Supplemental Appendix

The Aggregate Costs of Uninsurable Business Risk

Corina Boar[1] Denis Gorea[2] Virgiliu Midrigan[3]

Not for Publication

A Data

We use two datasets: Orbis and the Spanish Survey of Household Finances. In this section we describe each of them and show that our motivating facts in Section 2 hold for countries other than Spain.

A.1 Orbis

The main data source we use is the historical product of Orbis available at the NBER. Orbis is a large firm-level data set compiled by Moody's Bureau van Dijk Electronic Publishing (BvD). Bvd collects data from various sources, such as national business registries, and harmonizes them into an internationally comparable format. The Orbis data contain annual balance sheet and income statements together with information on the use of inputs and detailed industry identifiers for both private and public firms.

A.1.1 Data Extract

Starting from the Orbis data available at the NBER enclave (`hindgfr-panel.dta`), we first drop firms (`BVDIDNUMBER`) with missing sales (`OPRE`) or fixed assets (`FIAS`) in all years. Second, we drop firms with missing sector, incorporation year or negative firm age. We then pre-process the data as follows. First, if applicable, we correct for any issues arising from changes in the units of reporting over time (e.g., from millions to thousands). Second, as in Ottonello and Winberry (2020), for the main variables of interest, we linearly interpolate missing values whenever a missing observation is located between two periods with non-missing data. Third, following the recommendations in Kalemli-Özcan et al. (2024), we drop partial or entire time spells within a firm that present any of the following issues: (1) balance sheet components do not correctly add up, allowing for a 0.5% error threshold; (2) if total assets (`TOAS`) or sales (`OPRE`) exhibit large changes above 1000 or below 1/1000 times, or

[1]New York University and NBER, corina.boar@nyu.edu.
[2]Bank for International Settlements, denis.gorea@bis.org.
[3]New York University and NBER, virgiliu.midrigan@nyu.edu.

if there are changes above 50% that coincide with changes in reporting units; (3) if sales (OPRE), tangible assets (TFAS), employment (EMPL), current liabilities (CULI), or non-current liabilities (NCLI) are negative; (4) if employment (EMPL) is larger than that of Walmart (2 million) in any year. If only a subset of a firm's observations have these problems, we keep the last time spell (consecutive observations) of the firm that does not have these issues.[4]

A.1.2 Variable Definition

Our analysis uses the following variables:

1. **Capital**: we define physical capital as the sum of tangible assets (TFAS) and intangible assets (IFAS)

   $$\texttt{Capital = TFAS + IFAS}.$$

2. **Equity**: we define equity as the difference between total assets (TOAS) and total liabilities (CULI+NCLI)

   $$\texttt{Equity = TOAS - (CULI + NCLI)}.$$

3. **Wage bill**: we define the wage bill as the firm's total wage bill, inclusive of pension costs (STAF).

4. **Output**: we define output as value added, which we compute as the sum of after-tax profit and losses (PLAT), depreciation (DEPR), wage bill (STAF), net of financial profit and losses (FIPL). Because Orbis defines PLAT as the sum of operating revenues (OPRE) and financial profit and losses (FIPL), net of the costs of goods sold (COST), other operating expenses (OOPE) and taxes (TAXA), our definition of output amounts to

   $$\texttt{Output = PLAT + DEPR - FIPL + STAF = OPRE - COST - OOPE - TAXA + DEPR + STAF}.$$

   Because the two cost measures include the cost of labor as well as depreciation, we add these to the definition of output in order to capture the difference between operating revenues and the cost of production, excluding the cost of labor and depreciation.

5. **Profit**: we define profit in a way that is consistent with our model as output net of labor costs and the user cost of capital. We thus calculate profits in the Orbis data as

   $$\texttt{Profit = PLAT - FIPL - 0.02} \times \texttt{Capital = Output - STAF - DEPR - 0.02} \times \texttt{Capital}$$

   where the last term assumes an interest rate of 2%, as in the model, to calculate the interest cost of capital.

[4] We have conducted robustness checks to confirm that none of these choices influence our findings.

6. **Debt**: since our notion of debt in the model is financial liabilities net of financial assets, we define debt as the difference between capital `Capital` and firm equity `Equity`, whenever this difference is positive. Otherwise, we set it equal to zero.

   We deflate nominal variables using country-specific CPI deflators from the World Bank World Development Indicators, using 2015 as the base year.

A.1.3 Sample Selection

Given our interest in private businesses, we restrict attention to partnerships and private limited companies. Most of the private firms in Spain are private limited companies ("Sociedad de Responsabilidad Limitada" or S.L.). This is a separate legal entity, so its owners are only liable for the company's debt up to the amount of their capital contribution. A private limited company must have at least one shareholder, but there is no maximum limit. The remaining, much smaller share of firms in the sample, are general partnerships ("Sociedad Colectiva" or S.C.) whose owners face unlimited personal liability for the company's debt. A general partnership must have at least two partners, but there is no maximum limit.

We exclude firms that operate in finance, insurance and real estate, public administration, defense and education. Additionally, we exclude firms with negative values of output, capital, wage bill and depreciation, as well as firms with missing values of output capital, wage bill, equity, profits and depreciation. To minimize the concern that variables are measured with error, we exclude observations in the top and bottom 0.1% of the distribution of the debt-to-capital ratio, the capital-to-output ratio, the wage bill-to-output ratio, the profit-to-output ratio, as well as the annual growth rate of output, capital and the wage bill. Lastly, because we compute growth rates over horizons of up to 3 years, we restrict attention to firms that have at least 4 years of data. Table A.1 reports the impact these sample restrictions have on the sample size.

A.2 Spanish Survey of Household Finances (EFF)

The survey is conducted every three years and each wave samples approximately 6,000 households. A subset of these households is surveyed across multiple waves, creating a panel component. The sample for each wave is chosen to be representative of the population and it oversamples the wealthy to ensure that it adequately captures the distribution of wealth.

A.2.1 Variable Definition

Our analysis uses the following variables:

Table A.1: Sample Selection

	Number of observations	Percent of sample
Sample from data extract	13,103,529	100
Only private firms	11,449,985	87.4
No FIRE, public admin, defense and educ	10,005,535	76.4
No negative observations	9,227,697	70.4
No missing observations	7,036,146	53.7
Drop top and bottom 0.1%	6,972,670	53.2
At least 4 years	6,298,358	48.1

Notes: The table records the number of observations and the corresponding percentage of the full sample at every step of the sample selection. An observation is a firm-year pair.

1. **Entrepreneur**: Dummy variable equal to one when the household owns a business ran or managed by a member of the household, and the household reports positive business wealth. Specifically, a household is considered to run or manage a business if the reference person answers YES to the survey question: "*Does your household own (even if not fully) any business run or managed by a member of your household*?"[5]. To compute the business wealth of a household, we sum over the values of all the businesses that they own.

2. **Business wealth**: This variable is provided in the EFF summary data and is constructed as the sum of the value of each business related to self-employment owned by the household. The value of the business is the answer to the survey question: "*What is the current value of the business after deducting any outstanding debt associated with that business?*".

3. **Wealth (net worth)**: This variable is equal to the net worth variable provided in the EFF summary data plus the value of automobiles.

4. **Total income**: This variable is provided in the EFF summary data and is the total income that the household obtained in the year previous to the survey wave.

5. **Business income**: This variable is the annual profit from the business (net of the

[5]The survey question refers to businesses not listed on the stock market and in whose management one or more household members participate directly. They may be self-employed persons, owners or members of a family business, sole proprietors of a business, fee-earning professionals, or managing partners of a non-family jointly-owned business.

annual losses arising from the business), for each of the businesses owned by the household. It does not include the amount received by the household as remuneration for their work. The value of this variable is the answer to the survey question: "*What is the annual profit before tax provided by this business to your household*".

6. **Consumption**: This variable is the annualized value of average monthly spending on consumer goods reported by the household. Specifically, it is the annualized value of the answer to the survey question: "*What is your household's total average spending on consumer goods in a month including food?*"

A.2.2 Sample Selection

We use data for the 2008-2020 survey waves. We restrict the sample to households in which the reference household member is between 22 and 79 years old. We convert all nominal variables in 2020 Euros.

A.3 Evidence From Other Countries

In this section we revisit the main facts reported in Section 2 for five other countries for which the Orbis data has relatively good coverage: Italy, France, Norway, Portugal and Slovakia. We also report the main facts for the sample of public firms in Spain.

Profit Shares Fluctuate Considerably. Table A.2 reports moments of the distribution of profit share deviations for each country.

Firms Experience Large, Fat-Tailed and Transitory Changes in Output. Table A.3 reports the standard deviation, inter-quartile range and the kurtosis of the distribution of output growth rates.

Capital and Labor Do Not Track Output Closely. Table A.4 reports the slope coefficients of regressions of the growth rate of labor and capital againts the growth rate of output.

A.4 Additional Empirical Results

In this section we report additional statistics from the Orbis data. Table A.5 summarizes the distribution of output growth rates for young (age ≤ 5) and old (age > 5) firms.

Table A.2: Distribution of Profit Share Deviations in Other Countries

	p5	p10	p25	p50	p75	p90	p95
Spain	-0.33	-0.17	-0.05	0.01	0.11	0.27	0.47
Spain (public firms)	-0.35	-0.18	-0.05	0.02	0.12	0.29	0.50
Italy	-0.29	-0.15	-0.05	0.01	0.10	0.26	0.45
France	-0.23	-0.13	-0.04	0.01	0.07	0.17	0.29
Norway	-0.27	-0.15	-0.05	0.01	0.09	0.20	0.32
Portugal	-0.37	-0.19	-0.05	0.02	0.13	0.31	0.54
Slovakia	-0.33	-0.17	-0.05	0.01	0.11	0.27	0.41

Notes: All statistics are weighted by the time-series average of each firm's output. All the statistics, except for those in the row labeled "Spain (public firms)", are for the sample of private firms.

Table A.3: Distribution of Output Growth Rates in Other Countries

	s.d.	iqr	kurtosis
Spain	0.48	0.32	13.7
Spain (public firms)	0.43	0.25	17.7
Italy	0.53	0.33	13.2
France	0.36	0.24	16.0
Norway	0.41	0.27	16.1
Portugal	0.57	0.37	12.8
Slovakia	0.61	0.41	11.3

Notes: The table reports statistics of the distribution of output growth rates $\log y_{it}/y_{it-1}$. All the statistics, except for those in the row labeled "Spain (public firms)", are for the sample of private firms.

Tables A.6, A.7 and A.8 report our motivating facts on the deviations of profit shares, the distribution of output growth and the comovement between output and inputs for the five largest sectors in Spain in terms of total output.

Table A.9 shows that the comovement between output and inputs is similar for small and

Table A.4: Capital, Labor and Output Comovement in Other Countries

	$\Delta \log wl$	$\Delta \log k$
Spain	0.583	0.313
	(0.001)	(0.001)
Spain (public firms)	0.434	0.255
	(0.002)	(0.003)
Italy	0.655	0.273
	(0.001)	(0.001)
France	0.587	0.255
	(0.001)	(0.001)
Norway	0.528	0.224
	(0.002)	(0.005)
Portugal	0.453	0.319
	(0.001)	(0.003)
Slovakia	0.483	0.337
	(0.004)	(0.006)

Notes: The sample is restricted to observations for which $|\Delta \log y| < 0.5$. Standard errors in parentheses are clustered at the firm level. All the statistics, except for those in the row labeled "Spain (public firms)", are for the sample of private firms.

Table A.5: Distribution of Output Growth by Firm Age

	all	young	old
mean	0.012	0.110	-0.014
standard deviation	0.484	0.550	0.462
interquartile range	0.316	0.418	0.294
kurtosis	13.67	10.30	15.10

Notes: The column labeled "young" reports moments of the distribution of output growth for Spanish firms whose age is 5 years or younger. The column labeled "old" reports moments for firms that are older than 5 years.

Table A.6: Distribution of Profit Share Deviations by Sector

	p5	p10	p25	p50	p75	p90	p95
Manufacturing	-0.35	-0.18	-0.05	0.02	0.12	0.28	0.47
Construction	-0.43	-0.19	-0.04	0.02	0.13	0.35	0.60
Wholesale and retail trade	-0.32	-0.17	-0.06	0.02	0.11	0.26	0.43
Accommodation and food	-0.31	-0.17	-0.05	0.01	0.10	0.25	0.41
Professional activities	-0.33	-0.16	-0.05	0.01	0.09	0.27	0.46

Notes: All statistics weighted by the time-series average of each firm's output.

Table A.7: Distribution of Output Growth Rates by Sector

	s.d.	iqr	kurtosis
Manufacturing	0.40	0.27	17.1
Construction	0.60	0.43	10.0
Wholesale and retail trade	0.44	0.28	16.0
Accommodation and food	0.41	0.26	17.4
Professional activities	0.55	0.36	11.1

Notes: The table reports statistics of the distribution of output growth rates $\log y_{it}/y_{it-1}$.

large firms. We define a firm to be large if it is in the top decile of the distribution of value added, and small otherwise.

B Model

In this section, we discuss in more detail the model with flexible labor in Section 5 and the extension with unconstrained firms in Section 7. We also report results from alternative calibration strategies of the economies discussed in Section 5.

Table A.8: Capital, Labor and Output Comovement by Sector

	$\Delta \log wl$	$\Delta \log k$
Manufacturing	0.591	0.343
	(0.002)	(0.003)
Construction	0.546	0.325
	(0.001)	(0.003)
Wholesale and retail trade	0.668	0.330
	(0.002)	(0.004)
Accommodation and food	0.580	0.258
	(0.003)	(0.005)
Professional activities	0.593	0.267
	(0.003)	(0.005)

Notes: The sample is restricted to observations for which $|\Delta \log y| < 0.5$. Standard errors in parentheses are clustered at the firm level.

Table A.9: Capital, Labor and Output Comovement by Firm Size

	$\Delta \log wl$	$\Delta \log k$
Small	0.582	0.304
	(0.001)	(0.002)
Large	0.581	0.368
	(0.002)	(0.004)

Notes: The sample is restricted to observations for which $|\Delta \log y| < 0.5$. Standard errors in parentheses are clustered at the firm level.

B.1 A Model With Flexible Labor

When labor is chosen after the realization of productivity shocks, the entrepreneur equates the marginal product of labor to the wage,

$$(1-\alpha)\,\eta z_{it+1}\varepsilon_{it+1}\left(\left(k_{it+1}\right)^{\alpha}\left(l_{it+1}\right)^{1-\alpha}\right)^{\eta}\frac{1}{l_{it+1}} = W,$$

so in choosing its capital before productivity is realized it recognizes that labor is a function of the realization of productivity. Consequently, the first-order condition for capital is

$$\alpha\eta\left(\frac{(1-\alpha)\eta}{W}\right)^{\frac{(1-\alpha)\eta}{1-(1-\alpha)\eta}}(k_{it+1})^{\frac{\alpha\eta}{1-(1-\alpha)\eta}-1}\frac{\mathbb{E}_{it}\left(c_{it+1}\right)^{-\theta}\left(z_{it+1}\varepsilon_{it+1}\right)^{\frac{1}{1-(1-\alpha)\eta}}}{\mathbb{E}_{it}\left(c_{it+1}\right)^{-\theta}}=R+(1-\xi)\,\mu_{it},$$

so the risk wedge is a function of the covariance of consumption with $\left(z_{it+1}\varepsilon_{it+1}\right)^{\frac{1}{1-(1-\alpha)\eta}}$. The risk wedge is therefore

$$\tau_{it}=\left(1+\frac{\mathbb{COV}_{it}\left(c_{it+1}^{-\theta},\left(z_{it+1}\varepsilon_{it+1}\right)^{\frac{1}{1-(1-\alpha)\eta}}\right)}{\mathbb{E}_{it}c_{it+1}^{-\theta}\mathbb{E}_{it}\left(z_{it+1}\varepsilon_{it+1}\right)^{\frac{1}{1-(1-\alpha)\eta}}}\right)^{-1},$$

and the collateral wedge is

$$\omega_{it}=1+(1-\xi)\,\frac{\mu_{it}}{R}.$$

To calculate the aggregate productivity losses from misallocation, we follow the same approach as in the baseline model. Since the planner can also choose labor after the realization of productivity, the share of labor allocated to each firm is a function of the predetermined amount of capital

$$n_{it}^{l}=\frac{\left(z_{it+1}\varepsilon_{it+1}\right)^{\frac{1}{1-(1-\alpha)\eta}}\left(k_{it+1}\right)^{\frac{\alpha\eta}{1-(1-\alpha)\eta}}}{\int\left(z_{it+1}\varepsilon_{it+1}\right)^{\frac{1}{1-(1-\alpha)\eta}}\left(k_{it+1}\right)^{\frac{\alpha\eta}{1-(1-\alpha)\eta}}\mathrm{d}i}.$$

In turn, the share of capital allocated to each firm is

$$n_{it}^{k}=\frac{\left(\mathbb{E}_{it}\left(z_{it+1}\varepsilon_{it+1}\right)^{\frac{1}{1-(1-\alpha)\eta}}\right)^{\frac{1-(1-\alpha)\eta}{1-\eta}}}{\int\left(\mathbb{E}_{it}\left(z_{it+1}\varepsilon_{it+1}\right)^{\frac{1}{1-(1-\alpha)\eta}}\right)^{\frac{1-(1-\alpha)\eta}{1-\eta}}\mathrm{d}i}$$

and the resulting efficient level of productivity is

$$Z_{t+1}^{P}=\left(\int\left(\mathbb{E}_{it}\left(z_{it+1}\varepsilon_{it+1}\right)^{\frac{1}{1-(1-\alpha)\eta}}\right)^{\frac{1-(1-\alpha)\eta}{1-\eta}}\mathrm{d}i\right)^{1-\eta}.$$

The productivity losses from misallocation are therefore

$$\frac{Z_{t+1}}{Z_{t+1}^{P}}=\frac{\int\left(\tau_{it}\omega_{it}\right)^{-\frac{\alpha\eta}{1-\eta}}n_{it}^{k}\mathrm{d}i}{\left(\int\left(\tau_{it}\omega_{it}\right)^{-\frac{1-(1-\alpha)\eta}{1-\eta}}n_{it}^{k}\mathrm{d}i\right)^{\alpha\eta}\left(\int\left(\tau_{it}\omega_{it}\right)^{-\frac{\alpha\eta}{1-\eta}}n_{it}^{k}\mathrm{d}i\right)^{(1-\alpha)\eta}}$$

and depend on the distribution of risk and credit wedges, weighted by the efficient capital share of firms. Since labor is undistorted in this economy, the aggregate labor wedge is $\tau_t^L=1$. The aggregate capital wedge τ_t^K is

$$\tau_t^{K}=\frac{\int\left(\tau_{it}\omega_{it}\right)^{-\frac{\alpha\eta}{1-\eta}}n_{it}^{k}\mathrm{d}i}{\int\left(\tau_{it}\right)^{-\frac{1-(1-\alpha)\eta}{1-\eta}}n_{it}^{k}\mathrm{d}i},$$

and the aggregate output wage losses satisfy

$$\frac{Y_{t+1}}{Y_{t+1}^P} = \frac{W}{W^P} = \left(\tau_t^K\right)^{-\frac{\alpha\eta}{1-\alpha\eta}} \left(\frac{Z_{t+1}}{Z_{t+1}^P}\right)^{\frac{1}{1-\alpha\eta}}.$$

B.2 A Model With Unconstrained Firms

We assume that private businesses produce alongside a representative unconstrained firm with technology

$$Y_c = Z_c \left(K_c^\alpha L_c^{1-\alpha}\right)^\eta$$

who faces no frictions and therefore sets capital equal to

$$K_c = \left(\frac{\alpha\eta}{R}\right)^{\frac{1-(1-\alpha)\eta}{1-\eta}} \left(\frac{(1-\alpha)\,\eta}{W}\right)^{\frac{(1-\alpha)\eta}{1-\eta}} Z_c^{\frac{1}{1-\eta}}$$

and labor equal to

$$L_c = \left(\frac{\alpha\eta}{R}\right)^{\frac{\alpha\eta}{1-\eta}} \left(\frac{(1-\alpha)\,\eta}{W}\right)^{\frac{1-\alpha\eta}{1-\eta}} Z_c^{\frac{1}{1-\eta}},$$

so the output of the unconstrained sector is

$$Y_c = \left(\frac{\alpha\eta}{R}\right)^{\frac{\alpha\eta}{1-\eta}} \left(\frac{(1-\alpha)\,\eta}{W}\right)^{\frac{(1-\alpha)\eta}{1-\eta}} Z_c^{\frac{1}{1-\eta}}.$$

In contrast, the total capital and labor used by the entrepreneurial sector are[6]

$$K = \left(\frac{\alpha\eta}{R}\right)^{\frac{1-(1-\alpha)\eta}{1-\eta}} \left(\frac{(1-\alpha)\,\eta}{W}\right)^{\frac{(1-\alpha)\eta}{1-\eta}} \left(\tau^K\right)^{-\frac{1-(1-\alpha)\eta}{1-\eta}} \left(\tau^L\right)^{-\frac{(1-\alpha)\eta}{1-\eta}} Z^{\frac{1}{1-\eta}}$$

and

$$L = \left(\frac{\alpha\eta}{R}\right)^{\frac{\alpha\eta}{1-\eta}} \left(\frac{(1-\alpha)\,\eta}{W}\right)^{\frac{1-\alpha\eta}{1-\eta}} \left(\tau^K\right)^{-\frac{\alpha\eta}{1-\eta}} \left(\tau^L\right)^{-\frac{1-\alpha\eta}{1-\eta}} Z^{\frac{1}{1-\eta}},$$

so output produced in the entrepreneurial sector is

$$Y = \left(\frac{\alpha\eta}{R}\right)^{\frac{\alpha\eta}{1-\eta}} \left(\frac{(1-\alpha)\,\eta}{W}\right)^{\frac{(1-\alpha)\eta}{1-\eta}} \left(\tau^e\right)^{-1} Z^{\frac{1}{1-\eta}}.$$

We can write therefore write an aggregate production function that expresses aggregate output $Y_a = Y_c + Y$ as a function of the total amount of capital $K_a = K_c + K$ and labor $L_a = L_c + L$ as

$$Y_a = Z_a \left(K_a^\alpha L_a^{1-\alpha}\right)^\eta,$$

[6] We omit all time subscripts, as aggregate variable are time-invariant.

where aggregate productivity is

$$Z_a = \frac{Z_c^{\frac{1}{1-\eta}} + \left(\tau^K\right)^{-\frac{\alpha\eta}{1-\eta}} \left(\tau^L\right)^{-\frac{(1-\alpha)\eta}{1-\eta}} Z^{\frac{1}{1-\eta}}}{\left(Z_c^{\frac{1}{1-\eta}} + (\tau^K)^{-\frac{1-(1-\alpha)\eta}{1-\eta}} (\tau^L)^{-\frac{(1-\alpha)\eta}{1-\eta}} Z^{\frac{1}{1-\eta}}\right)^{\alpha\eta} \left(Z_c^{\frac{1}{1-\eta}} + (\tau^K)^{-\frac{\alpha\eta}{1-\eta}} (\tau^L)^{-\frac{1-\alpha\eta}{1-\eta}} Z^{\frac{1}{1-\eta}}\right)^{(1-\alpha)\eta}}.$$

The aggregate capital and labor wedges τ_a^K and τ_a^L are implicitly defined by

$$\alpha\eta\frac{Y_a}{K_a} = \tau_a^K R \qquad \text{and} \qquad (1-\alpha)\,\eta\frac{Y_a}{L_a} = \tau_a^L W$$

and are equal to

$$\tau_a^K = \frac{Z_c^{\frac{1}{1-\eta}} + \left(\tau^K\right)^{-\frac{\alpha\eta}{1-\eta}} \left(\tau^L\right)^{-\frac{(1-\alpha)\eta}{1-\eta}} Z^{\frac{1}{1-\eta}}}{Z_c^{\frac{1}{1-\eta}} + (\tau^K)^{-\frac{1-(1-\alpha)\eta}{1-\eta}} (\tau^L)^{-\frac{(1-\alpha)\eta}{1-\eta}} Z^{\frac{1}{1-\eta}}}$$

and

$$\tau_a^L = \frac{Z_c^{\frac{1}{1-\eta}} + \left(\tau^K\right)^{-\frac{\alpha\eta}{1-\eta}} \left(\tau^L\right)^{-\frac{(1-\alpha)\eta}{1-\eta}} Z^{\frac{1}{1-\eta}}}{Z_c^{\frac{1}{1-\eta}} + (\tau^K)^{-\frac{\alpha\eta}{1-\eta}} (\tau^L)^{-\frac{1-\alpha\eta}{1-\eta}} Z^{\frac{1}{1-\eta}}}.$$

B.3 A Model With Working Capital Constraints

In this section we consider a model in which a fraction ϑ of the wage bill must be paid in advance of production. To illustrate how this affects input choices, we discuss here the choices of an agent who is an entrepreneur in both periods t and $t+1$. The budget constraint of this entrepreneur is

$$c_{it} + \vartheta W l_{it+1} + k_{it+1} - b_{it+1} = z_{it}\varepsilon_{it}\left(k_{it}^{\alpha} l_{it}^{1-\alpha}\right)^{\eta} + (1-\delta)\,k_{it} - (1-\vartheta)\,W l_{it} - (1+r)\,b_{it}.$$

Letting $a_{it+1} = \vartheta W l_{it+1} + k_{it+1} - b_{it+1}$ denote the entrepreneur's net worth, including the working capital $\vartheta W l_{it+1}$, the budget constraint can be written as

$$c_{it} + a_{it+1} = (1+r)\,a_{it} + z_{it}\varepsilon_{it}\left(k_{it}^{\alpha} l_{it}^{1-\alpha}\right)^{\eta} - R k_{it} - (1+r\vartheta)\,W l_{it}.$$

Recall that in our empirical analysis we measured debt as the difference between the firm's physical capital and its equity. Thus, our notion of debt corresponds to the difference between financial liabilities net of financial assets, inventories, working capital, etc. Under this interpretation, if we were to treat the firm's payments to labor $\vartheta W l_{it+1}$ as working capital, as does Mendoza (2010), and assume that this is correctly recorded on the firm's balance sheet, the relevant notion of debt would be $b_{it+1} - \vartheta W l_{it+1}$ and therefore the credit constraint would be

$$b_{it+1} - \vartheta W l_{it+1} \leq \xi k_{it+1},$$

or, using the definition of net worth,

$$k_{it+1} \leq \frac{1}{1-\xi} a_{it+1}.$$

The optimal choice of labor would then satisfy

$$(1-\alpha)\,\eta \frac{\left(k_{it+1}^{\alpha} l_{it+1}^{1-\alpha}\right)^{\eta}}{l_{it+1}} \frac{\mathbb{E}_{it} c_{it+1}^{-\theta} z_{it+1} \varepsilon_{it+1}}{\mathbb{E}_{it} c_{it+1}^{-\theta}} = W\,(1+r\vartheta)$$

and would be unaffected by the credit constraint. Because the interest rate r is relatively low, assuming that the wage bill must be paid in advance simply increases the cost of labor by a relatively small factor, $r\vartheta$, leaving outcomes largely unchanged.

Alternatively, one could assume that the working capital is not recorded on the firm's asset side of the balance sheet, so the credit constraint is

$$b_{it+1} \leq \xi k_{it+1},$$

or, equivalently,

$$k_{it+1} \leq \frac{1}{1-\xi} \left(a_{it+1} - \vartheta W l_{it+1}\right).$$

Under this alternative interpretation, every additional unit of working capital reduces the firm's pledgeable net worth one-for-one and therefore distorts the labor choice. Specifically, the first order condition for labor is now

$$(1-\alpha)\,\eta \frac{\left(k_{it+1}^{\alpha} l_{it+1}^{1-\alpha}\right)^{\eta}}{l_{it+1}} \frac{\mathbb{E}_{it} c_{it+1}^{-\theta} z_{it+1} \varepsilon_{it+1}}{\mathbb{E}_{it} c_{it+1}^{-\theta}} = W\left(1+\left(r+\mu_{it}\right)\vartheta\right),$$

where recall that μ_{it} is the multiplier on the credit constraint.

We gauge the quantitative relevance of this alternative interpretation by following Mendoza (2010) and setting $\vartheta = 0.25$, so that a quarter of the wage bill has to be paid one year in advance, and evaluating the model at our baseline parameter values.[7] We find that the losses from misallocation are 11.1%, close to the 10.8% in the baseline. Importantly, risk continues to account for the bulk of these losses (9.4%), and credit constraints are once again less important (1.3%). Similarly, uninsurable risk accounts for the majority (13.4%) of the overall output losses (16.3%) from financial frictions.

B.4 The Importance of Risk: Alternative Estimation

In this section we present an alternative estimation of two of the three models discussed in Section 5. Here, we re-estimate the economies without fat-tailed and without transitory

[7] We note that implied moments are close to the data and our baseline model.

Table B.1: Targeted Moments: Alternative Models

	data	no fat tails	no transitory shocks
fraction entrepr	0.12	0.09	0.09
wealth to income entrepr	12.5	12.4	12.4
capital-output ratio, k/y	1.22	1.27	1.26
labor share, wl/y	0.72	0.78	0.75
profit share, π/y	0.13	0.07	0.10
iqr $wl_{it}/y_{it} - \overline{wl_{it}/y_{it}}$	0.15	0.12	0.04
s.d. $\log y_{it}$	1.32	1.31	1.32
s.d. $\log y_{it}/y_{it-1}$	0.48	0.43	0.39
s.d. $\log y_{it}/y_{it-2}$	0.60	0.58	0.58
s.d. $\log y_{it}/y_{it-3}$	0.69	0.70	0.73
iqr $\log y_{it}/y_{it-1}$	0.32	0.33	0.31
iqr $\log y_{it}/y_{it-2}$	0.46	0.51	0.51
iqr $\log y_{it}/y_{it-3}$	0.58	0.65	0.66
corr $\log y_{it}$, $\log y_{it-1}$	0.93	0.95	0.96
corr $\log y_{it}$, $\log y_{it-2}$	0.89	0.90	0.91
corr $\log y_{it}$, $\log y_{it-3}$	0.86	0.86	0.86
value of objective	–	0.027	0.038

Notes: The table reports the fit of the estimation procedure across alternative models when we target the exact same set of moments as in the estimation of the baseline model.

shocks to match the exact same moments as in the baseline estimation and revisit our decomposition of productivity, output and wage losses attributed to risk and credit wedges. The results we discuss in Section 5 are robust to this alternative estimation strategy. Tables B.1 and B.2 report the results of the estimation and Table B.3 shows the decomposition between risk and credit wedges.

Table B.2: Parameter Values: Alternative Models

		no fat tails	no transitory shocks
β	discount factor	0.971	0.961
α	capital elasticity	0.183	0.204
η	span of control	0.981	0.953
ρ	persistence z	0.974	0.966
σ_u	volatility z	0.023	0.036
σ_ε	volatility ε	0.118	–
s	relative volatility mixture	–	3.944
p	baseline probability mixture	–	0.950
ϕ	relative time endowment	0.984	0.968

Notes: The table reports the parameter estimates across alternative models when we target the exact same set of moments as in the estimation of the baseline model.

Table B.3: Aggregate Implications: Alternative Models

	no fat tails	no transitory shocks
misallocation, $-\log Z/Z^P$		
total	0.056	0.081
due to risk	0.004	0.001
due to credit	0.041	0.076
output losses, $-\log Y/Y^P$		
total	0.103	0.162
due to risk	0.013	0.006
due to credit	0.093	0.159
wage losses, $-\log W/W^P$		
total	0.126	0.172
due to risk	0.049	0.024
due to credit	0.093	0.159

Notes: The table reports the misallocation, output and wage losses across alternative models evaluated at the estimates obtained when we target the same set of moments as in the estimation of the baseline model.