Which Diseases Generate the Largest Epidemiological Externalities?

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Abstract: The existence of positive epidemiological externalities from vaccination are widely accepted and motivate a variety of public policies, but we lack a sense of what affects the relative magnitude of these externalities across diseases. We analyze vaccine externalities in an integrated economic and epidemiological model in which producers have market power. A result of our model is that the marginal externality from vaccination is non-monotonic in disease transmissibility, and peaks when the transmissibility rate is very close to the death rate in the population. We show that the minimum optimal vaccination subsidy is also non-monotonic in disease transmissibility, which in turn characterizes the diseases for which the potential welfare gains from vaccination subsidies would be largest. We extend our model to consider other market structures including perfect competition and Cournot competition.

Keywords: vaccine, epidemiology, externality, pharmaceutical

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1. Introduction

It is widely accepted that vaccines (as well as some drug treatments) generate positive externalities by reducing disease prevalence and thus opportunities for disease transmission. These positive externalities drive a wedge between the social welfare and private benefit from vaccines, thus implying a case for subsidizing use of existing vaccines as well as subsidizing research and development (R&D) on new vaccines. The question of subsidizing use of existing vaccines arises in the context of increasing immunization rates in developed and developing countries[1], as well as specifically in the context of how much international financial support should be given to campaigns such as the polio eradication effort.[2] The question of the appropriate form and focus of subsidies to vaccine R&D has also been an area of public policy interest, particularly in the context of vaccines for so-called "neglected" diseases concentrated in low income countries.[3] A key input into discussions over subsidizing vaccine use or R&D is a sense of the quantitative magnitude of these positive externalities and a sense of relative priorities across diseases.

In this paper, we build an integrated economic and epidemiological model which formalizes the positive externality associated with vaccines. We focus on providing a tractable model with closed-form solutions for the equilibrium of a vaccine market in which, on the demand side, consumers' rational expectations are derived from a dynamic epidemiological model; on the supply side, for clarify of exposition we focus on the case of a monopoly vaccine producer selling directly to consumers, but also consider the more general case of Cournot competition and also consider an extension to perfect competition. Tractability is preserved by considering the limiting case as the discount rate goes to zero, allowing a focus on steady states.

A key result from this model is that the externalities from vaccination are non-monotonic in a parameter representing the transmissibility of the disease. Specifically, we show that the marginal

[1] For example, in the United States the Vaccines for Children (VFC) program provides free vaccines to children who lack or have limited access to health insurance. Internationally, vaccines are subsidized through programs such as the World Health Organization (WHO)'s Expanded Programme on Immunization.
[2] On the Global Polio Eradication Initiative, see www.polioeradication.org.
[3] See, for example, the work of the Center for Global Development (CGD) on advance purchase contracts for vaccines for neglected diseases (www.cgdev.org/vaccine).

externality from one additional vaccination is largest when a parameter representing the transmissibility of a given disease is very close to the death rate in the population. Intuitively, if the disease transmissibility is sufficiently low, then an additional vaccination does not do much to protect other consumers since it was very unlikely that others would have contracted the disease from that consumer; on the other hand, as transmissibility increases, the less the vaccination of a marginal consumer protects others because they become increasingly likely to contract the disease from some other source anyways. Note that diseases with a transmissibility rate close to the death rate in the population will be rare in equilibrium, in the absence of a vaccine; thus, our work suggests that the externalities from vaccination are largest for rare diseases.

We also show that the minimal optimal subsidy for vaccination that a government would choose to provide is also non-monotonic in the transmissibility of the disease, which in turn characterizes the diseases for which the potential welfare gains from vaccination subsidies would be largest.

The point that vaccines may provide positive externalities and that these externalities may affect consumers' and firms' decisions is well understood from the literature (see, among others, Brito, Sheshinski, and Intrilligator 1991; Boulier 2006; Francis 1997; Geoffard and Philipson 1997; Gersovitz 2003; Gersovitz and Hammer 2004, 2005). Our model differs from most of the previous literature in that we derive explicit solutions for equilibrium, externalities, and optimal subsidies when producers have market power and demand is derived from a dynamic epidemiological model. In a related paper, Boulier, Datta, and Goldfarb (2007) use a standard epidemiological model alone (that is, not interacted with consumer decisions nor a supply-side model of firm behavior) to examine properties of vaccination externalities that arise when solely considering epidemiological concerns; they focus on providing some empirical calibrations for the cases of influenza and mumps. Also closely related to our paper is Geoffard and Philipson (1997). Using an epidemiological model similar to ours, the authors' main result is that a vaccine producer with market power will not choose to eradicate the disease in the steady state. Our work differs because we explicitly derive the optimal monopoly price and profits for vaccines, and we also extend their results by characterizing the

conditions under which externalities and subsidies will be greatest, and thus in which underinvestment and underproduction will be most severe.[4]

Sections 2 and 3 present our basic model, first outlining our epidemiological model and then integrating that into an economic model of vaccine supply and demand. In Section 4 we characterize externalities and optimal subsidies in our model, and also examines relative incentives facing firms choosing between developing vaccines and similarly effective drugs (without externalities). Section 5 extends our epidemiological model to include an inflow of foreign infections. Section 6 extends our results for monopoly to various oligopolistic market structures, including perfect competition and Cournot (quantity) competition among N homogenous firms, and Section 7 concludes.

2. Epidemiological Model

In this section, we will outline a standard epidemiological model into which, in Section 3, we will embed an economic model of vaccine supply and demand. Following standard epidemiological models (Bailey 1975, Anderson and May 1991), we will consider a non-fatal disease. Assume that consumers are born into the population at rate $\gamma > 0$ and die at rate γ as well. Since the disease is non-fatal, γ is the death rate for both infected and uninfected consumers. Together the assumptions imply that the total consumer population is constant over time, simplifying the analysis. Normalize the constant mass of consumers to unity.

We allow vaccines to be imperfectly effective. Let $\theta \in [0,1]$ be the probability that a vaccinated consumer is successfully immunized against the disease. The event that a vaccinated consumer is successfully immunized is a Bernoulli random variable independently and identically distributed in the population. If the vaccine turns out to be ineffective for a consumer, he or she can still contract and spread the disease. Assume that if the initial dose is not effective for a consumer, further doses will not be either.

[4] Mecholan (2007) provides some analysis of treatments (conditional on infection) for communicable diseases in the context of a monopolist manufacturer, but provides no analytical results – instead focusing on numerical simulations, primarily related to issues of drug resistance.

Let $V(t)$ be the proportion of the population that are successfully vaccinated at time t, $I(t)$ the proportion that are infected, and $S(t)$ the remaining proportion, the so-called susceptible population. The susceptible population includes consumers who are vaccinated but for whom the vaccine turns out to be ineffective. Since the categories of $V(t)$, $I(t)$, and $S(t)$ are mutually exclusive and exhaustive, and recalling that the mass of consumers is normalized to unity, we have

$$V(t) + I(t) + S(t) = 1. \tag{1}$$

Note that $V(t)$, $I(t)$, and $S(t)$ can be interpreted as either proportions or masses.

Susceptible consumers contract the disease from infected ones at rate $\beta I(t)$. Here, β embodies both the rate of contact with infected consumers and the rate at which those contacts lead to infection. The infection rate is linear in the number of infected consumers, $I(t)$.

Let $q(t)$ be the mass of newborns who are vaccinated, and thus $\theta q(t)$ the mass of newborns who are successfully vaccinated. Given the Poisson structure of the model, and hence the stationarity of consumers' lifecycles, assuming vaccines are purchased only by newborns is without loss of generality: we could equivalently have assumed that the vaccine is purchased by any arbitrary subset of susceptible consumers. For now we will take $q(t)$ as given; we will later solve for the equilibrium value of $q(t)$ using the economic model and substitute the value back into the epidemiological model.

The rate of change of the successfully vaccinated population is

$$\dot{V}(t) = \theta q(t) - \gamma V(t), \tag{2}$$

the number of new successful vaccinations minus the number of the successfully vaccinated who die. The rate of change of the infected population is

$$\dot{I}(t) = \beta I(t) S(t) - \gamma I(t), \tag{3}$$

the number of new infections from contact between the susceptible and infected populations minus the number of infected individuals who die (of natural causes).

Let variables without time *t* arguments (so *q*, *V*, *I*, *S*, *etc*.) denote steady-state values. Taking the flow of consumers *q* vaccinated in the steady state as given for now (we will solve for the equilibrium value of *q* later), the steady-state values of *V*, *I*, and *S* can be found by setting $\dot{V}(t) = \dot{I}(t) = 0$ in equations (2) and (3) and then forming a system of equations with (1). The solution to this system of equations depends on the value of *q*. If *q* is sufficiently high,

$$q \geq \frac{\gamma(\beta - \gamma)}{\beta\theta}, \tag{4}$$

then the solution is

$$I = 0 \qquad V = \frac{\theta q}{\gamma} \qquad S = 1 - \frac{\theta q}{\gamma}, \tag{5}$$

implying that the disease is eradicated in the steady state. If *q* is less than the threshold in equation (4), then the stable solution to the system of equations involves a positive infection level:[5]

$$I = 1 - \frac{\theta q}{\gamma} - \frac{\gamma}{\beta} \qquad V = \frac{\theta q}{\gamma} \qquad S = \frac{\gamma}{\beta}. \tag{6}$$

Before turning to the full economic analysis in Section 3, we can make provide a preliminary characterization of equilibrium at this stage. If $\beta \leq \gamma$, then equation (4) is satisfied for all $q \geq 0$. The analysis of the vaccine-market equilibrium is trivial in this case. There is no infection in the steady state and thus no vaccine sales. Equilibrium will involve a non-trivial infected population and positive vaccine sales when $\beta > \gamma$. This case will be the focus of the calculations in the next section.

3. Equilibrium

Part of the contribution of this paper is in characterizing equilibrium when vaccine manufacturers have market power. We will begin by analyzing the simplest case of market power, that of a profit-maximizing, monopoly manufacturer of the vaccine. In later extensions we will analyze alternative industrial structures ranging from Cournot to perfect competition. To allow us to focus on steady states and avoid having to

[5] Equation (5) continues to be a solution in this case as well, but it becomes unstable when (4) does not hold.

compute transition paths, we will consider the limiting case as the discount rate goes to zero. In the limit as the discount rate goes to zero, profit maximization reduces to maximizing steady-state flow profit.[6]

It is worth briefly describing why a model in which vaccine producers have market power is empirically relevant. In general, in a cross-sectional sense very few firms actively manufacture vaccines; for example, in 2002, only four firms were producing almost all of the recommended childhood vaccines for the US market (GAO 2002).[7] Market power is also empirically relevant when looking at time series data on subsequent entrants into vaccine markets over time; for several vaccines developed in the 1980s (such as for hepatitis B and varicella), no subsequent market entrants were approved until more than eight years after approval of the initial market entrant; for others (such as hepatitis A and haemophilus influenzae type b [Hib]) subsequent market entrants were approved within the first few years after approval of the initial entrant, but the number of firms entering either market was not more than two or three.[8]

For simplicity, we normalize the marginal cost of manufacturing and administering the vaccine to zero. Introducing marginal cost $c > 0$ complicates the equilibrium expressions below somewhat without adding much insight in the monopoly case.[9] We will also ignore fixed costs for now since they do not enter into the firm's pricing decisions.

Let $\Pi(t)$ denote flow profit from a vaccine, also the revenue given the assumption of zero costs, equal to the price $p(t)$ multiplied by the flow quantity sold $q(t)$. Let p and Π denote the corresponding steady-state values of price and flow profit. Let W denote the flow of social welfare from the vaccine in the steady state, measured as the difference between social welfare when the equilibrium quantity q of the vaccine is sold each instant and social welfare in the steady state in which no vaccine is available.

[6] In the limit as the discount rate goes to zero, there is no distinction between the closed-loop solution (in which the firm can commit to a price path) and the open-loop solution (in which the firm cannot commit and hence maximizes profit period by period).
[7] It is generally thought that one reason for vaccine firms having market power is that, relative to drugs, vaccine manufacturing processes are generally much less standardized and more difficult to replicate.
[8] These descriptive trends are based on a dataset compiled by Berndt *et al*. (2007) on the number and timing of entrants in the markets for vaccines developed in the 1980s. We here focus on defining subsequent market entrants as new firms entering a given vaccine market, rather than as new vaccines entering a given vaccine market (the difference being that sometimes the same firm may sell multiple vaccines in a given market).
[9] In the case of perfect competition, having a positive marginal cost makes for a more interesting calculation of optimal government subsidies, and so we will analyze both positive marginal costs and perfect competition in a later extension.

Vaccine demand is derived from the decisions of consumers who base their rational expectations about infection rates on the preceding epidemiological model. Assume consumers are risk neutral. Normalize the rate of harm *h* suffered by an individual who contracts the disease to unity. Since there is no discounting, an individual's expected total future burden from the disease conditional on being infected equals the unit harm times his or her expected lifespan $1/\gamma$. To see that expected lifespan is $1/\gamma$, note that the fact that the model has a Poisson structure together with the fact that an individual's hazard rate of dying is γ implies that lifespan is an exponentially distributed random variable with parameter γ, the mean of which is well known to be $1/\gamma$.

To derive steady-state vaccine demand, note that a consumer's maximum willingness to pay for a vaccine in a steady state equals the product of four factors: the harm from the disease *h*, the consumer's expected lifespan $1/\gamma$, the efficacy of the vaccine θ, and the consumer's probability of contracting the disease before he or she dies:

$$\int_0^\infty \beta I e^{-(\gamma+\beta I)t} dt = \frac{\beta I}{\gamma + \beta I}. \tag{7}$$

Thus if

$$p = \frac{\theta}{\gamma}\left(\frac{\beta I}{\gamma + \beta I}\right), \tag{8}$$

then consumers are indifferent between buying the vaccine and not each instant in the steady state. If *p* is less than the right-hand side of equation (8), then consumers strictly prefer to buy the vaccine, and if *p* exceeds it, then consumers strictly prefer not to buy. The right-hand side of (8) is increasing in *I*, indicating that consumers are willing to pay more for the vaccine the higher is the infection rate, and thus the greater the chance of being infected. Put another way, the vaccination of one consumer exerts a positive externality on unvaccinated consumers because this reduces the expected size of the infected population and hence the infection rate.

Inverting equation (8) yields a cutoff infection rate

$$\hat{I}(p) = \frac{\gamma^2 p}{\beta(\theta - \gamma p)}. \tag{9}$$

such that consumers are indifferent between buying the vaccine and not if $I = \hat{I}(p)$, strictly prefer to buy if $I > \hat{I}(p)$, and strictly prefer not to buy if $I < \hat{I}(p)$, yielding the following demand correspondence:

$$q = \begin{cases} 0 & I < \hat{I}(p) \\ [0, \gamma] & I = \hat{I}(p) \\ \gamma & I > \hat{I}(p) \end{cases}. \tag{10}$$

Demand correspondence (10) is graphed in Figure 1 as the solid curve. The demand correspondence is an implicit function of p as well as I and q because $\hat{I}(p)$ is a function of p. An increase in p is indicated as a rightward shift in the demand correspondence. The monopolist can thus shift the demand correspondence around by varying p. Figure 1 superimposes the other condition that must hold in steady-state equilibrium, namely the epidemiological correspondence, which is the first equation in (6), graphed as the dashed curve. The market demand curve is given by the intersection of the two correspondences given the price p set by the monopolist, yielding

$$q(p) = \frac{\gamma}{\theta}\left[1 - \frac{\theta\gamma}{\beta(\theta - \gamma p)}\right]. \tag{11}$$

The monopoly's optimal price p^* maximizes its profit $pq(p)$ subject to the inequality constraints that the quantity of vaccine sold be non-negative ($q \geq 0$) but no greater than the number of newborns ($q \leq \gamma$). The solution to this Kuhn-Tucker problem is provided by Proposition 1, proved in Appendix A.

Proposition 1. *In the benchmark model with a monopoly vaccine manufacturer, the steady-state equilibrium falls into one of three cases.*

Case (i): $\beta \in [0, \gamma]$. *No infection and thus no vaccine sales.*

Case (ii): $\beta \in (\gamma, \gamma/(1-\theta)^2]$. *The steady-state equilibrium price is* $p^* = (\theta/\gamma)(1 - \sqrt{\gamma/\beta})$, *flow quantity is* $q^* = (\gamma/\theta)(1 - \sqrt{\gamma/\beta})$, *flow profit is* $\Pi^* = (1 - \sqrt{\gamma/\beta})^2$, *and flow social welfare is* $W^* = 1 - \sqrt{\gamma/\beta}$.

Case (iii): $\beta \in (\gamma/(1-\theta)^2, \infty)$. *The steady-state equilibrium price is* $p^* = (\theta/\gamma)(1-\theta-\gamma/\beta)/(1-\theta)$, *flow quantity is* $q^* = \gamma$, *flow profit is* $\Pi^* = \theta(1-\theta-\gamma/\beta)/(1-\theta)$, *and flow social welfare is* $W^* = \theta$.

To provide the intuition behind Proposition 1, consider each of the three cases in turn. If $\beta \leq \gamma$, then as noted in Section 2 the disease disappears in the steady state even if no vaccine is sold. In the remaining cases, a positive amount of vaccine is sold in the steady-state equilibrium. Case (ii) corresponds to the vaccine's being sufficiently effective. To see this, note the condition $\beta \leq \gamma/(1-\theta)^2$ implies that the efficacy parameter, θ, exceeds $1-\sqrt{\gamma/\beta}$. If the vaccine is at least this effective, then the constraint that the firm cannot vaccinate more than the number of newborns ($q \leq \gamma$) does not bind. The firm can essentially replicate the outcome with a perfectly effective vaccine. It discounts the price of the imperfectly effective vaccine so that $1/\theta$ times more consumers purchase, leading to the exact same number of successful vaccinations as with a perfectly effective vaccine. Steady-state equilibrium profit and social benefit are identical to the case with a perfectly effective vaccine.

Case (iii) corresponds to the vaccine's being sufficiently ineffective. The firm would profit from successfully immunizing more consumers but is bound by the constraint that it cannot vaccinate more than the total population. In equilibrium, the firm vaccinates all consumers, a flow of γ newborns each instant. Substituting $q^* = \gamma$ into equation (6) shows that the infected population in the steady-state equilibrium is $I^* = 1-\theta-\gamma/\beta$, which is strictly positive under the condition from case (iii) that $\beta > \gamma/(1-\theta)^2$. Although all consumers are vaccinated, an infected population $I^* > 0$ remains because of the vaccine's limited efficacy. The first-best level of social welfare is attained in the steady-state equilibrium. To be precise, steady-state equilibrium attains the first-best level of social welfare, θ, conditional on having a vaccine of limited efficacy; the first best conditional on having a perfectly effective vaccine is greater at $1-\gamma/\beta$. The limited efficacy of the vaccine is so constraining in case (iii) that the firm would prefer to have even more successfully vaccinated consumers than in the first best.

The comparative-static properties of the equilibrium in Proposition 1 are straightforward to derive. The only complication is that we need to analyze three different cases from the proposition, although each case can be analyzed separately since all variables can be shown to be continuous in all the underlying parameters. Table 1 reports the results for three of the underlying parameters: θ, γ, and β.

Price, profit, and social welfare generally move in the same direction with parameter changes. An increase in effectiveness θ and an increase in the transmission rate β each lead to an increase in the value of the vaccine and thus lead to a weak increase in price, profit, and social benefit. A increase in γ has the opposite effect. Interpreting γ as the rate at which the population turns over, an increase in γ reduces the value of the vaccine as the average lifespan over which vaccine provides protection shrinks.

The comparative-static effects on steady-state equilibrium quantity, q^*, are slightly more subtle. An increase in the effectiveness, θ, of the vaccine either has no effect, as the firm vaccinates all newborns, or, for higher levels of effectiveness, reduces equilibrium quantity as the firm tries to maintain a certain disease prevalence. An increase in the birth rate, γ, has two opposing effects on q^*. On the one hand an increase in γ increases the number of newborns to be vaccinated. On the other hand, an increase in γ lowers consumer demand by reducing the prevalence of the disease in the population. Thus γ may increase or decrease q^*, depending on the parameters. An increase in transmission, β, increases consumer demand for the vaccine.

4. Externalities and Subsidies

The discussion in this section is divided into three subsections. The first subsection quantifies the positive externality associated with vaccines. The second subsection derives the government subsidy needed to offset underconsumption due to the positive externality (also due in part to the market power of the vaccine monopolist). The third subsection shows that externalities can provides a rationale for firms' alleged bias against developing vaccines in favor of similarly effective drugs. While a vaccine that prevents the individual from contracting the disease benefits others by preventing the spread of the disease to them, a drug treatment for the same disease may only mitigate the harm from symptoms without preventing

transmission. Thus drugs may not have the positive externalities associated with vaccines and may allow the firm to extract more surplus from consumers.

4.1. Marginal Externality

Vaccinating one individual exerts a positive externality on others by preventing the individual from contracting the disease and spreading it to others. In the epidemiology literature, this is called the herd immunity effect. One of the key focuses of this paper is to characterize the conditions under which this externality is greatest. In the presence of a positive externality, the quantity of vaccine sold will tend to be less than socially optimal. Knowing when the externality is greatest will help inform government policy regarding which diseases would be best to target with government subsidies to correct the underconsumption problem. Another reason to quantify vaccine externalities is that they are a source of surplus that cannot be appropriated even by a monopoly manufacturer. The presence of a large pool of unappropriable surplus will reduce the monopolist's incentive to invest in research, development, and capacity relative to the social optimum.

Let x^* denote the marginal externality from the vaccine, the difference between the social and private benefit from adding one vaccine dose to the equilibrium quantity sold each instant in the steady state.[10] We will examine the marginal social benefit and marginal private benefit in turn. Note as a preliminary step that if $\beta \leq \gamma$, there is no vaccine sold in steady-state equilibrium and thus no marginal externality. The calculations in this section will thus focus on the $\beta > \gamma$ case.

Marginal social welfare, dW / dq, can be shown to equal θ / γ. To see this, note that total social welfare equals the unit harm from the disease times the decrease in the steady-state infection rate due to the vaccine. From equation (6), the steady-state infection rate without a vaccine is

[10] The thought experiment behind the marginal externality of vaccinating one more consumer requires some discussion when $\theta < 1 - \sqrt{\gamma / \beta}$, corresponding to case (iii) in Proposition 1. In this case, the vaccine is sufficiently ineffective that the firm does not have to worry about restricting supply to maintain a certain infection rate; the firm sells the vaccine to every consumer. In this case x^* should be thought of as the marginal externality from vaccinating the last consumer rather than an additional consumer.

$1-\gamma/\beta$ and with a positive quantity of vaccine is $1-\theta q/\gamma-\gamma/\beta$. Hence the decrease in the infection rate—the difference between these two expressions—is $\theta q/\gamma$. Therefore $W=\theta q/\gamma$, and so $dW/dq=\theta/\gamma$.

To compute the marginal private benefit, note that the marginal consumer is indifferent between buying the vaccine in not in equilibrium. Since consumers are homogeneous, this means that a consumer's net private benefit must be zero in equilibrium. The firm extracts each consumer's net private benefit through the price. Thus price reflects the marginal private benefit from the vaccine.

Combining the calculations for marginal social and private benefits, we have $x^*=\theta/\gamma-p^*$. Substituting the value of p^* from the relevant case from Proposition 1 leads to the next proposition.

Proposition 2. *In the benchmark model with a monopoly vaccine producer, the marginal externality in the steady state equals*

$$x^*=\begin{cases}0 & \beta\in[0,\gamma] \\ \dfrac{\theta}{\sqrt{\gamma\beta}} & \beta\in\left(\gamma,\dfrac{\gamma}{(1-\theta)^2}\right] \\ \dfrac{\theta}{\beta(1-\theta)} & \beta\in\left(\dfrac{\gamma}{(1-\theta)^2},\infty\right),\end{cases} \tag{12}$$

an expression that is increasing in θ, decreasing in γ, and is non-monotonic in β, reaching a peak as β approaches γ from above.

A potentially puzzling result in the proposition is that x^* is positive (in particular, $x^*=\theta/\beta(1-\theta)$) when $\beta>\gamma/(1-\theta)^2$. This condition corresponds to case (iii) in Proposition 1. We saw in case (iii) that the first best is attained because all consumers are vaccinated in steady-state equilibrium. One might think that externalities should disappear in the first best. In our setting, if vaccines are imperfectly effective (i.e., $\theta<1$), externalities are still present in the first best. Though all consumers end up being vaccinated in

equilibrium, those for whom the vaccine is ineffective receive an external benefit from the reduction in disease prevalence from another's vaccination.

To gain some intuition for the comparative-static results in the proposition, consider each parameter in turn, starting with θ. The effect of an increase in vaccine efficacy θ on x^* depends on which case from Proposition 1 we are in. Recall that if the vaccine is sufficiently effective (in particular, if $\theta \geq 1 - \sqrt{\gamma / \beta}$, corresponding to case (ii) in Proposition 1), the firm reproduces the same market outcome (in terms of profit and infection rate) as with a perfectly effective vaccine. An increase in θ in this range thus has no effect on infection rates. It still scales x^* up because the marginal externality calculation is based on the addition of a physical dose of vaccine. An additional physical dose of vaccine will have a greater health impact the higher the efficacy of that added dose, and this increase in health impact increase x^* linearly. An additional, compounding effect arises if the vaccine is sufficiently ineffective (in particular, if $\theta < 1 - \sqrt{\gamma / \beta}$, corresponding to case (iii) in Proposition 1). Then all consumers receive the vaccine in equilibrium. An increase in θ reduces the prevalence of the disease, thereby reducing the private benefit from the vaccine and consequently increasing x^*. For sufficiently low θ, these reinforcing effects lead x^* to increase at an increasing rate in θ. Confirming this intuition, Figure 2.1.1 shows that x^* is convex in θ for low θ (the case (iii) region) and linear in θ for high θ (the case (ii) region).

Figure 2.1.2 confirms the comparative static result that x^* is nonincreasing in γ. More precisely, x^* is constant in γ for both low and high values of γ and is strictly decreasing for intermediate values. It is obvious that x^* does not vary with γ for $\gamma \geq \beta$: this corresponds to case (i) of Proposition 1 in which no vaccine is sold in the steady state and thus $x^* = 0$. What may be less clear is why x^* is independent of γ for low γ (i.e., for $\gamma < \beta(1-\theta)^2$, corresponding to case (iii) of Proposition 1). Recall that for these parameters all consumers are vaccinated in steady-state equilibrium, so the first best is attained, yet there is still an externality received by consumers who are vaccinated but for whom the vaccine proves to be

ineffective. The size of this unsuccessfully treated group depends only on the efficacy of the vaccine θ, not γ. Hence the marginal externality is independent of γ in this region of parameter space.

For intermediate values of γ (i.e., for $\gamma \in [\beta(1-\theta)^2, \beta)$, corresponding to case (ii) of Proposition 1) both the marginal social and marginal private benefits of vaccination are declining because of the reduction in the expected lifespan $1/\gamma$ over which the vaccine provides protection. The social benefit declines more slowly because of the offsetting effect that more people are exposed to the externality when the population turns over faster with the increased birth and death rate γ.

Figure 2.1.3 shows that x^* is non-monotonic in β, reaching a single peak as β approaches γ from above. This non-monotonicity is one of the central results in the paper and deserves some discussion. It is clear that $x^* = 0$ to the left of this peak because this corresponds to case (i) from Proposition 1 in which there are no vaccine sales in steady-state equilibrium. It is also clear that the marginal externality is positive as one moves out of case (i) into cases in which a positive amount of vaccine is sold, so that x^* jumps up as one moves from case (i) to case (ii). To complete the picture in Figure 2.1.3, it remains to be explained why x^* is strictly decreasing in β for $\beta > \gamma$. As transmissibility increases, the less the vaccination of a marginal consumer protects others because they becomes increasingly likely to contract the disease from some other source anyway.

The non-monotonicity in Figure 2.1.3 is not an artifact of the disappearance of the disease in the steady state when $\beta > \gamma$. In the extension of the model in Section 5 to allow for an inflow of infections from foreign sources, the disease never disappears in the steady state. In the limit as the foreign infection parameter ϕ approaches 0, the graph of the marginal externality converges smoothly to Figure 2.1.3. Thus, even in an extended model in which the disease does not disappear in the steady state, the marginal externality is small for low β. If the disease transmissibility is sufficiently low, the vaccination of a marginal consumer does not do much to protect others since it was very unlikely that they would have contracted the disease from that consumer.

4.2. Government Subsidies

The positive externality characterized the previous section will lead to too few consumers purchasing the vaccine relative to the social optimum. Consumers do not take into account the reduction in the spread of the disease to others from their own immunization. Compounding the externality problem is the problem of the deadweight loss from monopoly pricing. In this section we analyze subsidy policies the government could offset these problems and thereby boost vaccinations in the direction of the first best.

Assume the benevolent government maximizes social welfare, the equally weighted sum of consumer surplus (net of taxes) and producer surplus. Let s denote the per-dose subsidy, to which the government commits at the outset of the game. We will adopt the accounting convention that the subsidy is paid to the firm, so that p then reflects the price consumers pay and $p+s$ reflects the price the firm receives. Suppose the subsidy is paid for by non-distortionary taxation. Allowing for distortionary taxation via a positive cost of public funds is straightforward to model but would complicate finding closed-form solutions. With non-distortionary taxation, the benevolent government is willing to pay the firm an arbitrarily high subsidy and allow it to earn arbitrarily high profits as long as this moves vaccine quantity toward the first best. The subsidy is a frictionless transfer. As we will see, there is always a sufficiently high subsidy that the social optimum can be attained: either all consumers purchase the vaccine or the steady-state infection rate is driven to zero. We will look for the lowest possible subsidy attaining this objective.

The analysis with a per-unit subsidy is the same as in Section 3 except that the monopolist maximizes $(p+s)q(p)$ rather than $pq(p)$. The epidemiological model is unchanged, generating the same demand $q(p)$ as before in equation (11). The optimal subsidy can be found by backward induction. Given an arbitrary subsidy, one can solve the firm's maximization problem to compute the steady-state equilibrium price and quantity. Given the firm's response to the subsidy, one can compute the subsidy leading to the first best. The first best can be attained in one of two ways depending on vaccine efficacy θ. For low θ ($\theta < 1-\gamma/\beta$), the vaccine is sold to all consumers in the first best but the steady-state infection rate is still positive because of the ineffectiveness of the vaccine. For high θ ($\theta \geq 1-\gamma/\beta$), the steady-state infection

rate converges to zero in the first best. We take the minimum subsidy implementing the first best as the optimum, s^*. The following proposition, proved in Appendix A, characterizes s^*.

Proposition 3. *Suppose that in the benchmark model with a monopoly vaccine producer, the government can offer a subsidy funded by non-distortionary taxation. The minimum subsidy implementing the first best is*

$$s^* = \begin{cases} 0 & \beta \in [0, \gamma] \text{ or } \beta \in \left(\dfrac{\gamma}{(1-\theta)^2}, \infty \right) \\ \dfrac{\theta(\beta - \gamma)}{\gamma^2} & \beta \in \left(\gamma, \dfrac{\gamma}{1-\theta} \right] \\ \theta \left[\dfrac{1}{\beta(1-\theta)^2} - \dfrac{1}{\gamma} \right] & \beta \in \left(\dfrac{\gamma}{1-\theta}, \dfrac{\gamma}{(1-\theta)^2} \right], \end{cases} \tag{13}$$

which is nondecreasing in θ and non-monotonic in γ and β, reaching a peak in both cases for $\beta = \gamma/(1-\theta)$.

The comparative-static results can be read from Figure 2.2. The main difference between the graphs for s^* in Figure 2.2 and those for x^* in Figure 2.1 is that the marginal externality is positive even when all consumers are vaccinated because the immunization of the last consumer provides protection for consumers for whom the vaccine is ineffective. Though there is a positive marginal externality in this case (case (iii) from Proposition 1), no subsidy is needed because with all consumers receiving the vaccine, the first best is attained without a subsidy. When the externality on the vaccinated-but-unsuccessfully-immunized is subtracted from each panel in Figure 2.1, the graphs begin to look similar to Figure 2.2.

The main result of interest in Proposition 3 and Figure 2.2 is the non-monotonicity of s^* in γ and β, in particular the fact that the minimum optimal subsidy peaks for $\beta = \gamma/(1-\theta)$.

It is worth drawing a link from our results in this section to the epidemiology literature. Mathematical epidemiology models focus attention on what is called the basic reproductive rate, or R_0. Loosely speaking, R_0 is defined as the average number of secondary infections produced when one infected individual is introduced into a host population which is entirely successful (see Anderson and May 1991).

This R_0 parameter is defined in our notation as $\frac{\beta}{\gamma}$. A related concept, the effective reproductive rate, equals R_0 multiplied by the fraction of the host population which is susceptible. Epidemiologists have noted that diseases with high R_0 are likely more difficult to eradicate, because a large fraction of the susceptible population must be immunized in order for eradication to be achieved. For example, Anderson and May (1991) discuss how smallpox and polio have relatively small R_0, which may in part explain the success of the global eradication campaigns, and argue that the relatively high R_0 for pertussis (whooping cough) has likely contributed towards the difficulty of eradication effort. Our results on subsidies refine this statement in the sense that the relevant threshold in our model depends not only on the disease-specific parameters but also the efficacy of the vaccine.

4.3. Incentives to Develop Drugs Versus Vaccines

A perennial concern among commentators on the pharmaceutical industry is the potential bias firms have in favor of developing drugs rather than vaccines to treat diseases. There is a general feeling that drugs provide firms with more revenue, although a less concrete understanding of why. For example, a prominent author on the struggle to develop an AIDS vaccine writes:

> Private companies find vaccines less financially rewarding than drugs. In 2001, the global marketplace for therapeutic drugs exceeded $300 billion, whereas worldwide vaccine sales were only about $5 billion…. It is not hard to understand why major pharmaceutical companies, capable of developing drugs and preventive vaccines, generally invest in drugs that patients must take every day rather than shots given only occasionally. Drug company executives have investors to answer to, after all. (Thomas, 2002)

This account leaves unclear why consumers are not willing to pay the expected present value of the stream of benefits in an up-front lump sum for a vaccine, in which case the firm would earn the same revenue from a vaccine as from a drug.

The analysis in the preceding two sections provides an internally consistent argument for firm's bias against vaccines. By preventing individuals from becoming infected, vaccines interfere with the transmission of the disease among consumers, a positive externality which the firm cannot appropriate. On

the other hand, drugs often treat symptoms of the disease without curing it or reducing its transmission. Since there are no unappropriated externalities with drugs, the firm can potentially earn more revenue with them, and from an ex ante perspective would have more incentive to invest in developing a drug than similarly effective vaccine.

We can build upon the analysis from the previous two sections to quantify the monopoly manufacturer's bias toward a drug and against a vaccine. Consider a drug that is similar in all ways to the vaccine considered previously except that the drug does not interfere with disease transmission. In particular both are costless to manufacture and administer and have no side effects. Assume that, like the vaccine, the drug treatment is effective with probability θ. Assume that if the drug treatment is effective, it eliminates any harm from the symptoms experienced by infected individuals but does not prevent them from transmitting the disease to susceptible individuals.

Computing the steady-state equilibrium in the drug market is straightforward. The monopolist can sell the drug for a price equal to the avoided harm from the symptoms (recall normalized to unity) times the probability θ that the drug successfully treats the symptoms. The drug can be sold to the entire mass of infected consumers. If $\beta \leq \gamma$, then equation (5) implies that mass of infected consumers converges to zero in the steady state, so steady-state equilibrium drug quantity and profit equals zero. If $\beta > \gamma$, then substituting $q = 0$ (because the quantity of vaccine is zero if the firm develops a drug) into equation (6) implies that mass of infected consumers is $1 - \gamma / \beta$ in the steady state. Since production is costless, drug profit is the product of price and quantity: $\Pi^d = \theta(1 - \gamma / \beta)$. Letting W^d denote the flow of social welfare in the steady-state equilibrium with a drug, we have $W^d = \Pi^d = \theta(1 - \gamma / \beta)$ because the firm is able to extract all of consumer surplus.

Taking the difference between drug profit Π^d computed in the previous paragraph and vaccine profit Π^* leads immediately to the next proposition. (The proof in Appendix A fills in some remaining details.)

Proposition 4. *Consider the benchmark model with a monopoly producer. If $\beta \le \gamma$ then the disease disappears in the steady state without a vaccine or drug, so neither product generates profit or social welfare. If $\beta > \gamma$ then the following statements hold. Drug profit always exceeds vaccine profit, with the difference given by*

$$\Pi^d - \Pi^* = \begin{cases} \theta\left(\dfrac{\sqrt{\beta}+\sqrt{\gamma}}{\sqrt{\beta}-\sqrt{\gamma}}\right) & \beta \in \left(\gamma, \dfrac{\gamma}{(1-\theta)^2}\right] \\ \dfrac{(1-\theta)(\beta-\gamma)}{(1-\theta)\beta-\gamma} & \beta \in \left(\dfrac{\gamma}{(1-\theta)^2}, \infty\right), \end{cases} \tag{14}$$

an expression which is increasing in θ and non-monotonic in γ and β, reaching a peak at $\beta = (1+\theta)^2\gamma$. Steady-state equilibrium social welfare is strictly higher with a vaccine than a drug if $\beta > \gamma\theta^2/(1-\theta)^2$ and strictly lower if the reverse inequality holds.

The comparative-static effects of parameters θ, γ, and β on the profit difference $\Pi^d - \Pi^*$ are drawn in Figure 2.3. The graphs look similar to those for the minimum optimal subsidy in Figure 2.2.

Proposition 4 states that the firm strictly prefers to develop the drug than the similarly effective vaccine if there is a market for the products. However, for a range of parameters, in particular if $\beta > \gamma\theta^2/(1-\theta)^2$, social welfare is higher with the vaccine. Therefore, for sufficiently high β or sufficiently low θ or γ, the firm develops the "wrong" product. While a first-best quantity of the drug is sold, a given drug dose is socially inferior to a vaccine dose because the drug dose offers no positive externality for other consumers. Thus our discussion of externalities in the previous two sections provides a rationale for the commonly perceived bias of firms against developing vaccines and toward drugs. Proposition 4 quantifies the size of the bias and characterizes when it should be expected to be largest.

5. Foreign Infections

In this section, we extend the epidemiological model to allow for an inflow of infections from a foreign source. The source could be from foreigners immigrating or traveling to a home country or could be brought back home by domestic citizens traveling abroad. We have three motives behind this extension. First, we

can reassure the reader that our results on the non-monotonicity in β of the marginal externality, minimum optimal subsidy, and drug-vaccine profit difference in the benchmark model are not an artifact of the disappearance of the disease for low β. With foreign infections, the disease never disappears, but the non-monotonicity result persists. In the limit as the rate of foreign infections approaches zero, the equilibrium converges smoothly to the benchmark one; the comparative-static graphs converge smoothly to their benchmark analogues in Figure 2. A second motive is that adding foreign infections adds realism to the epidemiological model without sacrificing closed-form solutions for steady-state equilibrium (although the some of the closed-form solutions are considerably more complex). A third motive is that a model with foreign infections allows the analysis of policy-relevant questions such as the cost-effectiveness of reducing the domestic disease burden by pursuing health interventions abroad.

Let ϕ be the foreign infectiveness parameter, embodying both the rate of contact between foreign and domestic individuals as well as the rate of transmission from these contacts to susceptible individuals. The epidemiological model is identical to that in equations (1) through (3) except that a term $\phi S(t)$ is added to rate of change of the infected population in equation (3):

$$\dot{I}(t) = \beta I(t)S(t) + \phi S(t) - \gamma I(t). \tag{15}$$

The analysis follows the same steps as in the benchmark case without foreign infections. The result for steady-state equilibrium is quite similar to Proposition 1. As before, if the vaccine is sufficiently effective (the threshold value of θ now a more complicated expression depending on ϕ), the firm reproduces the outcome from a perfectly effective vaccine. The equilibrium price is exactly the same as it would be without foreign infections: $p^* = (\theta/\gamma)(1-\sqrt{\gamma/\beta})$. If the vaccine is less effective than the threshold, the firm vaccinates all consumers. The equilibrium price is higher than it would be without foreign infections because the steady-state infection level is greater.

We will relegate the details of the analysis and the exact expressions for steady-state equilibrium prices, quantities, profits, and welfare to Appendix B. Here we will content to provide a numerical example demonstrating that equilibrium with foreign infections converges smoothly to the benchmark case without

foreign infections. Figure 3 reproduces one of the more interesting panels from Figure 2, namely Figure 2.1.3, which graphs the effect of β on the marginal externality x^*. This graph was interpreted in the benchmark case as displaying non-monotonicity, but the discrete jump down for low values of β could be attributed to the disappearance of the vaccine market. Besides the discrete jump down to zero at $\beta = \gamma$, the rest of the graph is monotonic.

Figure 3 shows that the same non-monotonicity arises even when the vaccine market does not disappear, maintained by the constant inflow of foreign infections. The figure shows that the marginal externality graph converges smoothly to the benchmark case of $\phi = 0$ as ϕ is reduced from 0.1 to 0.001. Although difficult to see from the graphs for $\phi = 0.01$ and $\phi = 0.001$, at higher magnifications it can be seen that the marginal externality is indeed slightly positive for all $\beta > 0$. Even at the present magnification, the graph for $\phi = 0.1$ is clearly positive for all $\beta > 0$.

The intuition behind why the marginal externality is small for the lowest values of β in the model with foreign infections is straightforward. As β becomes very low, the disease becomes very rare, rare enough that the marginal consumer is unlikely to spread the disease to other individuals even if he or she is not vaccinated. Therefore his or her vaccination provides very little external benefit.

6. Alternative Market Structures

This section extends the previous results for monopoly to more competitive market structures including perfect competition and Cournot (quantity) competition among $n \geq 1$ homogenous firms. The analysis of perfect competition is valuable because it a standard assumption in the literature on the economics of vaccines, often providing particularly simple expressions for equilibrium variables. The analysis of Cournot competition is valuable because it is empirically relevant for many real world vaccine markets, in which a few capacity-constrained manufacturers supply the market. The Cournot model also nests the monopolist case analyzed previously by setting $n = 1$.

6.1. Perfect Competition

Under perfect competition, the equilibrium price equals marginal cost. Assuming costless production, the vaccine ends up being given away for free in steady-state equilibrium. The first best is obtained, and the disease is eradicated in the steady state.

The results that the first best is obtained and the disease is eradicated are artifacts of the assumption of costless production. Appendix C computes the steady-state equilibrium with a positive marginal production cost. For completeness, the appendix works out the equilibrium with positive marginal cost in both the monopoly and perfect-competition cases.

Because consumers do not internalize the positive externality associated with vaccination, for certain parameters laid out in the appendix (in particular, when the transmission rate β is sufficiently close to γ, in leading to a peak in the marginal externality as we saw from Proposition 2, or if the vaccine is sufficiently costly), too few consumers end up being vaccinated in the perfectly competitive equilibrium compared to the first best.

6.2. Cournot Competition

Next consider Cournot competition among $n \geq 1$ homogeneous firms. The analysis here generalizes our earlier monopoly results, in that the monopoly results can be recovered by setting n to 1.

The analysis is similar to that in Section 3. In particular, the demand correspondence is the same as in equation (10). The difference is that, rather than a single firm maximizing industry profit, each Cournot firm simultaneously sets its quantity to maximize its individual profit. A firm's objective function under Cournot competition is derived by first inverting demand to get an expression for price in terms of industry quantity, and multiplying this price by individual firm output to obtain individual firm profit (returning to the assumption of costless production for simplicity). Maximizing this objective function, and using symmetry to solve for equilibrium, leads to the following proposition.

Proposition 5. *Suppose the vaccine is produced by* $n \geq 1$ *Cournot competitors. Let*

$$\psi = \frac{\gamma}{\beta}\left(\frac{n-1}{2n}\right) + \sqrt{\left[\frac{\gamma}{\beta}\left(\frac{n-1}{2n}\right)\right]^2 + \frac{\gamma}{\beta}}. \tag{16}$$

The steady-state equilibrium falls into one of three cases.

Case (i): $\beta \in [0, \gamma]$. *No infection and thus no vaccine sales.*

Case (ii): $\beta \in (\gamma, (1-\theta+\theta/n)\gamma/(1-\theta)^2]$. *The steady-state equilibrium price is* $p^* = (\theta/\gamma)(1-\gamma/\psi\beta)$, *flow industry quantity is* $q^* = (\gamma/\theta)(1-\psi)$, *flow industry profit is* $\Pi^* = (1-\gamma/\psi\beta)(1-\psi)$, *flow social welfare is* $W^* = 1-\psi$, *and the marginal externality is* $x^* = \theta/\psi\beta$.

Case (iii): $\beta \in ((1-\theta+\theta/n)\gamma/(1-\theta)^2, \infty)$. *The steady-state equilibrium price is* $p^* = (\theta/\gamma)(1-\theta-\gamma/\beta)/(1-\theta)$, *flow industry quantity is* $q^* = \gamma$, *flow industry profit is* $\Pi^* = \theta(1-\theta-\gamma/\beta)/(1-\theta)$, *flow social welfare is* $W^* = \theta$, *and the marginal externality is* $x^* = \theta/\beta(1-\theta)$.

Proposition 5 nests both Propositions 1 and 2 because it characterizes the marginal externality as well as the other equilibrium variables. It is straightforward to see that the proposition gives the monopoly results upon substituting $n = 1$.

Although the expressions for the Cournot equilibrium are more complicated than for monopoly, the main results derived in the monopoly case continue to hold in this more general setting, including the non-monotonicity of the marginal externality x^* in the transmission rate β.

One wrinkle that arises in our vaccine context that does not arise in the typical Cournot analysis is that firms produce no more than the number of newborns γ each instant in equilibrium. If firms produced more than γ, then this excess supply would result in a zero price for the vaccine and zero profits for the firms. This possibility arises in case (iii) of Proposition 5. The division of equilibrium industry output among the *n* firms is not unique in this case; in fact, any partition among the *n* firms of equilibrium industry output (from the symmetric outcome of equal shares to the fully asymmetric outcome in which one firm produces all of industry output) can arise in an equilibrium.

Starting from any of these partitions, if any firm tried to deviate by increasing its output, the excess of supply over demand would drive price and profit to zero, so the deviation would not be strictly profitable. This wrinkle leads to a discontinuity in the equilibrium variables at the interface between cases (ii) and (iii) for $n > 2$.

7. Conclusion

Individuals who benefit from the positive externalities of vaccination do not compensate the firm for the benefits they receive from the vaccine, implying that vaccine developers capture a private benefit which is only a fraction of the social benefit of their innovation.

Our integrated economic and epidemiological model formalizes this externality, and yields the prediction that, the externalities from vaccination are non-monotonic in a parameter representing the transmissibility of the disease, and peak when the transmissibility of a given disease is very close to the death rate in the population. Note that diseases with a transmissibility rate close to the death rate in the population will be rare in equilibrium, in the absence of a vaccine; thus, our work suggests that the externalities from vaccination are largest for rare diseases. We also show that the minimal optimal subsidy for vaccination that a government would choose to provide is also non-monotonic in the transmissibility of the disease, which in turn characterizes the diseases for which the potential welfare gains from vaccination subsidies would be largest.

Previous work (Kremer and Snyder 2004) has argued that differences in the timing of the administration of drug treatments and vaccines allow drug manufacturers to extract more rent from consumers than vaccine manufacturers, thus driving a wedge between private and social incentives to invest in vaccine research and development R&D). Both this previous work and the current paper discuss market distortions which suggest a case for subsidies for vaccine R&D—in the case of our previous work, subsidies for vaccine R&D for sexually transmitted diseases beyond those for pharmaceutical R&D in general; in the context of this paper, subsidies for R&D on vaccines for rare diseases.

Appendix A: Proofs of Propositions

Proof of Proposition 1. As noted in Section 2, if $\beta \le \gamma$ the disease disappears in the steady state even without a vaccine, so no vaccine is sold in steady-state equilibrium. Therefore, suppose $\beta > \gamma$ in the remainder of the proof.

Following the logic of Figure 1, the epidemiological correspondence (first equation in (6)) and demand correspondence (equation (9)) can be combined to eliminate *I* and express *q* as a function of *p*:

$$\hat{I}(p) = \frac{\gamma^2 p}{\beta(\theta - p)} = 1 - \frac{\theta q}{\gamma} - \frac{\gamma}{\beta}. \tag{17}$$

Rearranging equation (17) yields the expression for market demand $q(p)$ in equation (11). The monopolist's problem is to choose *p* to maximize profit $pq(p)$ subject to the constraint that quantity of vaccine sold must be non-negative and cannot exceed the inflow of newborns; i.e., $0 \le q(p) \le \gamma$.

If the constraint $q(p) \le \gamma$ does not bind, then we arrive at the interior optimum in case (ii) of the proposition, namely

$$p^* = \frac{\theta}{\gamma}\left(1 - \sqrt{\frac{\gamma}{\beta}}\right). \tag{18}$$

Substituting this price into equation (11) yields equilibrium quantity $q^* = (\gamma/\theta)(1 - \sqrt{\gamma/\beta})$. If $\beta < \gamma/(1-\theta)^2$, then this value of q^* violates the constraint $q(p) \le \gamma$. We have a corner solution given by the value of p for which the right-hand side of (11) equals γ, implying $p^* = (\theta/\gamma)(1 - \theta - \gamma/\beta)/(1-\theta)$ and $q^* = \gamma$.

To complete the proof, profit Π^* can be found by multiplying p^* and q^*. Flow social welfare from the vaccine was shown in Section 4 to equal $W^* = \theta q^*/\gamma$. Substituting the relevant values of q^* into this expression for W^* gives the expressions listed in the statement of the proposition. *Q.E.D.*

Proof of Proposition 3. The first best is achieved without any subsidy if either $\beta \le \gamma$ (because the disease disappears in the steady state without a vaccine) or $\beta > \gamma/(1-\theta)^2$ (since all consumers are vaccinated in equilibrium without a subsidy). It remains to characterize the minimum optimal subsidy when $\beta \in (\gamma, \gamma/(1-\theta)^2]$.

The monopolist maximizes $(p+s)q(p)$, where $q(p)$ is given in equation (11). The profit-maximizing price as a function of the subsidy is

$$p^*(s) = \frac{\theta}{\gamma} - \sqrt{\frac{\theta(\theta+\gamma s)}{\beta\gamma}}. \tag{19}$$

Substituting this price into $q(p)$ and recognizing the constraint $q(p) \le \gamma$ yields the equilibrium quantity as a function of the subsidy:

$$q^*(s) = \min\left(\gamma, \frac{\gamma}{\theta} - \frac{\gamma}{\sqrt{\beta\theta\gamma(\theta+\gamma s)}}\right). \tag{20}$$

The minimum optimal subsidy either (a) forces the constraint that all consumers are vaccinated to bind or (b) forces the infection rate to zero in the steady state. Take these two possibilities in turn. Regarding possibility (a), treating the binding constraint as an equality implies $q^*(s) = \gamma$. Substituting for $q^*(s)$ from equation (20) implies

$$s^* = \theta\left[\frac{1}{\beta(1-\theta)^2} - \frac{1}{\gamma}\right]. \tag{21}$$

Regarding possibility (b), equation (4) implies that the the infection rate is zero if $q^*(s) = \gamma(\beta-\gamma)/\beta\theta$. Substituting for $q^*(s)$ from equation (20) in this last equality yields

$$s^* = \frac{\gamma(\beta-\gamma)}{\beta\theta}. \tag{22}$$

If $\beta < \gamma/(1-\theta)$, then (21) is less than (22) and vice versa if $\beta > \gamma/(1-\theta)$. It can be verified that Figure 3 accurately depicts the shape of s^* for arbitrary parameter values and that Figure 2.2 yields the comparative-static results stated in the proposition. *Q.E.D.*

Proof of Proposition 4. We will verify the last statement of the proposition, which compares social welfare from a vaccine and drug. The rest follow fairly immediately from the text before the proposition and the statement of the proposition itself. Suppose $\beta > \gamma$ throughout the proof.

The paragraph preceding the proposition states that social welfare from the drug is $W^d = \theta(1-\gamma/\beta)$. Proposition 1 states that social welfare from the vaccine is $W^* = 1-\sqrt{\gamma/\beta}$ if $\beta \le \gamma/(1-\theta)^2$ and $W^* = \theta$ if $\beta > \gamma/(1-\theta)^2$. The welfare difference is positive if $\beta > \gamma/(1-\theta)^2$: $W^* - W^d = \theta\gamma/\beta > 0$. If $\beta \le \gamma/(1-\theta)^2$ then

$$W^* - W^d = \left(1 - \sqrt{\frac{\gamma}{\beta}}\right)\left[1 - \theta\left(1 + \sqrt{\frac{\gamma}{\beta}}\right)\right], \tag{23}$$

which is positive if $\beta > \gamma\theta^2/(1-\theta)^2$. Thus the weakest condition under which $W^* - W^d > 0$ is $\beta > \gamma\theta^2/(1-\theta)^2$. *Q.E.D.*

Proof of Proposition 7. Assume that total industry output in the steady-state Cournot equilibrium does not violate the constraint $q \le \gamma$. We will return to this constraint at the end of the proof.

Equation (11) gives industry demand as a function of price. Inverting to give price as a function of industry output,

$$p = \theta\left[\frac{1}{\gamma} - \frac{\gamma}{\beta(\gamma - \theta q)}\right]. \tag{24}$$

Under the assumption of costless production, firm *i*'s profit is

$$pq_i = \theta\left\{\frac{1}{\gamma} - \frac{\gamma}{\beta[\gamma - \theta(q_i + q_{-i})]}\right\}q_i, \tag{25}$$

where we have written total industry output *q* as the sum of firm *i*'s output, q_i, plus the output of *i*'s rivals, q_{-i}. Taking the first-order condition with respect to q_i, imposing symmetry (*i.e.*, substituting $q_{-i} = (N-1)q_i$), and rearranging, we have

$$q_i^* = \frac{\gamma}{\theta}\left(\frac{1}{n} - \frac{\psi}{n^2}\right), \tag{26}$$

where ψ is defined in equation (16).

Multiplying equation (26) by *n* gives the q^* stated in case (ii) of the proposition:

$$q^* = \frac{\gamma}{\theta}\left(1 - \frac{\psi}{n}\right). \tag{27}$$

Substituting this value of q^* into equation (24) yields the p^* stated in case (ii) of the proposition. Substituting q_i^* from equation (26) and p^* into equation (25) and multiplying by *n* yields the Π^* stated in case (ii) of the proposition. Substituting q^* from equation (27) into the expression for social welfare derived in the proof of Proposition 1 ($W^* = \theta h q^*/\gamma$) yields the W^* stated in case (ii) of the proposition.

Return to the constraint ignored at the outset of the proof, $q \le \gamma$. Substituting q^* from equation (27) into the constraint yields the condition dividing case (ii) from case (iii):

$$\beta \le \left(1 - \theta + \frac{\theta}{n}\right)\frac{\gamma}{(1-\theta)^2}. \tag{28}$$

If condition (28) does not hold, then the constraint $q \le \gamma$ binds. Equilibrium industry output is thus $q^* = \gamma$. Any partition of this industry output among individual firms can be an equilibrium, including the symmetric outcome in which firms each produce $q_i^* = \gamma / N$ as well as asymmetric partitions. All of the equilibrium industry variables (quantity, price, profit, welfare) are unchanged from case (iii) of Proposition 1. *Q.E.D.*

Appendix B: Foreign Infections

Consider the extension of the benchmark model in Section 5 to allow for an inflow of foreign infections. The steady state of the epidemiological model is found by solving the system of equations (1), (2), and (3), substituting $\dot{V}(t) = 0$ into (2) and $\dot{I}(t) = 0$ into (3). The solution involves a positive steady-state infection level:

$$I = \frac{1}{2\gamma\beta}\left\{(\gamma - \theta q)\beta - \gamma(\gamma + \phi) + \sqrt{[(\gamma - \theta q)\beta - \gamma(\gamma + \phi)]^2 - 4\gamma\beta\phi(\gamma - \theta q)}\right\}. \tag{29}$$

As in the benchmark model, the steady-state vaccinated population is $V = \theta q / \gamma$ and susceptible population is the residual $S = 1 - V - I$. The marginal consumer's indifference between buying the vaccine and not implies

$$p = \frac{\theta}{\gamma}\left(\frac{\beta I + \phi}{\gamma + \beta I + \phi}\right), \tag{30}$$

analogous to equation (8) in the benchmark model. Inverting yields the cutoff infection level

$$\hat{I}(p) = \frac{\gamma^2 p}{\beta(\theta - \gamma p)} - \frac{\phi}{\beta}, \tag{31}$$

analogous to equation (9) in the benchmark model. The intersection between the epidemiological correspondence (29) and the demand correspondence (31) yields the demand curve

$$q(p) = \frac{\gamma}{\theta}\left[1 - \frac{\theta\gamma}{\beta(\theta - \gamma p)}\right] + \frac{\phi}{\beta p}. \tag{32}$$

analogous to equation (11) in the benchmark model.

The monopolist's optimal price maximizes profit $pq(p)$ subject to the constraints $q(p) \geq 0$ and $q(p) \leq \gamma$. For $\phi > 0$ it can be shown that constraint $q(p) \geq 0$ never binds and can be ignored. It is immediate that the addition of foreign infections does not change the interior solution the optimal price from equation (18) because the profit function only changes by the constant ϕ / β from the benchmark model. Substituting for price from equation (18) into the constraint $q(p) \leq \gamma$ and rearranging implies that the optimal price is an interior solution if and only if

$$(2-\theta)\sqrt{\beta\gamma} - (1-\theta)\beta - \gamma \geq \phi . \tag{33}$$

If condition (33) holds, then the equilibrium is

$$p^* = \frac{\theta}{\gamma}\left(1 - \sqrt{\frac{\gamma}{\beta}}\right) \tag{34}$$

$$q^* = \frac{\gamma}{\theta}\left(1 - \sqrt{\frac{\gamma}{\beta}}\right) + \frac{\gamma\phi}{\theta(\beta - \sqrt{\beta\gamma})} \tag{35}$$

$$\Pi^* = \left(1 - \sqrt{\frac{\gamma}{\beta}}\right)^2 + \frac{\phi}{\beta} \tag{36}$$

$$W^* = 1 - \sqrt{\frac{\gamma}{\beta}} + \frac{\phi}{\beta - \sqrt{\beta\gamma}} . \tag{37}$$

If condition (33) does not hold, then the equilibrium is

$$p^* = \frac{\beta(1-\theta) - (\gamma+\phi) + \sqrt{[\beta(1-\theta) - (\gamma+\phi)]^2 + 4\beta(1-\theta)\phi}}{2\gamma\beta(1-\theta)} \tag{38}$$

$$q^* = \gamma \tag{39}$$

$$\Pi^* = \frac{\beta(1-\theta) - (\gamma+\phi) + \sqrt{[\beta(1-\theta) - (\gamma+\phi)]^2 + 4\beta(1-\theta)\phi}}{2\beta(1-\theta)} \tag{40}$$

$$W^* = \theta . \tag{41}$$

Q.E.D.

Appendix C: Positive Marginal Cost

C.1. Existence of Vaccine Market Let $c \geq 0$ be the monopolist's marginal cost of producing and administering a dose of the vaccine. A consumer's gross surplus from the vaccine is bounded above by θ / γ, where θ / γ equals the benefit that a consumer who was certainly infected with the disease would receive from a cure with the same effectiveness as the vaccine (the cure is effective with probability θ, relieving the unit harm over the consumer's expected lifespan $1/\gamma$). For equilibrium to involve positive vaccine sales and thus be non-trivial, the marginal cost of a vaccine dose cannot exceed a consumer's gross benefit: $c < \theta / \gamma$, an assumption we will maintain throughout Appendix C.

C.2. Monopoly We will analyze how the addition of marginal-cost parameter *c* affects the expressions for steady-state monopoly equilibrium in Proposition 1. The epidemiological and demand correspondences are the same as in the proof of Proposition 1, yielding the same demand curve $q(p)$ as in equation (11). The only difference made by adding $c \geq 0$ is that rather than maximizing revenue the monopolist maximizes profit $(p-c)q(p)$ subject to constraint as before that $q(p) \leq \gamma$. We can immediately see that the problem is isomorphic to the monopolist's problem with a government subsidy analyzed in the proof of Proposition 3, where the monopolist maximized $(p+s)q(p)$.

Solving the analogous Kuhn-Tucker problem as in Proposition 3, it can be shown that if

$$\beta \leq \frac{\theta\gamma}{(1-\theta)^2(\theta - \gamma c)} \tag{42}$$

then constraint $q(p) \leq \gamma$ does not bind and the monopolist's optimal price is

$$p^* = \frac{\theta}{\gamma} - \sqrt{\frac{\theta(\theta - \gamma c)}{\beta\gamma}}, \tag{43}$$

the same as equation (19) once the substitution $s = -c$ is made. Analogous to equation (20), equilibrium monopoly quantity is

$$q^* = \min\left(\gamma, \frac{\gamma}{\theta} - \frac{\gamma}{\sqrt{\beta\theta\gamma(\theta - \gamma c)}}\right). \tag{44}$$

If condition (42) does not hold, then constraint $q(p) \leq \gamma$ binds. Then as in Proposition 1 we have $q^* = \gamma$ and $p^* = \theta h(1-\theta-\rho)/(1-\theta)\gamma$.

C.3. Perfect Competition Under perfect competition, equilibrium price for each of the $N \geq 2$ firms is $p_i^* = c$. Total vaccine quantity is

$$q^* = \min\left[\gamma, \left(\frac{\gamma}{\theta}\right)\left(1 - \frac{\theta\gamma}{\beta(\theta - \gamma c)}\right)\right]. \tag{45}$$

With a positive marginal cost, we need to derive a new expression for social welfare. Social welfare equals the expression derived in the proof of Proposition 1, $\theta q / \gamma$, minus the cost of production, *cq*:

$$W(q) = \frac{\theta q}{\gamma} - cq. \tag{46}$$

The first best is computed by maximizing equation (46) with respect to *q*. But (46) is linear in *q* and is increasing in *q* under the maintained assumption $c < \theta / \gamma$. Hence (46) is maximized for the highest feasible value of *q*. As discussed in the proof of Proposition 3, the highest feasible value of *q* either (a) forces the constraint that all consumers are vaccinated to bind or (b) forces the infection rate to zero in the steady state. Regarding (b), it can be shown that the infection rate can never be forced to zero in the steady state when $c > 0$. Thus we are left to investigate the condition under which, according to possibility (a), the first best is attained because $q^* = \gamma$.

Equation (46) can be manipulated to show that $q^* = \gamma$ if and only if

$$\beta \geq \frac{\theta\gamma}{(1 - \theta)(\theta - \gamma c)}. \tag{47}$$

If condition (47) is satisfied, the perfectly competitive equilibrium is socially inefficient. Condition (47) is satisfied if, among other conditions, β is sufficiently close to γ or c is sufficiently high.

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Table 1: Comparative Static Properties of Steady-State Equilibrium

Parameter change	p^*	q^*	Π^*	W^*
$\theta \uparrow$	+	−	+	+
$\gamma \uparrow$	−	?	−	−
$\beta \uparrow$	+	+	+	+

Notes: + indicates the starred variable is nondecreasing in the parameter, − indicates the starred variable nonincreasing in the parameter, 0 indicates the starred variable does not change with the parameter, and ? indicates that there are cases in which the starred variable increases and case in which it decreases with the parameter.

Figure 1. Solving for Equilibrium

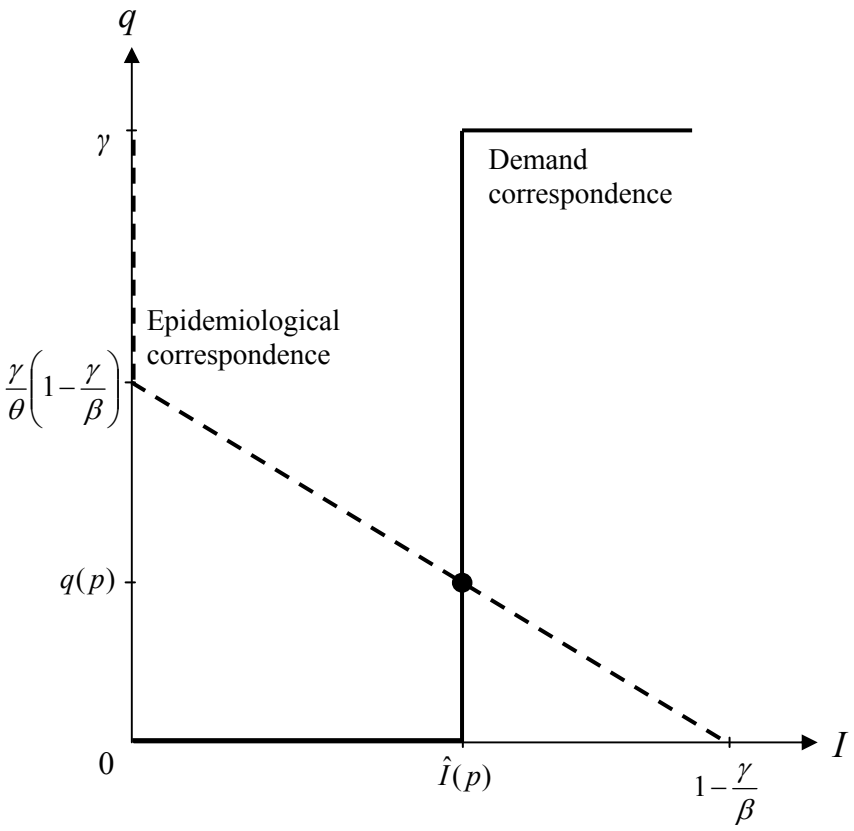
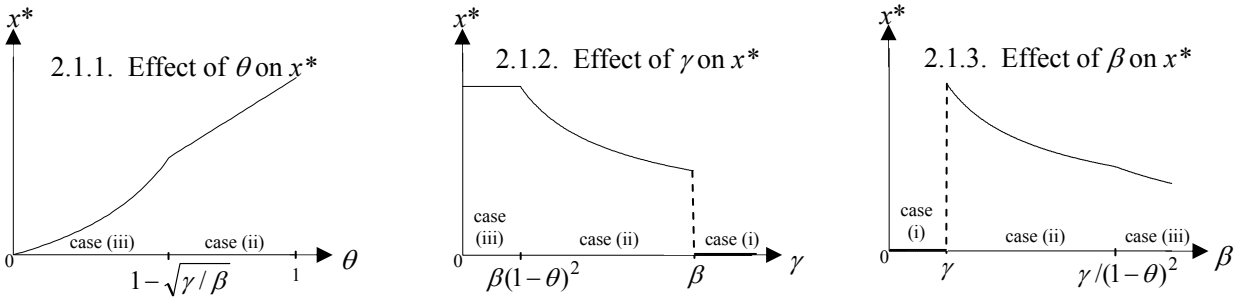
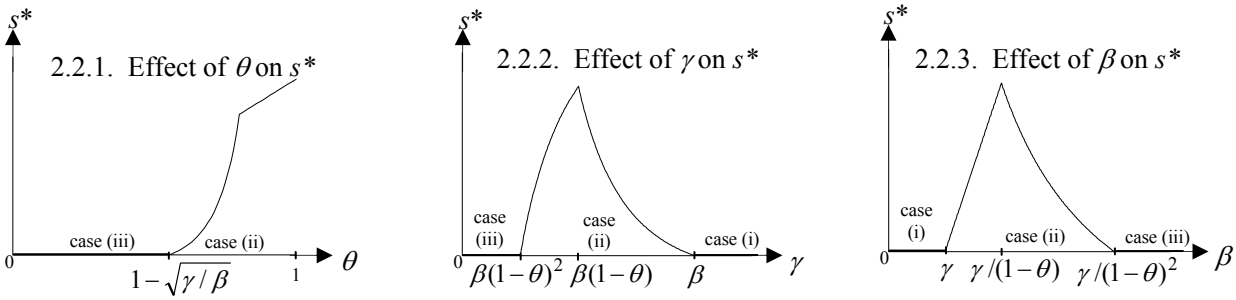


Figure 2. Comparative Statics

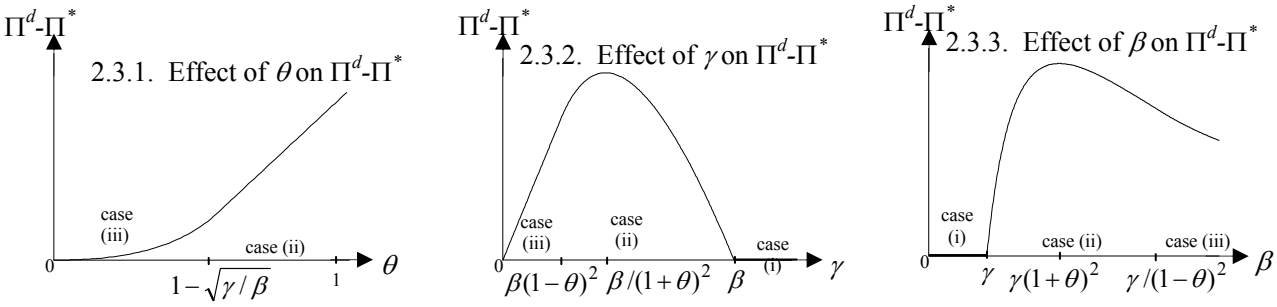
2.1. Effect of Parameters on Marginal Externality



2.2. Effect of Parameters on Minimum Optimal Subsidy



2.3. Effect of Parameters on Difference Between Drug and Vaccine Profit



Note: Cases refer to Proposition 1.

Figure 3. Marginal Externality in Foreign-Infections Model Converges to Benchmark