# Online Appendix

## *"How Costly Are Cartels?"*


## Flavien Moreau and Ludovic Panon

# A Data Appendix

## A.1 Institutional Background

Despite a strong tradition in industrial policy, antitrust regulation in France has a relatively short formal history. It can be roughly simplified into four periods, during which the competition regulator changed its name several times, and saw its mission successively specified and broadened. First established in 1953,[1] the French Technical Commission for Collusions and Dominant Positions' main goal was the fight against cartels and widespread price fixing in post-war France. In 1963, the Commission's objectives were extended to allow the formal investigation of cases of dominant positions.[2] In practice, this Commission would directly notify the Economic Ministry, which would then decide whether to impose fines.

Following the 1973 oil crisis, Prime Minister Raymond Barre and also an economics professor, advocated a stronger control of price fixing arising from anti-competitive behaviors. In 1977, the Commission became the Competition Commission (*Commission de la Concurrence*). In parallel of its mandate of detecting cartel and abuse of dominant positions, the Commission was to advise the French government on all competition-related matters, including on vertical and horizontal mergers and acquisitions.

The period 1986 to 2009 is important as it spans the beginning of our empirical analysis. Over this period, the Commission undergoes important transformations: its name is changed to the Competition Council (*Conseil de la Concurrence*) and the 1986 Ordinance introduces several changes. Companies can directly refer cases to the Council. Moreover, the antitrust body becomes more independent, better protects concerned parties' rights and is now able to directly fine the firms found guilty of anti-competitive practices, though this does not apply to merger projects. The 2001 New Economic Regulation Law further introduces leniency and transaction programs to better detect and fight cartels.[3]

Finally, as of 2008, the Competition Council turns into the Competition Authority (*Autorité de la Concurrence* or ADLC, henceforth). The 2008 Law on the Modernization of Economy not only gives the right to the Authority to review merger and acquisitions independently from the Minister of Economy, but also to investigate potential anti-competitive cases on its own.

There are two tools in ADLC's arsenal: fines and injunctions. Fines are set "ac-

[1] Décret n°53-704 du 9 août 1953.

[2] Loi n°63-628 du 2 juillet 1963.

[3] A firm part of a cartel can go to the authority and report it. Under specific circumstances the firm will receive a more lenient fine that the other members of the cartels or not be fined at all. Large cartels dismantled through a leniency program can be found here.

cording to the seriousness of the facts, the extent of the harm done to the economy, the individual situation of the company that has committed the infringement and of the group to which it belongs to, and whether it is an infringement that has been repeated or not".[4] Fines are capped at "10% of the global turnover of the group to which the company that is being fined belongs to" or at a maximum amount of the fine capped at 3 million euros if the infringement is committed by an entity other than a for-profit firm.[5] Alternatively the ADLC can issue injunctions to formally notify companies to stop anticompetitive behavior.

## A.2 Firm-level Database on Cartels

In order to extract information on the identity of the firms fined by the ADLC we proceed as follows. First, we scrape the website of the ADLC to recover all the decision files over the period 1994-2019. These PDF documents contain information on the situation of the market impacted by anti-competitive behaviors, the notification date of the case to the ADLC, the names of the firms fined for anti-competitive behaviors, the types of infraction they committed, their sales and the duration of the infraction. Some of these files contain information on when the firms were notified by the ADLC that an investigation is going to be launched. Extracting and getting data on the identity of these anti-competitive companies is straightforward to the extent that the layout is relatively similar across decision files. A salient and important example is that of the companies' name which always appears at the end of the PDF right after the word *Décide* ("Decides").

Second, we use Python's textual analysis tools to back out the name of these companies, their sales, the date when the ADLC was first notified of the infraction and the corresponding amount of the fine for each firm. This step requires some manual cleaning as some companies, numbers and cases are misreported. We therefore go through all the files to complement the information extracted from the textual analysis and double check that our newly created dataset is not missing anything that would appear in the original PDF files but that we would miss via the textual analysis exercise. At this stage, the dataset is informative about the identity (name) of the firms that were fined by the French Antitrust Authority, their sales, the case number of the decision, the amount of the fine for each firm and the notification date of the case to the ADLC.

Third, we make use of Orbis and Python to recover information on the identification number of the firms which will then allow us to match our database to the balance-sheet data. To do so, we upload our temporary database into the Batch

[4] French Commercial Code, L.420-1 or L.420-2.
[5] French Commercial Code, L.464-2.

Search engine of Orbis to look for the SIREN number of each firm given its name. We complement this information with a Python script that allows us to obtain the SIREN number of firms based on a Bing search of that firm's name.[6] Although these methods are imperfect, they facilitate the matching with FICUS.

Finally, before matching our database with FICUS, we manually verify that the SIREN numbers obtained from Orbis and from our scraping procedure are correct. We do so by making sure that the sales (in euros) of the firm in our database correspond to those reported in FICUS. For the firms that were not matched by any means in our third step, we manually search for them in FICUS using the information on their sales and add their SIREN number directly in our database.

## A.3 Sample

Our main sample consists of observations with strictly positive values for gross value-added, total and domestic sales, number of employees, labor compensation, expenditures on materials and tangible fixed assets. We keep firms that reported having more than two employees throughout the period from 1994 to 2007. To eliminate outliers, we exclude firm-year observations where the growth rate of total sales falls within the top 1% or bottom 1% of the growth rate distribution within each 2-digit sector. The final sample represents more than 80% of the total sales in France for each year from 1994 to 2007.

## A.4 List of Variables

We describe below the different variables used in our empirical framework.

- **APE code:** 4-digit industry code. Before 2008, APE codes are available in a 4-digit format corresponding to the NAF Rev. 1 classification. *Source: FICUS*

- **Capital:** Tangible fixed assets. This includes land, buildings, machinery, and other installations. The corresponding variable in FICUS is IMMOCOR. *Source: FICUS*

- **Cartel member:** Dummy variable that takes the value one if the firm engaged in anti-competitive practices in a given year. *Source: Moreau-Panon database*

- **Employment:** Total number of employees working in each firm (EFFSALM). *Source: FICUS*

- **Gross value-added:** This variable is directly available in FICUS (VAHT) and follows the accounting definition according to which it is equal to total sales

[6] We thank Arthur Guillouzouic Le Corff for sharing his code.

minus input expenses taking into account changes in inventories. *Source: FICUS*

- **Labor compensation:** This variable is the sum of two components separately available in the fiscal files (SALTRAI, CHARSOC): salaries and social benefits that are paid by the employer and that benefit the worker in the form of retirement funds, social security funds etc. *Source: FICUS*

- **Labor productivity:** This variable is the log of real value-added per employee, where real value-added is calculated by dividing gross value-added by 2-digit sector-specific deflators. *Source: FICUS and authors' calculation*

- **Market shares:** A firm's market share is defined at the 4-digit level. We compute market shares by dividing a firm's total sales by the total amount sold by all the firms operating in the same market at a point in time. *Source: FICUS and authors' calculation*

- **Markups:** Markups are estimated according to the methodology of De Loecker and Warzynski (2012). We further winsorize the top and bottom 5% of the markup distribution in each calendar year. *Source: Authors' calculation*

- **Materials:** Materials are defined as the sum of expenditures on raw materials and merchandises net of changes in stocks. The corresponding variables in FICUS are ACHAMPR, ACHAMAR, VARSTMP and VARSTMA. *Source: FICUS and authors' calculation*

- **NAF code:** 2-digit sector code according to the NACE Rev. 1 classification. Some sectors are pooled together, depending to the availability of sector-price deflators. *Source: FICUS*

- **Total sales:** Total sales (domestic sales plus export sales) reported by the firm in thousands of euros (CATOTAL). *Source: FICUS*

- **Wages:** Firm-level wages are obtained by dividing labor compensation by employment. *Source: FICUS and authors' calculation*

**Market definition.** We use both 2-digit and 4-digit industry classification. In the FICUS dataset, each firm is assigned a 4-digit principal activity code ("Code APE") by the INSEE and whose aim is to pin down in which industry the firm mostly operates. Because the precise breakdown of sales across products is not available, the relevant market of a firm is its 4-digit industry code. Therefore, throughout the paper, we will denote a firm's market share by its market share in the relevant 4-digit industry code. Our definition of a sector follows the NAF Rev. 1 classification.

# B Additional Evidence on Cartel Composition

## B.1 Empirical Literature

While we are not aware of any other empirical test of our results in Section I in the main text, some authors have found that the cumulative market share of cartel members is extremely large, which would arguably suggest that cartel members are the top producers in their industry. For instance, Combe and Monnier (2012) find that the average cartel market share in their sample is 80% and that two-thirds have a cumulative market share higher than 75%.[7] Similarly, Zimmerman and Connor (2005) report an average cartel market share of 85%, while Combe and Monnier (2012) report an average and a median cartel market share of 75%. Harrington Jr et al. (2015) document that the German cement cartel that operated from 1991 until 2002 was made up of the six largest cement firms which controlled 86% of the market. While these pieces of evidence from the literature do not rule out the possibility that cartels account for a large share of industry sales without strictly being formed by the most productive firms, they are nonetheless indicative of the fact that at least some of the top producers belong to cartels.

## B.2 Additional Results

Table E3 investigates the characteristics of both colluding firms and firms that classify as competitive. Colluding firms have a much higher market share —their market share averages 3% versus 0.0% for non-colluding firms, sell more, spend more on intermediate goods, have more employees, are more productive —as measured by labor productivity —, and are more capital-intensive.

## B.3 Theoretical Literature

As reviewed in Asker and Nocke (2021), the theoretical literature on the endogenous choice of cartel formation remains scarce with the recent exception of Bos and Harrington (2010) and Bos and Harrington (2015) who consider cartel formation across firms that are ex ante heterogenous in their capacities. The important result is that larger firms are more likely to find it profitable to join a cartel because of a trade-off: joining the cartel will allow them to increase their markups and prices but it will also lead to a decrease in their sales. The latter effect is larger for smaller firms with a low capacity, so that "we should not expect a cartel to include very

[7] This sample includes the 48 cartels for which they were able to compute the cumulative cartel market share.

small firms" (Bos and Harrington, 2010). Bos and Harrington (2015) include a competition authority that can detect and convict cartels. They find that antitrust enforcement deters small firms from joining a cartel.

# C Firm-level Markups

## C.1 De Loecker and Warzynski (2012)'s Approach

In seminal work, De Loecker and Warzynski (2012) show how to recover firm-level markups $\mu_{kt}$ using production data. Assuming that producers are cost-minimizing, one can show

$$\mu_{kt} = \frac{\beta^x_{kt}}{\alpha^x_{kt}}, \tag{1}$$

where $\beta^x_{kt} := \frac{\partial F(.)/\partial x_{kt}}{F(.)/x_{kt}}$ is the output elasticity of a flexible input $x$ and $\alpha^x_{kt} := \frac{p^x_{kt} x_{kt}}{p_{kt} q_{kt}}$ is that input's revenue share.

The denominator $\alpha^x_{kt}$ is typically available in standard production data while the output elasticity on the flexible input needs to be recovered, typically through production function estimation or through a cost share approach. The cost share approach is robust to the fact that revenue elasticities differ from output elasticities when markups vary across firms (Bond et al., 2021). We thus choose to rely on that approach, popularized by Foster et al. (2008), and implement Raval (2023a)'s estimator which we now explain briefly.

## C.2 Cost-share Approach

For the sake of simplicity, we drop the sectoral subscript $s$. Let us assume that production is CES with elasticity of substitution $\theta$ between labor $l_{kt}$, capital $k_{kt}$ and intermediates $m_{kt}$. Moreover, $b_{kt}$ is a labor-augmenting productivity term while $z_{kt}$ is Hicks-neutral. For the sake of simplicity, we omit the time subscript. We further assume a more realistic production function to estimate firm-level markups, allowing firms to use capital and intermediate inputs:

$$q_k = z_k \left[ (1 - \alpha_l - \alpha_m)\, k_k^{(\theta-1)/\theta} + \alpha_l \left(b_k l_k\right)^{(\theta-1)/\theta} + \alpha_m m_k^{(\theta-1)/\theta} \right]^{\theta/(\theta-1)}.$$

Taking factor prices as given and maximizing profits yields the following first-order conditions:

$$\frac{w l_k}{P_k q_k} = \frac{\beta^l_k}{\mu_k} = \frac{1}{\mu_k} \left( \frac{w_k}{\lambda_k z_k} \right)^{1-\theta} (\alpha_l)^{\theta} \left(b_k\right)^{\theta-1},$$

$$\frac{p_k^m m_k}{P_k q_k} = \frac{\beta_k^m}{\mu_k} = \frac{1}{\mu_k}\left(\frac{p_k^m}{\lambda_k z_k}\right)^{1-\theta} (\alpha_m)^\theta,$$

$$\frac{R k_k}{P_k q_k} = \frac{\beta_k^k}{\mu_k} = \frac{1}{\mu_k}\left(\frac{R}{\lambda_k z_k}\right)^{1-\theta} (1-\alpha_l-\alpha_m)^\theta,$$

where $\lambda_k$ is the marginal cost of production.

### C.2.1 Standard Approach

Let us assume that the production function is Cobb-Douglas, i.e. $\theta = 1$.

Focusing on materials, combining the first-order conditions yields:

$$\frac{\beta_k^m}{\beta_k^m + \beta_k^l + \beta_k^k} = \frac{p_k^m m_k}{w l_k + p_k^m m_k + R k_k}.$$

One can then recover the output elasticity of materials by further assuming that firms' output elasticities are the same within a given sector ($\beta_k^j = \beta_s^j$) for input $j$, and assuming that the degree of returns to scale is the same across sectors (RTS $:= \sum_j \beta_s^j$). In this case, the output elasticity in a given industry is given by:

$$\beta_s^m = \text{RTS} \times \frac{p_k^m m_k}{w l_k + p_k^m m_k + R k_k}. \tag{2}$$

Taking averages across firms within a 4-digit industry provides an estimate of the output elasticity of materials. For the sake of simplicity, we set RTS $= 1$.

### C.2.2 Accounting for Non-neutral Productivity Differences

Raval (2023b) shows that non-neutral technology can explain why markups estimated using different types of flexible inputs are negatively correlated and exhibit opposite time trends. Indeed, when $\theta \neq 1$, the factor-augmenting technology term $b_k$ affects output elasticities differently. Indeed, it affects the output elasticity indirectly through marginal costs $\lambda_k$ but also directly, through the $b_k^{\theta-1}$ term.

Taking the ratio of the first-order conditions for materials and labor yields:

$$b_k = \left(\frac{\alpha_l}{\alpha_m}\right)^{\frac{-\theta}{\theta-1}} \frac{w_k}{p_k^m} \left(\frac{w l_k}{p_k^m m_k}\right)^{\frac{1}{\theta-1}}.$$

Firms assigned to groups based on their labor to materials cost ratio will thus have similar values of $b_k$ and thus output elasticities. We follow Raval (2023a) by assigning firms to different quintiles within their 4-digit industry. Output elasticities are the input shares of total cost within a 4-digit industry-quintile. In other words, we take averages of the right-hand side of Equation (2) within each 4-digit industry-quintile pair.

### C.3 Estimation

To estimate markups from Equation (1), we first winsorize the revenue shares at the 5% level separately by calendar year and 2-digit sector to reduce the influence of outliers. We then define the cost share of materials as the ratio of expenditures on materials and merchandises accounting for variations in stocks (ACHAMPR, ACHAMAR, VARSTMP and VARSTMA) to total cost.[8] Total cost includes expenditures on materials, labor compensation, expenditures on service inputs (AUTACH), and capital expenditures. Note that we consider service inputs as a separate input, following Burstein et al. (2020). One may view capital in the production function as a bundle of service inputs and tangible assets, as service inputs are arguably less flexible.

The computation of capital expenditures is a bit more involved, and we turn to it now. Capital expenditures are defined as tangible fixed assets (IMMOCOR) times the capital rental rate, where we compute the rental rate as the sum of the real interest rate and the depreciation rate.[9] The real interest rate is defined as $R_t = \frac{i_t - \pi_t}{1+\pi_t}$, where $i_t$ denotes the nominal interest rate and $\pi_t$ denotes the inflation rate. Information on the nominal interest rate comes from 10-year government bond yields from the FRED of St. Louis database, while we use the World Bank national accounts data to recover the GDP deflator as our measure of inflation. Averaging over 1994-2007, we obtain a real interest rate of 1.68%. Finally, we use a depreciation rate of 5%, which corresponds to buildings in the classification of Boehm and Oberfield (2020).

We then compute the output elasticity of materials for each industry-quintile combination as detailed above. Finally, we winsorize the underlying distribution of firm-level markups at the 5% level, separately by calendar year.

# D Mathematical Appendix

## D.1 Model Derivations

This section derives equilibrium conditions when a subset of firms in each sector $s$ belong to a cartel $\mathcal{C}$: $\mathcal{C}_s \subseteq K_s$ and non-cartel members behave competitively. Firms

[8] Note that the definition of materials follows that of Burstein et al. (2020).

[9] Using tangible fixed assets to measure capital is consistent with the approach followed by De Ridder (2024).

do not internalize the effect of their decision on $c$ and $P$ and take wages and productivity levels as given.

The production side of the economy consists of a continuum of sectors indexed by $s \in [0,1]$. Final consumption $c$ is produced by a competitive firm that combines the outputs from all the sectors $y_s$ with a CES technology with elasticity $\eta$:

$$c = \left[ \int_0^1 y_s^{\frac{\eta-1}{\eta}} ds \right]^{\frac{\eta}{\eta-1}} . \tag{3}$$

The inverse demand function for each intermediate output from sector $s$ is given by:

$$\frac{P_s}{P} = \left( \frac{y_s}{c} \right)^{-\frac{1}{\eta}} , \tag{4}$$

where $P$, the price index for final consumption representing the "true cost of living", is a function of the sectoral prices:

$$P = \left[ \int_0^1 P_s^{1-\eta} ds \right]^{\frac{1}{1-\eta}} . \tag{5}$$

Each sector is populated by a finite number of firms $K_s$ indexed by $k$. Because each firm has a non-zero measure, it is therefore "large in the small but small in the large" (Neary, 2003), i.e., firms are small with respect to the economy but large in their own sector. The output of sector $s$ is a composite of the firms' outputs, combined with a CES technology with elasticity parameter $\rho$:[10]

$$y_s = \left[ \sum_{k=1}^{K_s} (q_{sk})^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}} . \tag{6}$$

The inverse demand functions within each sector are given by:

$$\frac{P_{sk}}{P_s} = \left( \frac{q_{sk}}{y_s} \right)^{-\frac{1}{\rho}} , \tag{7}$$

where the price index $P_s$ in sector $s$ is given by

$$P_s = \left[ \sum_{k=1}^{K_s} (P_{sk})^{1-\rho} \right]^{\frac{1}{1-\rho}} . \tag{8}$$

[10]Goods are imperfect substitutes, $\rho < \infty$, and more substitutable within than between sectors, $1 < \eta < \rho$.

### D.1.1 Cournot Competition

**Non-cartel members.** Competitive firms that do not belong to the cartel ($k \notin \mathcal{C}_s$) maximize their own profits. Their prices $P_{sk}$ and quantities $q_{sk}$ solve the following maximization problem:

$$\max_{q_{sk}} \left[ \left( P_{sk} - \frac{W}{z_{sk}} \right) q_{sk} \right], \quad \forall k \notin \mathcal{C}_s \tag{9}$$

subject to

$$\left( \frac{P_{sk}}{P} \right) = \left( \frac{q_{sk}}{y_s} \right)^{-\frac{1}{\rho}} \left( \frac{y_s}{c} \right)^{-\frac{1}{\eta}}. \tag{10}$$

Given the definition of sectoral output $y_s$ in Equation (6) and the inverse demand function of Equation (10), prices $P_{sk}$ can be rewritten as:

$$P_{sk} = Pc^{\frac{1}{\eta}} q_{sk}^{-\frac{1}{\rho}} y_s^{\frac{\eta-\rho}{\eta\rho}} = Pc^{\frac{1}{\eta}} q_{sk}^{-\frac{1}{\rho}} \left( \sum_{k=1}^{K_s} (q_{sk})^{\frac{\rho-1}{\rho}} \right)^{\frac{\eta-\rho}{\eta(\rho-1)}}. \tag{11}$$

Using the previous equation in the maximization problem detailed in Equation (9) yields:

$$\max_{q_{sk}} \left[ Pc^{\frac{1}{\eta}} q_{sk}^{\frac{\rho-1}{\rho}} \left( \sum_{k=1}^{K_s} (q_{sk})^{\frac{\rho-1}{\rho}} \right)^{\frac{\eta-\rho}{\eta(\rho-1)}} - \frac{W}{z_{sk}} q_{sk} \right], \quad \forall k \notin \mathcal{C}.$$

The first-order condition with respect to $q_{sk}$ yields:

$$P_{sk} \frac{\rho - 1}{\rho} + \frac{q_{sk}^{\frac{\rho-1}{\rho}}}{\sum_{j=1}^{K_s} q_{sj}^{\frac{\rho-1}{\rho}}} \frac{\eta - \rho}{\eta\rho} P_{sk} - \frac{W}{z_{sk}} = 0.$$

Given the CES inverse demand functions given in Equation (7), the market share of a firm in its sector $\omega_{sk}$ can be expressed as $\omega_{sk} = \frac{q_{sk}^{\frac{\rho-1}{\rho}}}{\sum_{j=1}^{K_s} q_{sj}^{\frac{\rho-1}{\rho}}}$. Using this expression and rearranging the first-order condition yields:

$$P_{sk} = \left[ 1 - \frac{1}{\rho}(1 - \omega_{sk}) - \frac{1}{\eta}\omega_{sk} \right]^{-1} \times \frac{W}{z_{sk}}. \tag{12}$$

Defining the demand elasticity as $\varepsilon(\omega_{sk}) := \left[ \frac{1}{\rho}(1 - \omega_{sk}) + \frac{1}{\eta}\omega_{sk} \right]^{-1}$ and rearranging the previous equation yields

$$P_{sk} = \mu_{sk} \frac{W}{z_{sk}}, \quad \forall k \notin \mathcal{C}, \tag{13}$$

where firm-level markups $\mu_{sk}$ are given by

$$\frac{1}{\mu_{sk}} = \frac{\rho - 1}{\rho} + \frac{\eta - \rho}{\eta \rho} \omega_{sk}, \quad \forall k \notin \mathcal{C}. \tag{14}$$

**Cartels and cross-ownership.** The simple form of collusion analyzed in the main text is meant to capture a large range of cartel arrangements, and is also consistent with the profit distortions created by cross-ownership. To see this, consider an industry with $K$ firms, let $\pi_k$ denote the profit function of firm $k$. Let $\beta_{jl}$ denote the share of firm $j$ which is owned by firm $l$ and $\gamma_{lj}$ firm $l$'s control or influence over firm $j$'s decisions. The financial profits accruing to firm $l$ correspond to the portfolio $\pi^l = \sum_j \beta_{jl} \pi_j$, where $\pi_l$ are the profits generated by firm $l$'s operations. However, because other firms can influence firm $k$'s operations, and that their shareholders' interests are not perfectly aligned, the managers of firm $k$ maximize a weighted average $\tilde{\pi}_k$ of the firm's shareholders portfolios, where the weights depend on the controlling shares. The objective function of firm $k$ is given by:

$$\tilde{\pi}_k = \sum_l \gamma_{kl} \pi^l = \sum_l \gamma_{kl} \sum_j \beta_{jl} \pi_j. \tag{15}$$

Taking $\pi_k$ out of the second summation and normalizing by $\sum_l \gamma_{kl} \beta_{kl}$ so as to isolate $\pi_k$, we can rewrite the objective function as (dropping the sectoral index $s$):

$$\tilde{\pi}_k \propto \pi_k + \sum_{j \in \mathcal{C} \setminus \{k\}} \frac{\sum_l \gamma_{kl} \beta_{jl}}{\sum_l \gamma_{kl} \beta_{kl}} \pi_j = \pi_k + \sum_{j \in \mathcal{C} \setminus \{k\}} \kappa_{kj} \pi_j, \tag{16}$$

where $\kappa_{kj} := \frac{\sum_l \gamma_{kl} \beta_{jl}}{\sum_l \gamma_{kl} \beta_{kl}}$ is the firm-specific weight assigned to other cartel members' profits. Equation (16) makes it clear that firm $k$ maximizes its own profits given by $\pi_{sk}$ *and* other firms' profits. Moreover, the profit weights are *firm*-specific.[11]

Cartel members solve the following maximization problem:

$$\max_{q_{sk}} \left[ \left( P_{sk} - \frac{W}{z_{sk}} \right) q_{sk} + \sum_{j \in \mathcal{C} \setminus \{k\}} \kappa_{kj} \left( P_{sj} - \frac{W}{z_{sj}} \right) q_{sj} \right], \quad \forall k \in \mathcal{C}_s, \tag{17}$$

[11] We note that these profit weights can be larger than one, in which case a firm values other firms' profits more than its own. Such a case is studied in Backus et al. (2021).

subject to the inverse demand function:

$$\left(\frac{P_{sk}}{P}\right) = \left(\frac{q_{sk}}{y_s}\right)^{-\frac{1}{\rho}} \left(\frac{y_s}{c}\right)^{-\frac{1}{\eta}}.$$

Their maximization problem can be written as:

$$\max_{q_{sk}} \left[ Pc^{\frac{1}{\eta}} q_{sk}^{\frac{\rho-1}{\rho}} \left( \sum_{k=1}^{K_s} (q_{sk})^{\frac{\rho-1}{\rho}} \right)^{\frac{\eta-\rho}{\eta(\rho-1)}} - \frac{W}{z_{sk}} q_{sk} \right.$$
$$\left. + \sum_{j \neq k} \kappa_{kj} \left( Pc^{\frac{1}{\eta}} q_{sj}^{\frac{\rho-1}{\rho}} \left( \sum_{k=1}^{K_s} (q_{sk})^{\frac{\rho-1}{\rho}} \right)^{\frac{\eta-\rho}{\eta(\rho-1)}} - \frac{W}{z_{sj}} q_{sj} \right) \right].$$

Taking the derivative of this equation with respect to $q_{sk}$ yields:

$$\frac{\partial \tilde{\pi}_{sk}}{\partial q_{sk}} = \frac{\partial \pi_{sk}(q_{sk}, q_{s-k})}{\partial q_{sk}} + \sum_{j \neq k} \kappa_{kj} \frac{\partial \pi_{sj}(q_{sk}, q_{s-k})}{\partial q_{sk}}.$$

The first term remains the same as for non-cartel members, while the second term is an additional component introduced by the cartel. This term reflects the firm's partial internalization of their decision on other members' profits. This can be rewritten as

$$\frac{\partial \tilde{\pi}_{sk}}{\partial q_{sk}} = \left[ 1 - \left\{ \frac{1}{\rho} + \left( \frac{1}{\eta} - \frac{1}{\rho} \right) \omega_{sk} \right\} \right] P_{sk} - \frac{W}{z_{sk}} + \sum_{j \neq k} \kappa_{kj} \frac{\partial P_{sj}}{\partial q_{sk}} q_{sj},$$

where

$$\frac{\partial P_{sj}}{\partial q_{sk}} q_{sj} = \left( \frac{1}{\rho} - \frac{1}{\eta} \right) P_{sk} \omega_{sj}. \tag{18}$$

Collecting the terms and rearranging yields

$$P_{sk}^{\mathcal{C}} = \mu_{sk}^{\mathcal{C}} \frac{W}{z_{sk}}, \quad \forall k \in \mathcal{C}, \tag{19}$$

where firm-level markups are given by

$$\frac{1}{\mu_{sk}^{\mathcal{C}}} = \frac{\rho-1}{\rho} + \frac{\eta-\rho}{\eta\rho} \left( \omega_{sk} + \sum_{j \in \mathcal{C} \setminus \{k\}} \kappa_{kj} \omega_{sj} \right), \quad \forall k \in \mathcal{C}. \tag{20}$$

The parameter $\kappa_{kj}$ controls the degree of symmetry of the cartel agreement. If $\kappa_{kj} = 1$ then a member of the cartel cares equally about her own-profits than that of other members of the cartel. In this extreme case, all the members of the cartels set the same markup, that depends only on the sum of the equilibrium market shares of the cartel members. Conversely, $\kappa_{kj} = 0$ corresponds to the competitive Nash-Cournot equilbrium.

**Full symmetric collusion.** Consider the case where the profit weights are equal to unity $\kappa_{kj} = 1$. This is the case, for example, when the share of two different rival firms $j$ and $k$ owned by investor $l$ is the same, *i.e.* $\beta_{jl} = \beta_{kl}$. This also arises when the control shares are the same across firms $\gamma_{kl} = \gamma_k$.[12] The case where the profit weights are equal to unity boils down to full collusion where firms maximize their joint profits and equally weight all cartel members' profits. Cartel member $k$'s markup is given by:

$$\frac{1}{\mu^{\mathcal{C}}_{sk}} = \frac{\rho - 1}{\rho} + \frac{\eta - \rho}{\eta\rho} \sum_{j \in \mathcal{C}} \omega_{sj}. \tag{21}$$

All colluding firms that belong to $\mathcal{C}$ charge the same markup that is governed by the combined market share $\sum_{j \in \mathcal{C}} \omega_{sj}$.

**Partial symmetric collusion.** Consider the case where the profit weights differ from unity but are *constant* across cartel members. This is the case when firms' ownership shares are constant across different firms so that $\beta_{jl} = \beta_j$ and $\beta_{kl} = \beta_k$. These shares can vary so that $\beta_j \neq \beta_k$ as long as certain parametric restrictions are satisfied. For instance, if $\beta_j \propto \kappa^{\zeta_j}$, $\beta_k \propto \kappa^{\zeta_k}$ and $\zeta_j - \zeta_k = 1$, the profit weights are equal to $\kappa$. We assume that $\kappa > 0$, $\zeta_j > 0$, $\zeta_k > 0$. In this case $\kappa_{kj} = \frac{\sum_l \gamma_{kl}\beta_{jl}}{\sum_l \gamma_{kl}\beta_{kl}} = \frac{\beta_j}{\beta_k} = \frac{\kappa^{\zeta_j}}{\kappa^{\zeta_k}} = \kappa$ where the last step follows from $\zeta_j - \zeta_k = 1$. Markups are given by:

$$\frac{1}{\mu^{\mathcal{C}}_{sk}} = \frac{\rho - 1}{\rho} + \frac{\eta - \rho}{\eta\rho} \left( \omega_{sk} + \kappa \sum_{j \in \mathcal{C} \setminus \{k\}} \omega_{sj} \right). \tag{22}$$

Equation (22) generates markup dispersion as each cartel member's decision's impact on other cartel members' profits is not fully internalized. As a result, markups depend on both the firm's own market share and the combined market share of the cartel. Markup dispersion across cartel members is higher in this case than in the full collusion case, as the weights assigned to other cartel members are not necessarily equal to one.

### D.1.2 Bertrand Competition

We can alternatively solve the model under the assumption that firms engage in a static game of Bertrand Competition. One can combine Equation (4) and Equa-

[12] The profit weights also equal unity in this case as $\sum_l \beta_{jl} = 1$.

tion (7):

$$q_{sk} = P_{sk}^{-\rho} \left( \sum_k P_{sk}^{1-\rho} \right)^{\frac{1}{\eta} - \frac{1}{\rho}} c P^{\eta}.$$

The firm chooses its prices subject to the above constraint. This yields the first-order condition:

$$q_{sk} + \left( P_{sk} - \frac{W}{z_{sk}} \right) \frac{\partial q_{sk}}{\partial P_{sk}} = 0. \tag{23}$$

The derivative of the constraint with respect to the firm's price gives:

$$\frac{\partial q_{sk}}{\partial P_{sk}} = -\rho \frac{q_{sk}}{P_{sk}} + (\rho - \eta)\omega_{sk} \frac{q_{sk}}{P_{sk}}.$$

Plugging this equation back into Equation (23) and rearranging yields:

$$P_{sk} = \frac{\rho - (\rho - \eta)\omega_{sk}}{\rho - (\rho - \eta)\omega_{sk} - 1} \frac{W}{z_{sk}}. \tag{24}$$

In the competitive Nash-Bertrand case, the demand elasticities are given by

$$\varepsilon_{sk}(\omega_{sk}) = \rho - (\rho - \eta)\omega_{sk}. \tag{25}$$

The elasticities are now sales-weighted arithmetic means, rather than sales-weighted harmonic means as in the Cournot case. They are thus at least as large as in the Cournot case. Firm-level markups in the Bertrand setting are thus typically smaller than in the Cournot setting. The demand elasticities of the cartel members are given by:

$$\varepsilon_{sk}^{\mathcal{C}}(\omega_{sk}) = \rho - (\rho - \eta) \left( \omega_{sk} + \sum_{j \in \mathcal{C} \setminus \{k\}} \kappa_{kj} \omega_{sj} \right). \tag{26}$$

## D.2 Proof of Proposition 1

**Proposition 1.** *Starting from the competitive equilibrium, symmetric collusion increases the sectoral price index. The increase is larger i) the greater the collusion intensity, $\Delta\kappa$ and ii) the larger the market share controlled by the cartel, $\omega_{s\mathcal{C}}$. In particular, the sectoral price increase is*

$$\hat{P}_s = \frac{1}{\rho - 1} \frac{1}{1 - \sum_k \omega_{sk} \Upsilon_{sk}} \sum_{k \in \mathcal{C}} \Upsilon_{sk} \left( \omega_{s\mathcal{C}} - \omega_{sk} \right) \Delta\kappa,$$

*where* $\Upsilon_{sk} := \frac{\omega_{sk}(\rho-1)\left(\frac{1}{\eta} - \frac{1}{\rho}\right)\mu_{sk}}{1+\omega_{sk}(\rho-1)\left(\frac{1}{\eta} - \frac{1}{\rho}\right)\mu_{sk}} \in (0,1)$ *represents the elasticity of the firm's own price with respect to the sectoral price index and* $\omega_{s\mathcal{C}} := \sum_{k \in \mathcal{C}} \omega_{sk}$ *is the cartel's combined market share.*

*Proof.* We study the economy as it transitions from the competitive Nash-Cournot equilibrium at time $t$ to a small level of collusion $\Delta\kappa$ at time $t + \Delta t$. For any variable $x_t$, we let $x$ denote the value of the variable in the initial equilibrium and $\hat{x}$ denote the log change between time $t$ and $t + \Delta t$, that is,

$$\hat{x}_{sk} := \log x_{sk,t+\Delta t} - \log x_{sk,t}. \tag{27}$$

We drop the time index henceforth to simplify notations. For non-cartel members, differentiating the markup equation around the competitive equilibrium (using a first-order approximation), we have

$$\hat{\mu}_{sk} = \mu_{sk}\left(\frac{1}{\eta} - \frac{1}{\rho}\right)\left(\omega_{sk}\hat{\omega}_{sk}\right). \tag{28}$$

Since $\omega_{sk}$ is the market share of firm $k$ in its sector $s$ and writes

$$\omega_{sk} := \frac{P_{sk}q_{sk}}{\sum_{j=1}^{K} P_{sj}q_{sj}} = \left(\frac{P_{sk}}{P_s}\right)^{1-\rho}, \tag{29}$$

the first-order response of market shares is equal to

$$\hat{\omega}_{sk} = (\rho - 1)\left(\hat{P}_s - \hat{P}_{sk}\right). \tag{30}$$

Because there are no shocks to fundamental productivity levels, prices change solely due to changes in markups: $\hat{P}_{sk} = \hat{\mu}_{sk}$. Using Equation (28), we obtain the price response of non-cartel members:

$$\hat{P}_{sk} = \mu_{sk}\left(\frac{1}{\eta} - \frac{1}{\rho}\right)\left(\omega_{sk}(\rho - 1)\left(\hat{P}_s - \hat{P}_{sk}\right)\right). \tag{31}$$

Collecting the terms we obtain

$$\hat{P}_{sk} = \Upsilon_{sk}\hat{P}_s, \tag{32}$$

where we define

$$\Upsilon_{sk} := \frac{\omega_{sk}(\rho - 1)\left(\frac{1}{\eta} - \frac{1}{\rho}\right)\mu_{sk}}{1 + \omega_{sk}(\rho - 1)\left(\frac{1}{\eta} - \frac{1}{\rho}\right)\mu_{sk}}, \tag{33}$$

to denote the umbrella pricing effect. Note that since this effect is of the form $x \to \frac{ax}{1+ax}$ with $a > 0$, this umbrella pricing effect is increasing with firm size.

The change in market shares for non-cartel firms can be expressed as:

$$\hat{\omega}_{sk} = (\rho - 1)\left(1 - \Upsilon_{sk}\right)\hat{P}_s. \tag{34}$$

Note that if the price level increases, all non-cartel members increase their market shares. This increase is higher for smaller firms, as $\Upsilon_{sk}$ increases with size.

For cartel members, the change in markups at the first-order is given by

$$\hat{\mu}_{sk}^{\mathcal{C}} = \mu_{sk}\left(\frac{1}{\eta} - \frac{1}{\rho}\right)\left(\omega_{sk}\hat{\omega}_{sk} + \Delta\kappa \sum_{j\in\mathcal{C}\setminus\{k\}} \omega_{sj} + \kappa \sum_{j\in\mathcal{C}\setminus\{k\}} \omega_{sj}\hat{\omega}_{sj}\right). \tag{35}$$

Since $\kappa = 0$ at $t$, we obtain

$$\begin{aligned} \hat{P}_{sk}^{\mathcal{C}} &= \mu_{sk}\left(\frac{1}{\eta} - \frac{1}{\rho}\right)\left(\omega_{sk}(\rho - 1)\left(\hat{P}_s - \hat{P}_{sk}\right) + \Delta\kappa \sum_{j\in\mathcal{C}\setminus\{k\}} \omega_{sj}\right) \\ &= \Upsilon_{sk}\hat{P}_s + \frac{1}{\rho - 1}\frac{\Upsilon_{sk}}{\omega_{sk}}\left(\omega_{s\mathcal{C}} - \omega_{sk}\right)\Delta\kappa, \end{aligned} \tag{36}$$

where $\omega_{s\mathcal{C}} := \sum_{j\in\mathcal{C}} \omega_{sj}$ is the total market share controlled by the cartel. Note that the first term is similar to that of non-cartel members. It captures the umbrella channel from higher prices in the sector, while the additional term captures the distortion arising from collusion. This distortion is larger the larger the cartel, and the more intense the collusion. The associated change in market shares is:

$$\hat{\omega}_{sk}^{\mathcal{C}} = (\rho - 1)\left(1 - \Upsilon_{sk}\right)\hat{P}_s - \frac{\Upsilon_{sk}}{\omega_{sk}}\left(\omega_{s\mathcal{C}} - \omega_{sk}\right)\Delta\kappa. \tag{37}$$

As we have seen, non-cartel firms are all gaining market shares. Therefore, by construction, some cartel members must be losing market shares (corollary 1).

Given the definition of the sectoral price index and using Equation (32) and Equation (36), the change in the sectoral price index is given by

$$\begin{aligned} \hat{P}_s &= \sum_k \omega_{sk}\hat{P}_{sk} \\ &= \sum_{k\notin\mathcal{C}} \omega_{sk}\hat{P}_{sk} + \sum_{k\in\mathcal{C}} \omega_{sk}\hat{P}_{sk} \\ &= \sum_{k\notin\mathcal{C}} \omega_{sk}\Upsilon_{sk}\hat{P}_s + \sum_{k\in\mathcal{C}} \omega_{sk}\Upsilon_{sk}\hat{P}_s + \sum_{k\in\mathcal{C}} \Upsilon_{sk}\frac{1}{\rho - 1}\left(\omega_{s\mathcal{C}} - \omega_{sk}\right)\Delta\kappa \\ &= \hat{P}_s \sum_k \omega_{sk}\Upsilon_{sk} + \sum_{k\in\mathcal{C}} \Upsilon_{sk}\frac{1}{\rho - 1}\left(\omega_{s\mathcal{C}} - \omega_{sk}\right)\Delta\kappa \\ &= \frac{1}{1 - \sum_k \omega_{sk}\Upsilon_{sk}}\frac{1}{\rho - 1}\sum_{k\in\mathcal{C}} \Upsilon_{sk}\left(\omega_{s\mathcal{C}} - \omega_{sk}\right)\Delta\kappa. \end{aligned} \tag{38}$$

The sectoral price change is a weighted average of the firms' overcharges. Since $0 < \Upsilon_{sk} < 1$ for all $s, k$, and $\sum_k \omega_{sk} = 1$, we have $0 < \sum_k \omega_{sk} \Upsilon_{sk} < 1$. Therefore the price change is positive, which proves Proposition 1.

The proof of corollary 1 follows from Equation (34) and Equation (37). Notice that, everything else equal, as $\omega_{sk}$ tends to 0 the first term tends to $(\rho - 1)\hat{P}_s > 0$ while the second term tends to $-\left(\frac{\rho}{\eta} - 1\right)\omega_{s\mathcal{C}}\Delta\kappa < 0$. Depending on the composition of the cartel, $\lim_{\omega_{sk}\to 0}\hat{\omega}^{\mathcal{C}}_{sk}$ can be either positive or negative.

□

## D.3 Proof of Proposition 2

**Proposition 2.** *Starting from the competitive equilibrium, symmetric collusion increases the markups of all firms within the cartelized industry. In particular, i) for cartel members, the markup increase declines with firm size ii) while for non-cartel members, the markup increase increases with firm size. For cartel members ($k \in \mathcal{C}$) in sector s, the markup change is given by*

$$\hat{\mu}^{\mathcal{C}}_{sk} = \underbrace{\Upsilon_{sk}\hat{P}_s}_{\text{Umbrella Pricing}} + \underbrace{\frac{1}{\rho - 1}\frac{\Upsilon_{sk}}{\omega_{sk}}\left(\omega_{s\mathcal{C}} - \omega^{\mathcal{C}}_{sk}\right)\Delta\kappa}_{\text{Cartel Overcharge}}.$$

*Proof.* The first part of the proposition follows from Equation (32) and Equation (36). As $P_s$ increases, all prices go up, both for cartel and non-cartel members.

The second part of the proposition follows from the fact that $\Upsilon_{sk} \in (0, 1)$ increases with firm size. This entails that the markup increase is larger for larger non-cartel members. For cartel members, notice that $\frac{\Upsilon_{sk}}{\omega_{sk}}$ decreases with size. To see this, let $a := (\rho - 1)\left(\frac{1}{\eta} - \frac{1}{\rho}\right)$ so that:

$$\frac{\Upsilon_{sk}}{\omega_{sk}} = \frac{a\mu_{sk}}{1 + a\omega_{sk}\mu_{sk}}. \tag{39}$$

Taking the derivative with respect to firm-level market shares yields

$$\begin{aligned} \frac{\partial \frac{\Upsilon_{sk}}{\omega_{sk}}}{\partial \omega_{sk}} &= \frac{a\frac{\partial \mu_{sk}}{\partial \omega_{sk}}(1 + a\omega_{sk}\mu_{sk}) - a\mu_{sk} \times a\left(\mu_{sk} + \omega_{sk}\frac{\partial \mu_{sk}}{\partial \omega_{sk}}\right)}{(1 + a\omega_{sk}\mu_{sk})^2} \\ &= \frac{a\left(\frac{\partial \mu_{sk}}{\partial \omega_{sk}} - a\mu^2_{sk}\right)}{(1 + a\omega_{sk}\mu_{sk})^2}. \end{aligned} \tag{40}$$

Now recall that $\frac{\partial \mu_{sk}}{\partial \omega_{sk}} = \mu_{sk}^2 \left(\frac{1}{\eta} - \frac{1}{\rho}\right)$ and therefore:

$$\frac{\partial \frac{\Upsilon_{sk}}{\omega_{sk}}}{\partial \omega_{sk}} = \frac{a\mu_{sk}^2 \left(\frac{1}{\eta} - \frac{1}{\rho}\right)(2-\rho)}{(1 + a\omega_{sk}\mu_{sk})^2} < 0. \tag{41}$$

Therefore $\frac{\Upsilon_{sk}}{\omega_{sk}}$ is decreasing with firm size if and only if $\rho > 2$. This is the case in all our quantitative analysis. □

## D.4 Proof of Proposition 3

**Proposition 3.** *The impact of collusion on sectoral productivity is given by*

$$\hat{z}_s = \rho Cov_\omega \left[\frac{\mu_s}{\mu_{sk}}, \hat{\mu}_{sk}\right],$$

*where $\mu_s$ is the sectoral markup, the sales-weighted average of $x_k$ is denoted by $\mathbb{E}_\omega[x_k] = \sum_k \omega_k x_k$ and the sales-weighted covariance of any two variables $x_k$ and $z_k$ writes $Cov_\omega[x_k, z_k] = \mathbb{E}_\omega[x_k z_k] - \mathbb{E}_\omega[x_k]\mathbb{E}_\omega[z_k]$.*

*Proof.* Sectoral productivity $z_s$ is given by the following firm-size-weighted harmonic average of firm-level technical efficiency:

$$z_s := \left(\sum_k \frac{q_{sk}}{y_s}\frac{1}{z_{sk}}\right)^{-1}. \tag{42}$$

Sectoral productivity can be rewritten as

$$\begin{aligned} z_s &= \left(\sum_k \frac{q_{sk}}{y_s}\frac{1}{z_{sk}}\right)^{-1} \\ &= \left(\sum_k \left(\frac{P_{sk}}{P_s}\right)^{-\rho} {z_{sk}}^{-1}\right)^{-1} \\ &= \left(\sum_k \left(\frac{\mu_{sk}}{\mu_s}\right)^{-\rho} {z_{sk}}^{\rho-1}\right)^{\frac{1}{\rho-1}} \\ &= \frac{\left(\sum_k \mu_{sk}^{1-\rho} z_{sk}^{\rho-1}\right)^{\frac{\rho}{\rho-1}}}{\sum_k \mu_{sk}^{-\rho} z_{sk}^{\rho-1}}, \end{aligned}$$

where the second line relies on the fact that $\frac{q_{sk}}{y_s} = \left(\frac{P_{sk}}{P_s}\right)^{-\rho}$ from the first-order condition for profit-maximization. In the third line, we use the fact that $P_{sk} = \mu_{sk}\frac{W}{z_{sk}}$

and $P_s = \mu_s \frac{W}{z_s}$. In the last line we use the expression for sectoral markups

$$\begin{aligned}
\mu_s &= \left( \sum_k \mu_{sk}^{-1} \omega_{sk} \right)^{-1} \\
&= \left( \sum_k \mu_{sk}^{-1} \frac{\mu_{sk}^{1-\rho} z_{sk}^{\rho-1}}{\sum_k \mu_{sk}^{1-\rho} z_{sk}^{\rho-1}} \right)^{-1} \\
&= \frac{\sum_k \mu_{sk}^{1-\rho} z_{sk}^{\rho-1}}{\sum_k \mu_{sk}^{-\rho} z_{sk}^{\rho-1}},
\end{aligned}$$

where we have used the expression for market shares $\omega_{sk}$ expressed as a function of productivity and firm-level markups.[13]

Taking a first-order approximation around the initial equilibrium, we get

$$\begin{aligned}
\hat{z}_s &= \frac{\rho}{\rho-1} \sum_k (1-\rho) \omega_{sk} \hat{\mu}_{sk} + \rho \sum_k \frac{\mu_{sk}^{-\rho} z_{sk}^{\rho-1}}{\sum_k \mu_{sk}^{-\rho} z_{sk}^{\rho-1}} \hat{\mu}_{sk} \\
&= -\rho \sum_k \omega_{sk} \hat{\mu}_{sk} + \sum_k \rho \frac{\mu_s}{\mu_{sk}} \omega_{sk} \hat{\mu}_{sk} \\
&= -\rho \sum_k \omega_{sk} (1 - \frac{\mu_s}{\mu_{sk}}) \hat{\mu}_{sk} \\
&= -\rho \mathbb{E}_\omega \left[ \left( 1 - \frac{\mu_s}{\mu_{sk}} \right) \hat{\mu}_{sk} \right] \\
&= -\rho \left( Cov_\omega \left[ 1 - \frac{\mu_s}{\mu_{sk}}, \hat{\mu}_{sk} \right] + \mathbb{E}_\omega \left[ 1 - \frac{\mu_s}{\mu_{sk}} \right] \mathbb{E}_\omega \left[ \hat{\mu}_{sk} \right] \right) \\
&= -\rho Cov_\omega \left[ 1 - \frac{\mu_s}{\mu_{sk}}, \hat{\mu}_{sk} \right] \\
&= \rho Cov_\omega \left[ \frac{\mu_s}{\mu_{sk}}, \hat{\mu}_{sk} \right].
\end{aligned}$$

where the sales-weighted —$\omega_i(s)$ —average of $x_i(s)$ is denoted by $\mathbb{E}_\omega[x_i] = \sum_i \omega_i x_i$ and the sales-weighted covariance of any two variables $x_i$ and $z_i$ writes

$$Cov_\omega \left[ x_i, z_i \right] = \mathbb{E}_\omega[x_i z_i] - \mathbb{E}_\omega[x_i] \mathbb{E}_\omega[z_i].$$

and by definition of $\mu_s$ we have $\mathbb{E}_\omega \left[ 1 - \frac{\mu_s}{\mu_{sk}} \right] = 0$

□

[13] Market shares can be expressed as $\omega_{sk} = \frac{\mu_{sk}^{1-\rho} z_{sk}^{\rho-1}}{\sum_j \mu_{sj}^{1-\rho} z_{sj}^{\rho-1}}$.

## D.5 Alternative Collusion Arrangements and Overcharges

As previously shown, in the case of symmetric collusion, distortions in both prices and quantities are larger for smaller cartel members. This occurs because, with a uniform collusion intensity, larger firms exert a relatively greater influence on the pricing decisions of smaller firms. This effect can also be observed by considering alternative forms of collusion. Recall that the inverse markup of cartel members is

$$\frac{1}{\mu_{sk}^{\mathcal{C}}} = \frac{\rho-1}{\rho} - \left(\frac{1}{\eta} - \frac{1}{\rho}\right)\left(\omega_{sk} + \kappa_k \sum_{j\in\mathcal{C}\setminus\{k\}} \omega_{sj}\right), \tag{43}$$

where $\kappa_k$ for $k \in \mathcal{C}$ is a collection of collusive intensities. For small given changes of collusion intensity $\Delta\kappa_k$ we thus have

$$\hat{\mu}_{sk}^{\mathcal{C}} = \mu_{sk}\left(\frac{1}{\eta} - \frac{1}{\rho}\right)\left(\omega_{sk}\hat{\omega}_{sk} + \Delta\kappa_k \sum_{j\in\mathcal{C}\setminus\{k\}} \omega_{sj}\right). \tag{44}$$

Price changes take the general form

$$\begin{cases} \hat{P}_{sk}^{\mathcal{C}} = \Upsilon_{sk}\hat{P}_s + \Theta_{sk} \\ \hat{\omega}_{sk}^{\mathcal{C}} = (\rho-1)\left(1-\Upsilon_{sk}\right)\hat{P}_s - (\rho-1)\,\Theta_{sk}, \end{cases} \tag{45}$$

where the overcharge is

$$\Theta_{sk} = \frac{1}{\rho-1}\frac{\Upsilon_{sk}}{\omega_{sk}}\left(\omega_{s\mathcal{C}} - \omega_{sk}\right)\Delta\kappa_k, \tag{46}$$

if firm $k$ joins the cartel and 0 otherwise.

Notice that the sectoral price change is a weighted average of overcharges, times a multiplier

$$\hat{P}_s = \frac{1}{1-\sum_k \omega_{sk}\Upsilon_{sk}} \sum_{k\in\mathcal{C}} \omega_{sk}\Theta_{sk}. \tag{47}$$

Consider a more general class of collusion arrangements of the form

$$\Delta\kappa_k = \psi(\omega_{sk})\psi_{\mathcal{C}}\Delta\kappa, \tag{48}$$

where $\Delta\kappa$ controls the intensive margin of collusion, $\psi(\omega_{sk})$ controls the "slope" of the effort sharing across members depending only on a member's initial market share, and $\psi_{\mathcal{C}}$ is a scaling factor common to all cartel members. For uniform symmetric collusions, $\psi(.) = 1$ and $\psi_{\mathcal{C}} = 1$. Now consider a specific collusion arrangement characterized by

$$\begin{cases} \psi(\omega_{sk}) = \frac{\omega_{sk}}{\Upsilon_{sk}(\omega_{s\mathcal{C}} - \omega_{sk})} \\ \psi_{\mathcal{C}} = \sum_{k \in \mathcal{C}} \Upsilon_{sk} \frac{(\omega_{s\mathcal{C}} - \omega_{sk})}{\omega_{s\mathcal{C}}}. \end{cases} \tag{49}$$

It follows from Equation (47) that such a cartel would increase the sectoral price level by exactly the same amount as a symmetric cartel with $\Delta\kappa$, as shown in Equation (38). As a result, it would have the exact same impact on non-cartel members. In addition, under such an arrangement, there will be less disparity in distortions within the cartel, as prices and quantities are now given by

$$\begin{cases} \hat{P}^{\mathcal{C}}_{sk} = \Upsilon_{sk} \hat{P}_s + \frac{1}{\rho - 1} \psi_{\mathcal{C}} \Delta\kappa \\ \hat{\omega}^{\mathcal{C}}_{sk} = (\rho - 1)\left(1 - \Upsilon_{sk}\right) \hat{P}_s - \psi_{\mathcal{C}} \Delta\kappa. \end{cases} \tag{50}$$

Under this arrangement, the change in prices for cartel members is now increasing with the size of the firm but the bulk of the overcharge, $\frac{1}{\rho-1}\psi_{\mathcal{C}}\Delta\kappa$, is the same across the cartel. Therefore, such cartels operate closer to the "fairness" principle considered in Bos and Harrington (2010).

## D.6 Collusion and Cartel Stability

While collusion can raise cartel members' profits, the presence of short gains from defecting from the cartel arrangement threaten the stability of the cartel. Cartels can nevertheless be stable in a repeated game settings when participants can credibly threaten to punish defection (Abreu, 1988). We show in this section that our framework lends itself to the canonical analysis of cartel stability in a repeated game and we derive conditions i) for profits to increase after joining the cartel and ii) for stability when firms are patient enough.

Consider the change in log profits after a cartel $\mathcal{C}$ is formed

$$\hat{\Pi}^{\mathcal{C}}_{sk} = \log \pi^{\mathcal{C}}_{sk,t+dt} - \log \pi_{sk,t}, \tag{51}$$

where there is no collusion at $t = 0$. We first show that there exist incentives to deviate, that is, $\hat{\Pi}^{\mathcal{C}\setminus\{k\}}_{sk} > \hat{\Pi}^{\mathcal{C}}_{sk}$. As there are no productivity shocks, and since profits can be written $\pi_{sk} = (\mu_{sk} - 1)\frac{W}{z_{sk}} q_{sk}$, after taking logs and differentiating, we obtain

$$\hat{\Pi}^{\mathcal{C}}_{sk} = \frac{\mu_{sk}}{\mu_{sk} - 1} \hat{\mu}_{sk} + \hat{q}_{sk} \tag{52}$$

On the other hand, combining Equation (4) and Equation (7) and taking log changes we obtain

$$\hat{q}_{sk} = \rho \left( \hat{P}_s^{\mathcal{C}} - \hat{P}_{sk} \right) - \eta \hat{P}_s^{\mathcal{C}}. \tag{53}$$

Therefore the change in profits is

$$\hat{\Pi}_{sk}^{\mathcal{C}} = \varepsilon_{sk} \hat{\mu}_{sk} + \rho \left( \hat{P}_s^{\mathcal{C}} - \hat{P}_{sk} \right) - \eta \hat{P}_s^{\mathcal{C}}. \tag{54}$$

Finally, as $\hat{\mu}_{sk} = \hat{P}_{sk}$ in the absence of technological shocks and using the notation for the overcharge introduced in Equation (46) we get

$$\hat{\Pi}_{sk}^{\mathcal{C}} = \left[ \rho - \eta - \Upsilon_{sk} \left( \rho - \varepsilon_{sk} \right) \right] \hat{P}_s^{\mathcal{C}} - \left( \rho - \varepsilon_{sk} \right) \Theta_{sk}. \tag{55}$$

Notice that, for a given sectoral price increase, the upper bound becomes tighter the larger the firm. This is because the term in brackets decreases with firm size.

**Participation constraint.** For non-cartel members, the change in log profits is always positive, as $\Upsilon_{sk} < 1$ and $\varepsilon_{sk} \in (\eta, \rho)$ for all $k$, we have

$$\hat{\Pi}_{sk} = \left[ \rho - \eta - \Upsilon_{sk} \left( \rho - \varepsilon_{sk} \right) \right] \hat{P}_s^{\mathcal{C}} > 0. \tag{56}$$

In addition, as both $\Upsilon_{sk}$ and $\rho - \varepsilon_{sk}$ are increasing with size, the term in brackets decreases with size, that is, smaller non-cartel firms exhibit a larger proportional increase in umbrella profits when the cartel forms.

For cartel members, recall that from Equation (55), the change in profits is given by

$$\hat{\Pi}_{sk}^{\mathcal{C}} = \left[ \rho - \eta - \Upsilon_{sk} \left( \rho - \varepsilon_{sk} \right) \right] \hat{P}_s^{\mathcal{C}} - \left( \rho - \varepsilon_{sk} \right) \Theta_{sk}.$$

Two channels are affecting profit changes: i) by joining the cartel, firm $k$ contributes to further raising the price level, increasing its profits; at the same time, ii) this increase comes at a personal cost in terms of lost market shares. The first term captures the first channel and is decreasing with size. Regarding the second term, the factor in front of the overcharge is increasing with size. Firm $k$ profits from joining the cartel compared to the baseline equilibrium if and only if

$$\Theta_{sk} < \left[ \frac{\rho - \eta}{\rho - \varepsilon_{sk}} - \Upsilon_{sk} \right] \hat{P}_s^{\mathcal{C}}. \tag{57}$$

Absent side-payments, non-monetary incentives, or threats, the participation constraint for a firm takes the form of an upper-bound on the overcharge $\Theta_{sk}$ it sets when joining the cartel. Note that the term in brackets is decreasing with size, that is, the constraint is less binding for smaller firms. The intuition is that, compared to the initial situation, smaller firms who are mostly price takers stand to gain from the increase in prices triggered by the cartel.

**Incentive compatibility.** However, firms also have an incentive to free-ride on the cartel, that is, benefit from the sectoral price level increase while not charging a collusive overcharge. The cartel is sustainable under a punishment trigger strategy if there exists $\delta$ such that

$$\frac{1}{1-\delta}\hat{\Pi}_{sk}^{\mathcal{C}} > \hat{\Pi}_{s}^{\mathcal{C}\setminus\{k\}} \tag{58}$$

To analyze the incentives for firms to join the cartel, suppose that that the cartel is not viable if firm $k$ does not join. Then the counterfactual is the initial oligopolistic equilibrium, that is, profits do not change. If the cartel is viable without firm $k$ joining, then the counterfactual profit is

$$\hat{\Pi}_{sk}^{\mathcal{C}\setminus\{k\}} = \left[\rho - \eta - \Upsilon_{sk}\left(\rho - \varepsilon_{sk}\right)\right]\hat{P}_{s}^{\mathcal{C}\setminus\{k\}}. \tag{59}$$

Notice that

$$\hat{\Pi}_{sk}^{\mathcal{C}} = \hat{\Pi}_{sk}^{\mathcal{C}\setminus\{k\}}\frac{\hat{P}_{s}^{\mathcal{C}}}{\hat{P}_{s}^{\mathcal{C}\setminus\{k\}}} - \left(\rho - \varepsilon_{sk}\right)\Theta_{sk}, \tag{60}$$

so that colluding profit incentives require

$$\Theta_{sk} < \left[\frac{\rho - \eta}{\rho - \varepsilon_{sk}} - \Upsilon_{sk}\right]\left[\hat{P}_{s}^{\mathcal{C}} - \left(1 - \delta\right)\hat{P}_{sk}^{\mathcal{C}\setminus\{k\}}\right], \tag{61}$$

where $\hat{P}_{sk}^{\mathcal{C}\setminus\{k\}}$ is the price level increase if all the other cartel members except for firm $k$ apply the overcharge.

This constraint is therefore always more binding than the participation constraint derived above. In fact this constraint converges to the participation constraint from below as cartel members become infinitely patient, i.e. $\delta \to 1$. This reflects the fact that, since larger firms have a larger price impact, the price level increase triggered by the cartel would be relatively much smaller if they opt to free-ride. On the other hand, for small firms with little market impact, profit incentives can be insufficient in themselves to induce them to join the cartel, which would be consistent with the use of threats or non-monetary incentives.

Conversely, if cartel members are perfectly impatient, the term in the right bracket reduces to the sectoral price increment due to member $k$ joining the cartel

$$\hat{P}_{sk}^{\mathcal{C}} - \hat{P}_{sk}^{\mathcal{C}\setminus\{k\}} = \frac{1}{\rho - 1}\frac{1}{1 - \sum_k \omega_{sk}\Upsilon_{sk}}\left[\Upsilon_{sk}\left(\omega_{s\mathcal{C}} - \omega_{sk}\right) + \omega_{sk}\sum_{j\in\mathcal{C}\setminus\{k\}}\Upsilon_{sj}\right]\Delta\kappa. \quad (62)$$

This additional price increase can be decomposed into two channels: i) the influence of other cartel members on firm $k$ and ii) the influence of firm $k$ on each other cartel member $j \in \mathcal{C} \setminus \{k\}$. Finally, notice that this upper bound is always strictly positive and that the constraint is no longer necessarily monotonous but will depend on how collusive effort is shared in the cartel.

## D.7 Consumption-Equivalent Welfare

The utility of the representative consumer in the cartelized economy is given by:

$$U\Big(C, L\Big) = \left(\ln C - \frac{L^{1+\psi}}{1+\psi}\right), \quad (63)$$

where $C$ denotes the consumption of the household, $L$ is its labor supply and $\psi$ is the inverse of the Frisch elasticity of labor supply. The aggregate production function is given by $Y = AL$.[14]

Edmond et al. (2023) show that one can obtain a simple static welfare formula that connects the level of the aggregate markup and aggregate productivity to welfare. Denoting $\mathcal{W}_{\text{cartel}}$ the level of consumption that solves $U(\mathcal{W}_{\text{cartel}}, 0) = U(C, L)$ for the allocation with collusion, while $\mathcal{W}_{\text{comp}}$ solves $U(\mathcal{W}_{\text{comp}}, 0) = U(C, L)$ for the competitive allocation. The consumption-equivalent losses from cartels is given by:

$$\frac{\mathcal{W}_{\text{cartel}}}{\mathcal{W}_{\text{comp}}} = \left(\frac{A_{\text{cartel}}}{A_{\text{comp}}}\right)\left(\frac{\mathcal{M}_{\text{cartel}}}{\mathcal{M}_{\text{comp}}}\right)^{-\frac{1}{1+\psi}}. \quad (64)$$

The consumption-equivalent losses from cartels compared to the efficient allocation is instead given by:

$$\frac{\mathcal{W}_{\text{cartel}}}{\mathcal{W}_{\text{eff}}} = \left(\frac{A_{\text{cartel}}}{A_{\text{eff}}}\right)\mathcal{M}_{\text{cartel}}^{-\frac{1}{1+\psi}}. \quad (65)$$

[14]This function, which abstracts from capital, is consistent —at least in the short run —with the low elasticities of substitution between capital and labor documented in Moreau (2019).

## D.8 Identification of $\eta$

The sum of the two slope parameters from equation (20) in the main text is:

$$a_1 + a_2 = \frac{1}{\rho} - \frac{1}{\eta}.$$

To see how the restriction reported in equation (22) in the main text also applies to non-cartel members, let us write the corresponding estimating equation for non-cartel members:

$$\frac{1}{\mu_{sk}} = b_0 + b_1 \omega_{sk} + \varepsilon_{sk},$$

where $b_0 = \frac{\rho-1}{\rho}$ and $b_1 := \frac{\eta-\rho}{\eta\rho}$.

We can see that:

$$b_1 = \frac{1}{\rho} - \frac{1}{\eta} = a_1 + a_2.$$

Given that $a_1 + a_2 = b_1$, one can express $\eta$ as a function of $b_1$ and $\rho$:

$$\eta = \left( \frac{1}{\frac{1}{\rho} - b_1} \right).$$

This shows that one can recover an estimate of $\eta$ conditional on $\rho$ by either regressing non-cartel members' markups on their own market shares to match the ratio $b_1$, or by regressing cartel members' markups on their own market shares and their cartel's market share to match $(a_1 + a_2)$. However, because non-cartel members may be cartel members that have not been detected by the competition authority, we choose to rely on equation (20) in the main text.

# E Additional Tables

Table E1: Summary Statistics

| | Mean | pc25 | pc50 | pc75 | SD | N |
|---|---|---|---|---|---|---|
| Market share (%) | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 8,314,703 |
| Markups | 1.14 | 0.97 | 1.08 | 1.24 | 0.27 | 8,314,699 |
| Sales (million euros) | 3.62 | 0.21 | 0.43 | 1.12 | 100 | 8,314,703 |
| Value-added (million euros) | 0.99 | 0.09 | 0.17 | 0.37 | 40.08 | 8,314,703 |
| Labor productivity (ln) | 3.48 | 3.15 | 3.49 | 3.84 | 0.63 | 8,314,703 |
| Labor | 18.07 | 3 | 5 | 9 | 516.80 | 8,314,703 |
| Wage (ln) | 3.26 | 3.00 | 3.28 | 3.56 | 0.49 | 8,314,703 |
| Capital/labor ratio (ln) | 2.78 | 2.11 | 2.86 | 3.53 | 1.19 | 8,314,703 |
| Intermediates (million euros) | 1.82 | 0.05 | 0.13 | 0.45 | 55.59 | 8,314,703 |
| Number of firms | | | 1,160,990 | | | |

*Notes:* This table presents summary statistics for the entire sample period (1994-2007). Sales, value-added, and expenditures on intermediates are reported in millions of euros.

Table E2: Characteristics of Cartels

| | Mean | SD | Median | Min | Max |
|---|---|---|---|---|---|
| Duration (years) | 4.49 | 5.74 | 3 | 1 | 47 |
| # Firms per cartel | 6.3 | 7.4 | 4 | 2 | 76 |
| Price fixing | 0.35 | 0.48 | 0 | 0 | 1 |
| Market allocation | 0.29 | 0.46 | 0 | 0 | 1 |
| Production quotas | 0.04 | 0.2 | 0 | 0 | 1 |
| Information sharing | 0.59 | 0.49 | 1 | 0 | 1 |
| Repeat offender | 0.08 | 0.27 | 0 | 0 | 1 |
| Bid rigging | 0.40 | 0.49 | 0 | 0 | 1 |
| Dominant leader | 0.04 | 0.2 | 0 | 0 | 1 |
| Abuse of dominant position | 0.03 | 0.18 | 0 | 0 | 1 |
| Guaranteed buy-backs | 0.07 | 0.25 | 0 | 0 | 1 |
| Exclusive dealing contracts | 0.18 | 0.38 | 0 | 0 | 1 |
| | | | | | |
| Number of cartels | | | 174 | | |
| Number of cartel members | | | 1,037 | | |

*Notes:* The table presents key characteristics of cartels. We only consider decision files involving at least two firms over the period 1994-2007. The duration of the cartel is expressed in years, with durations less than a year rounded up to one year. The variables (such as price fixing and market allocation) are dummy variables, taking values between 0 and 1.

Table E3: Mean Outcomes for Cartel Members and Competitive Firms

| | Cartel members | Competitive firms |
|---|---|---|
| Market share (%) | 0.03 | 0.00 |
| Markups | 1.18 | 1.14 |
| Sales (million euros) | 217 | 3.37 |
| Value-added (million euros) | 64.19 | 0.92 |
| Labor productivity (ln) | 3.89 | 3.48 |
| Labor | 764 | 17 |
| Wage (ln) | 3.60 | 3.26 |
| Capital/labor ratio (ln) | 3.47 | 2.78 |
| Intermediates (million euros) | 96.86 | 1.72 |
| | | |
| Observations | 9,461 | 8,305,242 |
| Number of firms | 807 | 1,160,183 |

*Notes:* This table presents summary statistics for cartel members and competitive firms over the sample period (1994-2007). Sales, value-added, and expenditures on intermediates are reported in millions of euros.

Table E4: Cartels in the Economy

| | Share of cartelized industries | Sales share of cartelized industries |
|---|---|---|
| 1994 | 0.12 | 0.23 |
| 1995 | 0.13 | 0.28 |
| 1996 | 0.09 | 0.21 |
| 1997 | 0.10 | 0.21 |
| 1998 | 0.10 | 0.21 |
| 1999 | 0.09 | 0.19 |
| 2000 | 0.10 | 0.20 |
| 2001 | 0.09 | 0.17 |
| 2002 | 0.09 | 0.21 |
| 2003 | 0.09 | 0.15 |
| 2004 | 0.09 | 0.19 |
| 2005 | 0.07 | 0.18 |
| 2006 | 0.06 | 0.16 |
| 2007 | 0.04 | 0.11 |
| | | |
| Mean | 0.09 | 0.20 |

*Notes:* This table illustrates the share of 4-digit industries that contain at least one cartel and the cumulative market share of these cartelized 4-digit industries for each year.

Table E5: Cartelized Industries are Larger: Robustness

| Dependent variable: | Industry market share | |
|---|---|---|
| | (1) | (2) |
| Cartelized industry | 0.196*** | 0.174*** |
| | (0.034) | (0.033) |
| Year FE | Yes | No |
| Two-digit Sector×Year FE | No | Yes |
| Observations | 9,134 | 9,134 |
| Adj. $R^2$ | 0.03 | 0.16 |

*Notes:* Standard errors clustered at the 4-digit industry level. * significant at 10%, ** significant at 5%, *** significant at 1%. The dependent variable is the sales share of a 4-digit industry. The independent variable is a dummy variable equal to one if a cartel operates in that industry in a given year.

Table E6: Cartel Premia: Markups

| Dependent variable: | $\mu_{kt}$ | |
|---|---|---|
| | (1) | (2) |
| Cartel member | 0.043*** | 0.040*** |
| | (0.010) | (0.009) |
| Year FE | Yes | No |
| Four-digit Industry $\times$ Year FE | No | Yes |
| Observations | 8,312,229 | 8,312,229 |
| Adj. $R^2$ | 0.00 | 0.06 |

*Notes:* Standard errors clustered at the firm level. * significant at 10%, ** significant at 5%, *** significant at 1%. The dependent variables is firm-level markups. The independent variable is a dummy variable that equals one if the firm is a cartel member. The periods during which firms collude have been excluded.

Table E7: Labor Productivity and Sales Dispersion: Non-Cartel versus Cartel Members

| Moment | Non-cartel members | | | Cartel members | | |
|---|---|---|---|---|---|---|
| | Mean | SD | IQ Range | Mean | SD | IQ Range |
| | (1) | (2) | (3) | (4) | (5) | (6) |
| *Panel A: Labor productivity* (ln) | | | | | | |
| Median | 3.79 | 0.48 | 0.52 | 4.44 | 1.02 | 1.36 |
| IQ range | 0.72 | 0.35 | 0.27 | 0.45 | 0.50 | 0.70 |
| 90-10 percentile range | 1.48 | 0.64 | 0.55 | 0.67 | 0.76 | 0.86 |
| 95-5 percentile range | 2.05 | 0.84 | 0.81 | 0.76 | 0.84 | 1.29 |
| *Panel B: Sales* | | | | | | |
| Median | 7.27 | 1.31 | 1.55 | 11.32 | 2.02 | 3.00 |
| IQ range | 1.98 | 0.96 | 0.88 | 1.21 | 1.16 | 1.62 |
| 90-10 percentile range | 3.75 | 1.40 | 1.61 | 1.79 | 1.38 | 2.78 |
| 95-5 percentile range | 4.76 | 1.61 | 1.87 | 1.89 | 1.48 | 3.04 |

*Notes:* This table summarizes firm-level labor productivity and total sales distribution moments across four-digit industries and cartels in 2007. Rows represent moments of within-industry and within-cartel producer productivity or total sales distributions, while columns show the mean and dispersion of these moments across industries and cartels. The IQ range refers to the interquartile range.

Table E8: Dispersion within the Manufacture of Plastic Components for Construction

| | Labor productivity (ln) | | Log sales | |
|---|---|---|---|---|
| | Non-cartel members | Cartel members | Non-cartel members | Cartel members |
| | (1) | (2) | (3) | (4) |
| Median | 4.76 | 5.59 | 7.87 | 11.10 |
| IQ range | 0.51 | 0.18 | 2.07 | 0.65 |
| 90-10 percentile range | 1.02 | 0.18 | 3.72 | 0.65 |
| 95-5 percentile range | 1.47 | 0.18 | 4.63 | 0.65 |

*Notes:* The industry considered is 252E, which corresponds to the “Manufacture of plastic components for construction.” In 2007, there were two cartels in this industry (Decisions “10D39” and “17D20”). The figures are obtained by calculating the firm mean of sales, value-added, and labor productivity. We then compute the relevant ratios for each cartel case. Labor productivity is defined as the ratio of value-added, deflated by 2-digit price indices, to the number of employees.

Table E9: Estimation of $\kappa$: Robustness

| Sample | All | All | Wins. at 3% | Wins. at 1% | Domestic |
|---|---|---|---|---|---|
| Dependent variable: | $\frac{Wl_{it}}{VA_{it}}$ | $\frac{VC_{it}}{pq_{it}}$ | $\frac{1}{\mu_{it}}$ | $\frac{1}{\mu_{it}}$ | $\frac{1}{\mu_{it}}$ |
| | (1) | (2) | (3) | (4) | (5) |
| Firm's market share | -0.187 | -0.032 | -0.162 | -0.192 | -0.212 |
| | (0.188) | (0.104) | (0.132) | (0.145) | (0.130) |
| Cartel's market share | -0.295*** | -0.153*** | -0.100** | -0.095* | -0.088* |
| | (0.058) | (0.036) | (0.049) | (0.055) | (0.047) |
| Year FE | Yes | Yes | Yes | Yes | Yes |
| Observations | 2,397 | 2,397 | 2,397 | 2,397 | 2,397 |
| Adj. $R^2$ | 0.08 | 0.05 | 0.03 | 0.02 | 0.04 |

*Notes:* Standard errors clustered at the firm level. * significant at 10%, ** significant at 5%, *** significant at 1%. The collusion intensity is estimated following equation (20) in the main text. The dependent variable is the firm's inverse markup. The cartel market share variable is the sum of the market shares of all firms that belong to the same cartel-4-digit industry pair. The dependent variable in Column 1 is firm-level labor shares, while in Column 2 it is variable costs over total sales. Column 3 (4) considers a markup distribution winsorized at the 3% (1%) level. Column 5 uses firm-level market shares defined using domestic sales instead of total sales.

Table E10: Robustness: Estimation of $\kappa$ using Initial Values as IV

| Dependent variable: | $\frac{1}{\mu_{it}}$ | |
|---|---|---|
| | (1) | (2) |
| Firm's market share | -0.111 | -0.126 |
| | (0.205) | (0.206) |
| Cartel's market share | -0.193*** | -0.174** |
| | (0.073) | (0.073) |
| Year FE | No | Yes |
| Instrument | $\omega_{it_0}, \omega^{\mathcal{C}}_{it_0}$ | |
| Implied $\kappa$ | 0.63 | 0.58 |
| Observations | 1,386 | 1,386 |
| Adj. $R^2$ | 0.02 | 0.02 |
| F statistic | 216.2 | 209.7 |

**Notes:** Standard errors clustered at the firm level. * significant at 10%, ** significant at 5%, *** significant at 1%. The collusion intensity is estimated following equation (20) in the main text. The dependent variable is the firm's inverse markup. The cartel market share variable is the sum of the market shares of all firms that belong to the same cartel-4-digit industry pair. Market shares in the first year in which cartel members appear in the sample are used as instrumental variables in both columns. The only difference between columns (1) and (2) is the inclusion of year fixed effects.

Table E11: Robustness: Estimation of $\kappa$ with Cartel Fixed Effects

| Dependent variable: | | $\frac{1}{\mu_{it}}$ | | |
|---|---|---|---|---|
| | (1) | (2) | (3) | (4) |
| Firm's market share | -0.184 | -0.004 | -0.006 | 0.003 |
| | (0.128) | (0.106) | (0.107) | (0.117) |
| Cartel's market share | -0.112** | -0.060 | -0.055 | -0.069 |
| | (0.044) | (0.055) | (0.056) | (0.063) |
| Year FE | Yes | No | Yes | No |
| Cartel FE | No | Yes | Yes | No |
| Cartel×Year FE | No | No | No | Yes |
| Implied $\kappa$ | 0.38 | 0.94 | 0.91 | 1.04 |
| Observations | 2,308 | 2,308 | 2,308 | 2,308 |
| Adj. $R^2$ | 0.04 | 0.33 | 0.33 | 0.33 |

**Notes:** Standard errors clustered at the firm level. * significant at 10%, ** significant at 5%, *** significant at 1%. The collusion intensity is estimated following equation (20) in the main text. The dependent variable is the firm's inverse markup. The cartel market share variable is the sum of the market shares of all firms that belong to the same cartel-4-digit industry pair.

Table E12: Robustness: Estimation of $\kappa$ with Sector Fixed Effects

| Dependent variable: | | $\frac{1}{\mu_{it}}$ | | |
|---|---|---|---|---|
| | (1) | (2) | (3) | (4) |
| Firm's market share | -0.217 | -0.041 | 0.002 | 0.046 |
| | (0.149) | (0.095) | (0.094) | (0.103) |
| Cartel's market share | -0.091** | -0.062 | -0.091* | -0.116 |
| | (0.046) | (0.052) | (0.051) | (0.071) |
| Year FE | Yes | No | Yes | No |
| Sector FE | No | Yes | Yes | No |
| Sector×Year FE | No | No | No | Yes |
| Implied $\kappa$ | 0.30 | 0.60 | 1.02 | 1.65 |
| Observations | 2,319 | 2,319 | 2,319 | 2,319 |
| Adj. $R^2$ | 0.04 | 0.17 | 0.18 | 0.22 |

**Notes:** Standard errors clustered at the firm level. * significant at 10%, ** significant at 5%, *** significant at 1%. The collusion intensity is estimated following equation (20) in the main text. The dependent variable is the firm's inverse markup. The cartel market share variable is the sum of the market shares of all firms that belong to the same cartel-4-digit industry pair.

Table E13: Slope Coefficient for Competitive Firms

| Dependent variable: | $\frac{1}{\mu_{it}}$ (1) | (2) |
|---|---|---|
| Firm's market share | -0.157*** | -0.159*** |
| | (0.018) | (0.018) |
| Year FE | No | Yes |
| Observations | 8,305,238 | 8,305,238 |
| Adj. $R^2$ | 0.00 | 0.00 |

*Notes:* Standard errors clustered at the firm level. * significant at 10%, ** significant at 5%, *** significant at 1%. The dependent variable is the firm's inverse markup. The sample consists of firms that do not officially classify as cartel members.

Table E14: Validation: Cartel does not Operate

| Dependent variable: | $\frac{1}{\mu_{it}}$ (1) | (2) |
|---|---|---|
| Firm's market share | -0.063 | -0.067 |
| | (0.098) | (0.098) |
| Cartel's market share | -0.065 | -0.060 |
| | (0.047) | (0.048) |
| Year FE | No | Yes |
| Observations | 7,064 | 7,064 |
| Adj. $R^2$ | 0.00 | 0.01 |

*Notes:* Standard errors clustered at the firm level. * significant at 10%, ** significant at 5%, *** significant at 1%. The dependent variable is the firm's inverse markup. The cartel market share variable is the sum of the market shares of all firms that belong to the same cartel-4-digit industry pair. The periods during which cartel members are officially operating have been excluded from the sample.

Table E15: Importance of the Umbrella Pricing Effect

| | Benchmark (1) | No umbrella pricing effect (2) |
|---|---|---|
| *Panel A: Aggregate productivity gains, in %* | | |
| $\mathcal{A}_{\text{cartel}} \rightarrow \mathcal{A}_{\text{comp}}$ | 0.21 | 0.22 |
| Distance to efficient allocation | -17.82 | -18.07 |
| | | |
| *Panel B: Aggregate welfare gains* | | |
| $\mathcal{M}_{\text{cartel}} \rightarrow \mathcal{M}_{\text{comp}}$ (in pp) | -0.82 | -0.81 |
| $\mathcal{W}_{\text{cartel}} \rightarrow \mathcal{W}_{\text{comp}}$ (in %) | 0.45 | 0.45 |

*Notes:* The table displays the aggregate productivity gains (rows 1 to 2), the change (in points) in the level of the aggregate markup (row 3), and the change in welfare (row 4) resulting from eliminating cartels. The figures are obtained by comparing the variables in the relevant equilibrium to that in the competitive Nash-Cournot equilibrium (rows 1, 3 and 4) and efficient allocation (row 2). The efficient allocation corresponds to the equilibrium without markup dispersion. In column 2, the markups of non-cartel members are held constant at their levels in the cartel equilibrium.

Table E16: Differences with Harberger (1954)

| | Benchmark (1) | Sectoral level (2) | Unit elastic demand (3) |
|---|---|---|---|
| $\mathcal{A}_{\text{cartel}} \rightarrow \mathcal{A}_{\text{comp}}$ (in %) | 0.21 | 0.06 | 0.12 |

*Notes:* The table displays the aggregate productivity gains from transitioning from the cartel equilibrium to the competitive Nash-Cournot equilibrium. Column 1 presents our benchmark results. Column 2 aggregates the model at the sectoral level, while Column 3 uses unit demand elasticities.

Table E17: Calibration: Robustness

| Panel A. Targets | | | | | | | |
|---|---|---|---|---|---|---|---|
| **Moments** | Baseline | Decile 8 | Top 20% | $\kappa = 0.83$ | Het. $\kappa$ | $\mathcal{M} = 1.1$ | Bertrand |
| Aggregate markup | 1.14 | 1.14 | 1.14 | 1.14 | 1.14 | 1.10 | 1.14 |
| Slope parameter | -0.24 | -0.24 | -0.24 | -0.24 | -0.24 | -0.24 | |
| Top-4 sales share, within-sector | 0.42 | 0.44 | 0.41 | 0.40 | 0.40 | 0.41 | 0.41 |
| Top-20 sales share, within-sector | 0.66 | 0.68 | 0.64 | 0.64 | 0.63 | 0.66 | 0.62 |
| Top-4 sales share, across-sector | 0.14 | 0.14 | 0.14 | 0.14 | 0.14 | 0.14 | 0.14 |
| Top-20 sales share, across-sector | 0.33 | 0.33 | 0.32 | 0.32 | 0.33 | 0.33 | 0.33 |
| Median # firms per sector | 178 | 175 | 175 | 178 | 168 | 178 | 178 |
| Median # members per cartel | 4 | 4 | 4 | 4 | 4 | 4 | 4 |
| Panel B: Parameters | | | | | | | |
| **Interpretation** | | | | | | | |
| Substitution within sectors | 12.06 | 11.55 | 11.98 | 12.03 | 11.49 | 17.91 | 9.73 |
| Substitution between sectors | 3.10 | 3.06 | 3.10 | 3.10 | 3.04 | 3.38 | 3.10 |
| Pareto shape, firms | 10.24 | 9.69 | 10.31 | 10.35 | 10.19 | 14.74 | 9.21 |
| Pareto shape, sectors | 2.81 | 2.80 | 2.83 | 2.86 | 2.76 | 3.18 | 2.83 |
| Geometric, firms | 0.004 | 0.004 | 0.004 | 0.004 | 0.004 | 0.004 | 0.004 |
| Geometric, cartel members | 0.16 | 0.16 | 0.15 | 0.19 | 0.20 | 0.15 | 0.16 |
| P25 $\kappa$ | | | | | 0.19 | | |
| P75 $\kappa$ | | | | | 0.63 | | |

*Notes:* Panel A reports the targeted moments generated by the model and their corresponding data counterparts. The parameters are chosen to minimize the distance between the model moments and the data moments, which are derived from French micro-data. Panel B reports the parameter values used to generate the model moments. The robustness checks are detailed in Table 8 in the main text.

Table E18: Calibration: Cartels in 1995 and 2007

| Panel A. Targets | | | | |
|---|---|---|---|---|
| **Moments** | Data (1995) | Model (1995) | Data (2007) | Model (2007) |
| Aggregate markup | 1.08 | 1.08 | 1.14 | 1.14 |
| Slope parameter | -0.24 | -0.24 | -0.24 | -0.24 |
| Top-4 sales share, within-sector | 0.38 | 0.38 | 0.41 | 0.41 |
| Top-20 sales share, within-sector | 0.57 | 0.62 | 0.60 | 0.64 |
| Top-4 sales share, across-sector | 0.13 | 0.13 | 0.14 | 0.14 |
| Top-20 sales share, across-sector | 0.31 | 0.27 | 0.33 | 0.33 |
| Median # firms per sector | 178 | 178 | 178 | 175 |
| Median # members per cartel | 4 | 4 | 4 | 4 |
| Panel B: Parameters | | | | |
| Substitution within sectors | | 19.62 | | 11.04 |
| Substitution between sectors | | 3.44 | | 3.03 |
| Pareto shape, firms | | 17.72 | | 9.82 |
| Pareto shape, sectors | | 4.16 | | 2.74 |
| Geometric, firms | | 0.004 | | 0.004 |
| Geometric, cartel members | | 0.18 | | 0.17 |
| Collusion intensity | | 0.41 | | 0.41 |
| Inverse Frisch elasticity | | 2 | | 2 |

*Notes:* Panel A reports the targeted moments generated by the model and their corresponding data counterparts. The parameters are chosen to minimize the distance between the model moments and the data moments, which are derived from French microdata. Panel B reports the parameter values used to generate the model moments. The results are detailed in Table 9 in the main text.

Table E19: Common Ownership

| Dependent variable: | | | $\frac{1}{\mu_{it}}$ | | | |
|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) |
| Firm's market share | -0.101*** | | 0.031 | 0.030 | -0.011 | -0.021 |
| | (0.018) | | (0.022) | (0.022) | (0.022) | (0.029) |
| Group's market share | | -0.106*** | -0.116*** | -0.124*** | -0.057*** | -0.026** |
| | | (0.009) | (0.011) | (0.011) | (0.010) | (0.012) |
| Year FE | No | No | No | Yes | No | No |
| Sector-Year FE | No | No | No | No | Yes | Yes |
| Firm FE | No | No | No | No | No | Yes |
| Observations | 641,712 | 641,712 | 641,712 | 641,712 | 641,712 | 641,712 |
| Adj. $R^2$ | 0.00 | 0.00 | 0.00 | 0.00 | 0.05 | 0.59 |

*Notes:* Standard errors clustered at the firm level. * significant at 10%, ** significant at 5%, *** significant at 1%. The collusion intensity is estimated following equation (20) in the main text. The dependent variable is the firm's inverse markup. The group market share variable is the sum of the market shares of all firms that belong to the same group-industry pair.

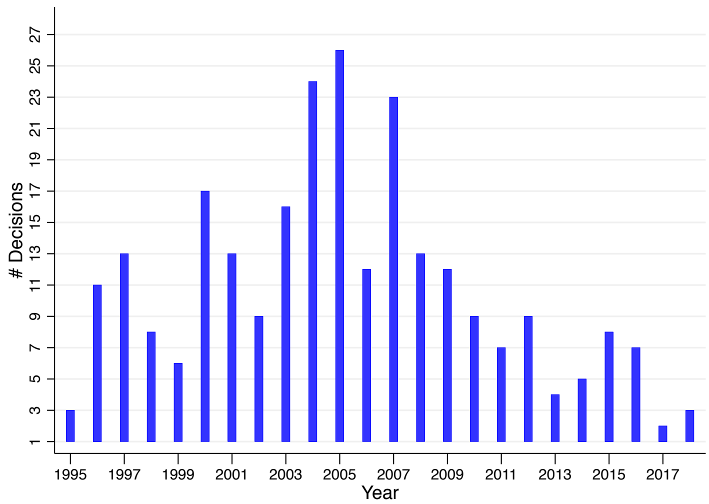
Table E20: Robustness: Common Ownership without Cartels

| Dependent variable: | | | $\frac{1}{\mu_{it}}$ | | | |
|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) |
| Firm's market share | -0.096*** | | 0.036 | 0.035 | -0.005 | -0.025 |
| | (0.019) | | (0.022) | (0.022) | (0.022) | (0.029) |
| Group's market share | | -0.104*** | -0.115*** | -0.123*** | -0.061*** | -0.030** |
| | | (0.009) | (0.011) | (0.011) | (0.010) | (0.012) |
| Year FE | No | No | No | Yes | No | No |
| Sector-Year FE | No | No | No | No | Yes | Yes |
| Firm FE | No | No | No | No | No | Yes |
| Observations | 636,035 | 636,035 | 636,035 | 636,035 | 636,035 | 636,035 |
| Adj. $R^2$ | 0.00 | 0.00 | 0.00 | 0.00 | 0.05 | 0.59 |

*Notes:* Standard errors clustered at the firm level. * significant at 10%, ** significant at 5%, *** significant at 1%. The collusion intensity is estimated following equation (20) in the main text. The dependent variable is the firm's inverse markup. The group market share variable is the sum of the market shares of all firms that belong to the same group-industry pair. Firms classified as cartel members have been removed from the sample.

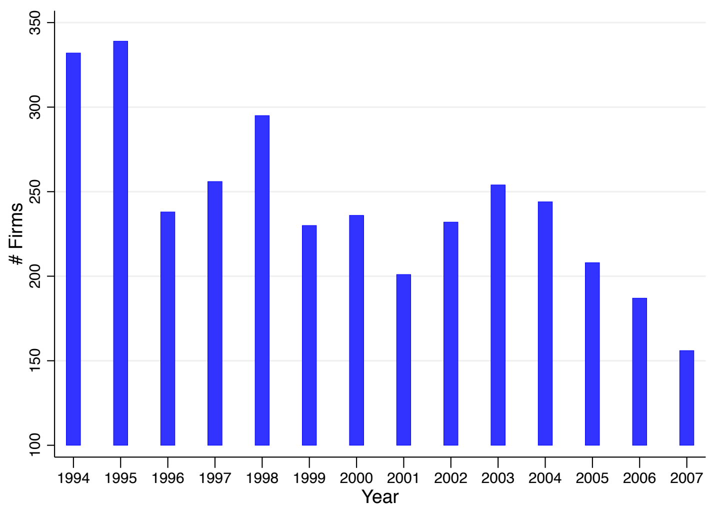
# F Additional Figures

Figure F1: Number of Decisions per Year




Data Source: Authors.

Figure F2: Number of Anti-competitive Firms per Year




Data Source: Authors.

## Figure F3: Example of Decision File (17d20): Firms' Identity

**DÉCISION**

**Article 1^er :** Il est établi que les sociétés Tarkett France, Tarkett, Tarkett AB et Tarkett Holding GmbH, Forbo Sarlino, Forbo Participations et Forbo Holding LTD, Gerflor SAS, Midfloor SAS et Topfloor SAS et le syndicat français des enducteurs calandreurs et fabricants de revêtements de sols et murs (SFEC) ont enfreint les dispositions de l'article L. 420-1 du code de commerce et du paragraphe 1 de l'article 101 du traité sur le fonctionnement de l'Union européenne en mettant en œuvre les pratiques visées par les trois griefs exposés au paragraphe 408.

**Article 2 :** À ce titre, sont infligées les sanctions pécuniaires suivantes :

- à la société Tarkett France, en tant qu'auteur et solidairement avec les sociétés Tarkett, Tarkett AB et Tarkett Holding GmbH, en leur qualité de sociétés mères, une sanction d'un montant de cent soixante-cinq millions d'euros (165 000 000 d'euros) ;

- à la société Forbo Sarlino, en tant qu'auteur et solidairement avec les sociétés Forbo Participations et Forbo Holding LTD, en leur qualité de sociétés mères, une sanction d'un montant de soixante-quinze millions d'euros (75 000 000 d'euros) ;

- à la société Gerflor SAS, en tant qu'auteur et solidairement avec les sociétés Midfloor SAS et Topfloor SAS en leur qualité de sociétés mères, une sanction d'un montant de soixante-deux millions d'euros (62 000 000 d'euros) ;

- au SFEC, en tant qu'auteur, une sanction d'un montant de de trois cent mille euros (300 000 euros).

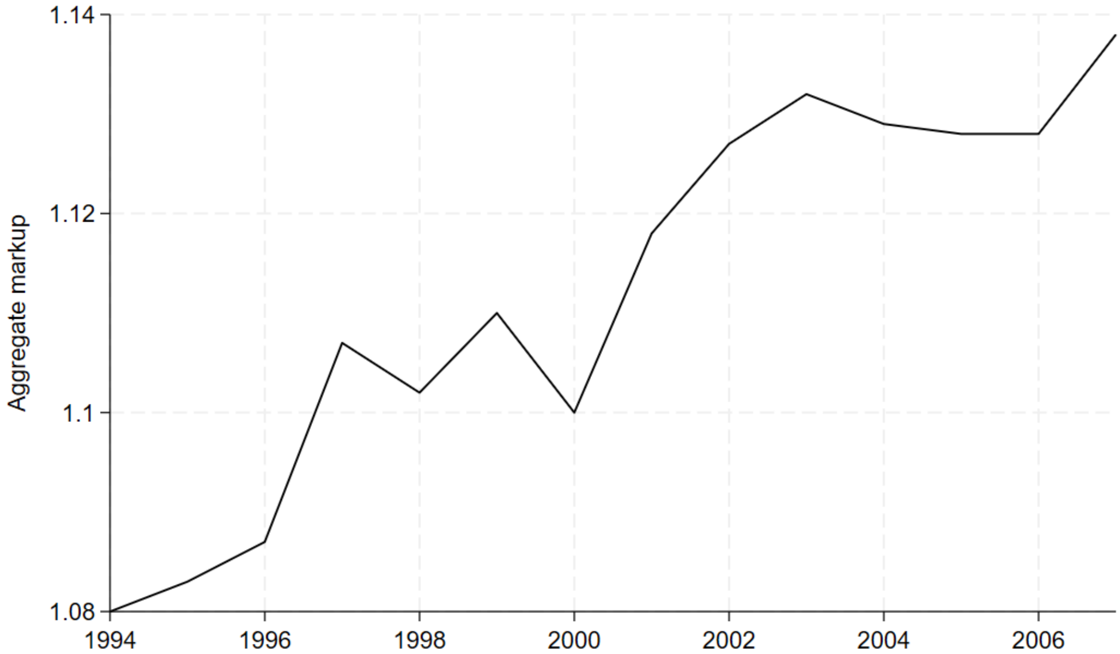
## Figure F4: Example of Decision File (17d20): Duration of Cartel

430. Ces accords et pratiques concertées constituent, par conséquent, une entente unique, complexe et continue dans le secteur de la fabrication et de la commercialisation des revêtements de sols résilients à laquelle Forbo, Gerflor et Tarkett ont participé, de manière continue, entre le 8 octobre 2001 et le 22 septembre 2011.

## Figure F5: Example of Decision File (17d20): Type of Infringement

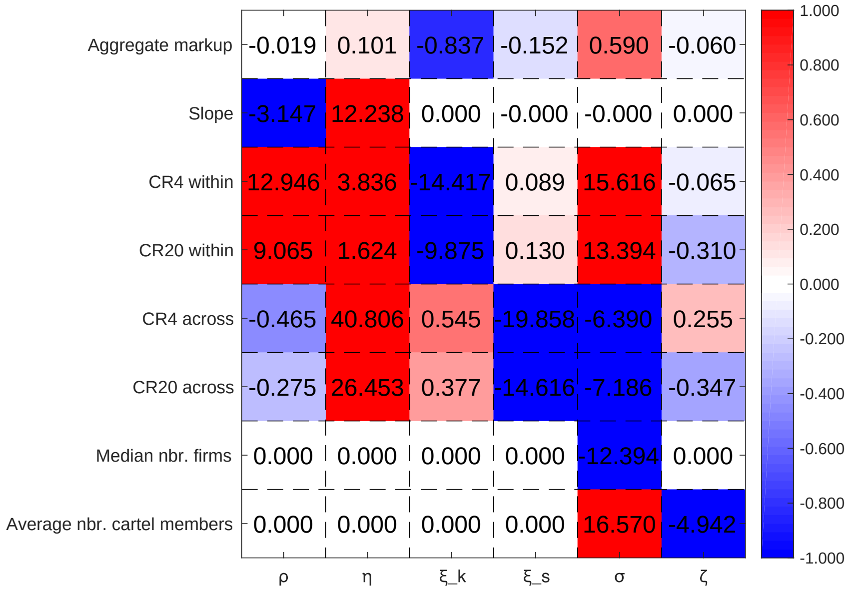
435. Il résulte de ce qui précède, que ces échanges d'informations, mis en œuvre entre 1990 et la fin de l'année 2013, ont été de nature à restreindre la concurrence, en violation du premier paragraphe de l'article 101 du TFUE et de l'article L. 420-1 du code de commerce.

Figure F6: Aggregate Markup in France



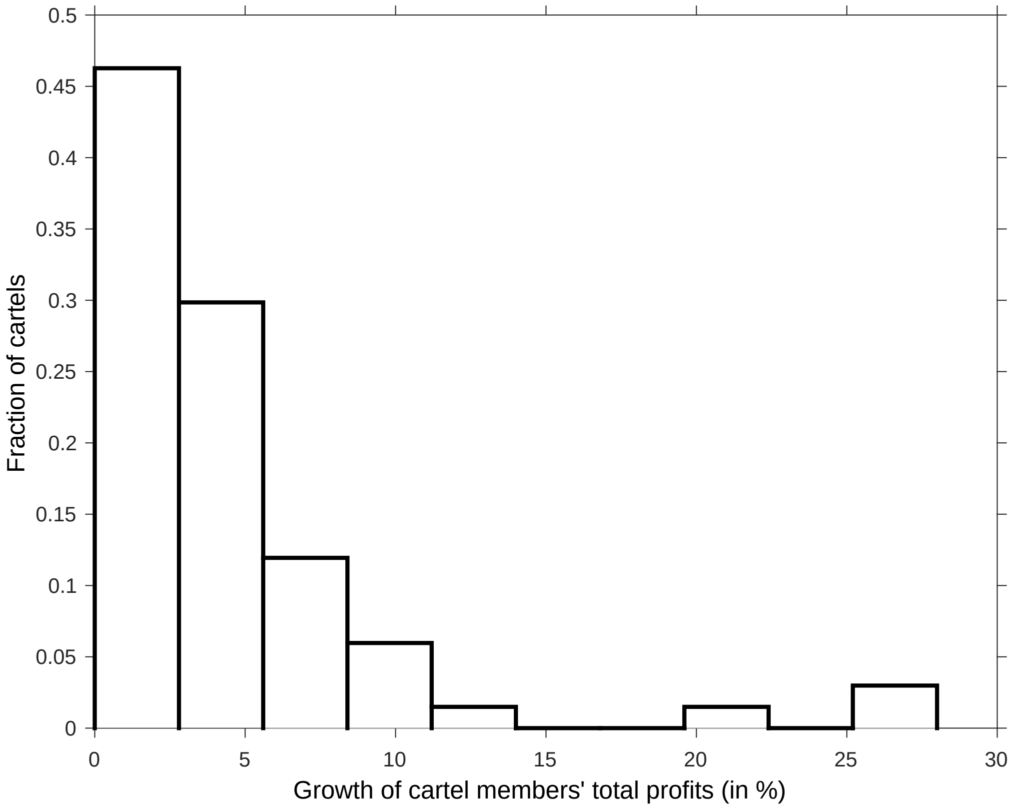

*Notes:* This figure reports the evolution of the aggregate markup in France over 1994-2007. The aggregate markup is computed as a sales-weighted harmonic average of firm-level markups, as defined in Appendix C.

Figure F7: Parameter Identification




*Notes:* This figure shows the sensitivity of moments to parameters. The numbers are obtained by computing the elasticity of moments with respect to parameters, evaluated at the calibrated parameters. Each entry in the matrix reports the percentage change in each moment following a ten percent increase in the value of each parameter. The average number of cartel members is reported instead of the median for illustrative purposes.

Figure F8: Distribution of Cartel Profit Growth




*Notes:* This figure illustrates the distribution of the growth rate of cartels' total profits. The growth rate is computed as the difference in total profits for each cartel before and after collusion.

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