Housing Prices, Property Taxes and Neighborhood Effects*

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Ethan Cohen-Cole, Enrique Martinez-Garcia and Jonathan Morse†

Federal Reserve Bank of Boston and Federal Reserve Bank of Dallas

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Abstract

We explore the links between demographic cycles, housing and human capital acquisition in order to understand the determinants of housing prices. We build an overlapping generations (OLG) model with idiosyncratic, uninsurable risk, where households differ on their ability to produce income as a result of their choice of school district.

A well known feature of the U.S. housing market is that property taxes are raised to finance the public education system. This potentially creates an incentive for households to invest on housing in neighborhoods with better funding and high quality schools, and naturally drives prices up. This paper explores these features built into an OLG model with population cycles to account for the pricing effects of the 'baby boom' generation and the impact of the subsequent 'baby echo'.

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†Ethan Cohen-Cole, Federal Reserve Bank of Boston. 600 Atlantic Ave, Boston, MA 02210. Phone: +1 (617) 973-3294. E-mail: ethan.cohen-cole@bos.frb.org. Enrique Martinez-Garcia, Federal Reserve Bank of Dallas. Correspondence: 2200 N. Pearl Street, Dallas, TX 75201. Phone: +1 (214) 922-5262. Fax: +1 (214) 922-5194. E-mail: enrique.martinez-garcia@dal.frb.org. Webpage: http://dallasfed.org/research/bios/martinez-garcia.html. Jonathan Morse, Federal Reserve Bank of Boston. 600 Atlantic Ave, Boston, MA 02210. Phone: +1 (617) 973-3646. E-mail: jonathan.morse@bos.frb.org.

1 Introduction

The 15 years leading up to 2007 saw one of the largest increases in housing prices in U.S. history, followed by a sharp drop in 2007-2008. With the increases came a panoply of justifications, laments, and dire predictions. There have been few structural interpretations, but none that have moved into general acceptance, particularly given the crash that followed. This paper presents a structural rationale for the path of housing prices that shies away from labeling it as an asset 'bubble,' from characterizations of irrationality amongst condo-flip type investors, and from dire forecasts related to the prevalence of subprime. We instead look at an incentive-based argument that links housing prices with two economic features, one demographic, and one of political economy. We believe that the combination of demographic and political economy factors can generate much of the path of housing prices.

The first feature is the demographic force of the 'baby boom' and its persistent low-frequency effects, often labeled the 'baby echo'. The baby boom, as is well known, was a large increase in births around mid-century. The increase was followed by a reversion towards birth levels somewhat lower than prior to the boom. Once grown, the baby boomers began having their own kids and though they did so over a wider range of years than the original boom, the number of children increased again towards the end of the century (peaking in the late 1980s and early 1990s).

We argue that housing demand from the baby boomers was an important factor in housing price increases over the past 50 years. However, since the original 'baby boom' was much larger than the second, one would expect that the peak for housing prices would be around 1989, as predicted by Mankiw and Weil (1989). Naturally, we would expect to see another peak in prices once the echo kids reach adulthood. In fact, we see a number of peaks. The first of these peaks is around 1972 and the second around 1989. A third 'peak' has likely just passed. See Figure 1.

`[Insert Figure 1 about here]`

The second feature takes a political economy dimension because it is related to the nature of school financing in the United States. Anyone that has purchased or considered the purchase of a house is aware that the house's school district association is an important component of the house's price. In many areas, two similar houses located within a block of each other, but on opposite sides of a district boundary will likely have markedly different prices. The reason is that school financing in the U.S. is a local affair. In most localities, furthermore, schools are financed using a property tax on a mark-to-market home value[1].

School district funding is important because (many believe that) the stream of revenues is related to educational outcome, tertiary educational opportunities, and thus lifetime income opportunities. There is a long debate about the degree to which better financing is related to educational outcomes; since the belief is sufficiently widespread, we use it as a benchmark. This creates a link between current housing location and investment, and a child's future income prospects[2]. In particular, it links the choice of housing to the

[1] Some locations now have revenue sharing agreements and/or taxes based on the purchase price of a house. The former will impact our discussion to some degree by ameliorating the local taxation effects.

[2] We leave this detail mostly in the background. However, it could be argued that higher funding produces better educated people with higher expected returns in the labor market. There may be other factors reinforcing this basic pattern. Over the past 50 years, the number of students attending college has increased dramatically from XX to YY. However, the number of slots at premier institutions, either elite publicly financed ones or private ones, has remained almost unchanged. This has led to cyclical changes in admission rates for these institutions, leaving the overflow of students to resort to other options. The

demographic patterns of the children rather than the parents. A rational, forward-looking parent with a strong bequest motive will choose to invest in a house that maximizes his child's future lifetime earnings; importantly, this decision will be made long before the child begins to earn his livelihood.

The baby boom demographic patterns and the institutional framework of elementary education interact and take shape a bit more clearly now. The first housing price peak occurred with the boomers themselves. The second occurred when the echo generation was of about age 5; and essentially continues until 2007. We continue to have an empirical problem because the first peak of young children was much larger than the second, but the second price increase has been much longer than the first. See Figure 2.

[Insert Figure 2 about here]

Our model suggests that households' home purchase decisions, made around the time that children are entering grade school, could be location-specific. The characteristics of the location, specially the funding choices for education, tend to skew the price distribution across school districts and drive a run-up in prices. The skewness of the distribution gets accentuated over time and pushes the mean price of housing much higher than demographic factors would otherwise suggest. It also has relevant implications for the wealth and income distributions within each neighborhood. The combination of these location features and the population pressures arising from the demographic shift due to the baby echo have led to the large increases in housing prices in the first half of this decade, and could have anticipated the current price decline.

Figure 3 shows the correlation of population growth rates by age and our housing price index. Notice the two 'peaks' on the graph for children under 5 and for individuals in their late twenties. The twenties peak are the children of the original baby boomers. The very young children are the baby boomers grandchildren.

[Insert Figure 3 about here]

Notice the distinction here from Mankiw and Weil (1989). They find that the 'baby bust' led to a paucity of individuals of house-buying age in the late 1980s, and predicted a consequent fall in house prices in the 1990s. For a few years in the late 1980s, their predictions were proven correct as real housing prices collapsed. However, they did not forecast the role of boomers kids in driving housing prices up after that.

2 Literature Review

It is widely believed that school quality at the secondary level is responsible for both tertiary opportunities and future income levels. As a result, the 'quality' of school districts has long been thought to play a role in the price of housing. Tiebout (1956) argues that individuals self-sort into communities based on public spending preferences, and property values are consequently determined by the demand for those public services such as schooling quality. Figlio and Lucas (2000) find a significant correlation between school-wide testing scores and housing prices. Scores are aggregated by 'letter grade' in Gainesville, Florida, and the authors find that an 'A' has a market value of seven percent over a 'B.'

probability of admissions has declined, and therefore the pressure of housing prices has increased as parents search for the best district to improve the chances that their kids would get into a top school. While we do not model the decision mechanism on an individual basis, an information problem lies in the background. Since the student's ability is not perfectly revealed, choosing a school that sends a signal of quality is a way to increase likelihood of admissions and thus lifetime earnings.

Chiodo *et al.* (2005) find that the relationship between school quality and housing prices is highly nonlinear; a premium is paid for housing in better school zones, but no penalty is paid for worse school zones. They also provide an excellent survey on the broader literature that finds strong correlations of school testing scores and housing prices (see, e.g., Black, 1999, Downes and Zabel, 2002, Kane *et al.*, 2006, and Bayer *et al.*, 2003). Brewer *et al.* (1999) and James *et al.* (1989) find a significant economic return to attending elite private colleges, even after selection effects have been controlled for.

The majority of the current literature regarding the baby boom's financial impact evaluates the effect of changing demographics on equity prices, with only a select few explicitly addressing the baby boom's relationship with housing prices. The literature shows a weak consensus that the bust/boom/bust cycle of birthrate and population causes an overall decrease in asset returns. However, there has been relatively little attention paid to the so-called baby-echo; since the boomers are now reaching retirement age and hold far more assets in their portfolios than their children, the effects of the latter generation have been viewed as being of secondary importance by comparison.

Munnell (2004) suggests that life in the U.S. will not return to normal following the passing of the baby boom due to the fertility boom, longer lifespans, and lower mortality rates. Martin (2005) explains that the baby boom's effect on the working population also translates into housing prices, but Mankiw and Weil's (1989) paper missed the mark with their predictions. They worked in a partial equilibrium environment and failed to anticipate the demographic shift's effect on the discount rate. As agents anticipate their children becoming adults, they are willing to borrow against these future increases in wealth and the bond market and interest rates evolve accordingly. When this is accounted for, Martin's general equilibrium model includes the demand effects created by the exogenous demographic structure, and his model is successful in replicating the long-term real interest rates (10-year treasury yield adjusted for inflation) for 37 of the past 55 years.

Poterba (2004) agrees that overall asset returns are reduced for baby boomers, but offers a standard model showing a sharp upturn of asset holdings of the population during their 30's and a slow decline throughout their retirement years, contradicting claims of a 'meltdown' between the years of 2020-2050 as the baby boomer generation retires. Poterba's (2001) model shows that demographic shocks affect asset prices and returns, and argues that a large cohort of population should expect lower returns, while a small cohort can expect higher returns.

A number of authors have used an overlapping generations framework (OLG) to look at asset price effects. Brooks (2002) develops one to show that baby boomers will earn approximately 100 basis points lower return on their assets, but will be better off than their children as higher investment returns will move with them throughout their working lives. In addition, the boomers tend to have fewer children and are therefore able to begin saving at an earlier age than previous generations. Geanakoplos *et al.* (2004) offer a complex OLG model that incorporates realistic age/income patterns and captures Social Security, bequest motives, and other factors. Their findings show a larger impact of demography on asset values than previous studies. Abel (2003) presents analytical results based on an OLG model with variable supply of capital. This shows that the baby boom increases overall saving and investment in the U.S., resulting in a price increase of capital when compared to an economy with a constant birthrate.

Abel (2001) also shows that bequest motives can impact the results of capital pricing while accounting for a baby boom. Yoo (1994) presents an OLG model that sees an overall decline in asset prices. Finally, Lim and Weil (2003) present a forward-looking macro-demographic model that shows that an exogenous demographic structure has the power to impact stock prices if their 'capital installation costs' are large, but conventional estimates are not enough to explain price movements in response to demographic changes. Other authors that have addressed the baby boom and housing prices include Ang and Maddaloni (2003), Glaeser *et al.* (2005), Lamont and Stein (1999), and Topel and Rosen (1988).

Features of the OLG Model. The OLG model, introduced by Samuelson (1958), is built around an economy that extends *ad infinitum*, populated by generations of households that are born at different dates and have a finite lifetime. We assume that each household has a constant probability of dying each period, and that the newborn households vary cyclically making the population time-varying too. This structure induces heterogeneity across households at each point in time, and allows for non-trivial life-cycle considerations for a given household across time as well as non-trivial population dynamics.

Furthermore, the model introduces a particular breakdown of the asset market structure. Other models of incomplete asset markets restrict the availability of financial assets, and therefore limit the savings and the opportunities to hedge risks available to households. The OLG model introduces market incompleteness independently of the asset availability. This is the result of incomplete markets for insurance purposes.

First, we allow for the possibility of private borrowing and lending (what used to be called 'inside money'). Households may die the next period and, therefore, it is 'riskier' to lend them money. In other words, the market for death insurance is missing. This feature of the model can generate substantial differences relative to models where households are infinitely-lived[3]. In extreme circumstances (e.g., when death rates are high) borrowing and lending may stop altogether because lenders are uncertain about how much of the debt would be repaid, and therefore interests sky-rocket.

Households can also allocate part of their 'liquid' wealth or savings in the form of housing capital. This investment has some value in the future because real estate trades are also intergenerational. Even though, riskiness is higher, real estate gives households a new asset instrument to recursively attain additional hedging opportunities. In any case, this may not be sufficient to replicate the complete markets outcome.

Second, the riskiness associated with savings on bonds has spill-over effects on other markets, in particular on real state, because this gives the asset a new role. In our model, housing is valued for three reasons: (a) it produces housing services, (b) it influences the returns on education (parents can influence the future income stream of their descendents down the family tree through the educational channel), and (c) it serves as a *deposit of value* and *means of exchange* to move resources intertemporally. If asset markets were complete, in the absence of housing services and returns to education there would be no effective demand for real estate. It would be a redundant asset because the returns to real estate could always be replicated with a combination of all other assets.

The heterogeneity of households and the role of market incompleteness is reinforced with our assumption that housing location is related to returns on education. Naturally, this introduces another missing market because there is simply no market insurance to protect young households from being residing into a neighborhood with taxes that are either too high or too low for their own 'needs'. And, even though location decisions may greatly influence their prospects in life, there is no insurance against the 'wrong' funding

[3] It is also worth noticing that the first welfare theorem does not apply to OLG models, so a competitive equilibrium may not be Pareto optimal. However, the potential inefficiency of the competitive equilibrium in the model has nothing to do with the fact that households overlap in a way that prevents them from getting together at date $t = 1$ and exploiting all possible gains from trade. That is, the results are not due to the fact that at date t the only feasible trades are between the generation born at time $t - 1$ and time t.

scheme for the school district.

3 The Model

We study a version of the overlapping generations model (henceforth, OLG model) that accounts for certain demographic features as well as for an important characteristic in the financing of public education across school districts in the U.S. This framework establishes a link between the housing market and the education system through property taxes. We use this model to explain the evolution over time (and the observed dispersion) of housing prices across neighborhoods. We also investigate the role that demographic features have on housing prices.

3.1 Population and Geography

The economy is populated by overlapping generations with a life-span of J periods. The world operates with one school district. We denote the school districts as q. In each period, households die with an exogenous probability. The probability $\pi_j = P\,(\text{alive at } j+1 \mid \text{alive at } j)$ denotes the survival rate at age $j+1$ conditional on having survived up to age j. We assume $\pi_j \in (0,1]$ for all $j \in \{1, 2, ..., J-1\}$ and $\pi_J = 0$ to indicate that no household within a given generation lives longer than a finite J number of periods. The survival rate is time independent, but varies with age. In other words, the survival rate varies depending on how old the individual is. Death is an exogenous random event but J is fixed, therefore 'involuntary' transfers of wealth will almost surely occur.

There is no altruistic bequest motive. These transfers are forced by the sudden death of a household, hence they are involuntary in nature. The 'involuntary' transfers of bonds are denoted T_t^b, and distributed as an equal lump-sum payment across all households currently alive. The 'involuntary' transfers of housing wealth are denoted T_t^h, and are redistributed as an equal lump-sum payment too. We can think of these transfers as an 'implicit' estate tax and redistribution scheme[4].

In each period, a new generation of households is also born. The mass of new households denoted n_t is exogenously given, and stochastic. The rate n_t captures cyclical variations of the population, and is assumed to be identical across all the different neighborhoods. The population mass n_t is interpreted as an aggregate shock to the economy. It allows us to explore the effects of population cycles on savings and housing choices.

[4] Since lifetime is uncertain, in order to avoid unintended bequest, some papers simply assume that a life insurance market is operative as in Yaari (1965) and Blanchard (1985). In particular, in each period individuals receive actuarially fair premia from competitive life insurance companies in exchange of their financial wealth when they pass away. Given the structure of the population, perfect competition in the insurance sector implies that the gross return on the insurance contract, incorporated in the consumer flow budget constraint, is equal to $\frac{1}{1-\pi_j}$.

3.2 Technologies[5]

Similar to Martin (2005), we include two final goods and one intermediate good in our model. One type of final good is the standard, perishable consumption good. The production of this good requires both labor and an intermediate good which we call education. Education itself is produced combining hours of study and public spending on education. The final output of the consumption good can be either consumed privately, spent on education or invested in the stock of housing stock (a durable good). The other type of final good available to households is housing services. Consumption of housing services comes from a flow derived from the stock of housing.

The production of consumption goods relies on a Cobb-Douglas technology function whose inputs are labor hours, x_t, and education, e_t,

$$y_t = a_t F\left(e_t, x_t\right) = a_t \left(e_t\right)^{\upsilon} \left(x_t\right)^{\zeta},$$

where a_t denotes an idiosyncratic productivity shock. For simplicity, we abstract entirely from capital. In the spirit of Ben-Porath (1967), we assume that the production of education is given by a Cobb-Douglas technology function whose inputs are study hours, v_t, and total spending on education, G_t,

$$\left(e_t\right)^{\upsilon} = \left(G_t\right)^{\phi_j} \left(v_t\right)^{1-\zeta}.$$

Education spending depends on housing because the school district is financed with property taxes[6]. Each household allocates his time simultaneously to production and education.

Every household within a school district believes it cannot influence public spending on education, hence G_t is treated as an externality. Education, in turn, is interpreted in a broad sense. It includes on-the-job learning, training and re-training, and spans over the entire life-cycle of the household. Education spending does not necessarily impact the education attainment equally over the life-cycle. The parameter ϕ_j is age-dependent, so the function can show varying returns to scale with age.

The variable for time spent in non-leisure activities, l_t, is an aggregate measure of the actual hours spent working and studying, respectively x_t and v_t. The Cobb-Douglas aggregator is,

$$l_t^{\theta_j} = \left(x_t\right)^{\zeta} \left(v_t\right)^{1-\zeta},$$

which is age-dependent in the parameter θ_j to account for decreasing marginal returns from labor and study

[5] For tractability, we implicitly assume that there is a continuum of identical firms fluctuating at the population rate to produce consumption goods, housing services and education. Each firm is entirely owned and operated by a single household to satisfy his own needs. We believe the model could incorporate also a general competitive framework where firms are independent profit-maximizers to which households supply labor and capital goods (if needed) on a competitive basis, such that firms make zero profits in equilibrium. The model could also be extended to allow for firm's stocks to be distributed across several households. However, neither one of these extensions is likely to add more light on housing prices.

[6] Consumption goods appear as an input in the production of education but, unlike Manuelli and Seshadri (2005), they appear in the form of government spending, G_t^q.

hours over the life-cycle. Then, the total production function of the household is summarized as[7],

$$y_t = a_t \left(G_t\right)^{\phi_j} \left(l_t\right)^{\theta_j}.$$

Final output is merely a function of an aggregate measure of time, l_t, and public spending on education, G_t. This structure greatly simplifies the model and suffices to introduce different returns to scale conditional on age, without altering significantly the performance of the model along the dimensions of interest.

We assume that there are no investment frictions (time-to-build, etc.), and therefore the investment in the stock of housing is utilized instantaneously. We denote the stock of housing (our durable good), h_t, and assume a proportional service flow, s_t,

$$s_t = m h_{t+1},$$

where m is a given constant. This equation maps the housing stock into flows of housing services through a constant scaling factor. Housing services today are derived from the stock of housing bought for tomorrow. This timing convention is also adopted by Martin (2005). Because the purchase of tomorrow's housing is decided today, households perceive some 'rents' from the moment of the purchase onwards.

In our model, this timing convention introduces an 'implicit' bequest motive that we cannot ignore. A household that survives up until age J realizes that he will die at the end of the period. However, he is still willing to invest in housing because of the services it extracts today. This ensures that there will always be a nonnegative amount of housing to be transferred to the next generation (and everybody else that stays alive). Hence, 'involuntary' transfers may not be all that involuntary after all.

The same technologies to produce consumption goods, housing services and education are available to each household.

3.3 Preferences

Households are endowed with no wealth of their own (neither bonds nor housing) when they are born, except for the 'involuntary transfers' redistributed from households exiting in that period. Each household is also endowed with his long-run productivity at birth. The first generation to be born in this economy at time $t = 0$ has size n_0. At that point no other prior generation is alive, and households are scattered uniformly across the school district.

Location affects the value of wealth transferred to the new generation because the housing markets are segmented and households must sell their housing stock in the market where they are born. Households maximize their lifetime discounted utility, which is a standard time-separable function of the form,

$$\sum\nolimits_{j=1}^{J} \pi_j \beta^{j-1} \mathbb{E}_t \left[u\left(c_{t+j}, s_{t+j}, l_{t+j}\right)\right],$$

where $\beta \in (0,1)$ denotes the corresponding time-discount factor. The period utility function is additively separable in consumption, c_t, housing services, s_t, and non-leisure time, l_t,

$$u\left(c_t, s_t, l_t\right) = \frac{c_t^{1-\sigma} - 1}{1-\sigma} + \kappa_s \frac{s_t^{1-\gamma} - 1}{1-\gamma} - \kappa_l \frac{l_t^{1+\varphi}}{1+\varphi}.$$

[7] We could add a productivity shock to the education function. However, that would be subsumed into the productivity shock for output after substituting the education function into the technology function.

Expectations are taken with respect to the stochastic processes governing the idiosyncratic productivity shock, a_t, the stochastic process driving the aggregate shock, n_t, and the survival rate, π_j.

3.4 Shocks

Each household faces an idiosyncratic productivity shock, a_t, which is not fully insurable, and an aggregate shock, n_t, that captures the size of a generation born at time t. We assume that $a_t \in \mathbb{A} = \{a_1, ..., a_{I^a}\}$ is a realization of the idiosyncratic stochastic process, which is independent and identically distributed across households. This process evolves according to a finite-state Markov chain with stationary transitions,

$$F^a(A \mid a) = P(a_{t+1} \in A \mid a_t = a).$$

We assume that $n_t \in \mathbb{N} = \{n_1, ..., n_{I^n}\}$ is a realization of the aggregate stochastic process. The aggregate population shock is independent from the idiosyncratic productivity shock. This process evolves according to a finite-state Markov chain with stationary transitions,

$$F^n(N \mid n) = P(n_{t+1} \in N \mid n_t = n).$$

We assume that F^a and F^n consist of only strictly positive entries. Hence, there exists a unique, strictly positive, invariant distribution for F^a and F^n, which we denote respectively $\overline{F}^a$ and $\overline{F}^n$.

All households receive at birth the long-run average of their idiosyncratic productivity shock, i.e. $\overline{a}$. The size of the population of newborns will be determined by the realization of the aggregate shock, n_t. The size of each birth cohort, obviously, does not change over time except by death until the entire generation completely disappears J periods later. Hence, n_t represents the population size at birth and once determined it remains fixed once and for all. However, the population size of a given generation still varies with age due to the positive probability that individual households will die out.

We compute the long-run average of the shocks as $\overline{a} = \sum_{a \in A} a\overline{F}^a$ and $\overline{n} = \sum_{n \in N} n\overline{F}^n$, where $\overline{a} \in A$ and $\overline{n} \in N$. From here, we derive the initial conditions for the idiosyncratic productivity shock, and assume that $\overline{n} = n_0 = 1$. In a simplistic way, this makes good the old saying that 'we are all equal at birth'. However, different realizations of the idiosyncratic shocks, the death rate and the endogenous decisions of households lead to cross-sectional income and wealth differences over the entire life-cycle. Property taxes and public spending also contribute to magnify these effects.

Conesa and Krueger (2006) and Conesa *et al.* (2007), among others, argue that in the absence of insurance markets for the idiosyncratic risks, the tax system can be administered to share some of those risks across households. This provides the government with a purpose and a tool to 'improve' the allocation of the households. Of course, in all these models, heterogeneity among households due to differences in education or innate abilities is entirely exogenous. In fact, households are born with certain ability level and it does not change over their lifetime. This means that differences in income and wealth arise due to idiosyncratic shocks even among households with the same education level.

In our model, instead, the education and skills that a household displays are not constant over time, but endogenously determined. Furthermore, property taxes affect the level of education that a household can attain (because they serve to finance the school district), and therefore his productivity. One important way in which households' income profile can change is simply through changes in funding for the school district.

In this context, school district-specific policies on taxation and public spending become an important source of heterogeneity for the population. Now, it becomes less obvious that taxes may be effective in sharing risks across households because they may simultaneously increase the heterogeneity across individuals and age groups.

3.5 Budget Constraints

The school districts are financed with a constant property tax, τ. Schools tax away a proportion of the value of housing owned by residents within their district (whether through direct purchases or 'involuntary' transfers). The same tax rate applies to all households, and all the revenue raised is used to finance education within the district. Borrowing and transfers across districts are ruled out, no other revenue sources are permitted, and school districts are required to balance their budgets every period.

The stock of housing, h_t, is a homogeneous, durable good. Households derive a flow of housing services from the housing stock they acquire today. However, they can also realize capital gains by buying and selling property in the housing market. In principle, all markets (for consumption goods, housing and bonds) are fully integrated across locations. The depreciation of the aggregate stock of housing, δ, reflects the impact of weather patterns that can affect the preservation of the property, crime rates, sanitation services, availability and quality of maintenance, etc., all of which influence the depreciation of the stock.

Frictions generated by heterogeneity of housing units, indivisibilities and the matching of buyers and sellers are ignored. We assume that housing units can be measured on a homogeneous scale through the use of some hedonic index (or a similar measure), that market transactions can be treated as if they occur in a frictionless world, and that housing markets operate otherwise efficiently, except perhaps because of the distortion induced by property taxes.

Households invest their savings in zero-net supply bonds, b_{t+1}, or the stock of housing, h_{t+1}. The risk-free returns of the zero-net supply bonds are denoted r_t, while the price of housing is p_t. Taking the consumption good as numeraire, we define the budget constraint of the household as,

$$c_t + b_{t+1} + p_t h_{t+1} = y_t + r_t \left(b_t + (\lambda_t)^{-1} T_t^b \right) + (1-\tau)(1-\delta) p_t \left(h_t + (\lambda_t)^{-1} T_t^h \right),$$

where the fraction the size of the entire population in the school district is denoted as,

$$\lambda_t = \int d\Gamma_t,$$

and Γ_t is the joint distribution of the state vector at time t (we describe this distribution later on). We use this notation for generality, but obviously total population would depend on size of all generations born within the last J periods adjusted according to the survival rate of each generation during the current period. Households pay taxes on the current value of the depreciated stock of housing, independently of whether this housing stock was bought by the household one period before or whether it comes from 'involuntary' transfers, T_t^h.

Household finances depend critically on how the purchase of a house can be financed. First, we impose a nonnegativity (or no short-selling) constraint on housing,

$$h_{t+1} \geq 0.$$

This restriction is trivially satisfied given our choice of preferences. But ensures (and emphasizes) that there is always some collateral asset available to repay debts. It is worth noticing that housing is a durable good and has intrinsic value, therefore it can effectively be used to repay some of the outstanding debts. Furthermore, this specification guarantees that some positive amount of housing is transferred to the ‘new’ generation.

Second, we impose a constraint on borrowing. One option would be to consider that housing is used as a collateral for all debts in this model. For that, we could impose a lower bound on wealth. We define the value at time t of the household’s wealth for time $t+1$ net of borrowing as $w_{t+1} \equiv b_{t+1} + p_t h_{t+1}$. For a household born at time t, it must hold true that,

$$w_{t+j+1} \geq -(\varpi_j - 1)\, p_{t+j} h_{t+j+1},\ \forall j \in \{1, 2, ..., J\},$$

or simply,

$$b_{t+j+1} \geq -\varpi_j p_{t+j} h_{t+j+1},\ \forall j \in \{1, 2, ..., J\}.$$

Moreover, $\varpi_j \geq 0$ for all $j \in \{1, 2, ..., J-1\}$ and $\varpi_J = 0$. The lower-bound on wealth is independent of location, but may vary with age. It requires that households do not make debt commitments that are too large relative to the acquisition price of their next period housing stock[8]. However, this specification may easily lead to anomalous behavior. Households, independently of their income, can buy as much housing stock as they wish borrowing on the basis of the house’s value. Hence, all households will tend to invest in as much housing as they possible can and the borrowing constraint will have no bite. Of course, this set-up is exposed to many ‘dangers’ during times of declining house prices. The current situation in the financial markets may be a reflection of that.

Instead, we calibrate the model based on a more conventional financial restriction. Usually, the financial restriction is defined as a maximum fraction of total income devoted to repay the loan. We model that idea as follows,

$$r_{t+j} b_{t+j} \geq -\varpi_j y_{t+j},\ \forall j \in \{1, 2, ..., J\},$$

where total income is determined by the production function. If the household is a lender (i.e., $b_{t+j} \geq 0$), this constraint is completely irrelevant. If the household is a borrower (i.e., $b_{t+j} \geq 0$), then the constraint becomes important. This constraint ensures that households cannot borrow excessively beyond their means. In other words, the loan repayment (including interest and principal) cannot exceed a fraction ϖ_j of their income. This fraction is age-dependent, positive and $\varpi_J = 0$.

3.6 Market Structure

We assume that households cannot insure themselves against idiosyncratic shocks to labor productivity through an insurance market. Annuity markets to hedge the idiosyncratic risk of death are also missing in this specification. The aggregate population shock is obviously also uninsurable. Households still can trade one-period risk-free bonds (as described above) to insure themselves against all idiosyncratic risks. Bonds

[8] A household of age J is expected to die by the end of the period, hence he cannot borrow at all. This is an intuitive restriction, but not one that comes naturally from the model. In fact, lenders may take the risk because they anticipate that the debt would be repaid by another household (or group of them) because those commitments would not be defaulted, instead they are transferred after death.

are in zero-net supply, hence this amounts to an intragenerational (and to some extent an intergenerational) borrowing and lending scheme. However, it is only of limited use because: (a) borrowing is tied down by the need to use housing as a collateral, which prevents them from dying with pending debt repayments too large relative to the value of their housing properties, and (b) the OLG framework also restricts intragenerational trading or trading across age groups. Nobody wants to lend money to households at age J. As discussed before, the effect of taxes as a government-sponsored instrument to share risks is also unclear.

4 The Recursive Competitive Equilibrium

In this section we define the competitive equilibrium and the stationary competitive equilibrium. We denote the state vector that characterizes each individual household as (a_t, w_t, j). Hence, at any given time t, households are characterized by a random idiosyncratic shock, a_t, by their net wealth, w_t, by their stock of housing, h_t, and by their age, j.

The distribution function Γ_t and the aggregate population shock n_t are sufficient to describe the aggregate state of the economy. The role of the aggregate state variables is to allow households to predict future housing prices and interest rates. The part of the law of motion that depends on the aggregate population shock is exogenous, and can be described with the transition matrix for n_t. The joint distribution of the state vector at time t is given by $\Gamma_t \equiv \Gamma_t (a_t, w_t, j)$, and its law of motion can be expressed as follows,

$$\Gamma_{t+1} = \Lambda (\Gamma_t; n_t, n_{t+1}; \eta),$$

where $\eta = (\tau, \delta)$ is the vector of (constant) district-specific property tax rates and depreciation rates. To ease our notation requirements we do not keep track of these parameters, since they remain unchanged. However, we acknowledge their impact and anticipate the important role they can play if taxation decisions are endogenized.

With Γ_t we calculate the aggregate stock of housing in the market[9], H_t, as,

$$H_t = \int h_t d\Gamma_t (a_t, w_t, , j).$$

Aggregate housing satisfies the following law of motion[10],

$$H_{t+1} = (1 - \delta) H_t + I_t. \tag{1}$$

The stock of durable goods diminishes due to the effects of depreciation, but depreciation differs across

[9] H_t is not an aggregate state variable because when markets are incomplete aggregation fails to deliver a sufficient statistic to predict future housing prices and interest rates.

[10] Notice that by construction aggregate investment comes from,

$$\begin{aligned} I_t &= \int h_{t+1} d\Gamma_{t+1} (a_{t+1}, w_{t+1}, j) - (1 - \delta) \int h_t d\Gamma_t (a_t, w_t, j) \\ &= \int h_{t+1} dH (\Gamma_t (a_t, w_t, j); n_t, n_{t+1}) - (1 - \delta) \int h_t d\Gamma_t (a_t, w_t, j). \end{aligned}$$

This shows that aggregate investment depends on the current distribution of shocks Γ_t, which is our sufficient aggregate state, as long as $H(\cdot)$ which is taken as given.

neighborhoods.

The maximization problem is described below in the language of dynamic programming. To further ease our demands on notation, we use a 'prime' to denote next period values and (if needed) a 'double prime' to denote values two-periods ahead. We write the household's problem recursively as follows[11],

$$\begin{aligned} V(a,w,j;\Gamma,n) = & \max_{c,l,b',h'} u(c,s,l) + \pi_j \beta \mathbb{E}\left[V(a',w',j+1;\Gamma',n') \mid a,n\right] \\ = & \max_{c,l,b',h'} u(c,s,l) + \pi_j \beta \int_{a',n'} V(a',w',j+1;\Gamma',n')\, dF^a(a' \mid a)\, dF^n(n' \mid n), \end{aligned} \tag{2}$$

subject to,

$$\begin{aligned} & c + b' + ph' = a(G)^{\phi_j} l^{\theta_j} + r\left(b + (\lambda_t)^{-1} T^b\right) + (1-\tau)(1-\delta)p\left(h + (\lambda_t)^{-1} T^h\right), \ \forall j \in \{1,...,J\}, \\ & s = mh', \\ & (c,l,h') \geq 0, \ rb \geq -\varpi_j a(G)^{\phi_j} l^{\theta_j}, \ \forall j \in \{1,...,J\}, \\ & \Gamma' = \Lambda(\Gamma; n, n'), \end{aligned}$$

where c stands for consumption, l denotes time spent in labor or studies, and h and s represent the stock of housing and the flow of housing services respectively. The function V denotes the value function, and $\left(T^b, T^h\right)$ represent the 'involuntary' transfers. The control variables are consumption, c, labor supply, l, demand for bonds, b', and demand for housing, h'.

The maximal utility of a household is computed as in (2), while after the terminal period J the value function is set to zero, i.e. $V(a,w,J+1;\Gamma,n) = 0$. Hence, the value function $V(a,w,j;\Gamma,n)$ follows in a straightforward way by backward induction. Markets are competitive. The optimal decision rules for the household's problem can be represented as,

$$\begin{aligned} c &= c(a,w,j,p,r;\Gamma,n), \\ l &= l(a,w,j,p,r;\Gamma,n), \\ b' &= b'(a,w,j,p,r;\Gamma,n), \\ h' &= h'(a,w,j,p,r;\Gamma,n), \end{aligned}$$

given the housing price and the risk-free interest rate.

In this economy, there are markets for consumption goods, housing, and bonds (labor is non-marketable). The consumption good is taken as the numeraire. By Walras' law, to find the equilibrium prices we only need to use the market clearing conditions for bonds and housing in each district. The market clearing conditions are discussed later as part of the Recursive Competitive Equilibrium (RCE) definition. Equilibrium prices, nonetheless, should be a function of the aggregate state only, so we argue that,

$$\begin{aligned} p &= p(\Gamma,n), \\ r &= r(\Gamma,n), \end{aligned}$$

[11] We do not keep track of individual households because the decision of households in the same individual state variable coordinates are always the same.

where the idiosyncratic shock and other individual state variables (the stock of net wealth and the age of the household) do not affect prices directly. Therefore, all the decision rules can be expressed uniquely as functions of the individual and aggregate state variables, i.e.

$$\begin{aligned} c &= c\left(a, w, j; \Gamma, n\right), \\ l &= l\left(a, w, j; \Gamma, n\right), \\ b' &= b'\left(a, w, j; \Gamma, n\right), \\ h' &= h'\left(a, w, j; \Gamma, n\right), \\ q' &= q'\left(a, w, j; \Gamma, n\right), \end{aligned}$$

since the equilibrium prices are a function of the aggregate states[12]. This notation should be viewed as a short-hand for the deeper relationships that exist.

Finally, to close down the model we need to specify the school district budget constraint as follows,

$$G = \tau\left(1-\delta\right)p\left(\sum\nolimits_{i=1}^{J}\int \pi_i 1_{\{j=i\}} h d\Gamma\left(a, b, h, q, j\right) + T^h\right), \tag{3}$$

where $1_{\{j=i\}}$ is an index function that takes the value of one if the individual's age is i and takes the value of zero otherwise. This structure implies that current school district revenues come from the property taxes they collect from the stock of housing net of depreciation owned by the residents of the district. We take this tax scheme to be exogenously given, hence not subject to the optimization of the school officials or to voting. A natural extension of the model would allow for property taxes being set by majority rule, where τ summarizes the control variables used to determine the policies of the district on funding for education. We believe that this set-up lends itself easily to discuss relevant issues of education financing and political economy. However, if we were to endogenize the choice of property taxes now, the problem would become much harder.

Let $a \in \mathbb{A} = \{a_1, ..., a_{I^a}\}$, $n \in \mathbb{N} = \{n_1, ..., n_{I^n}\}$, $w \in \mathbb{R}$, and $j \in \mathbb{J} = \{1, ..., J\}$. Let $\mathbb{S} = \mathbb{A} \times \mathbb{N} \times \mathbb{R} \times \mathbb{J}$, $\mathbb{B}\left(\mathbb{R}\right)$ be the Borel $\sigma-$algebra of $\mathbb{R}$, and $\mathbb{P}\left(\mathbb{A}\right)$, $\mathbb{P}\left(\mathbb{N}\right)$, $\mathbb{P}\left(\mathbb{J}\right)$ the power sets of $\mathbb{A}$, $\mathbb{N}$, and $\mathbb{J}$, respectively. Finally, let $\mathbb{M}$ be the set of all finite measures over the measurable space $\left(\mathbb{S}, \mathbb{P}\left(\mathbb{A}\right) \times \mathbb{P}\left(\mathbb{N}\right) \times \mathbb{B}\left(\mathbb{R}\right) \times \mathbb{P}\left(\mathbb{J}\right)\right)$. Based on our set-up, we define the recursive competitive equilibrium (RCE) as follows,

Definition 1 *Let us take as given the school district property tax and the depreciation rate* $\{\eta\}$*, the individual state variable initial conditions (zero-wealth at birth except for 'involuntary' transfers and long-run idiosyncratic shocks), and the initial conditions on the aggregate state variables* (Γ_0, n_0)*. A* ***Recursive Competitive Equilibrium*** *is a set of a value function* $\{V : \mathbb{S} \to \mathbb{R}_+\}$*, of decision rules for the households* $\{c, l, h' : \mathbb{S} \to \mathbb{R}_+\} \cup \{b' : \mathbb{S} \to \mathbb{R}\}$*, of public spending on education* $\{G : \mathbb{R}_+ \to \mathbb{R}_+\}$*, and of 'involuntary transfers'* $\left\{T^b, T^h : \mathbb{R} \to \mathbb{R}\right\}$*, as well as a set of prices* $\{p, r : \mathbb{M} \to \mathbb{R}_+\}$*, a measure* $\Gamma \in \mathbb{M}$ *and its corresponding law of motion* $\{\Lambda : \mathbb{M} \to \mathbb{M}\}$*, such that:*

(i) Given $\left\{p, r, T^b, T^h, \Gamma, \Lambda, n, \eta\right\}$ *and the initial conditions, the functions* $V\left(\cdot\right)$*,* $c\left(\cdot\right)$*,* $l\left(\cdot\right)$*,* $b'\left(\cdot\right)$ *and* $h'\left(\cdot\right)$ *solve the household's problem in* $(2) - ($**??**$)$*.*

[12] Notice that given the equilibrium prices $p\left(\cdot\right)$ and $r\left(\cdot\right)$ we can easily compute the household time allocations $l\left(a, w, j; \Gamma, n\right)$, as well as the aggregate allocation $L = \int l\left(a, w, j; \Gamma, n\right) d\Gamma\left(a, w, j\right)$. Using the budget constraint we can also compute the household consumption each period $c\left(a, w, j; \Gamma, n\right)$, and the aggregate consumption $C = \int c\left(a, w, j; \Gamma, n\right) d\Gamma\left(a, w, j\right)$.

(ii) The school district budget constraints defined in (3) *are satisfied,*

$$G=\tau\left(1-\delta\right)p\left(\Gamma,n\right)\left(\sum\nolimits_{i=1}^{J}\int\pi_{i}1_{\{j=i\}}hd\Gamma\left(a,w,j\right)+T^{h}\right).$$

(iii) 'Involuntary' transfers are given by,

$$\begin{aligned} T^{b\prime} &= \int\left(b'\left(a,w,j;\Gamma,n\right)-\sum\nolimits_{i=1}^{J}\pi_{i}1_{\{j=i\}}b'\left(a,w,j;\Gamma,n\right)\right)d\Gamma\left(a,w,j\right), \\ T^{h\prime} &= \int\left(h'\left(a,w,j;\Gamma,n\right)-\sum\nolimits_{i=1}^{J}\pi_{i}1_{\{j=i\}}h'\left(a,w,j;\Gamma,n\right)\right)d\Gamma\left(a,w,j\right). \end{aligned}$$

(iv) Prices clear the bond market (bonds are in zero net supply) as well as the housing market,

$$\begin{aligned} \int b'\left(a,w,j,p,r;\Gamma,n\right)d\Gamma\left(a,w,j\right) &= 0, \\ \int h'\left(a,w,j,p,r;\Gamma,n\right)d\Gamma\left(a,w,j\right) &= H'. \end{aligned}$$

(v) The law of motion for the stock of housing in each market is given by (1)*. The resource constraint is also satisfied,*

$$\begin{aligned} &\int\left[c\left(a,w,j;\Gamma,n\right)+p\left(\Gamma,n\right)h'\left(a,w,j;\Gamma,n\right)\right]d\Gamma\left(a,w,j\right) \\ &=\int a\left(G\right)^{\phi_{j}}l\left(a,w,j;\Gamma,n\right)^{\theta_{j}}d\Gamma\left(a,w,j\right)+\left(1-\delta\right)p\left(\Gamma,n\right)H. \end{aligned}$$

(vi) The law of motion $\Gamma'=\Lambda\left(\Gamma;n,n'\right)$ *is given by,*

(a) for all $j'>1$*,*

$$\Gamma'\left(a',w',j'\right)=\int\widehat{F}\left(a'\mid a,w,j;\Gamma,n\right)d\Gamma\left(a,w,j\right),$$

where

$$\widehat{F}\left(a'\mid a,w,j;\Gamma,n\right)=\begin{cases}\pi_{j}F^{a}\left(a'\mid a\right)\ \textit{if} & \begin{array}{c} b'\left(a,w,j;\Gamma,n\right)=\left(1-\alpha'\right)w' \\ h'\left(a,w,j;\Gamma,n\right)=\alpha'\frac{w'}{p} \\ q'\left(a,w,j;\Gamma,n\right)=q' \\ j'=j+1\in\mathbb{J} \end{array} \\ & 0\ \textit{otherwise,}\end{cases}$$

(b) for $j'=1$*,*

$$\Gamma'\left(a',w',\{1\}\right)=\begin{cases} n'\ \textit{such that} & \begin{array}{c} a'=\overline{a} \\ b'=T^{b\prime} \\ h'=T^{h\prime} \end{array} \\ & 0\ \textit{otherwise.}\end{cases}$$

Definition 2 *A stationary equilibrium is a* ***Recursive Competitive Equilibrium*** *in which endogenous variables and functions as well as prices and policy rules are constant, and time-invariant. Moreover, the distribution across individual states is also time-invariant.*

5 A Toy Model and Some Motivation

The question that we want to answer is mainly a quantitative one: how much of the behavior in housing prices can we explain based on the stylized features of a public education system financed with property taxes and with demographics? To assess how significant the discrepancy is between the data and theory, it helps us to start by positing a toy model that captures some of the relevant features of housing (similar to Mankiw and Weil, 1989, but in general equilibrium). Then, we can always conjecture that the gap between this toy model prediction and the observed housing prices gives us an implicit upper bound estimate for the magnitude of the model mispecification.

We are going to make several fundamental simplifications to the housing model. As in Martin (2005), we assume inifinitely-lived agents (instead of overlapping generations, i.e. $J = +\infty$), and the aggregate stock of housing is in fixed supply along the balanced growth path.[13] We also assume that property taxes are assessed, rather than charge on a mark-to-market basis. We view the productivity and the population dynamics as aggregate shocks, but still treat education expenses as an externality in the production function.

These assumptions allow us to build a bridge between the model of Martin (2005) and the type of models we want to explore more generally. Most importantly, these assumptions allow us to obtain analytical solutions. By looking at the problem of an inifinitely-lived household, we eliminate one source of market incompleteness. In this model the lack of insurance against 'death' is no longer a problem. The assumption that all shocks are aggregate shocks implies that there is no income and wealth heterogeneity between the households; that is, all households are identical. Only aggregate shocks are priced in the value of houses since markets are complete. In principle, we could allow for borrowing constraints, but it will become clear that in the particular version of the toy model that we solve here household's borrowing equals zero.

5.1 The Features of the Toy Model

We assume that households own capital and labor and they rent them to firms. We normalize the consumption price to be equal to one. Under constant returns to scale, there is only need for one firm whose problem can be described as a static, one-period profit maximization, i.e.

$$\max_{K_t, H_t} Z_t F\left(\widetilde{G}_t, K_t, L_t\right) - r_t K_t - w_t L_t, \ \forall t, \tag{4}$$

where the production function takes the following form, i.e.

$$F\left(\widetilde{G}_t, K_t, L_t\right) \equiv \widetilde{G}_t K_t^{\alpha} L_t^{1-\alpha}.$$

The expenditure in education, $\widetilde{G}_t = \frac{G_t}{H_t}$, is viewed as an externality to the production function (see, e.g., Barro and Sala-i-Martin, 1992). We assume that $\widetilde{G}_t$ is a function of the tax rate in the school district, τ_t^h, as well as other relevant features like the tax value of housing relative to its market value, $\frac{p_t}{p_t^*}$, the potential

[13] We interpret the housing stock as a variable along its balance growth path, and ignore the transition dynamics for the housing stock at this stage.

dispersion on house ownership, $\int_0^{N_t} h_{jt} \ln \frac{h_{jt}}{\overline{H}_t} dj$, and the per capita ownership, $\overline{H}_t$, i.e.

$$\widetilde{G}_t = g\left(\tau^h, p_t^*, \frac{p_t}{p_t^*}, \overline{H}_t, \int_0^{N_t} h_{jt} \ln \frac{h_{jt}}{\overline{H}_t} dj\right).$$

In the version of the model we contemplate here the generality of this specification reduces to a constant that depends exclusively on an invariant tax rate and property value assessment, $g\left(\tau^h, p^*\right)$. This is so because we assume that the market value of housing does not enter into the calculation of property taxes.

Aggregate capital and labor are denoted K_t and L_t, respectively, while $Z_t \equiv A_t e^{a_t}$ is an aggregate productivity. The aggregate productivity can be decomposed into two components, one deterministic and one stochastic. Exogenous technological progress is captured by $A_t \equiv (1+g_a)^t$, so the state of technology evolves deterministically over time at a growth rate of g_a. We normalize the initial condition on productivity to be equal to one. The cyclical component of productivity in logs, a_t, evolves according to the following law of motion,

$$a_{t+1} = \rho_a a_t + \varepsilon_{t+1}^a, \;\; |\rho_a| < 1, \tag{5}$$

where ε_{t+1}^a is normally distributed, with mean zero and variance σ_a^2. Here, in the specification of the aggregate productivity shock we conform with standard practice in the RBC literature.

There is a continuum of households in the interval $[0, N_t]$ indexed by j. The exogenous population size is captured by $N_t \equiv (1+g_n)^t$, so the state of the population size evolves deterministically over time at a growth rate of g_n. Household j chooses consumption, c_{jt}, labor, l_{jt}, and savings in the form of either bonds, b_{jt+1}, capital, k_{jt+1}, or housing stock (a durable good), h_{jt+1}, in order to maximize their expected discounted utility. The housing stock is viewed as a durable good from which households extract housing services. Subject to sequential budget constraints and the laws of motion for the housing and capital stocks, an individual household j born at time t maximizes,

$$\begin{aligned}
&\max_{\{c_{jt+\tau}, l_{jt+\tau}, k_{jt+\tau+1}, h_{jt+\tau+1}, b_{jt+\tau+1}\}_{\tau=0}^{+\infty}} \sum_{\tau=0}^{+\infty} \beta^\tau \mathbb{E}_t \left[U\left(c_{jt+\tau}, s_{jt+\tau}, l_{jt+\tau}\right)\right], \; 0 < \beta < 1, \\
&s.t. \; c_{jt} + i_{jt}^k + p_t i_{jt}^h + q_t b_{jt+1} \le w_t l_{jt} + r_t k_{jt} + b_{jt} - \tau^h p_t h_{jt}, \\
&k_{jt+1} = \left(1-\delta^k\right) k_{jt} + i_{jt}^k, \; \delta^k = 1, \\
&h_{jt+1} = \left(1+g^h\right) h_{jt} + i_{jt}^h, \\
&s_{jt} = m h_{jt}, \; m = 1, \\
&\left(k_{jt+1}, h_{jt+1}, b_{jt+1}\right) \ge \left(\underline{k}, \underline{h}, \underline{b}\right), \; 0 < l_t \le e^{n_t}.
\end{aligned} \tag{6}$$

The housing price within a period, p_t, is the same for the acquisition of new housing stock or for the re-selling of the depreciated stock. Bonds are in zero-net supply, promise one unit of the consumption good in the following period and can be bought at a price q_t. Households receive income from renting capital and labor services to the firms, w_t and r_t. By simplicity, we assume that the capital stock fully depreciates within a period, i.e. $\delta^k = 1$. The stock of housing grows at a rate g^h and through household investments too.

Households pay a property tax, τ^h, proportional to the re-sale value of their stock of housing which is used to finance the school district. The budget constraint of the school district is balanced in each period, i.e.

$$G_t = \tau^h p^* H_t. \tag{7}$$

We adopt an additively separable specification for individual preferences, i.e.

$$U(c_{jt}, s_{jt}, l_{jt}) \equiv \ln(c_{jt}) + \kappa_s \ln(s_{jt}) + \kappa_l \ln(1 - l_{jt}), \tag{8}$$

where the utility of consumption is affected by the government expenditure. Public goods are in principle not directly valued by agents, however they do provide an externality in the production function. We assume that the housing service flow, s_{jt}, is proportional to the housing stock, h_{jt}. The factor of proportionality between housing services and housing stock is normalized to one, since it does not otherwise affect the decisions of the individual household.

We also assume that $\kappa_l = 0$, which implies that labor is supplied inelastically. We assume that household labor supply is exogenously given, random and identical for all households. Hence, the total aggregate labor supply is equal to the population size times a cyclical component, i.e. $L_t = N_t e^{n_t}$. The cyclical component of labor supply in logs, n_t, follows an exogenous stochastic process,

$$n_{t+1} = \rho_n n_t + \varepsilon_{t+1}^n, \;\; |\rho_n| < 1, \tag{9}$$

where ε_{t+1}^n is normally distributed, with mean zero and variance σ_n^2. This variable could capture the changes in employment and labor supply due to demographic factors (e.g., changes in working age population) or economic conditions (e.g., unemployment). We leave the door open to the possibility that population size and productivity might be correlated, but at this stage we simply assume that $corr(\varepsilon_t^a, \varepsilon_t^n) = 0$.

Prices will adjust such that all markets clear. By Walras' law that implies that the bond market, the housing market and the consumption market clear, i.e.

$$\int_0^{N_t} b_{it} di = B_t = 0, \; \forall t, \tag{10}$$

$$\int_0^{N_t} h_{it+1} di = H_{t+1} = \left(1 + g^h - \tau^h\right) H_t, \; \forall t, \tag{11}$$

$$C_t + G_t + I_t^k + p_t I_t^h = Z_t F\left(g\left(\tau^h\right), K_t, L_t\right). \tag{12}$$

These market clearing conditions reflect the fact that bonds are in zero net supply, the aggregate stock of housing grows at a fixed rate, and that all resources produced in this economy are either consumed privately or spend in education or invested in capital or housing stock. Ours is a model along a balanced growth path, so the assumption that housing growth is exogenously fixed is translated into the assumption that housing capital expands along the balanced growth path, i.e.

$$\left(1 + g^h - \tau^h\right) = (1 + g_a)^{\frac{1}{1-\alpha}} (1 + g_n). \tag{13}$$

A more complex model with variable housing capital supply would essentially add a transition path for housing towards the same balanced growth path anyway. This assumption not only simplifies our later calculations, it also guarantees that the per capita housing stock will not drop to zero (that would create a problem since production in this economy depends on per household property taxes through the education externality). By consistency, aggregate capital and consumption must satisfy that,

$$\int_0^{N_t} c_{it} di = C_t, \ \forall t, \tag{14}$$

$$\int_0^{N_t} k_{it+1} di = K_{t+1}, \ \forall t, \tag{15}$$

and also the aggregate law of motion for capital and investment should hold true, i.e.

$$K_{j+1} = \left(1 - \delta^k\right) K_t + I_t^k, \ \delta^k = 1, \tag{16}$$

$$H_{t+1} = \left(1 + g^h\right) H_t + I_t^h. \tag{17}$$

Under the assumption that all households start with the same initial conditions, we would expect that $\{c_{jt+\tau}, k_{jt+\tau+1}, h_{jt+\tau+1}, b_{jt+\tau+1}\}_{\tau=0}^{+\infty} = \{c_{t+\tau}, k_{t+\tau+1}, h_{t+\tau+1}, b_{t+\tau+1}\}_{\tau=0}^{+\infty}$ for all j.

The First-Order Conditions. From the necessary and sufficient first-order conditions of the problem of the firm, we obtain that the real wage rate, w_t, and the real rental price of capital, r_t, are equal to,

$$w_t = (1 - \alpha) Z_t g\left(\tau^h\right) \left(\frac{K_t}{L_t}\right)^{\alpha}, \tag{18}$$

$$r_t = \alpha Z_t g\left(\tau^h\right) \left(\frac{K_t}{L_t}\right)^{\alpha - 1}, \tag{19}$$

for all t. Assuming constant returns to scale in capital and labor, implies also that there are no profits in equilibrium. Hence, we can safely ignore issues regarding the ownership of the firms.

To characterize an interior solution, we obtain the following first-order conditions for the problem of the households,

$$\begin{aligned}
\frac{1}{c_{jt}} - \lambda_{jt} &= 0, \\
-\lambda_{jt} + \beta \mathbb{E}_t \left[\lambda_{jt+1}\left(r_{t+1} + \left(1 - \delta^k\right)\right)\right] &= 0, \\
-\lambda_{jt} p_t + \beta \mathbb{E}_t \left[\kappa_s \frac{1}{h_{jt+1}} + \lambda_{jt+1}\left(1 + g^h - \tau^h\right) p_{t+1}\right] &= 0, \\
-\lambda_{jt} q_t + \beta \mathbb{E}_t \left[\lambda_{jt+1}\right] &= 0.
\end{aligned}$$

In particular, we want to emphasize the Euler equations that determine the pricing of bonds and housing in this model. We summarize these two equations as follows,

$$\begin{aligned}
p_t &= \beta \mathbb{E}_t \left[\kappa_s \frac{c_{jt}}{h_{jt+1}} + \frac{c_{jt}}{c_{jt+1}} (1 + g_a)^{\frac{1}{1-\alpha}} (1 + g_n) p_{t+1}\right], \\
q_t &= \beta \mathbb{E}_t \left[\frac{c_{jt}}{c_{jt+1}}\right],
\end{aligned}$$

where we have already used the implicit characterization of the growth rate of the housing stock, i.e. $\left(1 + g^h - \tau^h\right) = (1 + g_a)^{\frac{1}{1-\alpha}} (1 + g_n)$. Pricing housing and bonds in this model would very much depend on these equations.

5.2 The Pricing Equations

Given our assumptions on the evolution of the stock of housing capital, i.e. $H_{t+j} = \left((1+g_a)^{\frac{1}{1-\alpha}}\right)^j (1+g_n)^j H_t$, then after some algebra we can determine the pricing equations that would prevail in this environment. In other words, the initial condition on the housing stock, H_t, pins down the evolution of housing capital along the balanced growth path. Hence, it follows that,

$$\begin{aligned} p_t &= \frac{\beta \kappa_s}{(1+g_n)(1+g_a)^{\frac{1}{1-\alpha}}(1-\beta(1+g_n))} \frac{\widehat{C}_t}{H_t}, \\ \widehat{C}_t &= \frac{g\left(\tau^h\right)}{1+\beta(1+g_n)\alpha\gamma_2} e^{a_t+(1-\alpha)n_t} \widehat{K}_t^{\alpha}, \end{aligned}$$

where $\widehat{C}_t = \frac{C_t}{A_t^{\frac{1}{1-\alpha}} N_t}$, and C_t is aggregate consumption. Similarly, $\widehat{K}_t = \frac{K_t}{A_t^{\frac{1}{1-\alpha}} N_t}$, and K_t is aggregate capital. For more details, see the technical appendix.

In other words, the price of housing today would be determined by ratio of consumption (along the balanced growth path) relative to today's housing supply stock. We have determined explicitly what the consumption path, and the housing and capital path should be in this economy (see appendix). However, it follows immediately that in logs,

$$\ln p_t = c_1 + c_2 t + c_3 a_t + c_4 n_t, \tag{20}$$

gives a good approximation of the solution, where the time trend captures potential differences in the balanced growth path for housing and capital. Notice the similarity between the model-implied pricing equation, and the Mankiw and Weil (1989) housing price regression. The housing price equation can be expressed as a log-linear function of capital in this analytical solution. It is worth noticing, however, that the stock of housing also plays a role. Ceteris paribus, the more housing supply available, the lower the price of housing as expected.

We know the housing stock evolves exogenously around the balanced growth path. After a shock, capital would eventually converge towards the balanced growth path, but does not sit on it. Therefore, housing prices would be determined by the transition of capital in response to those shocks. If capital grows much higher than along the balanced growth path, the income effect we would observe on the transition path leads to higher housing prices. In a similar vein the direct effect of either productivity shocks or labor supply shocks is to increase the price of housing. Although we cannot analyze the transitions between two different regimes of property taxes, we conclude by exploring the pricing formula that the sign with which property taxes operate would be crucial. E.g., if $g'\left(\tau^h\right) > 0$, then higher taxes would lead to higher productivity, higher income and consequently higher housing prices.

We can estimate the equation above to infer the following result,

Regression Estimates

	Productivity	Working-Age Pop.	Time ($\times 10^{-3}$)	Constant	Std. Dev.
housing prices	0.028024 (0.003524)	7.099459 (0.760323)	-0.292 (3.02×10^{-5})	-0.889803 (0.094435)	0.639×10^{-3}

The numbers in parenthesis are the standard errors, while the R-squared is 0.794221 and the adjusted R-squared is 0.789509. The power and the limitations of this simple theory can better be scrutinized with just a look at figure 4.

`[Insert Figure 4 about here]`

This plot clearly indicates that the predictive power of the model significantly deteriorates in the early 1990s. While the model, obviously, misses the huge price increases of the last decade, also grossly overstates the housing prices over the prior decade. The model, as Mankiw and Weil (1989) would have suggested, performed rather well until the end of the 1980s. This type of findings reinforces our view that demographic factors alone cannot explain the changes in housing prices.

If we make asset markets incomplete and the productivity shocks are treated as idiosyncratic shocks to each household, then housing prices will not only have to reflect the impact of aggregate shocks but also the effect of incomplete insurance against those idiosyncratic shocks. It will also have to reflect the impact of potentially binding borrowing constraints for some agents, and the interaction of these constraints with wealth and income inequality. Then, we would be able to also explore the relationship between inequality and housing prices. Introducing overlapping generations creates new sources of market incompleteness (in intergenerational trade) that will also be reflected in prices.

The question we are asking at this stage is whether prices can be accounted for in this simpler setting. If not, as suggested by our estimates and figure 4, then we want to ask whether there is a connection between inequality and housing prices and what can we do to quantify it. Eventually, we also want to understand whether moving from one school district to another (i.e., relocation) would contribute to increase/decrease inequality and increase/decrease housing prices. Why do we want all the features of the baseline model in the first place? For two reasons mainly. On one hand,because we would like to see if the same mechanism that explains housing prices is also consistent with the observed pattern of inequality. In more broader terms, we want to be in a position to answer the question: can changes in the funding of our schools lead to less (or more) wealth/income inequality across neighborhoods?

On the other hand, we want to be able to answer the question: how important is wealth inequality for the overall behavior of housing prices? In fact one thing that should be evident is that any two model that result in a mean-preserving wealth distribution, will have zero correlation between wealth inequality and housing prices. Whatever policy you can design to improve risk-sharing and reduce wealth inequality for your citizens, will be completely orthogonal to the behavior of housing prices (assuming that you do not change the overall education funding system). Or course, if the change in the property taxes is not mean-preserving, the implications can be of great 'quantitative' importance.

6 Benchmark OLG Model

6.1 Calibration

Here, we discuss the parameterization of the model used in our quantitative exploration of the benchmark economy.

Demographics. Households enter the economy at age at age 1 and retire at age 65. Our model is intended to match the live-cycle patterns of households until retirement age, J. We do not have much to say after that, so we impose the simplifying assumption that all households die with certainty after age 65. This choice of J replicates the compulsory retirement age in the U.S. The life-cycle literature includes models with a fixed lifespan and mortality models. In the fixed lifespan model, each generation lives with certainty until the terminal data J.

In turn, our mortality model each household within a generation can die with an exogenous probability prior to that limiting age. The survival rate is chosen to fit a polynomial function using data from actuarial life tables (Bell and Miller, 2005). The maximum lifespan in the actuarial life tables is 100, so the survival rate needs to be appropriately censored at 65. In other words, the probability of dying at age 65 reflects the probability of dying at 65 or over. We are particularly interested in the contribution of the baby echo generation born around 1980, therefore we calibrate the survival rates based on the 1980s cohort. This calibration ensures that the mortality process is consistent with the data.

The population cycles are obtained after a linear time-trend is extracted from the population in logs. The long-run average population growth rate for the U.S. is $X.XX\%$. Our demographic parameters are summarized in Table 1. The maximum age, J, the population growth and the survival probabilities together determine the population cycles of the model. All exogenous demographic parameters are summarized in Table 1. The demographic patterns, however, need to be completed by adding a birth rate. The birth rate is interpreted as an aggregate random shock to the economy, and we describe it later.

Table 1. Demographic Parameters

Parameter	Value	Target
Min. age	1	First year of life (assumed)
Max. age (J)	65	Compulsory retirement (assumed)
Survival probability, age-dependent (π_j)	Bell and Miller (2005)	Data
Population growth (in a linear time trend)	$X.XX\%$	Data

Preferences. We choose the discount factor, β, so that the annual real rate of return in steady state is 3%. We fix the inverse of the intertemporal elasticity of substitution, σ, to 2 reflecting the common view that a value above 2 is necessary to calibrate a macro model to match aggregate data. For example, Kydland and Prescott (1982) select 1.5 in their seminal paper, while Lucas (1990) argues that even an inverse elasticity of 2 may be too low for the macro data. We assume the coefficient of relative risk aversion on housing services, γ, is just 2, and set the inverse of the Frisch elasticity, φ, to 0.47 based on the evidence provided by Rotemberg and Woodford (1998a, 1998b). For simplicity, the scaling factors on the utility of housing services and labor, κ_s and κ_l respectively, are set equal to 1. All preference parameters are summarized in Table 2.

Table 2. Preference Parameters		
Parameter	Value	Target
Discount rate (β)	0.971	3% annual real interest rate
Inverse of the intertemporal elasticity of substitution (σ)	2	Kydland and Prescott (1982), Lucas (1990)*
Coefficient of relative risk aversion on housing services (γ)	2	Assumed
Inverse of the Frisch elasticity of labor supply (φ)	0.47	Rotemberg and Woodford (1998a, 1998b)
Scaling factor on the utility of housing services (κ_s)	1	Assumed
Scaling factor on the disutility of labor (κ_l)	1	Assumed

Technologies. We assume a production function of the Cobb-Douglas type with labor share, θ_j. We choose a labor share of 0.67, independent of age, in accordance with the long-run share for the U.S. economy. We abstract from capital, but require the production function to satisfy constant returns to scale. Therefore, the education spending share in output, ϕ_j, must be equal to $\phi_j = 1 - \theta_j = 0.33$. The fact that the labor share is age-dependent allows us the possibility of experimenting with variants of this benchmark where the impact of education spending diminishes with age. That feature can be exploited to capture a simple stylized facts: direct spending on education seems to be more relevant during the formative years of the individual, because it is at that stage of their life when they spend more time building up their human capital.

We assume that the ratio of housing services over housing stock, m, is merely 0.15. All technological parameters are described in Table 3. Evidently, we should pay special attention to the idiosyncratic productivity shocks while referring to technology, but those would be discussed later.

Table 3. Technology Parameters		
Parameter	Value	Target
Labor share in output, age-dependent (θ_j)	0.67	Long-run U.S. Labor Share
Education spending share in output, age-dependent (ϕ_j)	0.33	By Constant Returns to Scale
Ratio of housing services over housing stock (m)	0.15	Assumed

Housing and Location Parameters. We assume that there is only one neighborhood identified as one. Neighborhood zero is presumed to have either low property taxes and low depreciation of the housing capital or the opposite. These parameterization is intended to give households a stark contrast between the underlying patterns in the two alternative scenarios. In a rudimentary way, the calibration reflects the believe that the depreciation of the housing stock and property taxes tends to be higher in wealthier neighborhoods.

We conjecture that the lower bound on borrowing using the housing stock as a collateral is a function of the life-cycle. Households early in life and near their retirement age cannot borrow more than their houses are worth. The income of households tends to pick when they reach the middle age in the data. We conjecture that borrowing practices follow a similar pattern, and allow the households between 19 and 54 to borrow up to 9% above the value of their houses with the peak at 36. At age 65 a household knows for a fact that his life is coming to an end at the end of the period and this deserves a special mention.

A 65 year old household has naturally incentives to borrow as much as he can. After death, the household will not have to repay the debt, but his assets and debts will be distributed among all living households through the 'estate tax'. Therefore, in a world where these debts cannot be defaulted, there will always be an amount of lending directed to these retirement-age households. In order to avoid a potential distortion on the borrowing and lending patterns of the model and to ensure that the net wealth re-distributed to all other living households is non-negative (which is the pattern that we observe on average), we assume that households that survive to be 65 years of age cannot borrow anything. All housing and location parameters are described in Table 4.

Table 4. Housing and Location Parameters

Parameter	Value	Target
Number of neighborhoods (K)	1	Assumed
Neighborhood identification (q)	$q = 1$	Assumed
Property tax rates (τ)	$\tau = \{0.1, 0.2\}$	Assumed
Depreciation rate of housing stock (δ)	$\delta = \{0.05, 0.1\}$	Assumed
Lower bound on borrowing relative to collateral, age-dependent (ϖ_j)	$\varpi_j = \begin{cases} 1, \text{ if } j = 1-18, \\ 1.05 \nearrow 1.09, \text{ if } j = 19-36, \\ 1.09 \searrow 1.05, \text{ if } j = 37-54, \\ 1, \text{ if } j = 55-64, \\ 0, \text{ if } j = 65. \end{cases}$	N.A.

Markov Process for the Shocks. We assume that there are two types of shocks: an idiosyncratic productivity shock, a_t, and an aggregate birth rate shock, n_t. We also assume that there are three states of nature for each shock. Both shocks, however, are assumed to be independent and identically distributed (iid). The transition matrix for productivity shocks has an equal probability of $\frac{1}{3}$. Similarly, the transition matrix for the birth rate shock has an equal probability of $\frac{1}{3}$.

The average productivity is one, so the good and bad times are represented by a 25% increase or drop in the productivity level. This is an idiosyncratic shock, therefore by the law of large numbers the average productivity level of the entire population should tend to one in each period. The choice of a high dispersion of the productivity shock naturally introduces a significative income dispersion among households which, in turn, generates much stronger incentives for households to relocate across neighborhoods as the realization of the shocks progresses.

The average birth rate is 500 for each population. This choice gives us a large enough number of households per each generation without being too burdensome computationally. Good and bad times in the population cycle are represented by 20% birth rate increases or decreases respectively. Over time, the average size of the generation of newborns tends to 500. However, the birth rate is an aggregate shock, therefore the birth rates can be very different from one period to the next. This specification is intended as an approximation for the empirically relevant baby boom-baby bust cycles in the U.S. observed during the post-World War II period. All parameters needed to describe the productivity and birth rate shocks are described in Table 5.

Table 5. Markov Processes Parameters

Parameter	Value	Target
Number of shocks (assumed independent) ($\#Z$)	2	Assumed
Number of states of nature for productivity shock ($\#I^a$)	3	Assumed
Number of states of nature for population shock ($\#I^n$)	3	Assumed
Number of total states of nature ($\#n_i + \#a_i$)	$\#I^a + \#I^n = 6$	
Probabilities in the transition matrix (F^a_{ip}, F^n_{ip})	$(\#I^a)^2 + (\#I^n)^2 = 18$	
States of nature for productivity shock (I^a)	$I^a = 0.75, 1, 1.25$	Assumed
States of nature for population shock (I^n)	$I^n = 400, 500, 600$	Assumed

6.2 Theoretical Results

The borrowing function of the young depends among other things on the characteristics of the location of their house (in other words, the school district policies). We think this makes sense because where you live now affects your educational outcome and, therefore, your future income stream. These young families are simply borrowing against future income. We would expect that a young family living in a good neighborhood is more likely to buy a house in the 'good' neighborhood because it anticipates it will also attain a better education and become more productive. Therefore, it is also more confident to take on higher debt while young. Alternatively, a young family in a 'bad' neighborhood knows that its chances of getting a high paying job are slim, so it would be less inclined to borrow large amounts now and also less inclined to invest more on housing.

Housing prices in the 'good' neighborhood tend to be higher than in the 'bad' neighborhood because incentives are pushing the demand higher. Access to borrowing certainly makes more people able to pay for higher housing prices. However, in a more general model that would allow for relocation, this competition translates into even higher housing prices in the good neighborhood since the more productive households will want to pay higher prices in order to sort themselves apart from the less productive households. They do so by driving the prices high enough to ensure that the low productive households hit their borrowing constraints and are forced to either relocate to the 'bad' neighborhood or remain there. The model would predict that most borrowing-constrained, cash-strapped households live in the 'bad' neighborhood.

Moreover, whenever the birth rate increases and the population subsequently grows up, we expect these trends often to be exacerbated. The reasons are complex and multiple. However, it should be noted that whenever population grows the demand for housing is also likely to increase. Higher demand drives prices further up. This implies higher prices across all different neighborhoods, but the population growth may interact with the educational channel in the model to reinforce the trends on housing prices. That results in a spike in the average housing price across all neighborhoods.

[More Results Coming Soon...]

7 Concluding Remarks

[TBA...]

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Appendix

A Description of the Dataset

A.1 Productivity and Working-Age Population

We collect all quarterly U.S. data spanning the post-Bretton Woods period from 1970q1 through 2008q3 (for a total of 155 observations per series). All data is seasonally adjusted. We rely on aggregate data obtained from Thomson Datastream. However, the U.S. civilian non-institutional population is from Haver Analytics.

Data Series. We collect data on real output (rgdp), employment (emp), and population size (n) for the U.S.

◦ Real output (rgdp). Data at quarterly frequency, transformed to millions of U.S. Dollars, at constant prices, and seasonally adjusted. Source: Bureau of Economic Analysis.

◦ Employment (emp). Data at quarterly frequency, expressed in thousands, and seasonally adjusted. Source: OECD's *Economic Outlook*.

◦ Working-age Population between 16 and 64 years of age (pop): Data at quarterly frequency, expressed in thousands, and seasonally adjusted. Source: Bureau of Labor Statistics. We take the difference between civilian non-institutional population 16 and over and civilian non-institutional population 65 and over. We also seasonally-adjust the resulting series with the multiplicative method X12.

Updating Procedure. The real output (rgdp), and employment (emp) are expressed in per capita terms dividing each one of these series by the population size (pop). We express all variables in logs and multiply them by 100. At this point, we also define the productivity term as $a_t \approx y_t - \left(\frac{2}{3}\right) l_t$. Finally, the productivity (a_t) and working-age population (pop) series are estimated to fit an $AR(1)$ process with a linear trend.

A.2 Other Features of the Population

All the population data comes from the decennial censuses. The housing price index (HPI) is based on Robert Gordon's measure, and the data comes from the Office of Federal Housing Enterprise Oversight (OFHEO). To get the national HPI we use a weighted average of all state's HPI by year. We also linearly detrend the data for analysis.

Average Top-tier Tuition. This variable is the average of the annual tuition from the top 14 schools (according to the 2007's U.S. News and World Report college rankings). The following schools were chosen for their prestige, history, and wealth of available information: Princeton University, Harvard University, Yale University, Stanford University, Massachusetts Institute of Technology, University of Pennsylvania, Duke University, Dartmouth College, Columbia University in the City of New York, University of Chicago, Cornell University (All Campuses), Washington University, Northwestern University, and Brown University.

The NSF's WebCASPAR (www.webcaspar.nsf.gov) service was used to find the historical enrollment and tuition data from these schools (as well as a number of large public universities). The information came from the IPEDS Enrollment and Institutional Characteristics surveys, but it was not without imperfections, and in a few cases creative measures were taken to smooth the data. The primary problem with the dataset was that we are looking for information from 1975-2005 (the years that we have an HPI to compare possible influences), but the tuition data exists only until 2001. To provide data for 2001-2005 a few websites from these schools had archived news articles where the historical tuitions could be found. For those schools where this information was not available, a chart was found on a University of Pennsylvania almanac website (http://www.upenn.edu/almanac/v49/n26/undergrad_charges.html) detailing the annual percentage change in tuition charges for undergrads in major universities in the U.S. Using the percentage increases in tuition found there, the tuition through 2004 could be found.

However, there exists another problem with the data - the tuition for some years simply didn't exist in the WebCASPAR database. For example, Harvard University had no tuition data reported for 1983 or 1984. To mend this, a simple linear extrapolation was performed by fitting the previous five years' tuition data to those years. The result is not perfectly accurate, but our purposes were served as we are only looking for the average tuition for all of these institutions across time.

Top-tier Freshmen. This was also taken from the WebCASPAR site. The IPEDS first-year freshman enrollment data was intact and none of these measures were necessary.

Total, Male and Female. These indicate the population by state and year for men and women separately (total is obviously the sum of the two). The data was taken from the U.S. Census Bureau website (census.gov) and was largely ready to use, aside from the 1970's data. To clean the data, a clever MATLAB routine was written to clean up the data into the desired format of single years of age.

B Shocks and Markov Chains

We assume that a given stochastic process, s, can take on i values, i.e. $s \in S = \{s_1, ..., s_i\}$, which are symmetrically and evenly spaced over the interval $[\mu - \vartheta, \mu + \vartheta]$. Implicitly, we assume that the shocks can be centered around any value μ. The shock, in turn, follows a first-order Markov process with a symmetric transition probability Π. The particular family of Markov processes that we discuss here can be used to approximate an $AR(1)$ process. Furthermore, under certain conditions, the stationary distribution behaves as a symmetric binomial. The symmetric binomial, in turn, offers a discretization that can approximate the normal distribution of the shock arbitrarily closely.

The two state member of this family is characterized by the following transition matrix,

$$\Pi_3 = \begin{bmatrix} p & (1-p) \\ (1-q) & q \end{bmatrix},$$

with stationary distribution $\left(\frac{1-q}{(1-p)+(1-q)}, \frac{1-q}{(1-p)+(1-q)}\right)$ and first-order serial correlation of $p + q - 1$. The three state Markov process is characterized by the following transition matrix,

$$\Pi_3 = \begin{bmatrix} p^2 & 2p(1-p) & (1-p)^2 \\ p(1-q) & pq + (1-p)(1-q) & q(1-p) \\ (1-q)^2 & 2q(1-q) & q^2 \end{bmatrix},$$

with stationary distribution $\left(\frac{(1-q)^2}{((1-p)+(1-q))^2}, \frac{2(1-p)(1-q)}{((1-p)+(1-q))^2}, \frac{(1-p)^2}{((1-p)+(1-q))^2}\right)$ and first-order serial correlation of $p + q - 1$. The Π_i case can be derived recursively from Π_{i-1} by applying the following procedure. First, compute the following $(i \times i)$ matrix,

$$p \begin{bmatrix} \Pi_{i-1} & \mathbf{0} \\ \mathbf{0}^T & 0 \end{bmatrix} + (1-p) \begin{bmatrix} \mathbf{0} & \Pi_{i-1} \\ 0 & \mathbf{0}^T \end{bmatrix} + (1-q) \begin{bmatrix} \mathbf{0}^T & 0 \\ \Pi_{i-1} & \mathbf{0} \end{bmatrix} + q \begin{bmatrix} 0 & \mathbf{0}^T \\ \mathbf{0} & \Pi_{i-1} \end{bmatrix},$$

where $\mathbf{0}^T$ is an $(i-1) \times 1$ row vector. Then, divide all but the top and bottom rows by two to restore the requirement that the conditional probabilities sum up to one. This characterization of the transition matrix preserve the property that the first-order serial correlation is determined by $p + q - 1$.

The variance of the shock s can easily be computed as,

$$\sigma_s^2 = \frac{\vartheta^2}{i-1}.$$

The fourth central moment of s is,

$$\mu_s^4 = \frac{(3i-5)\,\vartheta^4}{(i-1)^3},$$

so that the Kurtosis of the distribution is given by,

$$\frac{\mu_s^4}{\sigma_s^4} = 3 - \frac{2}{i-1}.$$

As $i \to \infty$, the kurtosis becomes 3. Therefore, for large i and choosing $\vartheta = \sigma \sqrt[2]{i-1}$, it is possible to obtain an arbitrarily close approximation to a normal distribution for the shocks with mean μ and variance σ^2, and with an autocorrelation of $p+q-1$. A special case is $p = q = \pi$, when the transition matrix is symmetric and characterized by one parameter only. It can be shown that in this special case the stationary distribution of the Markov process becomes independent of the single parameter π. The first-order autocorrelation of the process, i.e. $2\pi - 1$, and the mean and volatility of the distribution, μ and σ^2, are pined down with only three parameters.

Estimates on Productivity and Working-Age Population. The three state case gives us a rough approximation, but one that can be accomplished by choosing a small number of parameters. As it is usually done, we choose to center the shocks around a zero mean, i.e. $\mu = 0$. The productivity shock can be inferred from the data as $a_t = \ln(Y_t) - (1-\theta)\ln(K_t) - \theta \ln(L_t)$, where Y_t stands for output, K_t for capital and L_t for employment. Based on the findings of Gomme and Rupert (2007), however, we compute the productivity without capital. The labor supply variable, n_t, is computed as $\ln(PoP_t)$ where PoP_t stands for working-age population. We compute the $AR(1)$ processes from the data as,

$$\begin{aligned} a_t &= \rho_a a_{t-1} + \varepsilon_t,\ \varepsilon_t \sim N\left(0, \sigma_a^2\right) \\ n_t &= \rho_n n_{t-1} + \epsilon_t,\ \epsilon_t \sim N\left(0, \sigma_n^2\right). \end{aligned}$$

Our estimates indicate that,

Regression Estimates

	Lag	Time ($\times 10^{-3}$)	Constant	Std. Dev.
Productivity	0.947764	0.209	0.195497	0.006853
	(0.023862)	(9.14×10^{-5})	(0.088051)	
Working-age pop.	0.958744	0.00113	0.004879	1.39×10^{-5}
	(0.007416)	(2.32×10^{-7})	(0.000869)	

We calibrated the shocks based on these patterns. The numbers in parenthesis are the standard errors.

The term p^2 for the Markov transition matrix corresponding to the productivity shock, a, is the conditional probability of a realization $a_{t+1} = -\vartheta$ given that $a_t = -\vartheta$ and $(1-p)^2$ is the conditional probability of $a_{t+1} = \vartheta$ given that $a_t = -\vartheta$. Similarly, q^2 is the conditional probability of a realization $a_{t+1} = \vartheta$ given that $a_t = \vartheta$ and $(1-q)^2$ is the conditional probability of $a_{t+1} = -\vartheta$ given that $a_t = \vartheta$. The same description can apply to the transition matrix that characterizes the labor supply shock. Hence, p governs the probability of improving after a low realization of the productivity shock, and q defines the probability of worsening given a high realization of the shock. The choice of p and q affects the expected duration of contractions and expansions, as can be inferrer from the dominant role they play in the characterization of the first-order autocorrelation.

However, the frequency of the expansions and contractions may not be symmetric if we choose p and q to be different. In particular, choosing p to be smaller than q implies that bad shocks occur less frequently than good shocks. Moreover, the conditional probabilities of leaving the bad state are higher than the probabilities of leaving the good state, which reduces the expected duration of contractions in productivity (or labor supply) relative to expansions. If $p = q = \pi$, then the stationary distribution of the process is $\left(\frac{1}{4}, \frac{1}{2}, \frac{1}{4}\right)$ which is independent of the choice of π. Hence, the average time spent in the good and bad states is identical.

For $p < q$, the recessions become shorter than expansions. Also for a symmetric process where $\{\mu - \vartheta, \mu, \mu + \vartheta\}$, this would introduce conditional heteroskedasticity in the shocks.

C The Toy Model: The Solution

C.1 The Representative Household

Given that all households are effectively identical, aggregation is not a problem. It results in the following maximization program for a representative household,

$$\max_{\{C_{t+\tau},K_{t+\tau+1},H_{t+\tau+1},B_{t+\tau+1}\}_{\tau=0}^{+\infty}} \sum\nolimits_{\tau=0}^{+\infty} \beta^{\tau} N_{t+\tau} \mathbb{E}_t \left[\ln\left(\frac{C_{t+\tau}}{N_{t+\tau}}\right) + \kappa_s \ln\left(\frac{H_{t+\tau}}{N_{t+\tau}}\right)\right],\ 0<\beta<1, \quad (21)$$
$$s.t.\ C_t + I_t^k + p_t I_t^h + q_t B_{t+1} \leq w_t L_t + r_t K_t + B_t - \tau^h p_t H_t,$$
$$K_{t+1} = \left(1-\delta^k\right) K_t + I_t^k,\ \delta^k = 1,$$
$$H_{t+1} = \left(1+g^h\right) H_t + I_t^h,$$
$$w_t = (1-\alpha) Z_t g\left(\tau^h\right) \left(\frac{K_t}{L_t}\right)^{\alpha},$$
$$r_t = \alpha Z_t g\left(\tau^h\right) \left(\frac{K_t}{L_t}\right)^{\alpha-1}.$$

In equilibrium, we know that the optimal allocation implies that,

$$I_t^h = -\tau^h H_t,\ \forall t,$$
$$B_t = 0,\ \forall t.$$

Using these equilibrium conditions and the functional forms for real wages and the rental rate of capital implies that we can write the problem of the representative household as,

$$\max_{\{C_{t+\tau},K_{t+\tau+1}\}_{\tau=0}^{+\infty}} \sum\nolimits_{\tau=0}^{+\infty} \beta^{\tau} N_{t+\tau} \mathbb{E}_t \left[\ln\left(\frac{C_{t+\tau}}{N_{t+\tau}}\right) + \kappa_s \ln\left(\frac{H_{t+\tau}}{N_{t+\tau}}\right)\right],\ 0<\beta<1, \quad (22)$$
$$s.t.\ C_t + I_t^k \leq Z_t g\left(\tau^h\right) K_t^{\alpha} L_t^{1-\alpha},$$
$$K_{t+1} = \left(1-\delta^k\right) K_t + I_t^k,\ \delta^k = 1,$$
$$H_{t+\tau} = \left((1+g_a)^{\frac{1}{1-\alpha}}\right)^{\tau} (1+g_n)^{\tau} H_t,$$

where the sequence $g\left(\tau^h\right)$ is treated as completely exogenous and out of the control of the planner. The assumption of logarithmic preferences coupled with full capital depreciation ensures a Solow-type solution to the model with constant savings rates. Based on this model, it immediately follows that the individual consumption and savings decisions can be inferred as,

$$c_{jt} = \overline{C}_t = \frac{C_t}{N_t},$$
$$k_{jt+1} = \overline{K}_{t+1} = \frac{K_{t+1}}{N_t}.$$

These transformations would be important to characterize the solution of the pricing equations later.

At this stage, we want to be able to pin down the solution around the balanced growth path. For any variable, X_t, we define an alternative variable $\widehat{X}_t = \frac{X_t}{A_t^{\frac{1}{1-\alpha}} N_t}$. With this notation, the resource constraint and the law of motion for capital are given by,

$$\begin{aligned}\widehat{C}_t + \widehat{I}_t^k &\leq e^{a_t+(1-\alpha)n_t} g\left(\tau^h\right) \widehat{K}_t^{\alpha}, \\ \widehat{K}_{t+1} &= \frac{1-\delta^k}{(1+g_a)^{\frac{1}{1-\alpha}}(1+g_n)} \widehat{K}_t + \frac{1}{(1+g_a)^{\frac{1}{1-\alpha}}(1+g_n)} \widehat{I}_t^k, \ \delta^k = 1.\end{aligned}$$

Using the same transformation, preferences can be written as,

$$\sum_{\tau=0}^{+\infty} \left[\beta\left(1+g_n\right)\right]^{\tau} \mathbb{E}_t \left[\ln\left(\widehat{C}_{t+\tau}\left(\left(1+g_a\right)^{\frac{1}{1-\alpha}}\right)^{\tau}\right) + \kappa_s \ln\left(\widehat{H}_{t+\tau}\left(\left(1+g_a\right)^{\frac{1}{1-\alpha}}\right)^{\tau}\right)\right].$$

Since $H_{t+\tau} = \left(\left(1+g_a\right)^{\frac{1}{1-\alpha}}\right)^{\tau}\left(1+g_n\right)^{\tau} H_t$, then we can immediately write $\widehat{H}_{t+\tau} = H_t$. As a result, we determine the following objective for the representative household,

$$\begin{aligned}&\sum_{\tau=0}^{+\infty} \left[\beta\left(1+g_n\right)\right]^{\tau} \mathbb{E}_t \left[\ln\left(\widehat{C}_{t+\tau}\right) + \kappa_s \ln\left(H_t\right) + \left(1+\kappa_s\right) \ln\left(\left(\left(1+g_a\right)^{\frac{1}{1-\alpha}}\right)^{\tau}\right)\right] \\ &\sum_{\tau=0}^{+\infty} \left[\beta\left(1+g_n\right)\right]^{\tau} \mathbb{E}_t \left[\ln\left(\widehat{C}_{t+\tau}\right) + \kappa_s \ln\left(H_t\right)\right] + \left(\frac{1+\kappa_s}{1-\alpha}\right) \ln\left(1+g_a\right) \sum_{\tau=0}^{+\infty} \left[\beta\left(1+g_n\right)\right]^{\tau} \tau \\ &\sum_{\tau=0}^{+\infty} \left[\beta\left(1+g_n\right)\right]^{\tau} \mathbb{E}_t \left[\ln\left(\widehat{C}_{t+\tau}\right) + \kappa_s \ln\left(H_t\right)\right] + \left(\frac{1+\kappa_s}{1-\alpha}\right) \ln\left(1+g_a\right) \frac{\beta(1+g_n)}{(1-\beta(1+g_n))^2}.\end{aligned}$$

However, H_t is already given at the time the maximization is performed and the last term is merely a constant. Hence, the objective function of the representative household can be summarized with the logarithmic utility on consumption alone. Hence, the maximization problem reduces effectively to this one,

$$\begin{aligned}&\max_{\left\{\widehat{C}_{t+\tau}, \widehat{K}_{t+\tau+1}\right\}_{\tau=0}^{+\infty}} \sum_{\tau=0}^{+\infty} \left[\beta\left(1+g_n\right)\right]^{\tau} \mathbb{E}_t \left[\ln\left(\widehat{C}_{t+\tau}\right)\right], \ 0 < \beta\left(1+g_n\right) < 1, \qquad (23) \\ &s.t. \ \widehat{C}_t + \widehat{K}_{t+1} \leq g\left(\tau^h\right) e^{a_t+(1-\alpha)n_t} \widehat{K}_t^{\alpha},\end{aligned}$$

where we already use the fact that $\delta^k = 1$ and $\widehat{K}_{t+1} = \widehat{I}_t^k$.

The Planner's Problem. There is no possible insurance against aggregate shocks (productivity and labor supply) and those are the only sources of uncertainty in the model. Hence, there are no missing markets in this environment to worry about. Our interest shifts now to the planner's problem motivated by the fact that for this model economy the solution to the planner's problem is quite simple and reminiscent of the Solow model.

In that case, the social planner's problem can be expressed in the following form,

$$\begin{aligned}V\left(a, n, \widehat{K}\right) = &\max_{\widehat{C}, \widehat{K}'} \ln\left(\widehat{C}\right) + \beta\left(1+g_n\right) \mathbb{E}\left[V\left(a', n', \widehat{K}'\right) \mid a, n\right], \qquad (24) \\ &s.t. \ \widehat{C} + \widehat{K}' \leq g\left(\tau^h\right) e^{a+(1-\alpha)n} \widehat{K}^{\alpha}, \\ &a' = \rho_a a + \varepsilon_a', \ \left|\rho_a\right| < 1, \\ &n' = \rho_n n + \varepsilon_n', \ \left|\rho_n\right| < 1, \qquad (25)\end{aligned}$$

where $V\left(a, n, \widehat{K}\right)$ is the value function for the social planner. This problem is rather convenient because we can easily characterize an interior solution. Let us assume that an interior solution exists, i.e.

$$\begin{aligned}&V\left(a, n, \widehat{K}\right) = \max_{\widehat{K}'} \ln\left(g\left(\tau^h\right) e^{a+(1-\alpha)n} \widehat{K}^{\alpha} - \widehat{K}'\right) + \beta\left(1+g_n\right) \mathbb{E}\left[V\left(a', n', \widehat{K}'\right) \mid a, n\right], \qquad (26) \\ &a' = \rho_a a + \varepsilon_a', \ \left|\rho_a\right| < 1, \\ &n' = \rho_n n + \varepsilon_n', \ \left|\rho_n\right| < 1. \qquad (27)\end{aligned}$$

Logarithmic preferences is the same assumption that Krusell and Smith (1998) use in their calibrations, full capital depreciation is not but allows us to obtain an analytic solution.

C.2 The Solution

We employ the old-fashioned method of guess-and-verify to properly characterization the decision rules of the planner and its Bellman equation in order to pin down the solution of the model. Logarithmic preferences and full capital depreciation allows us to obtain an analytic solution comparable to the ones underlying the model of Krusell and Smith (1998).

1) I conjecture that the value function takes the following form,

$$\begin{aligned} V\left(a, n, \widehat{K}\right) &= \gamma_1 + \gamma_2 \ln\left(e^{\gamma_3 a + \gamma_4 n} \widehat{K}^{\alpha}\right) \\ &= \gamma_1 + \gamma_2 \alpha \ln\left(\widehat{K}\right) + \gamma_2 \gamma_3 a + \gamma_2 \gamma_4 n. \end{aligned}$$

With this conjecture of the Bellman equation we expect to find a decision rule for capital as follows,

$$\widehat{K}' = k\left(a, n, \widehat{K}\right) = sg\left(\tau^h\right) e^{a+(1-\alpha)n} K^{\alpha},$$

with a constant savings share $0 < s < 1$.

2) We substitute out the conjectured value function in the Bellman equation in order to write a simpler maximization problem as,

$$\begin{aligned} \widehat{K}' &\in \underset{\widehat{K}' \in \left[0, g(\tau^h) e^{a+(1-\alpha)n} \widehat{K}^{\alpha}\right]}{\arg\max} \ln\left(g\left(\tau^h\right) e^{a+(1-\alpha)n} \widehat{K}^{\alpha} - \widehat{K}'\right) + \beta\left(1+g_n\right) \mathbb{E}\left[V\left(a', n', \widehat{K}'\right) \mid a, n\right] \\ &= \underset{\widehat{K}' \in \left[0, g(\tau^h) e^{a+(1-\alpha)n} \widehat{K}^{\alpha}\right]}{\arg\max} \ln\left(g\left(\tau^h\right) e^{a+(1-\alpha)n} \widehat{K}^{\alpha} - \widehat{K}'\right) + \beta\left(1+g_n\right)\left[\gamma_1 + \gamma_2 \alpha \ln\left(\widehat{K}'\right) + \gamma_2 \mathbb{E}\left(\gamma_3 a' + \gamma_4 n' \mid a, n\right)\right] \\ &= \underset{\widehat{K}' \in \left[0, g(\tau^h) e^{a+(1-\alpha)n} \widehat{K}^{\alpha}\right]}{\arg\max} \ln\left(g\left(\tau^h\right) e^{a+(1-\alpha)n} \widehat{K}^{\alpha} - \widehat{K}'\right) + \beta\left(1+g_n\right)\left[\gamma_1 + \gamma_2 \alpha \ln\left(\widehat{K}'\right) + \gamma_2 \gamma_3 \rho_a a + \gamma_2 \gamma_4 \rho_n n\right]. \end{aligned}$$

3) We solve the simple maximization problem to obtain the following first-order condition,

$$-\frac{1}{g\left(\tau^h\right) e^{a+(1-\alpha)n} \widehat{K}^{\alpha} - \widehat{K}'} + \beta\left(1+g_n\right) \alpha \gamma_2 \frac{1}{\widehat{K}'} = 0.$$

From simple re-arranging it follows that the decision rule for new capital is,

$$\widehat{K}' = \frac{\beta\left(1+g_n\right) \alpha \gamma_2}{1 + \beta\left(1+g_n\right) \alpha \gamma_2} g\left(\tau^h\right) e^{a+(1-\alpha)n} \widehat{K}^{\alpha}.$$

It also naturally follows that the decision rule for consumption is,

$$\widehat{C} = g\left(\tau^h\right) e^{a+(1-\alpha)n} \widehat{K}^{\alpha} - \widehat{K}' = \frac{1}{1 + \beta\left(1+g_n\right) \alpha \gamma_2} g\left(\tau^h\right) e^{a+(1-\alpha)n} \widehat{K}^{\alpha},$$

which indicates that the consumption and savings shares are constant.

4) We can write the Bellman equation now as,

$$\begin{aligned} \gamma_1 + \gamma_2 \alpha \ln\left(\widehat{K}\right) + \gamma_2 \gamma_3 a + \gamma_2 \gamma_4 n &= \max_{\widehat{K}' \in [0, e^a K^{\alpha}]} \ln\left(g\left(\tau^h\right) e^{a+(1-\alpha)n} \widehat{K}^{\alpha} - \widehat{K}'\right) + \beta\left(1+g_n\right) \mathbb{E}\left[\gamma_1 + \gamma_2 \alpha \ln\left(\widehat{K}'\right) + \gamma_2 \gamma_3 \right. \\ &= \max_{\widehat{K}' \in [0, e^a K^{\alpha}]} \ln\left(g\left(\tau^h\right) e^{a+(1-\alpha)n} \widehat{K}^{\alpha} - \widehat{K}'\right) + \beta\left(1+g_n\right)\left[\gamma_1 + \gamma_2 \alpha \ln\left(\widehat{K}'\right) + \gamma_2 \gamma_3 \rho_a \right. \end{aligned}$$

or, simply,

$$\gamma_1 + \gamma_2 \alpha \ln\left(\widehat{K}\right) + \gamma_2\gamma_3 a + \gamma_2\gamma_4 n = \beta\left(1+g_n\right)\gamma_1 + \ln\left(\tfrac{1}{1+\beta(1+g_n)\alpha\gamma_2} g\left(\tau^h\right) e^{a+(1-\alpha)n}\widehat{K}^{\alpha}\right) +$$
$$+\beta\left(1+g_n\right)\gamma_2\alpha\ln\left(\tfrac{\beta(1+g_n)\alpha\gamma_2}{1+\beta(1+g_n)\alpha\gamma_2} g\left(\tau^h\right) e^{a+(1-\alpha)n}\widehat{K}^{\alpha}\right) + \beta\left(1+g_n\right)\gamma_2\gamma_3\rho_a a + \beta\left(1+g_n\right)\gamma_2\gamma_4\rho_n n,$$

After a little bit of algebra, I can re-write the right-hand side more compactly as follows,

$$\gamma_1 + \gamma_2 \alpha \ln\left(\widehat{K}\right) + \gamma_2\gamma_3 a + \gamma_2\gamma_4 n = \beta\left(1+g_n\right)\gamma_1 + \ln\left(\tfrac{1}{1+\beta(1+g_n)\alpha\gamma_2} g\left(\tau^h\right)\right) + \beta\left(1+g_n\right)\gamma_2\alpha\ln\left(\tfrac{\beta(1+g_n)\alpha\gamma_2}{1+\beta(1+g_n)\alpha\gamma_2} g\left(\tau^h\right)\right)$$
$$+\left(1+\beta\left(1+g_n\right)\gamma_2\alpha\right)\alpha\ln\left(\widehat{K}\right) + \left(1+\beta\left(1+g_n\right)\gamma_2\left(\alpha+\gamma_3\rho_a\right)\right)a + \left(\left(1-\alpha\right)+\beta\left(1+g_n\right)\gamma_2\left(\alpha\left(1-\alpha\right)+\gamma_4\rho_n\right)\right)n.$$

At this stage, we are left with the task of matching coefficients appropriately.

5) By the method of matching coefficients we obtain the following equivalences between the coefficients on the right- and left-hand side of the equation above,

$$\begin{aligned}
\gamma_1 &= \beta\left(1+g_n\right)\gamma_1 + \ln\left(\frac{1}{1+\beta\left(1+g_n\right)\alpha\gamma_2} g\left(\tau^h\right)\right) + \beta\left(1+g_n\right)\gamma_2\alpha\ln\left(\frac{\beta\left(1+g_n\right)\alpha\gamma_2}{1+\beta\left(1+g_n\right)\alpha\gamma_2} g\left(\tau^h\right)\right), \\
\alpha\gamma_2 &= \left(1+\beta\left(1+g_n\right)\gamma_2\alpha\right)\alpha, \\
\gamma_2\gamma_3 &= \left(1+\beta\left(1+g_n\right)\gamma_2\left(\alpha+\gamma_3\rho_a\right)\right), \\
\gamma_2\gamma_4 &= \left(\left(1-\alpha\right)+\beta\left(1+g_n\right)\gamma_2\left(\alpha\left(1-\alpha\right)+\gamma_4\rho_n\right)\right).
\end{aligned}$$

It follows from the second condition that,

$$\gamma_2 = \frac{1}{1-\beta\left(1+g_n\right)\alpha}.$$

Using this condition, therefore, we can re-write the third constraint as,

$$\frac{1}{1-\beta\left(1+g_n\right)\alpha}\gamma_3 = 1 + \frac{\beta\left(1+g_n\right)}{1-\beta\left(1+g_n\right)\alpha}\left(\alpha+\gamma_3\rho_a\right) = \frac{1}{1-\beta\left(1+g_n\right)\alpha} + \frac{\beta\left(1+g_n\right)}{1-\beta\left(1+g_n\right)\alpha}\gamma_3\rho_a,$$

and obtain that,

$$\gamma_3 = \frac{1}{1-\beta\left(1+g_n\right)\rho_a}.$$

Similarly, we can re-write the fourth constraint as,

$$\begin{aligned}
\frac{1}{1-\beta\left(1+g_n\right)\alpha}\gamma_4 &= \left(1-\alpha\right) + \frac{\beta\left(1+g_n\right)}{1-\beta\left(1+g_n\right)\alpha}\left(\alpha\left(1-\alpha\right)+\gamma_4\rho_n\right) \\
&= \left(\frac{1}{1-\beta\left(1+g_n\right)\alpha}\right)\left(1-\alpha\right) + \frac{\beta\left(1+g_n\right)}{1-\beta\left(1+g_n\right)\alpha}\gamma_4\rho_n,
\end{aligned}$$

and obtain that,

$$\gamma_4 = \frac{1-\alpha}{1-\beta\left(1+g_n\right)\rho_n}.$$

Replacing γ_2 into the first constraint we obtain that,

$$\begin{aligned}
(1-\beta(1+g_n))\gamma_1 &= \ln\left(\tfrac{1}{1+\beta(1+g_n)\alpha\gamma_2}\right) + \beta(1+g_n)\gamma_2\alpha\ln\left(\tfrac{\beta(1+g_n)\alpha\gamma_2}{1+\beta(1+g_n)\alpha\gamma_2}\right) + \left[1+\beta(1+g_n)\gamma_2\alpha\right]\ln\left(g\left(\tau^h\right)\right) \\
&= \ln\left(\tfrac{1}{1+\frac{\beta(1+g_n)\alpha}{1-\beta(1+g_n)\alpha}}\right) + \tfrac{\beta(1+g_n)\alpha}{1-\beta(1+g_n)\alpha}\ln\left(\tfrac{\frac{\beta(1+g_n)\alpha}{1-\beta(1+g_n)\alpha}}{1+\frac{\beta(1+g_n)\alpha}{1-\beta(1+g_n)\alpha}}\right) + \left[1+\tfrac{\beta(1+g_n)\alpha}{1-\beta(1+g_n)\alpha}\right]\ln\left(g\left(\tau^h\right)\right) \\
&= \ln\left(\tfrac{1}{\frac{1}{1-\beta(1+g_n)\alpha}}\right) + \tfrac{\beta(1+g_n)\alpha}{1-\beta(1+g_n)\alpha}\ln\left(\tfrac{\frac{\beta(1+g_n)\alpha}{1-\beta(1+g_n)\alpha}}{\frac{1}{1-\beta(1+g_n)\alpha}}\right) + \left(\tfrac{1}{1-\beta(1+g_n)\alpha}\right)\ln\left(g\left(\tau^h\right)\right) \\
&= \ln(1-\beta(1+g_n)\alpha) + \tfrac{\beta(1+g_n)\alpha}{1-\beta(1+g_n)\alpha}\ln(\beta(1+g_n)\alpha) + \tfrac{1}{1-\beta(1+g_n)\alpha}\ln\left(g\left(\tau^h\right)\right),
\end{aligned}$$

or, more compactly,

$$\gamma_1 = \frac{1}{1-\beta(1+g_n)}\left[\ln(1-\beta(1+g_n)\alpha) + \frac{\beta(1+g_n)\alpha}{1-\beta(1+g_n)\alpha}\ln(\beta(1+g_n)\alpha) + \frac{1}{1-\beta(1+g_n)\alpha}\ln\left(g\left(\tau^h\right)\right)\right].$$

This pins down completely the Bellman equation for our model.

6) To sum up, the conjecture has been verified and implies that the solution to this model is characterized by,

$$\begin{aligned}
V\left(a,n,\widehat{K}\right) &= \tfrac{1}{1-\beta(1+g_n)}\left[\ln(1-\beta(1+g_n)\alpha) + \tfrac{\beta(1+g_n)\alpha}{1-\beta(1+g_n)\alpha}\ln(\beta(1+g_n)\alpha)\right] + \tfrac{1}{1-\beta(1+g_n)\alpha}\left[\ln\left(g\left(\tau^h\right)^{\frac{1}{1-\beta(1+g_n)}} e^{\frac{1}{1-\beta(1+g_n)\rho_a}\cdots}\right.\right. \\
&= \tfrac{1}{1-\beta(1+g_n)}\left[\ln(1-\beta(1+g_n)\alpha) + \tfrac{\beta(1+g_n)\alpha}{1-\beta(1+g_n)\alpha}\ln(\beta(1+g_n)\alpha)\right] + \tfrac{1}{1-\beta(1+g_n)\alpha}\left[\tfrac{1}{1-\beta(1+g_n)}\ln\left(g\left(\tau^h\right)\right) + \alpha\ln\left(\widehat{K}\right) + \tfrac{\cdots}{1-\beta(1\cdots}\right. \\
\widehat{K}' = k\left(a,n,\widehat{K}\right) &= \tfrac{\frac{\beta(1+g_n)\alpha}{1-\beta(1+g_n)\alpha}}{1+\frac{\beta(1+g_n)\alpha}{1-\beta(1+g_n)\alpha}} g\left(\tau^h\right)e^{a+(1-\alpha)n}\widehat{K}^{\alpha} = \beta(1+g_n)\alpha g\left(\tau^h\right)e^{a+(1-\alpha)n}\widehat{K}^{\alpha}, \\
\widehat{C} &= \tfrac{1}{1+\frac{\beta(1+g_n)\alpha}{1-\beta(1+g_n)\alpha}} g\left(\tau^h\right)e^{a+(1-\alpha)n}\widehat{K}^{\alpha} = (1-\beta(1+g_n)\alpha)\, g\left(\tau^h\right)e^{a+(1-\alpha)n}\widehat{K}^{\alpha}.
\end{aligned}$$

Taking a look at this expression it becomes rather obvious why a log-linear law of motion for the aggregate state can be perceived as a good initial conjecture for this type of models as Krusell and Smith (1998) argue in their paper. Based on this characterization of the law of motion for aggregate capital we can precisely determined the steady state of the transformed $\widehat{K}$ where,

$$\begin{aligned}
\widehat{K}' &= \beta(1+g_n)\alpha g\left(\tau^h\right)e^{a+(1-\alpha)n}\widehat{K}^{\alpha}, \\
\widehat{K}'' &= \beta(1+g_n)\alpha g\left(\tau^h\right)e^{a'+(1-\alpha)n'}\left(\widehat{K}'\right)^{\alpha} = \left(\beta(1+g_n)\alpha g\left(\tau^h\right)\right)^{1+\alpha}e^{\alpha a+a'+(1-\alpha)\left(\alpha n+n'\right)}\widehat{K}^{\alpha^2} = (\beta\alpha)^{1+\alpha}e^{\alpha a+a'}K^{\alpha^2}, \\
\widehat{K}''' &= \beta(1+g_n)\alpha g\left(\tau^h\right)e^{a''+(1-\alpha)n''}\left(\widehat{K}''\right)^{\alpha} = \left(\beta(1+g_n)\alpha g\left(\tau^h\right)\right)^{1+\alpha+\alpha^2}e^{\alpha^2 a+\alpha a'+a''+(1-\alpha)\left(\alpha^2 n+\alpha n'+n''\right)}\widehat{K}^{\alpha^3}, \\
&\ldots \\
\widehat{K}^{(n)} &= \left(\beta(1+g_n)\alpha g\left(\tau^h\right)\right)^{\sum_{j=0}^{n-1}\alpha^j}e^{\sum_{j=0}^{n-1}\alpha^j a^{(n-1-j)}+(1-\alpha)\alpha^j n^{(n-1-j)}}\widehat{K}^{\alpha^n}.
\end{aligned}$$

In a deterministic steady state where the aggregate productivity and labor supply shocks are invariant (and normalized to be equal to their unconditional mean of zero), naturally $\widehat{K}^{(n)}$ converges towards a steady state value $\widehat{K}^*$ as $n\to+\infty$ which is a function of $\left(\beta,\alpha,g_n,\tau^h\right)$ alone, i.e.

$$\widehat{K}^* = \lim_{n\to\infty}\left(\beta(1+g_n)\alpha g\left(\tau^h\right)\right)^{\sum_{j=0}^{n-1}\alpha^j}\widehat{K}^{\alpha^n} = \left(\beta(1+g_n)\alpha g\left(\tau^h\right)\right)^{\sum_{j=0}^{+\infty}\alpha^j} = \left(\beta(1+g_n)\alpha g\left(\tau^h\right)\right)^{\frac{1}{1-\alpha}}.$$

This is an important feature of the model that we can exploit in many ways.

C.3 Pricing Bonds and the Housing Stock

Let us recall the Euler equations that help us price both bonds and assets in this economy, i.e.

$$\begin{aligned} p_t &= \beta \mathbb{E}_t \left[\kappa_s \frac{c_{jt}}{h_{jt+1}} + \frac{c_{jt}}{c_{jt+1}} (1+g_a)^{\frac{1}{1-\alpha}} (1+g_n) p_{t+1} \right], \\ q_t &= \beta \mathbb{E}_t \left[\frac{c_{jt}}{c_{jt+1}} \right]. \end{aligned}$$

We know that under the conditions implied by this model, the solution ought to be symmetric, i.e. $c_{jt} = c_t$ and $h_{jt+1} = h_{t+1}$ for all $j \in [0, N_t]$. Therefore, it must be the case that,

$$\begin{aligned} c_{jt} &= \frac{C_t}{N_t}, \\ h_{jt+1} &= \frac{H_{t+1}}{N_t}, \end{aligned}$$

implying that the Euler equations can be expressed in terms of aggregate variables instead,

$$\begin{aligned} p_t &= \beta \mathbb{E}_t \left[\kappa_s \frac{\frac{C_t}{N_t}}{\frac{H_{t+1}}{N_t}} + \frac{\frac{C_t}{N_t}}{\frac{C_{t+1}}{N_{t+1}}} (1+g_a)^{\frac{1}{1-\alpha}} (1+g_n) p_{t+1} \right] = \beta \mathbb{E}_t \left[\kappa_s \frac{\frac{C_t}{N_t}}{\frac{H_{t+1}}{N_t}} + (1+g_n) \frac{C_t}{C_{t+1}} (1+g_a)^{\frac{1}{1-\alpha}} (1+g_n) p_{t+1} \right], \\ q_t &= \beta \mathbb{E}_t \left[\frac{\frac{C_t}{N_t}}{\frac{C_{t+1}}{N_{t+1}}} \right] = \beta (1+g_n) \mathbb{E}_t \left[\frac{C_t}{C_{t+1}} \right]. \end{aligned}$$

We have transformed the variables such that for any aggregate variable, X_t, we define an alternative variable $\widehat{X}_t = \frac{X_t}{A_t^{\frac{1}{1-\alpha}} N_t}$. Therefore, we can once again re-write the pricing equations in the following terms,

$$\begin{aligned} p_t &= \beta \mathbb{E}_t \left[\kappa_s \frac{\frac{C_t}{(1+g_a)^{\frac{1}{1-\alpha}} A_t^{\frac{1}{1-\alpha}} N_t}}{\frac{(1+g_n) H_{t+1}}{A_{t+1}^{\frac{1}{1-\alpha}} (1+g_n) N_t}} + \frac{\frac{C_t}{(1+g_a)^{\frac{1}{1-\alpha}} A_t^{\frac{1}{1-\alpha}} N_t}}{\frac{C_{t+1}}{A_{t+1}^{\frac{1}{1-\alpha}} N_{t+1}}} (1+g_a)^{\frac{1}{1-\alpha}} (1+g_n) p_{t+1} \right] \\ &= \beta \mathbb{E}_t \left[\kappa_s \frac{\frac{\widehat{C}_t}{(1+g_a)^{\frac{1}{1-\alpha}}}}{(1+g_n) \widehat{H}_{t+1}} + \frac{\frac{\widehat{C}_t}{(1+g_a)^{\frac{1}{1-\alpha}}}}{\widehat{C}_{t+1}} (1+g_a)^{\frac{1}{1-\alpha}} (1+g_n) p_{t+1} \right] \\ &= \beta \mathbb{E}_t \left[\kappa_s \frac{\widehat{C}_t}{(1+g_n)(1+g_a)^{\frac{1}{1-\alpha}} \widehat{H}_{t+1}} + \frac{\widehat{C}_t}{\widehat{C}_{t+1}} (1+g_n) p_{t+1} \right], \\ q_t &= \beta \mathbb{E}_t \left[\frac{\frac{C_t}{(1+g_a)^{\frac{1}{1-\alpha}} A_t^{\frac{1}{1-\alpha}} N_t}}{\frac{C_{t+1}}{A_{t+1}^{\frac{1}{1-\alpha}} N_{t+1}}} \right] = \beta (1+g_a)^{-\frac{1}{1-\alpha}} \mathbb{E}_t \left[\frac{\widehat{C}_t}{\widehat{C}_{t+1}} \right]. \end{aligned}$$

Given our assumptions on the evolution of the stock of housing capital, i.e. $H_{t+\tau} = \left((1+g_a)^{\frac{1}{1-\alpha}} \right)^{\tau} (1+g_n)^{\tau} H_t$, then we can immediately determine that $\widehat{H}_{t+1} = H_t$ (which is the initial condition on the housing stock).

Therefore, the system of equations reduces to,

$$
\begin{aligned}
p_t &= \beta\left(1+g_n\right)\mathbb{E}_t\left[\frac{\widehat{C}_t}{\widehat{C}_{t+1}}p_{t+1}\right]+\frac{\beta\kappa_s}{\left(1+g_n\right)\left(1+g_a\right)^{\frac{1}{1-\alpha}}}\frac{\widehat{C}_t}{H_t},\\
q_t &= \beta\left(1+g_a\right)^{-\frac{1}{1-\alpha}}\mathbb{E}_t\left[\frac{\widehat{C}_t}{\widehat{C}_{t+1}}\right].
\end{aligned}
$$

We know what the aggregate decision rule on consumption is, so it can be inferred that,

$$
\begin{aligned}
p_t &= \beta\left(1+g_n\right)\mathbb{E}_t\left[\frac{e^{a_t+(1-\alpha)n_t}\widehat{K}_t^{\alpha}}{e^{a_{t+1}+(1-\alpha)n_{t+1}}\widehat{K}_{t+1}^{\alpha}}p_{t+1}\right]+\beta\kappa_s\frac{\left(1-\beta\left(1+g_n\right)\alpha\right)g\left(\tau^h\right)}{\left(1+g_n\right)\left(1+g_a\right)^{\frac{1}{1-\alpha}}}\frac{e^{a_t+(1-\alpha)n_t}\widehat{K}_t^{\alpha}}{H_t},\\
q_t &= \beta\left(1+g_a\right)^{-\frac{1}{1-\alpha}}\mathbb{E}_t\left[\frac{e^{a_t+(1-\alpha)n_t}\widehat{K}_t^{\alpha}}{e^{a_{t+1}+(1-\alpha)n_{t+1}}\widehat{K}_{t+1}^{\alpha}}\right],
\end{aligned}
$$

or, more precisely,

$$
\begin{aligned}
p_t &= \beta\left(1+g_n\right)\mathbb{E}_t\left[\frac{e^{a_t+(1-\alpha)n_t}\widehat{K}_t^{\alpha}}{e^{\rho_a a_t+\varepsilon_{t+1}^a+(1-\alpha)\left(\rho_n n_t+\varepsilon_{t+1}^n\right)}\left(\beta\left(1+g_n\right)\alpha g\left(\tau^h\right)e^{a_t+(1-\alpha)n_t}\left(\widehat{K}_t\right)^{\alpha}\right)^{\alpha}}p_{t+1}\right]+\beta\kappa_s\frac{\left(1-\beta\left(1+g_n\right)\alpha\right)g\left(\right.}{\left(1+g_n\right)\left(1+g_a\right)^{\frac{1}{1-}}}\\
q_t &= \beta\left(1+g_a\right)^{-\frac{1}{1-\alpha}}\mathbb{E}_t\left[\frac{e^{a_t+(1-\alpha)n_t}\widehat{K}_t^{\alpha}}{e^{\rho_a a_t+\varepsilon_{t+1}^a+(1-\alpha)\left(\rho_n n_t+\varepsilon_{t+1}^n\right)}\left(\beta\left(1+g_n\right)\alpha g\left(\tau^h\right)e^{a_t+(1-\alpha)n_t}\left(\widehat{K}_t\right)^{\alpha}\right)^{\alpha}}\right].
\end{aligned}
$$

More compactly, we obtain the following expressions,

$$
\begin{aligned}
p_t &= \frac{\beta\left(1+g_n\right)}{\left(\beta\left(1+g_n\right)\alpha g\left(\tau^h\right)\right)^{\alpha}}\mathbb{E}_t\left[\frac{e^{a_t+(1-\alpha)n_t}\widehat{K}_t^{\alpha}}{e^{\rho_a a_t+\varepsilon_{t+1}^a+(1-\alpha)\left(\rho_n n_t+\varepsilon_{t+1}^n\right)}e^{\alpha a_t+\alpha(1-\alpha)n_t}\left(\widehat{K}_t\right)^{\alpha^2}}p_{t+1}\right]+\beta\kappa_s\frac{\left(1-\beta\left(1+g_n\right)\alpha\right)g\left(\tau^h\right)}{\left(1+g_n\right)\left(1+g_a\right)^{\frac{1}{1-\alpha}}}\frac{e^{a_t+}}{}\\
q_t &= \frac{\beta\left(1+g_a\right)^{-\frac{1}{1-\alpha}}}{\left(\beta\left(1+g_n\right)\alpha g\left(\tau^h\right)\right)^{\alpha}}\mathbb{E}_t\left[\frac{e^{a_t+(1-\alpha)n_t}\widehat{K}_t^{\alpha}}{e^{\rho_a a_t+\varepsilon_{t+1}^a+(1-\alpha)\left(\rho_n n_t+\varepsilon_{t+1}^n\right)}e^{\alpha a_t+\alpha(1-\alpha)n_t}\left(\widehat{K}_t\right)^{\alpha^2}}\right],
\end{aligned}
$$

or simply,

$$
\begin{aligned}
p_t &= \frac{\beta\left(1+g_n\right)}{\left(\beta\left(1+g_n\right)\alpha g\left(\tau^h\right)\right)^{\alpha}}\mathbb{E}_t\left[e^{(1-\alpha-\rho_a)a_t+(1-\alpha)(1-\alpha-\rho_n)n_t}\widehat{K}_t^{\alpha(1-\alpha)}e^{-\varepsilon_{t+1}^a-(1-\alpha)\varepsilon_{t+1}^n}p_{t+1}\right]+\beta\kappa_s\frac{\left(1-\beta\left(1+g_n\right)\alpha\right)g\left(\tau^h\right)}{\left(1+g_n\right)\left(1+g_a\right)^{\frac{1}{1-\alpha}}}\frac{e^{\alpha}}{}\\
&= \frac{\beta\left(1+g_n\right)}{\left(\beta\left(1+g_n\right)\alpha g\left(\tau^h\right)\right)^{\alpha}}e^{(1-\alpha-\rho_a)a_t+(1-\alpha)(1-\alpha-\rho_n)n_t}\widehat{K}_t^{\alpha(1-\alpha)}\mathbb{E}_t\left[e^{-\varepsilon_{t+1}^a-(1-\alpha)\varepsilon_{t+1}^n}p_{t+1}\right]+\beta\kappa_s\frac{\left(1-\beta\left(1+g_n\right)\alpha\right)g\left(\tau^h\right)}{\left(1+g_n\right)\left(1+g_a\right)^{\frac{1}{1-\alpha}}}\frac{e^{\alpha}}{}\\
q_t &= \frac{\beta\left(1+g_a\right)^{-\frac{1}{1-\alpha}}}{\left(\beta\left(1+g_n\right)\alpha g\left(\tau^h\right)\right)^{\alpha}}\mathbb{E}_t\left[e^{(1-\alpha-\rho_a)a_t+(1-\alpha)(1-\alpha-\rho_n)n_t}\widehat{K}_t^{\alpha(1-\alpha)}e^{-\varepsilon_{t+1}^a-(1-\alpha)\varepsilon_{t+1}^n}\right]\\
&= \frac{\beta\left(1+g_a\right)^{-\frac{1}{1-\alpha}}}{\left(\beta\left(1+g_n\right)\alpha g\left(\tau^h\right)\right)^{\alpha}}e^{(1-\alpha-\rho_a)a_t+(1-\alpha)(1-\alpha-\rho_n)n_t}\widehat{K}_t^{\alpha(1-\alpha)}\mathbb{E}_t\left[e^{-\varepsilon_{t+1}^a-(1-\alpha)\varepsilon_{t+1}^n}\right]\\
&= \frac{\beta\left(1+g_a\right)^{-\frac{1}{1-\alpha}}}{\left(\beta\left(1+g_n\right)\alpha g\left(\tau^h\right)\right)^{\alpha}}e^{\frac{\sigma_a^2+(1-\alpha)^2\sigma_n^2}{2}+(1-\alpha-\rho_a)a_t+(1-\alpha)(1-\alpha-\rho_n)n_t}\widehat{K}_t^{\alpha(1-\alpha)}.
\end{aligned}
$$

This gives us a way to pin down precisely the price of a bond, which nicely corresponds with the log-linear approximation favored by Krusell and Smith (1998). It also gives us a single equation that would characterize the solution for the housing prices. An analytic solution may even be feasible in this environment.

A Characterization of the Housing Prices. The pricing equation for housing, i.e.

$$p_t = \beta\,(1+g_n)\,\mathbb{E}_t\left[\frac{\widehat{C}_t}{\widehat{C}_{t+1}}p_{t+1}\right] + \frac{\beta\kappa_s}{(1+g_n)\,(1+g_a)^{\frac{1}{1-\alpha}}}\mathbb{E}_t\left[\frac{\widehat{C}_t}{\widehat{H}_{t+1}}\right].$$

can be expressed recursively as,

$$\begin{aligned}
p_t &= \beta\,(1+g_n)\,\mathbb{E}_t\left[\frac{\widehat{C}_t}{\widehat{C}_{t+1}}\left(\beta\,(1+g_n)\,\mathbb{E}_{t+1}\left[\frac{\widehat{C}_{t+1}}{\widehat{C}_{t+2}}p_{t+2}\right] + \frac{\beta\kappa_s}{(1+g_n)(1+g_a)^{\frac{1}{1-\alpha}}}\frac{\widehat{C}_{t+1}}{\widehat{H}_{t+2}}\right)\right] + \frac{\beta\kappa_s}{(1+g_n)(1+g_a)^{\frac{1}{1-\alpha}}}\mathbb{E}_t\left(\frac{\widehat{C}_t}{\widehat{H}_{t+1}}\right) \\
&= (\beta\,(1+g_n))^2\,\mathbb{E}_t\left[\frac{\widehat{C}_t}{\widehat{C}_{t+1}}\mathbb{E}_{t+1}\left(\frac{\widehat{C}_{t+1}}{\widehat{C}_{t+2}}p_{t+2}\right)\right] + \frac{\beta\kappa_s}{(1+g_n)(1+g_a)^{\frac{1}{1-\alpha}}}\left[\beta\,(1+g_n)\,\mathbb{E}_t\left(\frac{\widehat{C}_t}{\widehat{H}_{t+2}}\right) + \mathbb{E}_t\left(\frac{\widehat{C}_t}{\widehat{H}_{t+1}}\right)\right] \\
&= (\beta\,(1+g_n))^2\,\mathbb{E}_t\left[\frac{\widehat{C}_t}{\widehat{C}_{t+2}}p_{t+2}\right] + \frac{\beta\kappa_s}{(1+g_n)(1+g_a)^{\frac{1}{1-\alpha}}}\left[\beta\,(1+g_n)\,\mathbb{E}_t\left(\frac{\widehat{C}_t}{\widehat{H}_{t+2}}\right) + \mathbb{E}_t\left(\frac{\widehat{C}_t}{\widehat{H}_{t+1}}\right)\right],
\end{aligned}$$

$$\begin{aligned}
p_t &= (\beta\,(1+g_n))^2\,\mathbb{E}_t\left[\frac{\widehat{C}_t}{\widehat{C}_{t+1}}\mathbb{E}_{t+1}\left[\frac{\widehat{C}_{t+1}}{\widehat{C}_{t+2}}\left(\beta\,(1+g_n)\,\mathbb{E}_{t+2}\left(\frac{\widehat{C}_{t+2}}{\widehat{C}_{t+3}}p_{t+3}\right) + \frac{\beta\kappa_s}{(1+g_n)(1+g_a)^{\frac{1}{1-\alpha}}}\frac{\widehat{C}_{t+2}}{\widehat{H}_{t+3}}\right)\right]\right] + \frac{\beta\kappa_s}{(1+g_n)(1+g_a)^{\frac{1}{1-\alpha}}}\Big[\beta\,(1 \\
&= (\beta\,(1+g_n))^3\,\mathbb{E}_t\left[\frac{\widehat{C}_t}{\widehat{C}_{t+2}}\mathbb{E}_{t+2}\left(\frac{\widehat{C}_{t+2}}{\widehat{C}_{t+3}}p_{t+3}\right)\right] + \frac{\beta\kappa_s}{(1+g_n)(1+g_a)^{\frac{1}{1-\alpha}}}\Big[(\beta\,(1+g_n))^2\,\mathbb{E}_t\left(\frac{\widehat{C}_t}{\widehat{H}_{t+3}}\right) + \beta\,(1+g_n)\,\mathbb{E}_t\left(\frac{\widehat{C}_t}{\widehat{H}_{t+2}}\right) + \mathbb{E}_t\Big(\\
&= (\beta\,(1+g_n))^3\,\mathbb{E}_t\left[\frac{\widehat{C}_t}{\widehat{C}_{t+3}}p_{t+3}\right] + \frac{\beta\kappa_s}{(1+g_n)(1+g_a)^{\frac{1}{1-\alpha}}}\left[(\beta\,(1+g_n))^2\,\mathbb{E}_t\left(\frac{\widehat{C}_t}{\widehat{H}_{t+3}}\right) + \beta\,(1+g_n)\,\mathbb{E}_t\left(\frac{\widehat{C}_t}{\widehat{H}_{t+2}}\right) + \mathbb{E}_t\left(\frac{\widehat{C}_t}{\widehat{H}_{t+1}}\right)\right],
\end{aligned}$$

...

$$p_t = (\beta\,(1+g_n))^j\,\mathbb{E}_t\left[\frac{\widehat{C}_t}{\widehat{C}_{t+j}}p_{t+j}\right] + \frac{\beta\kappa_s}{(1+g_n)\,(1+g_a)^{\frac{1}{1-\alpha}}}\left[\sum_{j=0}^{n-1}(\beta\,(1+g_n))^j\,\mathbb{E}_t\left(\frac{\widehat{C}_t}{\widehat{H}_{t+1+j}}\right)\right],\ \text{where } \beta\,(1+g_n) < 1.$$

Let us take j to infinity, then the no-bubbles solution to this equation must satisfy that,

$$p_t = \frac{\beta\kappa_s}{(1+g_n)\,(1+g_a)^{\frac{1}{1-\alpha}}}\left[\sum_{j=0}^{+\infty}(\beta\,(1+g_n))^j\,\mathbb{E}_t\left(\frac{\widehat{C}_t}{\widehat{H}_{t+1+j}}\right)\right],\ \beta\,(1+g_n) < 1,$$

which is a well-known present value formula. Given our assumptions on the evolution of the stock of housing capital, i.e. $H_{t+j} = \left((1+g_a)^{\frac{1}{1-\alpha}}\right)^j (1+g_n)^j H_t$, then we can immediately determine that $\widehat{H}_{t+j} = H_t$ for any j. In other words, the initial condition on the housing stock, H_t, pins down the evolution of housing capital along the balanced growth path. Hence, it follows that,

$$\begin{aligned}
p_t &= \frac{\beta\kappa_s}{(1+g_n)\,(1+g_a)^{\frac{1}{1-\alpha}}}\left[\sum_{j=0}^{+\infty}(\beta\,(1+g_n))^j\right]\frac{\widehat{C}_t}{H_t} \\
&= \frac{\beta\kappa_s}{(1+g_n)\,(1+g_a)^{\frac{1}{1-\alpha}}\,(1-\beta\,(1+g_n))}\frac{\widehat{C}_t}{H_t}.
\end{aligned}$$

In other words, the price of housing today would be determined by ratio of consumption (along the balanced growth path) relative to today's housing supply stock. We have determined explicitly what the consumption path should be in this economy, therefore it follows immediately that,

$$p_t = \frac{\beta\kappa_s\,(1-\beta\,(1+g_n)\,\alpha)\,g\left(\tau^h\right)}{(1+g_n)\,(1+g_a)^{\frac{1}{1-\alpha}}\,(1-\beta\,(1+g_n))}\frac{e^{a_t+(1-\alpha)n_t}\widehat{K}_t^{\alpha}}{H_t}.$$

Once again, the conjecture that housing prices can be expressed as a log-linear function of capital appears to be consistent with the analytical solution of this type of model. It is worth noticing, however, that the stock of housing also plays a role. Ceteris paribus, the more housing supply available, the lower the price of housing as expected.

We know the housing stock evolves exogenously around the balanced growth path. After a shock, capital would eventually converge towards the balanced growth path, but does not sit on it. Therefore, housing prices would be determined by the transition of capital in response to those shocks. If capital grows much higher than along the balanced growth path, the income effect we would observe on the transition path leads to higher housing prices. In a similar fashion the direct effect of either productivity shocks or labor supply shocks is to increase the price of housing. Although we cannot analyze the transitions between two different regimes of property taxes, we conclude by exploring the pricing formula that the sign with which property taxes operate would be crucial. E.g., if $g'\left(\tau^h\right) > 0$, then higher taxes would lead to higher productivity, higher income and consequently higher housing prices.

D The Baseline Model: The Computational Algorithm

With idiosyncratic uncertainty and a continuum of *ex ante* identical agents, there usually exists a steady state wealth distribution which can be computed. With aggregate uncertainty, however, a steady state wealth distribution will not exist in general, as stochastic aggregate shocks by construction do not cancel out in the aggregate. Hence, the distribution of wealth changes with the stochastic aggregate shock. This feature, noted by Krueger and Kubler (2004), makes it very difficult to approximate equilibria with many agents of different ages and aggregate uncertainty.

In the context of a model with infinitely-lived agents, Krusell and Smith (1998) recognize that each household only needs to forecast future interest rates, but not the future distribution of wealth. If interest rates can be forecasted with sufficient accuracy using a low-dimensional summary statistic of current endogenous variables (such as the current aggregate capital stock), one can simulate an approximate equilibrium allocation by just solving the individual problem given the approximate forecasting rule. Evidently, if this forecasting rule is sufficiently accurate, the dimension of the problem is independent of the number of agents in the economy.

Applying their algorithm to a model with a continuum of infinitely-lived agents (no heterogeneity with respect to age), borrowing constraints and uninsurable idiosyncratic labor income uncertainty Krusell and Smith (1998) find that the aggregate capital stock is sufficient for private agents to forecast future returns to labor and capital with high accuracy. Thus, although exact aggregation of saving behavior across households fails (households differ in their marginal propensity to save out of current wealth), 'quasi-aggregation' is obtained, in the sense that agents do not make large forecasting errors and do not suffer large welfare losses when behaving as if exact aggregation held.

The key to understanding why in the paper of Krusell and Smith the algorithm leads to good approximations even if only the first moment is used, is to realize that, apart from the (small number of) agents right at the borrowing constraint all agents in their model have approximately the same marginal propensity to save out of current wealth, so that quasi-aggregation is obtained. However, when agents differ with respect to their age as in OLG models, in the absence of operative bequest motives their propensity to save will vary greatly by age and thus quasi-aggregation is more likely to fail.

There are many cases where it is sufficient to describe the evolution of the capital stock as a function of last period's aggregate capital stock alone. In particular, if the law of motion for capital is not hit by stochastic shocks, the OLS coefficient estimates describe the evolution of the aggregate capital stock well, independently of the magnitude of the productivity shocks and the aggregate shocks. The reason why this result is likely to occur is that in this case the distribution of wealth does not change much along the equilibrium path. A bad aggregate shock influences both the income of the young, the income of the middle aged, and the income of the old. It is worth mentioning that Krueger and Kubler (2004) obtain better measures of fit (R^2) for TFP specifications that allow for larger-size technology shocks.

This suggests that the choice of the algorithm depends heavily on the application: when a model with large number of generations and heterogeneity within generations is desired, Krusell and Smith's approach appears to be the only feasible way to proceed, whereas for fairly large OLG models without intragenerational heterogeneity Krueger and Kubler (2004) provide a viable contender which maintains full rationality of agents.

D.1 The One-Neighborhood Model

Here, we outline the algorithm used to compute the numerical equilibrium of our model without relocation. The individual state variables are represented by the vector (a, w, j), which includes idiosyncratic productivity shocks, net wealth, and age. The endogenous aggregate state variable, $\Gamma \equiv \Gamma(a, w, j)$, is a high-dimensional object. The exogenous aggregate variable is the population size, n. The combined vector of state variables is $(a, w, j; \Gamma, n)$. A numerical solution of the dynamic programming problem is proposed in the spirit of Krusell and Smith (1998, 2006).

In models where prices are competitive, the current prices depend on aggregate capital. To know future prices, it is necessary to know how the total capital stock evolves. Individual savings decisions do not equal the aggregate whenever markets are incomplete and households are hit with idiosyncratic shocks. Then, total capital stock in the future is a nontrivial function of all the moments of the current distribution, and this distribution is not invariant. Krusell and Smith (1998) propose a log-linear approximation for the evolution of total capital which simplifies things and gives a good fit. In our model, this is no longer true because we cannot establish that aggregate housing stock is sufficient to determine prices (since housing markets are segmented).

Let us specify the structure of the problem that will be relevant for our computational simulation. Notice that we express the problem on the basis of the approximation suggested by Krusell and Smith (1998, 2006) for the high-dimensional distribution $\Gamma(a, w, j)$. Let us assume that there is just one neighborhood, $q \in \{1\}$. Let us assume that households forecast future prices as depending on only the first moments of the distribution, Γ, in addition to n[14]. We denote the first moments for forecasting as $\overline{M} \equiv \left(\overline{W}, ...\right)$, where $\overline{W}$ represents the net wealth held in each neighborhood by the average resident household (conditional on being located there).

We do not consider the mean productivity because by the law of large numbers idiosyncratic uncertainty from the productivity shock cancels out. For all practical purposes we ignore the aggregate bondholdings because by assumption bonds are an asset in zero-net supply. Hence, in equilibrium $\overline{B}$ should be equal to zero in every state. All markets are integrated.

We summarize all the aggregate information that the first moments provide in terms of population structure and net wealth with one aggregate variable: the mean net wealth across all ages, which we denote $\overline{W}$. In other words, $\overline{M} = \left(\overline{W}\right)$.

Population dynamics. Population fluctuations, in the absence of multiple neighborhoods and endogenous relocation, are purely exogenous and can be computed as follows,

$$\begin{aligned}
\lambda_1 &= [n_1], \\
\lambda_2 &= [n_2 + \pi_1 n_1], \\
&\dots \\
\lambda_{65} &= [n_{65} + \pi_1 n_{64} + \pi_2 n_{63} + \dots + \pi_{64} n_1], \\
\lambda_{66} &= [n_{66} + \pi_1 n_{65} + \pi_2 n_{64} + \dots + \pi_{64} n_2], \\
&\dots \\
\lambda_t &= \left[\sum\nolimits_{j=0}^{64} \pi_j n_{t-j}\right], \ \forall t \geq 65.
\end{aligned}$$

The survival rate is denoted π_j, and describes the probability of a household of age j surviving one more year. We assume that $\pi_j = 0$ for all $J \geq 65$ and $\pi_0 = 1$[15].

In our calibration, we assume that the aggregate birth rate shocks are $iid\left(\overline{n}, \sigma_n^2\right)$[16]. Hence, this means that total population $\lambda_t = \sum\nolimits_{j=0}^{64} \pi_j n_{t-j}$ can be viewed as an $MA(65)$ stochastic process. Let us assume that the survival rate takes the following geometric form, $\pi_j = \pi^j$ with $\pi \in (0, 1)$. Then, we may conjecture that the total population approximately takes the following form,

$$\widetilde{\lambda}_t = \overline{n} + \pi \widetilde{\lambda}_{t-1} + \widetilde{n}_t,$$

[14] Limiting households to a finite set of moments is an approximation because housing prices and interest rates do depend on the entire distribution.

[15] This last assumption implies that there are no deaths at birth. Otherwise, it can be interpreted as saying that the birt rate n_t only counts newborns that survive to age one.

[16] The idyosincratic productivity shocks are also $iid\left(\overline{a}, \sigma_a^2\right)$.

where $\widetilde{n}_t \equiv n_t - \overline{n}$. It is possible to re-write the conjecture $AR(1)$ process in $MA(\infty)$ form as,

$$\begin{aligned} \widetilde{\lambda}_t &= \frac{\overline{n}}{1-\pi} + \sum\nolimits_{j=0}^{64} \pi^j \widetilde{n}_{t-j} + \sum\nolimits_{j=65}^{\infty} \pi^j \widetilde{n}_{t-j} \\ &= \lambda_t + \pi^{65} \widetilde{\lambda}_{t-65}. \end{aligned}$$

Since the $AR(1)$ process $\widetilde{\lambda}_t$ is ergodic for the mean, it can be said that the population dynamics would follow a very close path to that marked by an $AR(1)$ process. As a result, even though we do not introduce persistence in the shocks, population dynamics will exhibit this feature. In fact, we know that,

$$\begin{aligned} \mathbb{E}\left(\widetilde{\lambda}_t\right) &= \frac{\overline{n}}{1-\pi}, \\ \mathbb{V}\left(\widetilde{\lambda}_t\right) &= \frac{\sigma_n^2}{1-\pi^2}, \\ \mathbb{CV}\left(\widetilde{\lambda}_t, \widetilde{\lambda}_{t-j}\right) &= \pi^j \frac{\sigma_n^2}{1-\pi^2}. \end{aligned}$$

Hence, we can deduce that,

$$\begin{aligned} \lambda_t &= \widetilde{\lambda}_t - \pi^{65} \widetilde{\lambda}_{t-65}, \\ \mathbb{E}(\lambda_t) &= \mathbb{E}\left(\widetilde{\lambda}_t\right) - \pi^{65} \mathbb{E}\left(\widetilde{\lambda}_{t-65}\right) = \overline{n}\frac{1-\pi^{65}}{1-\pi}, \\ \mathbb{V}(\lambda_t) &= \mathbb{V}\left(\widetilde{\lambda}_t\right) + \left(\pi^{65}\right)^2 \mathbb{V}\left(\widetilde{\lambda}_{t-65}\right) - 2\pi^{65} \mathbb{V}\left(\widetilde{\lambda}_t, \widetilde{\lambda}_{t-65}\right) \\ &= \left[1 + \left(\pi^{65}\right)^2\right] \frac{\sigma_n^2}{1-\pi^2} - 2\left(\pi^{65}\right)^2 \frac{\sigma_n^2}{1-\pi^2} = \sigma_n^2 \frac{1-\left(\pi^{65}\right)^2}{1-\pi^2}, \\ \mathbb{CV}(\lambda_t, \lambda_{t-j}) &= \mathbb{CV}\left(\widetilde{\lambda}_t, \widetilde{\lambda}_{t-j}\right) + \left(\pi^{65}\right)^2 \mathbb{CV}\left(\widetilde{\lambda}_{t-65}, \widetilde{\lambda}_{t-j-65}\right) - \pi^{65} \mathbb{CV}\left(\widetilde{\lambda}_t, \widetilde{\lambda}_{t-j-65}\right) - \pi^{65} \mathbb{CV}\left(\widetilde{\lambda}_{t-65}, \widetilde{\lambda}_{t-j}\right) \\ &= \left[1 + \left(\pi^{65}\right)^2\right] \pi^j \frac{\sigma_n^2}{1-\pi^2} - \pi^{65}\pi^{j+65} \frac{\sigma_n^2}{1-\pi^2} - \pi^{65}\pi^{65-j} \frac{\sigma_n^2}{1-\pi^2} = \sigma_n^2 \pi^j \frac{\left[1-\left(\pi^{65-j}\right)^2\right]}{1-\pi^2} \text{ if } j \leq 65, \\ &= \left[1 + \left(\pi^{65}\right)^2\right] \pi^j \frac{\sigma_n^2}{1-\pi^2} - \pi^{65}\pi^{j+65} \frac{\sigma_n^2}{1-\pi^2} - \pi^{65}\pi^{j-65} \frac{\sigma_n^2}{1-\pi^2} = 0 \text{ if } j > 65, \end{aligned}$$

which are the unconditional moments of the population dynamics in the model.

A comment on the budget constraint and the production function. Let us introduce a little bit of notation on the household's budget constraint which in sequential form can be expressed as,

$$\begin{aligned} c_t + p_t^r b_{t+1} + p_t i_t &= y_t + b_t - \tau(1-\delta) p_t h_t + \left[t_t + (1-\tau)(1-\delta) p_t t_t^h\right], \\ h_{t+1} &= (1-\delta) h_t + i_t, \end{aligned}$$

where i_t is investment, and p_t is the price of a riskless bond today that promises one unit of the consumption good tomorrow. By arbitrage, $p_t^r = r_t^{-1}$ where r_t is the riskless interest rate. Naturally, p_t is the price of housing in the neighborhood, t_t^b is the real transfer of bonds through the 'estate tax' in per capita terms, and t_t^h is the real transfer of housing capital through the 'estate tax' in per capita terms.

Putting aside the transfer due to the estate tax, this budget constraint simply says that all income net of property taxes combined with the returns on bonds ought to finance consumption, investment in housing capital and the demand for next period bonds. Transfers provide an external source of funding, which is out of the control of the individual households.

The property taxes are paid at the beginning of period. However, housing investment on the stock of capital was undertaken at the end of the previous period. At the point taxes are collected, the housing stock has already depreciated. Hence, taxes can only be collected on the depreciated value of the housing stock and do not take advantage of the investments that the households might choose during the current period. More compactly, we can combine these two constraints into one equation as follows,

$$c_t + p_t^r b_{t+1} + p_t h_{t+1} = y_t + \left(b_t + t_t^b\right) + (1-\tau)(1-\delta) p_t \left(h_t + t_t^h\right).$$

This is the standard budget constraint that we study in this paper.

We use w_t to define individual asset wealth at the beginning of period t, a variable which we identify as,

$$w_t \equiv b_t + (1-\tau)(1-\delta) p_t h_t.$$

Hence, we can re-write the budget constraint as,

$$c_t + p_t^r b_{t+1} + p_t h_{t+1} = y_t + w_t + t_t^b + (1-\tau)(1-\delta) p_t t_t^h.$$

Further manipulation of the budget constraint gives us that,

$$c_t + p_t^r b_{t+1} + p_t h_{t+1} = y_t + w_t + t_t^w,$$

where $t_t^w \equiv t_t^b + (1-\tau)(1-\delta) p_t t_t^h$ is the per capita transfer of net real wealth through the estate tax (conditional on location). The vector of prices that we seek to uncover is (p_t^r, p_t), all of which ought to be functions of (Γ, n) which we approximate as functions of $\left(\overline{M}, n\right)$.

Finally, we define the production function in this economy as,

$$y_t \equiv a_t (g_t)^{\phi_j} l_t^{\theta_j},$$

which is both a function of labor, l_t, and per capita government spending (conditional on location), g_t.

The problem of the individual household. The problem of an individual household born into this school district can be represented as follows,

$$V\left(a, w, j; \overline{M}, n\right) = \max_{c,l,b',h'} u(c, mh', l) + \pi_j \beta \sum\nolimits_{a',n'} V\left(a', w', j+1; \overline{M}', n'\right) F^a(a' \mid a) F^n(n' \mid n),$$

where,

$$u(c, mh', l) = \frac{c^{1-\sigma} - 1}{1-\sigma} + \kappa_s \frac{(mh')^{1-\gamma} - 1}{1-\gamma} - \kappa_l \frac{l^{1+\varphi}}{1+\varphi},$$

subject to,

$$\begin{aligned}
&c + p^r b' + ph' = a g^{\phi_j} l^{\theta_j} + w + t^w, \ \forall j \in \{1, ..., J\}, \\
&(c, l) \geq 0, \ w' \equiv b' + (1-\tau)(1-\delta) p' h' \geq -\overline{w}_j, \ \forall j \in \{1, ..., J\}, \\
&\overline{M}' = K\left(\overline{M}; n, n'\right).
\end{aligned}$$

There are effectively three control variables in practice, (l, b', h'), since consumption is pined down by the budget constraint.

The Aiyagari-style borrowing constraint implies that,

$$b' \geq -\overline{w}_j - (1-\tau)(1-\delta) p' h', \ \forall j \in \{1, ..., J\}.$$

For all intensive purposes, this is a borrowing constraint that implies housing is being used as a collateral. Alternatively, we could impose a more complex financing constraint in terms of the debt service ratio, which would takes the following form,

$$b' \geq -\varpi_j a g^{\phi_j} l^{\theta_j}, \ \forall j \in \{1, ..., J\}.$$

We may start with the first borrowing constraint because of its simplicity. Notice, however, that the way in which we define b' counts 'interests and principal', since these bonds are sold at a discount whenever they are issued.

Government spending and Estate Taxes. In order to close the model,we need to specify the government spending in education and the 'involuntary' transfers through the estate tax. In equilibrium, it must hold that the average net wealth satisfies that,

$$\begin{aligned}\overline{W} &\equiv \overline{B} + (1-\tau)(1-\delta)\, p\left(\overline{M}, n\right)\overline{H} \\ &= (1-\tau)(1-\delta)\, p\left(\overline{M}, n\right)\overline{H},\end{aligned}$$

where the second equality follows from the fact that bonds are in zero net-supply, i.e. $\overline{B} = 0$. Therefore, average net wealth should be proportional to the value of the average house in the neighborhood at the beginning of the period (net of taxes and depreciation). The implicit 'estate taxes' and the budget constraint for the school districts are expressed in aggregate terms as follows,

$$\begin{aligned}T^b &= \sum\nolimits_{i=1}^{J} (1-\pi_i) \int 1_{\{j=i\}} b d\Gamma, \\ T^h &= \sum\nolimits_{i=1}^{J} (1-\pi_i) \int 1_{\{j=i\}} h d\Gamma,\end{aligned}$$

and,

$$\begin{aligned}G &= \tau(1-\delta)\, p\left(\overline{M}, n\right)\left(\sum\nolimits_{i=1}^{J} \pi_i \int 1_{\{j=i\}} h d\Gamma + T^h\right) \\ &= \tau(1-\delta)\, p\left(\overline{M}, n\right)\left(\int h d\Gamma\right),\end{aligned}$$

where $J = 64$. We define the relevant measures of per capita real wealth transferred through the 'estate taxes' or converted into education units through government spending as follows,

$$t^b \equiv (\lambda)^{-1} T^b,\ t^h \equiv (\lambda)^{-1} T^h,\ g \equiv (\lambda)^{-1} G.$$

Transfers and government spending are defined in terms of the physical housing stock exchanged and the real borrowing. We can, therefore, re-express the per capita spending and net wealth transfers in each neighborhood as,

$$\begin{aligned}g &\equiv \tau(1-\delta)\, p\left(\overline{M}, n\right)\overline{H} \\ &= \frac{\tau(1-\delta)}{(1-\tau)(1-\delta)}\overline{W}, \\ t^w &\equiv t^b + (1-\tau)(1-\delta)\, p\left(\overline{M}, n\right) t^h,\end{aligned}$$

where $\overline{H}$ is the average housing capital in the neighborhood. Let us assume that the survival rate one more period is constant and independent of age (although it drops to zero at 65), i.e. $\pi_i = \pi$. Then, we can write the per capita net wealth transfer more compactly as,

$$\begin{aligned}t^w &\equiv (1-\pi)\left[\overline{B} + (1-\tau)(1-\delta)\, p\left(\overline{M}, n\right)\overline{H}\right] \\ &= (1-\pi)\left[(1-\tau)(1-\delta)\, p\left(\overline{M}, n\right)\overline{H}\right] \\ &= (1-\pi)\overline{W}.\end{aligned}$$

These are the only two additional expressions that we care about in order to close our model, and they can all be expressed in terms of average net wealth in each neighborhood. The specification with constant survival rates until the end of their lifetime greatly simplifies the model, and serves to make our economy more tractable.

NOTE: I believe there are already a number of good reasons why we should move towards a model in which the survival rate is constant (as expressed before). The loss of realism I believe would be compensated by greater tractability. However, we should be careful to apply this. It might be better to use exponential death rate as a more correct approximation.

D.2 Conjecture and Implementation

Conjecture for the pricing equations. Following in the spirit of Krusell and Smith (1998, 2006) we conjecture a log-linear approximation of the pricing equations $\left\{p^r\left(\overline{H},n\right),p\left(\overline{H},n\right)\right\}$ in these terms,

$$\begin{aligned}
\ln p^r &= p^r_{a0}+p^r_{a1}\ln\overline{H} \text{ if } n=n_a,\\
\ln p &= p_{a0}+p_{a1}\ln\overline{H} \text{ if } n=n_a,\\
\ln p^r &= p^r_{b0}+p^r_{b1}\ln\overline{H} \text{ if } n=n_b,\\
\ln p &= p_{b0}+p_{b1}\ln\overline{H} \text{ if } n=n_b,\\
\ln p^r &= p^r_{c0}+p^r_{c1}\ln\overline{H} \text{ if } n=n_c,\\
\ln p &= p_{c0}+p_{c1}\ln\overline{H} \text{ if } n=n_c.
\end{aligned}$$

Conjecture the law of motion for housing capital. We define the aggregate state in terms of housing capital, but we exploit the fact that the net wealth in each neighborhood is intimately related to the stock of housing in each neighborhood since in equilibrium,

$$\overline{W}=(1-\tau)(1-\delta)\,p\left(\overline{H},n\right)\overline{H}.$$

Slightly abusing notation, we redefine the vector of moments with which we approximate the distribution of this model as $\overline{M}=\left(\overline{H}\right)$ (instead of $\overline{M}=\left(\overline{W}\right)$). We also guess a parameterized functional form corresponding to the law of motion for $\overline{M}=\left(\overline{H}\right)$, which satisfies that,

$$\overline{M}'=K\left(\overline{M};n,n'\right).$$

Analogous to Krusell and Smith (1998), we choose the following log-linear approximation,

$$\begin{aligned}
\ln\overline{H}' &= a_0+a_1\ln\overline{H} \text{ if } n=n_a,\\
\ln\overline{H}' &= b_0+b_1\ln\overline{H} \text{ if } n=n_b,\\
\ln\overline{H}' &= c_0+c_1\ln\overline{H} \text{ if } n=n_c,
\end{aligned}$$

assuming that the aggregate population shock is characterized with 3 different states. That makes for a total of 6 free parameters. Moreover, notice that given this approximation, the steady state of housing capital takes the following form,

$$\begin{aligned}
\ln\overline{H}' &= \ln\overline{H}=\ln\overline{H}^*=\frac{a_0}{1-a_1} \text{ if } n=n_a,\\
\ln\overline{H}' &= \ln\overline{H}=\ln\overline{H}^*=\frac{b_0}{1-b_1} \text{ if } n=n_b,\\
\ln\overline{H}' &= \ln\overline{H}=\ln\overline{H}^*=\frac{c_0}{1-c_1} \text{ if } n=n_c,
\end{aligned}$$

which implies that the housing stock in each neighborhood can take up to three different values depending on the realization of the population shock. In a stationary equilibrium in which the housing stock is identical in each population state, the coefficients of the law of motion must satisfy the following constraint,

$$\frac{a_0}{1-a_1} = \frac{b_0}{1-b_1} = \frac{c_0}{1-c_1}.$$

This implies that we can reduce the number of free parameters by 2 imposing the following intercepts,

$$a_0 = \frac{b_0}{1-b_1}\left(1-a_1\right),\ c_0 = \frac{b_0}{1-b_1}\left(1-c_1\right).$$

NOTE: We may not want to impose the restrictions on the coefficients that we impose here and in our previous attempt at coding the model. It is too restrictive.

The implementation of the approximation. Equipped with all these conjectures we can specify the problem of an individual household at any point in time in the following terms,

$$\begin{aligned}
&V\left(a, w, j; \overline{H}, n\right) = \max_{l, b', h'} u\left(ag^{\phi_j} l^{\theta_j} + w + t^w - p\left(\overline{H}, n\right) b' - p\left(\overline{H}, n\right) h', mh', l\right) + \\
&+\pi_j \beta \sum\nolimits_{a', n'} V\left(a', b' + (1-\tau)(1-\delta) p\left(K\left(\overline{H}; n, n'\right), n'\right) h', j+1; K\left(\overline{H}; n, n'\right), n'\right) F^a\left(a' \mid a\right) F^n\left(n' \mid n\right),
\end{aligned}$$

where,

$$\begin{aligned}
u\left(c, mh', l\right) &= \frac{c^{1-\sigma}-1}{1-\sigma} + \kappa_s \frac{\left(mh'\right)^{1-\gamma}-1}{1-\gamma} - \kappa_l \frac{l^{1+\varphi}}{1+\varphi}, \\
g &\equiv \tau(1-\delta) p\left(\overline{H}, n\right) \overline{H}, \\
t^w &\equiv (1-\pi)(1-\tau)(1-\delta) p\left(\overline{H}, n\right) \overline{H},
\end{aligned}$$

subject to,

$$\begin{aligned}
&(c, l) \geq 0,\ b' + (1-\tau)(1-\delta) p\left(K\left(\overline{H}; n, n'\right), n'\right) h' \geq -\overline{w}_j,\ \forall j \in \{1, ..., J\}, \\
&\overline{H}' = K\left(\overline{H}; n, n'\right),\ p^r\left(\overline{H}, n\right),\ p\left(\overline{H}, n\right).
\end{aligned}$$

Solving this problem would give us the following set of policy rules,

$$\begin{aligned}
l &= l\left(a, w, j; \overline{H}, n\right), \\
b' &= b\left(a, w, j; \overline{H}, n\right), \\
h' &= h\left(a, w, j; \overline{H}, n\right).
\end{aligned}$$

But given the fact that we have conjectured the form of the pricing equations based on a pair of aggregate, it is not clear that we can guarantee that the markets clear. In other words, we cannot guarantee that our pricing equations are a solution to the general equilibrium model.

Instead, we are going to use a two-stage algorithm in the spirit of Krusell and Smith (2006). For the two-stage algorithm to work, we need to redefine the problem of the household with an intermediate step. We shall assume that the current prices for bonds and housing are given and that the households forecast future prices based on the conjectures for the pricing equations and the housing capital, i.e. $\overline{H}' = K\left(\overline{H}; n, n'\right)$, $p^r\left(\overline{H}, n\right)$, and $p\left(\overline{H}, n\right)$. Under these conditions, we can write the intermediate problem of the household as,

$$\begin{aligned}
&\widehat{V}\left(a, w, j; \overline{H}, n\right) = \max_{l, b', h'} u\left(ag^{\phi_j} l^{\theta_j} + w + t^w - p^r b' - p h', mh', l\right) + \\
&+\pi_j \beta \sum\nolimits_{a', n'} \widehat{V}\left(a', b' + (1-\tau)(1-\delta) p\left(K\left(\overline{H}; n, n'\right), n'\right) h', j+1; K\left(\overline{H}; n, n'\right), n'\right) F^a\left(a' \mid a\right) F^n\left(n' \mid n\right),
\end{aligned}$$

where,

$$
\begin{aligned}
u\left(c, mh', l\right) &= \frac{c^{1-\sigma}-1}{1-\sigma}+\kappa_s \frac{\left(mh'\right)^{1-\gamma}-1}{1-\gamma}-\kappa_l \frac{l^{1+\varphi}}{1+\varphi}, \\
g &\equiv \tau\left(1-\delta\right) p\overline{H}, \\
t^w &\equiv \left(1-\pi\right)\left(1-\tau\right)\left(1-\delta\right) p\overline{H},
\end{aligned}
$$

subject to,

$$
\begin{aligned}
&(c, l) \geq 0,\ w' \equiv b'+(1-\tau)(1-\delta) p\left(K\left(\overline{H}; n, n'\right), n'\right) h' \geq -\overline{w}_j,\ \forall j \in\{1, \ldots, J\}, \\
&\overline{H}' = K\left(\overline{H}; n, n'\right),\ p^r\left(\overline{H}, n\right),\ p\left(\overline{H}, n\right).
\end{aligned}
$$

Solving this problem would give us the following set of policy rules,

$$
\begin{aligned}
\widehat{l} &= \widehat{l}\left(a, w, j; p^r, p; \overline{H}, n\right), \\
\widehat{b}' &= \widehat{b}\left(a, w, j; p^r, p; \overline{H}, n\right), \\
\widehat{h}' &= \widehat{h}\left(a, w, j; p^r, p; \overline{H}, n\right),
\end{aligned}
$$

which now clearly depend on the current prices prevailing in the economy. These policy rules conditional on current prices are going to be essential to make sure that any algorithm that we design to implement this model guarantees that all markets clear.

Brief description of the algorithm. *Step 1.* Start with a conjecture for the functional form of $K\left(\overline{H}; n, n'\right)$, $p^r\left(\overline{H}, n\right)$, and $p\left(\overline{H}, n\right)$. Let me denote the vector of coefficients that characterizes the law of motion as Φ, and the vector of coefficients that characterize the pricing equations as Θ. Propose an initial guess for the coefficients of all these equations, i.e. Φ^0 and Θ^0.

Step 2. By backward induction, solve the household's intermediate problem described through the value function $\widehat{V}\left(a, w, j; \overline{H}, n\right)$. The procedure could be as in our previous version of the model. This should give us the following set of policy rules as a function of current prices,

$$
\begin{aligned}
\widehat{l} &= \widehat{l}\left(a, w, j; p^r, p; \overline{H}, n\right), \\
\widehat{b}' &= \widehat{b}\left(a, w, j; p^r, p; \overline{H}, n\right), \\
\widehat{h}' &= \widehat{h}\left(a, w, j; p^r, p; \overline{H}, n\right).
\end{aligned}
$$

Therefore, we need to extend the grid to include interest rates and house price.

Step 3. Simulate the economy using the policy functions obtained in step 2 in order to compute $\{B, H\}_{t=0}^{T}$. In each period, we compute the aggregate bondholdings and the aggregate housing capital in each neighborhood by adding up the individual rules of each household given a pair of current prices (p^r, p). We keep changing the current pricing vector until the bond and housing markets clear in that period.

(a) Bond market clearing requires that aggregate bondholding be equal to zero, i.e. $B = 0$, since the bonds are in zero-net supply.

(b) Housing market clearing requires that we aggregate the housing decisions of all agents residing in one neighborhood and divide by the total resident population in that neighborhood. This measure tells us the average housing capital for next period from individual decisions based on the current state of the economy and the current prices. Based on the current aggregate state and the law of motion for capital, we have a conjecture for what average housing capital in the neighborhood must be in the following period. Market clearing in the housing market is attained whenever the average of housing capital coming from individual decisions coincides with the aggregate state for capital according to the law of motion we conjectured.

NOTE: This approach would ensure that the markets clear in each period. Labor might be a problem that we would need to consider later on to make the model work, although in this problem there is no labor market *per se*. What makes this complex is that now there is a two-layer approximation. The approximation on prices might be influenced by the conjecture on the law of motion.

Step 4. Use the simulated data to obtain a new estimate for Φ^{j+1} and Θ^{j+1} in the following way. First, fix the pricing parameters at their initial guess Θ^0. Second, sort the data $\left\{n_t, \overline{H}_t\right\}$ according to the aggregate shock n_t: for all $n \in \{n_a, n_b, n_c\}$, if $n_t = n$, then $y_\tau = \ln \overline{H}_{t+1}$ and $x_\tau = \ln \overline{H}_t$. For all $n \in \{n_a, n_b, n_c\}$, obtain Φ^{j+1} as OLS estimates of regressions of the following form,

$$y_\tau = a_{n0} + a_{n1} x_\tau + \varepsilon_\tau,$$

where ε_τ is the error term of the regression. Also compute measures of fit for this regression (the standard error and the R^2 of the regression suffice). Repeat steps 2 and 3 until convergence is achieved, i.e. $\Phi^{j+1} \longrightarrow \Phi^*$. In a first pass, if convergence is not achieved, try with another initial guess for Θ^0.

Step 5. Use the simulated data to obtain a new estimate for Φ^{j+1} and Θ^{j+1} in the following way. First, fix the law of motion parameters at the values they converged to in step 4, i.e. Φ^*. Second, sort the data $\left\{n_t, p_t^r, p_t, \overline{H}_t\right\}$ according to the aggregate shock n_t: for all $n \in \{n_a, n_b, n_c\}$, if $n_t = n$, then $y_\tau^r = \ln p_t^r$, $y_\tau = \ln p_t$ and $x_\tau = \ln \overline{H}_t$. For all $n \in \{n_a, n_b, n_c\}$, obtain Θ^{j+1} as OLS estimates of regressions of the following form,

$$\begin{aligned} y_\tau^r &= p_{n0}^r + a_{n1}^r x_\tau + \varepsilon_\tau^r, \\ y_\tau &= p_{n0} + p_{n1} x_\tau + \varepsilon_\tau, \end{aligned}$$

where ε_τ^r and ε_τ are the error terms of the regression. Also compute measures of fit of these regressions (the standard error and the R^2 of the regression suffice). Repeat steps 2 and 3 until convergence is achieved, i.e. $\Theta^{j+1} \longrightarrow \Theta^*$.

Step 6. Using the converging parameters coefficients estimated in step 5, we fix Θ^* in step 4 and repeat the simulation. If the new parameters for the law of motion of capital after running step 4 a second time, i.e. Φ^{**}, coincide with the values derived in the first pass, i.e. Φ^*, then stop. If not, keep iterating until the parameters of the law of motion and the pricing equations are consistent with each other.

Tables and Figures

Table 1: Simulation Results for the Benchmark Calibration

T.B.A.

xxxxx

xxxxx

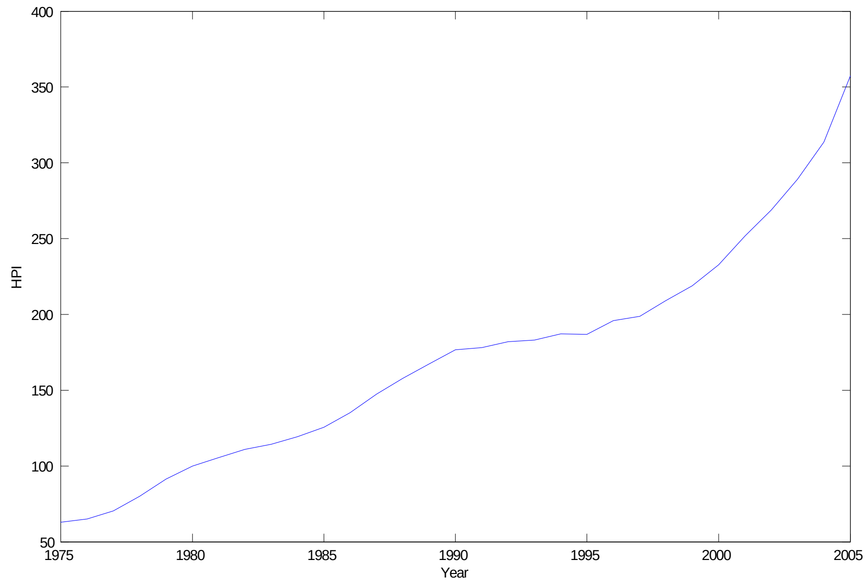
Table 2: Sensitivity Analysis for the Model

T.B.A.

XXXXX

XXXXX

Figure 1: Housing Prices in the U.S., 1975-2005



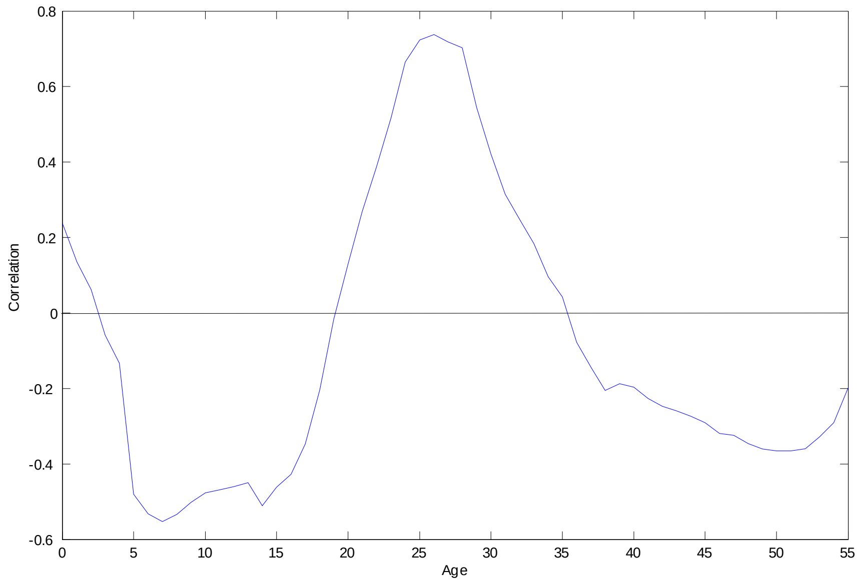
Note the peak in 1979 matches the peak population for 18 year olds. In 1985 housing prices reach their lowest value at a point when the population of 18-year olds is still declining. Note that the peak in 1978 is also at the nadir of the baby bust - at that point demographers were uncertain of a baby echo. As the baby echo began, one would not expect that housing prices would fall conditioned only on the number of 18-year olds. Individuals began to anticipate the increased future population and housing prices took account of this.

Figure 2: The Baby Boom Demographic Patterns

N.A.

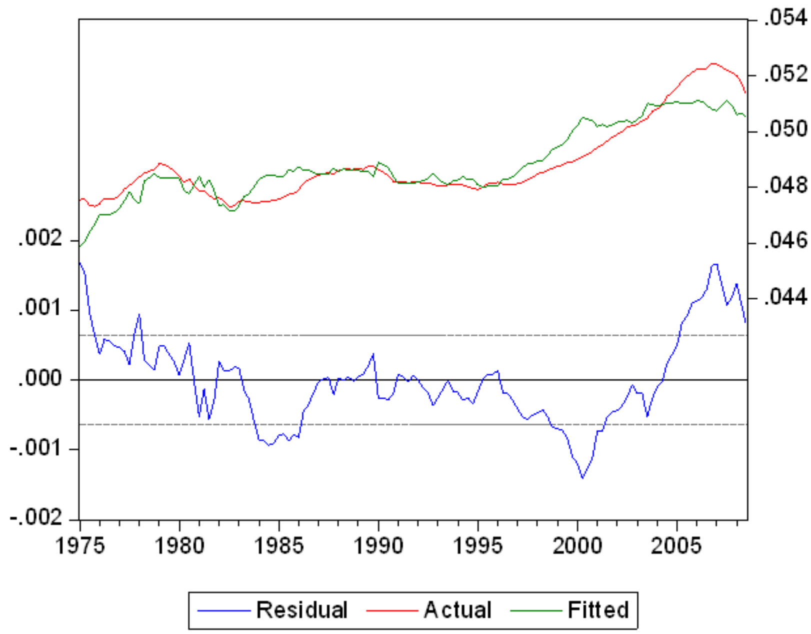
Note the peak in 1979 matches the peak population for 18 year olds. In 1985 housing prices reach their lowest value at a point when the population of 18-year olds is still declining. Note that the peak in 1978 is also at the nadir of the baby bust - at that point demographers were uncertain of a baby echo. As the baby echo began, one would not expect that housing prices would fall conditioned only on the number of 18-year olds. Individuals began to anticipate the increased future population and housing prices took account of this.

Figure 3: Correlation of Population Growth Rates by Age and the Housing Price Index



Note the peak in 1979 matches the peak population for 18 year olds. In 1985 housing prices reach their lowest value at a point when the population of 18-year olds is still declining. Note that the peak in 1978 is also at the nadir of the baby bust - at that point demographers were uncertain of a baby echo. As the baby echo began, one would not expect that housing prices would fall conditioned only on the number of 18-year olds. Individuals began to anticipate the increased future population and housing prices took account of this.

Figure 4: Actual and Fitted House Prices based on the Toy Model.



Note that the in-sample properties of the Toy model deteriorate from the 1990s, while the model performs relatively well until then (see also Mankiw and Weil, 1989). During the 90s, it overestimates the house prices, while afterwards it underestimates the price increase.