Online Appendix
Great Layoff, Great Retirement and Post-pandemic Inflation

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A. Derivation of TANK model equations

Households.. — To derive first-order conditions for the household problem, write the value function for type a as

$$\mathcal{V}_{a,t} = \max_{E_{a,t},D_{a,t}} \mathcal{U}_{a,t} + \beta \mathbb{E}_t \mathcal{V}_{a,t+1} \ . \tag{A.1}$$

Rewrite the budget constraint as

$$C_{a,t} = D_{a,t} - \frac{R_{t-1}}{\Pi_t} D_{a,t-1} + E_{a,t}\frac{W_t}{P_t} + b^u U_{a,t} + b^n_{a,t} N_{a,t} - \frac{\lambda}{2}(D_{a,t})^2 + \Omega_{a,t} \ , \tag{A.2}$$

where $\Omega_{a,t}$ collects all terms outside agents' control.[1] The first-order condition for optimal consumption choices are

$$\begin{aligned}\frac{\partial \mathcal{V}_{a,t}}{\partial D_{a,t}} &= \frac{\partial \mathcal{U}_{a,t}}{\partial D_{a,t}} + \beta \mathbb{E}_t \frac{\partial \mathcal{V}_{a,t+1}}{\partial D_{a,t}} = 0 \\ &= Z_t \frac{1}{C_{a,t} - \xi C_{a,t-1}} (1 - \lambda D_{a,t}) + \beta \mathbb{E}_t \left(-Z_{t+1} \frac{1}{C_{a,t+1} - \xi C_{a,t}} \frac{R_t}{\Pi_{t+1}} \right) = 0 \ ,\end{aligned}$$

yielding the Euler equation

$$R_t \mathbb{E}_t \left(Q^a_{t,t+1} \Pi^{-1}_{t+1} \right) + \lambda D_{a,t} = 1 \ . \tag{A.3}$$

Consider now the households' labor supply decision. The derivative with respect to employment is given by

$$\frac{\partial \mathcal{V}_{a,t}}{\partial E_{a,t}} = \frac{\partial \mathcal{U}_{a,t}}{\partial E_{a,t}} + \beta \mathbb{E}_t \frac{\partial \mathcal{V}_{a,t+1}}{\partial E_{a,t}} \ . \tag{A.4}$$

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[1] Specifically, $\Omega_{a,t}$ includes real dividends d_t, steady state transfers $tr/(1-\zeta)$ and $-tr/\zeta$ for old and young agents respectively, as well as net lump-sum taxes which are given by $T_{o,t} \equiv (U_t - U_{o,t})b^u - \zeta b^n_t N_{o,t} - \tau_t/(1-\zeta)$ for the old agents and $T_{y,t} \equiv (U_t - U_{y,t})b^u + (1-\zeta) b^n_t N_{o,t} + \tau_t/\zeta$ for the young agent.

Using that $\partial \mathcal{V}_{a,t+2}/\partial E_{a,t} = 0$, we have that

$$\frac{\partial \mathcal{V}_{a,t+1}}{\partial E_{a,t}} = \frac{Z_{t+1}}{C_{t+1} - \xi C_t} \frac{\partial C_{t+1}}{\partial E_{a,t}} + \phi (h_{a,t+1})^{-\nu_a} \frac{\partial h_{a,t+1}}{\partial E_{a,t}} \ . \tag{A.5}$$

By substituting Eq. (A.5) in Eq. (A.4) and computing $\frac{\partial \mathcal{U}_{a,t}}{\partial E_{a,t}}$ we have that

$$\begin{aligned}\frac{\partial \mathcal{V}_{a,t}}{\partial E_{a,t}} = {} & \frac{Z_t}{C_{a,t} - \xi C_{a,t-1}} \frac{\partial C_{a,t}}{\partial E_{a,t}} + \phi (h_{a,t})^{-\nu_a} \frac{\partial h_{a,t}}{\partial E_{a,t}} + \\ & + \beta \mathbb{E}_t \left(\frac{Z_{t+1}}{C_{a,t+1} - \xi C_{a,t}} \frac{\partial C_{a,t+1}}{\partial E_{a,t}} + \phi (h_{a,t+1})^{-\nu_a} \frac{\partial h_{a,t+1}}{\partial E_{a,t}} \right) \ .\end{aligned}$$

Rewriting the dynamics of employment as

$$E_t = (1-\rho_t) E_{t-1} + f_t S_t = (1-\rho_t) E_{t-1} + \frac{f_t}{1-f_t} U_t \ , \tag{A.6}$$

and computing derivatives

$$\begin{aligned}\frac{\partial C_{a,t}}{\partial E_{a,t}} &= \frac{W_t}{P_t} + \frac{b^u (1-f_t)}{f_t} - \frac{b^n_{a,t}}{f_t} \\ \frac{\partial h_{a,t}}{\partial E_{a,t}} &= (1-\alpha_h)(h_{a,t})^{-\frac{\alpha_h}{1-\alpha_h}} A^h_{a,t} \left(-1 - \Gamma \frac{1-f_t}{f_t} \right) \\ \frac{\partial C_{a,t+1}}{\partial E_{a,t}} &= -\frac{1-f_{t+1}}{f_{t+1}} (1-\rho_{t+1}) b^u + \frac{1-f_{t+1}}{f_{t+1}} (1-\rho_{t+1}) b^n_{a,t+1} \\ \frac{\partial h_{a,t+1}}{\partial E_{a,t}} &= (1-\alpha_h)(h_{a,t+1})^{-\frac{\alpha_h}{1-\alpha_h}} A^h_{a,t+1} \left(\Gamma \frac{1-f_{t+1}}{f_{t+1}} (1-\rho_{t+1}) \right) \ ,\end{aligned}$$

we can rewrite the equation for $\partial \mathcal{V}_{a,t}/\partial E_{a,t}$ as

$$\begin{aligned}\frac{\partial \mathcal{V}_{a,t}}{\partial E_{a,t}} = {} & \frac{Z_t}{C_{a,t} - \xi C_{a,t-1}} \left[\left(\frac{W_t}{P_t} + \frac{b^u (1-f_t)}{f_t} - \frac{b^n_{a,t}}{f_t} \right) - MRS_{a,t} \left(1 + \Gamma \frac{1-f_t}{f_t} \right) \right] + \\ & + \beta \mathbb{E}_t \left[\frac{Z_{t+1}}{C_{a,t+1} - \xi C_{a,t}} \left(\frac{1-f_{t+1}}{f_{t+1}} (1-\rho_{t+1}) (b^n_{a,t+1} - b^u + \Gamma MRS_{a,t+1}) \right) \right] \ ,\end{aligned} \tag{A.7}$$

where $MRS_{a,t} \equiv \frac{C_{a,t} - \xi C_{a,t-1}}{Z_t} \phi A^h_{a,t} (1-\alpha_h)(h_{a,t})^{-\frac{\alpha_h}{1-\alpha_h} - \nu_a}$ denotes the value of home-produced goods generated by the marginal non-participant in terms of market consumption goods. Setting the derivative of Eq. (A.7) to zero, we obtain the participation condition for households

$$\begin{aligned} & f_t \frac{W_t}{P_t} + (1-f_t) b^u + \mathbb{E}_t \left[Q^a_{t,t+1} f_t (1-\rho_{t+1}) \frac{1-f_{t+1}}{f_{t+1}} (\Gamma MRS_{a,t+1} - b^u + b^n_{a,t+1}) \right] = \\ & \qquad b^n_{a,t} + MRS_{a,t} \left[f_t + (1-f_t)\Gamma \right] \ .\end{aligned} \tag{A.8}$$

To provide an intuition for this equation, let us define the value of a marginal resource in the various states: searcher, employed, unemployed or out of the labor force. Note that all this values generally would imply a continuation value, but one: the value of the marginal searcher that can be defined as

$$V_{a,t}^S = f_t V_{a,t}^E + (1 - f_t) V_{a,t}^U, \tag{A.9}$$

because an individual is a searcher only at the beginning of the period and then immediately becomes either employed or unemployed within the period t. If employed, agents receive a wage in period t. In the following period $t+1$, they may either lose their job or remain employed. Separation occurs with probability ρ_{t+1}, in which case agents join the pool of job seekers and obtain a value V_{t+1}^S. Conversely, with probability $(1 - \rho_{t+1})$, they remain employed and receive a value V_{t+1}^E. Hence

$$V_{a,t}^E = \frac{W_t}{P_t} + \mathbb{E}_t \left[Q_{t,t+1}^a \left(\rho_{t+1} V_{a,t+1}^S + (1 - \rho_{t+1}) V_{a,t+1}^E \right) \right] . \tag{A.10}$$

The value of a marginal unemployed individual is

$$V_{a,t}^U = b^u + (1 - \Gamma) MRS_{a,t} + \mathbb{E}_t \left[Q_{t,t+1}^a V_{a,t+1}^S \right], \tag{A.11}$$

because the agent gets the unemployment benefit and can work at home a fraction $(1 - \Gamma)$ of her time getting $MRS_{a,t}$ in the current period t. In the next period $t + 1$ the agent is back in the pool of searchers thus getting $V_{a,t+1}^S$.

Finally, the value of a marginal individual out of the labor force is:

$$V_{a,t}^N = b_{a,t}^n + MRS_{a,t} + \mathbb{E}_t \left[Q_{t,t+1}^a V_{a,t+1}^N \right], \tag{A.12}$$

because individuals out of the labor force receive the pension and the full value of home production, $b_{a,t}^n + MRS_{a,t}$, plus the continuation value.

Substituting Eq. (A.9) into Eq. (A.11) we can write the value of being unemployed as

$$V_{a,t}^U = b^u + (1 - \Gamma) MRS_{a,t} + \mathbb{E}_t \left[Q_{t,t+1}^a \left(f_{t+1} V_{a,t+1}^E + (1 - f_{t+1}) V_{a,t+1}^U \right) \right]. \tag{A.13}$$

Eq. (A.8) describes the condition under which agents are indifferent between participating in the labor market and staying out of the labor force. At the optimum, the marginal value of these two choices must be equal.

The value of a marginal searcher is given by equations (A.9), (A.10) and (A.13):

$$\begin{aligned}
V_{a,t}^S &= f_t V_{a,t}^E + (1-f_t)V_{a,t}^U \\
&= f_t \left\{ \frac{W_t}{P_t} + \mathbb{E}_t \left[Q_{t,t+1}^a \left(\rho_{t+1} V_{a,t+1}^S + (1-\rho_{t+1}) V_{a,t+1}^E \right) \right] \right\} \\
&\quad + (1-f_t) \left\{ b^u + (1-\Gamma) MRS_{a,t} + \mathbb{E}_t \left[Q_{t,t+1}^a V_{a,t+1}^S \right] \right\} \\
&= f_t \frac{W_t}{P_t} + (1-f_t)(b^u + (1-\Gamma) MRS_{a,t}) \\
&\quad + \mathbb{E}_t Q_{t,t+1}^a \Big[f_t (\rho_{t+1} V_{a,t+1}^S + (1-\rho_{t+1}) V_{a,t+1}^E) + (1-f_t) V_{a,t+1}^S \Big] \\
&= f_t \frac{W_t}{P_t} + (1-f_t)(b^u + (1-\Gamma) MRS_{a,t}) \\
&\quad + \mathbb{E}_t Q_{t,t+1}^a \Big[(1 - f_t + f_t \rho_{t+1}) V_{a,t+1}^S + f_t (1-\rho_{t+1}) V_{a,t+1}^E \Big]
\end{aligned}$$

Given that at the FOC, the household must be indifferent, i.e., $V_{a,t}^S = V_{a,t}^N$ for all t, we have that

$$\begin{aligned}
f_t \frac{W_t}{P_t} &+ (1-f_t)(b^u + (1-\Gamma) MRS_{a,t}) \\
&+ \mathbb{E}_t Q_{t,t+1}^a \Big[(1 - f_t + f_t \rho_{t+1}) V_{a,t+1}^S + f_t (1-\rho_{t+1}) V_{a,t+1}^E \Big] \\
&= b_{a,t}^n + MRS_{a,t} + \mathbb{E}_t Q_{t,t+1}^a V_{a,t+1}^N
\end{aligned}$$

Since $V_{a,t+1}^N = V_{a,t+1}^S$, the expectation term simplifies:

$$\begin{aligned}
f_t \frac{W_t}{P_t} &+ (1-f_t)(b^u + (1-\Gamma) MRS_{a,t}) \\
&+ \mathbb{E}_t Q_{t,t+1}^a f_t (1-\rho_{t+1}) (V_{a,t+1}^E - V_{a,t+1}^N) = b_{a,t}^n + MRS_{a,t}
\end{aligned}$$

Now using Eq. (A.9),

$$V_{a,t+1}^E = \frac{V_{a,t+1}^S}{f_{t+1}} - \frac{1-f_{t+1}}{f_{t+1}} V_{a,t+1}^U$$

and since $V_{a,t+1}^N = V_{a,t+1}^S$, we obtain:

$$\begin{aligned}
f_t \frac{W_t}{P_t} &+ (1-f_t)(b^u + (1-\Gamma) MRS_{a,t}) \\
&+ \mathbb{E}_t Q_{t,t+1}^a f_t (1-\rho_{t+1}) \frac{1-f_{t+1}}{f_{t+1}} (V_{a,t+1}^N - V_{a,t+1}^U) = b_{a,t}^n + MRS_{a,t}
\end{aligned}$$

Finally, using Eqs. (A.12) and (A.11), we can write

$$V_{a,t+1}^N - V_{a,t+1}^U = b_{a,t+1}^n + \Gamma MRS_{a,t+1} - b^u$$

and recover Eq. (A.8):

$$f_t \frac{W_t}{P_t} + (1 - f_t)(b^u + (1 - \Gamma)MRS_{a,t}) \\ + \mathbb{E}_t Q^a_{t,t+1} f_t (1 - \rho_{t+1}) \frac{1 - f_{t+1}}{f_{t+1}} (b^n_{a,t+1} + \Gamma MRS_{a,t+1} - b^u) = b^n_{a,t} + MRS_{a,t}$$

In what follows, we derive the net benefit of having an additional member employed instead of unemployed, for a chosen level of participation, used in the bargaining process between union and firms. Let us define $V^w_{a,t} = V^E_{a,t} - V^U_{a,t}$ and rewrite Eq. (A.10) as

$$\begin{aligned} V^E_{a,t} &= \frac{W_t}{P_t} + \mathbb{E}_t \left[Q^a_{t,t+1} \left(\rho_{t+1} V^S_{a,t+1} + (1 - \rho_{t+1}) V^E_{a,t+1} \right) \right] \\ &= \frac{W_t}{P_t} + \mathbb{E}_t \left[Q^a_{t,t+1} \left(\rho_{t+1} \left(f_{t+1} V^E_{a,t+1} + (1 - f_{t+1}) V^U_{a,t+1} \right) + (1 - \rho_{t+1}) V^E_{a,t+1} \right) \right] \\ &= \frac{W_t}{P_t} + \mathbb{E}_t \left[Q^a_{t,t+1} \left(\left(\rho_{t+1} f_{t+1} + 1 - \rho_{t+1} \right) V^E_{a,t+1} + \rho_{t+1} (1 - f_{t+1}) V^U_{a,t+1} \right) \right]. \end{aligned} \tag{A.14}$$

Substituting Eqs. (A.14)–(A.11) in the definition of $V^w_{a,t}$ yields

$$V^w_{a,t} = V^E_{a,t} - V^U_{a,t} = \frac{W_t}{P_t} - b^u - (1 - \Gamma) MRS_{a,t} + \mathbb{E}_t \Bigg[Q^a_{t,t+1} \Big(\left(\rho_{t+1} f_{t+1} + 1 - \rho_{t+1} \right) V^E_{a,t+1} \\ + \rho_{t+1}(1 - f_{t+1}) V^U_{a,t+1} - f_{t+1} V^E_{a,t+1} - (1 - f_{t+1}) V^U_{a,t+1} \Big) \Bigg]$$

The first term in the equation captures the difference between the direct marginal benefit of being employed in period t, given by W_t/P_t, and that of being unemployed, given by $b^u + (1 - \Gamma)MRS_{a,t}$. The second term instead reflects the expected difference in future values associated with employment and unemployment. Specifically, an employed individual in period t may either lose her job with probability ρ_{t+1}—in which case she finds a new job with probability f_{t+1} or remains unemployed with probability $1 - f_{t+1}$—or remain employed with probability $1 - \rho_{t+1}$. Similarly, an unemployed individual in t may find a job in $t + 1$ with probability f_{t+1} or stay unemployed with probability $1 - f_{t+1}$. These different possible transitions determine the expected future values $V^E_{a,t+1}$ and $V^U_{a,t+1}$ entering the continuation term. Collecting terms yields

$$\begin{aligned} V^w_{a,t} &= \frac{W_t}{P_t} - b^u - (1 - \Gamma) MRS_{a,t} + \mathbb{E}_t \Bigg[Q^a_{t,t+1} (1 - \rho_{t+1}) (1 - f_{t+1}) \left(V^E_{a,t+1} - V^U_{a,t+1} \right) \Bigg] \\ &= \frac{W_t}{P_t} - b^u - (1 - \Gamma) MRS_{a,t} + \mathbb{E}_t \left[Q^a_{t,t+1} (1 - \rho_{t+1}) (1 - f_{t+1}) V^w_{a,t+1} \right], \end{aligned}$$

which is the expression for surplus $V^w_{a,t}$ used in the main text.

FINAL-GOOD PRODUCERS.. — Denote by $1-\delta$ the probability of resetting prices, and define the reset price of firm i as P_i^*. The downward sloping demand function, for any period $k \geq 0$ in which the firm does not reset its price, is given by

$$Y_{i,t+k} = \left(\frac{P_{i,t}^*}{P_{t+k}}\right)^{-\epsilon} Y_{t+k} \ , \tag{A.15}$$

where ϵ is the elasticity of substitution and Y_t is aggregate income. The optimization problem of the firm is given by

$$\max_{P_{i,t}^*} \sum_{k=0}^{\infty} \delta^k \mathbb{E}_t \left[Q_{t,t+k} \frac{1}{P_{t+k}} \left(\mu_{t+k} P_{i,t}^* Y_{i,t+k} - TC(Y_{i,t+k})\right)\right] \ , \tag{A.16}$$

where $Q_{t,t+k}$ is the stochastic discount factor described before, μ_{t+k} is a cost-push shock akin to the revenue tax mechanism introduced by Adjemian et al. (2008) and Abbate and Thaler (2019), and $TC(Y_{i,t+k})$ is the total cost in period $t+k$ as a function of output $Y_{i,t+k}$. Using the production function we have that $TC(Y_{i,t}) = P_t^X X_{i,t} = P_t^X Y_{i,t}$, and we can write firm's marginal costs as

$$MC_{i,t} = MC_t = P_t^X \ .$$

Differentiating Eq. (A.16) with respect to price $P_{i,t}^*$ we can derive the first-order condition

$$\begin{aligned}
&\sum_{k=0}^{\infty} \mathbb{E}_t \left[\delta^k Q_{t,t+k} \frac{1}{P_{t+k}} \left(\mu_{t+k} Y_{i,t+k} + \mu_{t+k} P_{i,t}^* \frac{\partial Y_{i,t+k}}{\partial P_{i,t}^*} - \frac{\partial TC}{\partial Y_{i,t+k}} \frac{\partial Y_{i,t+k}}{\partial P_{i,t}^*} \right) \right] = 0 \\
&\sum_{k=0}^{\infty} \mathbb{E}_t \left[\delta^k Q_{t,t+k} \frac{1}{P_{t+k}} \left(\mu_{t+k} Y_{i,t+k} + \frac{\partial Y_{i,t+k}}{\partial P_{i,t}^*} \left(\mu_{t+k} P_{i,t}^* - \frac{\partial TC}{\partial Y_{i,t+k}} \right) \right) \right] = 0 \\
&\sum_{k=0}^{\infty} \mathbb{E}_t \left[\delta^k Q_{t,t+k} \frac{1}{P_{t+k}} \left(\mu_{t+k} Y_{i,t+k} - \epsilon \left(\frac{P_{i,t}^*}{P_{t+k}} \right)^{-\epsilon-1} \frac{1}{P_{t+k}} Y_{t+k} \left(\mu_{t+k} P_{i,t}^* - MC_{t+k} \right) \right) \right] = 0 \\
&\sum_{k=0}^{\infty} \mathbb{E}_t \left[\delta^k Q_{t,t+k} \frac{1}{P_{t+k}} \left(\mu_{t+k} Y_{i,t+k} - \epsilon Y_{i,t+k} \frac{1}{P_{i,t}^*} (\mu_{t+k} P_{i,t}^* - MC_{t+k}) \right) \right] = 0 \\
&\sum_{k=0}^{\infty} \mathbb{E}_t \left[\delta^k Q_{t,t+k} Y_{i,t+k} \frac{1}{P_{t+k}} \left((1-\epsilon) \mu_{t+k} + \epsilon MC_{t+k} \frac{1}{P_{i,t}^*} \right) \right] = 0 \\
&\sum_{k=0}^{\infty} \mathbb{E}_t \left[\delta^k Q_{t,t+k} Y_{i,t+k} \frac{1}{P_{t+k}} \left(\mu_{t+k} P_{i,t}^* - \frac{\epsilon}{\epsilon-1} MC_{t+k} \right) \right] = 0 \\
&\sum_{k=0}^{\infty} \mathbb{E}_t \left[\delta^k Q_{t,t+k} Y_{i,t+k} \frac{1}{P_{t+k}} \left(\mu_{t+k} P_{i,t}^* - \frac{\epsilon}{\epsilon-1} RMC_{t+k} P_{t+k} \right) \right] = 0 \ ,
\end{aligned} \tag{A.17}$$

where $RMC_{t+k} \equiv P^X_{t+k}/P_{t+k}$ are real marginal costs. To derive a recursive formulation of the pricing equation, rewrite Eq. (A.17) as

$$\sum_{k=0}^{\infty} \mathbb{E}_t \left[\delta^k Q_{t,t+k} \left(\frac{P^*_{i,t}}{P_{t+k}} \right)^{-\epsilon} Y_{t+k} \frac{1}{P_{t+k}} \left(\mu_{t+k} P^*_{i,t} - \frac{\epsilon}{\epsilon-1} RMC_{t+k} P_{t+k} \right) \right] = 0$$

$$\sum_{k=0}^{\infty} \mathbb{E}_t \left[\delta^k Q_{t,t+k} \left(\frac{P_t}{P_{t+k}} \right)^{-\epsilon} \left(\frac{P^*_{i,t}}{P_t} \right)^{-\epsilon} Y_{t+k} \frac{P_t}{P_{t+k}} \left(\mu_{t+k} \frac{P^*_{i,t}}{P_t} - \frac{\epsilon}{\epsilon-1} RMC_{t+k} \frac{P_{t+k}}{P_t} \right) \right] = 0$$

$$\sum_{k=0}^{\infty} \mathbb{E}_t \left[\delta^k Q_{t,t+k} \left(\frac{P_t}{P_{t+k}} \right)^{1-\epsilon} \left(\frac{P^*_{i,t}}{P_t} \right)^{-\epsilon} Y_{t+k} \left(\mu_{t+k} \frac{P^*_{i,t}}{P_t} - \frac{\epsilon}{\epsilon-1} RMC_{t+k} \frac{P_{t+k}}{P_t} \right) \right] = 0$$

$$\sum_{k=0}^{\infty} \mathbb{E}_t \left[\delta^k Q_{t,t+k} Y_{t+k} \mu_{t+k} \left(\frac{P_t}{P_{t+k}} \right)^{1-\epsilon} \left(\frac{P^*_{i,t}}{P_t} \right)^{1-\epsilon} \right] =$$

$$\sum_{k=0}^{\infty} \mathbb{E}_t \left[\delta^k Q_{t,t+k} Y_{t+k} \left(\frac{P_t}{P_{t+k}} \right)^{-\epsilon} \left(\frac{P^*_{i,t}}{P_t} \right)^{-\epsilon} \frac{\epsilon}{\epsilon-1} RMC_{t+k} \right] .$$

Dividing both sides by $\left(\frac{P^*_{i,t}}{P_t} \right)^{-\epsilon}$ we get

$$\sum_{k=0}^{\infty} \mathbb{E}_t \left[\delta^k Q_{t,t+k} Y_{t+k} \mu_{t+k} \left(\frac{P_t}{P_{t+k}} \right)^{1-\epsilon} \frac{P^*_{i,t}}{P_t} \right] =$$

$$\frac{\epsilon}{\epsilon-1} \sum_{k=0}^{\infty} \mathbb{E}_t \left[\delta^k Q_{t,t+k} Y_{t+k} \left(\frac{P_t}{P_{t+k}} \right)^{-\epsilon} RMC_{t+k} \right] .$$

Factoring out and defining $\Pi_{t+k} \equiv P_{t+k}/P_t$ yields

$$\text{(A.18)} \qquad \frac{P^*_{i,t}}{P_t} \sum_{k=0}^{\infty} \mathbb{E}_t \left[\delta^k Q_{t,t+k} Y_{t+k} \mu_{t+k} \Pi^{\epsilon-1}_{t+k} \right] = \frac{\epsilon}{\epsilon-1} \sum_{k=0}^{\infty} \mathbb{E}_t \left[\delta^k Q_{t,t+k} Y_{t+k} \Pi^{\epsilon}_{t+k} RMC_{t+k} \right] .$$

Define the right-hand side of Eq. (A.18) as K_t, that is

$$K_t = \frac{\epsilon}{\epsilon-1} \sum_{k=0}^{\infty} \mathbb{E}_t \left[\delta^k Q_{t,t+k} Y_{t+k} \Pi^{\epsilon}_{t+k} RMC_{t+k} \right] ,$$

and take the first term of the summation out to get

$$K_t = \frac{\epsilon}{\epsilon-1} Y_t RMC_t + \frac{\epsilon}{\epsilon-1} \sum_{k=1}^{\infty} \mathbb{E}_t \left[\delta^k Q_{t,t+k} \Pi^{\epsilon}_{t+k} Y_{t+k} RMC_{t+k} \right] .$$

Noticing that K_{t+1} is given by

$$K_{t+1} = \frac{\epsilon}{\epsilon-1}\sum_{k=0}^{\infty}\mathbb{E}_t\left[\delta^k Q_{t+1,t+k}\Pi^{\epsilon}_{t+1+k}Y_{t+1+k}RMC_{t+1+k}\right] ,$$

we can then write K_t recursively as

$$K_t = \frac{\epsilon}{\epsilon-1}Y_t RMC_t + \delta\mathbb{E}_t\left[Q_{t,t+1}\Pi^{\epsilon}_{t+1}K_{t+1}\right] .$$

Similarly, defining F_t as

$$F_t = \sum_{k=0}^{\infty}\delta^k\mathbb{E}_t\left[Q_{t,t+k}\Pi^{\epsilon-1}_{t+k}Y_{t+k}\mu_{t+k}\right] ,$$

and writing F_t recursively in a similar fashion as

$$F_t = Y_t\mu_t + \delta\mathbb{E}_t\left[Q_{t,t+1}\Pi^{\epsilon-1}_{t+1}F_{t+1}\right] ,$$

we can finally use Eq. (A.18) write the optimal relative price in a symmetric equilibrium where $P^*_{i,t} = P^*_t$ for all resetting firms as

$$\frac{P^*_t}{P_t} = \frac{K_t}{F_t} . \tag{A.19}$$

In each period t, the average price level is given by the aggregator

$$P_t^{1-\epsilon} = \delta P_{t-1}^{1-\epsilon} + (1-\delta)(P^*_t)^{1-\epsilon} . \tag{A.20}$$

Dividing both sides of Eq. (A.20) by $P_t^{1-\epsilon}$ we get

$$1 = \delta\Pi_t^{\epsilon-1} + (1-\delta)\left(\frac{P^*_t}{P_t}\right)^{1-\epsilon} ,$$

so that inflation is linked to the optimal relative price according to

$$\Pi_t = \left(\frac{1-(1-\delta)\left(\frac{P^*_t}{P_t}\right)^{1-\epsilon}}{\delta}\right)^{\frac{1}{\epsilon-1}} . \tag{A.21}$$

Search frictions and equilibrium wages.. — Let $V^w_{a,t}$ denote the surplus of having an additional member employed rather than unemployed, for a chosen level of participation $L_t = \bar{L}$. In this case we have that employment is given by

$$E_t = (1-\rho_t)E_{t-1} + f_t S_t = (1-\rho_t)E_{t-1} + f_t L_t - f_t(1-\rho_t)E_{t-1} = (1-\rho_t)(1-f_t)E_{t-1} + f_t\bar{L} ,$$

and the surplus $V^w_{a,t}$ can be written as

$$\begin{aligned} V^w_{a,t} &= \frac{\partial \mathcal{V}_{a,t}}{\partial E_{a,t}} \frac{C_{a,t} - \xi C_{a,t-1}}{Z_t} = \frac{\partial \mathcal{U}_{a,t}}{\partial E_{a,t}} \frac{C_{a,t} - \xi C_{a,t-1}}{Z_t} + \beta \mathbb{E}_t \frac{\partial \mathcal{V}_{a,t+1}}{\partial E_{a,t}} \frac{C_{a,t} - \xi C_{a,t-1}}{Z_t} \\ &= \frac{W_t}{P_t} - b^u - (1-\Gamma) MRS_{a,t} + \mathbb{E}_t \left[Q^a_{t,t+1} (1-\rho_{t+1})(1-f_{t+1}) V^w_{a,t+1} \right] . \end{aligned} \tag{A.22}$$

When bargaining wages, the union considers average surplus given by $V^w_t \equiv \zeta V^w_{y,t} + (1-\zeta) V^w_{o,t}$. According to Nash bargaining, the optimal wage $(W_t/P_t)^*$ maximizes the Nash product, which balances each party's payoff according to their bargaining power:

$$\max_{W_t/P_t} (V^w_t)^{1-\eta} (V^J_t)^\eta ,$$

where η is the firm's relative bargaining power. The first-order condition for the problem above is

$$\frac{\partial (V^w_t)^{1-\eta}(V^J_t)^\eta}{\partial W_t/P_t} = (1-\eta)(V^w_t)^{-\eta} \frac{\partial V^w_t}{\partial W_t/P_t} (V^J_t)^\eta + (V^w_t)^{1-\eta} \eta (V^J_t)^{\eta-1} \frac{\partial V^J_t}{\partial W_t/P_t} = 0 .$$

The condition above can be simplified to yield the following constant (period-by-period) sharing rule

$$\begin{aligned} (1-\eta)(V^w_t)^{-\eta}(V^J_t)^\eta &= (V^w_t)^{1-\eta} \eta (V^J_t)^{\eta-1} \\ (1-\eta)(V^J_t)^\eta &= V^w_t \eta (V^J_t)^{\eta-1} \\ (1-\eta) V^J_t &= \eta V^w_t , \end{aligned} \tag{A.23}$$

which defines the optimal wage $(W_t/P_t)^*$. Under the assumed form of wage rigidity, the real wage is then given by

$$\frac{W_t}{P_t} = (1-\theta_w) \left(\frac{W_t}{P_t} \right)^* + \theta_w \frac{W_t}{P_{t-1}} \Pi_t^{-1} .$$

Market clearing.. — The aggregate production of the intermediate-good sector is given by $X_t = \int_0^1 X_{j,t} dj = A_t E_t$, while the standard aggregate resource constraint is given by $Y_t = C_t + kV_t$, where aggregate output is defined by the standard aggregator $Y_t = \left(\int_0^1 Y_{i,t}^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}$. Moreover, market clearing in the labor market requires $E_t = \int_0^1 E_{i,t} di$, where $E_{i,t}$ denotes labor used to produce $X_{i,t}$, in turn used for the production of $Y_{i,t}$. Using the production function of final-good firms we get $E_t = \int_0^1 \frac{Y_{i,t}}{A_t} di$, and substituting in the individual demand faced by firm i yields

$$E_t = \frac{1}{A_t} Y_t \int_0^1 \left(\frac{P_{i,t}}{P_t} \right)^{-\epsilon} di = \frac{1}{A_t} Y_t \Delta_t ,$$

where the last equality follows from the definition of price dispersion $\Delta_t = \int_0^1 \left(\frac{P_{i,t}}{P_t} \right)^{-\epsilon} di$. The aggregate production function can thus be written as $Y_t = X_t \Delta_t^{-1}$, while the law of motion for Δ_t

is

$$\begin{aligned}\Delta_t &= (1-\delta)\left(\frac{P_t^*}{P_t}\right)^{-\epsilon} + \delta \int_0^1 \left(\frac{P_{i,t-1}}{P_t}\right)^{-\epsilon} di \\ &= (1-\delta)\left(\frac{P_t^*}{P_t}\right)^{-\epsilon} + \delta \Pi_t^{\epsilon} \int_0^1 \left(\frac{P_{i,t-1}}{P_{t-1}}\right)^{-\epsilon} di \\ &= (1-\delta)\left(\frac{P_t^*}{P_t}\right)^{-\epsilon} + \delta \Pi_t^{\epsilon} \Delta_{t-1} \, .\end{aligned}$$

B. TANK model with CES technology

This section derives an extension of the baseline model by introducing a CES aggregator over age-specific intermediate goods, and letting labour-market object be age specific. Specifically, vacancies, tightness, job-filling and job-finding rates, Nash bargaining, wage stickiness and firm/worker values are duplicated by type. Matching efficiency ω is common across markets, as is the search cost Γ. The two age-specific intermediate goods are produced linearly in age-specific labour

$$X_{a,t} = A_{a,t} E_{a,t} \, , \tag{B.24}$$

with $a \in \{y, o\}$, and a representative final-good producer combines them through a CES bundle

$$X_t = \left[\gamma_y (X_{y,t})^{\frac{\sigma-1}{\sigma}} + \gamma_o (X_{o,t})^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \, . \tag{B.25}$$

The demand for input $X_{a,t}$ is given by

$$X_{a,t} = \left(\frac{P_{a,t}}{\gamma_a P_t^X}\right)^{-\sigma} X_t \, , \tag{B.26}$$

and the real price of the intermediate good P_t^X/P_t is

$$\frac{P_t^X}{P_t} = \left[\gamma_y^{\sigma} \left(\frac{P_{y,t}}{P_t}\right)^{1-\sigma} + \gamma_o^{\sigma} \left(\frac{P_{o,t}}{P_t}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \, . \tag{B.27}$$

The type-specific prices then enter the (otherwise unchanged) job-creation conditions and Nash bargaining FOCs by age type, translating into different wages and different employment outcomes across the two markets.

Data and estimation. The estimation uses U.S. quarterly data from 1998:Q1 to 2019:Q4 on real GDP growth, CPI inflation, the shadow policy rate from Wu and Xia (2016), changes in labor force participation and retirement rates, real average wage growth, real wage growth for prime age individuals, real consumption growth, and unemployment rates for individuals both below and above 55 years of age. The calibrated parameters are reported in Table 1 and they are set from the same targets as the baseline model, with the only additions being the two CES distribution parameters (γ_y is pinned down in steady state, γ_o is normalised to 1) and the wage-ratio target $\bar{R}_w$ used to identify γ_y.

The list of estimated parameters, information on the priors and the posterior modes are reported in Table 2. The posterior mode of the CES elasticiy σ is 3.85, while the remaining estimates are very close to the baseline paper, suggesting that adding the imperfect substitutability across young and old workers does not significantly change the estimated structural and policy parameters.

Table 1—. Calibrated parameters

	Param	Value	Target / Source
Discount factor	β	0.99	4% average real return
Elasticity of substitution between goods	ϵ	6	20% price markup
Home-production exponent	α_h	0.30	Campolmi–Gnocchi (2016)
Firms' bargaining power	η	0.4	Campolmi–Gnocchi (2016)
Elasticity of matches to searchers	γ	0.6	Campolmi–Gnocchi (2016)
Household search cost	Γ	0.368	ATUS
Mean separation (young, old)	$\bar{\rho}_y, \bar{\rho}_o$	0.12	quarterly separation rate
Vacancy cost / wage	$\bar{v}$	0.10	SS calibration of k
Target old/young wage ratio	$\bar{R}_w$	1.10	SS pin-down of γ_y
Target aggregate employment rate	E/L^{tot}	0.955	US data
Target aggregate participation	L^{tot}	0.63	US data
Target old participation	L^o	0.40	55+ participation
Job-filling rate (SS, both markets)	q^a	2/3	SS calibration of ω
Share of young population	w	0.63	Pop. shares (16–54 vs $\geq$55)
Portfolio cost	λ	0.01	Montoro–Ortiz (2023)
CES distribution: old	γ_o	1	normalisation
CES distribution: young	γ_y	SS-pinned	matches $\bar{R}_w$
Unemployment benefit	b	SS-pinned	$E/L^{\text{tot}} = 0.955$
Home-good scaling (young, old)	ϕ_y, ϕ_o	SS-pinned	Nash equations
Pension benefit	p	SS-pinned	old participation 0.40
Matching efficiency	ω	SS-pinned	$q = 2/3$

Counterfactual exercise. Figure B.1 reports the smoothed baseline (solid) and the counterfactual without the excess-retirement (dashed) for inflation, log GDP, retirement, the nominal wage, total participation and 55+ unemployment. Figure B.2 reports the implied contribution of excess retirement to the price level and to real GDP.

The results are qualitatively similar to those of the baseline specification. Quantitatively, however, the effect on inflation is stronger, with a cumulative contribution of approximately 4 percentage points, whereas the effect on output remains limited, in line with the baseline results.

Table 2—. Estimated parameters

	Param	Dist.	Prior mean	Prior SD	Mode	Mean	10%	90%
Intertemporal elasticity (young)	ν_y	IG	2	2	1.093	1.129	0.908	1.368
Intertemporal elasticity (old)	ν_o	IG	2	2	1.199	1.268	0.956	1.613
Consumption habits	ξ	B	0.5	0.06	0.743	0.740	0.670	0.811
Calvo parameter	δ	B	0.5	0.1	0.648	0.649	0.611	0.687
Wage stickiness (young)	θ_w^y	B	0.5	0.1	0.433	0.449	0.380	0.516
Wage stickiness (old)	θ_w^o	B	0.5	0.1	0.357	0.367	0.303	0.432
Pension scaling	ς	N	0.5	0.1	0.505	0.505	0.341	0.670
Monetary policy (inflation)	ϕ_π	N	1.5	0.1	1.549	1.572	1.413	1.731
Monetary policy (output gap)	ϕ_y	G	0.125	0.05	0.111	0.140	0.053	0.224
CES elasticity (young / old)	σ	G	3.5	1.0	3.849	4.149	2.995	5.270
Policy rate inertia	ρ_R	B	0.75	0.1	0.446	0.451	0.374	0.528
Productivity (young input)	$\rho_{A,y}$	B	0.5	0.1	0.744	0.711	0.595	0.830
Productivity (old input)	$\rho_{A,o}$	B	0.5	0.1	0.614	0.611	0.471	0.749
Home productivity (young)	$\rho_{Ah,y}$	B	0.5	0.1	0.506	0.515	0.348	0.683
Home productivity (old)	$\rho_{Ah,o}$	B	0.5	0.1	0.529	0.540	0.365	0.713
Preference	ρ_Z	B	0.5	0.1	0.726	0.721	0.660	0.783
Cost-push	ρ_{cp}	B	0.5	0.1	0.369	0.387	0.232	0.538
Tax-redistribution	ρ_τ	B	0.5	0.1	0.376	0.382	0.246	0.517
Separation (young)	$\rho_{\rho,y}$	B	0.5	0.1	0.765	0.750	0.661	0.842
Separation (old)	$\rho_{\rho,o}$	B	0.5	0.1	0.633	0.604	0.458	0.754
Productivity (young)	$\sigma_{A,y}$	IG	0.1	1	0.013	0.014	0.012	0.016
Productivity (old)	$\sigma_{A,o}$	IG	0.1	1	0.018	0.018	0.015	0.022
Home prod. (young)	$\sigma_{Ah,y}$	IG	0.1	1	0.042	0.055	0.021	0.091
Home prod. (old)	$\sigma_{Ah,o}$	IG	0.1	1	0.032	0.040	0.017	0.065
Preference	σ_Z	IG	0.1	1	0.036	0.037	0.030	0.045
Monetary	σ_{mp}	IG	0.1	1	0.016	0.016	0.014	0.019
Cost-push	σ_{cp}	IG	0.1	1	0.017	0.018	0.013	0.022
Separation (young)	$\sigma_{\rho,y}$	IG	0.1	1	0.023	0.024	0.019	0.029
Separation (old)	$\sigma_{\rho,o}$	IG	0.1	1	0.024	0.025	0.019	0.031
Tax-redistribution	σ_τ	IG	0.1	1	0.023	0.025	0.018	0.032
LFP change	σ_{Ltot}	IG	0.05	1	0.0055	0.0057	0.0049	0.0065
Real wage growth	σ_w	IG	0.05	1	0.0058	0.0059	0.0051	0.0067
Retirement change	σ_{Re}	IG	0.05	1	0.0049	0.0050	0.0044	0.0057
Consumption growth	σ_c	IG	0.05	1	0.0060	0.0061	0.0053	0.0069
Unemployment 55+	$\sigma_{o,u}$	IG	0.05	1	0.0057	0.0059	0.0050	0.0068
Unemployment <55	$\sigma_{y,u}$	IG	0.05	1	0.0068	0.0070	0.0058	0.0081
Young real wage growth	σ_{w_y}	IG	0.05	1	0.0058	0.0059	0.0051	0.0067

Note: Prior distribution, prior mean and SD, posterior mode and posterior summary statistics. Distributions: B Beta, IG: Inverse-Gamma, N Normal, G Gamma.

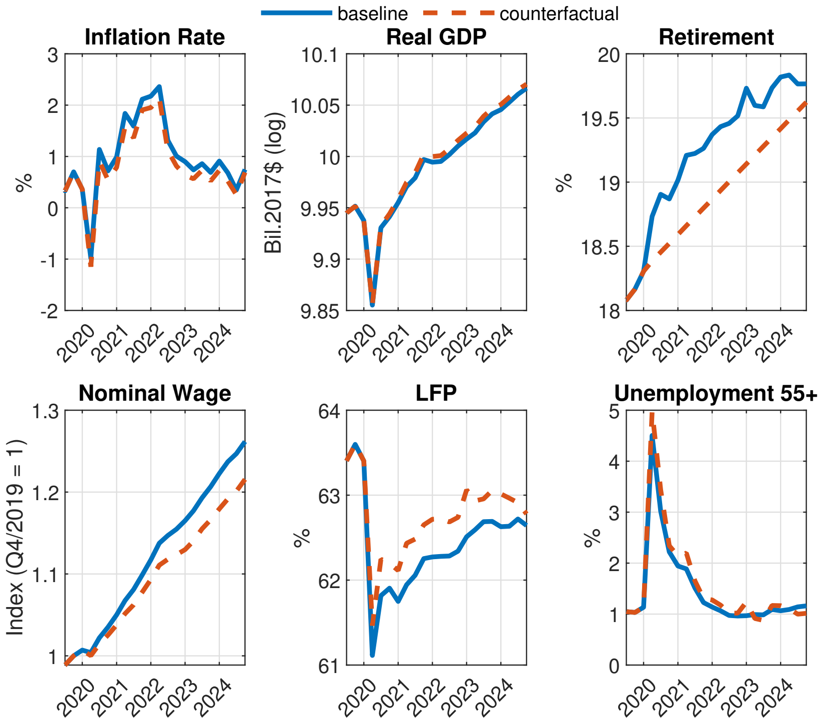

Figure B.1. . Smoothed baseline (solid) vs. counterfactual without the Great Retirement (dashed).

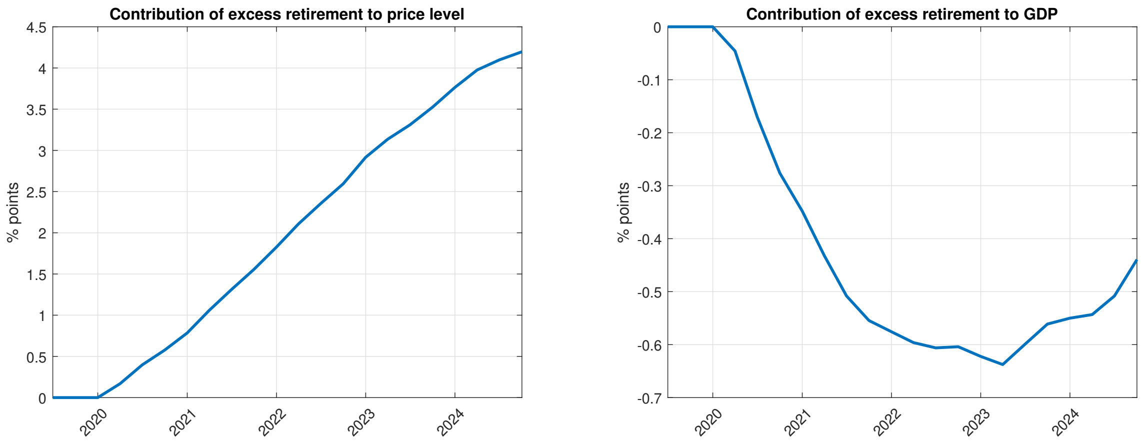

Figure B.2. . Contribution of the Great Retirement to prices and output.

Note: Left: Contribution to price level. Right: Contribution to real GDP.

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