# Marriage Market Equilibrium <br> and Bargaining in Marriage* 

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#### Abstract

This paper examines marriage market equilibrium when allocation is determined by bargaining in marriage rather than binding agreements between prospective spouses. We view sorting in the marriage market and allocation in marriage as a two-stage game in which prospective spouses, when they meet in the marriage market, foresee the allocation that would emerge from bargaining in marriage, but are powerless to alter it. Our analysis is consistent with a wide range of noncooperative and cooperative bargaining models as well as with Becker's altruist model, in which one spouse has the power to impose his or her preferred allocation. Our approach is in contrast to the now-standard Beckerian assumption that prospective spouses, when they meet in the marriage market, make binding agreements about allocation in marriage. With binding agreements in the marriage market, the appropriate mathematical framework for analyzing the marriage market is the Koopmans-Beckmann assignment model and marriage market equilibrium is Pareto efficient. With bargaining in marriage, the appropriate mathematical framework for analyzing the marriage market is the Gale-Shapley matching model. Even if bargaining leads to Pareto-efficient allocations within each marriage, the marriage market equilibrium -- who marries, and who marries whom-- need not be Pareto efficient.

The inability of prospective spouses to make binding agreements in the marriage market about allocation in marriage may explain behavior that otherwise appears anomalous. The standard Beckerian marriage market model predicts that in equilibrium there cannot be both unmarried men and unmarried women. Yet what demographers call the "European marriage pattern" is characterized by high rates of non-marriage for both men and women.


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We model sorting in the marriage market and allocation in marriage as a two-stage game. The first stage, the marriage market, determines who marries, and who marries whom. The second stage, bargaining in marriage, determines allocation within each marriage. Our analysis is consistent with a wide range of household allocation models, including noncooperative and cooperative bargaining models of marriage, Chiappori's collective approach, and Becker's altruist model. We assume that prospective spouses, when they meet in the marriage market, foresee the allocation that would emerge (e.g., from Nash bargaining) but are powerless to alter it.

Our critical premise is the inability of individuals to make binding agreements before marriage about allocation during marriage. This implies that, with transferable utility and no search frictions, an appropriate framework for analyzing marriage market equilibrium is the Gale-Shapley (1962) matching model which they introduced and analyzed in "College Admissions and the Stability of Marriage." We show that, without binding agreements in the marriage market, the implied marriage market equilibrium need not be Pareto efficient. The intuition is simple: even if marriage is productivity-enhancing, an individual may prefer to remain unmarried rather than enter a marriage in which his or her spouse has too much bargaining power.

The standard assumption in the marriage market literature is that prospective spouses, when they meet in the marriage market, can make binding agreements about allocation in marriage. Becker $(1973,1991)$ was the first to recognize that, with binding agreements in the marriage market, the appropriate framework for analyzing marriage market equilibrium is the Koopmans-Beckmann (1957) assignment model. The behavioral foundations of the Choo and Siow (2006) marriage matching functions also assume that all match-specific returns are divided at marriage. With binding agreements in the marriage market, transferable utility, and no search frictions, the implied marriage
market equilibrium is Pareto efficient. Iyigun and Walsh (2008) and Chiappori, Iyigun, and Weiss (2008) also propose and analyze models in which a dense and frictionless marriage market and transferable utility ensures that the equilibrium matching of men and women maximizes aggregate marital surplus.

We depart from the standard Beckerian assumption of binding agreements in the marriage market but, unless otherwise stated, we retain the assumptions of transferable utility and no search frictions. Instead of binding agreements in the marriage market, we assume that men and women in the marriage market know that, after they have married, they will play a well-specified bargaining game with their spouse that will divide up the realized surplus from the marriage. If the partners do not have the option of costless divorce and costlessly reentering the marriage market, the marriage market equilibrium corresponding to bargaining in marriage need not coincide with the marriage market equilibrium corresponding to binding agreements in the marriage market. Although we refer to this second-stage process as a "bargaining game," we construe bargaining broadly enough to include Becker's altruist model, as well as a wide range of noncooperative and cooperative bargaining models. We do require that the bargaining game have a unique equilibrium, because we need the equilibrium of the second-stage bargaining game to analyze the first-stage marriage market game: the equilibrium of the bargaining game determines each individual's utility payoff in each possible marriage. ${ }^{1}$ We begin by discussing non-marriage. The standard Beckerian marriage market model predicts that in equilibrium there cannot be both unmarried men and unmarried women, yet the "European marriage pattern" is characterized by the presence of both unmarried men and unmarried women, as is the African-American marriage market. In section 2 we discuss the Beckerian marriage market model. In section 3 we discussion allocation in marriage, emphasizing

[^0]the role of bargaining in marriage. In section 4 we examine the implications of bargaining in marriage for sorting in the marriage market. Section 5 is a brief conclusion.

## 1. Marriage Markets and the Prevalence of Non-marriage

Individuals marry because they expect to be better off married than single. Economic models posit a number of different sources of gains to marriage, including returns to specialization and exchange within the household and complementarities between men and women in production. If the output of a married couple household is greater than the sum of the outputs of single male and female households, and if the spouses can divide this output so that each partner receives part of the marital surplus, then this simple model predicts "universal marriage." Universal marriage implies that the maximum number of (monogamous) marriages are formed: with equal numbers of men and women, everyone marries. With unequal numbers, the number of marriages is equal to $\min \left\{\mathrm{N}_{\mathrm{m}}\right.$, $\mathrm{N}_{\mathrm{f}}$, where $\mathrm{N}_{\mathrm{m}}$ and $\mathrm{N}_{\mathrm{f}}$ denote the number of males and the number of females in the marriage market. Thus, the standard Beckerian marriage market model predicts that in equilibrium there cannot be both unmarried men and unmarried women.

Yet demographers have identified populations with substantial numbers of unmarried man and unmarried women. Hajnal (1965) describes the marriage pattern that persisted in Europe for two centuries up to 1940: "The distinctive marks of the 'European' pattern are (1) a high age at marriage and (2) a high proportion of people who never marry at all (p. 101)." Our concern is with the second feature identified by Hajnal: a high proportion of people who never marry. Hajnal himself acknowledged that the marriage pattern he identified might more accurately be described as the "Western European pattern." He writes: "The European pattern extended all over Europe to the west of a line running roughly from Leningrad (as it is now called) to Trieste." Coale and Treadway (1968) accept Hajnal's characterization of the European marriage pattern: "As John Hajnal (1965) showed, late marriage and a high proportion still single at age 50 have long been characteristics of
populations west of an imaginary line from Trieste to St. Petersburg (p. 47)." Lee and Feng (1999) contrast the Chinese marriage pattern with the European marriage pattern: "While 15 percent of Western females continue to remain unmarried at age 40, only 1 percent of Chinese females are unmarried at age 30 (pp. 67-68)." ${ }^{2}$ Many developed countries have experienced falling marriage rates and increased propensities to divorce in recent years, and this has been attributed to decreases in returns to marriage. Increases in the relative wages of women and changes in the technology of household production, for example, have reduced the benefits of household gender specialization. ${ }^{3}$ However, the inability of prospective spouses to make binding agreements in the marriage market about allocation in marriage may also explain changes and variations in marriage patterns that otherwise appears anomalous.

In Lundberg and Pollak (2007, p. 16), we suggest that the decline in marriage among lower socio-economic groups in the United States may be related to the weakening of traditional gender roles and the inability of individuals to make binding agreements in the marriage market. More specifically, the inability of prospective spouses to make binding agreements in the marriage market about their behavior during marriage (e.g., regarding sexual fidelity, the sharing of child care responsibilities, or expenditures on children) may explain both the European marriage pattern and more recent retreats from marriage. Incomplete contracting may be a particular problem in societies in which traditional gender norms (e.g., regarding the husband's role as the primary earner and the wife's role as the primary provider of child care) are eroding. If gender norms are sticky and lag behind changes in the relative earning capacities of men and women, then the retreat from marriage may be a disequilibrium phenomenon.

For example, Sevilla-Sanz (2005) focuses on countries with conservative gender norms in
${ }^{2}$ Table 2, page 68, shows the percentage of Chinese women never married by age 30 for various locations from the 17 th century to the present. The percentages range from 0 to 4 percent.
${ }^{3}$ Stevenson and Wolfers (2007) summarize recent changes in marriage and divorce in the U.S.
which women's education levels and wages have risen rapidly, so that household efficiency dictates a departure from the traditional division of child care and other household tasks. She argues that a weakening consensus regarding appropriate gender roles, together with the inability of prospective spouses to make binding agreements in the marriage market about allocation in marriage, is responsible for low marriage rates in Japan and Southern Europe. ${ }^{4}$

The African-American marriage market, in which many adults of both sexes remain unmarried, provides another example. Current Population Survey data indicates that, in 2007, 10 percent of white women aged 40 to 44 and 15 percent of white men had never married. In the same age group, 31 percent of black women and 30 percent of black men have never married. The reasons for the retreat from marriage and the increase in nonmarital fertility among blacks has been discussed and debated since the Moynihan (1967) report, "The Negro Family: The Case for National Action."5 6

Other social science research supports the notion that contracting problems may prevent marriages from occurring. Furstenberg (2001) points to a breakdown in consensus regarding appropriate gender roles as a barrier to marriage, particularly for African-Americans. Edin and
${ }^{4}$ An alternative explanation is that any set of agreements that would make marriage attractive to young women would make it unattractive to young men. If this is the case, then low marriage rates reflect an empty bargaining set, rather than the inability of prospective spouses to make binding agreements.
${ }^{5}$ See, for example, Tucker and Mitchell-Kernan (1995).
${ }^{6}$ More generally, the recent retreat from marriage and the increase in nonmarital fertility, concentrated among individuals with low educational attainment, also involves unmarried adults of both sexes. Willis (1999) and Neal (2004) propose models to account for this pattern. In our view, it would be easier to construct models that predict the opposite pattern -- a retreat from marriage and an increase in nonmarital fertility concentrated among individuals with high rather than low educational attainment. Such a model would emphasize both the greater ability of high-education women to provide for themselves and their children, and the greater need of low-education women to take advantage of economies of scale in consumption, living arrangements, and the sharing of child care.

Kefalas (2005, p. 118) report that poor women fear that marriage "activates traditional gender roles" and so choose to maintain their independence by not marrying the fathers of their children.

Ethnographic studies of unmarried mothers in the Fragile Families Study identify lack of trust and, in particular, female fear of male infidelity, as a major barrier to marriage (Gibson-Davis, Edin, and McLanahan, 2005). These reports suggest that, given community norms and peer effects on behavior, low-income unmarried parents are unable to negotiate and enforce agreements involving legal marriage that would make both parents better off than remaining unmarried.

## 2. The Beckerian Marriage Market Model

Following Becker and most of the subsequent marriage market literature, we assume that each possible marriage produces a homogeneous, transferrable commodity which determines the resource constraint that the couple would face if they were to marry. We denote the output produced by the couple consisting of man i and woman j by $\mathrm{a}_{\mathrm{ij}}$. The output produced by man i if he remains unmarried is $a_{i 0}$ and woman $j$ 's output if she remains unmarried is $a_{0 j}$. The corresponding utilities are $\mathrm{u}_{\mathrm{i} 0}$ and $\mathrm{u}_{\mathrm{j} 0}$.

We also assume that, in terms of production, marriage dominates remaining single. Becker writes, "Since the complementarity between men and women and the differences between their comparative advantages imply that both men and women are better off married, the row and column giving single outputs can be ignored and attention focused on the Nx N matrix of marital outputs (p. 110)." ${ }^{7}$ The "production dominance" assumption is

$$
a_{i j} \geq a_{i 0}+a_{0 j} \quad \text { for all } i, j
$$

where the difference between marital output and the sum of single outputs is the "surplus" associated

[^1]with marriage.
As we have suggested, Becker's conclusion that we can ignore the single outputs because "both men and women are better off married," requires not only the production dominance assumption, but also an assumption about the process that determines allocation within marriage. The standard assumption is that prospective spouses, when they meet in the marriage market, can make binding agreements about allocation in marriage such that each receives a share of the marital surplus.

Becker and Murphy (2000, Chapter 4) provide a clear statement of the requirement that spouses make binding agreements in the marriage market. In a section entitled "Equilibrium Sorting with Flexible Prices" they write: "In this situation, each person's utility function is assumed to depend only on his or her own marital income, and the marriage market allows ... [prospective spouses] to bid for different spouses by offering a larger or smaller share of the output they would produce together (p. 31)." In the Treatise (Chapter 4) Becker emphasizes that his marriage market analysis depends on the assumption of "flexible prices," and contrasts his analysis to that implied by the contrary assumption that the division of outputs is "inflexible" (p. 126) or "rigid" (p. 133). Thus, Becker's statement that "The analysis of equilibrium sorting developed in his chapter has assumed that all divisions of outputs between mates are feasible (p. 126)" is as an assumption about the ability of prospective spouses to make binding agreements in the marriage market about allocation within marriage.

Becker's marriage market model is essentially the Koopmans-Beckmann assignment model. Koopmans and Beckmann (1957) analyze the problem of a manager or social planner assigning industrial plants to locations so as to maximize total output or profit. The statement of the problem presupposes that the plants produce a homogeneous output or that profits are expressed in comparable units, so that maximization is meaningful. Applying the model to marriage requires a homogeneous output or commodity that can be transferred from one spouse to the other within the
household. Becker assumed "transferable utility," a simplifying assumption about preferences often made by economists and game theorists. ${ }^{8}$ Although transferable utility is a highly restrictive assumption, it is not the most problematic of the assumptions needed to justify the appeal to the Koopmans-Beckman assignment model. More dubious, in our opinion, is the assumption is that prospective spouses can make binding agreements about allocation in marriage.

We can interpret the marriage market analysis corresponding to the assignment model in three distinct ways. First, we can interpret it as a description of the maximizing behavior of an omniscient planner who assigns husbands to wives. ${ }^{9}$ The maximizing omniscient-planner interpretation avoids the need to discuss the choices made by men and women in the marriage market because they make no choices; they simply follow the planner's instructions. Second, we can reinterpret the assignment model as a description of equilibrium in a competitive marriage market in which prospective spouses meet each other instantaneously, negotiate binding agreements about allocation within marriage, and receive the utility payoffs corresponding to these allocations. Koopmans and Beckmann show that the output-maximizing pattern corresponding to the assignment model coincides with the equilibrium in a competitive market, where the prices correspond to "imputations" in the dual of the planner's maximization problem. These imputations can be interpreted as competitive equilibrium prices or shares, provided that prospective spouses can make binding agreements in the marriage market about allocation within marriage. When the distribution of characteristics among marriage market participation is "dense," then these imputations are unique; in general, however, they are not unique. Third, we can interpret the assignment model as a description of equilibrium in a competitive market in which spouses bargain during marriage, but

[^2]bargain in the shadow of costless divorce and costless reentry into the marriage market. By costless reentry into the marriage market, we mean that prospective spouses do not distinguish between previously-married individuals and never-married individuals. ${ }^{10}$ Becker contrasts the assumptions that support the assignment model of marriage to those underlying the Gale-Shapley matching model, writing: "These models...assume...that the division of outputs in any marriage is not determined by the marriage market and is completely rigid (p. 127)." Thus, Becker suggests a dichotomy: the division of outputs (i.e., allocation in marriage) is determined in the marriage market or, alternatively, the division of output is "completely rigid." This dichotomy is false. Becker's altruist model provides an example of a specification in which the division of output is neither "determined in the marriage market" nor "completely rigid." In the altruist model, one spouse, the altruist, has the power to impose his or her preferred allocation, subject to participation constraints. Other specifications in which allocation within marriage is neither determined in the marriage market nor completely rigid include virtually all noncooperative and cooperative bargaining models, including Nash bargaining. Yet Becker's examples in which "the division of output in any marriage were determined not in the marriage market but in other ways" assumes that "a person would receive the same fraction of output of all possible matches" (p. 126)."

Contrary to the assumption that prospective spouses make binding agreements in the marriage market, suppose that, as in Becker's altruist model, one spouse -- for definiteness, the husband -- has the power to determine allocation within marriage. In the altruist model, the husband need not give his wife more than her reservation level of utility -- the amount she would receive if she were to leave the marriage and divorce him. If the husband determines allocation in marriage subject to his wife's participation constraint, then the wife's utility in marriage depends on the couple's resource constraint ( $\mathrm{a}_{\mathrm{ij}}$ ), on the husband's preferences (e.g., how "altruistic" he is), and on

[^3]the wife's participation constraint.

## 3. Allocation in Marriage

Instead of introducing special notation for the altruist model, we introduce general notation that allows us to analyze marriage market equilibrium in any model in which bargaining in marriage determines allocation in marriage. ${ }^{11}$ We let S denote the "bargaining set" in the utility space. That is,

$$
\left(\mathrm{u}_{\mathrm{i}}, \mathrm{u}_{\mathrm{j}}\right) \in \mathrm{S}_{\mathrm{ij}}
$$

means that the utility pair $\left(u_{i}, u_{j}\right)$ is feasible in the marriage of man $i$ and women $j$ and that $u_{i}$ and $u_{j}$ satisfy the participation constraint. That is, for all

$$
\left(\mathrm{u}_{\mathrm{i},}, \mathrm{u}_{\mathrm{j}}\right) \in \mathrm{S}_{\mathrm{ij}}
$$

$\mathrm{u}_{\mathrm{i}} \geq \mathrm{u}_{\mathrm{i}-}$ and $\mathrm{u}_{\mathrm{j}} \geq \mathrm{u}_{\mathrm{j}-}$
where $\mathrm{u}_{\mathrm{i}-}\left(\mathrm{j}_{\mathrm{j}}\right)$ is the utility of man i (woman j ), if he (she) were to leave the ij marriage. If remarriage is impossible, then $u_{i-}$ and $u_{j-}$ represent the utilities associated with divorce and remaining unmarried forever. If remarriage is possible, then $u_{i-}$ and $u_{j-}$ reflect, with appropriate probability weights, the utility of remaining unmarried forever and the prospects of man i and woman j in the remarriage market.

Most bargaining models contain parameters that determine the "bargaining power" of each spouse; we denote these parameters by $\pi$. An "allocation rule," $\mathrm{G}(\mathrm{S}, \pi)$, maps $(\mathrm{S}, \pi)$ into an element of the bargaining set, S . That is, the allocation rule gives the utility pair implied by the bargaining model, given the bargaining set and the bargaining power of the spouses. We denote the corresponding utilities by
$\left(\mathrm{u}_{i^{*}}, \mathrm{u}_{\mathrm{j}^{*}}\right)$

[^4]$$
\left(\mathrm{u}_{\mathrm{i} *}, \mathrm{u}_{\mathrm{j}^{*}}\right)=\mathrm{G}^{\mathrm{ij}}\left(\mathrm{~S}_{\mathrm{ij}}, \pi\right)
$$
or, dropping some of the subscripts and superscripts
$$
\left(\mathrm{u}_{\mathrm{i}^{*}}, \mathrm{u}_{\mathrm{j}^{*}}\right)=\mathrm{G}(\mathrm{~S}, \pi) .
$$

For example, in the case of the altruist model, the allocation rule selects the point in the bargaining set that maximizes the altruist's utility; the requirement that the point selected lie in the bargaining set (rather than merely in the feasible set) implies that the allocations selected satisfy the participation constraint of the nonaltruist spouse.

The allocation rule allows us to finesse some but not all of the difficulties of modeling bargaining in marriage when our objective is to analyze marriage market equilibrium. We have adopted and adapted the idea of an allocation rule from Chiappori $(1988,1992)$ who proposed a "sharing rule." Chiappori's sharing rule is a Pareto efficient allocation rule which specifies the utility of the two spouses as a function of variables assumed to determine their bargaining power.

Allocation rules are especially useful in analyzing two-stage games because they allow us to analyze the first-stage game without getting bogged down in the details of the second-stage game. ${ }^{12} \mathrm{By}$ beginning with an allocation rule, we avoid not only the need to specify the bargaining game, but also the need to specify whether the game is cooperative or noncooperative. If bargaining in marriage is a cooperative game, then the implied allocation rule is Pareto efficient. If bargaining in marriage is a noncooperative game, then the implied allocation rule may or may not be Pareto efficient.

## 4. Sorting in the Marriage Market

Using the allocation rule, we now analyze marriage market equilibrium under the

[^5]assumption that allocation in marriage is determined by bargaining during marriage. In Becker's terminology, any allocation rule, including the allocation rule implied by the altruist model, implies a division of outputs in marriage that is, from the standpoint of prospective spouses in the marriage market, "inflexible" or "rigid." If man i and women $j$ marry, their utility payoffs are $\left(u_{i}{ }^{*}, u_{j^{*}}\right)=G(S$, $\pi$ ). Hence, the appropriate mathematical model for analyzing marriage market equilibrium is the Gale-Shapley matching model, not the Koopmans-Beckmann assignment model.

In a paper famously entitled "College Admissions and the Stability of Marriage," Gale and Shapley (1962) first proposed and analyzed matching models. ${ }^{13}$ Matching models of the marriage market assume that each man has a ranking of all the women, and that each woman has a ranking of all the men; remaining unmarried is treated as an additional alternative ranked by men and by women. Most matching models treat these rankings as primitives, reflecting individuals' idiosyncratic preferences. We depart from this treatment and assume instead that each individual's ranking of prospective spouses reflects the utility of the allocation that marriage market participants foresee resulting from bargaining within marriage.

The equilibrium concept for matching models, a "stable matching," is defined by two properties: (i) no married individual prefers being unmarried to his or her current assignment and (ii) no two individuals of opposite sexes prefer being married to each other to their current assignments (e.g., being married to their assigned spouse or being unmarried). Under relatively weak assumptions at least one stable matching exists. ${ }^{14}$ We assume that the stable matching is unique, although uniqueness requires strong assumptions. ${ }^{15}$

Thus, except in special cases, the standard Beckerian marriage market model is incompatible

[^6]with bargaining models of marriage, including divorce threat and separate spheres bargaining models, and also incompatible with Becker's altruist model. The leading special case in which binding agreements in the marriage market and bargaining in marriage imply the same marriage market equilibrium is costless divorce and costless reentry into the marriage market (i.e., prospective spouses do not distinguish between previously-married individuals and never-married individuals). ${ }^{16}$

Despite the prevalence of divorce and remarriage, costless divorce and costless reentry into the marriage market are implausible assumptions. An argument can be made that these assumptions are plausible for "starter marriages." Paul (2002) introduced the term in her book Starter Marriage and the Future of Matrimony; she defines a starter marriage as a first marriage lasting five years or less and ending without children. For divorced individuals with children -- that is, for fathers with child support obligations and mothers with custody of children -- the assumption of costless reentry into the marriage market is implausible. With costly divorce or costly reentry into the marriage market, an unmarried individual might reject marriage and remain single, even thought the same individual, if he or she were married, would prefer to remain married rather than divorce. This may be the case if divorce is costly or the status of divorced individuals (taking account of their remarriage prospects), is worse than that of never married individuals. ${ }^{17}$ Without costless divorce and costless reentry into the marriage market, different participation constraints are relevant for the decision to enter a marriage than for the decision to exit a marriage.

When the ex ante and ex post participation constraints on marital bargaining differ, and binding agreements in the marriage market are not possible, it is easy to construct examples with

[^7]both unmarried men and unmarried women in marriage market equilibrium. Indeed, we can construct examples in which no marriages take place even though, were individuals married, they would choose to remain married. More specifically, suppose that all men are identical, all women are identical, and that there are equal numbers of men and women. Suppose further that husbands have dictatorial power within marriage, and that husbands place little weight on the well-being of their wives. Finally, suppose that divorced women are much worse off than never-married women. Under these assumptions, women refuse to marry, and men derive no benefit from having dictatorial power in marriage because women refuse to marry them. Because prospective husbands lack the ability to make binding agreements, they cannot commit to refraining from exploiting their bargaining advantage even though doing so would be in their interest.

We now develop a simple numerical example based on divorce-threat Nash bargaining. Suppose each individual's von Neummann-Morgenstern utility function is equal to his or her consumption of a single private consumption good. (This implies the absence of household public goods; we assume that marriage is motivated by production efficiency.) Let the KoopmansBeckmann solution require that man $i$ marry woman $j$, where this marriage produces an output of $\mathrm{a}_{\mathrm{ij}}=24$. Suppose that, in the event of divorce, man $i$ produces $\mathrm{a}_{\mathrm{i}-}=12$ and woman $j$ produces $\mathrm{a}_{\mathrm{j}-}=4$. Hence, the marriage produces a surplus of 8 relative to divorce. We emphasize that, for divorce threat Nash bargaining, the surplus relative to divorce determines the bargaining set and bargaining power. Nash bargaining divides the surplus (8) equally between the spouses, so that the husband receives $12+4=16$ and the wife receives $4+4=8$. We now consider the conditions under which this allocation is consistent with man i and women j agreeing to marry each other. Recall that, if
man $i$ remains unmarried, he produces $\mathrm{a}_{\mathrm{i}}$; if woman j remains unmarried, she receives $\mathrm{a}_{0}$. Hence, a necessary condition for the spouses to enter the marriage is

$$
16 \geq \mathrm{a}_{\mathrm{i} 0} \text {. and } 8 \geq \mathrm{a}_{0 \mathrm{j}}
$$

${ }^{18}$ If $\mathrm{a}_{0 \mathrm{j}}>8$, then the marriage to man i is not individually rational for woman j . In this case, divorced women, due perhaps to poor remarriage prospects, are substantially worse-off than never-married women. Given the number in our example, for marriage to be ex ante attractive to women, they must receive more than half of the marital surplus. Since allocation within marriage is determined by divorce-threat Nash bargaining, women know they will receive only half, and therefore no marriages will take place.

A similar difficulty arises with the altruist model. Suppose that the altruist's "social welfare function" is Cobb-Douglas

$$
U_{i}=a_{i}^{\beta} a_{j}^{1-\beta}
$$

where $\beta=2 / 3$. He will therefore divide the output of the marriage so that he gets $2 / 3$ and his wife gets $1 / 3$. Suppose, as above, that the marriage produces an output of 24 , implying that the altruist would allocate 16 to himself and 8 to his wife. Now consider the (prospective) wife's decision in the marriage market. If she marries man j , she will receive 8 . If she remains unmarried, she receives $\mathrm{a}_{0 \mathrm{j}}$. Hence, she is unwilling to enter the marriage if $\mathrm{a}_{0 \mathrm{j}}>8$.

These examples imply the need to distinguish carefully between at least three distinct notions of "surplus." (1) The surplus relative to divorce is relevant after the couple has married. In most

[^8]bargaining models of marriage, divorce is an outside option; in some bargaining models, it is also a source of "bargaining power" (e.g., in divorce threat bargaining). (2) The surplus relative to remaining single is relevant before marriage. Individual rationality implies that no individual will enter a marriage that leaves him or her worse off than remaining single. ${ }^{19}$ (3) The surplus relative to the next best marriage plays a role in a competitive marriage market. As individuals become "dense" in the space of characteristics, the surplus over the next best marriage approaches 0 .

## 5. Conclusion

Contrary to the assumption of the standard Beckerian marriage market model, prospective spouses do not make binding agreements in the marriage market. Instead, bargaining in marriage determines allocation in marriage. We interpret bargaining broadly enough to include Becker's altruist model in which one spouse, the altruist, has the power to impose his or her preferred allocation subject to the appropriate participation constraint.

We view sorting in the marriage market and allocation in marriage as a two-stage game. The first stage, the marriage market, determines who marries, and who marries whom. The secondstage, bargaining in marriage, determines allocation within each marriage. Given a specification of the bargaining model, we can use backwards induction to solve the two-stage game. Because we are primarily interested in marriage market equilibrium, we finesse the need to specify and solve a bargaining model by using an "allocation rule."

With binding agreements in the marriage market and no search frictions, the appropriate mathematical framework for analyzing the marriage market is the Koopmans-Beckmann assignment

[^9]model and the implied marriage market equilibrium is Pareto efficient. ${ }^{20}$ With bargaining in marriage, the appropriate mathematical framework is the Gale-Shapley matching model and marriage market equilibrium need not be Pareto efficient.

Unlike the standard Beckerian model, our two-stage model does not imply "universal marriage" and thus is compatible with the "European marriage pattern" and with the marriage pattern observed in the African-American community. More specifically, if prospective spouses can make binding agreements in the marriage market and if marriage "production dominates" remaining single (i.e., any couple produces more than the sum of their outputs when single), then marriage market equilibrium implies "universal marriage." (i.e., the maximum number of marriages will form, so that with equal numbers of men and women, everyone marries). In contrast, our two-stage model with spouses bargaining in marriage is compatible with the simultaneous presence of unmarried men and unmarried women in marriage market equilibrium.

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[^0]:    ${ }^{1}$ A mixed strategy equilibrium satisfies this requirement, but set-valued solution concepts such as the core do not.

[^1]:    ${ }^{7}$ Here Becker assumes equal numbers of men and women; he discusses unequal numbers later in the chapter. All page citations to Becker's Treatise on the Family are to the 1991 enlarged edition, although virtually all of the material we cite appeared in the 1981 edition and, before that, much of it in journal articles.

[^2]:    ${ }^{8}$ Bergstrom (1997) provides an excellent discussion of transferable utility.
    ${ }^{9}$ It makes no difference whether we describe the problem as assigning husbands to wives or wives to husbands.

[^3]:    ${ }^{10}$ Costless divorce and costless reentry is the interpretation of Iyigun and Walsh (2007).

[^4]:    ${ }^{11}$ Not quite any model; we require uniqueness.

[^5]:    ${ }^{12}$ In Lundberg and Pollak (2003) we use an allocation rule to analyze the "two-earner couple location problem." Pezzin, Pollak, and Schone (2007) use an allocation rule to analyze a two-stage game in which adult children make choices that affect the living arrangements of their disabled elderly parents.

[^6]:    ${ }^{13}$ Roth and Sotomayor (1990) provide the definitive exposition and analysis of matching models. ${ }^{14}$ More precisely, if each individual's preferences are an ordering, where preferences are defined over all members of the opposite sex, together with the possibility of remaining unmarried.
    ${ }^{15}$ Eeckhout (2000) establishes a sufficient condition for uniqueness in matching models.

[^7]:    ${ }^{16}$ Divorce threat" bargaining models assume neither costless divorce nor costless reentry into the marriage market.
    ${ }^{17}$ This can also be the case if marriage changes individuals' preferences or if marriage provides information (e.g., about the spouses' type). We focus, however, on costly divorce or costly reentry into the marriage market

[^8]:    ${ }^{18}$ The Nash bargaining solution, even if it satisfies these two conditions, need not satisfy the further restrictions on imputations implied by the Koopmans-Beckmann assignment model. Production dominance implies that $24 . \geq a_{i 0}+a_{0 j}$ so at least one of these two inequalities must hold.

[^9]:    ${ }^{19}$ This presupposes that prospective spouses, rather than their parents or other relatives, are the decision makers in the marriage market.

[^10]:    ${ }^{20}$ The analysis of the standard Beckerian marriage market model is relatively simple because, with very little analysis, we can identify which sex will be redundant in equilibrium and which individual of that sex will be the marginal unmarried individual. This simplification is not available when allocation in marriage is determined by bargaining in marriage.

