Are Recessions Good for Young People*

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Abstract

We construct a stochastic overlapping generations model in which households of different ages are subject to aggregate shocks that affect both wages and asset prices. We use a realistically calibrated version of the model to assess the distributional consequences of severe recessions. More specifically, within the context of this model we ask whether young people can be better off if they become economically active in the midst of a large and persistent economic downtown. The key to the answer is the size of the decline in asset prices, relative to the decline in wages. If older generations have a strong incentive to sell their assets in the downturn to finance old-age consumption (e.g. because they strongly value smooth consumption profiles) then asset prices decline more strongly than wages in equilibrium. This in turn benefits younger generations that can buy these assets at low prices, more than compensating the fall in wages these generations experience. We demonstrate that for realistic parameter values this mechanism is indeed strong enough to generate welfare gains from recessions for young generations. Older cohorts, on the hand, face massive welfare losses from large economic downturns.

Keywords: Recession, Overlapping Generations, Asset Prices

JEL Codes: E21, D31, D58, D91

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1 Introduction

The current economic downturn is the most severe economic downturn since the great depression. Labor incomes of households have fallen significantly below trend and asset prices of real estate and stocks have plummeted massively. The distribution of labor income and wealth is very uneven across age cohorts, suggesting that the welfare consequences from a large and prolonged recession differ significantly across age cohorts.

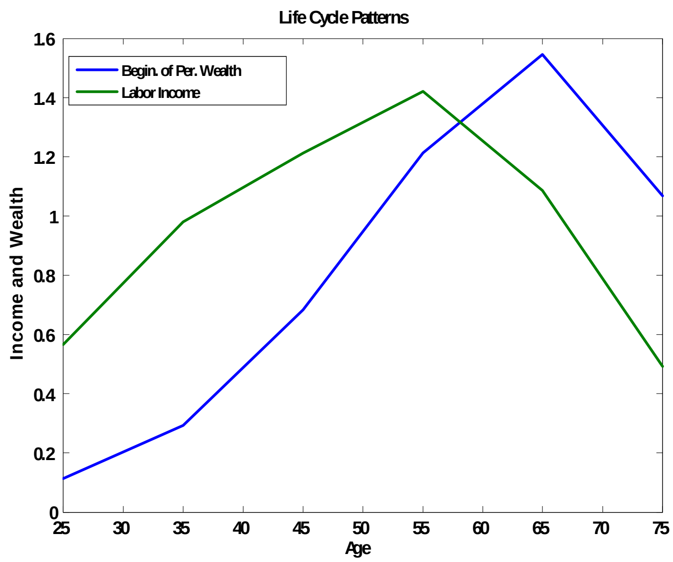

Figure 1: Life Cycle Labor Income and Wealth Profile

In figure 1 we display the life cycle profile of labor income and net worth measured from the Survey of Consumer Finances (SCF) 2007. In this plot average income is normalized to 1, and averages over 10 year age windows have been calculated.[1]

First we observe that the average level of wealth in the data is large (about eight times labor income), which is a well-known general finding from the SCF. Second, young households have little wealth, relative to their labor income. In addition, young households have most of their labor income in front of them. In contrast, older households are wealth-rich but are endowed with little remaining human wealth, measured as the present discounted value of future labor income. Thus it is likely that a deep recession that leads to labor income and asset price declines has substantially different welfare implications for younger and households.

[1]Therefore labor income at age 25 is the average labor income of households headed by individuals aged 20-29.

For wealth, we divide the corresponding numbers in the data by 10, first to express the income and wealth data in similar units, but second, and more importantly, to make the data comparable to our model in which one time period will correspond to 10 years. Since wealth is a stock and income a flow, the empirical wealth data need to be adjusted by the period length to conform to their model

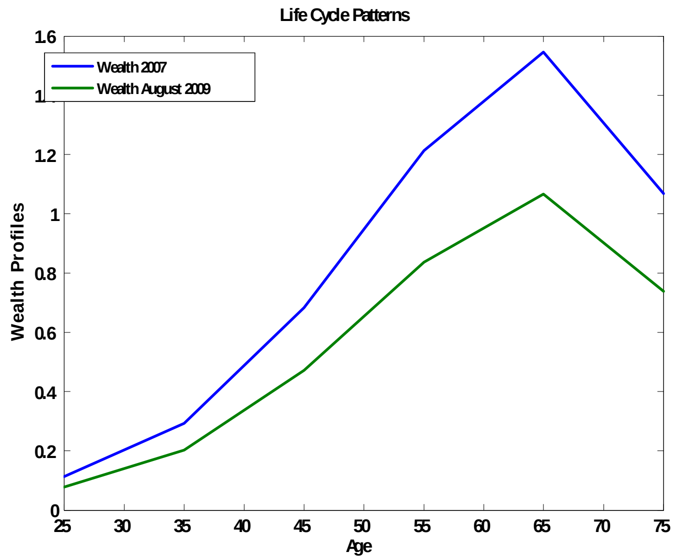

Figure 2: Life Cycle Profile of Wealth, Old and New Asset Prices

This conjecture is reinforced by figure 2. This figure displays the impact of the empirically observed declines in asset prices on the wealth profile of households. To construct this figure we use the age-specific wealth portfolios of households in the SCF in 2007 and re-value them with asset prices for broad asset categories from August 2009.[2] We observe that the implied wealth declines are large and concentrated among households in their 50s to their 70s. For the 65 year old age group the decline in their average net worth amounts to more than twice average labor income. The reason these households face a disproportionately larger decline in the value of their net worth is due to the fact that their portfolio is more strongly tilted towards stocks which have lost more in value between 2007 and 2009 than other assets, including owner-occupied houses.

These empirical facts suggest that the welfare losses from large economic downturns such as the one the U.S. is currently experiencing are unevenly distributed among different age cohorts in the population. In the remainder of this paper we therefore construct a stochastic overlapping generations model in which households of different ages are subject to aggregate shocks that affect both wages and asset prices. We use a realistically calibrated version of the model to assess the distributional consequences of severe recessions. More specifically, within the context of this model we ask whether young people can be better off if they become economically active in the midst of a large and

counterpart.

[2]For details see the data appendix. Of course this exercise ignores the endogenous response of household portfolios to changing asset prices between 2007 and 2009. Since the SCF is only available every three years a more precise evaluation of the impact of changing asset prices on the distribution of household wealth has to wait until the 2010 wave of the SCF is available.

persistent economic downtown.

The key to the answer is the size of the decline in asset prices, relative to the decline in wages. If older generations have a strong incentive to sell their assets in the downturn to finance old-age consumption (e.g. because they strongly value smooth consumption profiles) then asset prices decline more strongly than wages in equilibrium. This in turn benefits younger generations that can buy these assets at low prices, more than compensating the fall in wages these generations experience. We demonstrate that for realistic parameter values this mechanism is indeed strong enough to generate welfare gains from recessions for young generations. Older cohorts, on the hand, face massive welfare losses from large economic downturns.

The remainder of this paper is organized as follows. In the next subsection we briefly discuss the literature that is most relevant for our study. In section 2 we set up our model and define a recursive competitive equilibrium. In section 3 we analyze a simple example that can be characterized analytically and provides crucial insights into the key mechanism of the model. Section 4 is devoted to the calibration of the model and section 5 summarizes the main results. In section 6 we extend our analysis to an environment with multiple assets, before section 7 concludes. Details about the computational approach and the SCF data are relegated to the appendix.

1.1 Related Literature

We add to the quantitative literature on overlapping generations models with aggregate risk which includes Huffman (1987), Krueger and Kubler (2004, 2006), Rios-Rull (1996), Storesletten et al. (2004, 2007). Our analysis of the distribution of welfare consequences across different age cohorts of a large aggregate shock is similar in spirit to Doepke and Schneider's (2006a, b) study of the inflationary episode of the 1970's.

To be completed

2 The Model

We study an overlapping generations economy with aggregate risk that both affects wages and dividends and thus asset prices.

2.1 The Stochastic Structure

The current aggregate shock is denoted by $z \in Z$, where Z is a finite set. We assume that z follows a Markov process with transition matrix $\Gamma_{z,z'}$.

2.2 Technology

A representative firm owns a fixed factor (say land or capital) K, hires labor L in a competitive labor market and produces an amount Y of the single nonstorable output good according to the production function

$$Y = zK^\theta L^{1-\theta}$$

where $\theta \in (0,1)$ is the elasticity of output with respect to the fixed factor whose total amount we normalize to $K = 1$. A total number of 1 shares in the firm is traded on competitive markets. Each share entitles the owner to dividends $d(z)$. Let p denote the price of shares.

2.3 Endowments and Preferences

Households live for N periods, therefore there are N distinct age cohorts alive in the economy at any given point in time. In each period of their life these households are endowed with one unit of time which they supply to the market inelastically. Their age-dependent labor productivity profile is given by $\{\varepsilon_i\}_{i=1}^N$. We normalize units so that $\sum_{i=1}^N \varepsilon_i = 1$ so that aggregate labor supply is $L = 1$. Newborn households start with zero asset holding.

Households have standard time-separable preferences over stochastic consumption streams $\{c_i\}_{i=1}^N$ representable by

$$E \sum_{i=1}^N \prod_{j=1}^i \beta_j u(c_i)$$

where β_i is the time discount factor between age $i-1$ and i which we allow to be varying by age.[3] Expectations $E(.)$ are taken with respect to the underlying stochastic process governing aggregate risk. The period utility function is assumed to be of constant relative risk aversion variety

$$u(c) = \begin{cases} \frac{c^{1-\sigma}-1}{1-\sigma} & \sigma \neq 1 \\ \log(c) & \sigma = 1 \end{cases}$$

where the parameter $\frac{1}{\sigma}$ measures the intertemporal elasticity of substitution.

2.4 Recursive Competitive Equilibrium

We now state the definition of a competitive equilibrium and discuss how we compute it. The aggregate state of the economy is described by the current aggregate shock z and the distribution of wealth (shares) $S = (S_1, \ldots, S_N)$. Individual state variables are a household's age i and its individual share of wealth s.

Therefore the dynamic programming problem of the household reads as

[3] Unless otherwise noted we normalize $\beta_1 = 1$. Using some abuse of notation we let $\beta_1 = 0$ to signify that households do not value consumption in the first period of their lives (but in other periods).

$$
\begin{aligned}
v_i(s,z,S) &= \max_{c\geq 0, s'} \left\{ u(c) + \beta_{i+1} \sum_{z'\in Z} \Gamma_{z,z'} v_{i+1}(s',z',S') \right\} \\
c + s'p(z,S) &= \varepsilon_i w(z) + \left[p(z,S) + d(z)\right] s \\
S' &= G(z,S)
\end{aligned}
$$

where G is the aggregate law of motion. Let by $c_i(s,z,S)$ and $g_i(s,z,S)$ denote the optimal policy functions for consumption and share holdings.

We now can state the following

Definition 1 *A recursive competitive equilibrium are sequences of value and policy functions $\{v_i, c_i, g_i\}$, pricing functions $w, d, , p$ and an aggregate law of motion G such that*

1. *Given the pricing functions and the aggregate law of motion the value functions $\{v_i\}$ solve the recursive problem of the households and $\{c_i, g_i\}$ are the associated policy functions.*

2. *Wages and dividends satisfy*

$$
\begin{aligned}
w(z) &= (1-\theta)z \\
d(z) &= \theta z
\end{aligned}
$$

3. *Markets clear*

$$
\begin{aligned}
\sum_{i=1}^{N} \varepsilon_i &= L = 1 \\
\sum_{i=1}^{N} g_i(S_i,z,S) &= 1 \\
\sum_{i=1}^{N} c_i(S_i,z,S) &= z
\end{aligned}
$$

4. *Consistency*

$$
\begin{aligned}
S_1' &= 0 \\
S_{i+1}' &= G_i(z,S) = g_i(S_i,z,S)
\end{aligned}
$$

2.5 Computation

Even for moderate number of generations state space is large: $N-2$ continuous state variables (plus z). Since we want to deal with big shocks local methods (e.g. perturbation) may not be applicable. We use global methods based on sparse grids, as in Krueger and Kubler (2004, 2006). *To be Completed*

3 An Example

In order to clarify the main mechanism linking shocks to aggregate productivity, wages and asset prices we now study a simple example in which the recursive competitive equilibrium can be computed in closed form. Suppose that households live for three periods, $N = 3$. Households do not value consumption when young (i.e. $\beta_1 = 0$) and discount the future at a constant factor $\beta_2 = \beta_3 = \beta$. Households are only productive in the first period of their lives, i.e. $\varepsilon_1 = 1$ and $\varepsilon_2 = \varepsilon_3 = 0$. Thus by construction young households save everything and then face a simple consumption-saving problem when middle-aged. For simplicity we also assume that the aggregate shock takes only two values $Z = \{z_l, z_h\}$ values, although the existence of an analytical solution does not depend on this assumption.

Given that young households start their lives with zero asset holdings and that the total number of wealth shares has to sum to one, the crucial aggregate state variable in this simple example is the share of asset held by old households S_3 which we for simplicity denote $S_3 = S$. Consequently the share of assets owned by middle-age households is given by $S_2 = 1 - S$. The only households with a meaningful economic decision are the middle-aged households that choose the number of shares s' to be carried into old age.

This choice satisfies the standard intertemporal Euler equation

$$\begin{aligned} & p(z,S)u'\left[(1-S)(p(z,S)+\theta z) - s'p(z,S)\right] \\ = \; & \beta \sum_{z'} \Gamma_{z,z'} u'\left[s'(p(z',S')+\theta z')\right]\left[p(z',S')+\theta z'\right] \end{aligned} \tag{1}$$

where consistency requires that tomorrow's asset share of the old is equal to the number of shares purchased by the current middle-aged households: $S' = s'(z,S)$. In this expression marginal utility from consumption when middle aged, $c_m = (1-S)(p(z,S)+\theta z) - s'p(z,S)$ is equated to expected discounted marginal utility from old age consumption $c_o = s'(p(z',S')+\theta z')$, adjusted by the gross return on assets $\frac{p(z',S')+\theta z'}{p(z,S)}$. Given a pricing function $p(z,S)$ equation (1) defines the optimal policy function $s'(z,S)$.

The second functional equation determining the pricing and optimal policy function states that the equilibrium demand for shares of the young, $1-s'(z,S)$ equals the number of shares that can be purchased with total labor income of the young, $\frac{w(z)}{p(z)} = \frac{(1-\theta)z}{p(z)}$. Thus

$$[1 - s'(z,S)]p(z,S) = (1-\theta)z \tag{2}$$

Equations (1) and (2) form a pair of functional equations that jointly determine the unknown pricing and policy functions $p(z,S)$ and $s'(z,S)$. Given a solution for these functions consumption and welfare, as function of the aggregate state of the economy z

and the distribution of wealth S, are given by

$$\begin{aligned}
c_o(z,S) &= S(p(z,S)+\theta z) && (3)\\
c_m(z,S) &= (1-S)(p(z,S)+\theta z)-s'(z,S)p(z,S) && (4)\\
v_o(z,S) &= u(c_o(z,S)) && (5)\\
v_m(z,S) &= u(c_m(z,S))+\beta\sum_{z'}\Gamma_{z,z'}u(c_o(z',s'(z,S))) && (6)\\
v_y(z,S) &= \beta\sum_{z'}\Gamma_{z,z'}v_m(z',s'(z,S)) && (7)
\end{aligned}$$

We are mainly interested in conditions under which young households benefit from becoming economically active in an economic downturn $z=z_l$ relative to an expansion, $z=z_h$. Since young households do not consume when young, they mainly care about the current aggregate state of the world because it determines their cash at hand $cah(z,S)$ and thus their consumption possibility set when turning middle aged:

$$cah(z,S)=[p(z',S')+\theta z']\frac{w(z)}{p(z,S)}$$

If aggregate shocks are iid over time and the law of motion for the wealth distribution satisfies $S'(z,S)=s'(z,S)=s'(S)$, as will be the case if the utility function is logarithmic, then young households benefit from a recession if and only if

$$\frac{w(z_l)}{p(z_l,S)}>\frac{w(z_h)}{p(z_h,S)} \tag{8}$$

that is, if and only if (endogenous) asset prices in a recession fall more significantly than wages. Persistent effects of the aggregate shock (either through serial correlation of the exogenous Markov process or through endogenous persistence generated by the dynamics of the wealth distribution determined by the law of motion $S'=G(z,S)$) complicates the argument somewhat. However, as we will see in our quantitative analysis of a realistically calibrated version of the model the relative movement of wages and asset prices remains the crucial determinant of the welfare impact of recessions on young households.

3.1 Log-Utility

As well known at least since Huffman (1987) if the period utility function is logarithmic and households earn stochastic labor income only in the first period of their lives, then the recursive equilibrium can be determined in closed form. It is straightforward to verify (by simply checking that the proposed functions satisfy the system of functional equations (1) and (2)) that the recursive equilibrium, if $u(c)=\log(c)$, takes the form

$$
\begin{aligned}
p(z,S) &= zp(S) \\
s'(z,S) &= s'(S) \\
c_o(z,S) &= zc_o(S) \\
c_m(z,S) &= zc_m(S) \\
v_o(z,S) &= \log(z) + \Psi_o(S) \\
v_m(z,S) &= \log(z) + \beta E_z \log(z') + \Psi_m(S) \\
v_y(z,S) &= \beta E_z \log(z') + \beta^2 E_z E_{z'} \log(z'') + \Psi_y(S)
\end{aligned}
$$

where Ψ_i are known functions. Crucially and perhaps not surprisingly, given log-utility, for the optimal saving decision of the middle aged generation the income and substitution effect of falling asset prices cancel out exactly so that the optimal number of shares carried into old age is independent of the aggregate productivity shock z, and thus $s'(z,S) = s'(S)$. Note that this result also implies that the wealth dynamics in the model is nonstochastic since $S' = s'(z,S) = s'(S)$. Further inspection of the $s'(S)$ function reveals that from any initial condition $S_0 \in (0,1)$ the wealth share of the elderly converges monotonically to its unique positive steady state value. Furthermore with log utility the ratio of wages and asset prices $w(z)/p(z)$ is independent of z.

Given the previous result the welfare consequences of a recession for the different generations can be easily calculated as:

$$
\begin{aligned}
v_o(z_l,S) - v_o(z_h,S) &= \log\left(\frac{z_l}{z_h}\right) < 0 \\
v_m(z_l,S) - v_m(z_h,S) &= \log\left(\frac{z_l}{z_h}\right) + \beta\left(E_{z_l} - E_{z_h}\right)\log(z') \\
v_y(z_l,S) - v_y(z_h,S) &= \beta\left(E_{z_l} - E_{z_h}\right)\log(z') \\
&\quad + \beta^2\left(E_{z_l} - E_{z_h}\right)E_{z'}\log(z'')
\end{aligned} \tag{9}
$$

Thus old households always lose from a recession, not surprisingly since they simply consume the value of their assets which are worth less in a recession. For the middle aged and young generations the welfare consequences of a recession depend somewhat on the properties of the stochastic process driving aggregate risk.

The middle-aged generations are also unambiguous losers of recessions unless $\beta > 1$ and aggregate productivity is strongly negatively correlated so that $E_{z_l}\log(z') \gg E_{z_h}\log(z')$. For young generations, on the other hand, the model with log-utility hints at the possibility of benefits from recessions. If aggregate shocks are independent over time then, as equation (9) suggests, the young generations are indifferent between being (economically) born into good or bad aggregate circumstances. If these shocks are positively correlated, young households unambiguously lose from a recession. On the other hand, if aggregate shocks display sufficiently negative serial correlation young households may gain from an economic downturn.

The crucial property of the model with log-utility is that the extent to which middle-aged households sell shares to finance their consumption is independent of the aggregate shock, due to income and substitution effect cancelling out: $s'(z,S) = s'(S)$. This result in turn implies that in equilibrium asset prices are proportional to aggregate shocks and thus fall to the same extent as wages. This result also suggests that for preferences that feature an intertemporal elasticity lower than one middle-aged households have a stronger motive to sell shares to smooth consumption in light of temporary declines in asset payoffs (the income effect dominates the intertemporal substitution effect). This is turn may lead to a decline in asset prices that is steeper than the corresponding fall in wages and thus generate welfare gains from economic downturns for young households.

We now evaluate this conjecture by computing equilibria for varying degrees of the willingness to intertemporally substitute consumption by households $\frac{1}{\sigma}$ before turning to the full quantitative analysis of our general model.

3.2 General Intertemporal Elasticity of Substitution

For $\sigma \neq 1$ the recursive competitive equilibrium of the model cannot be solved analytically. The numerical solution of the system of functional equations (1) and (2) is straightforward, however. For the purpose of this simple example we assume a time discount factor of $\beta = 0.5$ (note that a model period amounts to about 20 years in the data), a capital share of $\theta = 0.3$, and let the aggregate shock by serially uncorrelated and take two values $z_l = 0.97, z_h = 1.03$. Thus a fall in aggregate technology leads to a decline of aggregate output in the order of about 6%, in line with the observed fall in output below trend in the most recent recession.

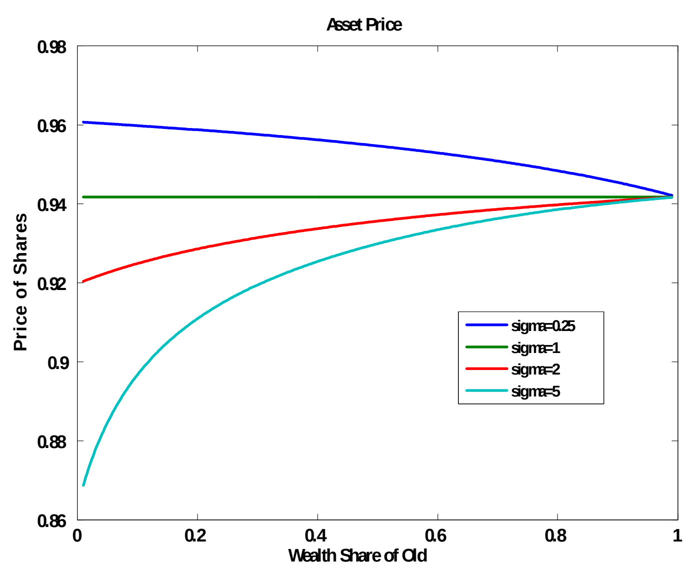

Figure 3: Fall in Asset Prices in Recession: $\frac{p(z_l,S)}{p(z_h,S)}$

Figure 3 plots equilibrium asset prices in recessions, relative to expansions $p(z_l, S)/p(z_h, S)$, as a function of the share of wealth held by the old generation, for various values of the intertemporal elasticity of substitution $1/\sigma$. Note that, as demonstrated above, for the logarithmic case $\sigma = 1$ we have $p(z_l, S)/p(z_h, S) = z_l/z_h = 0.942$, independent of the wealth distribution S.

This figure displays two key findings. First, relative to the unit elasticity case, the lower is the willingness of households to intertemporally substitute consumption (the higher is σ), the larger is the fall in asset prices in a recession. For example, for $\sigma = 5$ an output fall of 6% is associated with a corresponding fall of stock prices by up to 14%, in case most of the shares are held initially by the middle-aged generation. Second, the extent of the asset price movements are strongly affected by the wealth distribution S if preferences are not logarithmic. The larger is the share of wealth held by the middle aged, the stronger is the divergence between movements in fundamentals z_l/z_h and movements in asset prices. Old households have no choices to make and sell all their assets inelastically. Middle-aged households, on the other hand face a meaningful consumption-savings decision and thus have an elastic (with respect to aggregate shocks) asset demand (unless preferences are logarithmic). The larger the share of wealth in the hands of households with elastic demand, the more strongly do asset prices deviate from underlying technology shocks.[4]

The welfare consequences for young generations of starting their economic lives in a recession, relative to an expansion, are displayed in figure 4. We measure welfare consequences as the percentage increase in consumption in all periods of a household's life, under all state contingencies, that a household born in an expansion would require to be as well off as being born in a recession. Positive numbers thus reflect welfare gains from a recession whereas negative figures signify welfare losses. Again we plot these numbers as a function of the initial wealth distribution as measured by S, and for different degrees of intertemporal substitution.

The main message of the figure is that the welfare consequences from recessions for the young follow the relative movement of asset prices, relative to wages (productivity) rather closely. Thus these results confirm the intuition developed in the discussion surrounding equation (8): to a first approximation young households benefit from an economic downturn if and only if asset prices decline more strongly than wages, with the size of the welfare gains being roughly proportional to this relative decline. The larger the share of wealth held by the price-elastic middle aged, and the lower their intertemporal elasticity of substitution, the larger are the welfare gains of the young from a large recession.

The purpose of the simple model was to develop the intuition for the main drivers of

[4] In the economies for which $\sigma \neq 1$ the wealth holdings of the old at the start of a recession need not be the same as at the start of an expansion, on average. The figure displays results from a hypothetical thought experiment that traces out the differences between expansions and recessions, conditional on the same wealth distribution in the economy. In practice the differences in average wealth holdings of the old at the beginning of expansions and recessions is small even when aggregate shocks are highly persistent, and absent when shocks are independent over time.

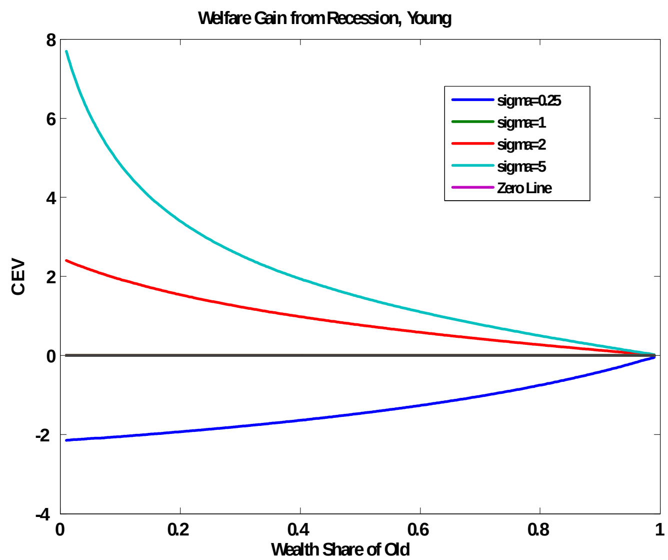

Figure 4: Welfare Consequences of Recession for Young Households

the welfare consequences of economic downturns. To do so we have deliberately stacked the deck in favor of delivering the desired result. In the simple model young households do not value consumption and thus suffer no adverse consequences from a decline in current aggregate consumption. Aggregate shocks are purely temporary so that asset prices will recover when these young households turn into their later years. We now want to investigate how robust our qualitative results for the simple model are in a somewhat more realistically calibrated economy in which the life cycle labor income and wealth profile of the model matches that observed in the 2007 SCF.

4 Calibration

Now households live for $N = 6$ periods, so that one period stands for ten years. This choice enables us to map life cycle statistics for labor income and wealth from the model into their empirical counterparts from the SCF. For the process driving technology shocks and the capital share we retain the assumptions $\theta = 0.3$, $z_l = 0.97, z_h = 1.03$ from the previous section to enable a more meaningful comparison of results. Aggregate shocks remain serially uncorrelated over time.[5] In lieu of documenting results for the full stochastic version of the model in this version of the paper we study the consequences of a one-time unexpected temporary negative shock z_l. That is, we trace out the transition path, starting from the deterministic steady state associated with a permanent technol-

[5] This is not an implausible assumption for a 10 year horizon. The restriction to these long time periods admittedly has the undesirable consequence of making recessions very long-lasting events, reminiscent of the lost decade on Japan in the 1990s'.

ogy level $z = z_h$, induced by a one-period unexpected drop to $z = z_l$, after which the technology recovers (predictably for all households) to $z = z_h$.

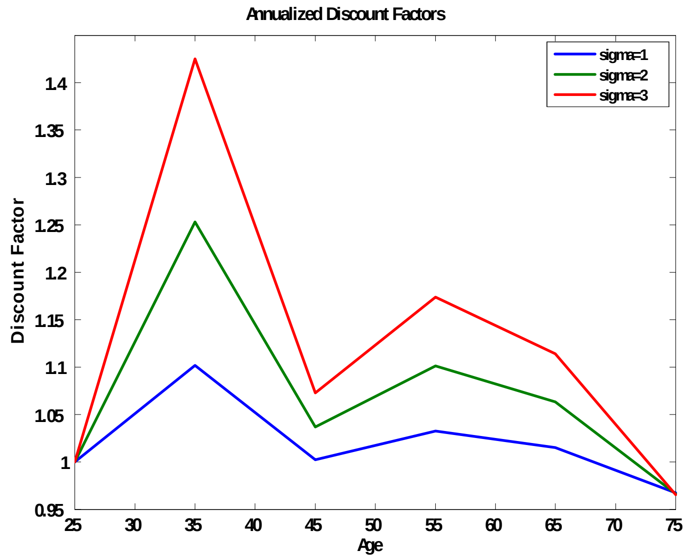

Figure 5: Annual Time Discount Factors, as a Function of σ

In order to assess the distribution of welfare consequences across age cohorts we view it as important that the model we use is consistent with the observed life cycle labor income and wealth profiles. We achieve this in a rather direct and perhaps somewhat brute force way. With respect to labor income we simply choose the age-dependent labor productivities $\{\varepsilon_i\}$ to match the life cycle labor earning profile from SCF. We then choose the age-dependent time discount factors $\{\beta_i\}$ so that the steady state of model matches the life cycle profile of assets from SCF documented in figure 2. Figure 5 displays the anualized time discount factors that are required to achieve this, as a function of the parameter σ. The model-implied steady state life cycle labor income, wealth[6] and conumption paths are displayed in figure 6.

Note that since the life cycle wealth profile target is independent of σ, and so is the life cycle labor income profile (and since aggregate wealth shares have to sum to 1), from the steady state household budget constraints and the market clearing conditions the implied consumption profile $\{c_i\}$ and equilibrium rate of return on assets R (the gross interest rate) is independent of σ, conditional on wanting to match the same empirical life cycle wealth profile. The required time discount factors are then implied by the steady state Euler equations

$$\beta_{i+1} R \left(\frac{c_{i+1}}{c_i} \right)^{-\sigma} = 1$$

[6]Since our current model is a general equilibrium model and does not include bequests we approximate the empirically observed asset holdings of households at the beginning of the life cycle with zero asset holdings.

and evidently do depend on the intertemporal elasticity of substitution $1/\sigma$. More precisely, the larger is σ, the more volatile over the life cycle do the time discount factors have to become in order to generate the same asset (and thus consumption) profile.

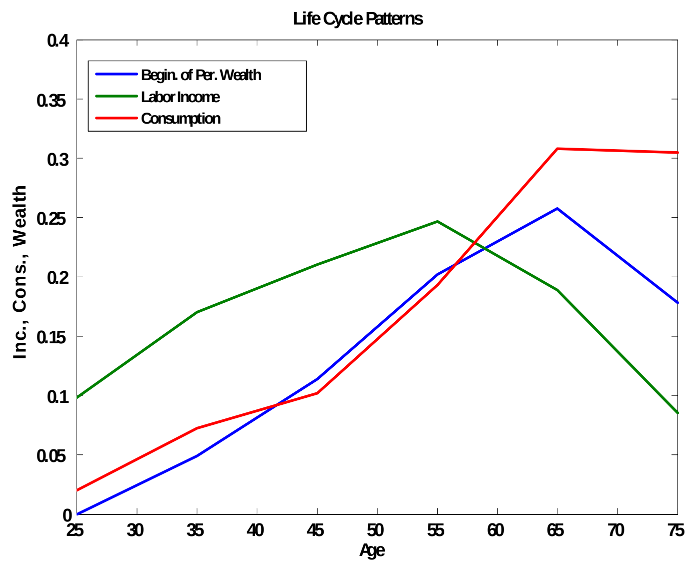

Figure 6: Steady State Life Cycle Profiles for Labor Income, Wealth and Consumption

The fact that the required time discount factors are larger than one for all but the last period is due to the fact that, in the data, household income and wealth life cycle profiles are initially upward sloping. Expecting higher income in the future and still accumulating more assets is a behavior consistent with a standard life cycle model such as our only if households are sufficiently patient early in their life. Also note that the β_i should not be interpreted only as capturing pure time preference but also, in a very reduced form, the effects of mortality risk and changes of family size on the marginal utility of consumption.[7]

5 Results

In this section we document the asset pricing and welfare implications from a large (and unexpected) recession. Figure 7 displays the transition path of asset prices implied by the one-time transitory negative technology shock, for $\sigma = 3$. We observe that relative to the underlying shock (z declines by 6%) the model-implied price decline is about twice

[7]The required large time discount factor (extreme patience) between the first and the second period of life is partially due to the fact that we let households start their economic lives with zero inital wealth rather than the empirically observed moderately positive wealth levels. This assumption strenthens the need, in the model, to accumulate wealth early on despite facing an upward sloping income profile and thus implies an especially large β_2.

as large. This finding is consistent with the intuition developed in section 3 that for an elasticity of substitution $1/\sigma$ smaller than one asset prices are more volatile than the underlying shocks. Also note that due to the endogenous wealth dynamics in the model asset prices overshoot and take several periods to return to their steady state values, despite the fact that the underlying shock is purely transitory.

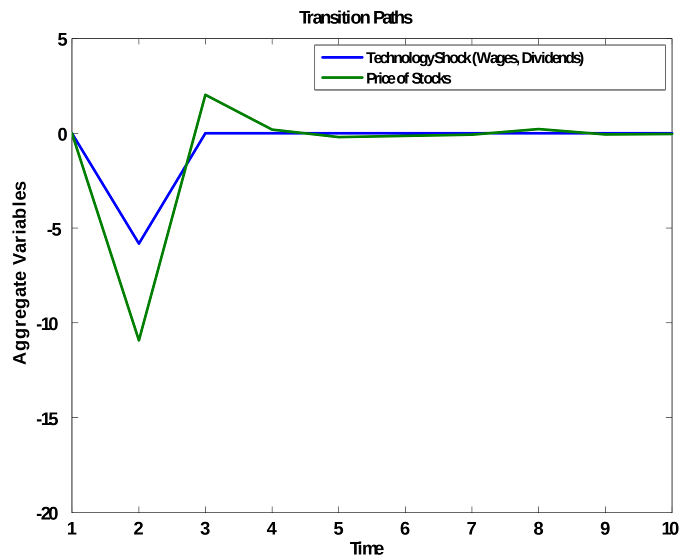

Figure 7: Transition Path for Asset Prices, $\sigma = 3$

In table I we summarize the main welfare implications from our analysis. The first row presents, for various σ, the percentage decline of asset prices, relative to the percentage fall of the underlying technology shock in period 2, the period in which the unexpected negative shock to technology hits the economy. Again consistent with the analysis in section 3 we see that the lower is the intertemporal elasticity of substitution the larger are relative asset price fluctuations. For the log-case asset prices decline exactly as much as output and technology.

Table I: Welfare Gain from Recession

age i	$\sigma = 1$	$\sigma = 2$	$\sigma = 3$
$\frac{\%\Delta(p_2/p_1)}{\%\Delta(z_2/z_1)}$	1.00	1.53	1.88
1	-0.76%	-0.03%	0.13%
2	-1.13%	-0.65%	-0.51%
3	-1.40%	-0.70%	-0.43%
4	-2.06%	-1.60%	-1.30%
5	-3.43%	-3.73%	-3.85%
6	-5.83%	-7.61%	-8.80%

The remaining rows of the table display the welfare consequences[8] from the large recession for different generations in the economy, with $i = 1$ denoting households that become economically active in the period in which the economy plunges into the recession. The table highlights three main observations. First, the welfare consequences are monotone in age, with older generations suffering significantly more from the recession. Second, the lower is the intertemporal elasticity of substitution and thus the more extreme are the asset price responses to the underlying shock, the more dispersed the welfare consequences become: with higher σ the old lose more and the young lose less. Third, for sufficiently low $1/\sigma$ households becoming economically active in the recession indeed benefit from it (whereas the oldest generations suffer large welfare losses in the order of almost 9% of consumption). These large welfare losses are due to the fact that for the oldest generations the temporary shock is a permanent shock (since these households die after period 2) and the asset prices fall significantly more than overall output and thus consumption. For the young, in contrast, the economy recovers for the rest of their lives and thus the price of the asset they have bought for cheap rebounds as well in the periods in which these households use it to finance consumption. Thus to answer the questions posed in the introduction: the young might quite plausibly gain from coming to age in a recession and the distributional impacts of large recessions across age groups are very substantial, with old-age households being at the losing end.

6 Multiple Assets and Realistic Portfolios

So far we restricted attention to a model in which households can only hold one asset, risky stocks. In the SCF data there is substantial heterogeneity in the composition of asset portfolios across age cohorts. In fact, the reason the consequences on net worth of asset price declines were asymmetric across age cohorts in figure 2 in the introduction was exclusively due to the fact that young households hold less risky assets and the price of stocks declined more between 2007 and 2009 than the price of other assets. Therefore we now investigate to what extent our results established so far are robust in a model that features heterogeneity with respect to portfolio allocations over the life cycle across broad asset categories. We now augment the model by a risk free bond that is in positive net supply and endogenous portfolio choice. The bond is issued by the firms which are therefore leveraged; this constructed has the added potential benefit that stock prices might become more volatile, relative to output, than in the one-asset economy.

Let the exogenous supply of bonds be denoted by γ, and the price of the bond be denoted by $q(z, S)$. Capital income now θz is now decomposed into dividends paid by

[8] Again welfare gains are measures as consumption equivalent variation: as the percent increase in consumption (in all periods of life) under the no-shock scenario needed to make households indifferent to living through the transition induced by the negative transitory shock. Positive numbers thus denote welfare gains, negative numbers welfare losses from the recession.

firms and the interest payments that these firms make to bondholders

$$\theta z \quad = \quad d(z) = \underbrace{[1 - q(z,S)]\,\gamma}_{\text{Interest Income}} + \underbrace{d(z) - [1 - q(z,S]\gamma}_{\text{Dividends}}$$

Let λ_i denote the share of a household of age i that is invested in risky assets.

6.1 Recursive Competitive Equilibrium

The aggregate state space of the model remains the exogenous aggregate shock z and the wealth distribution S. Total aggregate wealth is now given by the sum of the value of the firm plus the value of bonds in the economy:

$$[p(z,S) + d(z) + q(z,S)\gamma]\,.$$

S_i now denotes the share of this wealth owned by generation i. The individual state variable is the household share of wealth s, and the dynamic programming problem of the households reads as:

$$\begin{aligned}
v_i(s,z,S) &= \max_{c\geq 0, y, \lambda, s'(z')} \left\{ u(c) + \beta_{i+1} \sum_{z'\in Z} \Gamma_{z,z'} v_{i+1}(s'(z'), z', S') \right\} && (10) \\
c + y &= \varepsilon_i w(z) + [p(z,S) + d(z) + q(z,S)\gamma]\, s && (11) \\
s'(z') &= \frac{[p(z',S') + d(z') - (1 - q(z',S')\gamma]\,\frac{\lambda}{p(z,S)} y + \frac{1-\lambda}{q(z,S)} y}{p(z',S') + d(z') + q(z',S')\gamma} && (12) \\
S' &= G(z,S,z') && (13)
\end{aligned}$$

Households choose consumption, how much of labor income $\varepsilon_i w(z)$ plus gross capital income $[p(z,S) + d(z) + q(z,S)\gamma]\, s$ to save, y, and the share of these savings allocated to stocks, λ. Thus the number of shares purchased by the household today is given by $\frac{\lambda y}{p(z,S)}$ and the number of bonds is given by $\frac{(1-\lambda)y}{q(z,S)}$. Tomorrow the bonds pay out one unit of consumption per bond purchased, and stocks pay out $[p(z',S') + d(z') - (1 - q(z',S')\gamma]$ per share. Thus the share of wealth owned by the household tomorrow is given by equation (12). Let the optimal policy functions of the dynamic programming problem be denoted by $c_i(s,z,S), y_i(s,z,S), \lambda_i(s,z,S), g_i(z';s,z,S)$. We have the following

Definition 2 *A recursive competitive equilibrium are sequences of value and policy functions $\{v_i, c_i, y_i, \lambda_i, g_i\}$, pricing functions w, d, p, q and an aggregate law of motion G such that*

1. *Given the pricing functions and the aggregate law of motion the value functions $\{v_i\}$ solve the recursive problem of the households and $\{c_i, y_i, \lambda_i, g_i\}$ are the associated policy functions.*

2.

$$
\begin{aligned}
w(z) &= (1-\theta)z \\
d(z) &= \theta z
\end{aligned}
$$

3. Markets clear

$$
\begin{aligned}
\sum_{i=1}^{N} \varepsilon_i &= L = 1 \\
\sum_{i=1}^{N} c_i(S_i, z, S) &= z \\
\sum_{i=1}^{N} \lambda_i(S_i, z, S) y_i(S_i, z, S) &= p(z, S) \\
\sum_{i=1}^{N} (1 - \lambda_i(S_i, z, S)) y_i(S_i, z, S) &= \gamma q(z, S)
\end{aligned}
$$

4. Consistency

$$
\begin{aligned}
S'_{i+1} &= G_i(z, S, z') = g_i(z'; S_i, z, S) \textit{ for } i = i, \ldots, N-1 \\
S'_1 &= 0
\end{aligned}
$$

6.2 Calibration

To be completed

6.3 Results

To be completed

7 Conclusion

In this paper we have analyzed the distributional consequences of a large recession across different age cohorts. For a quantitative version of our stochastic overlapping generations economy restricted to match life cycle income and asset profiles from the SCF we find that older households suffer large welfare losses from a severe recession. Young households, in contrast, lose less and might even benefit from the economic downturn. The key statistic determining these welfare consequences is the price decline of assets, relative to the fall in wages and output. If households have low intertemporal elasticity of substitution then older households are pressed to sell their assets in the downturn in order to smooth consumption, putting additional pressure on asset prices, inducing larger welfare losses for older households and small welfare gains for households that become economically active in the recession.

Our model also has strong predictions how the recession affects the wealth distribution across households of different ages. Once the 2010 SCF is available we can evaluate whether the model predictions along this dimension are born out by the actual wealth data prior and after the great recession the U.S. economy is currently experiencing. We defer this to future work.

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A Computational Details

To be completed

B Data Appendix

To be completed