‘The Child is Father of the Man:’ Implications for the Demographic Transition

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Abstract

We propose a new theory of the demographic transition based on the evidence that body development during childhood is an important predictor of adult life expectancy. In an OLG framework, fertility, longevity and education all result from individual decisions. The model displays different regimes. In a Malthusian regime with no education and subsistence consumption, fertility increases with adult life expectancy. In a modern regime, life expectancy and fertility move in opposite directions due to the following mechanisms. Firstly, parents face a trade-off between the number of children they have and the spending they can afford on each of them in childhood, the latter determining children longevity. It is in this sense that we refer to Wordsworth's aphorism that “The Child is Father of the Man.” Parents face a second trade-off in allocating their time between increasing their own human capital and rearing children. The dynamics display the key features of the demographic transition, including the hump in both population growth and fertility, and replicate the observed rise in educational attainment and adult life expectancy. Consistent with the empirical evidence, a distinctive implication of our theory is that improvements in adult life expectancy precede the increase in education and the decline in fertility.

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1 Introduction

Providing children with appropriate hygienic conditions and good nutrition, as well as promoting good attitudes towards health during childhood are a very effective way of providing them a longer life.[1] Starting with Kermack, McKendrick, and McKinlay (1934), who showed that the first fifteen years of life were central in determining the longevity of the adult, the relationship between early development and late mortality within cohorts has been well-established. Another important contribution in the field is that of Barker and Osmond (1986) who related lower childhood health status to higher incidence of heart disease in later life. This idea also had an echo in the literary tradition as witnessed by the aphorism (with apologies to feminists) "The Child is Father of the Man" (Wordsworth 1802), meaning that the way a child is brought up determines what he or she will become in the future.

The main mechanism suggested in the literature to explain the link between childhood development and longevity is through improvements in nutrition and physiological status, as emphasized by Fogel (1994). Another mechanism stressed by epidemiologists links infections and related inflammations during childhood to the appearance of specific diseases in old age (Crimmins and Finch 2006). The contribution of this paper is to propose a new theory of the demographic transition based on the evidence that body development during childhood is an important predictor of life expectancy.

The key and novel mechanism we propose in this paper is that parents face a trade-off between the quantity of children they have and the amount they can afford to spend on each of them during childhood. Parents like to have children, but they also care about their longevity. By ensuring an appropriate physical development for their children and protecting them from infections, parents provide them with a longer life.[2] Such provision

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[1] A survey of the related epidemiology literature can be found in Harris (2001)).

[2] The state of the debate between nature and nurture may be synthesized by the following statement "The effects of genes depend on the environment," (Pinker 2004) where genes are associated with nature, and the environment as a short-hand for the effects of human behavior with nurture. In this paper, we are mainly interested in understanding how changes in human behavior, for a given distribution of genes, affect childhood development. In other words, by restricting the analysis to a representative cohort member, a standard assumption in OLG models, we do not pay much attention to within cohort differences in genes, but concentrate on how changes in nurture over time affect childhood development on average. The implicit assumption is that the complex interaction between nature and nurture shaping

is costly though, and its cost is increasing with the number of children. As a consequence, having many children prevents parents of spending much on their body development. The proposed quality/quantity trade-off makes longevity and fertility negatively related.

We are aware that longevity does not depend solely on childhood development. Adults' investment in health and government spending on the elderly also contribute considerably to reductions in mortality. However, adding these mechanisms into our setup would not alter the trade-off we want to put forward, and, therefore, we abstract from these additional mechanisms in order to streamline the argument.[3]

In this paper, the key mechanism relating demographics and education is the Ben-Porath hypothesis that longevity positively affects education, by extending an individual active life.[4] Following Boucekkine, de la Croix, and Licandro (2002), we assume that adults decide about their own optimal amount of education, and we take basic education, even if provided by parents, as being exogenously given. In addition to the trade-off between the number and development of children stressed above, adults face a trade-off between having children and improving their own education, which makes the number of children and schooling negatively related. This is similar to the trade-off faced by parents in a Beckerian world, where they care about the quantity and quality (education) of their offsprings. Letting parents to care also about the education of their children would certainty complicate the resolution of the model, obscuring the trade-off between the number of children and their childhood development, which is the key mechanism in our theory.

The dynamics of our model displays the key features of the demographic transition, including the hump in total net fertility rate and in population growth. In particular, it is able to replicate the observed rise in life expectancy and educational attainment, as well as the initial increase and then decline in fertility. If the mechanisms we describe predominate, the logic of the demographic transition could well be the reverse of that

the distribution of childhood development across cohort members in society has no significant effect on the mean over a period of time covering one or two centuries.

[3]We are also aware that there are at least two different types of health capital, as pointed out by Murphy and Topel (2006). One extends life expectancy so that individuals can enjoy consumption and leisure for longer; the other increases the quality of life, raising utility from a given quantity of consumption and leisure. In this paper, since we are mainly interested in longevity, we restrict the analysis to the first type of health capital.

[4]Conditions for this mechanism to hold in the presence of endogenous fertility are derived in Hazan and Zoabi (2006). In a recent controversial paper, Hazan (2006) argues that the Ben-Porath hypothesis holds only if longevity affects lifetime labor supply positively, which he claims is not consistent with US data. The main argument in our paper would also hold if, as in Hazan and Zoabi, childhood development affects education directly because healthier children perform better.

which is usually assumed: the key trade-off is not between fertility and education, with effects on longevity as a byproduct, but between fertility and longer living children, with a subsequent effect on education. This timing is evident in the data presented in the next section.

In our model, fertility, childhood development, longevity, and education all result from individual decisions. In this sense, this paper differs from the previous attempts in the literature to endogenize fertility and longevity simultaneously. Many papers have the standard education/fertility trade-off with exogenous longevity (for example, (Doepke 2004)). Other papers model health investment either by households (Chakraborty and Das (2005) and Sanso and Aisa (2006)) or by the government (Chakraborty (2004) and Aisa and Pueyo (2006)) but have exogenous fertility. A few treat both fertility and longevity as endogenous variables, but the mechanism leading to longer lives always relies on an externality: more aggregate human capital or more aggregate income leads to higher life expectancy (Blackburn and Cipriani (2002), Lagerloef (2003), Cervellati and Sunde (2005) and (2007) and Hazan and Zoabi (2006)).

Two recent papers have modeled the trade-off between the number and survival of children exclusively in the context of pre-modern societies. In Galor and Moav (2005)'s paper, there is an evolutionary trade-off (i.e. not faced by individuals but by nature), between the survival to adulthood of each offspring and the number of offspring that can be supported. Lagerloef (2007) suggested that agents chose how aggressively to behave, given that less aggressive agents stand a better chance of surviving long enough to have children, but gain less resources, so more of their children die early from starvation. In both cases, the trade-off is between the number and survival of children, while in our paper it is between the number of children and adult longevity.

This paper is organized as follows. Section 2 presents some evidence on the link between the improvement in childhood development and the demographic transition. In Section 3, we present the problem for individuals, and solve for the optimal allocation. Section 4 is devoted to the study of the dynamics of dynasties. Aggregate dynamics are studied in Section 5. A simulation of the demographic transition is proposed in Section 6. Implications for growth are studied in Section 7. Section 8 presents our conclusions.

2 Data on Childhood Development and Fertility

Height is a frequently used indicator in microeconomic studies of the relationship between health and income. Weil (2007) finds that the effect on wages of an additional centimeter of height ranges between 3.3% and 9.4%, depending on the data set used. In a second step, he exploits the correlation between height and direct measures of health such as the adult survival rate to evaluate health's role in accounting for income differences among countries; he finds that eliminating health variations would reduce world income variance by a third.

Height is a simple measure of childhood development, since both better nutrition and lower exposure to infections leads to increased height.[5] Height is constant after, say, the age of 18, but is still a good predictor of life expectancy and mortality in old age. According to Waaler (1984), the trend towards greater height found in the data means that younger cohorts, which have grown up with better nutrition, will develop better and live longer as adults.

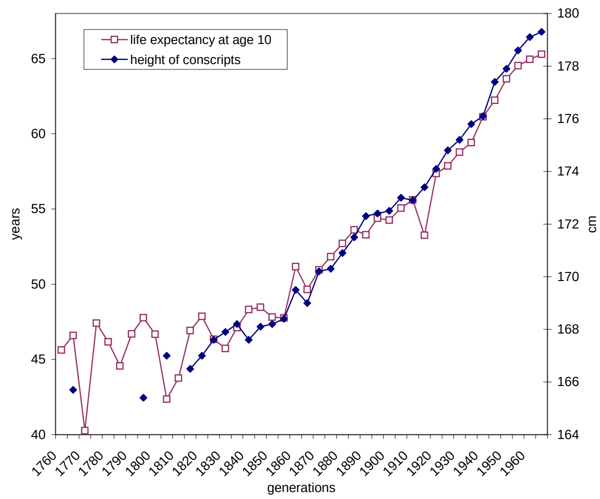
The height of conscripts has been systematically recorded by the Swedish army since 1820, which provides time-series information on changes in height throughout the demographic transition. Figure 1 presents data for the cohorts born between 1760 and 1960. The left panel shows that the height of soldiers (measured at approximately age 20) is highly correlated with the life expectancy of the same generation.[6] The right panel of Figure 1 shows that body height and years of schooling are positively correlated and, more importantly, that changes in height precede changes in education. Figure 2 completes the picture by reporting the net fertility rate for the same period. We observe that fertility and height are positively correlated over the period 1800-1830, uncorrelated between 1830 and 1900, and negatively correlated afterwards.

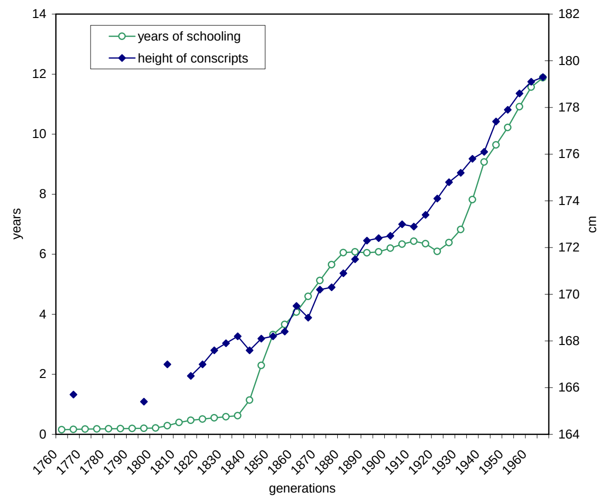
Further insights into the links between childhood development and fertility during the demographic transition can be gained by combining two data sets. Baten (2003) classified the former provinces of the German empire into six categories according to conscripts' height in 1906, i.e., for men born in the 1880s. In Figure 2 we retain the two extreme

[5] According to Silventoinen (2003), height is a good indicator of childhood living conditions (mostly family background), not only in developing countries but also in modern Western societies. In poor societies, the proportion of cross-sectional variation in body height explained by living conditions is larger than in developed countries, with lower heritability of height as well as larger socioeconomic differences in height.

[6] Notice that this strong correlation over time can also be established in a cross section of countries: Baten and Komlos (1998) regressed life expectancy at birth on adult height and explained 68% of the variance for a sample of 17 countries in 1860.

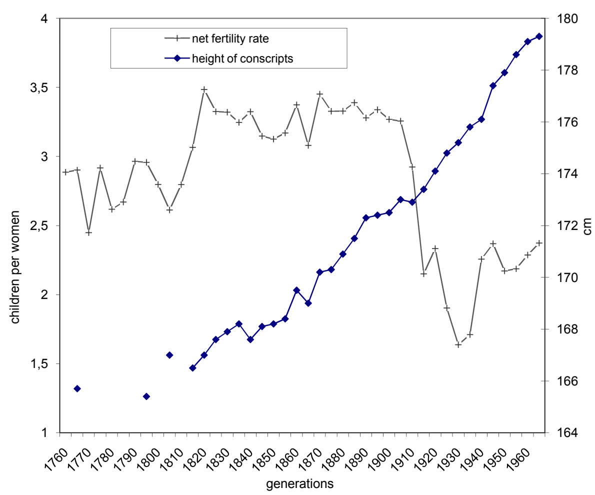
Figure 1: Height, Life Expectancy and Education in Sweden





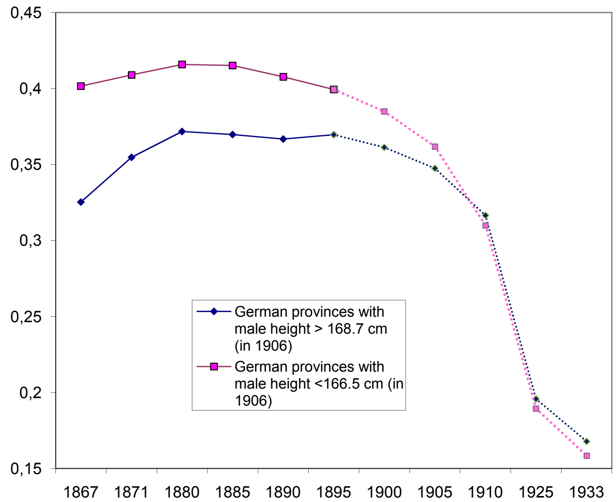
Sources: Sandberg and Steckel (1997) for height data from 1820; Floud (1984) for height data before 1820 from Denmark); The "Human Mortality Database" for life expectancy data; and de la Croix, Lindh, and Malmberg (2007) for education data.

Figure 2: Height and Net Fertility in Sweden



Net fertility is computed as the product of the fertility rate (Statistics Sweden) with the probability of survival until age 15 (Human Mortality Database).

Figure 3: Fertility Rates in Germany



Sources: Baten (2003) for height data; Knodel (1974) for fertility data. Fertility is the ratio of total fertility rate to a benchmark.

categories: provinces with the tallest (168.70 cm and more) and the shortest (166.50 cm and less) soldiers. The Princeton European Fertility Project provides information on fertility in these provinces for the years 1867-1933 (see Knodel (1974)). In the period 1870-1890, which is when the soldiers of 1906 were born, we can see that fertility rates were systematically higher in the provinces with shorter soldiers, which is consistent with the idea of a trade-off between the number of children and childhood development (as measured by adult height). Later on, fertility rates dropped and converged.

3 The Model: Individuals

Here we describe a continuous-time overlapping generations (OLG) model with endogenous fertility and mortality inspired by de la Croix and Licandro (1999) and Boucekkine, de la Croix, and Licandro (2002), who modeled the link between longevity and education in a framework where all the demographic variables are exogenous.

Let us denote by B the age of puberty, i.e., the age at which individuals acquire regular fertility. B is assumed to be constant. Individuals reaching puberty at time t are said to belong to cohort t, whose size is denoted by $P(t)$. Life expectancy at age B is denoted by $A(t)$, which is referred to below as life expectancy. We abstract from infant mortality, and assume that the survival law is rectangular, with mortality rates equal to zero for ages below $B+A$, and individuals dying with probability one at this age. Consequently, $B+A$ is life expectancy at birth. Consistently with the thesis developed by Aries (1962) according to which people become adults much earlier in old times and start making decisions themselves, we assume that choices are made by individuals reaching puberty.[7] Preferences are represented by (we have dropped the cohort index to ease the exposition)

$$\int_0^A c(t)\ \mathrm{d}t + \left(\beta \ln \hat{n} + \delta \ln \hat{A}\right). \tag{1}$$

We assume that individuals do not consume until they reach age B. $c(t)$ represents consumption at age $B+t$. Preferences in consumption are linear and the time preference parameter is assumed to be zero. Under this assumption, the equilibrium interest rate is zero and the marginal value of the intertemporal budget constraint, the associated Lagrange multiplier, is unity. In addition to their own consumption flow, individuals value the number of children, denoted by $\hat{n}$, as well as the life expectancy of their children, denoted by $\hat{A}$. Parameters β, $\beta > 0$, weight the marginal utility of children relative to adult consumption. The marginal utility of the quality of children is weighted by δ, $\delta > 0$.

Our assumption that utility is linear in consumption and logarithmic in children is highly consistent with the objective pursued by all living creatures. The first thing which matters is to survive. Imposing the constraint

$$C = \int_0^A c(t)\ \mathrm{d}t \geq 0$$

amounts to require that consumption is above the survival threshold; here 0 but could be easily generalized to any positive real number $\bar{C}$. Once this subsistence consumption is achieved, priority moves to reproduction. This is what quasi-linear preferences delivers. Reproduction covers two aspects: quantity of children and longevity. Both dimensions are important in order to optimize the transmission of genes. In nature, we observe different possible strategies; elephants favor quality of descendants, while jellyfishes favor

[7] Modeling family behavior is not a simple issue. As children grow, parents take child preferences into account more and more, but parents still have something so say as long as they support children financially until they find a job, leave home and become fully independent. Since modeling this complex process is beyond the objective of this paper, we assume that children become fully independent at one stroke.

quantity.

The technology producing human capital depends on the time allocated to education T:

$$h = \mu \left(\theta + T\right)^{\alpha}.$$

The productivity parameter μ and the parameter θ, which relates to schooling before puberty, are strictly positive, and $\alpha \in (0,1)$. θ ensures that human capital, and hence income, are positive even if individuals choose not to go to school after age B.

The budget constraint takes the form

$$\int_0^A c\,(t)\ \mathrm{d}t + \hat{n}\ \Psi(\hat{A}) = \mu \left(\theta + T\right)^{\alpha} \left(A - T - \phi \hat{n}\right). \tag{2}$$

The right hand side is the total flow of labor income. For simplicity, we assume that people have and raise their children immediately after finishing their studies and before becoming active in the labor market. This greatly simplifies the dynastic structure of the model. Raising a child takes a time interval of length $\phi > 0$, implying that individuals work for a period of length $A - T - \phi\hat{n}$. The wage per unit of human capital is unity. Parental expenditure on each child's development is

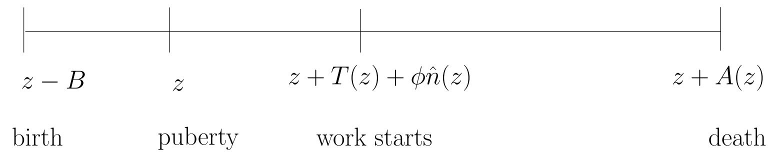
$$\Psi(\hat{A}) = \kappa\ \frac{1}{2}\frac{1}{A}\hat{A}^2, \tag{3}$$

which implies that the expenditure is quadratic in $\hat{A}$ and inversely related to A.[8] This formulation is consistent with the complex interaction between nurture and nature observed by biologists and psychologists. It stresses the difficulty of raising life expectancy above that of the parents, reflecting how the genes interact with human behavior (the environment) in building up a child body. The parameter $\kappa > 0$ measures the costs of developing children in a broad sense. Finally, note that the integral in Equation (2) may be substituted in Equation (1). The resulting objective function, depending on $\hat{A}$, $\hat{n}$ and T, is concave.

Figure 4 summarizes the life cycle of a representative individual of generation z, who borns at $z - B$, becomes independent at z, goes to school until $z + T$ and enters the labor market at $z + T + \phi\hat{n}$. Her children belong to generation $z + T + B$, since T is

[8] An example of a technology of childhood development is given by Dalgaard and Strulik (2006), where the metabolic energy to create a new cell is an exponential function of the body mass the individual wants to reach.

Figure 4: The life cycle



the age at which individuals have children, and children reach puberty after a period of length B. Individuals chose their own education T, the number of children $\hat{n}$ and the quality of children as measured by their life expectancy $\hat{A}$. Their choice depends on three types of parameters. First, those related to preferences, β and δ. Second, the parameters associated with child rearing and childhood development, ϕ, B and κ. Finally, the educational technology parameters θ, μ and α. The analysis of the demographic transition in Section 6 is built on letting some of these parameters change smoothly with time. To simplify the exercise, it will be assumed that all these parameters are cohort specific.

The maximization of utility (1), subject to the budget constraint (2) and to the positivity constraints $T \geq 0$ and $C \geq 0$, can be interior or corner. We make the following assumption about preferences:

Assumption 1 *Preferences satisfy $\delta < 2\beta$.*

Assumption 1 states that the preference weight attached to childhood development, δ, cannot exceed twice the weight attached to the number of children, β. The quantity/quality trade-off depends on the ratio of marginal utilities to marginal costs, which crucially depends on the factor two because of the quadratic form of the childhood development costs. A similar condition can be found in Moav (2005) and de la Croix and Doepke (2006), when parents face the standard fertility/education trade-off.

We make the following assumptions about education technology μ:

Assumption 2 *The productivity of education technology satisfies:*

$$\mu > \max\left[\frac{(\beta - \delta/2)\alpha^2}{\theta^{1+\alpha}(1+\alpha)}, \frac{\delta\alpha}{2\theta^{1+\alpha}}\right] \equiv \underline{\mu}.$$

This assumption requires the productivity coefficient μ to be large enough. Let us establish the main proposition on individual behavior.

Proposition 1 *Under Assumptions 1 and 2, there exist two thresholds* $\underline{A}$ *and* $\overline{A}$, $0 < \underline{A} < \overline{A}$, *such that:*
If $A \geq \overline{A}$, *there is a unique interior solution satisfying*

$$\hat{A}^2 = \frac{\delta}{\kappa \hat{n}} A, \tag{4}$$

$$T = \frac{\alpha}{1+\alpha}(A - \phi \hat{n}) - \frac{\theta}{1+\alpha}, \tag{5}$$

$$\hat{n} = \frac{\beta - \delta/2}{\mu \phi} (\theta + T)^{-\alpha}. \tag{6}$$

If $\underline{A} \leq A < \overline{A}$, *there is a unique corner solution with positive consumption satisfying*

$$\hat{A}^2 = \frac{\delta}{\kappa \hat{n}} A, \tag{7}$$

$$T = 0, \tag{8}$$

$$\hat{n} = \frac{\beta - \delta/2}{\mu \phi \theta^{\alpha}}. \tag{9}$$

If $0 < A < \underline{A}$, *there is a unique corner solution with zero consumption satisfying*

$$\hat{A}^2 = \frac{\delta}{\kappa} \frac{\mu \theta^{\alpha} \phi}{\beta - \delta/2} A, \tag{10}$$

$$T = 0, \tag{11}$$

$$\hat{n} = \frac{\beta - \delta/2}{\beta \phi} A. \tag{12}$$

Proof. Using the Kuhn-Tucker conditions for constrained optimization, we can identify the two thresholds $\underline{A}$ and $\overline{A}$ and characterize the different regimes. See Appendix A. ■

Restriction $A \geq \bar{A}$ in Proposition 1 states that parental life expectancy has to be large enough for schooling to be positive. At the interior solution, Equation (4) shows the trade-off faced by parents between the number and the life expectancy of their children. The relation is negative, since the total cost of providing children with a good body development increases as their number increases. Equation (5) is the standard Ben-Porath (1967) result, as described by de la Croix and Licandro (1999), where life expectancy positively affects the time allocated to education since it allows people to work for a longer time. The term $\phi \hat{n}$ in Equation (5) shows an additional trade-off of having children: parents expecting to have many children will postpone their entry into the labor market, reducing the incentives to take additional education. This trade-off also shows up in Equation (6).

When $\underline{A} \leq A < \overline{A}$, parental life expectancy A is not long enough to render optimal a positive investment in education. For lower levels of life expectancy, i.e. when $A < \underline{A}$, both education and consumption are zero.[9] Expected life time earnings are so low that parents use all their resources in bearing a limited number of children.

From now on, the interior solution, (4)-(6), and the corner solutions, (7)-(9) and (10)-(12), are referred to as $\hat{A} = f_A(A)$, $T = f_T(A)$ and $\hat{n} = f_n(A)$. The effect of an increase in A is given by Corollary 1:

Corollary 1 $f_A'(A) > 0$*;*

$f_n'(A) < 0$*, for* $A \geq \overline{A}$*,* $f_n'(A) = 0$*, for* $\underline{A} \leq A < \overline{A}$*, and* $f_n'(A) > 0$ *otherwise*

$f_T'(A) > 0$*, for* $A \geq \overline{A}$*, and* $f_T'(A) = 0$ *otherwise*

Proof. See Appendix A. ■

In the interior solution, increased life expectancy raises optimal schooling and human capital levels via the Ben-Porath effect. This increases the opportunity cost (time cost) of raising children. Hence, the optimal number of children drops as life expectancy increases.

In the corner solutions, since $T = 0$, a change in parental life expectancy does not affect education, canceling the Ben-Porath effect. In the corner regime (7)-(9) the number of children remains constant whatever the life expectancy, but childhood development is still positively affected by life expectancy, since the efficiency of body development activities depends positively on parental life expectancy. In the corner regime (10)-(12), when consumption C is zero, however, the effect of life expectancy on the quantity and quality of children reverses, since the number of children is directly determined by the $C = 0$ constraint, which allows for more children as life expectancy increases. This is a Malthusian effect. Consequently, for the study of the transition from Malthusian to modern growth in Section 6, it makes sense to initially put the economy into this last regime.

[9] Imposing a strictly positive minimum consumption level with $C \geq \bar{C}$ for the sake of realism would not change the results. The only difference is that $\underline{A}$ would be larger and increasing in $\bar{C}$.

4 Dynasties

Since individual decisions do not depend on aggregate variables, we can study the dynamics of life expectancy, fertility and education within dynasties separately, before the analysis of the aggregates.

Let us consider the dynamics of life expectancy first. At any point t, individuals reaching puberty belong to a representative dynasty with life expectancy $A(t)$. Let us denote as A_1 the life expectancy of the first generation of this dynasty. The operator f_A defines a difference equation governing the evolution of the dynasty's life expectancy since

$$A_{i+1} = \hat{A}_i = f_A(A_i),$$

where the index $i = 1, 2, 3, ...$, is associated with generations. From Proposition 1, for any initial value A_1 there exists a sequence of solutions $T_i = f_T^i(A_1)$, $\hat{A}_i = f_A^i(A_1)$ and $\hat{n}_i = f_n^i(A_1)$ for $i = 1, 2, 3, ...$, where X^i is the ith consecutive application of operator X. In the following, we characterize the dynamics of life expectancy. Once this has been done, the sequence $\{A_1, A_2, ...\}$ determines through the operators f_T and f_n the date and size of the following generations.

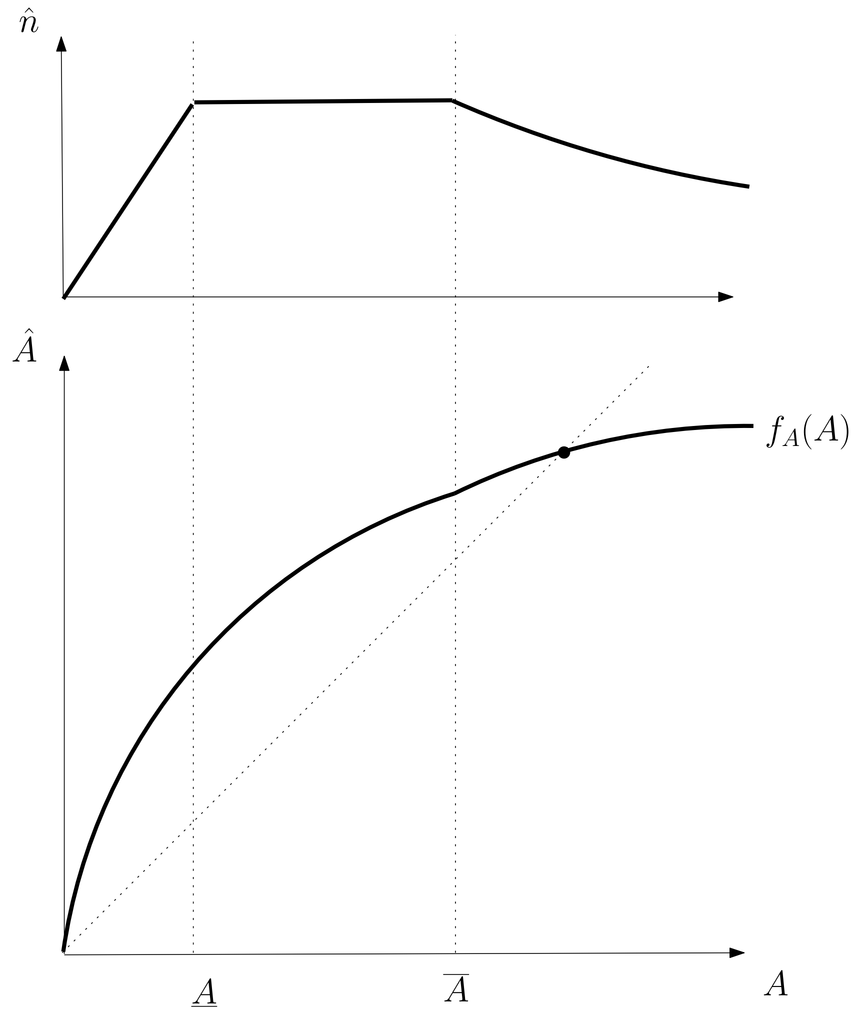
Proposition 2 *Under Assumptions 1 and 2, a stationary solution $A = f_A(A)$ exists, is unique and globally stable.*

Proof. See Appendix A. ■

Proposition 2 states that life expectancy converges to a constant value in the long run. Consequently, from Proposition 1, fertility and education also converge to a constant value. Demographic variables are then stationary, meaning that the demographic transition only occurs, as the name itself indicates, as a transitional phenomenon.

The results obtained so far allow us to assess some theoretical characteristics of the demographic transition in our model. Consider Figure 5. The lower panel plots the function $f_A(A)$. It describes a situation where the globally stable steady state is in the modern regime. The top panel shows fertility as a function of life expectancy. Suppose now that initial life expectancy is very low, below $\underline{A}$. The dynamics of life expectancy will be monotonic and converge to the steady state. A rise in life expectancy will first drive fertility up (as long as the economy is in the Malthusian regime $A < \underline{A}$), then fertility will peak in the zone where $\underline{A} \leq A < \overline{A}$ (i.e. where $T = 0$ but $C > 0$), and

Figure 5: A Steady State in the Interior Regime



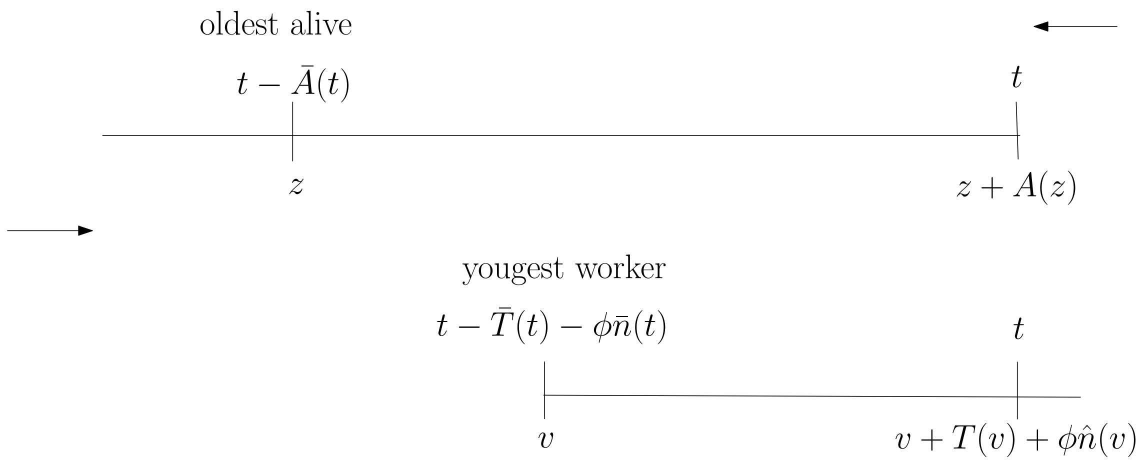
then decrease (in the interior regime). Schooling will be zero until we reach the modern regime and will then increase monotonically. This sharp characterization is very much in line with the stylized facts of the demographic transition as reported in Figures 1 and 2.

In this description of the theoretical dynamics, we have assumed a steady state in the interior regime. A condition for such a situation to occur is given by Proposition 3.

Proposition 3 *The steady state is in the modern regime if*

$$\kappa < \frac{4\alpha\delta\theta^{2\alpha}\mu^2\phi}{(2\beta-\delta)\left(\alpha[2\beta-\delta]+2\theta^{1+\alpha}\mu\right)} \equiv \overline{\kappa} \tag{13}$$

Figure 6: Life cycle (from left to right) and living cohorts (from right to left)



Proof. See Appendix A. ■

Condition (13) states that if the childhood development technology is cheap enough, there is an interior steady state with positive education. Changing the value of κ is a natural way of generating a demographic transition: if κ is initially high, the economy may be in one of the corner regimes. Once κ decreases, body development becomes more affordable, and the new steady state may move to the modern regime, with life expectancy converging monotonically.

5 Aggregates

Some definitions are useful to study the dynamics of the aggregate population, active population and human capital. In Figure 6, t and z represent time and cohort, respectively. Let us define $\tilde{A}\,(t)$ as the age of the oldest cohort still alive at time t, which then represents the life expectancy at time t of cohort $t - \tilde{A}(t)$. By definition, $A(z)$ is the life expectancy of cohort z. Then, given that generations z and $t - \tilde{A}(t)$ are the same, $A(z)$ has to be equal to $\tilde{A}(t)$. This is equivalent to introducing the variable change $z = t - \tilde{A}(t)$, implying that

$$\tilde{A}(t) = A\left(t - \tilde{A}\,(t)\right).$$

A similar argument applies to the functions $T(.)$ and $\hat{n}(.)$. Let us define $\tilde{T}\,(t)$ and $\tilde{n}\,(t)$ as the schooling time and the number of children of the youngest cohort entering the labor market at time t, i.e., cohort $v = t - \tilde{T}(t) - \phi\tilde{n}(t)$ in Figure 6. Since $\tilde{T}(t) = T(v)$ and $\tilde{n}(t) = \hat{n}(v)$,

$$\tilde{T}(t) = T(t - \tilde{T}(t) - \phi\tilde{n}(t)),$$

and

$$\tilde{n}(t) = \hat{n}(t - \tilde{T}(t) - \phi\tilde{n}(t)).$$

The total population is computed by integrating over all the living cohorts:

$$N(t) = \int_{t-\tilde{A}(t)}^{t+B} P(z)\ \mathrm{d}z, \tag{14}$$

from the oldest $t - \tilde{A}(t)$ to the youngest $t + B$. The cohort size $P(z)$ is given by

$$P(z + T(z) + B) = \hat{n}(z)\, P(z), \tag{15}$$

since members of cohort z have $\hat{n}(z)$ children at time $z + T(z)$, who belong to cohort $z + T(z) + B$.

The Balanced Growth Path

A balanced growth path is an equilibrium path where population grows at rate η and, the demographic variables T, n and A are all constant, as defined in Section 5. From Equation (15), the grow rate of cohorts' size is such that $\mathrm{e}^{\eta(T+B)} = n$, i.e.

$$\eta = \frac{\ln(n)}{T+B},$$

with $P(t) = P^{\star}\mathrm{e}^{\eta t}$, $P^{\star} > 0$. The population growth rate depends on the fertility rate n and on the age at child's birth $B + T$. At a given fertility rate, the smaller the age at birth, the larger the frequency of births and thus the growth rate of the dynasty.

Total population, as defined in Equation (14), evolves along a balanced growth path following

$$N(t) = N^{\star}\ \mathrm{e}^{\eta t} = P^{\star}\ \frac{\mathrm{e}^{\eta B} - \mathrm{e}^{-\eta A}}{\eta}\ \mathrm{e}^{\eta t},$$

with $N^{\star} > 0$. Population also grows at rate η and its size depends positively, as expected, on life expectancy. When η approaches zero, i.e., when population is constant, its size is given by $N(t) = P^{\star}(B + A)$, which is the product of the cohort size and life expectancy at birth. Along a balanced growth path, the active population is given by

$$E(t) = E^{\star} \, \mathrm{e}^{\eta t} = P^{\star} \, \frac{\mathrm{e}^{-\eta(T+\phi n)} - \mathrm{e}^{-\eta A}}{\eta} \, \mathrm{e}^{\eta t}.$$

Similarly as for total population, when η approaches zero $E(t)$ converges to $P^{\star}(A-T-\phi n)$, where the term in brackets is the length of active life.

6 Simulating the Demographic Transition

In this section, we investigate to what extent the trade-off between the number of children and their development can reproduce the key facts of the demographic transition. The transition is studied as the reaction to a change in the environment occurring after 1820 and leading the economy to a new balanced growth path characterized by longer lives. For this purpose, we implement a change in the parameter κ and analyze how the economy adjusts to this change, using numerical simulations.[10]

6.1 The Change in the Parameter κ

Before analyzing the demographic transition as a response to a change in parameter κ, we need to discuss briefly the interpretation to be given to such a change. Indeed, many would think that the period 1800-1870 does not contain major technology changes in health that could have increased life expectancy at puberty so much.

It is known that ancient ideas persisted a long time in modern Europe and the confidence of consumers in medicine was low. As a consequence some authors claim that the rise of life expectancy in early modern Europe did not relied on medical advances. Johansson (1999) argues against this therapeutic nihilism that tends to deny that medicine had any effectiveness before the end of the nineteenth century. First, in the period 1500-1800, medicine showed an increasingly experimental attitude: no improvement was effected on the grounds of the disease theory (which was still mainly based on traditional ideas), but significant advances were made based on practice and empirical observations. For example, although the theoretical understanding of how drugs work only came progressively in the nineteenth century with the development of chemistry (Weatherall 1996),

[10] A more sophisticated version of the model, in line with Galor and Weil (2000), would allow for an endogenous industrial revolution. This could be achieved by letting the body development technology, as measure by the inverse of κ, depend upon population size or density.

the effectiveness of the treatment of some important diseases was improved thanks to the practical use of new drugs coming from the New World. Second, the number of books containing lifestyle advice increasing significantly over the period 1750-1800, which provides some indirect evidence of the fact that lifestyle advice (concerning, for example, personal and domestic cleanliness) became popular. Third, Johansson (1999) reports that, as early as 1829, Dr.F.B. Hawkins wrote a book entitled *Elements of Medical Statistics*, in which he described what could be called an early modern epidemiological transition. He describes a set of diseases which were leading causes of death but can now (in 1829) be treated effectively: leprosy, plague, sweating sickness, ague, typhus, smallpox, syphilis and scurvy.

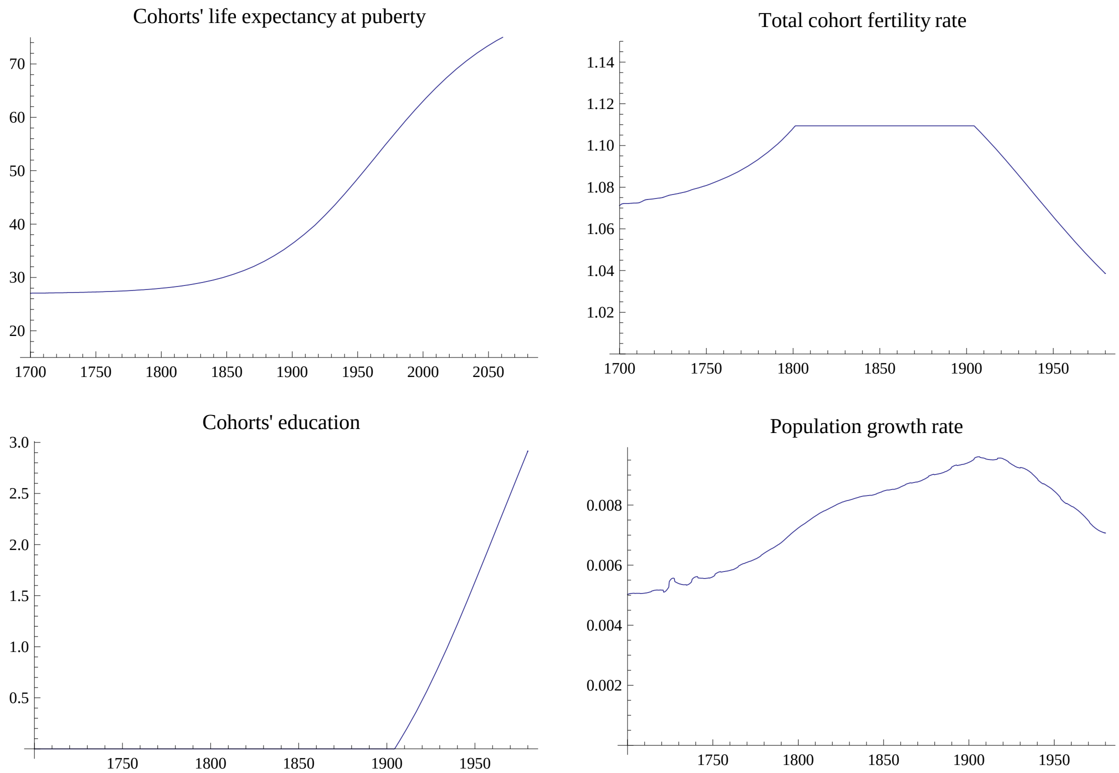
The cumulative effects of these improvements could have produced a net increase in the efficacy of medicine as early as in the eighteenth century (see de la Croix and Sommacal (2008) for further arguments).

6.2 The Demographic Transition over Time

A first set of parameters is set a priori. The age of puberty, B, is assumed equal to 13.5 (average between women, 12, and men, 15). The rearing cost per child, ϕ, is set to one year. The elasticity of human capital to schooling, α, is equal 1/6, which is a conservative value. Finally, the length of basic training, θ, is equal to six years. We next calibrate the remaining parameters to reproduce a steady state having the following properties in the pre-1820 balanced path: low life expectancy at age B ($A = 27$), no education after puberty ($T = 0$) and a population growth rate of 0.5% per year. We also set the parameters to obtain the thresholds $\underline{A} = 28$, which ensure that the economy is initially in the Malthusian regime and ends in the modern growth regime (interior regime). While μ, κ, δ and β have been computed to match the properties given above. This leads to the following results: $\mu = 0.7418$, $\kappa = 1.7954 \equiv \kappa_0$, $\delta = 53.7811$, and $\beta = 28$. With these values, notice that the life expectancy threshold leading from the corner regime with no education to the interior regime, $\overline{A}$, is 37.1095. Note also that for these values Assumption 1 holds. We can also compute the threshold for μ required by Assumption 2. It is equal to 0.5541, showing that Assumption 2 also holds in our example.

We assume that the Industrial Revolution produced a change in the cost of childhood development κ. We implement a drop in κ such that life expectancy is increased to 82.5 years at the new steady state. This requires us to divide κ by more than two, leading

Figure 7: Example of dynamics - drop in κ



to $\kappa = 0.6641 \equiv \kappa_1$ at the new steady state. We assume that this change takes place smoothly, following a logistic curve:

$$\kappa(t) = \kappa_0 + \frac{\kappa_1 - \kappa_0}{1 + e^{1890-t}/40}.$$

With such a function, 95% of the change takes place between 1792 and 2030.[11] We also assume that the parameter $\kappa(t)$ is specific to generation t. Hence any change only affects new generations, leaving past decisions unaffected.

Figure 7 depicts the simulation results.[12] We first observe that, following the drop in

[11]If, instead, the change were discrete, we would observe intervals of times with no births, corresponding to periods where everybody increases their length of schooling in a discrete way, giving rise to permanent replacement echoes which are typical of models with delays (Boucekkine, Germain, and Licandro 1997). In this case, the economy keeps fluctuating forever, moving from baby booms to baby busts. Non-monotonic convergence also occurs in the Galor and Weil model - see Lagerloef (2006).

[12]The simulation was performed using the method developed by Boucekkine, Licandro, and Paul (1997).

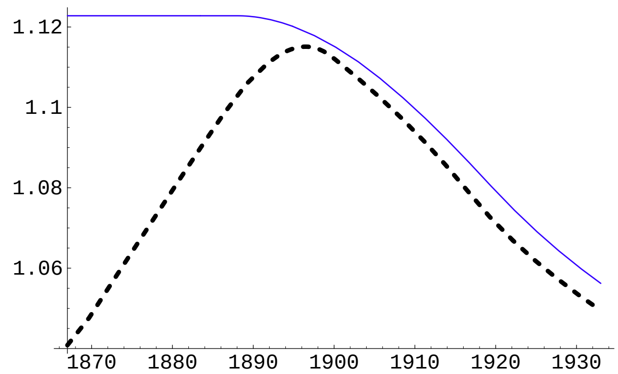
the cost of childhood development, life expectancy increases monotonically over time and converges to the new steady state, in accordance with the prediction of Proposition 2. Cohorts' education also increases monotonically, showing that the economy shifts from the Malthusian corner regime with no education to the interior regime with $T > 0$. Notice that the magnitude of the increase in T is about right, with schooling converging towards 3.5 years after puberty (13.5). Cohorts' fertility (per individual, to be multiplied by 2 to get fertility per women) first increases as long as the economy is in the Malthusian state, then peaks in the corner regime with positive consumption, then drops monotonically as a consequence of the trade-off between education and number of children in the interior regime. The interaction of fertility and longevity over time leads to a hump-shaped population growth rate, which is one important characteristic of observed demographic transitions.

6.3 Regional Variations

We conclude from the above simulation exercise that our model is able to reproduce the main features of the demographic transition. Another question is whether we can also shed some light on regional variations in the demographic transition. Considering the German data presented in Figure 3, we have seen that adult height (a proxy for childhood development) and fertility were negatively associated across provinces on the eve of the twentieth century. This is perfectly in line with the model when the economy is in the interior regime, i.e. in times of falling fertility. However, when fertility is rising, as it was the case in the 1860s in Germany, the economy is in the corner regime in which height and fertility are positively associated. It is thus not obvious *a priori* that the model can reproduce a world where shorter individuals have higher fertility at a time in when fertility is increasing.

One reason for different places exhibiting different fertility and height paths during the demographic transition is that the initial cost of body development could vary in different places, and the speed in the drop of the cost could be different. A pattern similar to that observed in Figure 3 can emerge if the place which initially had a higher cost κ benefited from a faster transition. Consider two economies with all parameters equal to those of the previous simulation except for κ. In place A, the initial κ is 10 % larger than in place B but it drops faster (the term e^{1840-t} is divided by 10 rather than by 5). The result of the simulation is shown in Figure 8. Fertility rates first rise then decline. There is initially a large gap between the two places. Place A, with the lower fertility

Figure 8: Total Fertility Rates in Places A (dashed) and B (solid)



in 1865-1895 also has higher childhood development in that period, as witnessed by the gap in life expectancy computed for the cohort reaching puberty in 1900 (born in 1885 and being in the military in 1906): life expectancy is 47.4 in place A and 45.2 in place B. Hence, both the cross-sectional and time-series aspect of the demographic transition can be captured if differential progress in body development technologies is allowed for.

7 Growth

The transition from a world of low economic growth with high mortality and high fertility to one with low mortality and fertility but sustained growth has been the subject of intensive research in recent years.[13] In this literature, the relation between growth and fertility results from the quantity/quality trade-off faced by parents between the number of children and their education. Indeed, the gradual increase in the observed level of human capital during the nineteenth century 'has led researchers to argue that the increasing role of human capital in the production process induced households to increase investment in the human capital of their offspring, ultimately leading to the onset of the demographic transition' (Galor 2005). If the rise is education was indeed driven by a stronger demand for skills from the industrial sector, one should have observed a rise in the skill premium during and following the industrial revolution.

[13] Rostow (1960) presents an early attempt to understand the transition from stagnation to growth. The first modern treatment of the issue is in the seminal paper by Galor and Weil (2000). Doepke (2006) contains a recent survey.

Looking for such evidence, Clark (2005) computes a skill premium over the period 1220-1990 in two different ways. First by measuring the relative wage of all skilled building workers [14] relative to all laborers and, second, by using only those observations in which there is a matched pair for the same place and year of wages for craftsmen and laborers. The two methods lead to the same conclusion: the skill premium did not rise during the Industrial Revolution. And Clark concludes that 'The market premium for skills, does not explain the increased investment in human skills evident after 1600.' Hence, we might wonder whether the human capital interpretations of the Industrial Revolution are based on the right trade-off. The new mechanism we develop above could be seen as an alternative to the usual one.

In order to introduce the possibility of growth into the model we first need to modify the technology producing human capital. It should now also depend on the average human capital per worker $\bar{H}$, reflecting for example the quality of the teacher or the cultural ambience in the society:

$$h = \mu \left(\theta + T\right)^{\alpha} \bar{H}.$$

Preferences should be modified as follows:

$$\int_0^A c\left(z\right) \; \mathrm{d}z + \bar{H} \left(\beta \ln \hat{n} + \delta \ln \hat{A}\right). \tag{16}$$

$\bar{H}$ now multiplies the term associated with children to keep utility balanced in a growing economy, implying that the value of children and their body development depends on the average human capital of the society.

Finally, parental expenditure on each child's development should also be indexed on average human capital:

$$\Psi(\hat{A}) = \kappa \, \frac{1}{2} \frac{\bar{H}}{A} \hat{A}^2, \tag{17}$$

This modifications do not affect the household decision problems and all the results above can be applied directly.

Aggregate human capital is defined by the human capital of active cohorts

$$H\left(t\right) = \int_{t-\tilde{A}(t)}^{t-\tilde{T}(t)-\phi\tilde{n}(t)} P\left(z\right) \underbrace{\mu \left(\theta + T\left(z\right)\right)^{\alpha} \bar{H}\left(z\right)}_{h(z)} \; \mathrm{d}z \tag{18}$$

[14] According to Clark, Skilled building workers typically acquired those skills by apprenticing themselves to a craftsman, with the traditional apprenticeship lasting up to seven years.

where average human capital per worker is given by

$$\bar{H}(t) = \frac{H(t)}{E(t)},$$

and total employment $E(t)$ is

$$E(t) = \int_{t-\tilde{A}(t)}^{t-\tilde{T}(t)-\phi\tilde{n}(t)} P(z)\ \mathrm{d}z.$$

The technology producing the consumption good, the only final good in this economy, is linear in aggregate human capital with productivity one, implying that the real wage per unit of human capital is unity. Output per capita is then $H(t)/N(t)$.

A balanced growth path is now an equilibrium path where the population grows at rate η, human capital grows at rate γ, and, the demographic variable T, n and A are all constant.

Finally, the growth rate of human capital γ satisfies at the balanced growth path

$$\gamma = \frac{P^\star}{E^\star}\,\mu(\theta+T)^\alpha\left(\mathrm{e}^{-\gamma(T+\phi n)} - \mathrm{e}^{-\gamma A}\right).$$

To understand this result better, let us differentiate, at the balanced growth path, the definition of $H(t)$ in Equation (18) with respect to time:

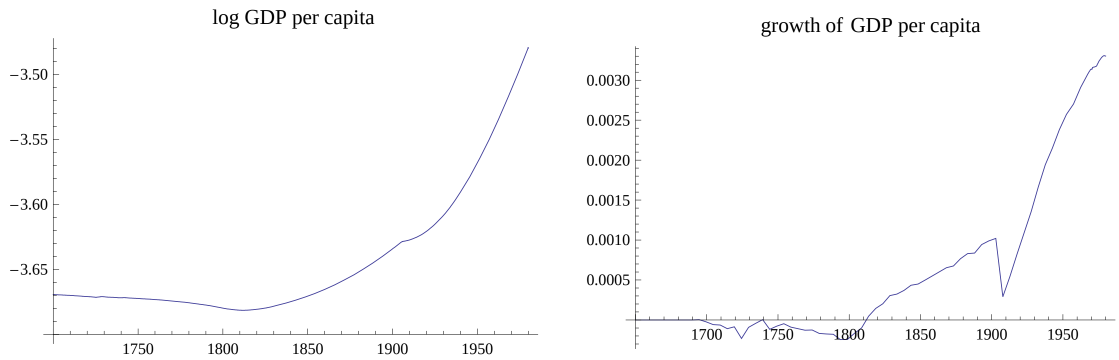
$$H'(t) = P(t-T-\phi n)h(t-T-\phi n) - P(t-A)h(t-A).$$

The change in aggregate human capital is the difference between the human capital of the youngest workers and that of the oldest. From the human capital technology, and using the balanced growth path assumption

$$\gamma = \frac{H'(t)}{H(t)} = \frac{P^\star}{E^\star}\,\mu(\theta+T)^\alpha\left(\mathrm{e}^{-\gamma(T+\phi n)} - \mathrm{e}^{-\gamma A}\right).$$

The first term on the r.h.s, $P^\star/E^\star$, derives directly from the assumption that per worker human capital affects the human capital of the current cohort. If, instead of normalizing total human capital by E, we normalized it by P, this term would vanish. It basically corresponds to the length of active life. The second term reflects the fact that both the oldest and the youngest cohort share the same human capital technology, with a common length of education. For this reason, the term $\mu(\theta+T)^\alpha$ is common. Finally, the last term

Figure 9: Example of dynamics - drop in κ - human capital



in brackets reflects the fact that aggregate human capital was not the same at the time the two cohorts were at school, the difference depending on the growth rate itself and the age difference between the cohorts. The vintage human capital nature of the model was pointed out by Boucekkine, de la Croix, and Licandro (2002): the human capital of new cohorts entering the labor market is larger than that of the retiring cohorts, because quality of schooling progressed with human capital accumulation.

The growth rate of per capita output is $\gamma - \eta$ at the balanced growth path. No theorem is available to assess the asymptotic behavior of the solutions of our dynamic system directly, and in particular, whether income per capita converges to its balanced growth path.[15] In the simulation below though, the solution converges asymptotically to the balanced growth path.

Figure illustrates the complex relationship between life expectancy and growth. As long as the economy is in the Malthusian regime, an increase in life expectancy increases fertility and the dependency ratio which reduces income per capita. In the corner regime with consumption above subsistence but still no education, increases in life expectancy promote growth, because it raises adult longevity and the number of workers per dependent (children). Finally, in the interior regime, the take-off of education generates an acceleration in growth and a switch to a balanced growth path with positive income growth. The sharp drop in 1900 is related to the fact that the first educated generation postpones its entry on the labor market. Notice that the model does not generate enough

[15] No direct stability theorem is available for delay differential systems with more than one delay since the stability outcomes depend on the particular values of the delays. See Mahaffy, Joiner, and Zak (1995).

growth compared to observations, as it relies only on population and human capital as the engine of growth.

8 Conclusion

The epidemiology literature stresses that life expectancy depends greatly on physical development during childhood. Both better nutrition and lower exposure to infections leads to increased body height and a longer life. We have proposed a theory of the demographic transition based on this fact. The novel mechanism of the model is that parents face a trade-off between the quantity of children they have and the amount they can afford to spend on childhood development of each of them. Parents like to have a lot of children, but they also care about their longevity as adults. Having many children prevents parents spending much on their body development. If its cost decreases, parents will increase their investment in their children's longevity. The number of children will first increase in the Malthusian regime as a consequence of higher lifetime income. As longevity rises, fertility starts falling as a result of the trade-off faced by parents between investing in their own human capital and spending time rearing children. Following the trade-off between the number of children and childhood development, adult longevity keeps increasing.

The model we have developed reproduces the characteristics of the demographic transition well, displaying the appealing features that longer education delays birth and reduces fertility. Our theory can be seen as an alternative to the one based on a rise in the return to human capital investment induced by economic progress, leading parents to substitute quality for quantity. A distinctive implication of our theory is that improvements in childhood development should precede the increase in education. Taking height as a proxy for childhood development, we have observed just such a pattern in Swedish historical data.

Our theory can also provide an explanation for the puzzling fact that height at age 18 is a strong predictor of education attained later in life (Magnusson, Rasmussen, and Gyllensten (2006) showed that Swedish men taller than 194 cm were two to three times more likely to obtain a higher education than men shorter than 165 cm), even after controlling for parental socioeconomic position, other shared family factors, and cognitive ability. A further test of our theory would consist of checking whether family size is related to childhood development as measured by average height on historical micro-data.

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A Proofs of Propositions

Proof of Proposition 1

After substituting the integral in (2) into (1) and diving by $\bar{H}$, the objective becomes

$$\left(\beta \ln \hat{n} + \delta \ln \hat{A}\right) + \underbrace{\mu \left(\theta + T\right)^{\alpha} \left(A - T - \phi \hat{n}\right) - \hat{n} \left(\frac{\kappa}{2} \frac{\hat{A}^2}{A}\right)}_{C}$$

which is maximized under the restrictions $T \geq 0$ and $C \geq 0$.

First order conditions to this problem are (omitting the Kuhn-Tucker conditions):

$$(1+\eta)\hat{A}^2 = \frac{\delta}{\kappa \hat{n}} A \tag{A.1}$$

$$(1+\eta)\alpha\mu \left(\theta + T\right)^{\alpha-1} \left(A - T - \phi\hat{n}\right) = (1+\eta)\mu \left(\theta + T\right)^{\alpha} - \lambda \tag{A.2}$$

$$\frac{1}{\hat{n}} \left(\beta - \frac{\delta}{2}\right) = (1+\eta)\mu \left(\theta + T\right)^{\alpha} \phi \tag{A.3}$$

where λ and η are the Kuhn-Tucker multipliers associated with the constraints $T \geq 0$ and $C \geq 0$, respectively. The interior solution (4)-(6) is (A.1)-(A.3) under $\eta = \lambda = 0$. The corner solution (7)-(9) results from the same system under $\eta = T = 0$, and finally, the corner solution (10)-(12) results from the first order conditions under $T = C = 0$. Under Assumption 2, $\eta = C = 0$ is not optimal.

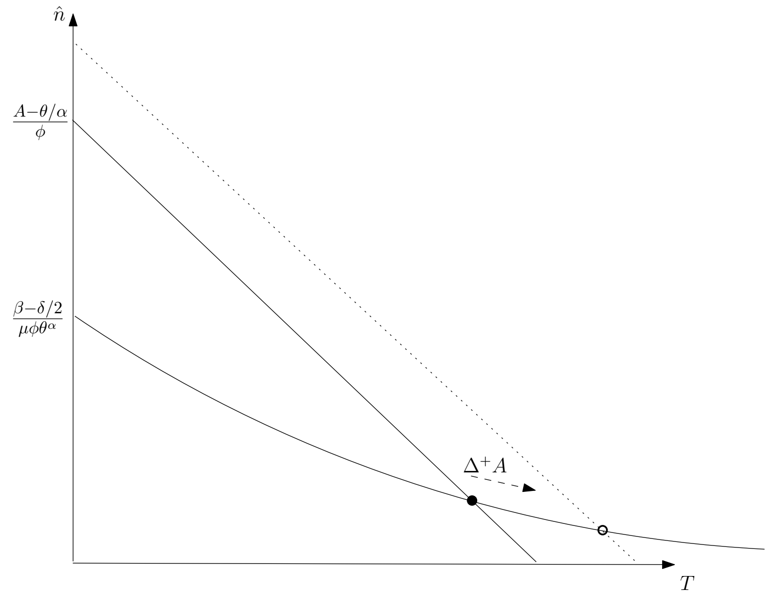
Interior Regime. The solution to the first order conditions (4)-(6) exists and is unique iff the loci in (5) and (6) cut once and only once for positive n and T, and $C \geq 0$ at the solution. The locus in (5) is a straight line with negative slope and cuts the $\hat{n}$ axes at $\frac{A-\theta/\alpha}{\phi} \equiv n_0$, see Figure A.1. The locus in (6) has a negative slope, is convex, and is such that $\hat{n}$ goes to zero when T goes to infinity and cuts the $\hat{n}$ axes at $\frac{\beta-\delta/2}{\mu\phi\theta^{\alpha}} \equiv n_1$. Comparing these two points and imposing $n_0 \geq n_1$ leads to the condition $A \geq \overline{A}$, where

$$\overline{A} = \frac{\beta - \frac{\delta}{2}}{\mu\theta^{\alpha}} + \frac{\theta}{\alpha}.$$

Substituting (4) and (5) in the definition of C gives

$$C = \frac{\mu}{\alpha}(\theta + T)^{1+\alpha} - \frac{\delta}{2},$$

Figure A.1: The interior solution



which is positive under Assumption 2 for all $T \geq 0$.

Corner regime $\underline{A} \leq A < \overline{A}$. If $A < \overline{A}$, the straight line is above the convex curve at $T = 0$ (see Figure A.1). A sufficient condition for these two curves not to intersect in the positive plane is that the straight line is steeper than the convex curve at zero. This is guaranteed by Assumption 2. In that case, there is no interior solution, since negative values for T are not feasible. Consequently, the solution must be corner with $T = 0$. From equations (7)-(9), at this corner solution

$$C = \mu\theta^\alpha \left(A - \frac{\beta - \delta/2}{\mu\theta^\alpha} \right) - \delta/2,$$

which is positive for $\underline{A} \leq A < \overline{A}$,with

$$\underline{A} \equiv \frac{\beta}{\mu\theta^\alpha}.$$

From Assumption 2, $\underline{A} < \overline{A}$. It is easy to see that the solution is unique.

Corner regime $0 < A < \underline{A}$. Finally, when $0 < A < \underline{A}$, the optimal solution is (10)-(12), with both inequality constraints being binding. Uniqueness is trivial.

Proof of Corollary 1

For the interior solution, we apply the implicit function theorem to (4)-(6), which leads to

$$f'_A = \mathrm{d}\hat{A}/\mathrm{d}A = \sqrt{\frac{A\delta}{\hat{n}\kappa}}\ \frac{(T(\alpha+1)+\theta+\alpha((A-\hat{n}\phi)\alpha+\theta))}{2A((\alpha+1)(T+\theta)-\hat{n}\alpha^2\phi)}.$$

The numerator is positive. Under Assumption 2, the denominator is also positive. The results for f'_n and f'_T can be proved using the same arguments. For the corner solutions, the result is straightforward.

Proof of Proposition 2

Let us denote the function $f_A(.)$ by $f_{A1}(.)$ when $A \geq \overline{A}$, $f_{A2}(.)$ when $\underline{A} \leq A < \overline{A}$, and $f_{A3}(.)$ when $0 < A < \underline{A}$. The dynamics of life expectancy following $A_{i+1} = f_A(A_i)$ are monotonic because f_A is continuous and non-decreasing.

Let us first prove the existence of a solution. From corollary 1, $f'_A(A) > 0$. It is easy to see that $\lim_{A\to 0} f_A(A) = \lim_{A\to 0} f_{A3}(A) > 0$. To prove the existence it is enough to show

$$\lim_{A\to\infty}\frac{f_A(A)}{A} = \lim_{A\to\infty}\frac{f_{A1}(A)}{A} = 0.$$

From (5) and (6)

$$\hat{n} = cte(A-\phi\hat{n})^{-\alpha}.$$

Substituting in (4), and dividing by A^2 gives

$$\left(\frac{\hat{A}}{A}\right)^2 = cte\frac{(A-\phi\hat{n})^{\alpha}}{A}.$$

Since $\lim_{A\to\infty} f_n(A) = 0$, it's now easy to see that $\lim_{A\to\infty}\frac{f_{A1}(A)}{A} = 0$.

Let us now prove its unicity. For $0 < A < \underline{A}$, the function $f_{A3}(.)$ is increasing and concave, with $f_{A3}(0) = 0$ and $f'_{A3}(0) = \infty$, implying that if it crosses the diagonal on the interval $(0, \underline{A})$, it crosses it only once.

Function $f_{A2}(.)$ is increasing and concave, with $f_{A2}(0) = 0$ and $f'_{A2}(0) = \infty$, implying that if it crosses the diagonal on the interval $[\underline{A}, \overline{A})$, it crosses it only once.

Finally, let us prove that $f'_{A1}(A) < 1$ for any fixed point of $f_{A1}(.)$ in $A \geq \overline{A}$. From the implicit function theorem applied to (4)-(6),

$$\frac{\mathrm{d}\hat{A}}{\hat{A}} / \frac{\mathrm{d}A}{A} = \frac{1}{2} \frac{(1+\alpha)(A - \phi\hat{n})}{A - (1+\alpha)\hat{n}}.$$

At a fixed point of f_{A1}, since Corollary 1 shows that $f'_A(A) > 0$ in this interval, the denominator must be strictly positive. It is then easy to see that $f'_A(A) < 1$ iff $A > \frac{1+\alpha}{1-\alpha}\phi\hat{n}$. Since, from Corollary 1, $f'_n(.) < 0$ in this interval, $f'_A(A) < 1$ for all $A \geq \overline{A}$ iff $\overline{A} > \frac{1+\alpha}{1-\alpha}\phi\hat{n}$, which holds under Assumption 2.

Global stability is then trivial, since f_A is above the diagonal before the unique steady state equilibrium and below it afterwards.

Proof of Proposition 3

A steady state for A in the interior regime exists iff there is a solution to the system (4)-(6) evaluated at the steady state. Eliminating A and n from Equation (5) using Equations (4) and (6) we find that the steady state T should satisfy:

$$T(1+\alpha) + \theta = \alpha \left(\frac{2\delta\phi\mu(T+\theta)^{\alpha}}{\kappa(2\beta - \delta)} - \frac{(2\beta - \delta)(T+\theta)^{-\alpha}}{2\mu} \right)$$

The left hand side is a linear increasing function of T. The right hand side is a concave function of T, with a slope going to zero as T goes to infinity. A sufficient condition for existence and uniqueness of a stationary solution is that the right hand side is larger than the left hand side at $T = 0$. This leads to Condition (13).