Online Appendix to Reserve Price Competition with Demand Uncertainty

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A. Proof of Proposition 1

Recall that for $R^1 = R^2 = R$, the consumer equilibrium is simply that all consumers with value above R choose either firm with probability $\frac{1}{2}$. Henceforth, we will assume, without loss of generality, that $R^1 > R^2$ holds. We aim to characterize the consumer equilibrium as given in Proposition 1. First, in Lemma A.1, we will specify the consumer equilibrium with $R^2 = 0$. Then, we will allow for $0 < R^2 < R^1$ in Lemma A.2. Let

$$\widehat{R} := \frac{\pi_H \alpha_H D^{-1}(\frac{1}{\alpha_H}) + \pi_L \alpha_L D^{-1}(\frac{1}{\alpha_L})}{\pi_H \alpha_H + \pi_L \alpha_L},$$

$$p^* := D^{-1}(\frac{1}{\alpha_H} + \frac{1}{\alpha_L}), \text{ and}$$

$$R^* := \frac{\pi_H \alpha_H p_H^c + \pi_L \alpha_L p^*}{\pi_H \alpha_H + \pi_L \alpha_L}.$$

A.1 Consumer equilibrium with $R^2 = 0$

Lemma A.1: *Assume that $R^2 = 0$ holds. For each $R^1 \geq 0$, $CS(R^1, 0)$ has a consumer equilibrium in exactly one of the five regimes, characterized as follows.*

1. *Regime 1:* $\widehat{R} \leq R^1$.

2. *Regime 2:* $R^* < R^1 < \widehat{R}$.

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3. *Regime 3:* $p^* \leq R^1 \leq R^*$.

4. *Regime 4:* $p_L^c < R^1 < p^*$.

5. *Regime 5:* $R^1 \leq p_L^c$.

Proof of Lemma A.1:

We will consider $R^2 = 0$ and reduce R^1 from b (upper limit of valuations) to 0.

Regime 1

If R^1 is high enough, consumers prefer to pay the price at firm 2 rather than the price R^1 at firm 1. Prices at firm 2 are given by

$$p_H^2 = D^{-1}(\frac{1}{\alpha_H}) \text{ and } p_L^2 = D^{-1}(\frac{1}{\alpha_L}).$$

All consumers choosing firm 2 constitutes a consumer equilibrium if and only if the expected price faced by consumers at firm 2 is weakly less than R^1, or

$$\pi_H \alpha_H D^{-1}(\frac{1}{\alpha_H}) + \pi_L \alpha_L D^{-1}(\frac{1}{\alpha_L}) \leq \pi_H \alpha_H R^1 + \pi_L \alpha_L R^1. \tag{A.1}$$

The reason is that (A.1) is necessary for a consumer that buys in both states to prefer firm 2. A consumer with a valuation less than p_H^2 finds choosing firm 2 to be even more beneficial, due to the option value of not purchasing in state H. The lowest R^1 consistent with Regime 1 occurs when (A.1) holds with equality. Thus, we have a consumer equilibrium in Regime 1 for

$$R^1 \geq \frac{\pi_H \alpha_H D^{-1}(\frac{1}{\alpha_H}) + \pi_L \alpha_L D^{-1}(\frac{1}{\alpha_L})}{\pi_H \alpha_H + \pi_L \alpha_L} \equiv \widehat{R}.$$

Regime 2

When R^1 is below $\widehat{R}$, some consumers will visit firm 1. In Regime 2, there is an interior cutoff valuation, $\overline{v}$, such that all consumers with $v > \overline{v}$ go to firm 1 and all consumers with $v < \overline{v}$ go to firm 2. Furthermore, there is excess supply at firm 1 in both states, so we have $p_H^1 = p_L^1 = R^1$. For this to be consistent with consumer equilibrium, we have $p_H^2 > R^1 > p_L^2$ and the indifference condition,

$$\pi_H \alpha_H p_H^2 + \pi_L \alpha_L p_L^2 = \pi_H \alpha_H R^1 + \pi_L \alpha_L R^1. \tag{A.2}$$

To see that (A.2) is required for a consumer equilibrium in Regime 2, if the right side of (A.2) exceeded the left side, then all consumers would prefer firm 2 and we would be in Regime 1. If the left side of (A.2) exceeded the right side, then all consumers with valuation greater than R^1 would prefer firm 1; however, market clearing at firm 2 would imply $p_H^2 < R^1$, contradicting the supposition that the left side of (A.2) exceeded the right side.

With condition (A.2), consumers who would purchase in both states at firm 2 are indifferent, and consumers who would would only purchase in state L at firm 2 strictly prefer firm 2, thereby justifying the consumer choices as sequentially rational. Given the cutoff valuation, market clearing prices at firm 2 are given by

$$\alpha_H D(p_H^2) - \alpha_H D(\overline{v}) = 1 \tag{A.3}$$

$$\alpha_L D(p_L^2) - \alpha_L D(\overline{v}) = 1 \tag{A.4}$$

Given R^1, (A.2), (A.3), and (A.4) can be solved for p_H^2, p_L^2 and $\overline{v}$. For these equations to characterize a consumer equilibrium within Regime 2, there must be excess supply at firm 1 in state H:

$$\alpha_H D(\overline{v}) < 1.$$

The lower limit of $\overline{v}$ consistent with Regime 2, which we denote by $\overline{v}^*$ therefore satisfies the condition that the measure of consumers at firm 1 in state H is exactly equal to firm 1's supply,[1]

$$\alpha_H D(\overline{v}^*) = 1. \tag{A.5}$$

As R^1 falls within Regime 2, $\overline{v}$, p_H^2, and p_L^2 all fall. At the threshold satisfying (A.5), from (A.3), we have

$$\alpha_H D(p_H^2) = 2,$$

so the lowest p_H^2 in Regime 2 is p_H^c. From (A.4) and (A.5), we see that the lowest p_L^2 in Regime 2, which we denote by p^*, satisfies

$$D(p^*) = \frac{1}{\alpha_L} + \frac{1}{\alpha_H}. \tag{A.6}$$

The lowest R^1 within Regime 2, which we denote by R^*, satisfies the indifference condition,

$$\pi_H \alpha_H p_H^c + \pi_L \alpha_L p^* = (\pi_H \alpha_H + \pi_L \alpha_L) R^*. \tag{A.7}$$

[1] It will be convenient to use the terminology "lowest" $\overline{v}$ consistent with Regime 2 as $\overline{v}^*$, even though, strictly speaking, it is a lower limit since the reserve price does not bind in both states when $\overline{v} = \overline{v}^*$. This should not cause confusion since Lemma A.1 is precisely stated.

It follows from (A.7), and the fact that market clearing prices are higher in state H than in state L, that $R^* < p^c_H$ holds. Also, from $\alpha_H D(p^c_H) = 2$ and (A.5), it follows that $\overline{v}^* > p^c_H$ holds.

Regime 3

At the cutoff, $\overline{v}^*$, satisfying (A.5), the measure of consumers choosing firm 1 in state H is exactly equal to firm 1's supply. Thus, any p^1_H between R^1 and $\overline{v}^*$ clears the market at firm 1 in state H. What will be the highest rejected bid when the cutoff is $\overline{v}^*$? The highest rejected bid would be $\overline{v}^*$ if a single consumer out of the continuum is not awarded a unit, and there would be no rejected bids if all consumers are awarded a unit. An important technical issue is that our application of the law of large numbers cannot resolve whether p^1_H should be R^1 or $\overline{v}^*$.

In Section A.3, we consider sequences of consumer equilibria of auctions with a large finite number of consumers, when $CE(R^1, 0)$ is in Regime 3. We show that the limiting equilibrium cutoff approaches $\overline{v}^*$ and the p^1_H solving (A.8) below is the limiting expected price at firm 1 in state H, as the number of consumers approaches infinity. In all these sequences, the excess demand or excess supply at firm 1 in state H, as a fraction of total supply, approaches zero, but uncertainty remains about whether demand (slightly) exceeds supply or supply (slightly) exceeds demand. This justifies our characterization of $CE(R^1, R^2)$ in which p^1_H is in between R^1 and $\overline{v}^*$, and satisfies the condition that a consumer with valuation $\overline{v}^*$ is indifferent between which firm to choose. We should think of p^1_H as the expected price at firm 1, conditional on state H.

As R^1 falls below R^*, $\overline{v}$ remains constant at $\overline{v}^*$, and p^1_H rises above R^*.[2] Since the cutoff remains at $\overline{v}^*$ for all R^1 in Regime 3, the prices at firm 2 are given by $p^2_H = p^c_H$ and $p^2_L = p^*$. The prices at firm 1 are given by $p^1_L = R^1$ and, for p^1_H, the solution to the indifference condition,

$$\pi_H \alpha_H p^c_H + \pi_L \alpha_L p^* = \pi_H \alpha_H p^1_H + \pi_L \alpha_L R^1. \tag{A.8}$$

The equation (A.8) guarantees that consumers who buy in both states are indifferent between firms, since the expected price at each firm is equated. At the upper boundary of Regime 3 (highest R^1), we have $p^1_H = R^*$.

What is the lower boundary of Regime 3 (lowest R^1)? Sequential rationality of all consumers with $v < \overline{v}^*$ choosing firm 2 requires $p^* \leq R^1$. Therefore, the lower boundary of Regime 3 occurs at $R^1 = p^*$ and the corresponding highest p^1_H consistent with $CE(R^1, R^2)$

[2] As R^1 crosses below R^*, one might think that there would be a consumer equilibrium in which $\overline{v}$ would adjust to fall below $\overline{v}^*$, but this is not the case. The reason is that the measure of consumers choosing firm 1 in state H would rise above the supply, so p^1_H would rise discontinuously from R^* to $\overline{v}^*$.

in Regime 3 is p^c_H.

Regime 4

For $R^1 < p^*$, we no longer have a cutoff equilibrium to the consumer subgame characterized by $\overline{v}$, above which consumers choose firm 1 and below which consumers choose firm 2. In Regime 4, there is a consumer equilibrium in which we have $p^1_L = p^2_L = R^1$ and $p^1_H = p^2_H = p^c_H$. Consumers with valuations greater than p^c_H choose each firm with probability one half, so we have the competitive, market clearing outcome in state H. Consumers with valuations between R^1 and p^c_H choose firm 2 with some probability, $\beta < 1$, such that the market clearing price at firm 2 is exactly R^1 in state L. Thus, β is determined by

$$\frac{1}{2}\alpha_L D(p^c_H) + \alpha_L \beta [D(R^1) - D(p^c_H)] = 1. \tag{A.9}$$

Firm 1 has excess supply in state L, so R^1 binds. Obviously, since the prices in each state are equated across firms, consumers' firm choices are sequentially rational.

For higher values of the reserve price, R^1, more consumers must be choosing firm 2. Therefore, the supremum of reserve prices consistent with Regime 4 occurs when all consumers with valuations between R^1 and p^c_H choose firm 2, $\beta = 1$. When this occurs, the market clearing condition at firm 2 in state L is

$$\frac{1}{2}\alpha_L D(p^c_H) + \alpha_L D(R^1) - \alpha_L D(p^c_H) = 1. \tag{A.10}$$

The first term on the left side of (A.10) reflects the fact that half of the consumers with valuations greater than p^c_H choose firm 2; the second and third terms reflect that fact that all consumers with valuations between R^1 and p^c_H choose firm 2. Equation (A.10) can be simplified to

$$D(R^1) = \frac{1}{\alpha_L} + \frac{1}{2} D(p^c_H). \tag{A.11}$$

Since, by definition, p^c_H satisfies $\alpha_H D(p^c_H) = 2$, we have

$$\begin{aligned} D(R^1) &= \frac{1}{\alpha_L} + \frac{1}{2} \cdot \frac{2}{\alpha_H}, \text{ or} \\ D(R^1) &= \frac{1}{\alpha_L} + \frac{1}{\alpha_H}. \end{aligned} \tag{A.12}$$

From (A.6), the R^1 solving (A.12) is exactly p^*, so p^* is the upper limit of R^1 consistent with Regime 4.

The lower limit of R^1 consistent with Regime 4 is p^c_L, which occurs when consumers with valuations between R^1 and p^c_H choose firm 2 with probability, $\beta = \frac{1}{2}$. To see this, when $\beta = \frac{1}{2}$ holds, it follows from (A.9) that $R^1 = p^c_L$ holds.[3] If $R^1 < p^c_L$ were to hold, (A.9) would require $\beta < \frac{1}{2}$. With more than half the consumers choosing firm 1, we would have $p^1_L > p^c_L > R^1$, so R^1 does not bind in state L, but this is a requirement for Regime 4. More to the point, we would have $p^2_L < p^c_L$, so prices differ across firms, clearly inconsistent with Regime 4.

Regime 5

For $R^1 \leq p^c_L$, there is a consumer equilibrium in which all consumers choose each firm with probability one half, essentially ignoring the reserve prices since they do not bind in either state. We refer to this regime, which occurs for the lowest reserve prices, as Regime 5.■

A.2 Consumer equilibrium for all (R^1, R^2)

Now, we characterize the consumer equilibrium for each $CS(R^1, R^2)$, where both reserve prices are positive. As above, we continue to assume without loss of generality that $R^1 > R^2$ holds. First, we establish Lemma A.2 to show that for $R^2 \leq p^*$ there is a consumer equilibrium which is identical to $CE(R^1, 0)$. Thereafter, we need only consider the cases where both $R^1 > R^2$ and $R^2 > p^*$ hold.

Lemma A.2: *If $R^2 < R^1$, and $R^2 \leq p^*$ hold, then there is a consumer equilibrium $CE(R^1, R^2)$ in which consumer behavior and prices are exactly as in $CE(R^1, 0)$.*

Proof of Lemma A.2: We show Lemma A.2 in two steps. First, we provide arguments that Lemma A.2 holds when R^1 is less than p^*. Next, we provide arguments for the case where $R^1 > p^*$ holds.

If $R^1 \leq p^*$ holds, then $CE(R^1, 0)$ is either in Regime 4 or Regime 5. In Regime 4, we have $p^1_L = p^2_L = R^1$ and $R^1 < p^1_H = p^2_H = p^c_H$. Thus, for R^1 in Regime 4 given $R^2 = 0$, even if R^2 were to be increased from 0 to a positive R^2 with $R^2 < R^1$ (as assumed), in the resulting $CE(R^1, R^2)$, such an R^2 would not bind in either state, and therefore not affect the CE relative to $CE(R^1, 0)$. Similarly, for R^1 in Regime 5 given $R^2 = 0$, we have $R^1 \leq p^c_L$ and R^1 does not bind in either state. Thus, replacing $R^2 = 0$ with a positive R^2 lower than R^1 does not affect the CE relative to $CE(R^1, 0)$ when $R^1 \leq p^*$ holds.

Next, we consider $R^1 > p^*$. Note that $R^1 > p^*$ implies $CE(R^1, 0)$ is in Regime 1, 2, or 3. We now show that $CE(R^1, 0)$ in Regimes 1, 2, or 3 yields $p^2_L \geq p^*$. Thus, replacing $R^2 = 0$

[3] In this boundary case, all consumers are mixing with probability one half, and markets clear at all firms in all states. Thus, this boundary case is in Regime 5 and not Regime 4, since R^1 does not bind.

with a positive R^2 weakly lower than p^* implies that in $CE(R^1, R^2)$ consumer behavior and prices are exactly as in $CE(R^1, 0)$.

First consider $CE(R^1, 0)$ with R^1 in Regime 1. Prices at firm 2 are given by

$$p_H^2 = D^{-1}(\frac{1}{\alpha_H}) \text{ and } p_L^2 = D^{-1}(\frac{1}{\alpha_L}),$$

and both $p_H^2 = D^{-1}(\frac{1}{\alpha_H})$ and $p_L^2 = D^{-1}(\frac{1}{\alpha_L})$ are greater than $p^* = D^{-1}(\frac{1}{\alpha_H} + \frac{1}{\alpha_L})$.

Next, consider $CE(R^1, 0)$ with R^1 in Regime 2. At the lowest $\overline{v}$ consistent with Regime 2, $\overline{v}^*$, we have $\alpha_H D(\overline{v}^*) = 1$, which implies that the lowest p_H^2 in Regime 2 is p_H^c, and the lowest p_L^2 in Regime 2 is p^*. Third, consider $CE(R^1, 0)$ with R^1 in Regime 3. The prices at firm 2 are given by $p_H^2 = p_H^c$ and $p_L^2 = p^*$. Thus, in $CE(R^1, 0)$ for R^1 in either of Regimes 1-3, p_L^2 is weakly greater than p^*. It follows from $R^2 \leq p^*$ that R^2 does not bind, and in $CE(R^1, R^2)$ consumer behavior and prices are exactly as in $CE(R^1, 0)$. ■

Now we consider consumer equilibrium for $CS(R^1, R^2)$ such that $R^1 > R^2 > p^*$ holds. It is clear that the regimes specified in the statement of Proposition 1 cover the entire (R^1, R^2) space, and that there is no overlap between any regimes. Thus, each (R^1, R^2) fits into exactly one of the regimes. The $CE(R^1, R^2)$ for all $CS(R^1, 0)$ has been specified in Lemma A.1. And Lemma A.2 has shown that for $R^2 \leq p^*$ the $CE(R^1, 0)$ specified in Lemma A.1 applies even for R^2 positive. The road-map of the following analysis is to consider each value of $R^1 > p^*$, and for each such value of R^1, consider each $R^2 \in [p^*, R^1)$. Note that there are no consumer equilibria satisfying $R^1 > R^2 \geq p^*$ in Regime 4 or Regime 5, so we will restrict attention to Regimes 1-3. We will show that the $CE(R^1, R^2)$ specified in Proposition 1 is indeed a consumer equilibrium for the conditions specified for each of these regimes in Proposition 1.

Regime 1 (with $R^2 \geq p^*$)

Recall that $CE(R^1, 0)$ is in Regime 1 if we have $\widehat{R} \leq R^1$. For $R^1 < \widehat{R}$, $CE(R^1, R^2)$ cannot be in Regime 1. Higher R^2 can only increase the attractiveness of firm 1, so it cannot be the case that all consumers choose firm 2. Thus, we consider $R^1 \geq \widehat{R}$.

Consider $R^1 \geq D^{-1}(\frac{1}{\alpha_H})$. In this case, for any R^2 such that $R^2 < R^1$ holds, $CE(R^1, R^2)$ is in Regime 1. To demonstrate this, we need to show

$$\pi_H \alpha_H p_H^2 + \pi_L \alpha_L p_L^2 \leq \pi_H \alpha_H R^1 + \pi_L \alpha_L R^1, \tag{A.13}$$

with $p_H^2 = max\{R^2, D^{-1}(\frac{1}{\alpha_H})\}$ and $p_L^2 = max\{R^2, D^{-1}(\frac{1}{\alpha_L})\}$. Inequality (A.13) holds, since by assumption we have $R^1 > R^2$ and $R^1 > D^{-1}(\frac{1}{\alpha_H})$, which also means $R^1 > D^{-1}(\frac{1}{\alpha_L})$.

Note that, for $R^1 \geq \widehat{R}$ and $R^2 \leq D^{-1}(\frac{1}{\alpha_L})$, the analysis is identical to $CE(R^1, 0)$, since firm 2's reserve price does not bind and firm 2 gets excess demand in both states. In this case, $CE(R^1, R^2)$ is in Regime 1.

Now consider $R^1 \in [\widehat{R}, D^{-1}(\frac{1}{\alpha_H})]$ and $R^2 > D^{-1}(\frac{1}{\alpha_L})$. Given $R^1 \in [\widehat{R}, D^{-1}(\frac{1}{\alpha_H})]$, $CE(R^1, R^2)$ is in Regime 1 for $R^2 \in (D^{-1}(\frac{1}{\alpha_L}), R^1)$ if and only if the expected price at firm 1 is higher. The condition is

$$\pi_H \alpha_H D^{-1}(\frac{1}{\alpha_H}) + \pi_L \alpha_L R^2 \leq \pi_H \alpha_H R^1 + \pi_L \alpha_L R^1.$$

Rearranging yields:

$$R^2 \leq R^1 - \frac{\pi_H \alpha_H}{\pi_L \alpha_L}(D^{-1}(\frac{1}{\alpha_H}) - R^1). \tag{A.14}$$

Thus, for $R^1 \in [\widehat{R}, D^{-1}(\frac{1}{\alpha_H})]$ and R^2 satisfying (A.14), $CE(R^1, R^2)$ is in Regime 1.[4]

Regime 2 (with $R^2 \geq p^*$)

For $CE(R^1, R^2)$ in Regime 2, firm 1 has excess supply with R^1 binding in both states, and there is a cutoff $\overline{v}$ such that each consumer with valuation above $\overline{v}$ goes to firm 1 and each consumer with valuation below $\overline{v}$ goes to firm 2. For all types with valuation such that they can purchase from any firm in any state, we have the following indifference condition:

$$\pi_H \alpha_H p_H^2 + \pi_L \alpha_L p_L^2 = \pi_H \alpha_H R^1 + \pi_L \alpha_L R^1. \tag{A.15}$$

Here p_H^2 is the maximum of R^2 and the solution to

$$\alpha_H D(p_H^2) - \alpha_H D(\overline{v}) = 1, \tag{A.16}$$

while p_L^2 is the maximum of R^2 and the solution to

$$\alpha_L D(p_L^2) - \alpha_L D(\overline{v}) = 1. \tag{A.17}$$

And finally we have

$$\alpha_H D(\overline{v}) < 1, \tag{A.18}$$

to ensure that firm 1 has excess supply. Note that R^2 cannot bind in both high and low states. This is because given $R^2 < R^1$, R^2 binding in both states would mean all consumers

[4] For $R^1 \in [\widehat{R}^1, D^{-1}(\frac{1}{\alpha_H})]$ and R^2 "close enough" to R^1, in the sense of not satisfying (A.14), $CE(R^1, R^2)$ is not in Regime 1, even though $CE(R^1, 0)$ is in Regime 1.

strictly prefer firm 2.

Case (a) in the statement of Proposition 1, part 2: Consider the case in which $R^1 \in [\widehat{R}^1, D^{-1}(\frac{1}{\alpha_H}))$ holds, so $CE(R^1, 0)$ is in Regime 1. If (A.14) holds, then for any cutoff $\overline{v}$, the left side of (A.15) is strictly less than the right side, so $CE(R^1, R^2)$ cannot be in Regime 2. However, if (A.14) does not hold, so we have

$$R^2 > R^1 - \frac{\pi_H \alpha_H}{\pi_L \alpha_L}(D^{-1}(\frac{1}{\alpha_H}) - R^1),$$

then if the cutoff is at the highest valuation, $\overline{v} = b$, prices at firm 2 are $p_H^2 = D^{-1}(\frac{1}{\alpha_H})$ and $p_L^2 = R^2$, so the left side of (A.15) is strictly greater than the right side. To be in Regime 2, it must also be the case that, if the cutoff is $\overline{v}^*$, the left side of (A.15) is less than the right side. By continuity, there would be a cutoff between $\overline{v}^*$ and b such that (A.15) is satisfied. With a cutoff of $\overline{v}^*$, the left side of (A.15) is $\pi_H \alpha_H p_H^c + \pi_L \alpha_L R^2$ if R^2 binds in state L, and even lower if R^2 does not bind in state L.

Therefore, if

$$R^2 \leq R^1 - \frac{\pi_H \alpha_H}{\pi_L \alpha_L}(p_H^c - R^1) \tag{A.19}$$

holds, then the left side of (A.15) is less than the right side and $CE(R^1, R^2)$ is in Regime 2.

Cases (b) and (c) in the statement of Proposition 1, part 2: Consider the case in which $R^1 \in [R^*, \widehat{R}^1)$ holds, so $CE(R^1, 0)$ is in Regime 2. Define $\hat{p}_L^2$ to be the corresponding value of p_L^2, solving (A.15)-(A.18) when $R^2 = 0$ holds. $CE(R^1, R^2)$ cannot be in Regime 1 for any R^2. Therefore, if the cutoff is at the highest valuation, $\overline{v} = b$, the left side of (A.15) is strictly greater than the right side, irregardless of whether or not (A.14) holds. However, we must still verify that, if the cutoff is $\overline{v}^*$, the left side of (A.15) is less than the right side. There are two subcases, depending on whether we have $R^1 \geq p_H^c$ or $R^1 < p_H^c$.

If we have $R^1 \in [R^*, \widehat{R}^1)$ and $R^1 \geq p_H^c$, and if the cutoff is $\overline{v}^*$, then the left side of (A.15) is less than the right side for any R^2. The reason is that we have $p_H^2 = p_H^c \leq R^1$ and $p_L^2 = \max[R^2, p^*] \leq R^1$.

If we have $R^1 \in [R^*, \widehat{R}^1)$ and $R^1 < p_H^c$, and if the cutoff is $\overline{v}^*$, then the left side of (A.15) is $\pi_H \alpha_H p_H^c + \pi_L \alpha_L R^2$ if R^2 binds in state L, and even lower if R^2 does not bind in state L. Therefore, if (A.19) holds, then the left side of (A.15) is less than the right side and $CE(R^1, R^2)$ is in Regime 2.

Regime 3 (with $R^2 \geq p^*$)

For $CE(R^1, R^2)$ in Regime 3, the threshold remains constant at $\overline{v}^*$, with consumers above $\overline{v}^*$ going to firm 1 and those below $\overline{v}^*$ going to firm 2. Note that it cannot be the case that

R^2 binds in both states, since otherwise the expected price at firm 2 is strictly lower, which makes a $\overline{v}^*$ cutoff equilibrium unsustainable. Thus, R^2 can bind only in the low state, if it binds at all, with $p^2_H = p^c_H$. We also require $p^1_H < p^c_H$, or else firm 2 would always have the lower price. Since the threshold is at $\overline{v}^*$, the demand at firm 1 is exactly equal to the supply, and $p^1_H \in [R^1, p^c_H]$ represents the expected price with a distribution over the realizations R^1 and $\overline{v}^*$. $CE(R^1, R^2)$ in Regime 3 is characterized by

$$\pi_H \alpha_H p^c_H + \pi_L \alpha_L p^2_L = \pi_H \alpha_H p^1_H + \pi_L \alpha_L R^1, \text{ where} \tag{A.20}$$

$$p^2_L = \max[R^2, p^*], \tag{A.21}$$

$$p^1_H \in [R^1, p^c_H]. \tag{A.22}$$

Case (a) in the statement of Proposition 1, part 3: Consider the case in which $R^1 \in [R^*, D^{-1}(\frac{1}{\alpha_H}))$ holds, so $CE(R^1, 0)$ is in Regime 1 or Regime 2 and the condition, $R^1 < p^c_H$, is satisfied. Notice that the right side of (A.20) is greater than the left side when we set $p^1_H = p^c_H$, because $R^1 \geq \max\{R^2, p^*\}$ holds. We also require the left side of (A.20) to be greater than the right side when we set $p^1_H = R^1$, If we can show that, then $CE(R^1, R^2)$ is in Regime 3, because by continuity some choice of $p^1_H \in [R^1, p^c_H]$ will cause (A.20) to hold. When R^2 does not bind in state L, the left side of (A.20) cannot be greater than the right side when we set $p^1_H = R^1$, since $CE(R^1, 0)$ is in Regime 1 or Regime 2. Thus, we require R^2 to bind, and to satisfy

$$\pi_H \alpha_H p^c_H + \pi_L \alpha_L R^2 > \pi_H \alpha_H R^1 + \pi_L \alpha_L R^1,$$

or equivalently,

$$R^2 > R^1 - \frac{\pi_H \alpha_H}{\pi_L \alpha_L}(p^c_H - R^1).$$

This is exactly the condition that (A.19) does not hold.

Case (b) in the statement of Proposition 1, part 3: Finally, consider the case in which $R^1 \in [p^*, R^*]$ holds, so $CE(R^1, 0)$ is in Regime 3. We will show that $CE(R^1, R^2)$ is in Regime 3 for any $R^2 < R^1$. The right side of (A.20) is greater than the left side when we set $p^1_H = p^c_H$, because $R^1 \geq \max[R^2, p^*]$ holds. When we set $p^1_H = R^1$, since $CE(R^1, 0)$ is in Regime 3, we have

$$\pi_H \alpha_H p^c_H + \pi_L \alpha_L p^* > \pi_H \alpha_H R^1 + \pi_L \alpha_L R^1.$$

It follows from $\max[R^2, p^*] \geq p^*$ that the left side of (A.20) is greater than the right side when we set $p_H^1 = R^1$ and, therefore, that $CE(R^1, R^2)$ is in Regime 3.■

A.3 Understanding Regime 3

In this subsection, we show that a consumer equilibrium in Regime 3, with p_H^1 (between R^1 and p_H^c) satisfying the required indifference condition, is the limit of consumer equilibria of the finite economy as the number of consumers approaches infinity. Along the sequence, the expected price at firm 1, conditional on state H, converges to p_H^1. We should interpret p_H^1 the same way in the continuum economy, because, although p_H^1 is one of the continuum of market clearing prices, it cannot be the highest rejected bid.

Consider a finite economy of "size" n, defined as follows. As in the continuum economy, there are two aggregate demand states, H and L, with prior probabilities π_H and π_L. Consumers demand either zero or one unit of the good. In state H, there are $\alpha_H n$ active consumers, with valuations drawn independently, such that the probability of receiving a valuation greater than or equal to v is $D(v)$. In state L, there are $\alpha_L n$ active consumers, with valuations drawn independently, such that the probability of receiving a valuation greater than or equal to v is $D(v)$. Again, we assume that the process that determines the activity and valuation of consumers is symmetric across "potential" consumers, so using Bayes' rule, the probability of state s, conditional on being an active consumer with valuation v, is given by (2). Each firm has a supply or capacity of n units. By the law of large numbers, the market clearing prices converge in probability to (3) and (4) as n approaches infinity.

Suppose we have reserve prices $(R^1, 0)$ in the large finite economy such that $CE(R^1, 0)$ is in Regime 3 in the continuum economy. That is, suppose we have $p^* < R^1 < R^*$.

Claim: Suppose we have $p^* < R^1 < R^*$. For all sufficiently small $\varepsilon > 0$, there is an N, such that $n > N$ implies there is a consumer equilibrium characterized by a cutoff, $\overline{v}^n$, where consumers with higher valuations choose firm 1 and consumers with lower valuations choose firm 2.

Proof of Claim. Fix a small ε and and consider cutoff strategies characterized by $\overline{v}$. Because $CE(R^1, 0)$ is in Regime 3 in the continuum economy, there is $p_H^1 \in (R^1, p_H^c)$ such that

$$\pi_H \alpha_H p_H^c + \pi_L \alpha_L p^* = \pi_H \alpha_H p_H^1 + \pi_L \alpha_L R^1 \tag{A.23}$$

holds. For the large finite economy, if we have $\overline{v} = \overline{v}^* - \varepsilon$, then for sufficiently large n, (i) the price at firm 2 in state H is almost surely less than p_H^c and the price at firm 2 in state L is almost surely less than p^*, and (ii) there will almost surely be excess demand at firm 1 in state H and excess supply in state L, so the price at firm 1 in state H is almost surely equal to $(\overline{v}^* - \varepsilon)$ and the price at firm 1 in state L is almost surely equal to R^1. Denote prices in the large finite economy with tildas. From (A.23) and $p_H^1 < \overline{v}^* - \varepsilon$, we have

$$E[\pi_H \alpha_H \widetilde{p}_H^2 + \pi_L \alpha_L \widetilde{p}_L^2] < E[\pi_H \alpha_H \widetilde{p}_H^1 + \pi_L \alpha_L \widetilde{p}_L^1]. \tag{A.24}$$

If we have $\overline{v} = \overline{v}^* + \varepsilon$, then for sufficiently large n, (i) the price at firm 2 in state H is almost surely greater than p_H^c and the price at firm 2 in state L is almost surely greater than p^*, and (ii) there will almost surely be excess supply at firm 1 in state H and in state L, so the price at firm 1 in state H and in state L is almost surely equal to R^1. From (A.23) and $p_H^1 > R^1$, we have

$$E[\pi_H \alpha_H \widetilde{p}_H^2 + \pi_L \alpha_L \widetilde{p}_L^2] > E[\pi_H \alpha_H \widetilde{p}_H^1 + \pi_L \alpha_L \widetilde{p}_L^1]. \tag{A.25}$$

By continuity and the fact that expected prices move monotonically with $\overline{v}$, there must be a unique cutoff, which we denote by $\overline{v}^n$, for which we have

$$E[\pi_H \alpha_H \widetilde{p}_H^2 + \pi_L \alpha_L \widetilde{p}_L^2] = E[\pi_H \alpha_H \widetilde{p}_H^1 + \pi_L \alpha_L \widetilde{p}_L^1]. \tag{A.26}$$

It also follows that $E(\widetilde{p}_H^2) > E(\widetilde{p}_H^1)$ and $E(\widetilde{p}_L^2) < E(\widetilde{p}_L^1)$ hold, so all consumers make sequentially rational choices and we have a unique consumer equilibrium (the notation suppresses the dependence on n).■

In the consumer equilibrium, the cutoff converges to the cutoff in the continuum economy, $\overline{v}^n \to \overline{v}^*$. Because the limiting cutoff is $\overline{v}^*$, by the law of large numbers, the prices, $\widetilde{p}_H^2$, $\widetilde{p}_L^2$, and $\widetilde{p}_L^1$ converge in probability: $\widetilde{p}_H^2 \to p_H^c$, $\widetilde{p}_L^2 \to p^*$, and $\widetilde{p}_L^1 \to R^1$. By (A.23) and (A.26), the expectation of $\widetilde{p}_H^1$ converges to p_H^1, $E(\widetilde{p}_H^1) \to p_H^1$. However, significant uncertainty about $\widetilde{p}_H^1$ remains when n is large. The law of large numbers tells us that the fraction of excess demand or excess supply is converging to zero, but $\widetilde{p}_H^1$ depends on whether there is a small amount of excess demand or excess supply. In the former case, $\widetilde{p}_H^1$ is approximately $\overline{v}^*$ (the valuation of the highest rejected bid), and in the later case, $\widetilde{p}_H^1$ is exactly R^1.

B. Linear demand

Consider the model with linear demand given by $D(v) = A - Bv$ for A, B strictly positive. This means $D^{-1}(q) = \frac{A-q}{B}$ for all demand measures q. To ensure $p_L^c \geq 0$, we assume $D^{-1}(\frac{2}{\alpha_L}) \geq 0$, or $A \geq \frac{2}{\alpha_L}$. Note that in this case, utilizing Proposition 1 and its proof, the key endogenous variables determined by the parameters are as follows:

$$
\begin{array}{c}
\widehat{R} = \frac{\pi_H\alpha_H}{\pi_H\alpha_H+\pi_L\alpha_L}\frac{A-\frac{1}{\alpha_H}}{B} + \frac{\pi_L\alpha_L}{\pi_H\alpha_H+\pi_L\alpha_L}\frac{A-\frac{1}{\alpha_L}}{B} \\
p_H^c = \frac{A-\frac{2}{\alpha_H}}{B};\ p_L^c = \frac{A-\frac{2}{\alpha_L}}{B};\ p^* = \frac{A-(\frac{1}{\alpha_H}+\frac{1}{\alpha_L})}{B} \\
\overline{v} = \frac{1}{B(\pi_H\alpha_H+\pi_L\alpha_L)} + R^1 \text{ if } R^2 \text{ does not bind} \\
\overline{v} = (R^1 + \frac{1}{B\alpha_H}) + \frac{\pi_L\alpha_L}{\pi_H\alpha_H}(R^1 - R^2) \text{ if } R^2 \text{ binds} \\
\overline{v}^* = \frac{[A-\frac{1}{\alpha_H}]}{B} \\
R^* = \frac{[A-\frac{1}{\alpha_H}]}{B} - \frac{1}{B(\pi_H\alpha_H+\pi_L\alpha_L)}
\end{array} \tag{A.27}
$$

Inspecting the terms shows that, for non-zero values of the parameters, we have the following ordering among the key values/prices:

$$p_L^c < p^* < R^* < p_H^c < \widehat{R} < \overline{v}^*. \tag{A.28}$$

B.1 Proof of Theorem 2

Now we proceed to stating the proof of Theorem 2 that deals with the linear demand setting. First, we fix $R^2 = 0$, and show that firm 1's payoff is strictly concave in R^1 when $(R^1, 0)$ is in either Regime 4 or Regime 2. In the proof of Theorem 1.1, we showed that firm 1's payoff in Regime 4, as a function of R^1, is given by

$$\wp^1(R^1; regime4) = \pi_H p_H^c + \pi_L\alpha_L R^1 D(R^1) - \pi_L R^1.$$

Under our linear specification, this becomes

$$\wp^1(R^1; regime4) = \pi_H p_H^c + \pi_L\alpha_L R^1(A - BR^1) - \pi_L R^1.$$

Therefore, we have

$$
\begin{aligned}
\frac{\partial\wp^1(R^1; regime4)}{\partial R^1} &= \pi_L\alpha_L(A - 2BR^1) - \pi_L \\
\frac{\partial^2\wp^1(R^1; regime4)}{\partial (R^1)^2} &= -\pi_L\alpha_L 2B < 0,
\end{aligned} \tag{A.29}
$$

so $\wp^1(R^1; regime4)$ is strictly concave.

In Regime 2, firm 1's profit is given by

$$\wp^1(R^1; regime2) = (\pi_H\alpha_H + \pi_L\alpha_L)R^1 D(\overline{v}).$$

When R^2 does not bind, using (A.27), this profit is:

$$\wp^1(R^1; R^2 = 0, \, regime2) = (\pi_H\alpha_H + \pi_L\alpha_L)R^1[A - \frac{1}{\pi_H\alpha_H + \pi_L\alpha_L} - BR^1]. \qquad \text{(A.30)}$$

This profit expression can be differentiated twice to see that $\wp^1(R^1; R^2 = 0, \, regime2)$ is strictly concave.

Next we show that if the derivative of $\wp^1$ is negative at the highest R^1 of Regime 4 , then the derivative of $\wp^1$ is negative at the lowest R^1 of Regime 2 (fixing $R^2 = 0$). That is, we claim

$$\frac{\partial\wp^1(R^1; regime4)}{\partial R^1} |_{p^*} < 0 \text{ implies } \frac{\partial\wp^1(R^1; R^2 = 0, regime2)}{\partial R^1} |_{R^*} < 0. \qquad \text{(A.31)}$$

From (A.29), we have

$$\begin{aligned} \frac{\partial\wp^1(R^1; regime4)}{\partial R^1} |_{p^*} &= \pi_L\alpha_L(A - 2Bp^*) - \pi_L \\ &= \pi_L\alpha_L[\frac{2}{\alpha_H} + \frac{1}{\alpha_L} - A]. \end{aligned}$$

Thus,

$$\frac{\partial\wp^1(R^1; regime4)}{\partial R^1} |_{p^*} < 0 \implies \frac{2}{\alpha_H} + \frac{1}{\alpha_L} < A. \qquad \text{(A.32)}$$

And

$$\frac{\partial\wp^1(R^1; R^2 = 0, regime2)}{\partial R^1} |_{R^*} = (\pi_H\alpha_H + \pi_L\alpha_L)[A - \frac{1}{\pi_H\alpha_H + \pi_L\alpha_L} - 2BR^*],$$

which is negative if and only if

$$A - \frac{1}{\pi_H\alpha_H + \pi_L\alpha_L} - 2BR^* < 0, \text{ or,}$$

inserting R^* from (A.27), the condition is

$$\frac{2}{\alpha_H} + \frac{1}{(\pi_H\alpha_H + \pi_L\alpha_L)} < A,$$

which clearly holds if (A.32) holds since $\frac{1}{(\pi_H\alpha_H + \pi_L\alpha_L)} > \frac{1}{\alpha_L}$ holds for $\pi_H \in (0, 1)$.

Now, let us consider the regions for A. It is easy to demonstrate that the thresholds are ordered such that

$$\frac{4}{\alpha_H} - \frac{1}{\pi_H\alpha_H + \pi_L\alpha_L} < \frac{2}{\alpha_H} + \frac{1}{\pi_H\alpha_H + \pi_L\alpha_L} < \frac{2}{\alpha_H} + \frac{1}{\alpha_L} < \frac{3}{\alpha_L}$$

holds.

Proof of part 1.

If we have

$$A \geq \frac{3}{\alpha_L},$$

then

$$\frac{\partial\wp^1(R^1; regime4)}{\partial R^1}\ |_{p_L^c} < 0$$

holds. By the concavity of $\wp^1(R^1; regime4)$, profits are decreasing in R^1 everywhere in Regime 4. It also follows that

$$\frac{\partial\wp^1(R^1; regime4)}{\partial R^1}\ |_{p^*} < 0$$

holds, so (A.31) and the concavity of $\wp^1(R^1; R^2 = 0, regime2)$ imply that profits are decreasing in R^1 everywhere in Regime 2. Since $\wp^1(R^*; R^2 = 0, regime2)$ equals firm 1's profits everywhere in Regime 3 holding $R^2 = 0$, which equals $\wp^1(p_L^*; regime4)$, it follows that $R^1 = R^2 = 0$ are SPE strategies of the reserve price game.

Proof of part 2.

If

$$\frac{2}{\alpha_H} + \frac{1}{\alpha_L} < A < \frac{3}{\alpha_L}$$

holds, then we have

$$\begin{aligned}\frac{\partial\wp^1(R^1; regime4)}{\partial R^1}\ |_{p_L^c} &> 0 \text{ and} \\ \frac{\partial\wp^1(R^1; regime4)}{\partial R^1}\ |_{p^*} &< 0.\end{aligned}$$

It follows that the best response to $R^2 = 0$ is in the interior of Regime 4, so there exists $R^1 \in (p_L^c, p^*)$ such that $(R^1, 0)$ are SPE strategies of the reserve price game (see Lemma 1 in the proof of Theorem 1.2). Using the first order condition for R^1 given $\wp^1(R^1; regime4)$, the equilibrium values are characterized by:

$$R^1 = \frac{1}{2B}[A - \frac{1}{\alpha_L}] \text{ and } R^2 < R^1.$$

Proof of part 3.

If

$$\frac{2}{\alpha_H} + \frac{1}{\pi_H\alpha_H + \pi_L\alpha_L} \leq A \leq \frac{2}{\alpha_H} + \frac{1}{\alpha_L}$$

holds, then we have

$$\begin{aligned} \frac{\partial \wp^1(R^1; regime4)}{\partial R^1} \,|_{p^*} &> 0 \text{ and} \\ \frac{\partial \wp^1(R^1; R^2 = 0, regime2)}{\partial R^1} \,|_{R^*} &< 0. \end{aligned}$$

Thus, the optimal R^1 in Regime 4 is $R^1 = p^*$ and the optimal R^1 in Regime 2 is $R^1 = R^*$. Since both choices yield the same profit for firm 1, it follows that $(p^*, 0)$ is optimal for firm 1, and indeed $(p^*, 0)$ are SPE strategies of the reserve price game.[5]

Proof of part 4.

If

$$A < \frac{2}{\alpha_H} + \frac{1}{\pi_H\alpha_H + \pi_L\alpha_L} \tag{A.33}$$

holds, then we have

$$\begin{aligned} \frac{\partial \wp^1(R^1; regime4)}{\partial R^1} \,|_{p^*} &> 0 \text{ and} \\ \frac{\partial \wp^1(R^1; R^2 = 0, regime2)}{\partial R^1} \,|_{R^*} &> 0. \end{aligned}$$

By concavity, there exists $R^1 > R^*$ such that R^1 is the unique best response to to $R^2 = 0$. The first order condition, resulting from differentiating (A.30) with respect to R^1 and setting the expression equal to zero, yields the candidate equilibrium reserve price for firm 1, which we denote by $R^{1,CE}$. We have

$$R^{1,CE} = \frac{A - \frac{1}{\pi_H\alpha_H + \pi_L\alpha_L}}{2B}.$$

[5]If firm 2 chooses to deviate to $R^2 \geq p^*$, $R^1 = p^*$ will not bind, and the above analysis will apply to firm 2, and so firm 2 will find it optimal to also choose R^2 in Regime 3, but that profit is lower than the profit from choosing $R^2 < p^*$.

Therefore, if $R^2 = 0$ is a best response to $R^{1,CE}$, then $(R^{1,CE}, 0)$ is an equilibrium.

If $R^2 = 0$ is not a best response to $R^{1,CE}$, to demonstrate that pure strategy SPE does not exist, it suffices to show that there cannot be a pure strategy SPE in which both reserve prices bind when (A.33) holds. The only possibilities for both reserve prices to bind in a pure strategy SPE are for (R^1, R^2) to be in either Regime 2 or Regime 3.

Ruling out a SPE in which (R^1, R^2) is in Regime 2 and both reserve prices bind: Suppose there is a SPE in which (R^1, R^2) is in Regime 2 with R^2 binding. Using the condition that the market clears at firm 2 in state H, the indifference condition becomes

$$\pi_H\alpha_H(\overline{v} - \frac{1}{B\alpha_H}) + \pi_L\alpha_L R^2 = (\pi_H\alpha_H + \pi_L\alpha_L)R^1,$$

from which we can solve for $\overline{v}$,

$$\overline{v} = (R^1 + \frac{1}{B\alpha_H}) + \frac{\pi_L\alpha_L}{\pi_H\alpha_H}(R^1 - R^2). \tag{A.34}$$

Profits for firm 1 are given by

$$(\pi_H\alpha_H + \pi_L\alpha_L)R^1 D(\overline{v}).$$

Using (A.34), we can express firm 1's profits as a function of the reserve prices as

$$\begin{aligned}
\wp^1(R^1; R^2, regime2) &= (\pi_H\alpha_H + \pi_L\alpha_L)R^1 D(R^1 + \frac{1}{B\alpha_H} + \frac{\pi_L\alpha_L}{\pi_H\alpha_H}(R^1 - R^2)) \\
&= (\pi_H\alpha_H + \pi_L\alpha_L)R^1[A - B(R^1 + \frac{1}{B\alpha_H} + \frac{\pi_L\alpha_L}{\pi_H\alpha_H}(R^1 - R^2))] \\
&= (\pi_H\alpha_H + \pi_L\alpha_L)R^1[A - B\{\frac{\pi_H\alpha_H + \pi_L\alpha_L}{\pi_H\alpha_H}R^1 + \frac{1}{B\alpha_H} - \frac{\pi_L\alpha_L}{\pi_H\alpha_H}R^2\}].
\end{aligned} \tag{A.35}$$

Thus, we have

$$\begin{aligned}
&\frac{\partial\wp^1(R^1; R^2, regime2)}{\partial R^1} = \\
&(\pi_H\alpha_H + \pi_L\alpha_L)[A - B\{\frac{\pi_H\alpha_H + \pi_L\alpha_L}{\pi_H\alpha_H}R^1 + \frac{1}{B\alpha_H} - \frac{\pi_L\alpha_L}{\pi_H\alpha_H}R^2\} - B\frac{\pi_H\alpha_H + \pi_L\alpha_L}{\pi_H\alpha_H}R^1] \\
&= (\pi_H\alpha_H + \pi_L\alpha_L)[A - B\{2\frac{\pi_H\alpha_H + \pi_L\alpha_L}{\pi_H\alpha_H}R^1 + \frac{1}{B\alpha_H} - \frac{\pi_L\alpha_L}{\pi_H\alpha_H}R^2\}].
\end{aligned} \tag{A.36}$$

It is immediate that this profit function is strictly concave in R^1.

Firm 2's profit is given by

$$\pi_H\alpha_H p_H^2[D(p_H^2) - D(\overline{v})] + \pi_L\alpha_L R^2[D(R^2) - D(\overline{v})]$$

$$= \pi_H p_H^2 + \pi_L \alpha_L R^2[A - BR^2 - A + B\overline{v}]$$

$$= \pi_H p_H^2 + \pi_L \alpha_L B R^2[\overline{v} - R^2].$$

Using the condition that the market clears at firm 2 in state H, and using (A.34), we can express firm 2's profits as a function of the reserve prices as

$$\wp^2(R^2; R^1, regime2) =$$

$$\pi_H[R^1 + \frac{\pi_L\alpha_L}{\pi_H\alpha_H}(R^1 - R^2)] + \pi_L\alpha_L B R^2[R^1 + \frac{1}{B\alpha_H} + \frac{\pi_L\alpha_L}{\pi_H\alpha_H}(R^1 - R^2) - R^2]. \qquad \text{(A.37)}$$

So, we have

$$\frac{\partial\wp^2(R^2; R^1, regime2)}{\partial R^2} =$$

$$= -\frac{\pi_L\alpha_L}{\alpha_H} + \pi_L\alpha_L B[R^1 + \frac{1}{B\alpha_H} + \frac{\pi_L\alpha_L}{\pi_H\alpha_H}(R^1 - R^2) - R^2 - \frac{\pi_L\alpha_L + \pi_H\alpha_H}{\pi_H\alpha_H}R^2]$$

$$= -\frac{\pi_L\alpha_L}{\alpha_H} + \pi_L\alpha_L B[R^1 + \frac{1}{B\alpha_H} + \frac{\pi_L\alpha_L}{\pi_H\alpha_H}R^1 - 2\frac{\pi_L\alpha_L + \pi_H\alpha_H}{\pi_H\alpha_H}R^2] \qquad \text{(A.38)}$$

It follows immediately that firm 2's profit is strictly concave in R^2.

If R^2 binds and we are in Regime 2, it must be characterized by equating (A.38) to zero, which can be greatly simplified to

$$R^2 = \frac{R^1}{2}. \qquad \text{(A.39)}$$

Substituting (A.39) into (A.36), simplifying, and solving for R^1 yields

$$R^1 = \frac{2\pi_H\alpha_H}{4\pi_H\alpha_H + 3\pi_L\alpha_L}\frac{(A - \frac{1}{\alpha_H})}{B}. \qquad \text{(A.40)}$$

Thus, the unique candidate equilibrium in Regime 2 with both reserve prices binding is

$$(\frac{2\pi_H\alpha_H}{4\pi_H\alpha_H + 3\pi_L\alpha_L}\frac{(A - \frac{1}{\alpha_H})}{B}, \frac{\pi_H\alpha_H}{4\pi_H\alpha_H + 3\pi_L\alpha_L}\frac{(A - \frac{1}{\alpha_H})}{B}).$$

Next, let $\widetilde{R}^2(R^1)$ denote the minimum value of R^2 value that binds (which depends on R^1). This occurs at the market clearing price in state L,

$$\overline{v} - \frac{1}{B\alpha_L},$$

and since $\overline{v}$ is given by

$$\overline{v} = \frac{1}{B(\pi_H\alpha_H + \pi_L\alpha_L)} + R^1,$$

we have

$$\widetilde{R}^2(R^1) = \frac{1}{B(\pi_H\alpha_H + \pi_L\alpha_L)} + R^1 - \frac{1}{B\alpha_L}.$$

Therefore, at the candidate equilibrium, we must have:

$$\begin{aligned} \frac{1}{B(\pi_H\alpha_H + \pi_L\alpha_L)} + R^1 - \frac{1}{B\alpha_L} &\leq \frac{R^1}{2}, \text{ or} \\ \frac{R^1}{2} &\leq \frac{1}{B\alpha_L} - \frac{1}{B(\pi_H\alpha_H + \pi_L\alpha_L)}. \end{aligned}$$

Substituting (A.40) yields

$$\frac{\pi_H\alpha_H}{4\pi_H\alpha_H + 3\pi_L\alpha_L}\frac{(A - \frac{1}{\alpha_H})}{B} \leq \frac{1}{B\alpha_L} - \frac{1}{B(\pi_H\alpha_H + \pi_L\alpha_L)}.$$

Thus, an equilibrium where R^2 binds requires

$$A \leq \frac{1}{\alpha_H} + [\frac{1}{\alpha_L} - \frac{1}{(\pi_H\alpha_H + \pi_L\alpha_L)}]\frac{4\pi_H\alpha_H + 3\pi_L\alpha_L}{\pi_H\alpha_H}. \tag{A.41}$$

Adding (A.33) and (A.41) rearranging yields

$$2A \leq \frac{3}{\alpha_H} + \frac{1}{\pi_H\alpha_H + \pi_L\alpha_L} + [\frac{1}{\alpha_L} - \frac{1}{(\pi_H\alpha_H + \pi_L\alpha_L)}][4 + \frac{3\pi_L\alpha_L}{\pi_H\alpha_H}].$$

This can be simplified to

$$2A \leq \frac{4}{\alpha_L}.$$

Thus, we have

$$A < \frac{2}{\alpha_L},$$

which implies $p_L^c < 0$, a contradiction.

Ruling out any possibility of a Regime 3 equilibrium with both R^1 and R^2 binding. First we show that, for any (R^1, R^2) in Regime 3 with $R^1 > R^2$, firm 1's profit that is independent of R^1 as long as we remain in Regime 3. If R^2 binds, or, in other words, $R^2 > p^*$ holds, firm 1's profit expression is

$$[\pi_H\alpha_H p_H^1 + \pi_L\alpha_L R^1]D(\overline{v}^*),$$

where p_H^1 is determined by the indifference condition,

$$p_H^1 = p_H^c + \frac{\pi_L\alpha_L}{\pi_H\alpha_H}[R^2 - R^1],$$

and $\overline{v}^*$ is given by

$$\overline{v}^* = \frac{[A - \frac{1}{\alpha_H}]}{B}.$$

Substituting p_H^1 and $\overline{v}^*$ into firm 1's profit expression, and simplifying, yields profits as a function of the reserve prices,

$$\wp^1(R^1; R^2, regime3) = [\pi_H \alpha_H \frac{A - \frac{2}{\alpha_H}}{B} + \pi_L \alpha_L R^2] \frac{1}{\alpha_H}.$$

While $\wp^1(R^1; R^2, regime3)$ depends on R^2, it does not depend on R^1.

Next, we show that, when firm 2 sets a reserve price slightly lower than R^1, firm 2 receives higher profits than firm 1. Firm 2's profit expression is given by

$$\pi_H p_H^c + \pi_L \alpha_L R^2 [D(R^2) - D(\overline{v}^*).$$

Substituting $\overline{v}^*$ into firm 2's profit expression, and simplifying, yields profits as a function of the reserve prices,

$$\wp^2(R^2; R^1, regime3) = \pi_H \frac{A - \frac{2}{\alpha_H}}{B} + \pi_L \alpha_L R^2 [A - BR^2 - \frac{1}{\alpha_H}].$$

We now show that, for ε sufficiently close to zero, we have

$$\wp^2(R^2 = R^1 - \epsilon; R^1, regime3) > \wp^1(R^1; R^2 = R^1 - \varepsilon, regime3). \qquad \text{(A.42)}$$

That is, we will show that (A.42) holds for $\varepsilon = 0$, or

$$\pi_H \frac{A - \frac{2}{\alpha_H}}{B} + \pi_L \alpha_L R^1 [A - BR^1 - \frac{1}{\alpha_H}] > [\pi_H \alpha_H \frac{A - \frac{2}{\alpha_H}}{B} + \pi_L \alpha_L R^1] \frac{1}{\alpha_H}.$$

This condition can be simplified to an equivalent condition,

$$\begin{aligned} [A - BR^1] &> \frac{2}{\alpha_H}, \text{ or} \\ R^1 &< \frac{A - \frac{2}{\alpha_H}}{B}. \end{aligned} \qquad \text{(A.43)}$$

Since the right side of (A.43) is equal to p_H^c and $R^1 < p_H^c$ must hold when we are in Regime 3, (A.43) holds, so firm 2 receives higher profit than firm 1 for ε sufficiently close to zero.

We are now able to rule out an equilibrium in Regime 3 with both reserve prices binding. If (R^1, R^2) were such an equilibrium, firm 1 is not best responding to R^2. By reducing R^1 to just above R^2, neither firm's profits change. Note that we remain in Regime 3 since $R^2 > p^*$ holds. However, if firm 1 slightly undercuts R^2, we remain in Regime 3 but now firm 1 is the firm with the lower reserve price and the higher profit. Thus, (R^1, R^2) is inconsistent with equilibrium.

We now show that a sufficient condition for the candidate being an equilibrium is (11). First, we show that the condition,

$$A < \frac{4}{\alpha_H} - \frac{1}{\pi_H \alpha_H + \pi_L \alpha_L},$$

implies $R^{1,CE} > p_H^c$. The immediate implication is that Regime 3 is impossible, so for all $R^2 < R^{1,CE}$, the corresponding consumer subgame is in Regime 2. To show $R^{1,CE} > p_H^c$, we must show

$$\frac{A - \frac{2}{\alpha_H}}{B} < \frac{A - \frac{1}{\pi_H \alpha_H + \pi_L \alpha_L}}{2B},$$

or, equivalently,

$$2A - \frac{4}{\alpha_H} < A - \frac{1}{\pi_H \alpha_H + \pi_L \alpha_L}.$$

However, this is equivalent to

$$A < \frac{4}{\alpha_H} - \frac{1}{\pi_H \alpha_H + \pi_L \alpha_L},$$

which is the right inequality of (11).

We now rule out deviations by firm 2 within Regime 2. Recall that (i) $\wp^2(R^2; R^1 = R^{1,CE}, regime2)$ is concave in R^2, that (ii) an interior solution to the problem of maximizing $\wp^2(R^2; R^1 = R^{1,CE}, regime2)$ is

$$R^2 = \frac{R^{1,CE}}{2},$$

and that (iii) R^2 "just binds" at

$$\widetilde{R}^2(R^{1,CE}) = \frac{1}{B(\pi_H \alpha_H + \pi_L \alpha_L)} + R^{1,CE} - \frac{1}{B\alpha_L}.$$

Therefore, it suffices to show that

$$\frac{R^{1,CE}}{2} < \frac{1}{B(\pi_H \alpha_H + \pi_L \alpha_L)} + R^{1,CE} - \frac{1}{B\alpha_L}$$

holds or, after substituting for $R^{1,CE}$ and simplifying,

$$\frac{1}{\alpha_L} < \frac{1}{(\pi_H\alpha_H + \pi_L\alpha_L)} + \frac{A - \frac{1}{\pi_H\alpha_H+\pi_L\alpha_L}}{4}.$$

Simplifying further, we have the required condition,

$$\frac{4}{\alpha_L} - \frac{3}{(\pi_H\alpha_H + \pi_L\alpha_L)} < A, \tag{A.44}$$

which is the left inequality of (11).[6]

A sufficient condition for nonexistence of pure strategy equilibrium is for (A.45) and

$$\frac{4}{\alpha_H} - \frac{1}{\pi_H\alpha_H + \pi_L\alpha_L} \quad < \quad A \text{ and} \tag{A.45}$$

$$\wp^2(R^2 = R^{1,CE} - \varepsilon; R^{1,CE}, regime3) \quad > \quad \wp^2(R^2 = 0; R^{1,CE}, regime2) \tag{A.46}$$

to hold. Inequality (A.45) ensures that firm 2's deviation to $R^2 = R^{1,CE} - \varepsilon$ puts us in Regime 3, and (A.46) ensures that the deviation is profitable for small enough ε. This can be calculated for $\varepsilon = 0$ as

$$\frac{1}{B(\pi_H\alpha_H + \pi_L\alpha_L)} + R^{1,CE} - (\frac{\pi_H}{B\alpha_H} + \frac{\pi_L}{B\alpha_L}) < \pi_H\frac{A - \frac{2}{\alpha_H}}{B} + \pi_L\alpha_L R^{1,CE}[A - BR^{1,CE} - \frac{1}{\alpha_H}].$$

As an example, the economy $D(v) = 1 - v$, $\pi_L = \pi_H = 0.5$, $\alpha_H = 3.2$, and $\alpha_L = 2$ satisfies (A.45), and one can verify computationally that there is no pure strategy SPE in this economy.[7]■

C. Reserve price competition with known demand in distinct states: $\widetilde{\Gamma}$

Proposition 2:

(a) Suppose $e_D(p_L^c) \geq \frac{1}{2}$ and $e_D(p^*) > 1$, so (by Theorem 1.1) Γ has an equilibrium in Regime 5, characterized by $R^1 = R^2 = 0$. Then $\widetilde{\Gamma}$ has an outcome-equivalent equilibrium.

[6] The following is an example of a SPE in Regime 2 satisfying (11), $D(v) = 1 - v$, $\alpha_H = 2.6$, $\alpha_L = 2.2$, $\pi_H = \pi_L = 0.5$.

[7] Finding necessary and sufficient conditions for nonexistence is equivalent to ruling out all deviations by firm 2, and not just $R^2 = R^{1,CE} - \varepsilon$. There are two other deviations to check. We must check whether firm 2 wants to deviate within Regime 2, which by concavity holds if and only if profits are decreasing when R^2 just binds. We must also check whether firm 2 wants to deviate to an interior point in Regime 3 when $R^{1,CE} > \overline{v}^*/2$ holds, in which case the optimal deviation by firm 2 within Regime 3 is $R^2 = \overline{v}^*/2$.

(b) Suppose $e_D(p_L^c) < \frac{1}{2}$ and $e_D(p^*) > 1$, so (by Theorem 1.2) Γ has an equilibrium in Regime 4, characterized by some $R^1 \in (p_L^c, p^*)$. Then $\widetilde{\Gamma}$ has an outcome-equivalent equilibrium.

Proof of part (a): If the equilibrium to Γ is in Regime 5, $p_H^1 = p_H^2 = p_H^c$ and $p_L^1 = p_L^2 = p_L^c$ hold, and as shown in Peck (2018, online appendix) that when the opposing firm sets a reserve price below the market clearing price (in this case the opposing firm is setting reserve price to 0), the first firm will set a nonbinding reserve price for both states if $e_D(p_s^c) \geq \frac{1}{2}$ for $s \in \{H, L\}$. We are given that $e_D(p_L^c) \geq \frac{1}{2}$ holds (and therefore $e_D(p_H^c) \geq \frac{1}{2}$ also holds under Assumption 1), thus $\widetilde{\Gamma}$ has an outcome equivalent equilibrium to $R^1 = R^2 = 0$.

Proof of part (b): Consider the consumer stage of $\widetilde{\Gamma}$ in state s with reserve prices, (R^1, R^2) and suppose without loss of generality that $R^1 \geq R^2$ holds. Then sequentially rational consumer behavior falls into exactly one of three regimes. If R^1 exceeds the auction price at firm 2 when all consumers choose firm 2, then all consumers choose firm 2. If $R^1 \leq p_s^c$ holds, then the reserve prices do not bind and consumers choose each firm with probability one half, leading to prices $p_s^1 = p_s^2 = p_s^c$. For intermediate values of R^1, consumers mix between firms so that firm 1's reserve price binds, firm 2 sells all its capacity, and prices are given by $p_s^1 = p_s^2 = R^1$.

Now consider the reserve price stage of $\widetilde{\Gamma}$ in state H. We claim that it is sequentially rational for each firm to set a non-binding reserve price, which occurs if the reserve prices are the same as in the equilibrium to Γ, which we denote by $(R^1, 0)$. It is shown in Peck (2018, online appendix) that when the opposing firm sets a reserve price below the market clearing price (in this case p_H^c), the first firm will set a nonbinding reserve price if and only if $e_D(p_H^c) \geq \frac{1}{2}$. But, we have $e_D(p^*) > 1$. Since $p^* < p_H^c$ holds, under Assumption 1 (elasticity rises with price), we have $e_D(p_H^c) > 1$, so $e_D(p_H^c)$ is obviously greater than one half. Since $R^1 < p^*$ holds, in state H, both firms are best responding to each other by setting non-binding reserve prices. Therefore, in the resulting consumer stage, half the consumers with valuation above p_H^c choose each firm, and prices are p_H^c.

Now consider the reserve price stage of $\widetilde{\Gamma}$ in state L. Since $R^1 > p_L^c$ holds, firm 1's reserve price binds. $e_D(p_L^c) < \frac{1}{2}$ holds, so firm 1 will want to set a binding reserve price in state L, but we must show that it is exactly the reserve price from the equilibrium to Γ. In $\widetilde{\Gamma}$, firm 1's profits conditional on observing state L, as a function of the binding reserve price R^1, are given by

$$R^1[\alpha_L D(R^1) - 1], \tag{A.47}$$

where the term in brackets is the quantity sold by firm 1 (total market demand minus firm 2's capacity). The optimal R^1 in Regime 4 of Γ maximizes (15), so the solution is clearly the same in both games.■