Online Appendix to *The Effect of Mergers on Innovation*

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Appendix C Alternative definition of the informational effect

Consider an alternative definition of the strength of the informational effect in (A.45). Lemma 5* implies that thus defined strength of the informational effect is decreasing in p_0.

Lemma 5*. *Given a prior belief $p_0 > \bar{p}$ and threshold $p^\dagger \in (\underline{p}, \bar{p})$, the strength of the informational effect* (A.45) *is equal to*

$$\frac{1}{\frac{p_0}{1-p_0} - \frac{p^\dagger}{1-p^\dagger}} \int_{p^\dagger}^{\bar{p}} \left(\frac{1}{1-p} - \frac{1}{1-p^\dagger}\right) \frac{1}{2(1-p)p} \left(\frac{1}{x^*(p)} - 1\right) dp. \tag{C.1}$$

Proof. Similar to (A.49), we calculate:

$$\begin{aligned}
\mathbb{E}\left[T \mid T < \hat{T}\right] &= \left(\frac{p(0)}{1-p(0)} - \frac{p(\hat{T})}{1-p(\hat{T})}\right)^{-1} \int_{p(\hat{T})}^{p(0)} \frac{T(p)}{(1-p)^2}\, \mathrm{d}p \\
&\overset{(4)}{=} \left(\frac{p(0)}{1-p(0)} - \frac{p(\hat{T})}{1-p(\hat{T})}\right)^{-1} \int_{p(\hat{T})}^{p(0)} \left(\int_{p}^{p(0)} \frac{\mathrm{d}s}{X(s)(1-s)s}\right) \frac{\mathrm{d}p}{(1-p)^2} \\
&= \left(\frac{p(0)}{1-p(0)} - \frac{p(\hat{T})}{1-p(\hat{T})}\right)^{-1} \int_{p(\hat{T})}^{p(0)} \left(\int_{p(\hat{T})}^{s} \frac{\mathrm{d}p}{(1-p)^2}\right) \frac{\mathrm{d}s}{X(s)(1-s)s} \\
&= \left(\frac{p(0)}{1-p(0)} - \frac{p(\hat{T})}{1-p(\hat{T})}\right)^{-1} \int_{p(\hat{T})}^{p(0)} \left(\frac{1}{1-s} - \frac{1}{1-p(\hat{T})}\right) \frac{\mathrm{d}s}{X(s)(1-s)s},
\end{aligned} \tag{C.2}$$

which implies (C.1). □

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Lemma 6* implies that the statement of Lemma 6 holds for the alternative definition of the strength of the informational effect (A.45), and so, Theorem 5 follows.

Lemma 6*. *Consider any parameter z of the model (except the discount rate r). If the competing firms' equilibrium research intensity $x^*(p)$ increases in z for all $p \in (\underline{p}, \bar{p})$, then for any prior belief $p_0 > \bar{p}$ and threshold $p^\dagger \in (\underline{p}, \bar{p})$, (C.1) decreases in z.*

Proof. The proof is analogous to the proof of Lemma 6. □

Appendix D Comparative statics in the green region

Theorem 6. *Suppose that*

$$c \leq \frac{(\lambda+\phi-1)\pi}{r}. \tag{D.1}$$

Then, the strength of the appropriability effect decreases in λ and increases in ϕ. The strength of the informational effect decreases in λ and in ϕ.

Proof. This case corresponds to case (iii) in Proposition 2 and case (iii)b in Proposition 4. Hence, the difference from case (B) of Theorem 5 is only in the stopping threshold $\check{p}$ for the merged entity — it is now defined in (9). Since the expressions for thresholds $\bar{p}$ and $\underline{p}$ do not change, the comparative statics of the strength of the informational effect is the same as in case (B) of Theorem 5.

The strength of the appropriability effect is given in (A.47). Substituting $\check{p}$ from (9) and $\underline{p}$ from (23) and differentiating (A.47) with respect to λ yields

$$-\frac{1-p_0}{p_0}\cdot\frac{2\pi}{(2+r)rc}\cdot\frac{\underline{p}-\check{p}}{(1-\underline{p})(1-\check{p})}\cdot\left(\frac{\underline{p}}{1-\underline{p}}+\frac{\check{p}}{1-\check{p}}\right)<0, \tag{D.2}$$

while differentiating (A.47) with respect to ϕ yields

$$\frac{1-p_0}{p_0}\cdot\frac{\pi^2}{(2+r)^2r^2c^2}\cdot\left(\underbrace{\left(1-\frac{cr}{\pi}\right)}_{>0 \text{ by (D.1)}}\left(\sqrt{2}-1\right)r+\frac{\sqrt{2}}{3+2\sqrt{2}}\underbrace{\left(\lambda-\frac{cr}{\pi}\right)}_{>0 \text{ by (D.1)}}+\left(2-\sqrt{2}\right)\underbrace{(\lambda-\phi)}_{\geq 0}\right)\cdot\frac{\check{p}\underline{p}}{(1-\check{p})(1-\underline{p})}\cdot\left(\frac{\underline{p}}{1-\underline{p}}+\frac{\check{p}\sqrt{2}}{1-\check{p}}\right)>0. \tag{D.3}$$

□

Appendix E Consumer surplus

In this section, we derive consumer surplus — which is identical to consumer welfare in our parametrization — using the Stackelberg game with differentiated products presented in Appendix B.[1]

[1]The linear demand system in Appendix B is derived from the quadratic utility function:

$$m+Q(q_1+q_2)-\frac{1}{2}\left(q_1^2+2\theta q_1q_2+q_2^2\right), \tag{E.1}$$

E.1 Flow consumer surplus

One innovation

Demand $p(q) = Q - q$ corresponds to consumer surplus

$$Qq - \frac{q^2}{2} - pq \overset{p=Q-q}{=} \frac{q^2}{2}. \tag{E.3}$$

When the firm chooses $q = Q/2$, consumer surplus (E.3) becomes

$$\text{CS}_1 = \frac{Q^2}{8}. \tag{E.4}$$

Two innovations

Demand $p_i(q_1, q_2) = Q - q_i - \theta q_{-i}$ corresponds to consumer surplus

$$Qq_1 - \frac{q_1^2}{2} + Qq_2 - \frac{q_2^2}{2} - \theta q_1 q_2 - p_1 q_1 - p_2 q_2 \overset{p_i=Q-q_i-\theta q_{-i}}{=} \frac{q_1^2 + q_2^2}{2} + \theta q_1 q_2. \tag{E.5}$$

When the firms choose q_1 and q_2 according to (B.2) and (B.3), consumer surplus (E.5) becomes

$$\text{CS}_2 = \left(1 + \frac{(4-3\theta)\theta^3}{8(2-\theta^2)^2}\right)\frac{Q^2}{4}. \tag{E.6}$$

Comparison

Consumers always benefit from the additional innovation:

$$\text{CS}_2 > \text{CS}_1, \tag{E.7}$$

as (E.6) is greater than $Q^2/4$, which in turn is greater than (E.4).

E.2 Cumulative consumer surplus

Consumer surplus for the merged entity

Lemma 9 derives the consumer surplus in the second stage, i.e., after the first innovation arrives.

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where m is the consumption of the outside good with its price normalized to 1. Maximizing this utility subject to the budget constraint $m + p_1 q_1 + p_2 q_2 \leq I$ is equivalent to maximizing

$$Q(q_1 + q_2) - \frac{1}{2}\left(q_1^2 + 2\theta q_1 q_2 + q_2^2\right) + I - p_1 q_1 - p_2 q_2. \tag{E.2}$$

Thus, the optimal consumer demand q_1 and q_2 is independent of income I whenever this income is sufficiently large for an interior optimum. The absence of income effect implies that the consumer surplus is an exact measure of consumer welfare.

Lemma 9. *Suppose that the first innovation has been produced and the firms are merged. Then, the consumer surplus is*

$$CS_{2M} = \frac{CS_1}{r} \quad \text{if (2) does not hold,} \quad CS_{2M} = \frac{CS_1 r + 2CS_2}{r(2+r)} \quad \text{if (2) holds.} \tag{E.8}$$

Proof. If (2) does not hold, then there will be no second innovation, and so consumer surplus is

$$\mathrm{CS}_{2M} = \mathrm{CS}_1 \int_0^{+\infty} e^{-rs}\,\mathrm{d}s = \frac{\mathrm{CS}_1}{r}. \tag{E.9}$$

If (2) holds, then the merged entity invests in research at full intensity $X = 2$ and the instantaneous consumer surplus is

$$\underbrace{\mathrm{CS}_1\,\mathrm{d}t}_{\substack{\text{flow payoff}\\ \text{before 2nd innovation}}} + \underbrace{X\,\mathrm{d}t}_{\text{prob of 2nd innovation}} \times \underbrace{\mathrm{CS}_2 \int_0^{+\infty} e^{-rs}\,\mathrm{d}s}_{\text{payoff after 2nd innovation}} = \left(\mathrm{CS}_1 + X\frac{\mathrm{CS}_2}{r}\right)\mathrm{d}t. \tag{E.10}$$

By analogy with (A.2), the consumers' expected discounted continuation payoff is $(1 - X\,\mathrm{d}t - r\,\mathrm{d}t)\,\mathrm{CS}_{2M}$. Hence, the overall consumer payoff is

$$\mathrm{CS}_{2M} = \left(\mathrm{CS}_1 + X\frac{\mathrm{CS}_2}{r}\right)\mathrm{d}t + (1 - X\,\mathrm{d}t - r\,\mathrm{d}t)\mathrm{CS}_{2M}, \tag{E.11}$$

which implies

$$\mathrm{CS}_{2M} = \frac{\mathrm{CS}_1 r + X\mathrm{CS}_2}{r(X+r)}. \tag{E.12}$$

□

Lemma 10 derives the consumer surplus before the first innovation arrives, for any prior belief. We assume that the prior belief is sufficiently high for research to take place, i.e., the prior belief is above the stopping threshold.

Lemma 10. *Suppose that the firms are merged. Given a prior belief p_0, the overall consumer surplus is*

$$CS_M = 0 \tag{E.13}$$

if (5) holds,

$$CS_M = p_0\left(1 - \left(\frac{\check{p}(1-p_0)}{(1-\check{p})p_0}\right)^{1+r/2}\right)\frac{2CS_1}{r(2+r)} \quad \text{with } \check{p} \text{ defined in (7)} \tag{E.14}$$

if (6) holds,

$$CS_M = p_0\left(1 - \left(\frac{\check{p}(1-p_0)}{(1-\check{p})p_0}\right)^{1+r/2}\right)\frac{2CS_1 r + 4CS_2}{r(2+r)^2} \quad \text{with } \check{p} \text{ defined in (9)} \tag{E.15}$$

if (8) *holds.*

Proof. By analogy with (A.6), the differential equation for the consumer surplus as a function of belief is

$$0 = \left(\mathrm{CS}_{2M} - \mathrm{CS}_M(p) - (1-p)\mathrm{CS}'_M(p)\right) pX(p) - r\mathrm{CS}_M(p). \tag{E.16}$$

If $X(p) = 0$, then $\mathrm{CS}_M(p) = 0$. If $X(p) = 2$, then by analogy with (A.9),

$$\mathrm{CS}_M(p) = p\left(\frac{\mathrm{CS}_{2M}}{1+r/2} + C_M\left(\frac{1-p}{p}\right)^{1+r/2}\right). \tag{E.17}$$

The constant of integration is pinned down by the boundary condition $\mathrm{CS}_M(\check{p}) = 0$, where $\check{p}$ is the merged entity's stopping threshold:

$$\mathrm{CS}_M(p) = p\left(1 - \left(\frac{\check{p}(1-p)}{(1-\check{p})p}\right)^{1+r/2}\right)\frac{\mathrm{CS}_{2M}}{1+r/2}. \tag{E.18}$$

□

Consumer surplus for the competing firms

Lemma 11 derives the consumer surplus in the second stage for the competing firms setup.

Lemma 11. *Suppose that the first innovation has been produced and the firms compete. Then, the consumer surplus is*

$$CS_{2C} = \frac{CS_1}{r} \quad \textit{if } (10) \textit{ holds,} \tag{E.19}$$

$$CS_{2C} = \frac{CS_1 r + CS_2}{r(1+r)} \quad \textit{if } (12) \textit{ holds,} \quad CS_{2C} = \frac{CS_1 r + 2CS_2}{r(2+r)} \quad \textit{if } (14) \textit{ holds.} \tag{E.20}$$

Proof. By analogy with Lemma 9, if (10) holds, then there will be no second innovation, and so consumer surplus is CS_1/r. If (12) holds, then only the follower invests in research at full intensity, and so the overall consumer payoff is equal to (E.12), with $X = 1$. If (14) holds, then both firms invest in research at full intensity, which means that $X = 2$. □

Lemma 12 derives the consumer surplus before the first innovation arrives, assuming that the prior belief is sufficiently high — that is, above threshold $\hat{p}$ for case (17), and above threshold $\bar{p}$ for case (19).

Lemma 12. *Suppose that the firms compete. Given a prior belief p_0, the overall consumer surplus is*

$$CS_C = 0 \tag{E.21}$$

if (16) *holds,*

$$CS_C = p_0\left(1 - \left(\frac{\hat{p}(1-p_0)}{(1-\hat{p})p_0}\right)^{1+r/2}\right)\frac{2CS_1}{r(2+r)} \quad \textit{with } \hat{p} \textit{ defined in } (18) \tag{E.22}$$

if (17) *holds,*

$$CS_C = p_0\left(1+\left(\int_{\underline{p}}^{\bar{p}} \frac{2+r}{2\bar{p}(1-q)}\exp\left(-\int_q^{\bar{p}} \frac{r+2sx^*(s)}{2(1-s)sx^*(s)}\,ds\right)dq-1\right)\left(\frac{\bar{p}(1-p_0)}{(1-\bar{p})p_0}\right)^{1+r/2}\right)$$
$$\times\frac{2CS_1r+2CS_2}{r(1+r)(2+r)} \quad \textit{with } x^* \textit{ defined in } (20),\ \underline{p} \textit{ defined in } (21), \textit{ and } \bar{p} \textit{ defined in } (22) \tag{E.23}$$

if (12) *holds,*

$$CS_C = p_0\left(1+\left(\int_{\underline{p}}^{\bar{p}} \frac{2+r}{2\bar{p}(1-q)}\exp\left(-\int_q^{\bar{p}} \frac{r+2sx^*(s)}{2(1-s)sx^*(s)}\,ds\right)dq-1\right)\left(\frac{\bar{p}(1-p_0)}{(1-\bar{p})p_0}\right)^{1+r/2}\right)$$
$$\times\frac{2CS_1r+4CS_2}{r(2+r)^2} \quad \textit{with } x^* \textit{ defined in } (20),\ \underline{p} \textit{ defined in } (23), \textit{ and } \bar{p} \textit{ defined in } (24) \tag{E.24}$$

if (14) *holds.*

Proof. The proof for cases (16) and (17) is analogous to the proof of Lemma 10.

Suppose that (19) holds. The analog of (E.16) holds here:

$$0=\left(\mathrm{CS}_{2C}-\mathrm{CS}_C(p)-(1-p)\mathrm{CS}'_C(p)\right)pX(p)-r\mathrm{CS}_C(p). \tag{E.25}$$

If $p<\underline{p}$, then $X(p)=0$ and so $\mathrm{CS}_C(p)=0$. If $\underline{p}<p<\bar{p}$, then $X(p)=2x^*(p)$ and so

$$\mathrm{CS}_C(p)=C_2\exp\left(\int_p^1 \frac{r+2sx^*(s)}{2(1-s)sx^*(s)}\,\mathrm{d}s\right)-\mathrm{CS}_{2C}\int_p^1 \frac{1}{1-q}\exp\left(\int_p^q \frac{r+2sx^*(s)}{2(1-s)sx^*(s)}\,\mathrm{d}s\right)\mathrm{d}q. \tag{E.26}$$

The constant of integration is pinned down by the boundary condition $\mathrm{CS}_C(\underline{p})=0$:

$$\mathrm{CS}_C(p)=\mathrm{CS}_{2C}\int_{\underline{p}}^{p} \frac{1}{1-q}\exp\left(-\int_q^p \frac{r+2sx^*(s)}{2(1-s)sx^*(s)}\,\mathrm{d}s\right)\mathrm{d}q. \tag{E.27}$$

If $p>\bar{p}$, then $X(p)=2$ and by analogy with (E.17),

$$\mathrm{CS}_C(p)=p\left(\frac{\mathrm{CS}_{2C}}{1+r/2}+C_1\left(\frac{1-p}{p}\right)^{1+r/2}\right). \tag{E.28}$$

The constant of integration is pinned down by continuity of $\mathrm{CS}_C(p)$ at $p=\bar{p}$:

$$\mathrm{CS}_C(p)=p\left(1+\left(\frac{\mathrm{CS}_C(\bar{p})}{\mathrm{CS}_{2C}}\frac{1+r/2}{\bar{p}}-1\right)\left(\frac{\bar{p}(1-p)}{(1-\bar{p})p}\right)^{1+r/2}\right)\frac{\mathrm{CS}_{2C}}{1+r/2}. \tag{E.29}$$

□

Appendix F Relaxing the additive payoff assumption

F.1 Generalization

In this section we generalize the main model by relaxing the additive payoff assumption. We assume that after the second innovation, the flow payoff of the merged entity is $\Gamma\pi$, where Γ satisfies (31). Moreover, in the competing firms setup, if both innovations come from the same firm, this firm's flow payoff also becomes $\Gamma\pi$.

In the merged entity setup, the payoff generalization necessitates replacing the sum $\lambda + \phi$ in (2), (3), (6), (8) and (9) with Γ in Propositions 1 and 2.

Proposition 2*.

(i) If (5) *holds, then the merged entity does not undertake research.*

(ii) If

$$\frac{(\Gamma-1)\pi}{r} < c < \frac{\pi}{r}, \tag{F.1}$$

then the merged entity undertakes research at full intensity as long as its current belief $p(t)$ is above threshold $\check{p}$ defined in (7). *Once an innovation arrives or the belief falls to $\check{p}$, the merged entity completely aborts research efforts.*

(iii) If

$$c \leq \frac{(\Gamma-1)\pi}{r}, \tag{F.2}$$

then the merged entity undertakes research at full intensity as long as its current belief $p(t)$ is above threshold

$$\check{p} = \frac{cr}{\pi}\frac{2+r}{2\Gamma+r-2rc/\pi}. \tag{F.3}$$

Once the belief falls to $\check{p}$, the merged entity aborts research efforts. If an innovation arrives before that, the merged entity undertakes research at full intensity until the second innovation arrives.

In the competing firms setup, in the second stage, a new case could now emerge. If Γ is sufficiently high, while ϕ is sufficiently low, then only the leader has incentives to invest in the second innovation. More specifically, Proposition 3 becomes

Proposition 3*.

(i) If

$$\max\{\phi, \Gamma-1\}\frac{\pi}{r} < c, \tag{F.4}$$

then both firms abort research after the first innovation and the second innovation never arrives. The potential follower's and the leader's expected payoffs are given in (11).

(ii) If

$$\frac{\phi\pi}{r} < c \leq \frac{(\Gamma-1)\pi}{r}, \tag{F.5}$$

then the leader undertakes research at full intensity until the second innovation arrives, while the other firm aborts research after the leader produced the first innovation. The firms' expected payoffs *are*

$$V_F = 0, \quad V_L = \frac{\pi}{r} + \frac{1}{1+r}\left(\frac{(\Gamma-1)\pi}{r} - c\right). \tag{F.6}$$

(iii) If

$$\frac{\pi}{r}\left(\Gamma - 1 + \frac{1-\lambda}{1+r}\right) < c \leq \frac{\phi\pi}{r}, \tag{F.7}$$

then the follower undertakes research at full intensity until the second innovation arrives, while the leader aborts research after producing the first innovation. The potential follower's and the leader's expected payoffs are given in (13).

(iv) If

$$c \leq \min\left\{\phi, \Gamma - 1 + \frac{1-\lambda}{1+r}\right\}\frac{\pi}{r}, \tag{F.8}$$

then both the leader and the follower undertake research at full intensity until the second innovation arrives. The potential follower's and the leader's expected payoffs are

$$V_F = \frac{1}{2+r}\left(\frac{\phi\pi}{r} - c\right), \quad V_L = \frac{\lambda+r}{1+r}\frac{\pi}{r} + \frac{1}{2+r}\left(\frac{\pi}{r}\left(\Gamma - 1 + \frac{1-\lambda}{1+r}\right) - c\right). \tag{F.9}$$

Consequently, Proposition 4 becomes

Proposition 4*.

(i) If (16) *holds, then neither firm undertakes research.*

(ii) If

$$\max\{\phi, \Gamma-1\}\frac{\pi}{r} < c < \frac{\pi}{r}, \tag{F.10}$$

then the firms undertake research at full intensity as long as their current belief $p(t)$ is above threshold $\hat{p}$ defined in (18). *Once an innovation arrives or the belief falls to $\hat{p}$, the firms completely abort research efforts.*

(iii) If

$$\frac{\phi\pi}{r} < c \leq \frac{(\Gamma-1)\pi}{r}, \tag{F.11}$$

then the firms undertake research at full intensity as long as their current belief $p(t)$ is above threshold

$$\hat{p} = \frac{cr}{\pi}\frac{1+r}{\Gamma + r - rc/\pi}. \tag{F.12}$$

Once the belief falls to $\hat{p}$, the firms completely abort research efforts. Once an innovation arrives, only the firm that produced the first innovation undertakes research at full intensity until the second innovation arrives.

(iv) If (19) *holds, then the firms undertake research at full intensity as long as their current belief $p(t)$ is above threshold $\bar{p}$. Once the belief falls to $\bar{p}$, the firms undertake research at intensity $x_1(t) = x_2(t) = x^*(p(t))$, defined on $p \in (\underline{p}, \bar{p})$ as in* (20). *This intensity decreases over time, from 1 at $p(t) = \bar{p}$ to 0 at $t \to +\infty$. In the absence of innovation, the belief approaches threshold $\underline{p}$ at $t \to +\infty$.*

(a) If (F.7) *holds, then once an innovation arrives, only the firm that did not innovate undertakes research at full intensity until the second innovation arrives. Threshold* $\underline{p}$ *is defined in* (21)*. Threshold* $\bar{p} \in [\underline{p}, 1)$ *is defined as a unique solution to* (22)*.*

(b) If (F.8) *holds, then once an innovation arrives, both firms undertake research at full intensity until the second innovation arrives. Threshold* $\underline{p}$ *is defined as*

$$\underline{p} = \frac{cr}{\pi}\left(\frac{\lambda + r}{1+r} + \frac{1}{2+r}\left(\Gamma - 1 + \frac{1-\lambda}{1+r} - \frac{cr}{\pi}\right)\right)^{-1}. \tag{F.13}$$

Threshold $\bar{p} \in [\underline{p}, 1)$ *is defined as a unique solution to* (24)*.*

Then, Theorems 1, 2 and 3 become

Theorem 1* (The merger has no impact on innovations)**.** *If*

$$\max\{\phi, \Gamma - 1\}\frac{\pi}{r} < c, \tag{F.14}$$

then the merger has no impact on the number and timing of innovations.

Theorem 2* (The merger has a positive impact on innovations)**.** *If*

$$c < \frac{(\Gamma - 1)\pi}{r}, \tag{F.15}$$

then the merger has an unambiguously positive impact: while it does not block the second innovation, it increases the probability that the first innovation arrives. Moreover, if

$$\frac{\phi\pi}{r} < c < \frac{(\Gamma - 1)\pi}{r}, \tag{F.16}$$

then the merger brings the second innovation forward in time, whereas if

$$c < \min\{\phi, \Gamma - 1\}\frac{\pi}{r}, \tag{F.17}$$

then the merger brings the first innovation forward in time.

Theorem 2* implies that the appropriability effect can operate alone, without the informational effect. This occurs if (F.11) holds, so that in the competing firms setup, the second innovation always comes from the same firm that produced the first innovation. In this case, although competition prompts firms to abort research earlier in the first stage, it does not result in free-riding, since the discovery that the research avenue is good is of no use to the potential follower, who finds it suboptimal to continue research after the first innovation arrives.[2] Moreover, since one of the competing firms does not undertake R&D, it takes the competing firms longer to produce the second innovation, compared to the merged entity which utilizes all its research intensity towards producing the second innovation.

[2] If (F.11) holds, then the merger increases the probability of the first innovation because threshold $\hat{p}$ defined in (F.12) is higher than threshold $\check{p}$ defined in (F.3): $\hat{p} - \check{p} = \frac{cr^2}{\pi}\frac{\Gamma - 1 - cr/\pi}{(\Gamma + r - rc/\pi)(2\Gamma + r - 2rc/\pi)} > 0$.

Theorem 3* (The merger has an ambiguous impact on innovations). *If*

$$\frac{(\Gamma-1)\pi}{r} < c < \frac{\phi\pi}{r}, \tag{F.18}$$

then the merger blocks the second innovation but increases the probability that the first innovation arrives and, moreover, brings it forward in time.

Theorem 7 analyzes how the appropriability and informational effects change with Γ.

Theorem 7. *As Γ increases, the appropriability effect tends to strengthen if the merged entity introduces the second innovation after the first, but may weaken if the merger blocks the second innovation:*

(i) If (F.16) holds, then the strength of the appropriability effect increases in Γ.

(ii) If (F.17) holds and[3]

$$\Gamma \leq 2\lambda + \left(\sqrt{2}-1\right)\phi, \tag{F.19}$$

then the strength of the appropriability effect increases in Γ.

(iii) If (F.18) holds, then the strength of the appropriability effect

(a) is independent of Γ if (F.7) holds;

(b) decreases in Γ if (F.8) holds.

Moreover, whenever present, the informational effect either weakens or is unaffected by Γ:

(i) If (F.7) holds, then the strength of the informational effect is independent of Γ.

(ii) If (F.8) holds, then the strength of the informational effect decreases in Γ.

Proof. Suppose that (F.16) holds. Then, the strength of the appropriability effect is given by (A.47) with the competing firms' stopping threshold $\underline{p} = \hat{p}$ defined in (F.12) and the merged entity's stopping threshold $\check{p}$ defined in (F.3). Differentiating this expression with respect to Γ yields

$$\frac{1-p_0}{p_0}\cdot\left(\frac{\hat{p}}{1-\hat{p}}\cdot\frac{\check{p}}{1-\check{p}}\right)^2\cdot\left(\frac{\pi(3+r)}{cr(1+r)^2(2+r)^2}\left(\frac{(\Gamma-1)\pi}{c}-r\right)^2\right.$$
$$\left.+\frac{\pi^3}{c^3r^2(1+r)(2+r)}\left(2\underbrace{\left(\Gamma-1-\frac{cr}{\pi}\right)}_{>0\text{ by (F.16)}}\overbrace{(2-\Gamma)}^{\geq 0\text{ by (31)}}+(2-\Gamma)^2\right)\right)>0. \tag{F.20}$$

Suppose that (F.17) holds. Then, the strength of the appropriability effect is given by (A.47) with the competing firms' stopping threshold $\underline{p}$ defined in (F.13) and the merged entity's stopping threshold $\check{p}$ defined in (F.3). Differentiating this expression with respect to Γ yields

[3]Condition (F.19) is rather mild and, for instance, holds for the parametrization from Appendix F.2 — see (F.30).

$$\frac{1-p_0}{p_0}\cdot\left(\underbrace{\left(1-\frac{cr}{\pi}\right)}_{>0 \text{ by (F.17)}}\left(\sqrt{2}-1\right)r+\frac{\sqrt{2}}{3+2\sqrt{2}}\underbrace{\left(\phi-\frac{cr}{\pi}\right)}_{>0 \text{ by (F.17)}}+\left(2-\sqrt{2}\right)\left(2\lambda+(\sqrt{2}-1)\phi-\Gamma\right)\right)$$
$$\cdot\frac{\pi^2}{(2+r)^2r^2c^2}\cdot\frac{\check{p}\underline{p}}{(1-\check{p})(1-\underline{p})}\cdot\left(\frac{\underline{p}}{1-\underline{p}}+\frac{\check{p}\sqrt{2}}{1-\check{p}}\right), \quad \text{(F.21)}$$

which is positive if (F.19) holds.

Suppose that (F.18) holds. Then, the merged entity's stopping threshold $\check{p}$ is defined in (7) and thus independent of Γ. If (F.7) holds, then the competing firms' stopping threshold $\underline{p}$ is defined in (21) and thus also independent of Γ. Hence, if (F.7) holds, the strength of the appropriability effect is independent of Γ. However, if (F.8) holds, then the competing firms' stopping threshold $\underline{p}$ is defined in (F.13), which is decreasing in Γ. Hence, if (F.8) holds, the strength of the appropriability effect decreases in Γ.

By Lemma 6, the comparative statics of the informational effect move in the opposite direction to that of the equilibrium intensity $x^*(p)$ on the free-riding region $(\underline{p}, \bar{p})$. If (F.7) holds, then thresholds $\underline{p}$ and $\bar{p}$ are given in (21) and (22), respectively. In this case, the threshold — and therefore $x^*(p)$ defined in (20) as well — are independent of Γ. Thus, if (F.7) holds, the strength of the informational effect is independent of Γ. In contrast, if (F.8) holds, then thresholds $\underline{p}$ and $\bar{p}$, defined in (F.13) and (24), vary with Γ. By Lemma 13, the equilibrium intensity $x^*(p)$ increases in Γ, implying that the strength of the informational effect decreases in Γ.

Lemma 13. *Given thresholds $\underline{p}$ defined in* (F.13) *and $\bar{p}$ defined in* (24)*, the equilibrium research intensity $x^*(p)$ defined in* (20) *increases in Γ.*

Proof. By (A.50) and (A.51), the equilibrium research intensity $x^*(p)$ defined in (20) decreases in $\bar{p}$ and in $\underline{p}$.

Clearly, threshold $\underline{p}$ defined in (F.13) decreases in Γ.

Threshold $\bar{p}$ defined in (24) also decreases in Γ. Indeed, the right-hand side of (24) is independent of both Γ and $\bar{p}$. The left-hand side of (24) increases in $\bar{p}$ by (A.52). Moreover, the left-hand side of (24) depends on Γ only through $\underline{p}$, which, by (F.13), decreases in Γ. Thus, since by (A.53) the left-hand side of (24) decreases in $\underline{p}$, it increases in Γ. □

□

If the merged entity introduces the second innovation after the first, then an increase in Γ provides it with additional incentives to pursue the first innovation because a higher Γ increases the profit from common ownership of both innovations. While an increase in Γ has a similar impact on the competing firms, this impact is weaker than on the merged entity because even in the absence of competition, producing the second innovation takes longer time and hence is more costly for a single firm, which has only one unit of research intensity, than for the merged entity, which has two units of research intensity. Thus, as Theorem 7 states, as Γ increases, the merger brings about an ever higher increase in the probability of the first innovation.

However, if the merger blocks the second innovation, an increase in Γ has no impact on the merged entity's profit. At the same time, a higher Γ increases the leader's profit from common ownership of both innovations. Hence, if the leader undertakes research to produce the second innovation — that is, if (F.8) holds — as Γ increases, the competing firms produce the first innovation with ever higher probability, while that probability for the merged entity remains unchanged. Thus, as Theorem 7 states, the appropriability affect weakens with Γ.

The informational effect is present only if the potential follower, armed with the knowledge that the research avenue is good, chooses to pursue the second innovation. If the leader does not compete for the second innovation — i.e., if (F.7) holds — then no firm ever acquires both innovations, and thus Γ has no impact on the competing firms' incentives to free-ride. However, if both firms pursue the second innovation — i.e., if (F.8) holds — then a higher Γ increases the leader's payoff from the second innovations without affecting the follower's. Hence, a higher Γ makes the leader's role more attractive, thereby mitigating free-riding and, as Theorem 7 states, weakening the informational effect.

F.2 Microfoundations for Γ

In this section, we extend Appendix B to express Γ as a function of the degree of substitutability between the first and second innovations.

Two innovations from the same firm

When both products belong to a single firm facing demand $p_i(q_1,q_2) = Q - q_i - \theta q_{-i}$, this firm maximizes

$$\max_{\substack{q_1\geq 0\\ q_2\geq 0}} q_1 p_1(q_1,q_2) + q_2 p_2(q_1,q_2) = q_1\left(Q - q_1 - \theta q_2\right) + q_2\left(Q - q_2 - \theta q_1\right) \tag{F.22}$$

and thus chooses

$$q_1 = q_2 = \frac{Q}{2(1+\theta)}. \tag{F.23}$$

Hence, the firm's payoff is

$$\Gamma\pi = \frac{2}{1+\theta}\frac{Q^2}{4}. \tag{F.24}$$

Interpretation of parameters

Since by (B.1) $\pi = Q^2/4$, (F.24) implies that

$$\Gamma(\theta) = \frac{2}{1+\theta}. \tag{F.25}$$

When products are independent ($\theta = 0$), the single ownership of both innovations does not provide any benefit and the profit equals twice the monopoly profit from each individual product:

$$\Gamma(0) = \lambda(0) + \phi(0) = 2. \tag{F.26}$$

As the degree of substitutability between the products, θ, increases, the profit from owning both innovations decreases but less so than the joint profit of the competing firms:

$$\lambda'(\theta) + \phi'(\theta) = -\frac{2}{(1+\theta)^2} - \frac{\theta\left((1-\theta^2)(6-2\theta+\theta^3)+2\right)}{(1+\theta)^2(2-\theta^2)^3} < -\frac{2}{(1+\theta)^2} = \Gamma'(\theta) < 0. \tag{F.27}$$

Results (F.26) and (F.27) justify restriction (31).

It is intuitive that for perfect substitutes ($\theta = 1$), adding the second product does not change the firm's profit:

$$\Gamma(1) = 1. \tag{F.28}$$

Note also that for this parametrization,

$$\phi(\theta) - (\Gamma(\theta) - 1) = \frac{\left(14 - \theta^2\right)(1-\theta)\theta + 5\theta^2(2+\theta)(1-\theta)^2 + 2\theta}{4\left(2-\theta^2\right)^2(1+\theta)} > 0 \tag{F.29}$$

and

$$\begin{aligned} 2\lambda(\theta) + \left(\sqrt{2}-1\right)\phi(\theta) - \Gamma(\theta) = &\frac{\left(\sqrt{2}-1\right)\left(2 + 20\theta(1-\theta)^2 + 10\sqrt{2}\theta(1-\theta)^3 + \left(2+3\sqrt{2}\right)(1-\theta)^5\right)}{4\left(2-\theta^2\right)^2(1+\theta)} \\ &+ \frac{1-\theta}{4\left(2-\theta^2\right)^2(1+\theta)}\left(1 + \frac{10}{3\sqrt{2}+4} + 6\theta(1-\theta)^2 + (1-\theta)^4\right) > 0. \end{aligned} \tag{F.30}$$

F.3 Consumer surplus

In this section, we extend Appendix E to derive consumer surplus in the presence of price effects.

Flow consumer surplus in the presence of two innovations

We differentiate two cases, when innovations come from different firms and when they come from the same firm. To reflect the difference, we reserve notation CS_{2c} for the flow consumer surplus in the first case and CS_{2m} for the second case. Surplus CS_2 derived in (E.6) corresponds to the first case:

$$\mathrm{CS}_{2c} = \mathrm{CS}_2. \tag{F.31}$$

For the second case, substituting (F.23) into consumer surplus (E.5) yields

$$\mathrm{CS}_{2m} = \frac{Q^2}{4(1+\theta)}. \tag{F.32}$$

Comparison

If products are independent ($\theta = 0$), then

$$\mathrm{CS}_{2c}(0) = \mathrm{CS}_{2m}(0) = \frac{Q^2}{4} = 2\mathrm{CS}_1, \tag{F.33}$$

so that each innovation brings the same surplus, irrespective of whether they come from the same or different firms.

As θ increases, the second innovation brings less surplus if it comes from the same firm as the first innovation:

$$\mathrm{CS}_{2m}'(\theta) = -\frac{Q^2}{4(1+\theta)^2} < 0. \tag{F.34}$$

In contrast, if innovations come from different firms, competitive price reduction ensures that the second innovation brings more surplus as θ increases:

$$\mathrm{CS}'_{2c}(\theta) = \frac{\theta^2\left(6(1-\theta)+\theta^2\right)Q^2}{8(2-\theta^2)^3} > 0. \tag{F.35}$$

If products are perfect substitutes ($\theta = 1$), then the second innovation brings no surplus if it comes from the same firm as the first innovation:

$$\mathrm{CS}_{2m}(1) = \frac{Q^2}{8} = \mathrm{CS}_1. \tag{F.36}$$

Results (F.33), (F.34), (F.35) and (F.36) imply that for any $\theta \in (0,1)$ consumers benefit from more innovations, and more so if these innovations come from different firms:

$$\mathrm{CS}_{2c} > \mathrm{CS}_{2m} > \mathrm{CS}_1. \tag{F.37}$$

Consumer surplus for the merged entity

In light of Proposition 2*, since the flow consumer surplus with two innovations from the merged entity is equal to CS_{2m}, Lemma 10 generalizes as follows.

Lemma 10*. *Suppose that the firms are merged. Given a prior belief p_0, the overall consumer surplus is* (E.13) *if* (5) *holds,* (E.14) *if* (F.1) *holds, and*

$$CS_M = p_0\left(1-\left(\frac{\check{p}(1-p_0)}{(1-\check{p})p_0}\right)^{1+r/2}\right)\frac{2CS_1 r + 4CS_{2m}}{r(2+r)^2} \quad \text{with } \check{p} \text{ defined in (F.3)} \tag{F.38}$$

if (F.2) *holds.*

Consumer surplus for the competing firms

Lemma 11 is generalizes as follows.

Lemma 11*. *Suppose that the first innovation has been produced and the firms compete. Then, the consumer surplus is*

$$CS_{2C} = \frac{CS_1}{r} \quad \text{if (F.4) holds,} \quad CS_{2C} = \frac{CS_1 r + CS_{2m}}{r(1+r)} \quad \text{if (F.5) holds,} \tag{F.39}$$

$$CS_{2C} = \frac{CS_1 r + CS_{2c}}{r(1+r)} \quad \text{if (F.7) holds,} \quad CS_{2C} = \frac{CS_1 r + CS_{2m} + CS_{2c}}{r(2+r)} \quad \text{if (F.8) holds.} \tag{F.40}$$

Proof. By analogy with Lemma 9, if (F.4) holds, then there will be no second innovation, and so consumer surplus is CS_1/r. If (F.5) holds, then only the leader invests in research at full intensity, and so the overall consumer payoff is equal to (E.12), with $X = 1$ and $\mathrm{CS}_2 = \mathrm{CS}_{2m}$. If (F.7) holds, then the second innovation comes from the other firm, which means that CS_2 has to be replaced with CS_{2c}. If (F.8) holds, then both firms invest in research at full intensity, which means the overall consumer payoff is equal to (E.12), with $X = 2$ and CS_2 replaced with $(\text{CS}_{2m} + \text{CS}_{2c})/2$ since the second innovation may come from either firm with equal probabilities. □

As in Lemma 12, in Lemma 12* assumes that the prior belief is sufficiently high — that is, above threshold $\hat{p}$ for cases (F.10) and (F.11), and above threshold $\bar{p}$ for case (19).

Lemma 12*. *Suppose that the firms compete. Given a prior belief p_0, the overall consumer surplus is* (E.21) *if* (16) *holds,* (E.22) *if* (F.10) *holds,*

$$CS_C = p_0\left(1-\left(\frac{\hat{p}(1-p_0)}{(1-\hat{p})p_0}\right)^{1+r/2}\right)\frac{2CS_1 r + 2CS_{2m}}{r(1+r)(2+r)} \quad \text{with } \hat{p} \text{ defined in (F.12)} \tag{F.41}$$

if (F.11) *holds,*

$$CS_C = p_0\left(1+\left(\int_{\underline{p}}^{\bar{p}} \frac{2+r}{2\bar{p}(1-q)}\exp\left(-\int_{q}^{\bar{p}}\frac{r+2sx^*(s)}{2(1-s)sx^*(s)}\,ds\right)dq - 1\right)\left(\frac{\bar{p}(1-p_0)}{(1-\bar{p})p_0}\right)^{1+r/2}\right)$$
$$\times\frac{2CS_1 r + 2CS_{2c}}{r(1+r)(2+r)} \quad \text{with } x^* \text{ defined in (20), } \underline{p} \text{ defined in (21), and } \bar{p} \text{ defined in (22)} \tag{F.42}$$

if (F.7) *holds,*

$$CS_C = p_0\left(1+\left(\int_{\underline{p}}^{\bar{p}} \frac{2+r}{2\bar{p}(1-q)}\exp\left(-\int_{q}^{\bar{p}}\frac{r+2sx^*(s)}{2(1-s)sx^*(s)}\,ds\right)dq - 1\right)\left(\frac{\bar{p}(1-p_0)}{(1-\bar{p})p_0}\right)^{1+r/2}\right)$$
$$\times\frac{2CS_1 r + 2CS_{2m} + 2CS_{2c}}{r(2+r)^2} \quad \text{with } x^* \text{ defined in (20), } \underline{p} \text{ defined in (F.13), and } \bar{p} \text{ defined in (24)} \tag{F.43}$$

if (F.8) *holds.*

Appendix G Correlated avenues

G.1 Setup

So far, we have assumed that there is only one method of conducting research — that is, a single research avenue. In this section, we extend our model to allow for two research avenues. Each firm has access to one avenue only and is restricted to conducting research along that avenue — i.e., firm i chooses intensity x_i to allocate to avenue i. This assumption reflects technological specialization and sunk investment in complementary assets, which constrain firms' ability to flexibly reallocate R&D efforts across different research methods.

Each research avenue is either *good* (capable of producing innovations) or *bad* (incapable of producing innovations), and it is possible that both of them are good:

$$\Pr\begin{pmatrix}\text{both avenues}\\ \text{are good}\end{pmatrix} > 0, \quad \Pr\begin{pmatrix}\text{only avenue 1}\\ \text{is good}\end{pmatrix} > 0, \quad \Pr\begin{pmatrix}\text{only avenue 2}\\ \text{is good}\end{pmatrix} > 0. \tag{G.1}$$

Let p_i denote the firms' common belief that avenue i is good, that is,

$$p_i = \Pr\begin{pmatrix}\text{only avenue } i\\ \text{is good}\end{pmatrix} + \Pr\begin{pmatrix}\text{both avenues}\\ \text{are good}\end{pmatrix}. \tag{G.2}$$

To capture the correlation between the avenues, we denote

$$\xi = \frac{\Pr(\text{both avenues are good}) - p_1 p_2}{\Pr(\text{only avenue 1 is good}) \cdot \Pr(\text{only avenue 2 is good})}. \tag{G.3}$$

By assumption (G.1), variable ξ is well-defined and must be greater than or equal to -1. The sign of ξ coincides with the sign of the correlation between the avenues. Using ξ is handy because, in contrast to the correlation, as shown in Mayskaya and Nikandrova (2024), prior to the arrival of the first innovation, the expression for ξ in (G.3) remains constant regardless of the R&D investment strategy of the firms.[4]

As in the single-avenue benchmark model, we assume that there can be, at most, two sequential innovations in the market. The payoff assumptions also remain unchanged.

G.2 Merged entity

Stage 2: After the first innovation

Without loss of generality, suppose that the first innovation was produced by avenue 1.

Proposition 5. *If (2) holds, then merged entity continues research along both avenues at full intensity until either the second innovation arrives or the merged entity's belief p_2 falls to*

$$\hat{\hat{p}} = \frac{c(1+r)}{c + (\lambda + \phi - 1)\pi}. \tag{G.4}$$

For beliefs $p_2 \leq \hat{\hat{p}}$, the merged entity undertakes research only along the first avenue until the second innovation arrives. If (2) does not hold, then the merged entity immediately aborts research after producing the first innovation. The merged entity's expected payoff from the second stage is

$$V_M(p_2) = \frac{\pi}{r} + \frac{2}{2+r}\left(\frac{(\lambda+\phi-1)\pi}{r} - c\right) - \frac{c(1-p_2)}{(1+r)(2+r)}\left(\frac{(\lambda+\phi-1)\pi}{c} + 2 - \left(\frac{\hat{\hat{p}}(1-p_2)}{p_2(1-\hat{\hat{p}})}\right)^{r+1}\right) \tag{G.5}$$

[4]In contrast to ξ, the correlation between the avenues, $\frac{\Pr(\text{both avenues are good}) - p_1 p_2}{\sqrt{p_1(1-p_1)p_2(1-p_2)}}$, varies as the firms undertake research to get the first innovation. In particular, as cumulative investment in each avenue grows, they eventually become so pessimistic about both avenues that the correlation gets close to 0.

if (2) *holds and* $p_2 > \hat{\hat{p}}$, *and*

$$V_M(p_2) = \frac{\pi}{r} + \frac{1}{1+r}\max\left\{\frac{(\lambda+\phi-1)\pi}{r} - c, 0\right\} \tag{G.6}$$

otherwise.

Proof. See Appendix G.5.1. □

Stage 1: Before the first innovation

For the sake of simplicity, we focus on the symmetric prior: $p_1 = p_2 \equiv p$. As in the one-avenue benchmark model, the stopping threshold is equal to c/V_M. If (2) does not hold, then $V_M = \pi/r$, and so, the solution coincides with the one-avenue case. However, if (2) holds, V_M depends on the stopping threshold through the posterior belief after the arrival of the first innovation:

$$h \equiv \Pr(\text{avenue } i \text{ is good} \mid \text{avenue } j \text{ is good}) = \frac{p_{\text{both}}}{p}, \tag{G.7}$$

where p_{both} is the probability that both avenues are good. For symmetric beliefs, the expression (G.3) for ξ becomes

$$\xi = \frac{p_{\text{both}} - p^2}{(p - p_{\text{both}})^2}. \tag{G.8}$$

Solving (G.7) and (G.8) for p_{both} and p yields

$$p = \frac{h}{1 + (1-h)^2\,\xi}. \tag{G.9}$$

Hence, the conditional belief which corresponds to the stopping threshold solves

$$c = \frac{\check{h}}{1 + \left(1-\check{h}\right)^2\xi} V_M\left(\check{h}\right). \tag{G.10}$$

As shown in Appendix G.5.1 in the proof of Proposition 5, function $V_M(h)$ is weakly increasing. The ratio (G.9) is also increasing:

$$\frac{\partial}{\partial h}\left(\frac{h}{1+(1-h)^2\,\xi}\right) = \frac{h^2 + (1+\xi)\left(1-h^2\right)}{\left(1+(1-h)^2\,\xi\right)^2} > 0. \tag{G.11}$$

Hence, the right-hand side of (G.10) is increasing in $\check{h} \in (0,1)$ from 0 to $V_M(1)$. When (2) holds — or equivalently, when (8) holds — $V_M(1)$ is greater than c, which means that (G.10) has a unique solution.

Proposition 6.

(i) If (5) *holds, then the merged entity does not undertake research.*

(ii) *If* (6) *holds, then the merged entity undertakes research at full intensity along both avenues as long as its current belief* $p(t)$ *is above threshold* $\check{p}$ *defined in* (7)*. Once an innovation arrives or the belief falls to* $\check{p}$*, the merged entity completely aborts research efforts.*

(iii) *If* (8) *holds, then the merged entity undertakes research at full intensity along both avenues as long as its current belief* $p(t)$ *is above threshold*

$$\check{p} = \frac{c}{V_M\left(\check{h}\right)}, \tag{G.12}$$

where $\check{h} \in (0,1)$ *is defined as a unique solution to* (G.10)*. Once the belief falls to* $\check{p}$*, the merged entity aborts research efforts. If an innovation arrives before that, the merged entity undertakes research at full intensity along both avenues until either the second innovation arrives, or the belief about the uncertain avenue drops to* $\mathring{\check{p}}$ *defined in* (G.4)*. In the latter case, the merged entity continues research along the avenue that produced the first innovation until the second innovation arrives.*

Proof. See Appendix G.5.2. □

Formula (G.12) hides two cases, $\check{h} < \mathring{\check{p}}$ and $\check{h} > \mathring{\check{p}}$. If $\check{h} < \mathring{\check{p}}$, then $V_M\left(\check{h}\right)$ is constant and defined in (G.6). In this case, (G.12) gives an explicit expression for threshold $\check{p}$, and this expression does not depend on ξ. If $\check{h} > \mathring{\check{p}}$, then $V_M\left(\check{h}\right)$ is increasing in $\check{h}$ and defined in (G.5). In this case, $\check{p}$ defined in (G.12) varies with ξ together with $\check{h}$. Lemma 14 derives the condition, which differentiates these two cases.

Lemma 14. *Suppose that* (8) *holds. Then, if*

$$\xi \leq \frac{((\lambda+\phi-1)\pi+c)((2-\lambda-\phi)\pi(1+r)+2((\lambda+\phi-1)\pi-cr))}{r((\lambda+\phi-1)\pi-cr)^2}, \tag{G.13}$$

$\check{p}$ *defined in* (G.12) *is equal to*

$$\check{p} = \frac{cr}{\pi}\frac{1+r}{\lambda+\phi+r-rc/\pi}. \tag{G.14}$$

If (G.13) *does not hold, then* $\check{p}$ *defined in* (G.12) *is decreasing in* ξ *and equal to* (9) *at the limit* $\xi \to +\infty$ *where the research avenues are perfectly positively correlated — i.e., as in our benchmark model.*

Proof. See Appendix G.5.3. □

G.3 Competing firms

Stage 2: After the first innovation

Proposition 7.

(i) *If* (10) *holds, then both firms abort research after the first innovation and the second innovation never arrives. The potential follower's and the leader's expected payoffs are given in* (11)*.*

(ii) If (12) holds, then, while the leader aborts research immediately after producing the first innovation, the follower undertakes research at full intensity until either the second innovation arrives or the firms' belief p_2 falls to

$$p_F = \frac{cr}{\phi\pi}; \tag{G.15}$$

The follower's and the leader's expected payoffs are

$$V_F(p_2) = \frac{1}{1+r}\left(p_2\frac{\phi\pi}{r} - c\right) - \frac{c(1-p_2)}{r(1+r)}\left(1 - \left(\frac{p_F(1-p_2)}{p_2(1-p_F)}\right)^r\right), \tag{G.16}$$

$$V_L(p_2) = \frac{\pi}{r} - \frac{(1-\lambda)\pi}{r(1+r)}p_2\left(1 - \left(\frac{p_F(1-p_2)}{p_2(1-p_F)}\right)^{r+1}\right). \tag{G.17}$$

if $p_2 > p_F$ and are given in (11) for $p_2 < p_F$.

(iii) If

$$\frac{(\lambda+\phi-1)\pi}{r} < c \le \frac{\pi}{r}\left(\phi - \frac{(1-\lambda)r}{1+r}\right), \tag{G.18}$$

then both the leader and the follower undertake research at full intensity until either the second innovation arrives or the belief p_2 falls to p_L; starting from belief p_L, only the follower continues research and aborts it at belief p_F. Threshold p_F is defined in (G.15); threshold $p_L > p_F$ is defined as a unique solution to

$$\frac{p_L}{1+r}\left(1 - \left(\frac{p_F(1-p_L)}{p_L(1-p_F)}\right)^{r+1}\right) = \frac{r}{1-\lambda}\left(\frac{c}{\pi} - \frac{\lambda+\phi-1}{r}\right). \tag{G.19}$$

The potential follower's and the leader's expected payoffs are

$$\begin{aligned} V_F(p_2) = {} & \frac{1}{2+r}\left(p_2\frac{\phi\pi}{r} - c\right) + \frac{c(1-p_2)}{r(1+r)} \cdot \frac{p_L(1-p_2)}{p_2(1-p_L)}\left(\frac{p_F(1-p_2)}{p_2(1-p_F)}\right)^r \\ & - \frac{c(1-p_2)}{(1+r)(2+r)}\left(1 - \left(\frac{p_L(1-p_F)}{p_F(1-p_L)} - \frac{2+r}{r}\right)\left(\frac{p_L(1-p_2)}{p_2(1-p_L)}\right)^{r+1}\right), \end{aligned} \tag{G.20}$$

$$\begin{aligned} V_L(p_2) = {} & \frac{\pi}{r} + \frac{1}{2+r}\left(\frac{(2\lambda+\phi-2)\pi}{r} - c\right) \\ & + (1-p_2)\frac{\phi\pi/r + (1-\lambda)\pi - c}{(1+r)(2+r)}\left(1 - \frac{p_L}{1+p_L+r}\left(\frac{p_L(1-p_2)}{p_2(1-p_L)}\right)^{r+1}\right) \\ & + \frac{(1-\lambda)\pi}{r(1+r)} \cdot \frac{p_L(1-p_2)}{1+p_L+r}\left(\frac{p_L+r}{1-p_L}\left(\frac{p_F(1-p_2)}{p_2(1-p_F)}\right)^{r+1} + \left(\frac{p_L(1-p_2)}{p_2(1-p_L)}\right)^{r+1}\right) \end{aligned} \tag{G.21}$$

for $p_2 > p_L$, (G.16) and (G.17) for $p_F < p_2 < p_L$, and (11) for $p_2 < p_F$.

(iv) If

$$c \le \frac{(\lambda+\phi-1)\pi}{r}, \tag{G.22}$$

then both the leader and the follower undertake research at full intensity until either the second innovation arrives or the belief p_2 falls to p_F defined in (G.15); starting from belief p_F, only the leader continues research until the second innovation arrives. The potential follower's and the leader's expected payoffs are

$$V_F(p_2) = \frac{1}{2+r}\left(p_2 \frac{\phi\pi}{r} - c\right) - \frac{c(1-p_2)}{(1+r)(2+r)}\left(1 - \left(\frac{p_F(1-p_2)}{p_2(1-p_F)}\right)^{r+1}\right), \tag{G.23}$$

$$V_L(p_2) = \frac{\pi}{r} + \frac{1}{2+r}\left(\frac{(2\lambda+\phi-2)\pi}{r} - c\right) + (1-p_2)\frac{\phi\pi/r + (1-\lambda)\pi - c}{(1+r)(2+r)}\left(1 + \frac{p_F}{1-p_F}\left(\frac{p_F(1-p_2)}{(1-p_F)p_2}\right)^{r+1}\right) \tag{G.24}$$

for $p_2 > p_F$ and

$$V_F(p_2) = 0, \quad V_L(p_2) = \frac{\pi}{r} + \frac{1}{1+r}\left(\frac{(\lambda+\phi-1)\pi}{r} - c\right) \tag{G.25}$$

for $p_2 < p_F$.

Proof. See Appendix G.5.4. □

Stage 1: Before the first innovation

As in the merged entity's setting, here we also focus on the symmetric prior. Due to high complexity of the analysis, we focus mostly on $\xi \leq 0$ and present the following result as a conjecture.

Proposition 8 (Conjecture)**.**

(i) If (16) holds, then neither firm undertakes research.

(ii) If (17) holds, then the firms undertake research at full intensity as long as their current belief $p(t)$ is above threshold $\hat{p}$ defined in (18). Once an innovation arrives or the belief falls to $\hat{p}$, the firms completely abort research efforts.

(iii) If (19) holds and $\xi \leq 0$, then the firms undertake research at full intensity as long as their current belief $p(t)$ is above threshold $\underline{p}$. Once the belief falls to $\underline{p}$, both firms completely abort research efforts. Threshold $\underline{p}$ is equal to $\bar{\tilde{p}}$ defined in Proposition 2.

Leveraging the results from the one-avenue benchmark model, we conjecture that the stopping threshold for the competing firms should be defined as

$$\underline{p} = \frac{c}{V_L\left(\underline{h}\right)}, \tag{G.26}$$

where $\underline{h} \in (0,1)$ solves

$$c = \frac{\underline{h}}{1 + \left(1-\underline{h}\right)^2 \xi} V_L\left(\underline{h}\right). \tag{G.27}$$

Moreover, intuitively, there might be an indifference region $(\underline{p}, \bar{p})$ with an intermediate intensity. A necessary condition for such region to appear in equilibrium is that, whenever the firms are about to abandon research — i.e., whenever the common current belief $p(t)$ is close to $\underline{p}$ — once one firm innovates, learning that the competitor's research avenue is good encourages the potential follower to undertake more research along its own avenue.

According to Proposition 7, if (10) holds, neither firm undertakes research after the first innovation. Hence, there is no indifference region. Since $V_L = \pi/r$, the stopping threshold (G.26) is equal to cr/π. If (16) holds, then this expression is greater than 1, which means that neither firm undertakes research — case (i) in Proposition 8. If (17) holds, then we get case (ii) in Proposition 8.

According to cases (ii) and (iii) in Proposition 7, if either (12) or (G.18) holds, then, if the first innovation arrives whenever the common current belief $p(t)$ is close to $\underline{p}$, neither firm undertakes research if $\underline{h}$, which solves (G.27), is less than p_F. If $\underline{h} < p_F$, then $V_L\left(\underline{h}\right) = \pi/r$, and so, the right-hand side of (G.27) becomes

$$\frac{\underline{h}}{1+\left(1-\underline{h}\right)^2 \xi}\frac{\pi}{r}. \tag{G.28}$$

Since, by (G.11), (G.28) is increasing in $\underline{h}$, $\underline{h} < p_F$ only if

$$c < \frac{p_F}{1+(1-p_F)^2 \xi}\frac{\pi}{r}, \tag{G.29}$$

which is equivalent to

$$0 < \frac{p_F}{1+(1-p_F)^2 \xi}\frac{\pi}{r} - c \overset{(G.15)}{=} \frac{c(1-p_F)^2}{1+(1-p_F)^2\xi}\left(\frac{1-\phi}{\phi(1-p_F)^2} - \xi\right), \tag{G.30}$$

which gives us

$$\xi < \frac{1-\phi}{\phi(1-p_F)^2}. \tag{G.31}$$

Hence, if either (12) or (G.18) holds and, in addition, condition (G.31) holds, then there is no indifference region, $V_L\left(\underline{h}\right) = \pi/r$ and the stopping threshold (G.26) is equal to cr/π — i.e., it is equal to $\check{p}$ defined in (7).

Case (iv) is similar to cases (ii) and (iii). The only difference is that if $\underline{h} < p_F$, then $V_L\left(\underline{h}\right)$ is defined in (G.25), and so, the analog of (G.30) is

$$\begin{aligned} 0 &< \frac{p_F}{1+(1-p_F)^2 \xi}\left\{\frac{\pi}{r} + \frac{1}{1+r}\left(\frac{(\lambda+\phi-1)\pi}{r} - c\right)\right\} - c \\ &\overset{(G.15)}{=} \frac{c(1-p_F)^2}{1+(1-p_F)^2\xi}\left(\frac{1-\phi}{\phi(1-p_F)^2} + \frac{\lambda+\phi-1-cr/\pi}{(1+r)\phi(1-p_F)^2} - \xi\right), \end{aligned} \tag{G.32}$$

which gives us

$$\xi < \frac{1-\phi}{\phi(1-p_F)^2} + \frac{\lambda+\phi-1-cr/\pi}{(1+r)\phi(1-p_F)^2}. \tag{G.33}$$

Hence, if (G.22) holds and, in addition, condition (G.33) holds, then there is no indifference region and $V_L\left(\underline{h}\right)$ is defined in (G.25). Hence, the stopping threshold (G.26) is equal to $\check{p}$ defined in (G.14). Condition $\xi \leq 0$ ensures that both (G.33) and (G.13) hold, which means that in this case, the stopping thresholds in the competing firms setting and in the merged entity's setting coincide.

G.4 Comparison

In this section, we compare the merged entity's optimal strategy (Propositions 5 and 6) and the equilibrium in the competing firms setting (Propositions 7 and 8).

Theorem 1 remains unchanged for all values of ξ. Indeed, under condition (25), the imperfect correlation between the avenues does not change the solution.

In contrast, Theorems 2 and 3 significantly change if $\xi \leq 0$. According to Propositions 6 and 8, the stopping thresholds of the competing firms and of the merged entity are the same. Hence, there is no appropriability effect. Moreover, since in the competing firms setting, there is no indifference region with an intermediate intensity, there is no informational effect.

Theorem 8. *If either* (26) *or* (28) *holds, then the merger has an unambiguously negative impact: while it does not change the probability and timing of the first innovation, it may delay or completely block the second innovation.*

In particular, if (26) holds, then after the first innovation arrives, the second innovation arrives with certainty in both settings because both the leader and the merged entity undertakes research along the avenue which produces the first innovation. However, along the other avenue, the potential follower may undertake research for longer than the merged entity because

$$\hat{\hat{p}} - p_F \overset{\text{(G.4) and (G.15)}}{=} \frac{c(1+r)}{c+(\lambda+\phi-1)\pi} - \frac{cr}{\phi\pi} = \frac{c\big(\overbrace{(\lambda+\phi-1)\pi - cr}^{>0 \text{ by } (26)} + (1+r)(1-\lambda)\pi\big)}{\phi\pi\,(c+(\lambda+\phi-1)\pi)} > 0. \quad \text{(G.34)}$$

If (28) holds, then the merged entity never produces the second innovation. In contrast, the competing firms may produce the second innovation if the first innovation arrives sufficiently early so that, despite zero or negative correlation between the avenues, learning that one avenue is good does not discourage the potential follower from undertaking research along the other avenue.

G.5 Proofs

G.5.1 Proof of Proposition 5

Let $V_M(p_2)$ be the merged entity's expected payoff from the second stage. Following the logic similar to the one behind (A.3) and (A.5), we get

$$V_M(p_2) = \max_{\substack{0\le x_1\le 1\\ 0\le x_2\le 1}} \Big\{ \pi\,\mathrm{d}t - c(x_1+x_2)\,\mathrm{d}t + \frac{(\lambda+\phi)\pi}{r}(x_1+x_2p_2)\,\mathrm{d}t$$
$$+ (1 - x_1\,\mathrm{d}t - x_2p_2\,\mathrm{d}t - r\,\mathrm{d}t)\,V_M(p_2) - V_M'(p_2)x_2(1-p_2)p_2\,\mathrm{d}t \Big\}, \quad \text{(G.35)}$$

which yields the Hamilton-Jacobi-Bellman equation

$$0=\max_{\substack{0\le x_1\le 1\\ 0\le x_2\le 1}}\left\{\pi+\left(\frac{(\lambda+\phi)\pi}{r}-c-V_M(p_2)\right)x_1\right.$$
$$\left.+\left(\frac{(\lambda+\phi)\pi}{r}p_2-c-p_2V_M(p_2)-p_2(1-p_2)V_M'(p_2)\right)x_2-rV_M(p_2)\right\}. \quad \text{(G.36)}$$

Naturally, if it is optimal to undertake research along avenue 2, which type is uncertain, then it is also optimal to undertake research along avenue 1, which is known to be good. Hence, it is sufficient to consider three cases: when no research is optimal, when only avenue 1 is explored, and when both avenues are explored.

Region with no research. If $x_1=x_2=0$, then, by (G.36), $V_M(p_2)=\pi/r$. According to (G.36), choosing $x_1=x_2=0$ is optimal only if

$$c\ge\max\left\{p_2\left(\frac{(\lambda+\phi)\pi}{r}-V_M(p_2)-(1-p_2)V_M'(p_2)\right),\frac{(\lambda+\phi)\pi}{r}-V_M(p_2)\right\}. \quad \text{(G.37)}$$

Since $V_M(p_2)=\pi/r$, condition (G.37) can be simplified as

$$c\ge\frac{(\lambda+\phi-1)\pi}{r}. \quad \text{(G.38)}$$

Region with research only along avenue 1. If $x_1=1$ and $x_2=0$, then, by (G.36),

$$V_M(p_2)=\frac{\pi}{r}+\frac{1}{1+r}\left(\frac{(\lambda+\phi-1)\pi}{r}-c\right). \quad \text{(G.39)}$$

According to (G.36), choosing $x_1=1$ and $x_2=0$ is optimal only if

$$p_2\left(\frac{(\lambda+\phi)\pi}{r}-V_M(p_2)-(1-p_2)V_M'(p_2)\right)\le c\le\frac{(\lambda+\phi)\pi}{r}-V_M(p_2). \quad \text{(G.40)}$$

Since $V_M(p_2)$ is defined in (G.39), condition (G.40) can be simplified as

$$c\le\frac{(\lambda+\phi-1)\pi}{r} \quad \text{(G.41)}$$

and

$$p_2\le\frac{c(1+r)}{c+(\lambda+\phi-1)\pi}. \quad \text{(G.42)}$$

Region with research along both avenues. For the region of beliefs, in which $x_1=x_2=1$, we can solve the differential equation (G.36):

$$\pi+\frac{(\lambda+\phi)\pi}{r}-2c+p_2\left(\frac{(\lambda+\phi)\pi}{r}-V_M(p_2)-(1-p_2)V_M'(p_2)\right)=(1+r)V_M(p_2), \quad \text{(G.43)}$$

for $V_M(p_2)$ explicitly up to a constant of integration:

$$V_M(p_2) = \frac{\pi}{r} + \frac{2}{2+r}\left(\frac{(\lambda+\phi-1)\pi}{r} - c\right) - (1-p_2)\left(\frac{(\lambda+\phi-1)\pi+2c}{(1+r)(2+r)} - C_M\left(\frac{1-p_2}{p_2}\right)^{r+1}\right). \quad \text{(G.44)}$$

According to (G.36), choosing $x_1 = x_2 = 1$ is optimal only if

$$c \le \min\left\{p_2\left(\frac{(\lambda+\phi)\pi}{r} - V_M(p_2) - (1-p_2)V_M'(p_2)\right), \frac{(\lambda+\phi)\pi}{r} - V_M(p_2)\right\}. \quad \text{(G.45)}$$

Equation (G.43) allows rewriting condition (G.45) as

$$\frac{\pi}{r} + \frac{1}{1+r}\left(\frac{(\lambda+\phi-1)\pi}{r} - c\right) \le V_M(p_2) \le \frac{(\lambda+\phi)\pi}{r} - c. \quad \text{(G.46)}$$

No-research solution. Choosing $x_1 = x_2 = 0$ is optimal for all beliefs $p_2 \in (0,1)$ only if condition (G.38) holds, which is equivalent to the statement that condition (2) does not hold.

Full-intensity solution. Suppose that there exists $\mathring{\hat{p}} \in (0,1)$ such that $x_1 = 1$ and $x_2 = 0$ for $p_2 \in [0, \mathring{\hat{p}})$ and $x_1 = x_2 = 1$ for the beliefs just above $\mathring{\hat{p}}$. Then the value-matching and smooth-pasting properties for (G.39) and (G.44) at $p_2 = \mathring{\hat{p}}$ give the expression for $\mathring{\hat{p}}$ and the constant of integration:

$$\mathring{\hat{p}} = \frac{c(1+r)}{c+\pi(\lambda+\phi-1)}, \quad C_M = \frac{c}{(1+r)(2+r)}\left(\frac{\mathring{\hat{p}}}{1-\mathring{\hat{p}}}\right)^{r+1}. \quad \text{(G.47)}$$

Observe that for thus defined $\mathring{\hat{p}}$, condition (G.42) holds for all $p_2 \le \mathring{\hat{p}}$. Condition (2) is necessary for the full-intensity solution to exist because it is equivalent to (G.41) and, moreover, it ensures that $\mathring{\hat{p}} \le 1$. Condition (2) is also sufficient because (G.46) holds for all $p_2 > \mathring{\hat{p}}$. Indeed, at $p_2 = \mathring{\hat{p}}$, $V_M(p_2)$ is equal to (G.39) — i.e., the lower bound in (G.46). Moreover, differentiating (G.44) twice yields

$$V_M''(p_2) = C_M \frac{(1+r)(2+r)}{p_2^3}\left(\frac{1-p_2}{p_2}\right)^r, \quad \text{(G.48)}$$

which is positive because $C_M > 0$. Hence, since $V_M'(\mathring{\hat{p}}) = 0$ by smooth-pasting property, $V_M(p_2)$ is increasing on $p_2 \in (\mathring{\hat{p}}, 1)$. At $p_2 = 1$, $V_M(p_2)$ is still below the upper bound in (G.46):

$$V_M(1) = \frac{\pi}{r} + \frac{2}{2+r}\left(\frac{(\lambda+\phi-1)\pi}{r} - c\right) < \frac{\pi}{r} + \left(\frac{(\lambda+\phi-1)\pi}{r} - c\right). \quad \text{(G.49)}$$

G.5.2 Proof of Proposition 6

Given the research intensities x_1 and x_2, the merged entity's instantaneous payoff is

$$\sum_{i=1,2} x_i \, \mathrm{dt}\left(p_i V_M\left(\frac{\Pr(\text{both avenues are good})}{p_i}\right) - c\right), \quad \text{(G.50)}$$

where V_M denotes the merged entity's second-stage payoff, as derived in Proposition 5, and where p_i and Pr(both avenues are good) represent the merged entity's current beliefs.

Following Mayskaya and Nikandrova (2024), in addition to ξ, we introduce the variables

$$q_1 = \frac{\Pr(\text{only avenue 2 is good})}{\Pr(\text{both avenues are good})}, \quad q_2 = \frac{\Pr(\text{only avenue 1 is good})}{\Pr(\text{both avenues are good})}. \tag{G.51}$$

By assumption (G.1), variables q_1 and q_2 are well-defined and can take any positive values. There is a one-to-one correspondence between (q_1, q_2, ξ) and the beliefs:

$$\Pr(\text{both avenues are bad}) = \frac{(1+\xi)q_1q_2}{1+q_1+q_2+(1+\xi)q_1q_2}, \tag{G.52}$$

$$\Pr(\text{both avenues are good}) = \frac{1}{1+q_1+q_2+(1+\xi)q_1q_2}, \tag{G.53}$$

$$\Pr(\text{only avenue 2 is good}) = \frac{q_1}{1+q_1+q_2+(1+\xi)q_1q_2}, \tag{G.54}$$

$$\Pr(\text{only avenue 1 is good}) = \frac{q_2}{1+q_1+q_2+(1+\xi)q_1q_2}. \tag{G.55}$$

Since before the arrival of the first innovation, ξ remains constant regardless of the investment strategy, vector (q_1, q_2) can serve as a state variable for the first-stage optimization problem. Investment in avenue i increases q_i while keeping the other state variable constant:

$$q_i'(t) = q_i x_i. \tag{G.56}$$

Let $\hat{W}(q_1, q_2)$ denote the merged entity's value function in the first stage. Given the instantaneous payoff (G.50) and the belief transition formulas (G.52) to (G.55), we can now write down the Hamilton-Jacobi-Bellman (HJB) equation:

$$\begin{aligned} 0 = \max_{\substack{0\le x_1\le 1\\ 0\le x_2\le 1}} \Bigg\{ & x_1\left(\frac{1+q_2}{1+q_1+q_2+(1+\xi)q_1q_2}V_M\left(\frac{1}{1+q_2}\right)-c\right) \\ & + x_2\left(\frac{1+q_1}{1+q_1+q_2+(1+\xi)q_1q_2}V_M\left(\frac{1}{1+q_1}\right)-c\right) \\ & -\left(\frac{(1+q_2)x_1+(1+q_1)x_2}{1+q_1+q_2+(1+\xi)q_1q_2}+r\right)\hat{W}(q_1,q_2)+\frac{\partial \hat{W}(q_1,q_2)}{\partial q_1}q_1x_1+\frac{\partial \hat{W}(q_1,q_2)}{\partial q_2}q_2x_2\Bigg\}. \end{aligned} \tag{G.57}$$

We focus on the diagonal $q_1 = q_2 \equiv q$ where the HJB equation becomes

$$\begin{aligned} r\hat{W}(q,q) = \max_{\substack{0\le x_1\le 1\\ 0\le x_2\le 1}} \Bigg\{ & (x_1+x_2)\left(\frac{1+q}{1+2q+(1+\xi)q^2}\left(V_M\left(\frac{1}{1+q}\right)-\hat{W}(q,q)\right)-c\right) \\ & + \frac{\partial \hat{W}(q,q)}{\partial q_1}qx_1+\frac{\partial \hat{W}(q,q)}{\partial q_2}qx_2\Bigg\}. \end{aligned} \tag{G.58}$$

Since by symmetry $\frac{\partial \hat{W}(q,q)}{\partial q_1} = \frac{\partial \hat{W}(q,q)}{\partial q_2}$, we conclude that along the diagonal, there exists a symmetric optimal strategy, i.e., an optimal strategy with $x_1 = x_2$.

If $x_1 = x_2$, then the ratio q_1/q_2 stays constant:

$$\frac{\mathrm{d}}{\mathrm{dt}} \frac{q_1(t)}{q_2(t)} \overset{(G.56)}{=} \frac{q_1}{q_2}(x_1 - x_2) \overset{x_1=x_2}{=} 0. \tag{G.59}$$

Hence, under any symmetric strategy, the state variable (q_1, q_2) stays on the diagonal. This observation allows us to move from the two-dimensional state space to a one-dimensional state space.

Denote $W\left(\frac{q_1+q_2}{2}, \frac{q_1}{q_2}\right) = \hat{W}(q_1, q_2)$. Then, the derivative of W with respect to its first argument is equal to

$$\frac{2}{q_1 + q_2}\left(q_1 \frac{\partial \hat{W}(q_1,q_2)}{\partial q_1} + q_2 \frac{\partial \hat{W}(q_1,q_2)}{\partial q_2}\right), \tag{G.60}$$

which means that the HJB equation on the diagonal $q_1 = q_2$ can be rewritten as

$$rW(q) = \max_{0 \le x \le 1} \left\{ 2x \left(\frac{1+q}{1+2q+(1+\xi)q^2} \left(V_M\left(\frac{1}{1+q}\right) - W(q) \right) - c \right) + W'(q)qx \right\}. \tag{G.61}$$

where we omit the second argument of W.

The rest of the proof follows closely the proof of Proposition 2.

Region with no research. If $x = 0$, then, by (G.61), $W(q) = 0$. According to (G.61), choosing $x = 0$ is optimal on the diagonal only if

$$c \ge \frac{1+q}{1+2q+(1+\xi)q^2}\left(V_M\left(\frac{1}{1+q}\right) - W(q)\right) + \frac{1}{2}W'(q)q. \tag{G.62}$$

Since $W(q) = 0$, condition (G.62) can be simplified as

$$c \ge \frac{1+q}{1+2q+(1+\xi)q^2} V_M\left(\frac{1}{1+q}\right) \equiv F(q). \tag{G.63}$$

Region with full intensity along both avenues. For the region of $x = 1$ we can solve the differential equation (G.61):

$$rW(q) = \frac{2(1+q)}{1+2q+(1+\xi)q^2}\left(V_M\left(\frac{1}{1+q}\right) - W(q)\right) - 2c + W'(q)q \tag{G.64}$$

for $W(q)$ up to a constant of integration:

$$W(q) = \frac{4q^{4+2r}}{\left(1+2q+(1+\xi)q^2\right)^2}\left(C_M + \int_q^{+\infty} \frac{\left(1+2s+(1+\xi)s^2\right)^2}{s^{5+2r}}(F(s) - c)\,\mathrm{d}s\right), \tag{G.65}$$

where function $F(s)$ is defined in (G.63). According to (G.61), choosing $x = 1$ is optimal only if

$$c \leq \frac{1+q}{1+2q+(1+\xi)q^2}\left(V_M\left(\frac{1}{1+q}\right)-W(q)\right)+\frac{1}{2}W'(q)q. \tag{G.66}$$

Equation (G.64) allows rewriting condition (G.66) as

$$W(q) \geq 0. \tag{G.67}$$

No-research solution. Suppose that the no-research region covers all $q > 0$. Since choosing zero intensity is optimal only if condition (G.63) holds and since function F is decreasing in q, the no-research region covers all $q > 0$ if and only if (G.63) holds for $q = 0$:

$$c \geq F(0) = V_M(1) \overset{\text{Proposition 5}}{=} \frac{\pi}{r} + \frac{2}{2+r}\max\left\{\frac{(\lambda+\phi-1)\pi}{r} - c, 0\right\}, \tag{G.68}$$

which is equivalent to (5). Hence, the no-research solution corresponds to case (i) in Proposition 6.

Full-intensity solution. Since the limit $q \to +\infty$ of (G.65) is either $\pm\infty$ — which is infeasible — or finite and negative — which contradicts (G.67) — $q = +\infty$ belongs to the region with no research.

Suppose that there exists $\check{q} > 0$ such that there is no research for $q > \check{q}$, while q just below $\check{q}$ belong to the full-intensity region. Then the value-matching and smooth-pasting properties, $W(\check{q}) = W'(\check{q}) = 0$, together with (G.65), give the expression for the constant of integration:

$$C_M = -\int_{\check{q}}^{+\infty} \frac{\left(1+2s+(1+\xi)s^2\right)^2}{s^{5+2r}}(F(s)-c)\,\mathrm{d}s, \tag{G.69}$$

and the equation for $\check{q}$:

$$c = F(\check{q}). \tag{G.70}$$

A necessary condition for the full-intensity solution to exist is

$$c < F(0) = V_M(1), \tag{G.71}$$

which is a necessary and sufficient condition for equation (G.70) to have a solution. Equation (G.70) implies that $c < F(q)$ for all $q < \check{q}$, and so the expression in the brackets in (G.65) is decreasing in q. Hence, since this expression is equal to 0 at $q = \check{q}$, it is positive — and so $W(q) > 0$ — for all $q < \check{q}$. Since condition (G.67) holds as strict inequality for all $q < \check{q}$, the full intensity region ends at $q = 0$. Moreover, condition (G.71) is sufficient for the full-intensity solution to exist.

The full-research solution corresponds to cases (ii) and (iii) in Proposition 6. In particular, if (6) holds, then, by Proposition 5, $V_M\left(\frac{1}{1+q}\right) = \frac{\pi}{r}$ and thus equation (G.70) becomes

$$\frac{1+\check{q}}{1+2\check{q}+(1+\xi)\check{q}^2} = \frac{cr}{\pi}, \tag{G.72}$$

which gives threshold (7) by the transition formulas (G.53) to (G.55). If (8) holds, then $\check{q}$ defined by equation (G.70) is equal to $1/\check{h}-1$, where $\check{h}$ solves (G.10).

G.5.3 Proof of Lemma 14

As we show below, condition (G.13) is equivalent to the condition that the solution $\check{h}$ to equation (G.10) is less or equal to $\hat{\check{p}}$ defined in (G.4). If $\check{h} \leq \hat{\check{p}}$, then the value of $V_M\left(\check{h}\right)$ is given in (G.6), which implies that $\check{p}$ defined in (G.12) is equal to (G.14).

To see why condition (G.13) is equivalent to $\check{h} \leq \hat{\check{p}}$, recall that the right-hand side of (G.10) is increasing in $\check{h}$. Hence, the solution $\check{h}$ to equation (G.10) is less or equal to $\hat{\check{p}}$ if and only if the value of the right-hand side of (G.10) at $\check{h} = \hat{\check{p}}$ is greater or equal to c:

$$\begin{aligned} 0 &< \frac{\hat{\check{p}}}{1+\left(1-\hat{\check{p}}\right)^2 \xi} V_M\left(\hat{\check{p}}\right) - c \\ &= \frac{c(1-\hat{\check{p}})^2}{1+(1-\hat{\check{p}})^2\xi}\left(\frac{((\lambda+\phi-1)\pi+c)((2-\lambda-\phi)\pi(1+r)+2((\lambda+\phi-1)\pi-cr))}{r\left((\lambda+\phi-1)\pi-cr\right)^2}-\xi\right), \end{aligned} \tag{G.73}$$

which gives us condition (G.13).

If condition (G.13) does not hold, then $\check{h} > \hat{\check{p}}$, and so, the value of $V_M\left(\check{h}\right)$ is given in (G.5). In this case, function $V_M\left(\check{h}\right)$ is increasing. Hence, to prove that $\check{p}$ defined in (G.12) is decreasing in ξ, it is sufficient to show that $\check{h}$ is increasing in ξ. The solution $\check{h}$ to equation (G.10) is indeed increasing in ξ because the right-hand side of (G.10) is increasing in $\check{h}$ and decreasing in ξ, while the left-hand side of (G.10) is independent of $\check{h}$ and ξ.

At the limit $\xi \to +\infty$, the solution $\check{h}$ to equation (G.10) is equal to 1. Hence, $\check{p}$ defined in (G.12) is equal to $c/V_M(1)$. Substituting the expression for $V_M(1)$ from Proposition 5 gives (9).

G.5.4 Proof of Proposition 7

Let $V_F(p_2)$ be the potential follower's expected payoff from the second stage. Then, given the research intensity $x_L(p_2)$ of the leader, we get

$$\begin{aligned} V_F(p_2) = \max_{x_F\in[0,1]} \Bigg\{ &-cx_F\,\mathrm{d}t + \frac{\phi\pi}{r}x_F p_2\,\mathrm{d}t + (1-x_F p_2\,\mathrm{d}t - x_L(p_2)\,\mathrm{d}t - r\,\mathrm{d}t)\,V_F(p_2) \\ &- V_F'(p_2)x_F(1-p_2)p_2\,\mathrm{d}t \Bigg\}, \end{aligned} \tag{G.74}$$

which yields the Hamilton-Jacobi-Bellman equation

$$(r+x_L(p_2))\,V_F(p_2) = \max_{x_F\in[0,1]}\left(\frac{\phi\pi}{r}p_2 - c - p_2V_F(p_2) - p_2(1-p_2)V_F'(p_2)\right)x_F. \tag{G.75}$$

Similarly, for the leader's expected payoff $V_L(p_2)$ we have

$$V_L(p_2) = \max_{x_L\in[0,1]} \Big\{ \pi\,\mathrm{dt} - c x_L\,\mathrm{dt} + \frac{(\lambda+\phi)\pi}{r} x_L\,\mathrm{dt} + \frac{\lambda\pi}{r} x_F(p_2)p_2\,\mathrm{dt}$$
$$+ (1 - x_F(p_2)p_2\,\mathrm{dt} - x_L\,\mathrm{dt} - r\,\mathrm{dt})\,V_L(p_2) - V_L'(p_2)x_F(p_2)(1-p_2)p_2\,\mathrm{dt} \Big\}, \quad \text{(G.76)}$$

and so

$$r\left(V_L(p_2) - \frac{\pi}{r}\right) + x_F(p_2)p_2\left(V_L(p_2) - \frac{\lambda\pi}{r} + (1-p_2)V_L'(p_2)\right) = \max_{x_L\in[0,1]}\left(\frac{(\lambda+\phi)\pi}{r} - c - V_L(p_2)\right)x_L. \quad \text{(G.77)}$$

Region with no research. If $x_L = x_F = 0$, then (G.75) and (G.77) give $V_F(p_2) = 0$ and $V_L(p_2) = \pi/r$. According to (G.75) and (G.77), choosing $x_L = x_F = 0$ is optimal only if

$$c \geq \max\left\{ p_2\left(\frac{\phi\pi}{r} - V_F(p_2) - (1-p_2)V_F'(p_2)\right), \frac{(\lambda+\phi)\pi}{r} - V_L(p_2)\right\}. \quad \text{(G.78)}$$

Since $V_F(p_2) = 0$ and $V_L(p_2) = \pi/r$, condition (G.78) can be simplified as

$$c \geq \max\left\{ \frac{\phi\pi}{r}p_2, \frac{(\lambda+\phi-1)\pi}{r}\right\}. \quad \text{(G.79)}$$

Region with full intensity for both firms. For the region of beliefs, in which $x_L = x_F = 1$, we can solve the differential equation (G.75):

$$(1+r)V_F(p_2) + p_2\big(V_F(p_2) + (1-p_2)V_F'(p_2)\big) = \frac{\phi\pi}{r}p_2 - c, \quad \text{(G.80)}$$

for $V_F(p_2)$ explicitly up to a constant of integration:

$$V_F(p_2) = \frac{1}{2+r}\left(p_2\frac{\phi\pi}{r} - c\right) - \frac{c(1-p_2)}{(1+r)(2+r)} + C_F\left(\frac{1-p_2}{p_2}\right)^{r+1}(1-p_2). \quad \text{(G.81)}$$

Similarly, the differential equation (G.77):

$$(1+r)V_L(p_2) + p_2\big(V_L(p_2) + (1-p_2)V_L'(p_2)\big) = \pi + \frac{(\lambda+\phi)\pi}{r} + \frac{\lambda\pi}{r}p_2 - c, \quad \text{(G.82)}$$

gives

$$V_L(p_2) = \frac{\pi}{r} + \frac{1}{2+r}\left(\frac{(2\lambda+\phi-2)\pi}{r} - c\right) + \left(\frac{\phi\pi/r + (1-\lambda)\pi - c}{(1+r)(2+r)} + C_L\left(\frac{1-p_2}{p_2}\right)^{r+1}\right)(1-p_2). \quad \text{(G.83)}$$

According to (G.75) and (G.77), choosing $x_L = x_F = 1$ is optimal only if

$$c \leq \min\left\{ p_2\left(\frac{\phi\pi}{r} - V_F(p_2) - (1-p_2)V_F'(p_2)\right), \frac{(\lambda+\phi)\pi}{r} - V_L(p_2)\right\}. \quad \text{(G.84)}$$

Equations (G.80) allows rewriting condition (G.84) as

$$V_F(p_2) \geq 0 \tag{G.85}$$

and

$$V_L(p_2) \leq \frac{(\lambda+\phi)\pi}{r} - c. \tag{G.86}$$

Region where only the leader undertakes research. If $x_L = 1$ and $x_F = 0$, then (G.75) and (G.77) give (G.25). According to (G.75) and (G.77), choosing $x_L = 1$ and $x_F = 0$ is optimal only if

$$p_2\left(\frac{\phi\pi}{r} - V_F(p_2) - (1-p_2)V_F'(p_2)\right) \leq c \leq \frac{(\lambda+\phi)\pi}{r} - V_L(p_2). \tag{G.87}$$

Substituting (G.25) into (G.87) yields

$$\frac{\phi\pi}{r}p_2 \leq c \leq \frac{(\lambda+\phi-1)\pi}{r}. \tag{G.88}$$

Region where only the follower undertakes research. For the region of beliefs, in which $x_L = 0$ and $x_F = 1$, we can solve the differential equation (G.75):

$$rV_F(p_2) + p_2\left(V_F(p_2) + (1-p_2)V_F'(p_2)\right) = \frac{\phi\pi}{r}p_2 - c, \tag{G.89}$$

for $V_F(p_2)$ explicitly up to a constant of integration:

$$V_F(p_2) = \frac{1}{1+r}\left(p_2\frac{\phi\pi}{r} - c\right) - \frac{c(1-p_2)}{r(1+r)} + C_f\left(\frac{1-p_2}{p_2}\right)^r(1-p_2). \tag{G.90}$$

Similarly, the differential equation (G.77):

$$rV_L(p_2) + p_2\left(V_L(p_2) + (1-p_2)V_L'(p_2)\right) = \pi + \frac{\lambda\pi}{r}p_2, \tag{G.91}$$

gives

$$V_L(p_2) = \frac{\pi}{r} - \frac{(1-\lambda)\pi}{r(1+r)}p_2 + C_l\left(\frac{1-p_2}{p_2}\right)^r(1-p_2). \tag{G.92}$$

According to (G.75) and (G.77), the choice $x_L = 0$, $x_F = 1$ is optimal only if

$$\frac{(\lambda+\phi)\pi}{r} - V_L(p_2) \leq c \leq p_2\left(\frac{\phi\pi}{r} - V_F(p_2) - (1-p_2)V_F'(p_2)\right). \tag{G.93}$$

Equation (G.89) allows rewriting condition (G.93) as

$$V_L(p_2) \geq \frac{(\lambda+\phi)\pi}{r} - c \tag{G.94}$$

and

$$V_F(p_2) \geq 0. \tag{G.95}$$

Conditions (G.79) and (G.88) are incompatible and thus give rise to two possibilities: $cr > (\lambda + \phi - 1)\pi$ when the leader cannot undertake research alone, and $cr < (\lambda + \phi - 1)\pi$ when at least one firm always undertakes research.

No-research equilibrium. Suppose that the no-research region covers the whole belief interval, $p_2 \in (0,1)$. Since choosing $x_L = x_F = 0$ is optimal only if condition (G.79) holds, this is an optimum if and only if

$$c > \frac{\phi\pi}{r}, \tag{G.96}$$

that is, condition (10) holds. Hence, the no-research equilibrium corresponds to case (i) in Proposition 7.

Follower-research equilibrium. Suppose that there exists $p_F \in (0,1)$ such that there is no research for $p_2 \in [0, p_F)$ and the beliefs just above p_F belong to the region where only the follower undertakes research. Then $V_F(p_F) = V_F'(p_F) = 0$, together with (G.90), give the expression for p_F and the constant of integration:

$$p_F = \frac{cr}{\phi\pi}, \quad C_f = \frac{c}{r(1+r)}\left(\frac{p_F}{1-p_F}\right)^r. \tag{G.97}$$

Moreover, $V_L(p_F) = \pi/r$ and (G.92) give

$$C_l = \frac{(1-\lambda)\pi}{r(1+r)}\left(\frac{p_F}{1-p_F}\right)^{r+1} \tag{G.98}$$

Substituting the constants of integration to (G.90) and (G.92) gives us the expressions (G.16) and (G.17) for $V_F(p_2)$ and $V_L(p_2)$ for all $p_2 > p_F$ within the only-follower-research region.

A necessary condition for the follower-research equilibrium to exist is

$$c \leq \frac{\phi\pi}{r}, \tag{G.99}$$

which ensures that p_F is below 1. The choice $x_L = 0$ and $x_F = 1$ is optimal only if conditions (G.94) and (G.95) hold. Function $V_F(p_2)$, as defined in (G.16), is convex:

$$V_F''(p_2) = \frac{c}{p_2^2(1-p_2)}\left(\frac{p_F(1-p_2)}{p_2(1-p_F)}\right)^r > 0. \tag{G.100}$$

Hence, since $V_F(p_F) = V_F'(p_F) = 0$, function $V_F(p_2)$ is positive for all $p_2 > p_F$ — which means that condition (G.95) holds as strict inequality. Function $V_L(p_2)$, as defined in (G.17), is decreasing for all p_2:

$$V_L'(p_2) = -\frac{(1-\lambda)\pi}{r(1+r)}\left(1 + \frac{p_2+r}{1-p_2}\left(\frac{p_F(1-p_2)}{p_2(1-p_F)}\right)^{r+1}\right) < 0, \tag{G.101}$$

and at $p_2 = 1$ equal to

$$V_L(1) = \frac{(\lambda+\phi)\pi}{r} - \frac{\pi}{r}\left(\phi - \frac{(1-\lambda)r}{1+r}\right). \tag{G.102}$$

Hence, we have two possibilities.

First, condition (G.94) holds for all $p_2 > p_F$:

$$c > \frac{\pi}{r}\left(\phi - \frac{(1-\lambda)r}{1+r}\right). \tag{G.103}$$

Together, conditions (G.99) and (G.103) are equivalent to (12). Note that (12) ensures that (G.79) holds for all $p_2 < p_F$. This equilibrium corresponds to case (ii) in Proposition 7.

Second, condition (G.94) holds only for p_2 close enough to p_F — that is, condition (G.18) holds. In this case, condition (G.94) holds for all $p_2 \in (p_F, p_L)$, where p_L is defined as a unique solution to (G.19). As p_2 increases above p_L, condition (G.94) fails, which means that for the leader, it becomes optimal to undertake research. Hence, we conjecture that all $p_2 \in (p_L, 1)$ belong to the both-firms-research region. At $p_2 = p_L$, both $V_F(p_2)$ and $V_L(p_2)$ must be continuous. Equalizing (G.16) and (G.81) at $p_2 = p_L$ yields

$$C_F = \frac{c}{(1+r)(2+r)}\left(\frac{p_L(1-p_F)}{p_F(1-p_L)} - \frac{2+r}{r}\right)\left(\frac{p_L}{1-p_L}\right)^{r+1} + \frac{c}{r(1+r)}\frac{p_L}{1-p_L}\left(\frac{p_F}{1-p_F}\right)^{r}. \tag{G.104}$$

Similarly, equalizing (G.17) and (G.83) at $p_2 = p_L$ yields

$$\begin{aligned} C_L = {} & \frac{\pi(1-\lambda)(p_L+r)p_L}{r(1+r)(1+p_L+r)(1-p_L)}\left(\left(\frac{p_F}{1-p_F}\right)^{r+1} + \frac{1-p_L}{p_L+r}\left(\frac{p_L}{1-p_L}\right)^{r+1}\right) \\ & - \frac{p_L(\phi\pi/r + (1-\lambda)\pi - c)}{(1+r)(2+r)(1+p_L+r)}\left(\frac{p_L}{1-p_L}\right)^{r+1} \\ & + \frac{\pi(1-\lambda)}{r(1-p_L)(1+p_L+r)}\underbrace{\left(\frac{r}{1-\lambda}\left(\frac{c}{\pi} - \frac{\lambda+\phi-1}{r}\right) - \frac{p_L}{1+r}\left(1 - \left(\frac{p_F(1-p_L)}{p_L(1-p_F)}\right)^{r+1}\right)\right)}_{=0 \text{ by (G.19)}}\left(\frac{p_L}{1-p_L}\right)^{r+1}. \end{aligned} \tag{G.105}$$

Substituting these constants into (G.81) and (G.83) gives (G.20) and (G.21). As expected, $V_L(p_2)$ is continuously differentiable at $p_2 = p_L$. Moreover, $V_L(p_2)$ is convex for all $p_2 > p_L$:

$$\begin{aligned} V_L''(p_2) = {} & \frac{(1-\lambda)\pi p_L^2}{p_2^3 r(1-p_L)(1+p_L+r)}\left(\left(\frac{p_L(1-p_2)}{p_2(1-p_L)}\right)^r + \frac{(2+r)p_F(p_L+r)}{p_L(1-p_F)}\left(\frac{p_F(1-p_2)}{p_2(1-p_F)}\right)^r\right) \\ & + \frac{p_L^2}{p_2^3(1-p_L)(1+p_L+r)}\underbrace{\left(c - \frac{(\lambda+\phi-1)\pi}{r}\right)}_{>0 \text{ by (G.18)}}\left(\frac{p_L(1-p_2)}{p_2(1-p_L)}\right)^r > 0. \end{aligned} \tag{G.106}$$

Hence, since by construction of p_L, $V_L(p_L)$ is equal to the right-hand side of (G.94) — i.e., (G.86) holds as equality at $p_2 = p_L$ — (G.86) holds for all $p_2 \in [p_L, 1]$ if and only if it holds at $p_2 = 1$. The latter is true under condition (G.18):

$$V_L(1) = \frac{(\lambda+\phi)\pi}{r} - c - \frac{1+r}{2+r} \underbrace{\left(\frac{\pi}{r}\left(\phi - \frac{(1-\lambda)r}{1+r}\right) - c\right)}_{\geq 0 \text{ by (G.18)}}. \tag{G.107}$$

As for condition (G.85), it holds for all $p_2 \in [p_L, 1]$. First, it holds at $p_2 = p_L$. Second, if at some point $p_2 > p_L$ the value of $V_F(p_2)$ becomes 0, its derivative $V_F'(p_2)$ at this point has to be positive:

$$V_F'(p_2) \overset{(G.80)}{=} \frac{\phi\pi p_2/r - c}{p_2(1-p_2)} \overset{(G.15)}{=} \frac{\phi\pi}{r}\frac{p_2 - p_F}{p_2(1-p_2)} > 0. \tag{G.108}$$

This equilibrium corresponds to case (iii) in Proposition 7.

Both-firms-research equilibrium. Suppose that $p_2 = 0$ belongs to the region where only the leader undertakes research. Choosing $x_L = 1$ and $x_F = 0$ is optimal only if condition (G.88) holds. This condition is equivalent to (G.22) and $p_2 < p_F$, where p_F is defined as in (G.15). For p_2 just above p_F, the follower also finds it optimal to undertake research, so that for such beliefs the firms' payoffs are defined in (G.81) and (G.83). At $p_2 = p_F$, both $V_F(p_2)$ and $V_L(p_2)$ must be continuous. Equalizing $V_F(p_2)$ in (G.25) and (G.81) at $p_2 = p_F$ yields

$$C_F = \frac{c}{(1+r)(2+r)}\left(\frac{p_F}{1-p_F}\right)^{r+1}, \tag{G.109}$$

so that $V_F(p_2)$ becomes (G.23). Equalizing $V_L(p_2)$ in (G.25) and (G.83) at $p_2 = p_F$ yields

$$C_L = \frac{\phi\pi/r + (1-\lambda)\pi - c}{(1+r)(2+r)}\left(\frac{p_F}{1-p_F}\right)^{r+2}, \tag{G.110}$$

so that $V_L(p_2)$ becomes (G.24). As expected, $V_F'(p_F) = 0$, which means that the smooth-pasting property holds.

Choosing $x_L = x_F = 1$ is optimal only if conditions (G.85) and (G.86) hold. Following the same reasoning as for the equilibrium in case (iii) — that is, appealing to (G.80) to argue that $V_F(p_2)$ cannot cross 0 from above on the region $p_2 > p_F$ — we conclude that condition (G.85) holds for all $p_2 > p_F$. Condition (G.86) also holds for all $p_2 > p_F$ because it holds at $p_2 = p_F$:

$$V_L(p_F) = \frac{(\lambda+\phi)\pi}{r} - c - \underbrace{\frac{(\lambda+\phi-1)\pi - cr}{1+r}}_{\geq 0 \text{ by (G.22)}} \tag{G.111}$$

and at $p_2 = 1$:

$$V_L(1) = \frac{(\lambda+\phi)\pi}{r} - c - \frac{1+r}{2+r}\underbrace{\left(\frac{(\lambda+\phi-1)\pi}{r} - c\right)}_{\geq 0 \text{ by (G.22)}} - \frac{(1-\lambda)\pi}{r(2+r)} \tag{G.112}$$

and function $V_L(p_2)$ is convex:

$$V_L''(p_2) = \frac{1}{p_2(1-p_2)^2}\left(\frac{(1-\lambda)\pi(1+r)}{r} + \underbrace{\frac{(\lambda+\phi-1)\pi}{r} - c}_{\geq 0 \text{ by (G.22)}}\right)\left(\frac{p_F(1-p_2)}{p_2(1-p_F)}\right)^{r+2} > 0. \quad \text{(G.113)}$$

This equilibrium corresponds to case (iv) in Proposition 7.

References

Mayskaya, Tatiana and Arina Nikandrova, "The effect of correlation and payoff interdependence on R&D," Working Paper, 2024. Available at https://www.tmayskaya.com.