-1}))] \lambda_{g_t} - \mu_{4,t} &= 0, \\
[3] \quad g_t^{-1/\sigma_g} - \mu_{4,t} &= 0,
\end{aligned}$$

$$\begin{aligned}
\mu_{1,t}\Gamma_{\tau_t} + [\mu_{2,t} - \mu_{2,t-1} (1 + r_{t-1}^* + p(d_{t-1}))] \lambda_{\tau_t} - \mu_{3,t}c_t + \mu_{4,t}c_t &= 0, \\
[4] \quad \mu_{1,t} \frac{1}{(1 - l_t)c_t^{-1/\sigma_c}} - [\mu_{2,t} - \mu_{2,t-1} (1 + r_{t-1}^* + \psi^c [\exp(d_{t-1} - \bar{d}) - 1])] \frac{c_t^{-1/\sigma_c}}{(1 + \tau_t)^2} & \\
- \mu_{3,t}c_t + \mu_{4,t}c_t &= 0,
\end{aligned}$$

$$\begin{aligned}
-\mu_{2,t}p'(d_t)\beta E_t\lambda_{t+1} + \mu_{3,t} - \beta (p'(d_t)d_t + 1 + r_t^* + p(d_t)) E_t\mu_{3,t+1} &= 0, \\
[5] \quad -\mu_{2,t}\psi \exp(d_t - \bar{d}) \beta E_t\lambda_{t+1} + \mu_{3,t} & \\
-\beta (\psi \exp(d_t - \bar{d}) d_t + 1 + r_t^* + \psi [\exp(d_t - \bar{d}) - 1]) E_t\mu_{3,t+1} &= 0,
\end{aligned}$$

$$\begin{aligned}
\mu_{4,t} - \beta (p'(d_t^g)d_t^g + 1 + r_t^* + p(d_t^g)) E_t\mu_{4,t+1} &= 0, \\
[6] \quad \mu_{4,t} - \beta (\psi_g \exp(d_t^g - \bar{d}^g) d_t^g + 1 + r_t^* + \psi_g [\exp(d_t^g - \bar{d}^g) - 1]) E_t\mu_{4,t+1} &= 0,
\end{aligned}$$

$$\begin{aligned} \Gamma_t - A_t &= 0, \\ [7] \quad \frac{1 + \tau_t}{(1 - l_t)c_t^{-1/\sigma_c}} - A_t &= 0, \end{aligned}$$

$$[8] \quad \lambda_t - (1 + r_t^* + \psi [\exp(d_t - \bar{d}) - 1]) \beta E_t \lambda_{t+1} = 0,$$

$$[9] \quad d_t - (1 + r_{t-1}^* + \psi [\exp(d_{t-1} - \bar{d}) - 1]) d_{t-1} + A_t l_t - (1 + \tau_t) c_t = 0,$$

$$[10] \quad d_t^g - (1 + r_{t-1}^* + \psi_g [\exp(d_{t-1}^g - \bar{d}^g) - 1]) d_{t-1}^g + \tau_t c_t - g_t = 0,$$

$$[11] \quad y_t = A_t l_t,$$

$$[12] \quad \lambda_t = \frac{c_t^{-1/\sigma_c}}{1 + \tau_t},$$

$$[13] \quad r_t = r_t^* + \psi [\exp(d_t - \bar{d}) - 1],$$

$$[14] \quad \ln(A_t/\bar{A}) = \rho_A \ln(A_{t-1}/\bar{A}) + \varepsilon_t^A, \quad \varepsilon_t^A \sim NIID(0; \sigma_A^2),$$

$$[15] \quad \ln(R_t^*/\bar{R}^*) = \rho_{R^*} \ln(R_{t-1}^*/\bar{R}^*) + \varepsilon_t^{R^*}, \quad \varepsilon_t^{R^*} \sim NIID(0; \sigma_{R^*}^2),$$

with associated 13 parameters

$$\{\sigma_c, \sigma_g, \beta, \psi, \bar{d}, \psi_g, \bar{d}^g, \rho_A, \sigma_A^2, \rho_{r^*}, \sigma_{r^*}^2, \bar{A}, \bar{r}^*\}.$$

8.2.8 Steady state

The steady state is a system of 15 equations with 28 unknowns, comprised of 15 steady-state variables

$$\{c, \tau, l, g, d, d^g, r, y, \lambda, \mu_1, \mu_2, \mu_3, \mu_4, A, r^*\},$$

and 13 parameters $\{\sigma_c, \sigma_g, \beta, \psi, \bar{d}, \psi_g, \bar{d}^g, \rho_A, \sigma_A^2, \rho_{r^*}, \sigma_{r^*}^2, \bar{A}, \bar{r}^*\}$:

$$[1] \quad c^{-1/\sigma_c} + \mu_1 \frac{1 + \tau}{\sigma_c(1 - l)c^{1-1/\sigma_c}} + \mu_2 [r^* + \psi^c [\exp(d - \bar{d}) - 1]] \frac{c^{-1/\sigma_c-1}}{\sigma_c(1 + \tau)} - \mu_3(1 + \tau) + \mu_4\tau = 0,$$

$$[2] \quad -\frac{1}{1-l} + \mu_1 \frac{1+\tau}{(1-l)^2 c^{-1/\sigma_c}} + \mu_3 A = 0,$$

$$[3] \quad g^{-1/\sigma_g} - \mu_4 = 0,$$

$$[4] \quad \mu_1 \frac{1}{(1-l)c^{-1/\sigma_c}} + \mu_2 [r^* + \psi^c [\exp(d - \bar{d}) - 1]] \frac{c^{-1/\sigma_c}}{(1+\tau)^2} - \mu_3 c + \mu_4 c = 0,$$

$$[5] \quad -\mu_2 \psi \exp(d - \bar{d}) \beta \lambda + \mu_3 - \beta (\psi \exp(d - \bar{d}) d + 1 + r^* + \psi [\exp(d - \bar{d}) - 1]) \mu_3 = 0,$$

$$[6] \quad \mu_4 - \beta (\psi_g \exp(d^g - \bar{d}^g) d^g + 1 + r^* + \psi_g [\exp(d^g - \bar{d}^g) - 1]) \mu_4 = 0,$$

$$[7] \quad \frac{1+\tau}{(1-l)c^{-1/\sigma_c}} - A = 0,$$

$$[8] \quad \lambda - (1 + r^* + \psi [\exp(d - \bar{d}) - 1]) \beta \lambda = 0,$$

$$[9] \quad d - (1 + r^* + \psi [\exp(d - \bar{d}) - 1]) d + Al - (1 + \tau) c = 0,$$

$$[10] \quad d^g - (1 + r^* + \psi_g [\exp(d^g - \bar{d}^g) - 1]) d^g + \tau c - g = 0,$$

$$[11] \quad y = Al,$$

$$[12] \quad r = r^* + \psi [\exp(d - \bar{d}) - 1],$$

$$[13] \quad \frac{c^{-1/\sigma_c}}{1+\tau} = \lambda,$$

$$[14] \quad \ln(A/\bar{A}) = \rho_A \ln(A/\bar{A})$$

$$[15] \quad \ln(R^*/\bar{R}^*) = \rho_{R^*} \ln(R^*/\bar{R}^*)$$

This system may be reduced to the following 8 equations:

$$\begin{aligned}
[1] \quad c^{-1/\sigma_c} + \mu_1 \frac{1 + \tau}{\sigma_c(1-l)c^{1-1/\sigma_c}} + \mu_2 r^* \frac{c^{-1/\sigma_c-1}}{\sigma_c(1+\tau)} - \mu_3(1+\tau) + \mu_4\tau &= 0, \\
[2] \quad -\frac{1}{1-l} + \mu_1 \frac{1+\tau}{(1-l)^2 c^{-1/\sigma_c}} + \mu_3 \bar{A} &= 0, \\
[3] \quad g^{-1/\sigma_g} - \mu_4 &= 0, \\
[4] \quad \mu_1 \frac{1}{(1-l)c^{-1/\sigma_c}} + \mu_2 r^* \frac{c^{-1/\sigma_c}}{(1+\tau)^2} - \mu_3 c + \mu_4 c &= 0, \\
[5] \quad -\mu_2 \psi \beta \frac{c^{-1/\sigma_c}}{1+\tau} + \mu_3 - \beta(\psi \bar{d} + 1 + r^*) \mu_3 &= 0, \\
[6] \quad \frac{1+\tau}{(1-l)c^{-1/\sigma_c}} - \bar{A} &= 0, \\
[7] \quad \bar{d} - (1+r^*) \bar{d} + \bar{A}l - (1+\tau)c &= 0, \\
[8] \quad d^g - (1+r^*)d^g + \tau c - g &= 0,
\end{aligned}$$

with the following 9 unknowns

$$\{c, \tau, l, g, \bar{d}, \mu_1, \mu_2, \mu_3, \mu_4\}$$

and the following equation added to close the system

$$(\bar{d} + d^g)/\bar{A}l = DtoY,$$

where $DtoY$ is calibrated to 1.34 (see Table 1).

Note that this system can be further simplified by setting $d^g = 0$. To see this, start from equation [6], repeated here for convenience,

$$[6] \quad \mu_4 - \beta(\psi_g \exp(d^g - \bar{d}^g) d^g + 1 + r^* + \psi_g [\exp(d^g - \bar{d}^g) - 1]) \mu_4 = 0,$$

Assume that $d^g = \bar{d}^g$ in the steady state. So, this is reduced to

$$\mu_4 - \beta(\psi_g d^g + 1 + r^*) \mu_4 = 0$$

Since the government's budget constraint is binding (satisfied with equality), it must be that $\mu_4 \neq 0$ in the steady state. Therefore,

$$\begin{aligned}\beta(\psi_g d^g + 1 + r^*) &= 1 \\ \psi_g d^g &= \frac{1}{\beta} - (1 + r^*) \\ \psi_g d^g &= \frac{1}{\beta} - (1 + r^*) \\ d^g &= \frac{\frac{1}{\beta} - (1 + r^*)}{\psi_g}\end{aligned}$$

In the incomplete asset market model, this result implies:

$$d^g = 0,$$

since we use the standard assumption that $\frac{1}{\beta} = (1 + r^*)$.

After introducing this further simplification and calibrating some of the parameters in this model (see text for details), we solve this system numerically.

8.3 Small open economy with complete asset markets

8.3.1 Households' problem

The household's problem is given by:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, g_t, l_t),$$

subject to:

$$\begin{aligned}E_t d_{t+1} r_{t+1} &= d_t + A_t l_t - (1 + \tau_t) c_t, \\ y_t &= A_t l_t, \\ \ln(A_t / \bar{A}) &= \rho_A \ln(A_{t-1} / \bar{A}) + \varepsilon_t^A, \quad \varepsilon_t^A \sim NIID(0, \sigma_A^2),\end{aligned}$$

and a no-Ponzi condition.

8.3.2 Lagrangian

The Lagrangian is given by

$$= E_0 \sum_{t=1}^{\infty} \beta^t U(c_t, g_t, l_t) + \beta^t \lambda_t [d_t + A_t l_t - (1 + \tau_t)c_t - d_{t+1} r_{t+1}].$$

The first-order conditions are given by

$$\begin{aligned} [c_t] &: \beta^t U_{c_t}(c_t, g_t, l_t) - \beta^t \lambda_t (1 + \tau_t) = 0, \\ [l_t] &: \beta^t U_{l_t}(c_t, g_t, l_t) + \beta^t \lambda_t A_t = 0, \\ [d_{t+1}] &: -\beta^t \lambda_t E_t r_{t+1} + \beta^{t+1} E_t \lambda_{t+1} = 0. \end{aligned} \tag{54}$$

These first-order conditions can be simplified to

$$\begin{aligned} U_{c_t}(c_t, g_t, l_t) &= \lambda_t (1 + \tau_t), \\ U_{l_t}(c_t, g_t, l_t) &= -\lambda_t A_t, \\ \lambda_t E_t r_{t+1} &= \beta E_t \lambda_{t+1}. \end{aligned}$$

8.3.3 Governments' flow budget constraint

The government's flow budget constraint is given by

$$E_t d_{t+1}^g r_{t+1} = d_t^g - g_t + \tau_t c_t.$$

8.3.4 Implementability conditions

Following Schmitt-Grohé and Uribe (2003),

$$\frac{U_{c_t}}{1 + \tau_t} = \lambda_t = \Psi_{CAM} \tag{55}$$

Thus, the implementability conditions are given by

$$\begin{aligned} A_t &= -\frac{U_{l_t}(c_t, g_t, l_t)}{\Psi_{CAM}} = \Gamma_t \equiv \Gamma(c_t, g_t, l_t), \\ \frac{U_{c_t}(c_t, g_t, l_t)}{1 + \tau_t} &= \Lambda_t = \Lambda(c_t, g_t, l_t, \tau_t) = \Psi_{CAM} \end{aligned}$$

From conditions (54) and (55), it follows that

$$E_t r_{t+1} = \beta$$

Multiplying on both sides by debt:

$$\begin{aligned} E_t r_{t+1} d_{t+1} &= \beta E_t d_{t+1}, \\ E_t r_{t+1} d_{t+1}^g &= \beta E_t d_{t+1}^g. \end{aligned}$$

Then, following Schmitt-Grohé and Uribe (2003) we can define,

$$\begin{aligned} s_{t+1} &\equiv E_t r_{t+1} d_{t+1}, \\ s_{t+1}^g &\equiv E_t r_{t+1} d_{t+1}^g. \end{aligned}$$

8.3.5 Ramsey problem

The Ramsey problem's Lagrangian takes the form:

$$\mathcal{L} = E_0 \left\{ \begin{array}{l} \sum_{t=1}^{\infty} \beta^t U(c_t, g_t, l_t) \\ + \beta^t \mu_{1,t} [\Gamma_t - A_t] \\ + \beta^t \mu_{2,t} [\Lambda_t - \Psi_{CAM}] \\ + \beta^t \mu_{3,t} [d_t + A_t l_t - (1 + \tau_t) c_t - s_t] \\ + \beta^t \mu_{4,t} [d_t^g - g_t + \tau_t c_t - s_t^g] \end{array} \right\}$$

The first-order conditions are given by:

$$\begin{aligned}
[c_t] &: U_{c_t} + \mu_{1,t}\Gamma_{c_t} + \mu_{2,t}\Lambda_c - \mu_{3,t}(1 + \tau_t) + \mu_{4,t}\tau_t = 0, \\
[l_t] &: U_{l_t} + \mu_{1,t}\Gamma_{l_t} + \mu_{2,t}\Lambda_l + \mu_{3,t}A_t = 0, \\
[g_t] &: U_{g_t} + \mu_{1,t}\Gamma_{g_t} + \mu_{2,t}\Lambda_g - \mu_{4,t} = 0, \\
[\tau_t] &: \mu_{2,t}\Lambda_\tau - \mu_{3,t}c_t + \mu_{4,t}c_t = 0, \\
[d_{t+1}] &: -\mu_{3,t} + E_t\mu_{3,t+1} = 0, \\
[d_{t+1}^g] &: -\mu_{4,t} + E_t\mu_{4,t+1} = 0, \\
[\mu_{1,t}] &: \Gamma_t - A_t = 0, \\
[\mu_{2,t}] &: \Lambda_t - \Psi_{CAM}, \\
[\mu_{3,t}] &: d_t + A_t l_t - (1 + \tau_t)c_t - s_{t+1} = 0, \\
[\mu_{4,t}] &: d_t^g - g_t + \tau_t c_t - s_{t+1}^g = 0.
\end{aligned}$$

8.3.6 Parameterization

Preferences are given by

$$\begin{aligned}
U(c_t, g_t, l_t) &= \frac{c_t^{1-1/\sigma_c} - 1}{1 - 1/\sigma_c} + \frac{g_t^{1-1/\sigma_g} - 1}{1 - 1/\sigma_g} + \log(1 - l_t), \\
U_{c_t} &= c_t^{-1/\sigma_c}; \quad U_{g_t} = g_t^{-1/\sigma_g}; \quad U_{l_t} = -\frac{1}{1 - l_t}.
\end{aligned}$$

The derivatives of Λ_t are given by

$$\begin{aligned}\Lambda_t &= \frac{U_{c_t}(c_t, g_t, l_t)}{1 + \tau_t} = \frac{c_t^{-1/\sigma_c}}{1 + \tau_t}, \\ \Lambda_{c_t} &= -\frac{c_t^{-\frac{1}{\sigma_c}-1}}{\sigma_c(1 + \tau_t)}, \quad \Lambda_{l_t} = 0, \\ \Lambda_{g_t} &= 0; \quad \Lambda_{\tau_t} = -\frac{c_t^{-\frac{1}{\sigma_c}}}{(1 + \tau_t)^2}.\end{aligned}$$

The derivatives of Γ_t are given by

$$\begin{aligned}\Gamma_t &= -\frac{U_{l_t}(c_t, g_t, l_t)}{\Psi_{CAM}} = \frac{1}{(1 - l_t)\Psi_{CAM}}, \\ \Gamma_{c_t} &= 0, \quad \Gamma_{l_t} = \frac{1}{(1 - l_t)^2\Psi_{CAM}}, \\ \Gamma_{g_t} &= 0.\end{aligned}$$

8.3.7 Dynamic system

The dynamic system consists of 13 endogenous and 1 exogenous variable, given by

$$\{c_t, l_t, g_t, \tau_t, d_t, d_t^g, s_t, s_t^g, \mu_{1,t}, \mu_{2,t}, \mu_{3,t}, \mu_{4,t}, y_t\} \text{ and } A_t, \text{ respectively.}$$

that are determined by 14 equations:

$$\begin{aligned}U_{c_t} + \mu_{1,t}\Gamma_{c_t} + \mu_{2,t}\Lambda_{c_t} - \mu_{3,t}(1 + \tau_t) + \mu_{4,t}\tau_t &= 0, \\ [1] \quad c_t^{-1/\sigma_c} - \mu_{2,t}\frac{c_t^{-\frac{1}{\sigma_c}-1}}{\sigma_c(1 + \tau_t)} - \mu_{3,t}(1 + \tau_t) + \mu_{4,t}\tau_t &= 0,\end{aligned}$$

$$\begin{aligned}U_{l_t} + \mu_{1,t}\Gamma_{l_t} + \mu_{2,t}\Lambda_{l_t} + \mu_{3,t}A_t &= 0, \\ [2] \quad -\frac{1}{1 - l_t} + \mu_{1,t}\frac{1}{(1 - l_t)^2\Psi_{CAM}} + \mu_{3,t}A_t &= 0,\end{aligned}$$

$$\begin{aligned}
U_{g_t} + \mu_{1,t}\Gamma_{g_t} + \mu_{2,t}\Lambda_g - \mu_{4,t} &= 0, \\
[3] \quad g_t^{-1/\sigma_g} - \mu_{4,t} &= 0,
\end{aligned}$$

$$\begin{aligned}
\mu_{2,t}\Lambda_\tau - \mu_{3,t}c_t + \mu_{4,t}c_t &= 0, \\
[4] \quad -\mu_{2,t}\frac{c_t^{-1/\sigma_c}}{(1+\tau_t)^2} - \mu_{3,t}c_t + \mu_{4,t}c_t &= 0,
\end{aligned}$$

$$[5] \quad -\mu_{3,t} + E_t\mu_{3,t+1} = 0,$$

$$[6] \quad -\mu_{4,t} + E_t\mu_{4,t+1} = 0,$$

$$\begin{aligned}
\Gamma_t - A_t &= 0, \\
[7] \quad \frac{1}{(1-l_t)\Psi_{CAM}} - A_t &= 0,
\end{aligned}$$

$$\begin{aligned}
\Lambda_t - \Psi_{CAM} &= 0, \\
[8] \quad \frac{c_t^{-1/\sigma_c}}{(1+\tau_t)} - \Psi_{CAM} &= 0,
\end{aligned}$$

$$[9] \quad d_t + A_t l_t - (1+\tau_t)c_t - s_{t+1} = 0,$$

$$[10] \quad d_t^g - g_t + \tau_t c_t - s_{t+1}^g = 0,$$

$$[11] \quad s_{t+1} = \beta E_t d_{t+1},$$

$$[12] \quad s_{t+1}^g = \beta E_t d_{t+1}^g,$$

$$[13] \quad y_t - A_t l_t = 0,$$

$$[14] \quad \ln(A_t/\bar{A}) = \rho_A \ln(A_{t-1}/\bar{A}) + \varepsilon_t^A, \quad \varepsilon_t^A \sim NIID(0, \sigma_A^2),$$

with associated 6 parameters

$$\{\sigma_c, \sigma_g, \beta, \rho_A, \sigma_A^2, \bar{A}, \Psi_{CAM}\}.$$

8.3.8 Steady state

The steady state is a system of 15 equations with 20 unknowns, comprised of 15 steady-state variables

$$\{c, l, g, \tau, d, d^g, \mu_1, \mu_2, \mu_3, \mu_4, A, y, s, s^g, \Psi_{CAM}\}$$

and 5 parameters $\{\sigma_c, \sigma_g, \beta, \rho_A, \bar{A}\}$:

$$[1] \quad c^{-1/\sigma_c} - \mu_2 \frac{c^{-\frac{1}{\sigma_c}-1}}{\sigma_c(1+\tau)} - \mu_3(1+\tau) + \mu_4\tau = 0,$$

$$[2] \quad -\frac{1}{1-l} + \mu_1 \frac{1}{(1-l)^2 \Psi_{CAM}} + \mu_3 A = 0,$$

$$[3] \quad g^{-1/\sigma_g} - \mu_4 = 0,$$

$$[4] \quad -\mu_2 \frac{c^{-1/\sigma_c}}{(1+\tau)^2} - \mu_3 c + \mu_4 c = 0,$$

$$[5] \quad \frac{1}{(1-l)\Psi_{CAM}} - A = 0,$$

$$[6] \quad \frac{c^{-1/\sigma_c}}{(1+\tau)} - \Psi_{CAM} = 0,$$

$$[7] \quad d + Al - (1+\tau)c - \beta d = 0,$$

$$[8] \quad d^g - g + \tau c - \beta d^g = 0,$$

$$[9] \quad \frac{d^g + d}{Al} - DtoY = 0,$$

$$[10] \quad s = \beta d,$$

$$[11] \quad s^g = \beta d^g,$$

$$[12] \quad y = Al,$$

$$[13] \quad A = \bar{A},$$

$$[14] \quad d^g = 0,$$

$$[15] \quad \mu_3 = \Psi_{CAM},$$

where we have added equations [9] and [14] so as to make the steady state consistent with that of the incomplete asset markets case: $d^g = 0$ and $DtoY = 1.34$. We also add equation [15], which is derived following the same procedure that supports condition (55). After calibrating the remaining parameters (see text for details), we solve this system numerically.

8.4 Small open economy with incomplete asset markets and labor tax

8.4.1 Households' problem

The household's problem is given by:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, g_t, l_t),$$

subject to

$$\begin{aligned} d_t &= (1 + r_{t-1}) d_{t-1} - A_t l_t (1 - \tau_t) + c_t, \\ y_t &= A_t l_t, \\ r_t &= r_t^* + p(\tilde{d}_t), \\ \ln(A_t/\bar{A}) &= \rho_A \ln(A_{t-1}/\bar{A}) + \varepsilon_t^A, \quad \varepsilon_t^A \sim NIID(0, \sigma_A^2), \\ \ln(R_t^*/\bar{R}^*) &= \rho_{R^*} \ln(R_{t-1}^*/\bar{R}^*) + \varepsilon_t^{R^*}, \quad \varepsilon_t^{R^*} \sim NIID(0, \sigma_{R^*}^2). \end{aligned}$$

and a no-Ponzi condition.

Note that, in equilibrium, $\tilde{d}_t = d_t$. This is not internalized by the household, but is internalized by the Ramsey planner (see below).

8.4.2 Lagrangian

The Lagrangian is given by

$$\mathcal{L} = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U(c_t, g_t, l_t) + \beta^t \lambda_t [d_t - (1 + r_{t-1}) d_{t-1} + A_t l_t (1 - \tau_t) - c_t] \right\}.$$

The first-order conditions are given by

$$\begin{aligned}
[c_t] &: \beta^t U_c(c_t, g_t, l_t) - \beta^t \lambda_t = 0, \\
[l_t] &: \beta^t U_l(c_t, g_t, l_t) + \beta^t \lambda_t A_t (1 - \tau_t) = 0, \\
[d_t] &: \beta^t \lambda_t - (1 + r_t) \beta^{t+1} E_t \lambda_{t+1} = 0.
\end{aligned}$$

The first-order conditions reduce to

$$\begin{aligned}
U_c(c_t, g_t, l_t) &= \lambda_t, \\
U_l(c_t, g_t, l_t) &= -\lambda_t A_t (1 - \tau_t), \\
\lambda_t &= (1 + r_t) \beta E_t \lambda_{t+1}.
\end{aligned}$$

8.4.3 Government's flow budget constraint

The government's flow budget constraint is given by

$$\begin{aligned}
d_t^g &= (1 + r_{t-1}^g) d_{t-1}^g - \tau_t A_t l_t + g_t, \\
r_t^g &= r_t^* + p(d_t^g).
\end{aligned}$$

8.4.4 Implementability conditions

The implementability conditions are given by

$$\begin{aligned}
A_t &= -\frac{U_l(c_t, g_t, l_t)}{U_c(c_t, g_t, l_t)} \frac{1}{(1 - \tau_t)} = \Gamma_t \equiv \Gamma(c_t, g_t, l_t, \tau_t), \\
U_c(c_t, g_t, l_t) &= \lambda_t \equiv \lambda(c_t, g_t, l_t) = (1 + r_t) \beta E_t \lambda(c_{t+1}, g_{t+1}, l_{t+1}).
\end{aligned}$$

8.4.5 Ramsey problem

The Ramsey problem takes the form:

$$= E_0 \left\{ \begin{array}{l} \sum_{t=0}^{\infty} \beta^t U(c_t, g_t, l_t) \\ + \beta^t \mu_{1,t} (\Gamma_t - A_t) \\ + \beta^t \mu_{2,t} [\lambda_t - (1 + r_t^* + p(d_t)) \beta E_t \lambda_{t+1}] \\ + \beta^t \mu_{3,t} [d_t - (1 + r_{t-1}^* + p(d_{t-1})) d_{t-1} + A_t l_t (1 - \tau_t) - c_t] \\ + \beta^t \mu_{4,t} [d_t^g - (1 + r_{t-1}^* + p(d_{t-1}^g)) d_{t-1}^g + \tau_t A_t l_t - g_t] \end{array} \right\}.$$

The first-order conditions are given by

$$\begin{aligned} [c_t] &: U_{c_t} + \mu_{1,t} \Gamma_{c_t} + \mu_{2,t} \lambda_{c_t} - \mu_{2,t-1} (1 + r_{t-1}^* + p(d_{t-1})) \lambda_{c_t} - \mu_{3,t} = 0, \\ [l_t] &: U_{l_t} + \mu_{1,t} \Gamma_{l_t} + \mu_{2,t} \lambda_{l_t} - \mu_{2,t-1} (1 + r_{t-1}^* + p(d_{t-1})) \lambda_{l_t} + \mu_{3,t} A_t (1 - \tau_t) + \mu_{4,t} A_t \tau_t = 0, \\ [g_t] &: U_{g_t} + \mu_{1,t} \Gamma_{g_t} + \mu_{2,t} \lambda_{g_t} - \mu_{2,t-1} (1 + r_{t-1}^* + p(d_{t-1})) \lambda_{g_t} - \mu_{4,t} = 0, \\ [\tau_t] &: \mu_{1,t} \Gamma_{\tau_t} - \mu_{3,t} A_t l_t + \mu_{4,t} A_t l_t = 0, \\ [d_t] &: -\mu_{2,t} p'(d_t) \beta E_t \lambda_{t+1} + \mu_{3,t} - \beta (p'(d_t) d_t + 1 + r_t^* + p(d_t)) E_t \mu_{3,t+1} = 0, \\ [d_t^g] &: \mu_{4,t} - \beta (p'(d_t^g) d_t^g + 1 + r_t^* + p(d_t^g)) E_t \mu_{4,t+1} = 0, \\ [\mu_{1,t}] &: \Gamma_t - A_t = 0, \\ [\mu_{2,t}] &: \lambda_t - (1 + r_t^* + p(d_t)) \beta E_t \lambda_{t+1} = 0, \\ [\mu_{3,t}] &: d_t - (1 + r_{t-1}^* + p(d_{t-1})) d_{t-1} + A_t l_t (1 - \tau_t) - c_t = 0, \\ [\mu_{4,t}] &: d_t^g - (1 + r_{t-1}^* + p(d_{t-1}^g)) d_{t-1}^g + \tau_t A_t l_t - g_t = 0. \end{aligned}$$

8.4.6 Parameterization

Preferences are given by:

$$\begin{aligned} U(c_t, g_t, l_t) &= \frac{c_t^{1-1/\sigma_c} - 1}{1 - 1/\sigma_c} + \frac{g_t^{1-1/\sigma_g} - 1}{1 - 1/\sigma_g} + \ln(1 - l_t), \\ U_{c_t} &= c_t^{-1/\sigma_c}; \quad U_{g_t} = g_t^{-1/\sigma_g}; \quad U_{l_t} = -\frac{1}{1 - l_t}. \end{aligned}$$

The derivatives of Γ_t and λ_t are given by, respectively,

$$\begin{aligned}\Gamma_t &= \frac{U_l(c_t, g_t, l_t)}{U_c(c_t, g_t, l_t)} \frac{1}{1 - \tau_t} = \frac{1}{(1 - \tau_t)(1 - l_t)c_t^{-1/\sigma_c}}, \\ \Gamma_{c_t} &= \frac{1}{\sigma_c(1 - l_t)c_t^{1-1/\sigma_c}(1 - \tau_t)}; \quad \Gamma_{l_t} = \frac{1}{(1 - l_t)^2 c_t^{-1/\sigma_c}(1 - \tau_t)}; \\ \Gamma_{g_t} &= 0; \quad \Gamma_{\tau_t} = \frac{1}{(1 - l_t)c_t^{-1/\sigma_c}(1 - \tau_t)^2}; \\ \lambda_t &= U_c(c_t, g_t, l_t) = c_t^{-1/\sigma_c}, \\ \lambda_{c_t} &= -\frac{c_t^{-1/\sigma_c-1}}{\sigma_c}; \quad \lambda_{l_t} = 0; \quad \lambda_{g_t} = 0; \quad \lambda_{\tau_t} = 0.\end{aligned}$$

Following Uribe and Schmitt-Grohé (2003), the debt-elastic specification for the household is given by:

$$\begin{aligned}p(d_t) &= \psi^c [\exp(d_t - \bar{d}) - 1], \\ p'(d_t) &= \psi^c \exp(d_t - \bar{d}),\end{aligned}$$

and for the government:

$$\begin{aligned}p(d_t^g) &= \psi^g [\exp(d_t^g - \bar{d}^g) - 1], \\ p'(d_t^g) &= \psi^g \exp(d_t^g - \bar{d}^g).\end{aligned}$$

8.4.7 Dynamic system

The dynamic system consists of 13 endogenous and 2 exogenous variables, respectively:

$$\{c_t, \tau_t, l_t, g_t, d_t, d_t^g, r_t, y_t, \mu_{1,t}, \mu_{2,t}, \mu_{3,t}, \mu_{4,t}, \lambda_t\} \text{ and } \{A_t, r_t^*\},$$

that are determined by 15 equations:

$$\begin{aligned}
U_{c_t} + \mu_{1,t}\Gamma_{c_t} + [\mu_{2,t} - \mu_{2,t-1} (1 + r_{t-1}^* + p(d_{t-1}))] \lambda_{c_t} - \mu_{3,t} &= 0, \\
[1] \quad c_t^{-1/\sigma_c} + \mu_{1,t} \frac{1}{\sigma_c(1-l_t)c_t^{1-1/\sigma_c}(1-\tau_t)} & \\
- [\mu_{2,t} - \mu_{2,t-1} (1 + r_{t-1}^* + \psi^c [\exp(d_{t-1} - \bar{d}) - 1])] \frac{c_t^{-1/\sigma_c-1}}{\sigma_c} & \\
- \mu_{3,t} &= 0,
\end{aligned}$$

$$\begin{aligned}
U_{l_t} + \mu_{1,t}\Gamma_{l_t} + [\mu_{2,t} - \mu_{2,t-1} (1 + r_{t-1}^* + p(d_{t-1}))] \lambda_{l_t} + \mu_{3,t}A_t(1 - \tau_t) + \mu_{4,t}A_t\tau_t &= 0, \\
[2] \quad \frac{-1}{1-l_t} + \mu_{1,t} \frac{1}{(1-l_t)^2c_t^{-1/\sigma_c}(1-\tau_t)} + \mu_{3,t}A_t(1 - \tau_t) + \mu_{4,t}A_t\tau_t &= 0,
\end{aligned}$$

$$\begin{aligned}
U_{g_t} + \mu_{1,t}\Gamma_{g_t} + [\mu_{2,t} - \mu_{2,t-1} (1 + r_{t-1}^* + p(d_{t-1}))] \lambda_{g_t} - \mu_{4,t} &= 0, \\
[3] \quad g_t^{-1/\sigma_g} - \mu_{4,t} &= 0,
\end{aligned}$$

$$\begin{aligned}
\mu_{1,t}\Gamma_{\tau_t} - \mu_{3,t}A_t l_t + \mu_{4,t}A_t l_t &= 0, \\
[4] \quad \mu_{1,t} \frac{1}{(1-l_t)c_t^{-1/\sigma_c}(1-\tau_t)^2} - \mu_{3,t}A_t l_t + \mu_{4,t}A_t l_t &= 0,
\end{aligned}$$

$$\begin{aligned}
-\mu_{2,t}p'(d_t)\beta E_t \lambda_{t+1} + \mu_{3,t} - \beta (p'(d_t)d_t + 1 + r_t^* + p(d_t)) E_t \mu_{3,t+1} &= 0, \\
[5] \quad -\mu_{2,t}\psi \exp(d_t - \bar{d}) \beta E_t \lambda_{t+1} + \mu_{3,t} - & \\
\beta (\psi \exp(d_t - \bar{d}) d_t + 1 + r_t^* + \psi [\exp(d_t - \bar{d}) - 1]) E_t \mu_{3,t+1} &= 0,
\end{aligned}$$

$$\begin{aligned}
\mu_{4,t} - \beta (p'(d_t^g)d_t^g + 1 + r_t^* + p(d_t^g)) E_t \mu_{4,t+1} &= 0, \\
[6] \quad \mu_{4,t} - \beta (\psi_g \exp(d_t^g - \bar{d}^g) d_t^g + 1 + r_t^* + \psi_g [\exp(d_t^g - \bar{d}^g) - 1]) E_t \mu_{4,t+1} &= 0,
\end{aligned}$$

$$\begin{aligned}
& \Gamma_t - A_t = 0, \\
[7] \quad & \frac{1}{(1-l_t)c_t^{-1/\sigma_c}(1-\tau_t)} - A_t = 0, \\
[8] \quad & \lambda_t - (1+r_t^* + \psi [\exp(d_t - \bar{d}) - 1]) \beta E_t \lambda_{t+1} = 0, \\
[9] \quad & d_t - (1+r_{t-1}^* + \psi [\exp(d_{t-1} - \bar{d}) - 1]) d_{t-1} + A_t l_t (1-\tau_t) - c_t = 0, \\
[10] \quad & d_t^g - (1+r_{t-1}^* \psi^g [\exp(d_{t-1}^g - \bar{d}^g) - 1]) d_{t-1}^g + \tau_t A_t l_t - g_t = 0, \\
[11] \quad & y_t = A_t l_t, \\
[12] \quad & \lambda_t = c_t^{-1/\sigma_c}, \\
[13] \quad & r_t = r_t^* + \psi [\exp(d_t - \bar{d}) - 1], \\
[14] \quad & \ln(A_t/\bar{A}) = \rho_A \ln(A_{t-1}/\bar{A}) + \varepsilon_t^A, \quad \varepsilon_t^A \sim NIID(0; \sigma_A^2), \\
[15] \quad & \ln(R_t^*/\bar{R}^*) = \rho_{R^*} \ln(R_{t-1}^*/\bar{R}^*) + \varepsilon_t^{R^*}, \quad \varepsilon_t^{R^*} \sim NIID(0; \sigma_{R^*}^2),
\end{aligned}$$

with associated 13 parameters

$$\{\sigma_c, \sigma_g, \beta, \psi, \bar{d}, \psi^g, \bar{d}^g, \rho_A, \sigma_A^2, \rho_{r^*}, \sigma_{r^*}^2, \bar{A}, \bar{r}^*\}.$$

8.4.8 Steady state

The steady state is a system of 15 equations with 28 unknowns, comprised of 15 steady-state variables

$$\begin{aligned}
[1] \quad & c^{-1/\sigma_c} + \mu_1 \frac{1}{\sigma_c(1-l)c^{1-1/\sigma_c}(1-\tau)} + [\mu_2 (r^* + \psi^c [\exp(d - \bar{d}) - 1])] \frac{c^{-1/\sigma_c-1}}{\sigma_c} - \mu_3 = 0, \\
[2] \quad & \frac{-1}{1-l} + \mu_1 \frac{1}{(1-l)^2 c^{-1/\sigma_c}(1-\tau)} + \mu_3 A(1-\tau) + \mu_4 A\tau = 0,
\end{aligned}$$

$$[3] \quad g^{-1/\sigma_g} - \mu_4 = 0,$$

$$[4] \quad \mu_1 \frac{1}{(1-l)c^{-1/\sigma_c}(1-\tau)^2} - \mu_3 Al + \mu_4 Al = 0,$$

$$[5] \quad -\mu_2 \psi \exp(d - \bar{d}) \beta \lambda + \mu_3 - \beta (\psi \exp(d - \bar{d}) d + 1 + r^* + \psi [\exp(d - \bar{d}) - 1]) \mu_3 = 0,$$

$$[6] \quad \mu_4 - \beta (\psi_g \exp(d^g - \bar{d}^g) d^g + 1 + r^* + \psi_g [\exp(d^g - \bar{d}^g) - 1]) \mu_4 = 0,$$

$$[7] \quad \frac{1}{(1-l)c^{-1/\sigma_c}(1-\tau)} - A = 0,$$

$$[8] \quad \lambda - (1 + r^* + \psi [\exp(d - \bar{d}) - 1]) \beta \lambda = 0,$$

$$[9] \quad d - (1 + r_{t-1}^* + \psi [\exp(d_{t-1} - \bar{d}) - 1]) d_{t-1} + Al(1 - \tau) - c = 0,$$

$$[10] \quad d^g - (1 + r_{t-1}^* + \psi_g [\exp(d_{t-1}^g - \bar{d}^g) - 1]) d_{t-1}^g + Al\tau - g = 0,$$

$$[11] \quad y = Al,$$

$$[12] \quad \lambda = c^{-1/\sigma_c},$$

$$[13] \quad r = r^* + \psi [\exp(d - \bar{d}) - 1],$$

$$[14] \quad \ln(A/\bar{A}) = \rho_A \ln(A/\bar{A})$$

$$[15] \quad \ln(R^*/\bar{R}^*) = \rho_{R^*} \ln(R^*/\bar{R}^*)$$

This system may be reduced to the following 8 equations:

$$\begin{aligned}
[1] \quad c^{-1/\sigma_c} + \mu_1 \frac{1}{\sigma_c(1-l)c^{1-1/\sigma_c}(1-\tau)} + [\mu_2(r^* + \psi^c [\exp(d - \bar{d}) - 1])] \frac{c^{-1/\sigma_c-1}}{\sigma_c} - \mu_3 &= 0, \\
[2] \quad \frac{-1}{1-l} + \mu_1 \frac{1}{(1-l)^2 c^{-1/\sigma_c}(1-\tau)} + \mu_3 A(1-\tau) + \mu_4 \bar{A} \tau &= 0, \\
[3] \quad g^{-1/\sigma_g} - \mu_4 &= 0, \\
[4] \quad \mu_1 \frac{1}{(1-l)c^{-1/\sigma_c}(1-\tau)^2} - \mu_3 \bar{A} l + \mu_4 \bar{A} l &= 0, \\
[5] \quad -\mu_2 \psi \beta c^{-1/\sigma_c} + \mu_3 - \beta(\psi \bar{d} + 1 + r^*) \mu_3 &= 0, \\
[6] \quad \frac{1}{(1-l)c^{-1/\sigma_c}(1-\tau)} - \bar{A} &= 0, \\
[7] \quad \bar{d} - (1+r^*) \bar{d} + \bar{A} l (1-\tau) - c &= 0, \\
[8] \quad d^g - (1+r^*) d^g + \tau \bar{A} l - g &= 0,
\end{aligned}$$

with the following 9 unknowns

$$\{c, \tau, l, g, \bar{d}, \mu_1, \mu_2, \mu_3, \mu_4\}$$

and the following equation added to close the system

$$(\bar{d} + d^g)/\bar{A} l = DtoY,$$

where $DtoY$ is calibrated to 1.34 (see Table 1).

As explained for the Incomplete Asset Markets with consumption tax case, note that this system can be further simplified by setting $d^g = 0$. After introducing this further simplification and calibrating some of the parameters in this model (see text for details), we solve this system numerically.

9 Adding Keynesian Ingredients

This section reports the details of extending our framework to incorporate downward wage rigidities, following the work by Bianchi et. al (2023) and Schmitt-Grohé and Uribe (2016). For tractability, we do so in a simple two period version of the dynamic model considered

above, expanded to include nontradable goods.³ This allows us to strike a balance between the simplicity of the static model presented first and the dynamic features of the DSGE model presented in previous sections, obtaining closed form solutions. As in the baseline model, we study the equilibrium dynamics in the model under various degrees of asset market incompleteness and intensity in financial frictions.

Results indicate that, under some circumstances, adding such new ingredients to the model allow it to generate countercyclicality in government expenditures akin to the empirical evidence presented in the Introduction. Crucially, our main findings are consistent with these new results, in that the cyclicity in government expenditure continues to be determined by the degree of asset market incompleteness and the strength of financial frictions. In the new expanded framework that we consider, while countercyclicality in government expenditure is now optimal in equilibrium in the wake of negative shocks, such result hinges on assuming relatively low levels of financial frictions and market incompleteness. In the presence of stronger financial frictions or the extreme case of financial autarky, public expenditures turn procyclical. Like the previous cases studied, acyclicity in government expenditure is obtained under complete markets.

9.1 Two-period Model with Wage Rigidities

Households. The representative household has preferences over private and public consumption in the two periods:

$$\sum_{t=1}^2 \beta^{t-1} U(c_t, g_t), \quad (56)$$

where, as before, $U(c, g) = (c^{1-1/\sigma_c} - 1)/(1 - 1/\sigma_c) + (g^{1-1/\sigma_g} - 1)/(1 - 1/\sigma_g)$. Private consumption is now assumed to be a composite of tradable (c^T) and nontradable (c^N) goods, with a Cobb-Douglas aggregation technology $c = (c^T)^\gamma (c^N)^{1-\gamma}$.

Following Schmitt-Grohé and Uribe (2016), we assume that the representative house-

³The presence of T-NT goods is required for countercyclicality to be an equilibrium feature. In the presence of negative domestic shocks (e.g. TFP) under a one good model, government expenditure would simply increase the trade deficit.

hold supplies inelastically \bar{h} hours to the labor market in each period. Because of the presence of downward nominal wage rigidity –to be explained below– households may not be able to sell all the hours they supply. As a result, they take employment, $h_t \leq \bar{h}$, as exogenously given.

Our baseline scenario features incomplete asset markets where, as before, the representative household has access to a one-period, internationally traded, state-noncontingent bond denominated in tradables. We let d_t denote the level of debt assumed in period t and due in period $t + 1$, and r_t the interest rate on debt held between t and $t + 1$. The sequential budget constraint of the household in each period is given by

$$P_t^T c_t^T + P_t^N c_t^N + P_t^T d_{t-1}(1 + r) = (1 - \tau)P_t^T Q_t^T + W_t h_t + \Pi_t + P_t^T d_t. \quad (57)$$

where P^T denotes the nominal price of tradable goods, P^N the nominal price of nontradable goods, Q^T the endowment of tradable goods, τ the predetermined tax rate on tradable income, W the nominal wage rate, h hours worked, and Π nominal profits from the ownership of firms that produce nontradable goods. Households are assumed to begin their life without assets or debt ($d_0 = 0$), and satisfy their lifetime budget constraint without engaging in Ponzi schemes or leaving assets at the end of their life ($d_2 = 0$). We assume that the law of one price holds for tradables. Specifically, letting P^{T*} denote the foreign currency price of tradables, the law of one price implies that $P^T = EP^{T*}$ where E is the nominal exchange rate defined as the domestic currency price of one unit of foreign currency. Furthermore, we assume for simplicity that the foreign currency price of tradables is constant and normalized to unity, $P^{T*} = 1$. Thus, we have that the nominal price of tradables equals the nominal exchange rate, $P^T = E$.

Households choose plans for consumption in both periods to maximize (56) subject to (57) for each period. Letting $p = P^T/P^N$ denote the relative price of nontradables in terms of tradables, using the fact that $P^T = E$, and assuming that $\beta(1 + r) = 1$, the optimality conditions associated with this problem are

$$\frac{\gamma}{1-\gamma} \frac{c_t^N}{c_t^T} = \frac{1}{p_t}, \quad (58)$$

$$c_1^T = c_2^T, \quad (59)$$

Firms. Nontraded output (Q^N) is produced by perfectly competitive firms. Each firm operates a production technology that uses labor services as the sole input and is given by $Q^N = h^\alpha$, $0 < \alpha \leq 1$. Profits of firms each period are given by

$$\Pi_t = P_t^N h_t^\alpha - W_t h_t. \quad (60)$$

The optimal choice of employment for a firm is pinned down when the value of the marginal product of labor equates the wage rate, $P^N \alpha h^{-1} = W$. Dividing both sides by P^T , and using the fact that $P^T = E$, yields a supply schedule of nontradable goods

$$p_t h_t^{-1} = W_t / E_t, \quad (61)$$

Wage Rigidity. The central new friction that we add to the analysis is downward nominal wage rigidity (DNWR). As in the seminal contribution of Barro and Grossman (1971) and, more recently, in Schmitt-Grohé and Uribe (2016) and Bianchi et. al (2023), we assume that there exists a minimum wage in nominal terms (\bar{W}) such that

$$W_t \geq \bar{W}. \quad (62)$$

The existence of a minimum wage gives rise to a non-Walrasian labor market where aggregate hours worked are the minimum of labor demand and labor supply, $h_t = \min(\bar{h}, h_t^d)$, and hence the labor market will in general not clear. Involuntary unemployment, given by $\bar{h} - h$, will thus be a regular feature of this economy. Because actual employment must

satisfy $h_t \leq \bar{h}$ at all times, at any point in time, wages and employment must satisfy the slackness condition

$$(W_t - \bar{W})(\bar{h} - h_t) = 0. \quad (63)$$

This condition states that periods of unemployment ($h_t < \bar{h}$) must be accompanied by a binding wage constraint. It also states that when the wage constraint is not binding ($W_t > \bar{W}$), the economy must be in full employment ($h_t = \bar{h}$).

Government. The government sets public spending, which we assume is only in non-tradable goods, and receives the tax revenue on the endowment of tradable goods (τQ^T). The government also has access to the one-period, internationally traded, state-non-contingent bond denominated in tradables. We let d_t^g denote the level of debt assumed by the government in period t and due in period $t + 1$. Its sequential budget constraint is given by

$$P_t^N g_t + P_t^T d_{t-1}^g (1 + r) = \tau P_t^T Q_t^T + P_t^T d_t^g. \quad (64)$$

The government is also assumed to begin without assets or debt ($d_0^g = 0$), and satisfy its lifetime budget constraint without engaging in Ponzi schemes or leaving assets at the end of the second period ($d_2^g = 0$). In terms of monetary policy, we assume that the government follows a fixed exchange rate policy, $\epsilon_t = E_t/E_{t-1} = 1$. In an extension, we will nonetheless also consider the case where some flexibility in the exchange rate is allowed, but not enough as to fully eliminate the slack in the labor market. This leaves fiscal policy as the central instrument of macroeconomic stabilization.

Equilibrium. In equilibrium, the market for nontradable goods must clear at all times:

$$c^N + g = h^\alpha. \quad (65)$$

Combining this condition, the production technology for nontradables, the budget constraint of households, profits of firms, and the intertemporal F.O.C of households, we obtain the equilibrium condition for tradable goods as a function of the stream of tradable endowment and the exogenous interest rate

$$c_1^T = c_2^T = \frac{1+r}{2+r} [Q_1^T + Q_2^T / (1+r)]. \quad (66)$$

Combining the intratemporal F.O.C of the household with the equilibrium conditions in the traded and nontradable goods yields a demand schedule of nontradable goods:

$$\frac{1-\gamma}{\gamma} \frac{c_t^T}{h_t - g_t} = p_t, \quad (67)$$

For a given level of the endowment stream of T-goods and fiscal policy (g_1, g_2) , the supply and demand schedules in (61) and (67) determine the equilibrium in the labor market. This is illustrated in Figure A1, for the simpler case of $\alpha = 1$ where the intersection between the supply schedule of N-goods, defined as a horizontal line along $p = W/E$, and the negatively sloped demand schedule determine the equilibrium level of employment h_t . The figure illustrates the case where the equilibrium A is achieved at full employment and $h_t = \bar{h}$.

Negative Shocks and Fiscal Policy. For a given level of public expenditure, negative shocks to the endowment process of tradable goods will raise equilibrium unemployment. This is illustrated in Figure A1 by considering a fall in Q_1^T . This lowers lifetime income in T-goods which, because of the imperfect substitutability between T and N goods, lowers the demand for N goods through the intratemporal F.O.C (58). In the presence of DNWR,

the fall in demand will translate into a fall of equilibrium employment from \bar{h} to h_1 in point B of the Figure. In the absence of DNWR, wages would fall to the point of restoring full employment.

The fall in employment can be fully or partially offset by countercyclical fiscal policy through an increase in g_1 . The transmission of fiscal policy is the one highlighted in Bianchi et. al (2023). Because public and private consumption goods are imperfect substitutes, the increase in public spending counteracts the fall in demand for nontradables from households thereby reducing the downward pressure on prices. This, in turn, reduces the pressure on firms to lay off labor. This is illustrated in Figure A1 in equilibrium point C , where g_1 increases to partially offset the impact of the fall in Q_1^T on unemployment. The extent to which fiscal policy can act countercyclically in the presence of adverse shocks may be partially or fully constrained by financial frictions and asset market incompleteness. We will quantitatively study this in the next subsections.

Market Incompleteness and Financial Frictions. As in the baseline model, we study also deviations from the incomplete assets market case. In particular we study again the polar cases of financial autarky and complete markets. Under financial autarky, households and the government cannot trade the one-period bond with the rest of the world and can only do so among themselves, implying that $d_1 = -d_1^g$ is a new equilibrium condition and the interest rate is now endogenous. Also the modified F.O.C of households (59), $c_1^T = \beta(1/1 + r)c_2^T$, no longer implies a constant path of consumption. Under complete markets, we modify the model so that the endowment of tradable goods in the first period is stochastic and follows a binomial distribution for the two states of nature, H and L , where $Q_1^T = Q_1^{T,H}$ with probability p , and $Q_1^T = Q_1^{T,L}$ with probability $1 - p$, with $Q_1^{T,L} < Q_1^{T,H}$, and $E[Q_1^T] = 1$. We denote by b_1^H and b_1^L the number of claims purchased by households in period 1 that promise to pay one unit of the good in the second period in state of nature H and L , respectively. The price of such claims is q^H and q^L , respectively. Crucially, we continue to assume that the prices of such state contingent claims are actuarially fair such that $q^H/q^L = p/(1 - p)$.

Within the incomplete markets case that features a one period, non-state contingent

bond, we also document the role of financial frictions akin to those in the DSGE model. Concretely, we consider two dimensions. First, we postulate a debt-elastic interest rate whereby the interest rates that households and the government face includes a spread over the world rate that is determined by $\Psi[\exp(d^g - \bar{D}) - 1]$. Second, we consider a more extreme case where the government cannot issue external debt ($d^g \leq 0$). In the paper we only discuss the more extreme case.

Alternative Policies. This section describes the full array of policies studied. We consider three types of fiscal policies in the presence of negative shocks. A first type of policy does not react to adverse shocks, the level of expenditure in the first period (g_1) is exogenously given, and expenditure in the second period (g_2) is pinned down by the lifetime budget constraint of the government. A second policy sets g_1 in order to minimize unemployment in period 1, in a similar vein to Schmitt-Grohé and Uribe (2016).⁴ Lastly, we consider a Ramsey planner that sets g_1 and g_2 to maximize the utility of households (56) subject to (57)-(65). The combination of these policies with various degrees of financial frictions determine the seven cases studied next.

9.1.1 Results

Table A1 summarizes the results of the full set of cases that we consider. This set includes the ones presented in the main paper as well as additional ones that were excluded for brevity. Throughout the analysis we focus on the following simple calibration. We set $\sigma_c = \sigma_g = 1$, and normalize $\bar{h} = 1$. The world interest rate is set to $r = 0.01$, implying $\beta = 0.99$. We set τ and ϕ equal to $\frac{1}{2}$. As in Bianchi et al. (2023), we consider the linear case where $\alpha = 1$. The gross devaluation rate (ϵ) is set to 1 consistent with a pegged regime, except for Case 3. Lastly, we consider $\Psi = 0$ except in case 5 where we set it to 1.0 in line with the estimated value by Garcia-Cicco et al. (2010) and set $\bar{D} = 0$. The remaining parameters will be described in the cases explained below.

⁴Schmitt-Grohé and Uribe (2016) studied an optimal exchange rate policy where the devaluation is set at every period such that unemployment is null. In our case, we deviate from their work by considering a government that uses fiscal expenditure instead, and is concerned only about period 1 unemployment.

Case 0. No shock. This case serves as our baseline and assumes no shocks to the endowment of tradables such that $Q_1^T = Q_2^T = 1$, and no DNWR exists ($\bar{W} = 0$). Full employment in equilibrium ($h_t = 1$) is consistent with flexible wages. Fiscal policy is set exogenously in the first period and is calibrated so that $g_1/Q_1^T = 0.25$ while g_2 is pinned down so that the lifetime budget constraint of the government is satisfied. Given this, at the (exogenous) level of interest rate, households find it optimal to borrow in the first period in order to smooth out tradable consumption and the government is a net creditor abroad which allows it to increase its second period public consumption.⁵ Because of full employment, higher public expenditure in the second period increases the relative price of N goods, which depresses private consumption of N relative to the previous period.

Case 0. Shock. A large negative shock of -80% is assumed to the first period endowment of T ($Q_1^T = 0.2$) while the second period remains constant ($Q_2^T = 1$). Crucially, fiscal policy in the first period is assumed to stay constant at the same level as in the previous case in the absence of shocks ($g_1 = 1/4$). This shock implies that the government runs a fiscal deficit that needs to be financed with external debt issuance. Households also need to increase their external borrowing to smooth out the shock. Since no DNWR is assumed, equilibrium is reached via a large fall (40%) in real wages and the relative price of N -goods. In turn, this allows to preserve full employment. The shock and the fall in relative prices bring about a fall in GDP, defined as $pQ^N + Q^T$, together with a fall in welfare. The latter is accounted entirely by the fall in T -consumption of households.

Case 1. DNWR and Constant Public Expenditure. The same shock to Q_1^T is assumed but now DNWR is in place, together with a still constant fiscal policy ($g_1 = 1/4$). Wages are assumed downward rigid at the same level as the no-shock equilibrium. Given this, equilibrium is restored via a large increase in unemployment of 30% ($h_1 = 0.7$). This depresses GDP too, which nonetheless falls less than in the previous no-DNWR case, because it is measured in terms of N goods and p_1 is supported by the DNWR, it falls relatively less than in the previous no-DNWR case. The relatively higher relative price of N -goods (compared to the previous case) generates less need to issue debt abroad by households.

⁵The fact that net private debt equals the public assets is a coincidence.

In contrast, this calls for the government to issue more debt to support its consumption of NT goods. DNWR also affects the equilibrium in the second period, where unemployment raises even more to 35%. This is entirely driven by the fact that g_2 needs to fall to satisfy the government's lifetime budget constraint. With wages in the N sector not being able to further adjust, equilibrium needs to be restored via further increases in unemployment. Welfare falls more relative to the previous case and the (Lucas) cost of the business cycle nearly quadruples via a vis that with no DNWR.⁶

Case 2. U_1 Optimal Policy. In this case, g_1 is set so as to minimize unemployment in period 1. This amounts to a *strongly countercyclical policy*, nearly doubling the flow of period-one public consumption relative to the previous cases considered ($g_1 = 0.45$), reducing unemployment from 30% observed with a constant fiscal policy to 11%. To a large extent, this is enabled via the much higher external debt issuance by the government. The trade off of conducting fiscal policy in this manner is that government consumption collapses in the second period to repay the debt incurred in period 1.⁷ This, in turn, largely depresses the N sector in the second period and, in the presence of DNWR, unemployment skyrockets to 55%. Welfare is also greatly reduced, largely driven by the non-linearity driven by the log function and the low levels of g_2 .

Case 3. U_1 Optimal Policy and Max. Devaluation. As in the previous case, g_1 is set to minimize unemployment in period 1. The additional policy lever considered is that fiscal policy is complemented with a devaluation. We assume that there is a maximum devaluation rate of 10% that the central bank can engineer and that, due to non-modelled elements like dollar liabilities, etc. the devaluation cannot surpass that threshold. While the DNWR friction continues to make wages rigid in the same levels as in the previous cases, the devaluation allows relative prices to decouple from wages and drop more. This produces

⁶The Lucas-type cost is computed as in Schmitt-Grohé et.al (2022), by solving for λ in the compensation equation:

$$\ln[(1 + \lambda)c_1^{c^X}] + \beta \ln[(1 + \lambda)c_2^{c^X}] = \ln c_1^{c^0} + \beta \ln c_2^{c^0}, \quad (68)$$

where c^{c^X} is consumption under case X and c^{c^0} is that related to the case without shocks in Q_1^T

⁷We assume g must be strictly positive.

two effects that reinforce one another. First, there is a direct effect in the supply of N goods which provides incentives for firms to employ more labor reducing unemployment. Second, the lower relative price of N goods allows for a stronger countercyclical fiscal response which further reduces unemployment. Both forces combined reduce unemployment to only 2% in the first period. It also allows for unemployment to slightly fall in period 2 relative to the previous case.

Case 4. Ramsey Policy. This case considers setting g_1 in order to maximize lifetime welfare of households (56), subject to the additional constraints and F.O.C. The key consideration here is that, because g_1 and g_2 enter into the computation of welfare, the Ramsey planner will try to smooth the path of public expenditures, moving away from the previous cases where g_1 was considerably increased, to the detriment of g_2 . Now the Ramsey planner actually decides to act procyclically by lowering g_1 relative to the case of a constant g_1 , and setting it equal to g_2 at 0.22. This smooths out unemployment to 33% in both periods. This, in turn, marginally improves welfare relative to Case 1.

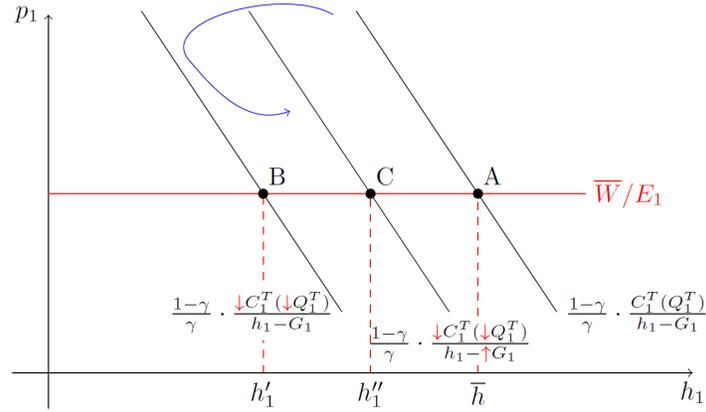
Case 5. U_1 Optimal Policy with Financial Frictions. This case explores the extent to which the countercyclicality of public expenditure when trying to minimize unemployment in the first period is affected by the presence of financial frictions. In the first case considered through a debt elastic interest rate, the fact that households save while the government borrows implies a considerable spread in the rates associated to the two assets with government debt being considerably more expensive than the return that households get for their savings. This somehow limits the ability of government expenditure to act countercyclically to $g_1 = 0.4$, thus unemployment increases to 27% relative to the case without this feature (11%). Still, the countercyclical response continues to be large despite the strong increase in interest rates for public external debt. In contrast, the second more extreme case does deliver a procyclical response as government cannot issue debt and, confronted with the large shock in its revenue, needs to cut its expenditure. This impacts unemployment which increases to 48%. The inability to issue debt, on the other hand, implies that government expenditure can rise in the second period which helps to dampen unemployment to 18%.

This case echoes more closely the result in Bianchi et.al (2023) which delivers a procyclical fiscal response in the presence of sovereign default risk.

Case 6. Financial Autarky. Since both households and the government want to issue debt, much pressure is added onto the (now endogenous) rates, reaching 400%. With such tight constraint, g_1 falls thereby behaving in a procyclical manner. It is, however, slightly less procyclical than in the previous case with no public debt because the government does borrow a minor share from households. This case is most traumatic for unemployment, which reaches 72%, mainly because T consumption falls forcing a drop in N consumption that brings about a big fall in employment amid DNWR and procyclical fiscal policies. In the second period, when the shock dissipates, unemployment recedes to 15%, without the need for fiscal stimulus as the main driver is the strong recovery in T consumption.

Case 7. Completes Markets. This case recovers the same result than in the DSGE: fiscal policy is acyclical.

Figure A1: Nontradable Goods Market with Downward Wage Rigidities



Notes: The plot illustrates the equilibrium in the nontradable goods market in the presence of wage rigidities and a nominal exchange rate peg. Point A is the initial equilibrium. Following a negative income shock in the tradable endowment and no policy reaction, the new equilibrium becomes point B with an increase in unemployment. Point C captures the case where government expenditure reacts in a countercyclical manner in response to the shock.

Table A1: Equilibrium Allocations in the Expanded Model with Downward Wage Rigidities

		Incomplete Asset Markets								Case 6	Case 7
		Baseline Case 0	Case 1	Case 2	Case 3	Case 4	Case 5		- Financial	- Complete	
		No Shock	Shock + No DNWR + Constant G	DNWR - Constant G	U1 - Optimal G	U1 - Optimal G + Deval.	Ramsey G	U1 Optimal G with Fin. Frictions		Autarky	Markets
								DEIR	No Pub. Debt		
1st Period	Q1	1.00	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20
	G1	0.25	0.25	0.25	0.45	0.49	0.22	0.40	0.08	0.13	0.25
	h1	1.00	1.00	0.70	0.89	0.98	0.67	0.73	0.52	0.28	1.00
	W1	1.33	0.80	1.33	1.33	1.33	1.33	1.33	1.33	1.33	1.33
	Wbar	0.00	0.00	1.33	1.33	1.33	1.33	1.33	1.33	1.33	0.00
	p1	1.33	0.80	1.33	1.33	1.21	1.33	1.33	1.33	1.33	1.33
	GDP1	2.33	1.00	1.13	1.39	1.39	1.10	1.17	0.90	0.57	1.53
	Dev1	0.00	0.00	0.00	0.00	10.00	0.00	0.00	0.00	0.00	0.00
	C1T	1.00	0.60	0.60	0.60	0.60	0.60	0.53	0.60	0.20	1.00
	C1N	0.75	0.75	0.45	0.45	0.49	0.45	0.39	0.45	0.15	0.75
	D1 / GDP1	0.07	0.30	0.15	-0.07	-0.07	0.18	-0.02	0.44	-0.13	0.57
D1g / GDP1	-0.07	0.10	0.21	0.35	0.35	0.18	0.30	0.00	0.13	0.23	
R1	1.01	1.01	1.01	1.01	1.01	1.01	0.98/1.43	1.01	5.05	1.01	
2nd Period	Q2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	G2	0.40	0.40	0.20	0.00	0.00	0.22	0.00	0.37	0.10	0.40
	h2	1.00	1.00	0.65	0.45	0.49	0.67	0.40	0.82	0.85	1.00
	W2	1.67	1.00	1.33	1.33	1.33	1.33	1.33	1.33	1.33	1.67
	p2	1.67	1.00	1.33	1.33	1.21	1.33	1.33	1.33	1.33	1.67
	GDP2	2.67	2.00	1.86	1.60	1.60	1.90	1.53	2.10	2.13	2.67
	C2T	1.00	0.60	0.60	0.60	0.60	0.60	0.53	0.60	1.00	1.00
	C2N	0.60	0.60	0.45	0.45	0.49	0.45	0.39	0.45	0.75	0.60
Welfare		-2.69	-3.20	-4.30	-8.70	-8.84	-4.28	-8.20	-4.87	-6.24	-2.69
Lambda (Lucas)		0.00	0.26	0.81	3.02	3.09	0.80	2.77	1.10	1.79	0.00

Note: The Table reproduces the equilibrium allocation for the two period economy with downward wage rigidities. The first (second) panel contains results for the first (second) period. The last two rows present the result in terms of welfare and the Lucas cost of business cycle. Each column represents one of the cases considered. See the main text for further details.