Supplemental Appendix for: Durables and the Marginal Propensity to Spend

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This online appendix contains a detailed description of the quantitative model and its numerical solution, as well as additional results for the article "Durables and the Marginal Propensity to Spend."

Any references to sections preceded by "I.," "II.," "III.," "IV.", or "V." refer to the main article. Similarly, any references to equations or figures not preceded by "A." refer to the main article.

A Consumption and Investment Problem

In this appendix, we discuss how to solve the households' consumption and investment problem. Section A.1 states the problem recursively. Section A.2 explains how we solve the model numerically. Finally, Section A.3 provides computational details.

A.1 Recursive Formulation

We now state the households' problem recursively in a way that lends itself to numerical implementation. Relative to Section I.D, it is more convenient to keep track of the number of credit installments already paid since the last adjustment (ξ) instead of the stock of oustanding credit (b) itself.[1] All prices are expressed relative to the non-durable good, anticipating our general equilibrium analysis in Section IV.C. Households are indexed by four idiosyncratic states: their stock of durables d, their cash-on-hand m, their outstanding credit b, and the permanent component of their income process ω. In addition, they draw two transitory shocks or innovations: a transitory income shock ζ and, subsequently, a utility cost ϵ of adjusting their stock of durables. In the following, we let $\mathbf{y} \equiv (\omega, \zeta)$ denote the two components of the income process and $\mathbf{x} \equiv (d, m, b, \boldsymbol{y})$ denote the households' characteristics that determine their adjustment, consumption, saving and investment choices. We do not have to keep track of the transitory utility cost of adjustment ϵ since our formulation allows for closed form expressions when integrating over that shock.

Continuation values. The continuation values $\{V_t(\cdot)\}$ can be characterized recursively.[2]

1. **Discrete choice.** The household chooses whether to adjust its stock of durables or not. The value associated with this discrete choice problem is

$$V_t(d, m, \xi, \mathbf{y}; \epsilon) = \max\left\{V_t^{\text{adjust}}(d, m, \xi, \mathbf{y}) - \epsilon, V_t^{\text{not}}(d, m, \xi, \mathbf{y})\right\}, \tag{A.1}$$

   where $V_t^{\text{adjust}}(d, m, \xi, \mathbf{y})$ is the value of adjusting the stock of durables, $V_t^{\text{not}}(d, m, \xi, \mathbf{y})$ is the value of not adjusting, and ϵ is a random utility cost which follows a logistic distribution $\mathcal{E}$ whose mean and variance are controlled by $\kappa > 0$ and $2\eta^2 \geq 0$,

[1] This formulation allows us to use a coarser grid for ξ than would be required if we kept track of b with the same degree of numerical accuracy. As we explain below, the outstanding credit b can be backed out using the current stock of durables d and the number of installments already paid ξ.

[2] The terminal condition for $\mathcal{V}_{t+1}(\cdot)$ in expression (A.14) is either an initial guess when solving for the stationary equilibrium, or the stationary values without stimulus checks when solving for transitions.

respectively. As usual with logistic shocks, the expected value $\mathcal{V}_t(d, m, \xi, \mathbf{y}) \equiv \int V_t(d, m, \xi, \mathbf{y}; \epsilon) \, \mathrm{d}\mathcal{E}(\epsilon)$ takes the form

$$\mathcal{V}_t(\cdot) = \eta \log\left(\exp\left(\frac{V_t^{\text{adjust}}(d, m, \xi, \mathbf{y}) - \kappa}{\eta}\right) + \exp\left(\frac{V_t^{\text{not}}(d, m, \xi, \mathbf{y})}{\eta}\right)\right) \tag{A.2}$$

and the adjustment hazard associated with this discrete choice problem is

$$\mathcal{S}_t(d, m, \xi, \mathbf{y}) = \frac{\exp\left(\frac{V_t^{\text{adjust}}(d,m,\xi,\mathbf{y})-\kappa}{\eta}\right)}{\exp\left(\frac{V_t^{\text{adjust}}(d,m,\xi,\mathbf{y})-\kappa}{\eta}\right) + \exp\left(\frac{V_t^{\text{not}}(d,m,\xi,\mathbf{y})}{\eta}\right)} \tag{A.3}$$

The continuation values are given by

$$V_t^{\text{adjust}}(d, m, \xi, \mathbf{y}) \equiv W_t^D\left(\hat{m}^D(d, m, \xi, \mathbf{y}), \omega\right) \tag{A.4}$$

$$V_t^{\text{not}}(d, m, \xi, \mathbf{y}) \equiv W_t^C\left((1 - (1 - \iota)\delta) d, \hat{m}^C(d, m, \xi, \mathbf{y}), \xi + 1, \omega\right), \tag{A.5}$$

where $\hat{m}^D(d, m, \xi, \mathbf{y})$ and $\hat{m}^C(d, m, \xi, \mathbf{y})$ denote the household's cash-on-hand conditional on adjusting its stock of durables or not. The household gets to choose a new stock of durables if it decides to adjust, and undertakes the mandatory maintaince otherwise by offsetting a share ι of the depreciation. Cash-on-hand conditional on adjusting or not is

$$\begin{aligned}
\hat{m}_t^D(d, m, \xi, \mathbf{y}) &\equiv \hat{m}_t(d, m, \xi, \mathbf{y}) + \Delta_t^D(\xi) d \\
\hat{m}_t^C(d, m, \xi, \mathbf{y}) &\equiv \hat{m}_t(d, m, \xi, \mathbf{y}) - \Delta_t^C(\xi) d,
\end{aligned}$$

where $\hat{m}_t(d, m, \xi, \mathbf{y})$ is cash-on-hand before the adjustment decision and the terms Δ_t^D and Δ_t^C capture debt repayments and maintenance costs conditional on adjustment or not. Cash-on-hand before the adjustment decision is

$$\begin{aligned}
\hat{m}_t(d, m, \xi, \mathbf{y}) \equiv \mathcal{Y}_t(\mathbf{y}) &+ \frac{1 + r_{t-1}^m}{1 + \pi_t} m \\
&- r_{t-1}^b \left(\frac{1 - \varphi}{1 - (1 - \iota)\delta}\right)^{\xi} (1 - \theta) \hat{P}_t^d(\xi) d + \frac{T_t}{\prod_{s=0}^t (1 + \pi_s)},
\end{aligned} \tag{A.6}$$

where household's income net of taxes is

$$\mathcal{Y}_t(\mathbf{y}) \equiv (1 - \tau_t) \exp(\omega + \zeta) Y_t^{\text{inc}}, \tag{A.7}$$

with τ_t denoting the tax rate. The inflation rate π_t accounts for the fact that the budget constraints are expressed in real terms, i.e., all prices are expressed relative to the price of the non-durable good. Stimulus checks T_t are expressed in nominal terms and deflated accordingly. The outstanding credit decreases by a share φ with every installment already paid ξ. The amount originally borrowed with credit corresponds to the share $1-\theta$ of the durable purchase that the household did not pay down. The value of this purchase can be recovered using the current stock of durables d after accounting for ξ periods of depreciation and maintenance, as well as the durable price

$$\hat{P}_t^d(\xi) \equiv \frac{P_{t-\xi-1}^d}{\prod_{s=0}^{\xi}(1+\pi_{t-s})} \tag{A.8}$$

that prevailed when the household bought the durable deflated to express it in current real terms. The term

$$\Delta_t^D(\xi) \equiv (1-\delta) P_t^d - \left(\frac{1-\varphi}{1-(1-\iota)\delta}\right)^{\xi} (1-\theta) \hat{P}_t^d(\xi) \tag{A.9}$$

captures the net profit that the household makes when selling its old durable after repaying its outstanding credit, per unit of durable d held. In turn,

$$\Delta_t^C(\xi) \equiv \left[(1-\varphi)^{\xi} - (1-\varphi)^{\xi+1}\right] \frac{(1-\theta)\hat{P}_t^d(\xi)}{(1-(1-\iota)\delta)^{\xi}} + \iota\delta P_t^d \tag{A.10}$$

captures the credit repayment on the principal, as well as the maintenance cost incurred when the household does not adjust its stock of durable. In the fully state-dependent limit $\eta \to 0$, the value (A.2) and the hazard (A.3) become

$$\mathcal{V}_t(d, m, \xi, \mathbf{y}) = \max\left\{V_t^{\text{adjust}}(d, m, \xi, \mathbf{y}) - \kappa, V_t^{\text{not}}(d, m, \xi, \mathbf{y})\right\} \tag{A.11}$$

and

$$\mathcal{S}_t(d, m, \xi, \mathbf{y}) = \begin{cases} 1 & \text{if } V_t^{\text{adjust}}(d, m, \xi, \mathbf{y}) - \kappa > V_t^{\text{not}}(d, m, \xi, \mathbf{y}) \\ 0 & \text{otherwise} \end{cases} \tag{A.12}$$

2. **Durable adjustment.** If the household decides to adjust its stock of durables, it

chooses how much durables to purchase

$$W_t^D(m,\omega) = \max_{d',m'} W_t^C(d',m',\xi',\omega) \tag{A.13}$$
$$\text{s.t. } \theta P_t^d d' + m' \leq m \quad \text{and} \quad \xi' = 0,$$

where m is cash-on-hand before the household purchases its new durables. Adjusting the stock of durables resets the number of installments already paid to $\xi' = 0$. The continuation value W_t^C reflects the subsequent optimal consumption-saving choice that occurs in the same period given the remaining cash-on-hand m'.

3. **Consumption-saving.** Finally, the household chooses how much to consume and save in liquid asset

$$W_t^C(d,m,\xi,\omega) = \max_{c,m'} u(c,d) + \beta \int\int \mathcal{V}_{t+1}(d,m',\xi,\omega',\zeta') \Gamma_\zeta(\mathrm{d}\zeta') \Gamma_\omega(\mathrm{d}\omega';\omega) \tag{A.14}$$
$$\text{s.t. } P_t^c c + m' \leq m \quad \text{and} \quad m' \geq 0,$$

where m is the household's real cash-on-hand before it chooses non-durable consumption c, and m' is cash-on-hand for next period.

A.2 Solving Households' Problem

We proceed as follows to solve numerically for the value functions defined above:

1. **Guess.** Fix a value

$$\mathcal{V}_{t+1}^\star(d,m,\xi,\omega) \equiv \int\int \mathcal{V}_{t+1}(d,m,\xi,\omega',\zeta') \Gamma_\zeta(\mathrm{d}\zeta') \Gamma_\omega(\mathrm{d}\omega';\omega) \tag{A.15}$$

for the terminal period.

2. **Consumption-saving.** Fix the states (d,ξ,ω). If the household's borrowing constraint $m' \geq 0$ is not binding, a necessary condition for an optimum to (A.14) is

$$u_c(c,d) = \beta P_t^c \partial_m \mathcal{V}_{t+1}^\star(d,m',\xi,\omega), \tag{A.16}$$

together with the budget constraint $c = (m - m')/P_t^c$. This condition is not suf-

ficient, however, since the problem is typically non-convex.[3,4] To recover policy functions, i.e., maps $m \mapsto (c, m')$, we proceed as follows. We first obtain a map $m' \mapsto m$ using the condition above and the endogenous grid method (EGM) of Carroll (2006). The generalized inverse of this map (as a function of m) might contain several points for m' since the problem is non-convex. We proceed as Druedahl and Jørgensen (2017) and split the map $m' \mapsto m$ into monotonic segments, i.e., either increasing or decreasing. Fixing some m on the grid of interest, we interpolate linearly the value of m' at m using each segment and add $\max\{m', 0\}$ to the set of candidates.[5] We also add the borrowing constraint $m' = 0$ to this set of candidates. The consumptions associated with these candidates are given by the budget constraint above. The optimum is found by comparing the values of the objective (A.14) associated with each candidate. The policy function $m \mapsto m'$ is given by the candidate that provides the highest value of the objective, and $m \mapsto c$ is obtained using the budget constraint. Using the resulting policy function $m'_t(\cdot)$, we compute the value $W_t^C(d, m, \xi, \omega)$ using (A.14), and the marginal values

$$\partial_d W_t^C(d, m, \xi, \omega) = u_d\left(\left(m - m'_t(\cdot)\right)/P_t^c, d\right) + \beta \partial_d \mathcal{V}_{t+1}^\star\left(d, m'_t(\cdot), \xi, \omega\right) \tag{A.17}$$

$$\partial_m W_t^C(d, m, \xi, \omega) = 1/P_t^c u_c\left(\left(m - m'_t(\cdot)\right)/P_t^c, d\right) \tag{A.18}$$

for the durable and the liquid assets. We then repeat this procedure for all states (d, ξ, ω).

3. **Durable adjustment.** Fix the state ω. A necessary condition for an optimum to (A.13) is

$$\partial_d W_t^C\left(d', m', \xi', \omega\right) - \theta P_t^d \partial_m W_t^C\left(d', m', \xi', \omega\right) = 0 \tag{A.19}$$

[3] The reason is that the continuation value involves the upper envelope (A.1). Random utility costs of adjustment, i.e., the smooth hazard (A.3), can make continuation values *smooth* (with no kinks) but not necessarily *concave*.

[4] Condition (A.16) is necessary for an optimum. The argument is similar to the one in Clausen and Strub (2012). Consider a simplified version of the problem of interest: $\max_c f(c) + G(-c)$ with $f(\cdot)$ and $G(\cdot)$ smooth except for a convex kink in $G(\cdot)$ at $-\bar{c} \in \mathbb{R}$ corresponding to a discrete choice. Suppose by contradiction that the optimizer is $\bar{c}$. By optimality, $f'(\bar{c}) \geq G'_+(-\bar{c})$ and $f'(\bar{c}) \leq G'_-(-\bar{c})$. However, $G'_+(-\bar{c}) > G'_-(-\bar{c})$ since $G(\cdot)$ admits a convex kink at $-\bar{c}$. This leads to the desired contradiction. Therefore, the optimizer cannot be the point where the kink occurs, and condition (A.16) is necessary. The argument generalizes to multiple kinks and multiple assets.

[5] An alternative approach is to focus only on the couples (m'_0, m'_1) such that m lies in the corresponding interval (m_0, m_1). In principle, this approach might miss the correct value of m' if the grid for liquid asset is too coarse. Our approach does not suffer from this problem, but it requires considering a few more candidates for m'.

with $\xi' = 0$ and

$$m' = m - \theta P_t^d d' \tag{A.20}$$

Again, (A.19) is typically not sufficient for an optimum.[6] We therefore define a set of candidates d' that satisfy (A.19). We compute the value (A.13) for each candidate. The policy function $m \mapsto d'$ is given by the candidate that provides the highest value of the objective. In turn, the policy function $m \mapsto m'$ is given by the budget constraint above. We compute the value $W^D(m, \omega)$ using (A.13), and the marginal value

$$\partial_m W_t^D(m, \omega) = \partial_m W_t^C\left(d_t'(\cdot), m_t'(\cdot), \xi', \omega\right) \tag{A.21}$$

with $\xi' = 0$. We then repeat this procedure for all states ω.

4. **Continuation values.** Compute the values $V_t^{\text{adjust}}(d, m, \xi, \mathbf{y})$ and $V_t^{\text{not}}(d, m, \xi, \mathbf{y})$ using (A.4)–(A.5) and the marginal values

$$\partial_d V_t^{\text{adjust}}(d, m, \xi, \mathbf{y}) = \left\{ -r_{t-1}^b \left(\frac{1-\varphi}{1-(1-\iota)\delta} \right)^{\xi} (1-\theta) \hat{P}_t^d(\xi) + \Delta_t^D(\xi) \right\} \partial_m W_t^D(\cdot) \tag{A.22}$$

$$\begin{aligned} \partial_d V_t^{\text{not}}(d, m, \xi, \mathbf{y}) = {} & (1-(1-\iota)\delta)\, \partial_d W_t^C(\cdot) \\ & - \left\{ r_{t-1}^b \left(\frac{1-\varphi}{1-(1-\iota)\delta} \right)^{\xi} (1-\theta) \hat{P}_t^d(\xi) + \Delta_t^C(\xi) \right\} \partial_m W_t^C(\cdot) \end{aligned} \tag{A.23}$$

for the durable stock, with $\Delta_t^D(\xi)$ and $\Delta_t^C(\xi)$ defined by (A.9)–(A.10), and

$$\begin{aligned} \partial_m V_t^{\text{adjust}}(d, m, \xi, \mathbf{y}) &= \frac{1 + r_{t-1}^m}{1 + \pi_t} \partial_m W_t^D(\cdot) \\ \partial_m V_t^{\text{not}}(d, m, \xi, \mathbf{y}) &= \frac{1 + r_{t-1}^m}{1 + \pi_t} \partial_m W_t^C(\cdot) \end{aligned}$$

for the liquid asset.

5. **Discrete choice.** Compute the value $\mathcal{V}_t(d, m, \xi, \mathbf{y})$ using (A.2) and the marginal values

$$\begin{aligned} \partial_z \mathcal{V}_t(d, m, \xi, \mathbf{y}) = {} & \mathcal{S}_t(d, m, \xi, \mathbf{y})\, \partial_z V_t^{\text{adjust}}(d, m, \xi, \mathbf{y}) \\ & + [1 - \mathcal{S}_t(d, m, \xi, \mathbf{y})]\, \partial_z V_t^{\text{not}}(d, m, \xi, \mathbf{y}) \end{aligned} \tag{A.24}$$

[6] The solution is necessarily interior, however, since $d' = 0$ cannot be optimal due to Inada conditions.

for the durable stock and the liquid asset $z \in \{d, m\}$, where $\mathcal{S}_t$ is the adjustment hazard (A.3).

6. **Update.** Compute the expected utility

$$\mathcal{V}_t^\star(d, m, \xi, \omega) \equiv \int\int \mathcal{V}_t(d, m, \xi, \omega', \zeta') \, \Gamma_\zeta(\mathrm{d}\zeta') \, \Gamma_\omega(\mathrm{d}\omega'; \omega), \tag{A.25}$$

Similarly, compute the marginal utilities

$$\partial_z \mathcal{V}_t^\star(d, m, \xi, \omega) \equiv \int\int \partial_z \mathcal{V}_t(d, m, \xi, \omega', \zeta') \, \Gamma_\zeta(\mathrm{d}\zeta') \, \Gamma_\omega(\mathrm{d}\omega'; \omega) \tag{A.26}$$

for the durable stock and the liquid asset $z \in \{d, m\}$. Finally, iterate on Steps 2–6 until convergence when solving for the stationary equilibrium, or until $t = 0$ when solving for transitions.

A.3 Computational Details

Numerical parameters. We use two grids with 120 exponentially-spaced points each for the stock of durables d and the liquid asset m. We use a grid with 7 unevenly-spaced points for the number of installments already paid.[7] We discretize the persistent component of the income process ω on a 7-point grid using the method of Rouwenhorst (1995), and we discretize the transitory component ζ on a 3-point grid using a standard Gauss-Hermite quadrature. The model therefore features approximately 2.12 million idiosyncratic states. We use stochastic simulations that allow us to track the same households over time, which is necessary to compute several moments that we are interested in. When simulating the model, we use the policy functions (Section A.2) together with the income process Γ_ζ and Γ_ω and we randomly assign households between adjustment and no adjustment according the hazard (A.3). The hazard and the policy functions are interpolated linearly between grid points. In particular, the number of installments paid is not restricted to fall on the corresponding grid.[8] We simulate 13,000 households over 1,200 quarters after a burn period of 400 quarters. This collection of households' states constitutes the stationary distribution that we use to compute the MPX, for instance.

Smoothing the responses. In Sections II-IV, we compare the properties of our model to a purely state-dependent version of our model with $\eta \to 0$. To obtain slightly smoother

[7] We truncate that grid at 15 years, at which point we assume that households repay their minuscule outstanding credit in full.

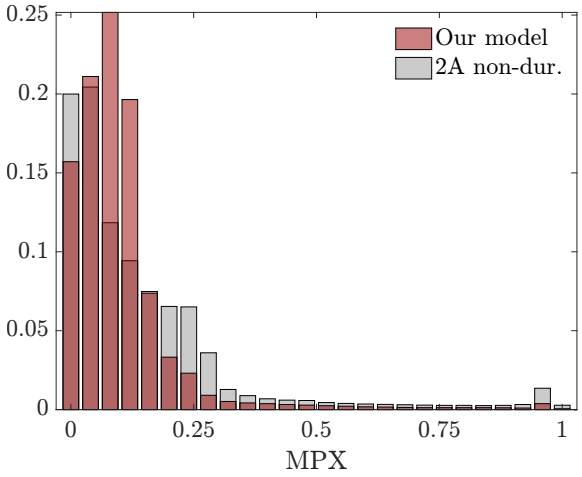
[8] The persistent component of the income process must lie on its corresponding grid, however.

responses, we suppose that the adjustment cost actually follows a logistic distribution with location parameter κ and a very small scale parameter $\eta = 0.001$, which is more than two orders of magnitudes smaller compared to $\eta \simeq 0.32$ in our model.

B Quantitative Results

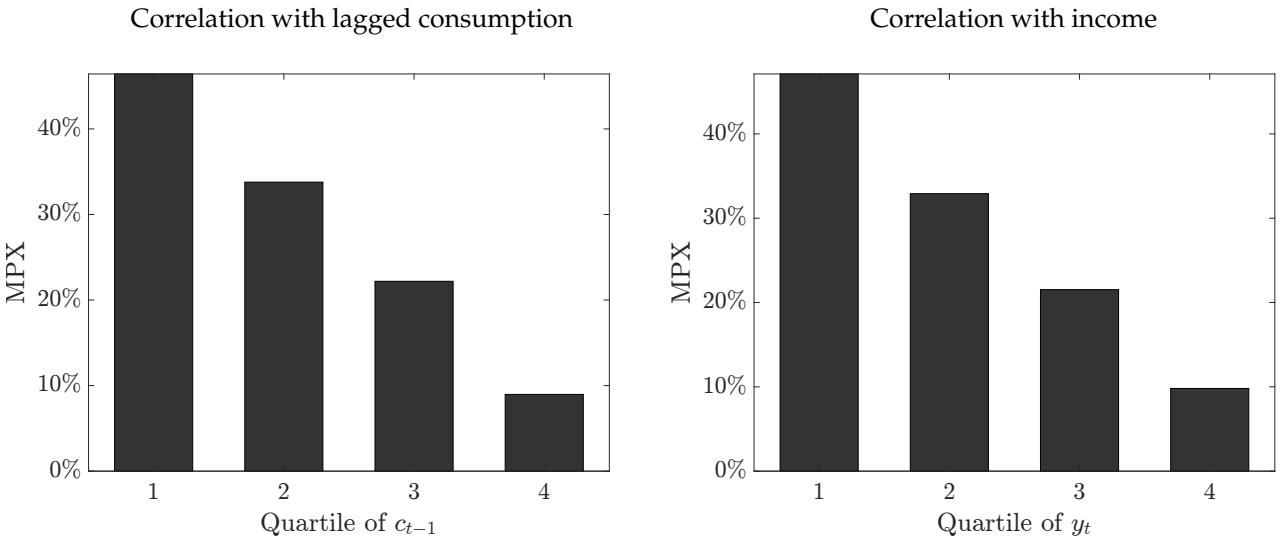
B.1 Additional Figures

Figure A.1: Unconditional distribution of MPXs on non-durables



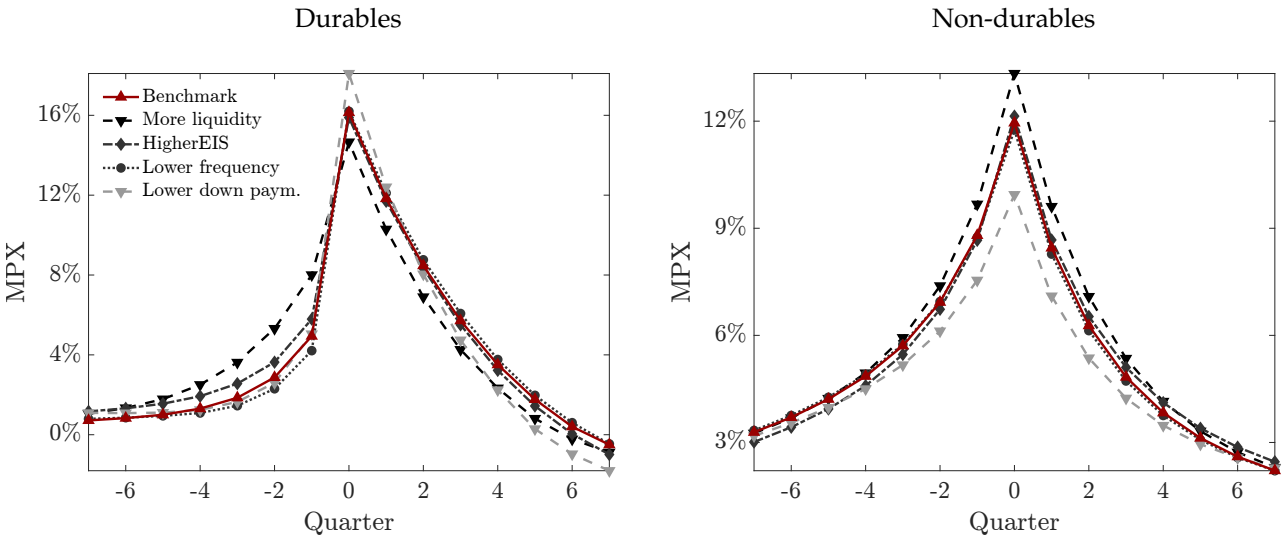
Notes: This figure plots the unconditional distribution of quarterly MPXs on non-durables out of $500 in our model (in red) and in the two-asset model of non-durable spending (in gray).

Figure A.2: MPX heterogeneity



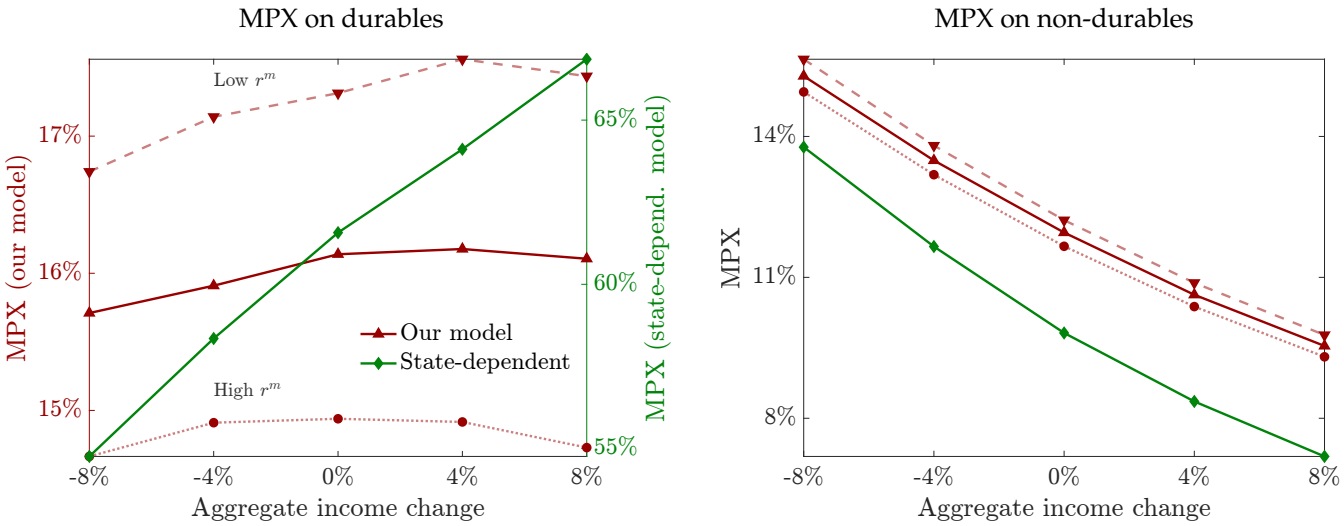
Notes: The left panel plots the quarterly total MPX in our model for each quartile of the distribution of lagged non-durable consumption. The right panel plots the same for the distribution of current income.

Figure A.3: Intertemporal MPXs (alternative calibrations)



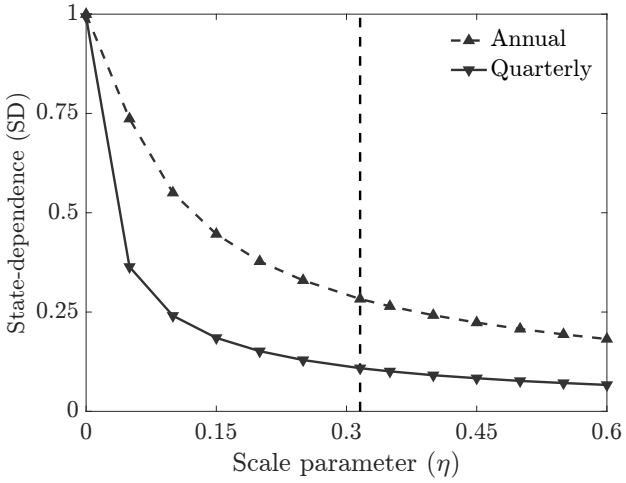
Notes: The left panel plots the intertemporal MPXs on durables out of \$500 in our model (in red) and in four alternative calibrations with more liquidity (39% of average annual income instead of 26%), a higher elasticity of intertemporal substitution ($\sigma = 1.5$ instead of $\sigma = 2$), a lower frequency of adjustment (30% instead of 38%), and a smaller required down payment ($\theta = 30\%$ instead of $\theta = 41\%$). All parameters are re-calibrated to match the other targets discussed in Section II.B. The right panel plots the same for non-durables.

Figure A.4: Aggregate conditions



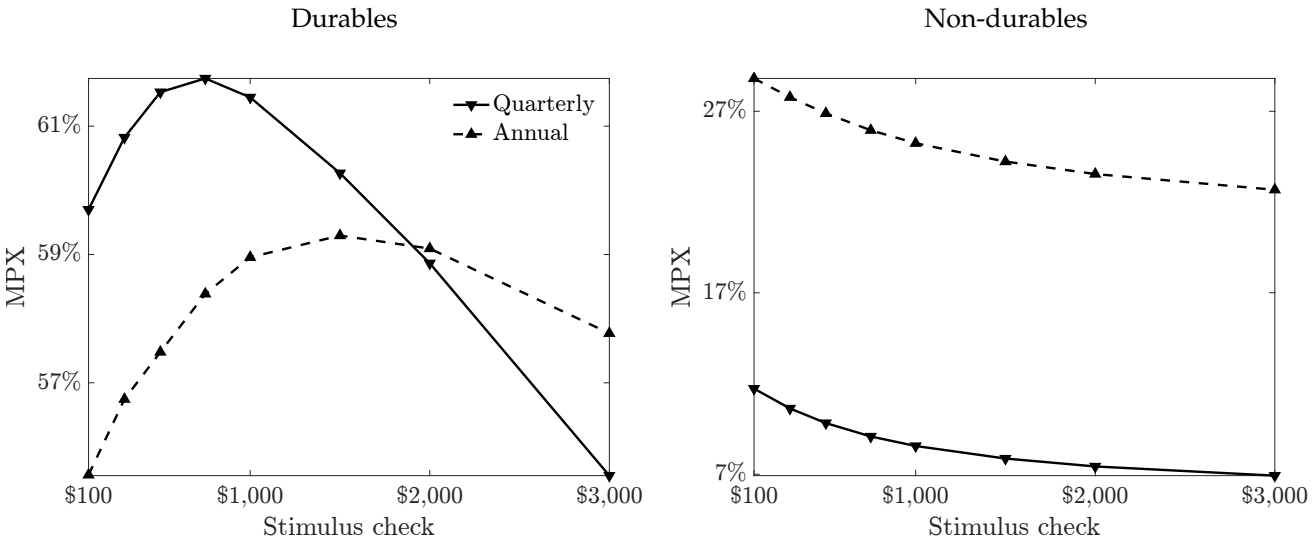
Notes: This figure plots the quarterly MPX on durables and non-durables out of $500 at various points of the business cycle (in red). We assume that the economy experiences three quarters of constant expansion (or contraction), followed by a linear mean-reversion over eight quarters. We compute the MPX in response to an unanticipated check sent in the third quarter. The left panel plots the MPX on durables. The left axis corresponds to our model, and the right axis to a purely state-dependent version with $\eta \to 0$. In turn, the right panel plots the MPX on non-durables.

Figure A.5: State- *vs.* time-dependent adjustments



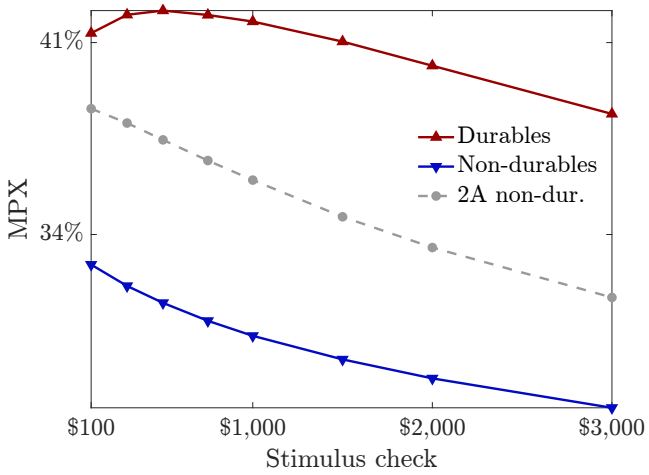
Notes: This figure plots our state-dependence index SD given by expression (7) as a function of the scale parameter η. All other parameters are re-calibrated to match the targets discussed in Section II.B. We compute this index at the quarterly frequency (solid curve) and annual frequency (dashed curve). The vertical dashed line indicates our calibrated value $\eta \simeq 0.32$.

Figure A.6: Size-dependence in the MPX in the purely state-dependent model



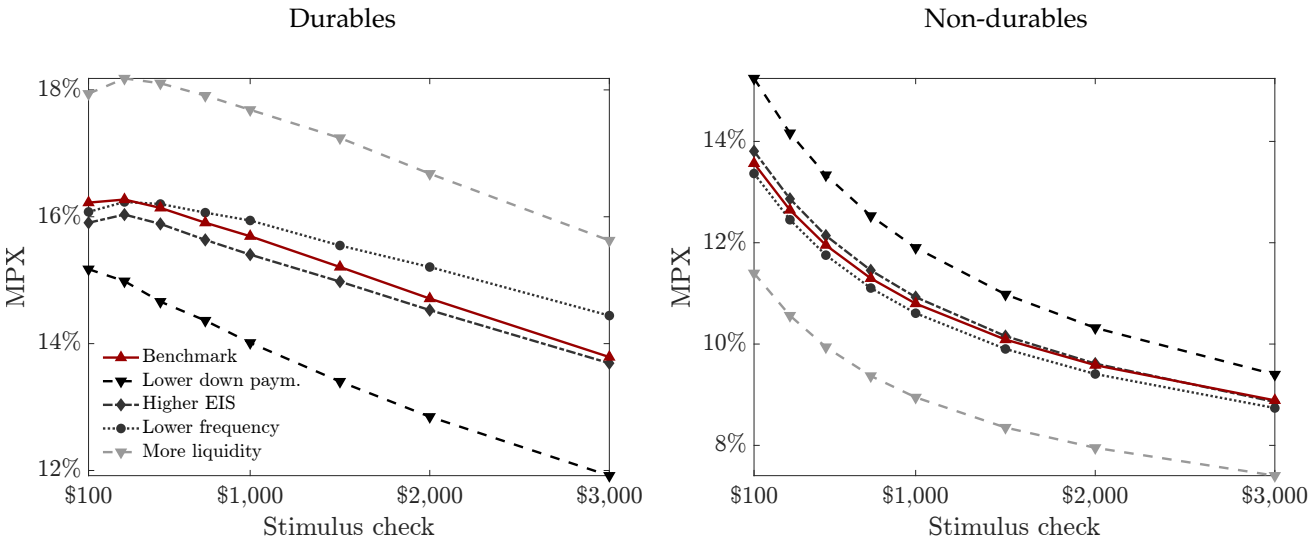
Notes: The left panel plots the MPX on durables in the purely state-dependent model with $\eta \to 0$, both at the quarterly frequency (solid curve) and annual frequency (dashed curve), as a function of the size of stimulus checks. The right panel plots the same for non-durables.

Figure A.7: Size-dependence in the annual MPX



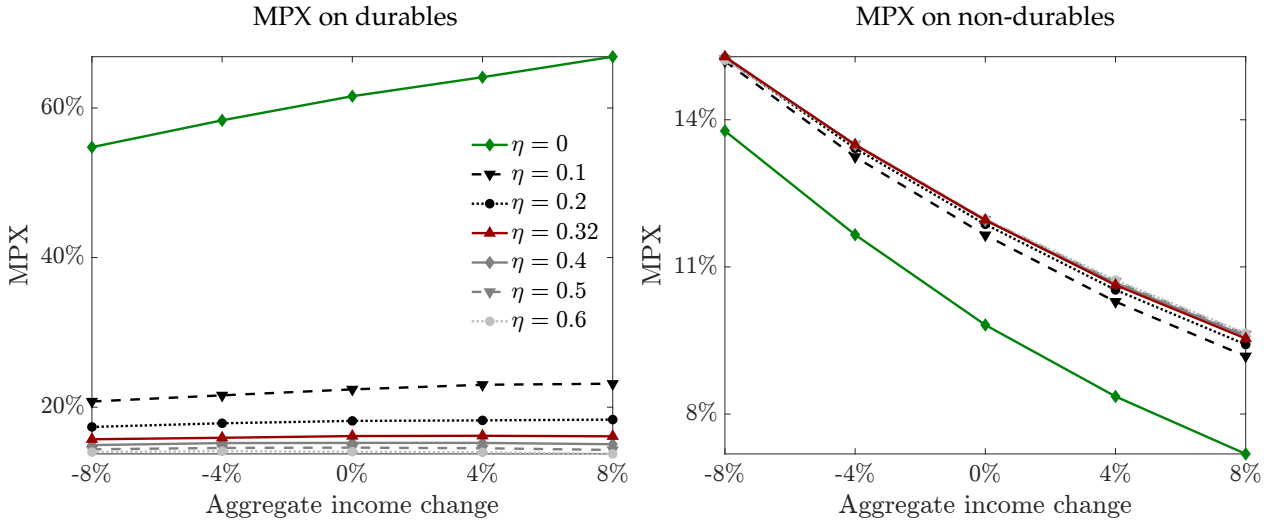
Notes: This figure plots the annual MPX on durables (in red) and non-durables (in blue) in our model as a function of the size of stimulus checks. We also plot the annual MPX in a two-asset model of non-durable spending (in gray).

Figure A.8: Size-dependence in the MPX (alternative calibrations)



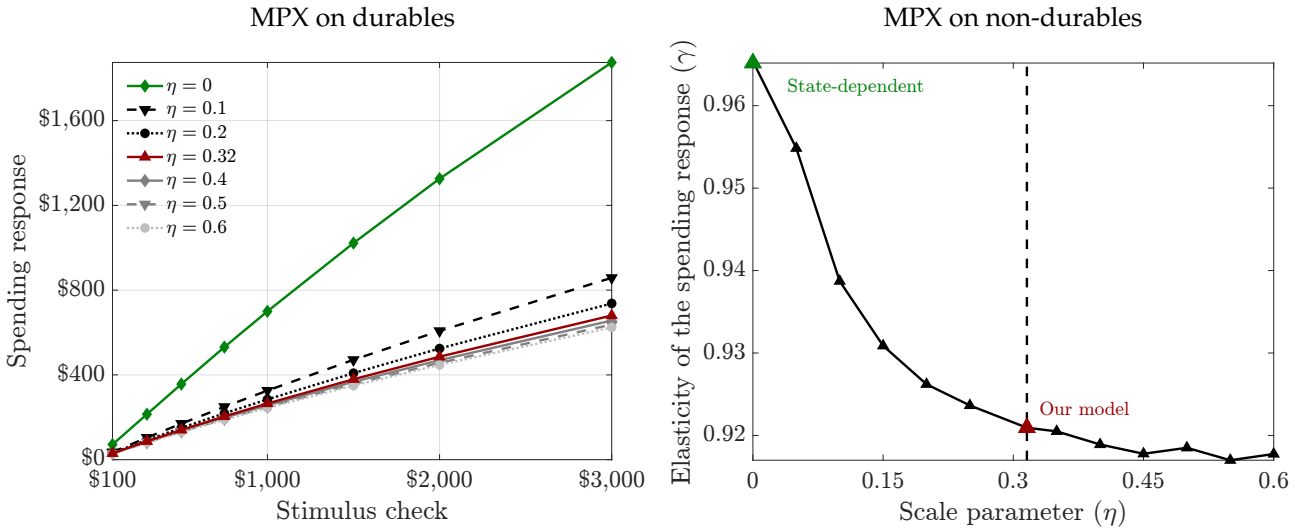
Notes: The left panel plots the quarterly MPX on durables as a function of the size of stimulus checks in our model and in four alternative calibrations with more liquidity (39% of average annual income instead of 26%), a higher elasticity of intertemporal substitution ($\sigma = 1.5$ instead of $\sigma = 2$), a lower frequency of adjustment (30% instead of 38%), and a smaller required down payment ($\theta = 30\%$ instead of $\theta = 41\%$). All parameters are re-calibrated to match the other targets discussed in Section II.B. The right panel plots the same for non-durables.

Figure A.9: Aggregate conditions (sensitivity)



Notes: This figure plots the quarterly MPX on durables and non-durables out of $500 at various points of the business cycle, for various values of the scale parameter η. All other parameters are re-calibrated to match the targets discussed in Section II.B. We assume that the economy experiences three quarters of constant expansion (or contraction), followed by a linear mean-reversion over eight quarters. We compute the MPX in response to an unanticipated check sent in the third quarter. The left panel plots the response of durable spending and the right panel plots the response of non-durable spending.

Figure A.10: Size-dependence in total spending (sensitivity)



Notes: The left panel plots the total spending response over one quarter as a function of the size of stimulus checks, for various values of the scale parameter η. All other parameters are re-calibrated to match the targets discussed in Section II.B. The right panel plots the elasticity γ defined in Section IV.B as a function of η.

B.2 Extensive and Intensive Margins

This appendix defines formally the extensive and intensive margins of adjustment that we discuss in Section IV.A. The MPX on durables in Section I.E can be expressed as

$$\text{MPX}^d(T) = \underbrace{\frac{\int\int\int\int \{\mathcal{S}(m+T,d,b,\mathbf{y}) - \mathcal{S}(m,d,b,\mathbf{y})\} \times x(m,d,b,\mathbf{y}) \times \mathrm{d}\mu(m,d,b,\mathbf{y})}{T}}_{\text{Extensive margin}}$$
$$+ \underbrace{\frac{\int\int\int\int \mathcal{S}(m,d,b,\mathbf{y}) \times \{x(m+T,d,b,\mathbf{y}) - x(m,d,b,\mathbf{y})\} \times \mathrm{d}\mu(m,d,b,\mathbf{y})}{T}}_{\text{Intensive margin}} + \text{res}$$

The extensive margin captures the change in the *number of households who adjust* in response to stimulus checks, i.e., changes in the adjustment hazard $\mathcal{S}$, holding fixed the policy function x conditional on adjustment. In turn, the intensive margin captures the response of the *number of units* and their *size* conditional on adjustment, i.e., changes in the policy function, holding the hazard fixed. Finally, the residual "res" captures the non-linear interaction between these two margins. The total effect corresponds to moving both the hazard and the policy function to the left by the amount T in Figure 2 or, equivalently, moving the distribution of liquid assets to the right; the extensive and intensive margins correspond to moving only the hazard or the policy function, respectively; and the non-

linear residual picks up the difference. Through the lens of the model, the purchase of an additional unit of durable (e.g., a computer or an appliance) is counted in the extensive margin if the household had decided not to purchase *any* other durable absent stimulus check; otherwise, it is counted in the intensive margin.

The left panel of Figure A.11 implements this decomposition in our model. The right panel does the same in the purely state-dependent model with $\eta \to 0$. Starting with the latter, the extensive margin is extremely strong and accounts for most of the response in durable spending. As expected, the intensive margin declines with the size of checks due to the usual precautionary savings motive. Perhaps surprisingly, the extensive margin also declines with the size of checks. By definition, it captures two effects as checks get larger: more households decide to adjust their stock of durables but less and less so in proportion to the size of checks (Figure A.12); and these marginal households are poorer and hence $x\left(m^{\star}\left(d,b,\mathbf{y}\right)-T,d,b,\mathbf{y}\right)$ is smaller for them, where $m^{\star}\left(d,b,\mathbf{y}\right)$ still denotes the adjustment threshold in the purely state-dependent model.[9] Finally, the non-linear residual captures a selection effect: because they are poorer, the marginal households who adjust have a larger marginal propensity to spend than those who had adjusted so far, and they bring the average MPX up. The increase in the non-linear residual is sufficiently strong that it dominates in the purely state-dependent model and the MPX on durables rises with the size of checks. Our model dampens the extensive margin of adjustment and hence softens this increase in the residual. The MPX declines slowly with the size of checks as a result.

C Alternative Models

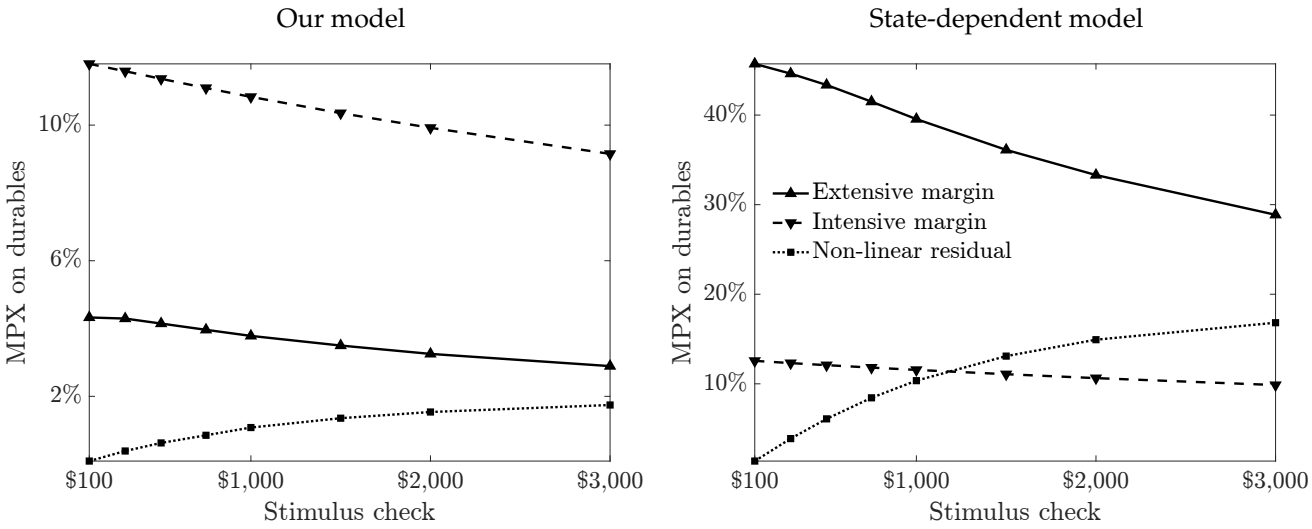
This appendix describes the two alternative models that we discuss in the paper.

C.1 Two-Asset Model of Non-Durable Spending

In Sections III–IV, we compare the predictions of our model to those of a two-asset model of non-durable spending similar to Kaplan and Violante (2022). This appendix states the households' problem recursively and discusses the calibration. Households are indexed

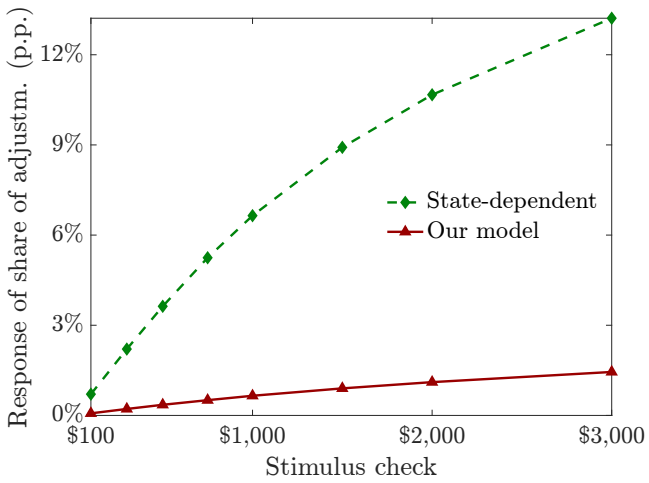
[9] How many households decide to adjust at the extensive margin depends on several factors, including the shape of the density of the distribution of liquid assets m conditional on $(d,b,\mathbf{y})$, the distribution of these three states, and how far the adjustment threshold $m^{\star}\left(d,b,\mathbf{y}\right)$ is from the lower bound $m=0$ for each of these states. In particular, the extensive margin decreases with the size of checks if most households that adjust in response to small checks have a small threshold $m^{\star}\left(d,b,\mathbf{y}\right)$, e.g., if there are many hand-to-mouth households that wish to replace their old durables.

Figure A.11: Extensive and intensive margins



Notes: The left panel decomposes the quarterly MPX on durables in our model into its extensive and intensive margins and a non-linear residual as as a function of the size of stimulus checks. The right panel does the same for the purely state-dependent model with $\eta \to 0$.

Figure A.12: Response of the share of adjustments (quarterly)



Notes: This figure plots the response of the share of households who adjust their stock of durables in the quarter when they receive a stimulus check as a function of the size of this check. The red curve is our model. The green curve is the purely state-dependent model with $\eta \to 0$.

by four idiosyncratic states $\mathbf{x} \equiv (b, m, \boldsymbol{y})$, i.e., their stock of illiquid financial asset b, their holdings of liquid asset m, and the two components of their income process $\mathbf{y} \equiv (\omega, \zeta)$.

Continuation values. The continuation values $\{V_t(\cdot)\}$ can be characterized recursively.

1. **Discrete choice.** The household chooses whether to adjust its stock of illiquid asset or not. The value associated with the discrete choice problem is

$$V_t(\mathbf{x}) \equiv \max \left\{ V_t^{\text{adjust}} \left(\hat{m}_t(b, m, \mathbf{y}) + \frac{b}{1+\pi_t}, \omega \right) - \kappa, \right. \\ \left. V_t^{\text{not}} \left(\frac{b}{1+\pi_t}, \hat{m}_t(b, m, \mathbf{y}), \omega \right) \right\} \tag{A.27}$$

   where V^{adjust} is the continuation value of adjusting the stock of illiquid assets, V^{not} is the continuation value of not adjusting, and $\kappa > 0$ is the adjustment cost. Cash-on-hand before the adjustment decision is

$$\hat{m}_t(b, m, \mathbf{y}) \equiv \mathcal{Y}_t(\mathbf{y}) + \frac{1 + r_{t-1}^m}{1+\pi_t} m + \frac{r_{t-1}^b}{1+\pi_t} b + \frac{T_t}{\prod_{s=0}^{t}(1+\pi_s)} \tag{A.28}$$

   where earnings $\mathcal{Y}_t(\mathbf{y})$ are given by expression (A.7), r_{t-1}^m is the return on the liquid asset, r_{t-1}^b is the return on the illiquid asset, and π_t is inflation. As in our model, stimulus checks T_t are expressed in nominal terms and deflated accordingly.

2. **Illiquid asset adjustment.** If the household decides to adjust its stock of illiquid assets, it chooses its new stock

$$V_t^{\text{adjust}}(m, \omega) = \max_{b', m'} V_t^{\text{not}}(b', m', \omega) \\ \text{s.t. } b' + m' \leq m \ , \ b' \geq 0, \tag{A.29}$$

   where m is real cash-on-hand before the household chooses its new stock of illiquid assets. The continuation value V_t^{not} reflects the subsequent optimal consumption-saving choice that occurs in the same period given the remaining cash-on-hand m'.

3. **Consumption-saving.** If the household decides not to adjust its stock of illiquid asset, it chooses how much to consume and save in liquid asset

$$V_t^{\text{not}}(b, m, \omega) = \max_{c, m'} u(c) + \beta \int \int V_{t+1}(b, m', \omega', \zeta') \, \Gamma_\zeta(\mathrm{d}\zeta') \, \Gamma_\omega(\mathrm{d}\omega'; \omega) \\ \text{s.t. } c + m' \leq m \ , \ m' \geq 0, \tag{A.30}$$

where m is the household's cash-on-hand before it chooses non-durable consumption c, and m' is cash-on-hand for next period.

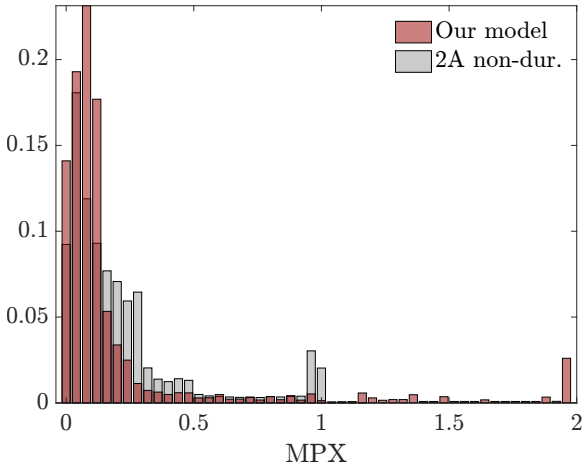
Calibration. The calibration strategy follows Kaplan and Violante (2022) closely. We set $u(c) = 1/(1-\sigma)\, c^{1-\sigma}$ with inverse elasticity of intertemporal substitution $\sigma = 2$, as in our model. We set the (real) return on cash to -2% per year as in Kaplan and Violante (2022). We discipline internally three parameters: the discount factor (β); the excess return on the illiquid asset $(\Delta r \equiv r_t^b - r_t^m)$; and the adjustment cost (κ). We calibrate these parameters to match an average stock of liquid assets of 26% as a share of average annual income as in our model, a share of total hand-to-mouth households of 41%, and a share of poor hand-to-mouths with zero holdings of b of 14% as in Kaplan and Violante (2022). To obtain slightly smoother responses, we suppose that the adjustment cost actually follows a logistic distribution with location parameter κ and a very small scale parameter.

Alternative formulation. The formulation above assumes that the return on the illiquid asset is paid to the liquid asset, as in Bayer et al. (2019) and Auclert et al. (2021). In this case, the return can be interpreted as dividends on stocks, interest rates on bonds, rental income on properties, returns on private equity, payments on annuities, etc. Alternatively, the return on the illiquid asset can be paid to the illiquid asset itself, as in Kaplan and Violante (2014) and Kaplan et al. (2018). In this case, the return can be interpreted as capital gains, house price appreciation, etc. Alves et al. (2020) explore how these two formulations differ in response to monetary policy shocks. Auclert et al. (2020) allow for a hybrid formulation.

We show below that the main properties that we focus on are unchanged in the alternative scenario where the return on the illiquid asset is paid to the illiquid asset itself. Figure A.13 plots the unconditional distribution of MPXs in our model (red) and in the alternative model of non-durable spending (in grey). The conclusions are the same as in Section III.A: the model of non-durable spending lacks large and lumpy durable purchases which generate more dispersion in the right tail. In particular, "scaling up" the responses from the model of non-durable spending would generate a significant mass of households with an MPX close to 45% or so, which is not the case in our model. In turn, Figure A.14 plots the intertemporal MPXs on durables (in red) and non-durables (in blue) in our model and in the alternative model of non-durable spending (in grey). Again, the conclusions are the same as in Section III.B: the MPX on durables declines much faster over time compared to the model of non-durable spending. Finally, Figure A.15 plots the size-dependence in the MPX on durables (in red) and non-durables (in blue) in our model and in the alternative model of non-durable spending (in grey). Once more, the conclu-

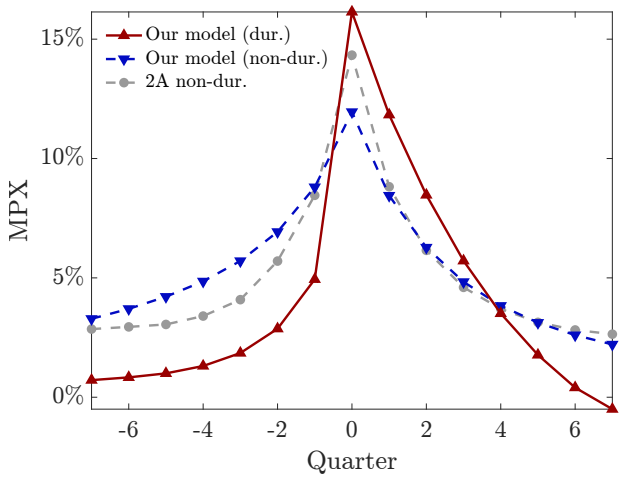
sions are the same as in Section IV.A: the MPX declines faster in the model of non-durable spending compared to our model.

Figure A.13: Unconditional distribution of MPXs



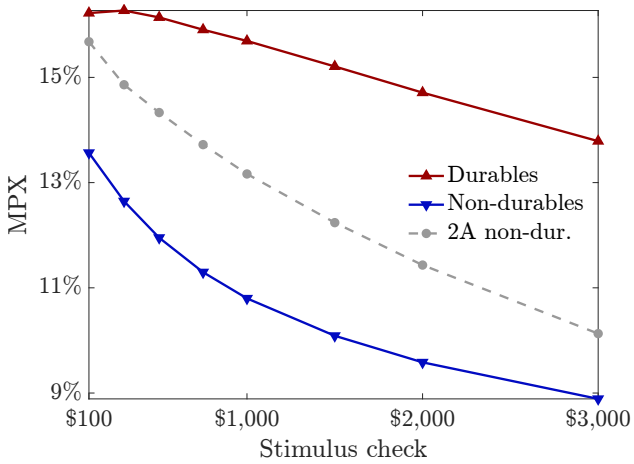
Notes: This figure plots the unconditional distribution of quarterly MPXs out of $500 in our model (in red) and in the two-asset model of non-durable spending where the return on the illiquid asset is paid to the illiquid asset itself (in gray). The height of the rightmost bar indicates the mass of households in the tail that is truncated on the plot.

Figure A.14: Intertemporal MPXs on durables and non-durables



Notes: This figure plots the intertemporal MPXs on durables (in red) and non-durables (in blue) out of $500 in our model. We consider both the dynamic effect in periods $s \geq 0$ following an unanticipated shock in $t = 0$ and the anticipatory effect in periods $s < 0$ from an anticipated shock in $t = 0$. We report the same intertemporal MPXs in the two-asset model of non-durable spending where the return on the illiquid asset is paid to the illiquid asset itself (in gray).

Figure A.15: Size-dependence in the MPX



Notes: This figure plots the quarterly MPX on durables (in red) and non-durables (in blue) in our model as a function of the size of stimulus checks. We also plot the quarterly MPX in the two-asset model of non-durable spending where the return on the illiquid asset is paid to the illiquid asset itself (in gray).

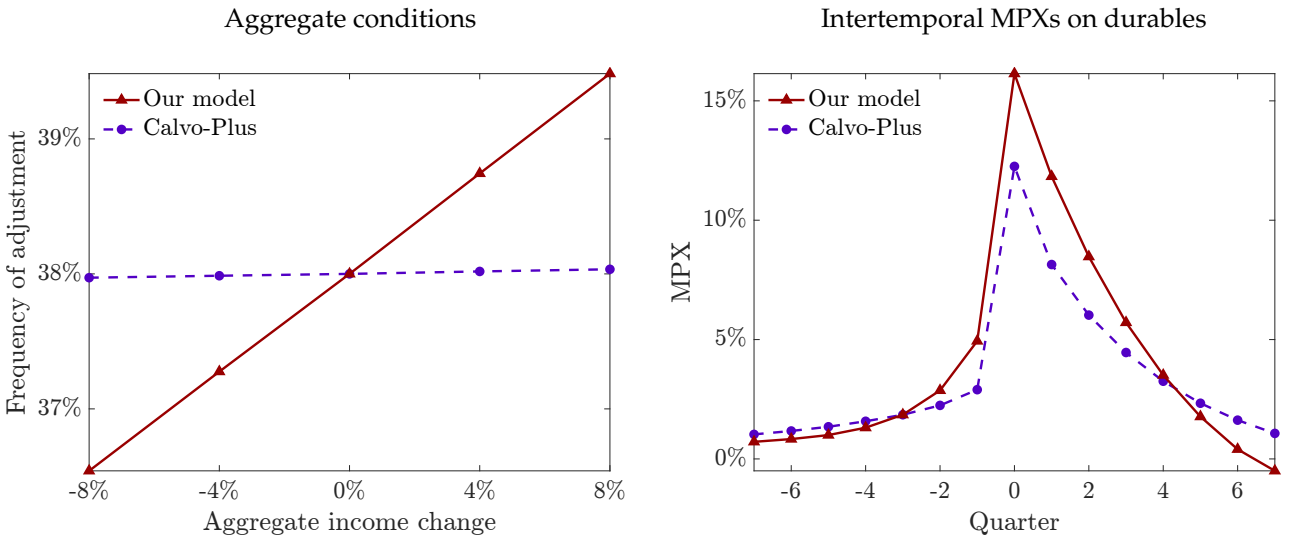
C.2 Calvo-Plus Model

In Section III.F, we discuss a Calvo-Plus variant of our model. Our calibration of this model follows McKay and Wieland (2021) who introduce exogenous adjustments to dampen the elasticity of durable spending to changes in interest rates (or the user cost). Accordingly, we choose the probability of free adjustments to dampen the user cost elasticity and match the same short-run price elasticity of durable purchases as in our model (Figure 6). All other parameters are re-calibrated to match the targets discussed in Section II.B.

We start by computing the size-dependence index (7) using this Calvo-Plus variant. The model behaves essentially as a purely time-dependent model with a flat adjustment hazard, i.e., a standard Calvo model where adjustments are purely exogenous. Indeed, the state-dependence index is 2.3% at the annual level, which is much lower than in our model (28%). The reason is that the Calvo-Plus model requires a large share of exogenous adjustments to dampen the otherwise large price elasticity induced by the deterministic fixed cost (Section III.C). The left panel of Figure A.16 shows that this extreme form of time-dependence implies that the frequency of adjustments is virtually acyclical, whereas it is clearly procyclical in the data and in our model (Section III.D). The right panel shows that this strong time-dependence has a meaningful effect on the MPX on durables.[10] The response is more muted on impact, and much more persistent over time relative to our model. Overall, the annual MPX on durables is 31% in the Calvo-Plus variant, and roughly 1/3 larger in our model (42%).

[10] The MPX on non-durables is comparable to our model so we omit it for concision.

Figure A.16: Calvo-Plus model



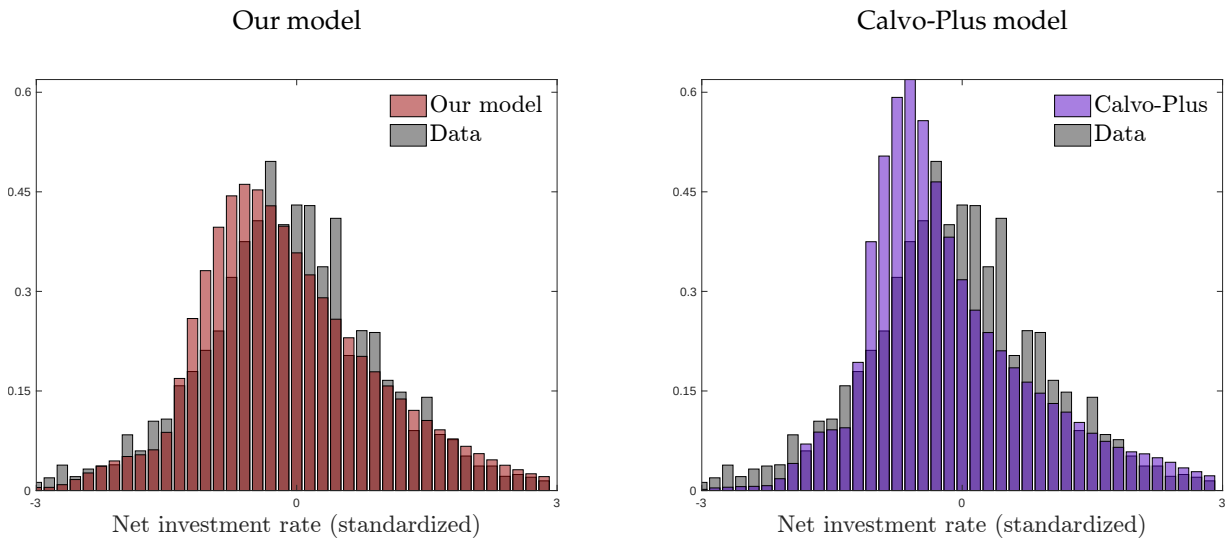
Notes: The left panel plots the frequency of durable adjustment in our model at various points of the business cycle (in red). We assume that the economy experiences three quarters of constant expansion (or contraction), followed by a linear mean-reversion over eight quarters. We report the frequency of adjustment in the third quarter and annualize this frequency by scaling it so that it is equal to its annual counterpart when there is no aggregate shock. We also plot this frequency in the Calvo-Plus model (in purple). The right panel plots the intertemporal MPXs in our model on durables (in red). We consider both the dynamic effect in periods $s \geq 0$ following an unanticipated shock in $t = 0$ and the anticipatory effect in periods $s < 0$ from an anticipated shock in $t = 0$. We report the same intertemporal MPXs on durables in the Calvo-Plus model (in purple).

Next, we inspect the distribution of net investment rates in steady state in these two models. Households are constantly subject to idiosyncratic income shocks and adjust their stock of durables as a result. Some of these shocks are much larger than typical stimulus checks — some even amount to several thousand dollars.[11] The distribution of net investment rates conditional on adjustment reflects the distance between households' current stock of durables and their ideal stock, as well as their propensity to adjust in response to these shocks. As such, this distribution is informative about the shape of the adjustment hazard and the underlying distribution of utility costs.

Figure A.17 shows the distribution of net investment rates in our model and in the Calvo-Plus variant. As usual, this distribution is standardized (Alvarez et al., 2016). That is, we substract its mean and divide by its standard deviation excluding the top and bottom 1% extreme values. This explains why the mode, which corresponds to small adjustments, can take a negative value. For comparison, we also report the empirical distribution of net investment rates that we measure between consecutive PSID waves

[11] For instance, the transitory component of our income process has a standard deviation of 10% (Section II.A). We assume that mean annual income is $67,000 in steady state as in Kaplan and Violante (2022).

Figure A.17: Distribution of net investment rates



Notes: The left panel plots the distribution of net investment rates across two consecutive PSID waves between which households adjusted their stock of durables. The distribution is standardized as in Alvarez et al. (2016), i.e., we substract its mean and divide by its standard deviation excluding the top and bottom 1% extreme values. The red histogram is our calibrated model ($\eta \simeq 0.32$). The black histogram plots the empirical distribution for cars that we measure in the PSID. The right panel plots the same for the Calvo-Plus variant.

conditional on adjustment. We compute this distribution for vehicles, which is the only durable stock other than housing that is observed in the PSID. We provide empirical details in Supplemental Appendix D. Our model fits this empirical distribution relatively closely. It produces a hump as in the data; and it matches the tails of the distribution quite well — an important moment in models with lumpy adjustment (Alvarez et al., 2016). In contrast, the Calvo-Plus variant produces a larger share of small adjustments, i.e., there is more mass around the mode, and a shorter left tail where households downsize their stock of durables.

D Empirical Moments

In this appendix, we explain how we use the Panel Study of Income Dynamics (PSID) to compute the frequency of adjustment for cars (Section II.B) and the distribution of net investment rates for cars (Sections III.F and C.2) that we discuss in the paper.

D.1 Durable Adjustments in the PSID

We restrict our sample to household heads or reference persons who are male, aged 21 or older, and own at least one vehicle in at least three PSID waves between 1999 and 2019. We use w to denote a bi-annual PSID wave. An adjustment ($\text{Adj}_w = 1$) occurs in two cases. Either the number of vehicles owned by the household changes. Or the household reports that the vehicle that was last purchased (vehicle "#1") was acquired more recently than the one reported in the previous wave $\text{Purchase}_w^{\#1} > \text{Purchase}_{w-1}^{\#1}$, and at most two years before the interview date $\text{Purchase}_w^{\#1} \geq t_w - 2$.[12]

D.2 Frequency of Adjustment for Cars

To compute the frequency of adjustment for cars, we start by estimating the empirical distribution π_k of the duration k between vehicle purchases. To do so, we compute $\text{Duration}_w = \text{Purchase}_w^{\#1} - \text{Purchase}_{w-1}^{\#1}$ whenever an adjustment occurs ($\text{Adj}_w = 1$) and the household reports the same number of cars or more than in the previous wave. In turn, we set $\text{Duration}_w = (\text{Year}_w - 1) - \text{Purchase}_{w-1}^{\#1}$ whenever an adjustment occurs ($\text{Adj}_w = 1$) but the household reports fewer vehicles, with $\text{Year}_w - 1$ denoting the median year between two consecutive PSID waves where the downward adjustment took place. We restrict attention to the first purchase observed for a given household. The reason is that subsequent purchases, if observed in the PSID's relatively short time dimension, are more likely to be of shorter duration which would bias our estimates. Focusing on the first adjustment allows us to circumvent this issue. This procedure yields an empirical probability distribution π_k over durations $k = 1, 2, \ldots$ expressed in years. The empirical frequency of adjustment is the inverse of the average duration $1/\sum_{k\geq 0} k\pi_k$. We obtain an annual frequency of adjustment for cars of 23% which is in line with the rest of the literature (McKay and Wieland, 2021).

D.3 Distribution of Net Investment Rates for Cars

We measure the net investment rate conditional on adjustment as $\log\left(d_w^{\text{net}}\right) - \log\left(d_{w-1}^{\text{net}}\right)$ when $\text{Adj}_w = 1$, where d_w^{net} is the real value of the stock of vehicles *net* of liabilities

[12] Households sometimes skip a PSID wave. In this case, we say that $\text{Adj}_w = 1$ if one of the two changes explained above occurs since the previous wave where the household is observed. We also experimented with a version where we restricted the sample to households who never skip a wave while they appear in the PSID. This approach mechanically shortens the period over which households are observed. We obtained very similar results in this case.

reported by the household in a PSID wave.[13] Lastly, we standardize the distribution of net investment rates by de-meaning it and normalizing it by its standard deviation (Alvarez et al., 2016). We trim the top and bottom 1% of the distribution when standardizing.

We discretize our model-simulated series into PSID waves and treat those identically to the actual data. We divide time into years, as our model is set up quarterly. The value of the stock of durables in the simulated PSID wave w is $d_{T(w)}$, where $T(w)$ is the last quarter in that wave. The value *net* of credit is

$$d_w^{\text{net}} \equiv (1-\delta)\, P^d d_{T(w)} - \left(1+r^b\right)\left(\frac{1-\varphi}{1-(1-\iota)\,\delta}\right)^{\xi}(1-\theta)\, P^d\left(\xi\right) d_{T(w)} \tag{A.31}$$

in the model, where the first term corresponds to the undepreciated value of the stock of durables and the second term corresponds to the outstanding credit (Section A.1). Prices and the return on credit are constant since we compute the distribution of net investment rates in steady state.

E General Equilibrium

In this appendix, we describe the general equilibrium model that we use in Section IV.C. Section E.1 presents the environment and Section E.2 discusses the parametrization. We provide further details about fiscal policy in Supplemental Appendix E.3. Section E.4 discusses our numerical implementation. Finally, Section E.5 contains additional numerical results.

E.1 Environment

The economy has two sectors: the first produces a non-durable good; and the second an investment good. The non-durable good can be used for consumption or as an intermediate for producing the investment good. In turn, the investment good can be used to build up the stock of durables. The non-durable good is produced with labor. The investment good is produced exclusively with non-durables as in McKay and Wieland (2021).

Households. The household block of the economy is identical to the one introduced in Section I in the main text and Section A.1 in this appendix.

[13] We do not attempt to back out the gross value of the stock by using imperfect information on liabilities, which would add another layer of measurement error. Instead, we directly compute the changes in the stock net of liabilities and we treat the model-generated data identically. We deflate the nominal stocks using the price indices for motor vehicles and parts from the Bureau of Economic Analysis (2019).

Non-durable goods. A firm produces non-durables using labor. Inflation in the price of the domestic non-durable good (π_t) is given by a Phillips curve

$$\pi_t = \lambda \log\left(\frac{Y_t^{\text{dom}}}{Y}\right) + \beta \pi_{t+1}, \tag{A.32}$$

where Y_t^{dom} is the aggregate demand for non-durables, Y is steady state output in that sector, and $\lambda > 0$ is the slope of the Phillips curve.[14]

Investment goods. A firm can produce $F(M) = A_0 M^{\frac{\chi}{1+\chi}}$ units of the investment good using M units of non-durables as in McKay and Wieland (2021, 2022), where $A_0 > 0$ is the productivity and $1/\chi > 0$ governs the decreasing returns in production and hence the inverse supply elasticity. Profit maximization implies that, in equilibrium, the relative price of the domestic investment good is

$$P_t^d \equiv \left(\frac{X_t^{\text{dom}}}{X}\right)^{1/\chi}, \tag{A.33}$$

where X_t^{dom} is the aggregate demand for durables produced domestically, and X is steady state output in that sector. The productivity A_0 is chosen so that $X \equiv \left(\frac{\chi}{1+\chi}\right)^{\chi} A_0^{1+\chi}$, which ensures that the relative price of durables is equal to 1 in steady state.

Policy. The government sends nominal stimulus checks to eligible households in the first period. We assume that households who earned less than \$75,000 in the previous year are eligible to receive a check. The size of the check decreases linearly with income after that and reaches zero at \$80,000.[15] The government's flow budget constraint is

$$B_t^g + P_t^c G_t + \frac{T_t}{\prod_{s=0}^{t}(1+\pi_s)} = \frac{1+r_{t-1}^m}{1+\pi_t} B_{t-1}^g + \mathcal{T}_t + \Sigma_t, \tag{A.34}$$

where B_t^g are real asset holdings, G_t is government spending on non-durables, T_t are nominal stimulus checks, r_t^m is the return on the liquid asset, and $\mathcal{T}_t$ and Σ_t are the tax revenues and the credit payments from households (Appendix E.3). The government

[14] This Phillips curve can result from sticky prices or wages. Distinguishing between the two would require taking a stance on workers' labor supply, which is not the focus of this paper.

[15] This distribution of checks mimics the one observed in 2020–2021. Again, we assume that mean annual income is \$67,000 in steady state as in Kaplan and Violante (2022).

maintains a constant ratio of debt to output at the stationary equilibrium. Its real spending $G_t = G > 0$ on domestic goods balances the budget (A.34) in steady state given $\mathcal{T}_t$ and Σ_t. In period $t = 0$, the government sends a one-time nominal stimulus check to eligible households. These checks are deficit-financed. In later periods, the government maintains a constant spending G and repays its new debt over time by raising the tax rate τ_t as we explain in Section E.2.

Monetary policy follows a standard rule $r_t^m = r^m + \phi_\Pi \pi_t$ where r^m is the steady state interest rate on the liquid asset and ϕ_Π is the coefficient on inflation.

Aggregate income. Households' aggregate income before interest and tax payments is

$$Y_t^{\text{inc}} = P_t^c \left(C_t + G_t\right) + P_t^d X_t \tag{A.35}$$

where C_t and X_t are the households' aggregate demands for the non-durable and investment good, respectively.

E.2 Parametrization

As in our baseline calibration (Section II), the real interest rate is $r^m = 1\%$ at the stationary equilibrium, the government maintains a constant ratio of debt to annual aggregate income of 26%, and the tax rate τ is 30% in the long-run. The slope of the Phillips curve is $\lambda = 0.0031$ based on the evidence of Hazell et al. (2022).[16] We suppose that $\chi \equiv 1/0.049$ as in McKay and Wieland (2021, 2022) so the relative price of durables is mildly procyclical.[17] The Taylor coefficient on inflation is $\phi_\Pi = 1.5$. Finally, the government finances the stimulus checks by issuing debt, and repays it slowly by raising the tax intercept uniformly over 15 years and letting it decay to its long-run value over the next 5 years.

E.3 Fiscal Policy

Budget constraint. The government's budget constraint is given by (A.34). The tax revenues are

$$\mathcal{T}_t \equiv \tau_t \int\int\int\int\int \exp\left(\omega + \zeta\right) \mathrm{d}\mu_{t-1}\left(d, m, \xi, \omega, \zeta\right) Y_t^{\text{inc}}, \tag{A.36}$$

[16] Hazell et al. (2022) estimate that this slope is -0.0062 in terms of unemployment since 1980. The semi-elasticity of unemployment with respect to output is roughly -0.5 over that period.

[17] Empirically, the relative price of new consumer durables is relatively acyclical, even when using transaction prices (Gavazza and Lanteri, 2021) as in CPI data (McKay and Wieland, 2021; Cantelmo and Melina, 2018).

where μ_{t-1} denotes the distribution of households' idiosyncratic states.[18] The government claims the net credit payments from households

$$\Sigma_t \equiv \int\int\int\int\int \left\{ r^b_{t-1} \left(\frac{1-\varphi}{1-(1-\iota)\,\delta} \right)^{\xi} (1-\theta)\, \hat{P}^d_t(\xi)\, d \right. \\ \left. + \mathcal{S}_t(\mathbf{x})\, \hat{\Delta}^D_t(\mathbf{x}) + \{1-\mathcal{S}_t(\mathbf{x})\}\, \hat{\Delta}^C_t(\mathbf{x}) \right\} \mathrm{d}\mu_{t-1}(d, m, \xi, \omega, \zeta), \tag{A.37}$$

where $\mathbf{x} \equiv (d, m, \xi, \omega, \zeta)$ denotes a household's states and

$$\hat{\Delta}^D_t(\mathbf{x}) = \left(\frac{1-\varphi}{1-(1-\iota)\,\delta} \right)^{\xi} (1-\theta)\, \hat{P}^d_t(\xi)\, d - (1-\theta)\, P^d_t d'(\mathbf{x}) \tag{A.38}$$

$$\hat{\Delta}^C_t(\mathbf{x}) = \varphi \left(\frac{1-\varphi}{1-(1-\iota)\,\delta} \right)^{\xi} (1-\theta)\, \hat{P}^d_t(\xi)\, d \tag{A.39}$$

denote the net *outflows* of credit from households' perspective in case of adjustment or no adjustment, respectively.

Taxe rates. The tax rate is $\tau_t = \bar{\tau} + \Psi_t \hat{\tau}$, where $\bar{\tau} = 0.3$ is the tax rate in steady state and $\{\Psi_t\}$ takes the value 1 over 15 years and then decreases linearly to 0 over the next 5 years (Section E.2). The change $\hat{\tau}$ ensures that the government's intertemporal budget constraint holds, so

$$\tau_t = \bar{\tau} - \Psi_t \frac{\sum_{s=0}^{\infty} Q_s \Omega_s + \frac{1+r_{-1}}{1+\pi_0} B^g_{-1}}{\sum_{s=0}^{\infty} Q_s \Psi_s \int\int\int\int\int \exp(\omega+\zeta)\, \mathrm{d}\mu_{s-1}(d, m, \xi, \omega, \zeta)\, Y^{\text{inc}}_s} \tag{A.40}$$

where

$$\Omega_t \equiv \bar{\tau} \int\int\int\int\int \exp(\omega+\zeta)\, \mathrm{d}\mu_{t-1}(d, m, \xi, \omega, \zeta)\, Y^{\text{inc}}_t \\ + \Sigma_t - P^c_t G_t - \frac{T_t}{\prod_{s=0}^{t}(1+\pi_s)} \tag{A.41}$$

and $Q_t \equiv \prod_{s=1}^{t} \left\{ (1+\pi_s) / \left(1+r^m_{s-1}\right) \right\}$ is the stochastic discount factor.

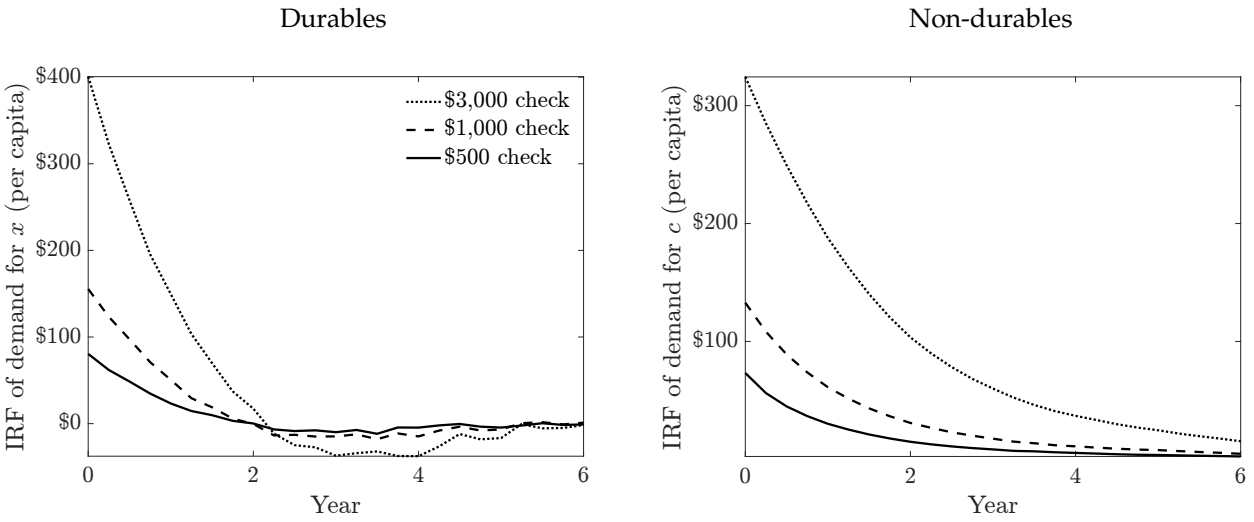
[18] As a reminder, it is equivalent to track households' credit b or the number of installments that they have already paid ξ (Section A.1).

E.4 Computational Details

We sample 200,000 households from the stationary distribution (Section A.3) and simulate them over a burn period of 400 quarters. We then solve for the transitions non-linearly over 125 quarters, for each size of stimulus check. Given sequences of incomes, interest rates, prices and tax rates, we iterate backwards on households' values starting from the steady state as described in Section A.2. We simulate households forward using the associated policies. We then use the equilibrium conditions laid out above to update the sequences of incomes, interest rates, prices and tax rates. We update these sequences slowly using weights that decay over time according to $v \exp(-0.15t)$ where t is measured in years and we use $v \equiv 0.6$ for quantities and tax rates and $v = 0.3$ for prices. We repeat this procedure until convergence.

E.5 Additional Results

Figure A.18: General equilibrium responses of sectoral demands to stimulus checks



Notes: This figure plots the general equilibrium impulse responses of aggregate demands for each good, per capita, over time following stimulus checks of various sizes sent in $t = 0$. The left panel corresponds to durables, and the right panel corresponds to non-durables.

References

Alvarez, Fernando, Hervé Le Bihan, and Francesco Lippi, "The Real Effects of Monetary Shocks in Sticky Price Models: A Sufficient Statistic Approach," *American Economic Review*, October 2016, *106* (10), 2817–51.

Alves, Felipe, Greg Kaplan, Benjamin Moll, and Giovanni L. Violante, "A Further Look at the Propagation of Monetary Policy Shocks in HANK," *Journal of Money, Credit and Banking*, 2020, *52* (S2), 521–559.

Auclert, Adrien, Bence Bardóczy, Matthew Rognlie, and Ludwig Straub, "Using the Sequence-Space Jacobian to Solve and Estimate Heterogeneous-Agent Models," *Econometrica*, September 2021, *89* (5), 2375–2408.

_ **, Matthew Rognlie, and Ludwig Straub**, "Micro Jumps, Macro Humps: Monetary Policy and Business Cycles in an Estimated HANK Model," NBER Working Papers 26647 January 2020.

Bayer, Christian, Ralph Luetticke, Lien Pham-Dao, and Volker Tjaden, "Precautionary Savings, Illiquid Assets, and the Aggregate Consequences of Shocks to Households," *Econometrica*, 2019, *87* (1), 255–290.

Bureau of Economic Analysis, "NIPA Table 1.5.4. Price Indexes for Gross Domestic Product, Expanded Detail (2015–2019)," https://apps.bea.gov/iTable/?reqid=19&step=2&isuri=1&categories=survey#eyJhcHBpZCI6MTksInN0ZXBzIjpbMSwyLDNdLCJkYXRhIjpbWyJjYXRlZ29yaWVzIiwiU3VydmV5Il0sWyJOSVBBX1RhYmxlX0xpc3QiLCIzNCJdXX0= (last accessed March 31, 2026), Department of Commerce 2019.

Cantelmo, Alessandro and Giovanni Melina, "Monetary policy and the relative price of durable goods," *Journal of Economic Dynamics and Control*, 2018, *86* (C), 1–48.

Carroll, Christopher D., "The method of endogenous gridpoints for solving dynamic stochastic optimization problems," *Economics Letters*, June 2006, *91* (3), 312–320.

Clausen, Andrew and Carlo Strub, "Envelope theorems for non-smooth and non-concave optimization," ECON - Working Papers 062, Department of Economics - University of Zurich April 2012.

Druedahl, Jeppe and Thomas Høgholm Jørgensen, "A general endogenous grid method for multi-dimensional models with non-convexities and constraints," *Journal of Economic Dynamics and Control*, 2017, *74* (C), 87–107.

Gavazza, Alessandro and Andrea Lanteri, "Credit Shocks and Equilibrium Dynamics in Consumer Durable Goods Markets," *The Review of Economic Studies*, 03 2021, *88* (6), 2935–2969.

Hazell, Jonathon, Juan Herreño, Emi Nakamura, and Jón Steinsson, "The Slope of the Phillips Curve: Evidence from U.S. States," *The Quarterly Journal of Economics*, 2022, *137* (3), 1299–1344.

Kaplan, Greg and Giovanni L. Violante, "A Model of the Consumption Response to Fiscal Stimulus Payments," *Econometrica*, July 2014, *82* (4), 1199–1239.

_ **and** _ , "The Marginal Propensity to Consume in Heterogeneous Agent Models," *Annual Review of Economics*, 2022, *14* (1), 747–775.

_ **, Benjamin Moll, and Giovanni L. Violante**, "Monetary Policy According to HANK," *American Economic Review*, March 2018, *108* (3), 697–743.

McKay, Alisdair and Johannes F. Wieland, "Lumpy Durable Consumption Demand and the Limited Ammunition of Monetary Policy," *Econometrica*, November 2021, *89* (6), 2717–2749.

_ **and** _ , "Forward Guidance and Durable Goods Demand," *American Economic Review: Insights*, March 2022, *4* (1), 106–22.

Rouwenhorst, Geert, *Asset Pricing Implications of Equilibrium Business Cycle Models'*, Vol. Chapter 10, Frontiers of Business Cycle Research, Princeton University Press, 1995.