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HARNESSING WINDFALL REVENUES:

Optimal policies for resource-rich developing economies[#]

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Abstract

A windfall of natural resource revenue (or foreign aid) faces government with choices of how to manage public debt, investment, and the distribution of funds for consumption, particularly if the windfall is both anticipated and temporary. We show that the permanent income hypothesis prescription of an ever-lasting increase in consumption financed by borrowing ahead of the windfall and then accumulating a Sovereign Wealth Fund (SWF) is not optimal for capital-scarce developing economies. Such countries should accumulate public and private capital to accelerate their development and, only if the windfall is large relative to initial foreign debt, is it optimal to build a SWF. The optimal time profile of consumption is biased towards the near future, as compared to the permanent income hypothesis. Outcomes depend on instruments available to government. We study cases where the government can make lump-sum transfers to consumers; where such transfers are impossible so optimal policy involves cutting distortionary taxation in order to raise investment and wages; and where Ricardian consumers can borrow against future revenues, in which case the policy response to possible over-consumption is a high level of investment in infrastructure.

Keywords: natural resource**,** windfall public revenues, risk premium on foreign debt, public infrastructure, private investment, credit constraints, optimal fiscal policy, debt management, Sovereign Wealth Fund, asset holding subsidy, developing economies.

JEL codes: E60, F34, F35, F43, H21, H63, O11, Q33

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1. Introduction

Over the period 2000-05 exports of hydro-carbons and minerals accounted for more than 50% of goods exports in 36 countries. In 18 of these, revenues from natural resources contributed more than half of total fiscal revenue (IMF 2007). These earnings figures increased enormously during the commodity boom of 2006-08, before falling back. At the same time new countries have made major resource discoveries, such as oil in Brazil, Ghana, and Uganda. A temporary windfall of natural resource revenues (or foreign aid) poses numerous policy challenges. Should the revenues be used for government investment in public infrastructure to stimulate economic activity? Should the government use the windfall to reduce government debt and thereby lower interest rates and boost private sector investment? Should the extra income be used to provide more education, health care and other public goods to improve the quality of life or transferred directly to citizens through tax cuts or citizen dividends? Alternatively, revenues could be used to transform exhaustible resource assets into interest-earning foreign assets by setting up a Sovereign Wealth Fund (SWF) for future generations. This is a bewildering array of policy options and the appropriate choice depends on the stage of development of the economy and the constraints it faces.

Our objective is to provide a rigorous analysis of these policy choices in relatively poor countries which are capital scarce and have less than perfect access to capital markets. We focus on welfare-maximizing government choices between three broad options: using the windfall for private (or public) consumption; investment in public assets that raise income and the marginal productivity of private investment; and altering the country's foreign asset/ debt position. We look at outcomes with different sets of policy instruments available and in a series of increasingly complex economic environments. These options provide different time-profiles of ultimate consumption benefit and elicit different private sector investment responses. While we focus on responses to windfalls, our analysis of these choices is of more general interest for policy formulation, particularly in developing economies.

Central to our analysis are several features that we think are important in many developing countries. The first is that the country is capital scarce, with low capital-labour ratio, little investment in public infrastructure, low wages and per capita income, and a high domestic interest rate. We model this by assuming that capital-scarce countries have a low level of initial assets. They can add to domestic capital by borrowing on international capital markets, but may face an interest premium the size of which depends on the level of foreign debt. The premium might be a consequence of the perceived likelihood of default, although we do not model this explicitly. In the absence of a foreign exchange windfall, developing economies are on a trajectory of capital accumulation, debt reduction, and rising consumption, and we examine how the windfall can be optimally used to alter this trajectory.

The second feature concerns the behaviour of households in the economy. In many countries households find it hard to borrow against future wage income so Ricardian debt neutrality is unlikely to hold. To capture this, we initially suppose that households live entirely from current wage income and government transfers, having no access to capital markets. The presence of credit-constrained households means that there is a role for government to smooth consumption by varying taxes paid by or subsidies given to these households. In a final section we remove this assumption, and allow households access to capital markets. However, they may not internalise other imperfections in the economy, so government policy has to address possible over-consumption from the resource boom.

The existing literature offers various prescriptions for use of a windfall, of which the best known is the permanent income hypothesis (PIH), familiar from the tax smoothing literature (Barro, 1979) or the optimal use of the current account (e.g. Sachs 1981). This and alternative recommendations are shown figure 1.1, which illustrates a case in which a flow of windfall revenue, (N, given by the step function) is discovered at date $t = 0$ and flows at a constant rate between dates T_0 and T_1, after which it ceases. Other lines on the figure give the increment in consumption, ΔC, under alternative prescriptions. The horizontal line is the PIH, giving a constant and permanent increase. In its simplest form, this involves borrowing ahead of the revenue flow, saving during the period of flow, and building up a SWF large enough for interest on the fund to maintain the consumption increment in perpetuity. These arguments underlie much of the advice for the setting up of a SWF proffered by the International Monetary Fund (e.g. Davis et al., 2002; Barnett and Ossowski, 2003; Segura, 2006; Leigh and Olters, 2006; Olters, 2007; Basdevant, 2008).[1]

A more conservative approach is the 'bird-in-hand' hypothesis (Bjerkholt, 2002; Barnett and Ossowski, 2003), under which all revenue is put in the SWF and incremental consumption is restricted to the interest earned on the fund. This extremely conservative strategy can be interpreted as being equivalent to the PIH, but with the windfall not valued until it has been banked.[2]

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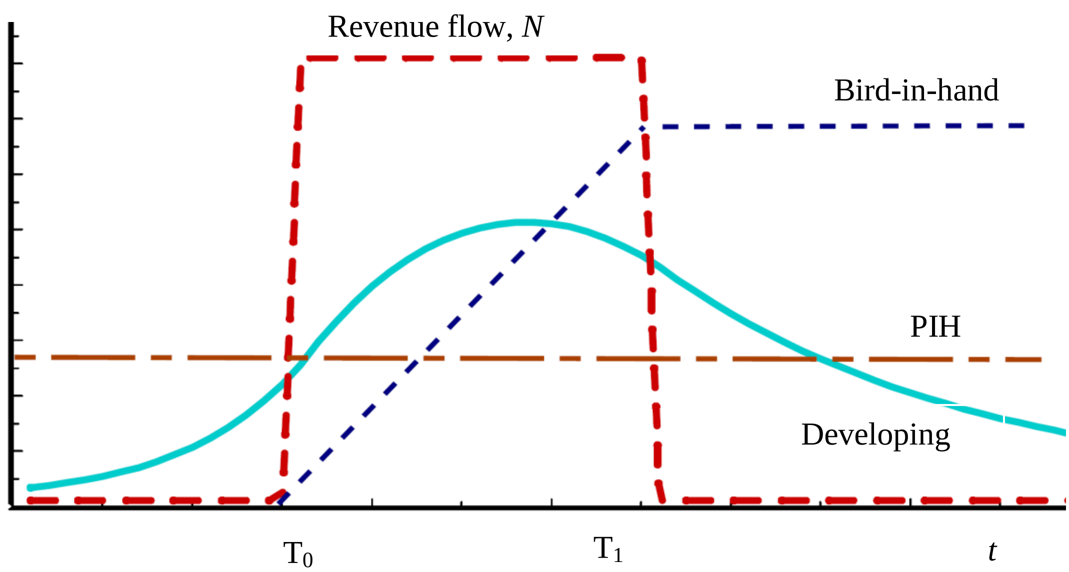
[1] Flight capital as a policy choice (e.g., formation of a SWF) has been discussed before in a different context (Collier, Hoeffler and Pattillo, 2001; Collier and Gunning, 2005). Although the analysis of genuine saving and a build-up of a SWF in a small open economy whilst at the same time optimally depleting an exhaustible resource and the consequent Hartwick and Hotelling rules have been analysed before (van der Ploeg, 2010), but suffers from being restricted to a Rawlsian social welfare function.

[2] This is sometimes known as the Norwegian model, although it is only after the recommendations for a bird-in-hand strategy by the Tempo Committee in 1983 and the recommendations for setting up a financial hydrocarbon fund in combination with sophisticated financial instruments by the Steigum Committee in 1988 that the Norwegians implemented their 4% bird-in-hand rule in 2001 (Harding and van der Ploeg, 2009). The rule says that 4% of the value of the Fund at the end of the previous year can be used to fund the general government deficit.

Figure 1.1: Alternative prescriptions; revenue flow and incremental consumption.



Both the PIH and the bird-in-hand strategies have the effect of transferring much of the consumption increment to future generations. While this may be appropriate for high income countries, it ignores the essential features of developing economies in which there is capital scarcity, current incomes are low, and there is a potential process of rapid growth and convergence. The curve labelled 'developing' is the consumption increment for such an economy, as computed in a later section of this paper. It indicates three phases of optimal policy. First, there is some borrowing to finance consumption following discovery, but this is small compared to the PIH. Second, during the period of revenue flow, consumption is higher than under the PIH, although a substantial part of revenue is saved; saving goes into debt reduction and building domestic assets rather than building an SWF. As a consequence, in the third phase once revenue flows have ceased, consumption converges back to the level that it would have attained on the optimal growth path but in the absence of the windfall. However, consumption along this path is higher, at all dates, than it would have been without the windfall. Consumption is in any case on a rising path and optimal use of the windfall involves bringing forwards the growth of consumption by accelerating development, rather than accumulating foreign assets to increase the consumption of far future generations.

Remaining sections of the paper are devoted to setting up a family of models in which we derive optimal policy rules for a resource-rich developing country. Section 2 sets up the benchmark for a country in which the home interest rate is pegged to the world interest rate and whose citizens are unable to smooth consumption but whose government can do it for them. This yields the permanent income hypothesis, in which a temporary windfall is best used to give an immediate and permanent boost to citizen dividends and private consumption, this being paid for by borrowing ahead of the windfall and then by the interest from a SWF accumulated during the windfall.

Section 3 provides some empirical evidence on the interest rates resource-rich developing economies have to pay on their debt. We find support for the hypothesis that highly indebted countries face a higher interest premium. The evidence that natural resource revenues reduce the interest premium on debt is mixed.

Section 4 analyses the best way to harness an anticipated windfall in capital-scarce developing countries which face an interest rate above the world interest rate. We initially assume that non-oil income is fixed, there is no physical capital, and the choice is simply between consumption and foreign debt/ asset management. As is suggested by figure 1.1, full consumption smoothing is no longer optimal. Instead, consumption is relatively more skewed towards current (poorer) generations than is the case with the permanent income hypothesis benchmark. With small windfalls, the economy's growth path is accelerated, but no SWF is built up. Only if windfalls are large relative to initial debt will it be optimal to also build an SWF and use part of the windfall to support a permanent increase in consumption.

Section 5 develops a richer model of the non-resource economy and of the policy options faced by government, adding private capital, public infrastructure and income taxation. The government now chooses not just how much to save, but also the mix of saving in domestic capital (infrastructure) versus financial debt/ assets. Infrastructure raises output directly, and also raises the return to private capital, so increasing the capital intensity and wage rate in the economy. We look at cases where lump-sum transfers are possible, and where government has to use distortionary income tax to finance infrastructure. Private investment is then influenced by both the level of infrastructure and the tax rate. The windfall permits government to promote private investment through both channels, cutting the tax rate and investing more in public infrastructure.

Section 6 further enriches the model to allow domestic consumers access to credit markets. This creates the possibility that, however prudent government may be, Ricardian consumers may over-expand consumption once the windfall is known. Government can respond by an asset holding subsidy or, if this is not available, by committing a higher level of expenditure to public infrastructure rather than putting resource revenue in a SWF.

To focus on the main public finance issues at hand, we abstract from many important elements of the problem. We use a single-sector model in which there are no problems in absorbing expenditure, either from an appreciation of the real exchange rate and its adverse impact on the traded sector (the Dutch disease, Corden and Neary, 1982; van Wijnbergen, 1984; Sachs and Warner, 1997) or from supply bottlenecks in particular domestic sectors such as construction. We abstract from political economy concerns. And most critically, we work in an environment of certainty, so that resource revenue volatility and associated precautionary motives are ignored. The final section of the paper discusses possible extensions based on these issues, also places our work in the context of the savings and spending decisions of countries that have experienced resource booms.

2. Benchmark: the permanent income hypothesis

We first consider a small open economy that can borrow or lend unlimited amounts at the world interest rate. This economy has exogenous and constant non-resource output *Y*. Consumers have no assets and simply receive a lump-sum transfer or citizen dividend *T* from the government so their consumption is $C = Y + T$. The government is the only agent in the economy that has access to the international capital market, so foreign debt *F* corresponds to public debt. It chooses transfers *T* and public consumption *G* to maximise utility of its citizens, $U \equiv \int_0^\infty \left(\frac{C^{1-1/\sigma} + \psi G^{1-1/\sigma}}{1-1/\sigma} \right) \exp(-\rho t) \mathrm{d}t,$ where $\psi \geq 0$ is the weight given to public consumption, σ is the elasticity of intertemporal substitution and the rate of time preference is ρ. Maximisation is subject to the budget constraint $\dot{F} = r^* F + G + C - Y - N$, with fixed initial debt F_0 and exogenous world interest rate r^*, assumed to equal the rate of time preference, $\rho = r^*$. *N* stands for the flow of windfall revenue from the sale of resource or foreign aid, all of which accrues to government.

The conditions for optimal government policy are familiar.[3] The intratemporal efficiency condition requires that public and private consumption are in fixed proportion, $G = \psi^\sigma C$. The intertemporal efficiency condition states that consumption of government and its citizens are smoothed over time, so $\dot{G} = \dot{C} = 0$. The levels of *C* and *G* come from the budget constraint, incorporating the value of resource revenues. The present value of the windfall at the date of discovery, $t = 0$, is $V(0) \equiv \int_0^\infty N(t) e^{-r^* t} dt$, the permanent income flow from this, $r^* V(0)$, so the permanent level of consumption is $C + G = Y + r^*\left(V(0) - F_0\right)$. The split between the levels of permanent private and public consumption depends on the weight given to public consumption in social welfare, ψ:

$$C = \left[Y + r^*\left(V(0) - F_0\right)\right] / (1 + \psi^\sigma), \qquad G = \left[Y + r^*\left(V(0) - F_0\right)\right] \psi^\sigma / (1 + \psi^\sigma).$$

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[3] We provide an explicit solution of a more general version of this problem in the next section.

If the flow of resource revenue is not constant through time then permanent consumption levels are maintained by changing the level of debt/ assets held according to the flow budget constraint. This requires that the non-windfall deficit must equal the return on resource wealth, because $\dot{F} + N = r^*V$ ensures that $V - F$ and thus C and G are held constant over time. For an anticipated temporary windfall (as illustrated in figure 1.1) there is borrowing (a current account deficit) during the interval before revenue flow, $t \in [0, T_0]$. During the period of revenue flow, $t \in [T_0, T_1]$ there is asset accumulation, paying off any foreign debt and subsequently building a SWF by running a current-account surplus. The foreign assets that are built up at the end of the windfall, $-\left[F(T_1) - F_0\right] = V(0)$, generate just sufficient interest revenue to finance the permanent rise in total consumption, that is $r^*V(0) = \Delta(C+G)$. This policy of borrowing, then saving and finally living of the return on the SWF thus transforms an anticipated temporary windfall revenue into a permanent increase in public and private consumption.

3. Capital scarcity and the interest premium

The benchmark of using debt to smooth consumption may be applicable for countries able to borrow or lend unlimited amounts at a given world interest rate. Yet many resource-rich developing economies are capital scarce and have high domestic interest rates. They are unable to remedy this by international borrowing, as they are likely to face a high and increasing interest premium on such borrowing. Table 3.1 indicates that resource-rich countries such as the Ukraine, Ecuador, the Russian Federation, Argentine and Côte d'Ivoire have to pay a premium varying from 11.2 to 18.5 percentage points on an annual basis.

Table 3.1: Interest spreads for resource-rich developing economies

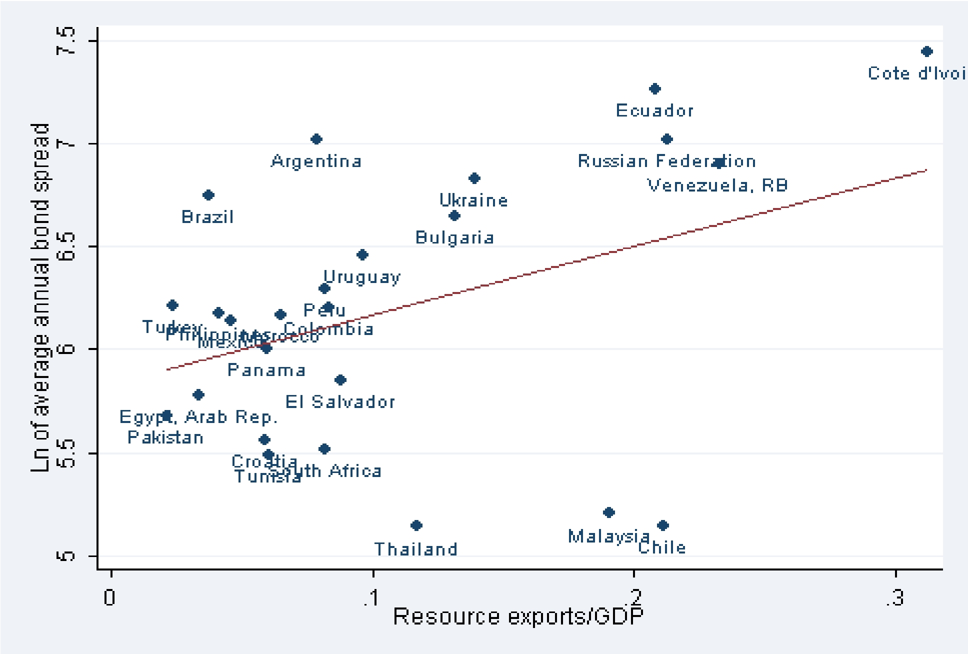
Lowest five spreads	Highest five spreads
Chile (1999-2003): 173.06 (21.1)	Ukraine (2000-2003): 1125.67 (13.9)
Thailand (1997-2003): 190.68 (11.7)	Ecuador (1995-2003): 1563.45 (20.8)
Malaysia (1996-2003): 220.04 (19.1)	Russian Federation (1998-2003): 1566.46 (21.3)
Tunisia (2002-2003): 248.17 (6.0)	Argentina (1994-2003): 1778.50 (7.8)
South Africa (1995-2003): 269.54 (8.2)	Côte d'Ivoire (1998-2003): 1847.93 (31.3)

Key: In brackets average resource exports as percentage of GDP.

Inspecting the webpage of the Sovereign Wealth Institute, we see that the resource-rich countries with the highest spreads, Ukraine, Ecuador, Argentina and Côte d'Ivoire do not have a SWF. The Russian Federation only instituted its SWF, the National Welfare Fund, in

2008. Resource-rich Chile, Thailand, Malaysia and Tunisia have instituted a SWF. Capital-scarce countries thus seem less likely to set up a SWF, which will be one of the main analytical insights of subsequent sections of this paper. Many other resource-rich developing economies are capital scarce and pay high interest rates. Indeed, figure 3.1 suggests that there is a positive correlation between countries being resource rich and capital scarce.

Figure 3.1: Interest spreads and natural resource dependence



There is empirical support for the hypothesis that interest spreads are higher in countries with high public and publicly guaranteed debt and low foreign reserves (Akitobi and Stratmann, 2008). Figure 3.2(a) shows indeed a positive relationship between interest rate spreads and the ratio of public and publicly guaranteed debt to GNI. Estimating the interest rate spreads (not reporting country fixed effects and time dummies, reporting standard errors in parentheses), we obtain that interest spreads are significantly higher if the ratio of PPG debt to GNI is high, foreign reserves are low and the probability of default is high[4]:

$$\begin{aligned} \text{Ln(spreads)} = {} & \underset{(0.54)}{1.89^{***}}\ \text{PPG debt/GNI} - \underset{(1.72)}{4.14^{**}}\ \text{reserves/GDP} + \underset{(0.036)}{0.056}\ \text{inflation} \\ & - \underset{(0.015)}{0.0458^{***}}\ \text{output gap} + \underset{(0.096)}{0.296^{***}}\ \text{Ln(default)} + \underset{(0.000124)}{0.0000866}\ \text{regional spread} \end{aligned}$$

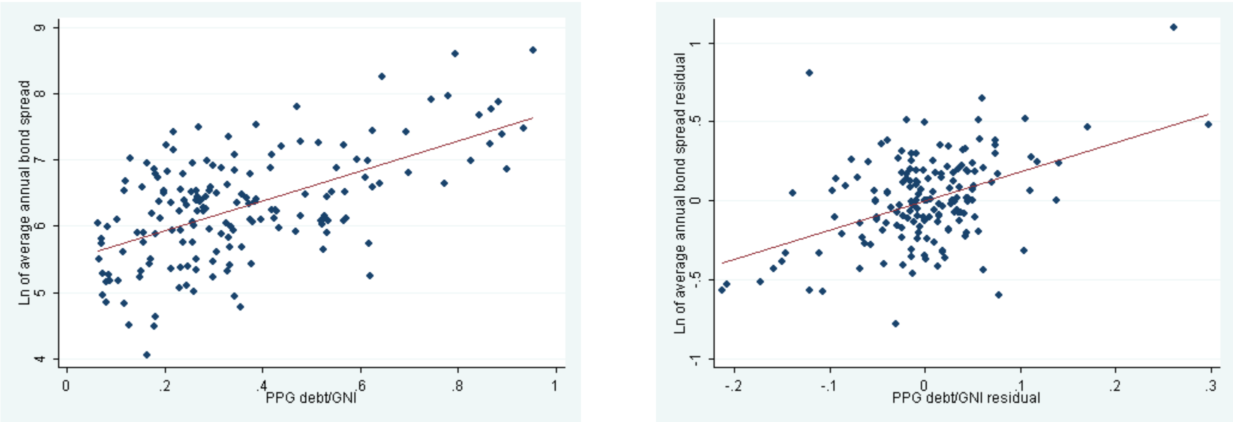
25 countries, 165 observations, within $R^2 = 0.732$, * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

[4] Apart from the public and publicly guaranteed debt and GNI variables which we obtained from World Bank Development Indicators (April 2008), we use exactly the same years and sample of countries and explanatory variables as Akitobi and Stratmann (2008). When we use their definition of PPG debt and GNI, we can exactly replicate their regression results but get different standard errors as we report standard errors that are not only robust to heteroskedasticity but also to serial correlation (Arellano, 1987; Stock and Watson, 2008).

Figure 3.2(b) gives the conditional effect of PPG debt on interest rate spreads. Compared with Akitobi and Stratmann (2008), we find a stronger a stronger effect on debt on interest spreads; they find a coefficient of about unity.

Figure 3.2: Interest rate spreads and public and publicly guaranteed debt

(a) Unconditional* (b) Conditional**



* The slope coefficient corresponding to the unconditional correlation for the pooled regression with N=165 and 25 countries is 2.270 with standard error 0.250, which is significantly different from zero at the 1% level, and the adjusted R^2 is 0.332.
** The slope coefficient corresponding to the conditional correlation after controlling for country and time fixed effects, reserves/GDP, ln(inflation), output gap, in-default dummy and regional spread is somewhat smaller, namely 1.855 with standard error 0.536 which is also significant at the 1% level. Within-R^2 = 0.732.

To test whether discoveries of natural resources induce a sharp downwards shift in the interest spread schedule, we include natural resource exports as a fraction of GDP as an explanatory variable. We then obtain the following regression result:

$$\begin{aligned} \text{Ln(spreads)} = {} & \underset{(0.68)}{1.48^{**}}\ \text{PPG debt/GNI} - \underset{(1.74)}{3.94^{**}}\ \text{reserves/GDP} + \underset{(0.031)}{0.036}\ \text{inflation} \\ & - \underset{(0.013)}{0.0443^{***}}\ \text{output gap} + \underset{(0.104)}{0.316^{***}}\ \text{Ln(default)} + \underset{(0.000132)}{0.000138}\ \text{regional spread} \\ & + \underset{(2.303)}{3.132}\ \text{resource exports/GDP} \end{aligned}$$

25 countries, 165 observations, within R^2 = 0.732, * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Although the effects of PPG debt, reserves, the probability of default and the output gap on interest spreads remain significant[5], we do not find a statistically significant effect of natural resource exports on interest rate spreads. To investigate robustness of our results, we tried various other specification and also some corrections for correct for endogeneity of PPG debt and resource exports. The qualitative nature of our results survived, albeit that we sometimes obtained a significantly positive effect of resource exports on interest spreads.

Hence, if anything, our regression results suggest that resource exports have a positive effect on interest spreads, as already suggested by the partial correlation indicated in figure 3.1, and therefore might, somewhat surprisingly, negatively affect credit worthiness. This could suggest that the discovery of natural resources increases political turmoil, conflict and even the risk of civil war (e.g., Collier and Hoeffler, 2004; Fearon, 2005) and thus in this way increases funding costs.

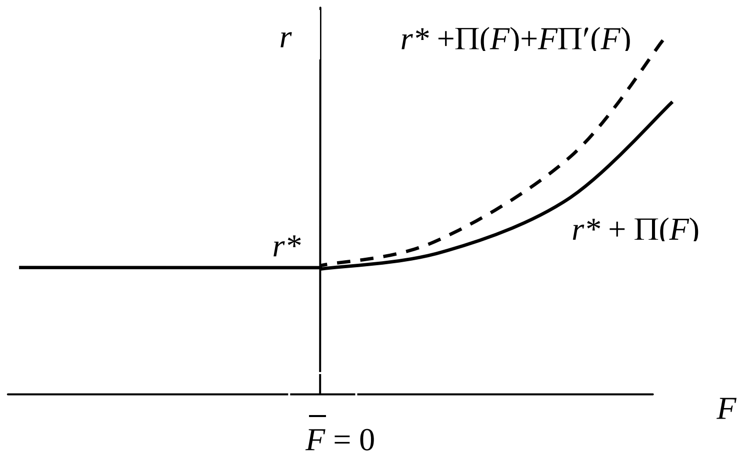
We capture the above empirical evidence with a supply schedule of foreign debt, where for low values of foreign indebtedness the home interest rate equals the world interest rate and for high levels of indebtedness the home interest rate rises above the world rate. To capture this relationship, we assume that the domestic interest rate, r, is determined by:

(1) $$r = r^* \text{ for } F \leq \bar{F} \text{ and } r = r^* + \Pi(F) > r^* \text{ for } F > \bar{F} \geq 0,$$

where $\Pi(F)$ is the interest rate premium and $\bar{F}$ the debt threshold below which the country is price taker at the world rate of interest. For simplicity, we take this threshold to be zero, so $\Pi(0) = 0$ and $\Pi'(F) > 0, F > 0$. Figure 3.3 portrays this supply schedule. One can interpret $\Pi(F)$ as an international premium on foreign debt to capture the risk of default, but we do not model that. In the macroeconomics literature (e.g. Turnovsky, 1997, section 2.6) it is common to close small open economy models by specifying a supply schedule of foreign debt which slopes upwards for all F.[6] Although this is analytically convenient, it has the unattractive feature of implying a unique steady-state value of F at which the domestic interest rate equals the world rate and to which development converges. This level is independent of windfall revenue, and therefore inconsistent with the permanent income hypothesis under which, as we saw in section 2, countries choose their steady-state value of F by, for example, building a SWF. It is to capture both the interest premium and the endogeneity of the steady-state value of F that we suppose that economies face a premium, $\Pi(F) > 0$, above some threshold level of indebtedness while below that level countries are price takers at r^*.

[5] If we use the debt variable of Akitobi and Stratmann (2008), it is no longer statistically significant.
[6] Most small open economy models with incomplete asset markets have steady states that depend on initial conditions and furthermore have equilibrium dynamics with a random walk component. To ensure stationarity and a unique steady state, one often postulates an upward-sloping supply schedule of foreign debt. Alternatives are to have an endogenous discount rate, convex portfolio adjustment costs or asset markets with a complete menu of state-contingent claims (Schmitt-Grohé and Uribe, 2003), but we do not explore these as a debt-elastic risk premium seems relevant for developing

Figure 3.3: The cost of foreign borrowing:



In section 4.4 we will discuss what happens if resource revenues affect credit worthiness and the interest premium that has to be paid on debt.

4. Developing economies: departure from the permanent income hypothesis

4.1. Optimal policy

We now derive optimal policy for an economy facing an interest premium, as described above. As in section 2, the government maximises utility of its citizens subject to its budget constraint, and this now takes the form $\dot{F} = [r^* + \Pi(F)]F + G + C - Y - N$, $F(0) = F_0$. The Hamiltonian for this problem is

$$\mathrm{H}(C, G, F, \lambda) \equiv \frac{C^{1-1/\sigma} + \psi G^{1-1/\sigma}}{1-1/\sigma} + \lambda\left[\left(r^* + \Pi(F)\right)F + G + C - Y - N\right]$$

with costate λ for debt. Optimality conditions are

(2) $\mathrm{H}_C = C^{-1/\sigma} + \lambda = 0$, $\mathrm{H}_G = \psi G^{-1/\sigma} + \lambda = 0$, $\rho\lambda - \dot{\lambda} = \mathrm{H}_F = \lambda\left[r^* + \Pi(F) + F\Pi'(F)\right]$,

and the transversality condition $\lim_{t\to\infty}[\exp(-r^* t)\lambda(t)F(t)] = 0$.

As before, intratemporal efficiency requires that G and C are in fixed proportion $G = \psi^{\sigma} C$. This can be used to write the current-account dynamics as

economies. The alternative of portfolio adjustment costs when asset holdings are different from some long-run level gives qualitatively similar conclusions.

(3) $$\dot{F} = [r^* + \Pi(F)]F + (1+\psi^{\sigma})C - Y - N, \quad F(0) = F_0.$$

The intertemporal efficiency condition (the first-order condition for the optimal consumption path) can be derived from equations (2) as,

(4) $$\dot{C} = \sigma C\left[\Pi(F) + F\Pi'(F) + r^* - \rho\right]$$

In contrast to the PIH, perfect consumption smoothing is no longer optimal, since (even with $r^* = \rho$) the marginal cost of borrowing is not equal to the pure time preference rate. Consumption is rising – and therefore initially low – if the marginal cost of foreign borrowing (or marginal return to accumulating foreign assets) exceeds the rate of pure time preference. The marginal cost of foreign borrowing now includes the premium $\Pi(F)$ and the value of any change in the premium, $F\Pi'(F)$. At this higher rate a country with $F > 0$ has an incentive to postpone consumption and save.

Figure 4.1 portrays the phase-plane diagram corresponding to (3)-(4). Looking first at the lower part of the figure, the $\left[\dot{F} = 0\right]_{N=0}$ is the locus of stationary values of F in the absence of windfall revenues. The locus slopes downwards; above it consumption is high and foreign debt increases and below it foreign debt declines over time. $\dot{C} = 0$ wherever $\Pi(F) + F\Pi'(F) = 0$, i.e., for all values of $F \leq 0$. Countries with foreign debt $F > 0$ face high domestic interest rates and have rising consumption, while countries with $F \leq 0$ have constant consumption. Combining the F and C stationaries gives a set of stationary points, the line S–S. For an economy which finds itself with $F \leq 0$ this line segment is unstable, so C must jump to S–S; this is precisely the permanent income hypothesis. For an economy with $F > 0$ there is a unique saddlepath, illustrated by the dashed line (see appendix 1).

Our focus is on a developing country, which is initially indebted and which starts out at point E_0 on figure 4.1. In the absence of a windfall the economy simply moves along the saddlepath with relatively high saving and growing consumption. As it pays off its foreign debt the domestic interest rate falls so that the propensity to save and the growth of consumption decline. In the long run the economy has paid off its foreign debt ($F = 0$), the domestic interest rate has fallen to the world interest rate, and private and public consumption have risen to their steady-state values.

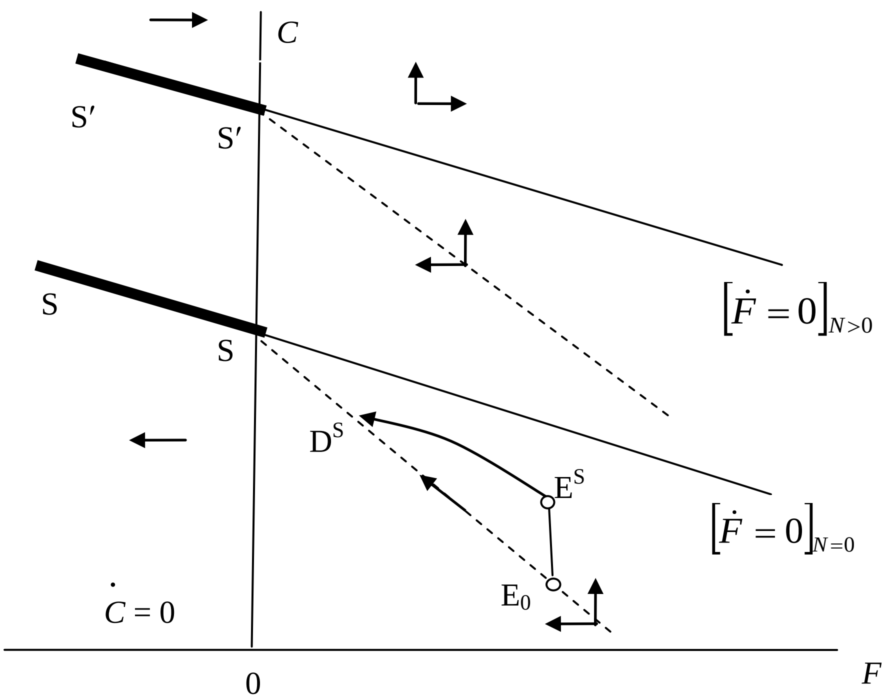
4.2. A small temporary windfall

We look at cases in which the flow of resource revenue, N, takes a constant value $N > 0$, while the resource is being extracted, $t \in [T_0, T_1]$, and zero outside this interval, $t < T_0$ and $t >$

T_1. As is clear from equation (3), this leads to two different *F* stationaries. They are illustrated on figure 4.1, with the upper line, $\left[\dot{F}=0\right]_{N>0}$, simply being an upwards shift of the original locus. This sets the dynamic during the period when $N > 0$, and the associated set of stationary points is S′-S′.

Our first case is a windfall that is known to be temporary, but for which there is no lag between discovery and extraction, so $N > 0$ at the date of discovery, $T_0 = 0$. Furthermore, the windfall is ‘small’ in a sense which we will make precise later on. The dynamics are illustrated on fig 4.1. Prior to the windfall the economy is on the saddlepath, at point E_0. During the period of revenue flow dynamics are subject to $\left[\dot{F}=0\right]_{N>0}$, but must return to the original saddlepath at date $t = T_1$. This involves an upwards jump in consumption at $t = 0$, followed by movement along the line $E^S D^S$. The size of the jump (distance $E_0 E^S$) is determined so that the path regains the initial saddlepath when the windfall ceases at $t = T_1$.

Figure 4.1: A small temporary windfall



We establish two sets of results concerning this path with an unanticipated, temporary windfall. First, comparing the situation with the windfall to that without, proposition 1 of appendix 1 proves that with a temporary windfall of size $N > 0$ lasting from time 0 to time T_1, the initial jump in total consumption equals $\Delta C(0)+\Delta G(0)=\left[1-\exp(-\lambda_u T_1)\right]N>0,$ where

$\lambda_u = \frac{1}{2} r^* + \frac{1}{2}\sqrt{r^{*2} + 8\sigma\Pi' Y} > r^* > 0$ and Δ indicates deviations from the benchmark trajectories. The initial jumps in private and public consumption thus go up one-for-one with the magnitude of the temporary windfall and are larger if the windfall is more prolonged (higher T_1). A more elastic supply of foreign debt (higher Π'), a higher elasticity of intertemporal substitution (higher σ) and a higher level of non-windfall endowment income (higher *Y*) imply a smaller initial increase in consumption in response to a temporary windfall of foreign exchange of given magnitude. A windfall thus leads to smaller upwards jumps in consumption in a capital-scarce economy (with $\Pi' > 0$). Proposition 2 of appendix 1 establishes that during the windfall the country runs a surplus and reduces debt and after the windfall it runs a deficit so that the debt level gradually increases back to its original level. This proposition also proves that the magnitude of the reduction in debt at the end of the windfall increases with the size (*N*) and duration of the windfall (T_1), that is

$\Delta F(T_1) = -\left[\frac{1-\exp\left((\lambda_s - \lambda_u)\mathrm{T}_1\right)}{\lambda_u - \lambda_s}\right] N < 0,$ where $\lambda_s = \frac{1}{2} r^* - \frac{1}{2}\sqrt{r^{*2} + 4\Sigma(1+\psi^{\sigma})} < 0,$ and

that the incremental change in total consumption at the end of the windfall is less than the change in initial total consumption $0 < \Delta C(T_1) + \Delta G(T_1) = -\lambda_u \Delta F(\mathrm{T}_1) < \Delta C(0) + \Delta G(0).$ Comparing the situation with the windfall to that without, we can establish from the phase-plane diagram that consumption is higher at all dates and converges asymptotically to its previous path; debt is lower at all dates, and converges asymptotically to its previous level: the point D^S (and all subsequent points on the saddlepath) are attained sooner than they otherwise would have been. These results establish that optimal use of the windfall does not involve raising consumption in perpetuity, but instead using it to bring forward the development path of the economy.

The second set of results compares the path to the prescriptions of the PIH, i.e. increasing consumption at all dates by the annuity value of the windfall.[7] The maximum jump in total consumption occurs when the windfall lasts indefinitely and starts immediately, that is $\Delta C(0) + \Delta G(0) = N.$ This states that with an unanticipated *permanent* windfall, increase in total consumption simply equals the increase in windfall revenue. This is the same result as would prevail under the PIH. However, proposition 1 of appendix 1 indicates that, with an unanticipated *transitory* windfall, the initial jump in total consumption in a capital-scarce economy is *bigger* than suggested by the PIH, $\Delta C(0) + \Delta G(0) = \left[1 - \exp(-\lambda_u \mathrm{T})\right] N$ $> \left[1 - \exp(-r^* \mathrm{T})\right] N$ as $\lambda_u > r^*$, especially if the supply of foreign debt is highly elastic, intertemporal substitution is easy and non-windfall income is substantial. Proposition 3 of appendix 1 establishes that the incremental increase in consumption at the end of the windfall is less than the initial incremental increase in consumption, but still larger than that would prevail under the permanent income hypothesis, $\Delta C(0)+\Delta G(0)>\Delta C(T_1)+\Delta G(T_1)=$ $\left(\frac{\lambda_u}{\lambda_u-\lambda_s}\right)\left[1-\exp\left((\lambda_s-\lambda_u)\mathrm{T}_1\right)\right]N \; >\left[1-\exp(-r*\mathrm{T}_1)\right]N.$ Following the discovery (and revenue flow) consumption jumps up and then falls, all the time during the windfall staying above the value that would prevail if the economy followed the PIH. After the windfall is finished, incremental consumption continues to fall eventually below the value that prevails under the PIH and asymptotically vanishes completely.

With a small windfall, there is no rationale for building a SWF. Debt is higher (assets lower) than under the PIH for all $t > 0$. This is because the economy is initially poor and is on a rising consumption path. It is optimal to skew consumption (relative to the PIH) towards the present poor generation rather than transfer too much to future relative rich generations.

To draw out results clearly, we have in this sub-section assumed that revenue flows from the date of discovery, $T_0 = 0$. It is straightforward to extend this to the case where there is a lag between discovery and revenue flow, $T_0 > 0$. Comparing the situation with the windfall to that without, proposition 1 of appendix 1 proves that the initial jump in total consumption equals $\Delta C(0)+\Delta G(0)=\exp(-\lambda_u\mathrm{T}_0)\left[1-\exp\left(-\lambda_u\left(\mathrm{T}_1-T_0\right)\right)\right]N>0.$ The initial jumps in total consumption thus go up one-for-one with the magnitude of the temporary windfall and are larger if the windfall is more prolonged and starts earlier. As before, a more elastic supply of foreign debt (higher Π') implies a smaller initial increase in consumption in response to a temporary windfall of foreign exchange of given magnitude. In a capital-scarce economy the initial increase in consumption in response to a windfall is less pronounced. In terms of the phase diagram, there are three stages. At date of discovery consumption jumps up from E_0 and during $t \in [0, T_0]$ the system remains under the influence of $\left[\dot{F}=0\right]_{N=0}$ so both C and F increase, i.e. debt is being incurred to finance increasing consumption. From $t = T_0$ onwards the dynamics of F are controlled by $\left[\dot{F}=0\right]_{N=0}$, and a path similar to $E^S D^S$ is followed, with consumption rising and debt being paid down. Proposition 3 of appendix 1 establishes that the longer the anticipation period T_0, the bigger the reduction in debt at the end of the windfall. The height of the initial jump is determined by the condition that the trajectory hits the original saddlepath when the revenue flow ceases, at date $t = T_1$. These phases give a figure qualitatively similar to that in figure 1.1, and are explored in further detail in later sections.

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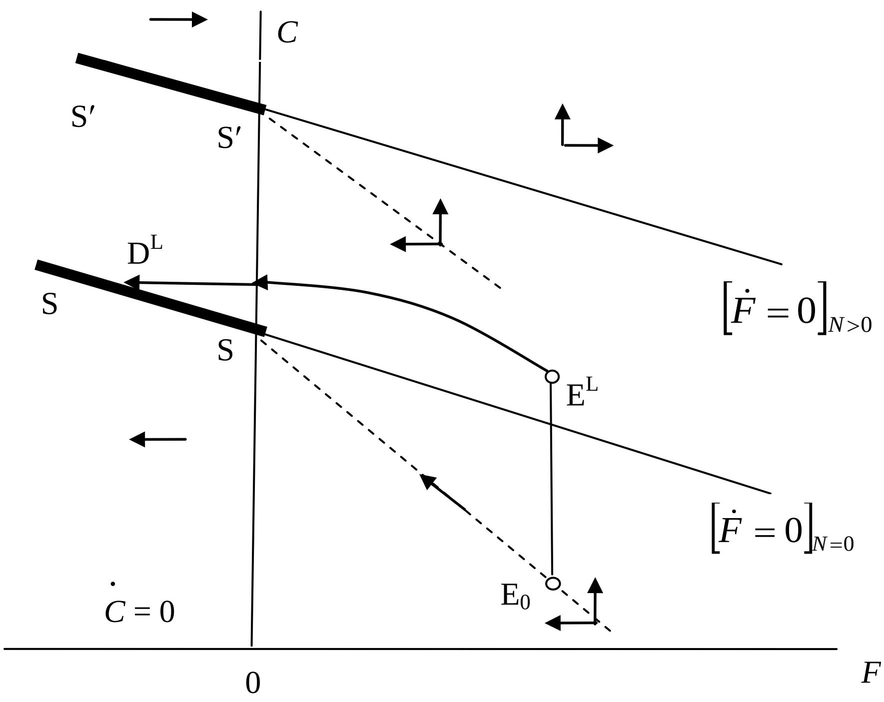
[7] This now calculated using the time varying interest rate r, see section 3.5.

4.3. Large windfalls: encompassing the permanent income hypothesis

In the analysis of figure 4.1 optimal policy returned the economy to the original saddlepath (point D^S) at date T_1. The larger and longer is the period of revenue flow the larger is the initial jump in consumption and the closer is D^S to the stationary. For a large enough windfall the path becomes that illustrated in figure 4.2. The initial jump in consumption is to point E^L. As with the small-windfall case, during the period of extraction t ϵ [T_0, T_1] consumption is higher and debt is being repaid, but now it is optimal to repay all debt (reach $F = 0$) before the resource is depleted, date $t = T_1$. Consumption increases until this point is reached and the marginal cost of capital reaches the rate of time preference. Once consumption has reached its permanent value, foreign assets continue to be built up to the level sufficient to sustain this consumption in perpetuity. At the date the resource runs out (point D^L), the economy becomes stationary. The size of the jump to E^L is determined by the requirement that the economy reaches the original stationary, S–S, when the resource is depleted and the revenue flow stops, date $t = T_1$. The size of the long-run SWF is now endogenously determined. For example, a lower initial debt or a larger and more protracted windfall will increase the size of the terminal SWF, as will a starting point with lower initial debt.

Figure 4.2: A large temporary windfall



The boundary between the ‘large’ and ‘small’ windfall is when points D^S and D^L coincide at foreign debt level $F = \bar{F} = 0$. For the linearised solution derived in appendix 1, we have a 'small' windfall and thus no building up of a SWF and no long-run increase in consumption if $F_0 + \Delta F(T_1) = F_0 - \left[\dfrac{1-\exp\left((\lambda_s - \lambda_u)\mathrm{T}_1\right)}{\lambda_u - \lambda_s}\right] N > 0,$ that is a windfall is 'small' if the initial level of debt is high, the windfall is small in magnitude and does not last very long. However, if the windfall is 'large' in the sense that the stream of revenues is sufficiently substantial and long-lasting to pay off all debt and completely eliminate the risk premium on foreign debt, a SWF will be built up and a long-run increase in consumption will be feasible. If this moment occurs at time T_1', we have $F(\mathrm{T}_1') = \bar{F} = 0$) and from that moment on the economy starts accumulating sovereign wealth and follows the PIH prescription.

4.4. Resource wealth and the cost of capital

Our analysis to this point assumes that the windfall brings down interest rates only in so far as it is associated with lower foreign debt. However, it is possible that the windfall has a direct positive effect on creditworthiness, lowering the borrowing rate at given *F*. While we find no support for this in the statistical evidence reported in section 3, it might hold for particular countries.[8] We do find very weak support for an adverse effect of a resource discovery on credit worthiness. Suppose that the interest premium is a function of both debt and the present value of the resource windfall, so $r = r^* + \Pi(H),$ where $H \equiv F - \omega V$ and ω is a constant. The present value of resources remaining at each date, $V(t) \equiv \int_t^\infty N(z)\exp\left[-\int_t^z r(v)dv\right]dz$, is changing through time according to $\dot{V} = rV - N$. The economy's flow budget constraint, equation (3), is $\dot{F} = \left[r^* + \Pi(H)\right]F + (1+\psi^\sigma)C - Y - N.$ The change H is related to the change in F by $\dot{H} = \dot{F} - \omega\dot{V} = \dot{F} - \omega(rV - N),$ so the system can be written as

$$\text{(3}'\text{)} \qquad \dot{H} = \left[r^* + \Pi(H)\right]H + (1+\psi^\sigma)C - Y - (1-\omega)N,$$

$$\text{(4}'\text{)} \qquad \dot{C} = \sigma C\left[\Pi(H) + H\Pi'(H)\right].$$

This pair of differential equations is analogous to that analysed above and can be studied with similar phase-plane diagrams, but with *H* replacing *F* on the horizontal axis. Analysis of the effect of the windfall is altered in two ways. First, whereas *F*(0) has given initial value F_0, credit worthiness $-H$ jumps upwards by the amount $\omega V(0)$ at the date of discovery, $t = 0$, provided $0 < \omega < 1$. In case, resource discoveries induce conflict and adversely affect credit worthiness, $\omega < 0$ and $-H$ jumps downwards at the time of the discovery. Second, the upwards shift of the $\dot{H}=0$ stationary is smaller than the shift in the $\dot{F}=0$ stationary if $0 < \omega < 1$, does not shift at all if $\omega = 1$, and is larger if $\omega < 0$. Looking at the case with $\omega = 1$, the economy remains on an unchanged (original) saddlepath at all dates. The effect of the windfall is to abruptly reduce $H(0)$, this causing an upwards jump in C to stay on this saddlepath. Consumption is higher and debt lower at all dates than they would be without the windfall, and they converge to the non-windfall level as the economy converges to the steady-state. Compared to the PIH, the increment to consumption is once again skewed towards near generations and (for a small windfall) no SWF is constructed.

Summing up the results of these sub-sections, we have shown that with an interest premium on foreign borrowing, perfect smoothing of public and private consumption is no longer optimal. Instead of raising long-run consumption, optimal policy accelerates progress towards this long-run value. While consumption will jump up at the date of announcement (this involving borrowing if announcement precedes the revenue flow), consumption does not jump the whole way to its steady-state value because the marginal cost of debt exceeds the rate of pure time preference. Whereas the permanent income hypothesis suggests that a SWF should be built up in response to a temporary windfall, this is no longer true, unless the windfall is so large that it moves the economy out of the regime in which it faces a premium on its foreign debt. If the windfall of foreign exchange directly improves credit worthiness of the economy and thus reduces the interest premium, consumption jumps up and is higher at all times than in the absence of the windfall.

5. Public infrastructure and domestic production with foreign-owned capital

The previous section made the point that, in a developing economy with interest rate greater than the rate of time preference and with growing consumption, it is optimal to use revenue to accelerate the growth of consumption towards its steady-state value, rather than to increase that value through investment in a SWF. However, the government could invest only in foreign assets, either by debt reduction or construction of an SWF. We now turn to the next question. If there are domestic assets – private and public capital stock – as well as foreign, how should optimal policy combine current consumption, debt reduction, public investment, and incentives to private investment?

To answer these questions we make non-resource output endogenous by including private capital and public infrastructure. Non-resource domestic income is given by a production function with constant returns to scale with respect to private capital and labour,

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[8] Ghana issued a $500million international bond on the basis of its discovery of oil.

expressed as $Y = \mathrm{f}(K, S)$ where the labour force is normalised at unity, K denotes the private capital stock and S is the stock of public infrastructure. Given the exogenous supply of labour, the function f(.) exhibits decreasing returns in K and S together, to rule out ever-increasing growth. Infrastructure can be thought of as consisting not only of seaports, airports, roads and railroads, but also of education, health or any other public investment that boosts the productivity of private production.

We retain for the moment the assumption that there are no private domestic asset holders. Public infrastructure is owned by government, while private capital is rented from abroad from foreign owners who face the world interest rate, r^*. They are subject to host country income taxation at a proportional rate τ. Profit maximisation requires that the after-tax marginal product of capital, net of depreciation δ_K, equals the world interest rate, so that

$$(5) \qquad (1-\tau)\,\mathrm{f}_K(K, S) = r^* + \delta_K$$

The equilibrium capital stock follows from (5), and can be written as $K = \mathrm{K}(S, \tau)$, this giving wages $\mathrm{W}(S,\tau) \equiv (1-\tau)\left[\mathrm{f}(K(S,\tau),S) - K(S,\tau)\mathrm{f}_K(K(S,\tau),S)\right]$ and production income $\mathrm{Y}(S,\tau) \equiv \mathrm{f}(\mathrm{K}(S,\tau), \mathrm{S})$. Note that we use roman letters throughout to indicate functions and italics to indicate variables. Capital stock, wages and income are all increasing S and decreasing in τ. Differentiating (5) and $\mathrm{W}(S, \tau)$, the effect of a tax change on the wage rate, W_τ, satisfies $\mathrm{W}_\tau = -Y$; given the fixed supply price of capital, the first order effect of a reduction in income tax is to transfer income to workers.

In this structure the only debt is that of government, D. It is held entirely by foreigners, and the interest rate becomes $r = r^* + \Pi(D)$.[9] The dynamics of government debt come from the government budget constraint in the familiar way:

$$(6) \qquad \dot{D} = \left[r^* + \Pi(D)\right]D + G + I_S + T - \tau Y - N$$

where I_S is spending on infrastructure investment and the final terms are lump-sum transfers to consumers, income taxation and resource revenues. The stock of infrastructure evolves according to $\dot{S} = I_S - \delta_S S$ where δ_S is the depreciation rate. Analysis is simplified by

[9] We revert from the approach of section 4.5 to having the interest premium depend only on foreign claims on government. Section 6 looks at the case where there are private domestic asset holders, and the interest premium is determined by and applies to both public and private liabilities.

working with net government assets defined as the stock of public infrastructure minus government debt, $B \equiv S - D$.[10] The budget constraint is then

(6′) $$\dot{B} = [r^* + \Pi(S-B)](B-S) + N + \tau Y - G - T - \delta_S S, \quad B(0) = B_0$$

with the initial value of net government assets fixed at B_0. The no-Ponzi game condition must be satisfied, $\lim_{t\to\infty} B(t)\exp\left(-\int_0^t r(v)\mathrm{d}v\right) = 0$, so that initial net government assets *plus* the present value of the stream of future income taxes and resource revenue must cover the present value of the stream of future spending on public consumption, government transfers and infrastructure services.

The government's problem is now to choose the public capital stock S, public consumption G, together with the rate of income taxation τ and transfers to households T, where households' consumption is $C = W + T$. Its objective is social welfare, as before, $U \equiv \int_0^\infty \left(\frac{C^{1-1/\sigma} + \psi G^{1-1/\sigma}}{1-1/\sigma}\right)\exp(-\rho t)\mathrm{d}t$ and the constraints are the budget equation (6′), together with initial conditions, the no-Ponzi condition and equilibrium levels of private capital (and hence wages and income) as captured by $\mathrm{K}(S, \tau)$, $\mathrm{W}(S, \tau)$ and $\mathrm{Y}(S,\tau)$. The Hamiltonian is defined as:

(7) $$\mathrm{H}(\tau,T,G,S,\mu) \equiv \left(\frac{[\mathrm{W}(S,\tau)+T]^{1-1/\sigma} + \psi G^{1-1/\sigma}}{1-1/\sigma}\right) + \mu\left[\{r^* + \Pi(S-B)\}(B-S) + N + \tau\mathrm{Y}(S,\tau) - G - T - \delta_S S\right]$$

with co-state for net government assets μ. This yields the optimality conditions

(7.1) $$\mathrm{H}_\tau = C^{-1/\sigma}\mathrm{W}_\tau + \mu(Y + \tau\mathrm{Y}_\tau) = 0,$$

(7.2) $$\mathrm{H}_T = C^{-1/\sigma} - \mu = 0$$

(7.3) $$\mathrm{H}_G = \psi G^{-1/\sigma} - \mu = 0$$

(7.4) $$\mathrm{H}_S = C^{-1/\sigma}\mathrm{W}_S + \mu\left[\tau\mathrm{Y}_S - \{r^* + \Pi(S-B) + (S-B)\Pi'(S-B) + \delta_S\}\right]$$

(7.5) $$r^*\mu - \dot{\mu} = \mathrm{H}_B = \mu\left[r^* + \Pi(S-B) + (S-B)\Pi'(S-B)\right]$$

and the transversality condition

[10] Asset market equilibrium implies that the private and public capital stocks that are not owned by the

(7.6) $$\lim_{t\to\infty}\left[\exp(-r^*t)\mu(t)B(t)\right]=0.$$

We analyse this system in two stages, first looking at the case in which lump-sum transfers – the instrument T – are possible, and then in section 5.2 removing this instrument.

5.1. Policy with lump-sum taxes/transfers

In order to focus on the role of public infrastructure investments, we first suppose that government can make lump-sum transfers to consumers. It is then optimal to set the income tax rate τ at zero (from (7.1) and (7.2) together with $W_\tau = -Y$). Infrastructure is set optimally to satisfy

(7.4′) $$W_S(S,0)=r^*+\Pi(S-B)+(S-B)\Pi'(S-B)+\delta_S\,.$$

The marginal value of infrastructure is simply its effect on national income which, absent income taxation and given foreign ownership of private capital, is its effect on the wage. Its marginal cost is the full marginal cost of public borrowing including the marginal cost of the interest premium, so that the optimal level of infrastructure will be lower in a capital-scarce economy.

The optimal path of consumption is as in section (3); using (7.2) in (7.5),

(7.5′) $$\dot{C}=\sigma C\left[\Pi(S-B)+(S-B)\Pi'(S-B)\right] \text{ for } S-B>0 \text{ and } \dot{C}=0 \text{ otherwise.}$$

This private consumption path is supported by transfers $T = C - W(S,0)$. The optimal level of public consumption follows from (7.2) and (7.3) as $G=\psi^\sigma C.$ Debt is $D = S - B$ and the dynamics of net government assets B are, from equation (6′):

(6″) $$\dot{B}=\left[r^*+\Pi(S-B)\right](B-S)+N+W(S,0)-C-\delta_S S\,,\quad B(0)=B_0$$

The dynamic system (6″) and (7.5′) in B and C is qualitatively similar to the system (3) and (4) in F and C illustrated in figures 4.2 and 4.3, although S is now endogenous through equation (7.4′), and K(S,0), Y(S,0) and W(S,0) are now all changing along the optimal path. It is straightforward to extend the analytical results of appendix 1 to this case. Since this

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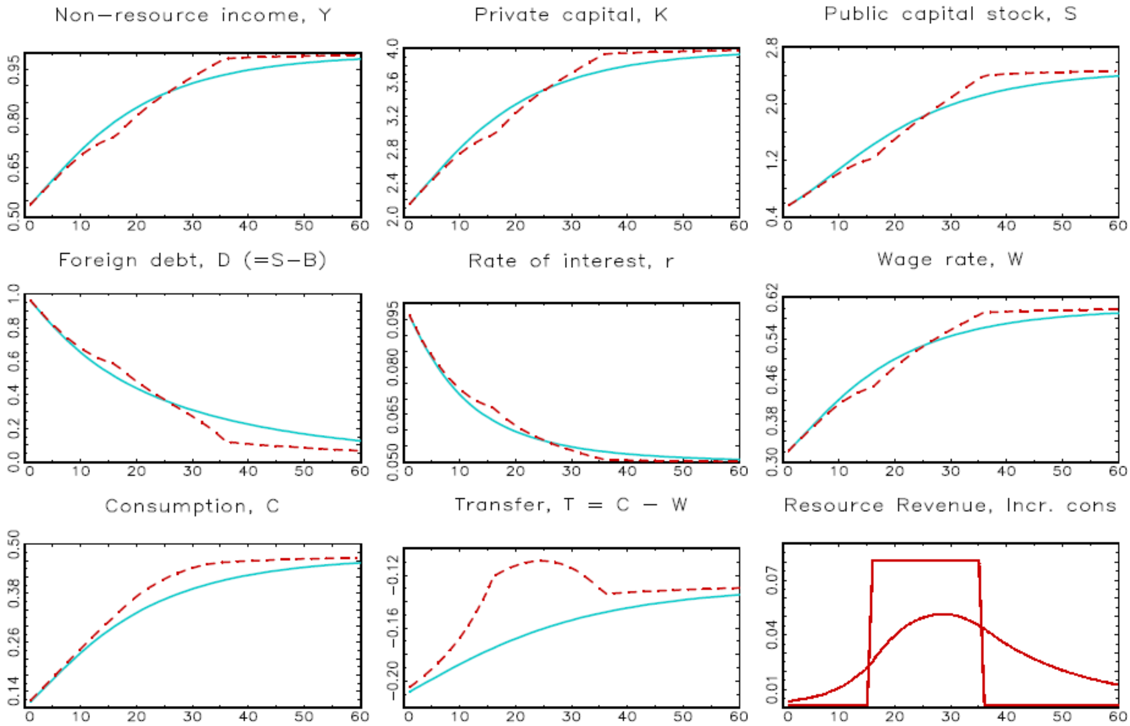
government are owned by foreigners, so that foreign liabilities are given by $F = K + S - B$.

becomes a bit cumbersome, we illustrate our model with endogenous production, private capital rented from abroad and public infrastructure by simulation of an example presented in the panels of figure 5.1 (and from which figure 1.1 was drawn).

Time is on the horizontal axis, and scaling of the vertical axes is achieved by having the long-run stationary value of income equal to unity. Production is Cobb-Douglas, $Y = AK^{\alpha} L^{1-\alpha} S^{\gamma}$, where A scales long-run output to unity, and parameters are set to $\alpha = 0.4$, $\gamma = 0.25$, $\rho = r^* = 0.05$, $\sigma = 0.75$, $\psi = 0$ and $\delta_K = \delta_S = 0.05$. In figures 5.1 and 5.2 $\Pi = \rho(F_+)^2$. Simulations are done with a reverse multiple shooting algorithm with a horizon of $t^* = 130$ and using the computer package GAUSS. We assume that $\overline{F} = \overline{D} = 0$ and that the time dimension is scaled such that the horizontal axis can be (loosely) interpreted as years.

Figure 5.1: Optimal development with lump-sum transfers



Key: Solid lines without a windfall and dashed lines with an anticipated temporary windfall.

The solid lines in figures 5.1 and 5.2 give the path of an economy which starts out with national wealth, B_0, set at half its long-run value, and which experiences no shocks. The economy has positive initial foreign debt and converges smoothly to its stationary value with accumulation of assets, decumulation of foreign debt, falling interest rates, and rising income and consumption. The effect of a temporary anticipated 'small' windfall is given by the dashed lines. Initial asset values are as in the base case, but at year zero a flow of resource revenue between years 16 and 35 is announced. The flow is equal to 8% of long-run stationary non-oil income.

At the date of announcement it is optimal to increase consumption, and there is a (small) upward jump in transfers *T* and consumption *C*. Transfers are then on a steeply rising path during the period prior to resource revenue flow. (Notice that overall lump-sum transfers to households are negative, because of the need to finance public infrastructure). Additional transfers in the interval before resource revenues start to flow have to be financed by foreign borrowing, and the downwards path of *D* flattens, leaving debt above what it otherwise would have been. Higher foreign debt translates into higher *r* and lower *S*, *K* and non-resource income, *Y*.

Once the revenue flow comes on stream debt *D* is paid off more rapidly than was the case absent the windfall, with the associated rapid fall in *r* and increase in *K*, *S*, *Y* and *W*. Rapidly rising wages mean that consumption growth can be maintained with *T* falling back. All the variables describing the production side of the economy cross their non-windfall path during the period of revenue flow, including the public capital stock. At the date when the windfall revenue ceases (t = 35, at the kink) domestic public and private capital stocks are something over 10% higher than they otherwise would have been, and foreign debt at half the level. At this date the economy reverts to its previous path, but earlier than would otherwise have been the case. Thus, the dashed lines converge to the same value as the solid ones, but are shifted to the left by some 30 time periods. The bottom right panel of figure 5.1 compares incremental consumption with the resource revenue flow (step function) and illustrates clearly the periods of borrowing, saving, and then higher consumption, this because the economy's development has been brought forward, and not because of perpetual income from an SWF.[11]

In summary, optimal use of the windfall involves increased consumption from the date at which the resource is discovered, and faster asset accumulation (debt decumulation) from the date windfall revenue flows. Higher public and private investment brings forward the economy's development path, but does not lead to the formation of a SWF.

5.2. Second best: No lump-sum taxes and subsidies

The possibility of lump-sum transfers makes it easy for government to control the level of private consumption. Without such transfers consumption can only be controlled indirectly via the wage rate, and two instruments affect this. More public infrastructure raises wages directly and also by attracting private investment; lower distortionary taxation attracts private investment and raises wages. Of course, these instruments are linked by the budget constraint. Resource revenues relax this constraint, and the ensuing second-best optimal policy response is outlined below.

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[11] If the windfall was large enough for the interest rate to fall to r^* during the period of revenue flow then the economy would commence construction of a SWF, as in section 3.4.

The optimal policy is found from the first-order conditions above, but with $T = 0$, so instead of first order condition (7.2) we have simply $C = \mathrm{W}(S, \tau)$. It is helpful to define the marginal cost of public funds as the shadow price of public funds relative to the marginal utility of private consumption $\phi \equiv \mu / C^{-1/\sigma}$. In the preceding subsection $\phi = 1$, but the fact that the government now has to raise funds by distortionary taxes means that $\phi > 1$. The relationship between the optimal income tax and the marginal cost of public funds is given by equation (7.1) (using (7.2) and $\mathrm{W}_\tau = -Y$) as

$$\tau = \left(\frac{1-\phi}{\phi}\right)\left(\frac{Y}{\mathrm{Y}_\tau}\right) \text{ or } \phi = \frac{1}{1+\tau \mathrm{Y}_\tau / Y}. \tag{7.1'}$$

Since $\mathrm{Y}_\tau < 0$ a positive tax rate is associated with marginal cost of funds greater than unity. A higher cost of funds depresses the demand for public relative to private consumption:

$$G = \left(\frac{\psi}{\phi}\right)^\sigma \mathrm{W}(S,\tau). \tag{7.3'}$$

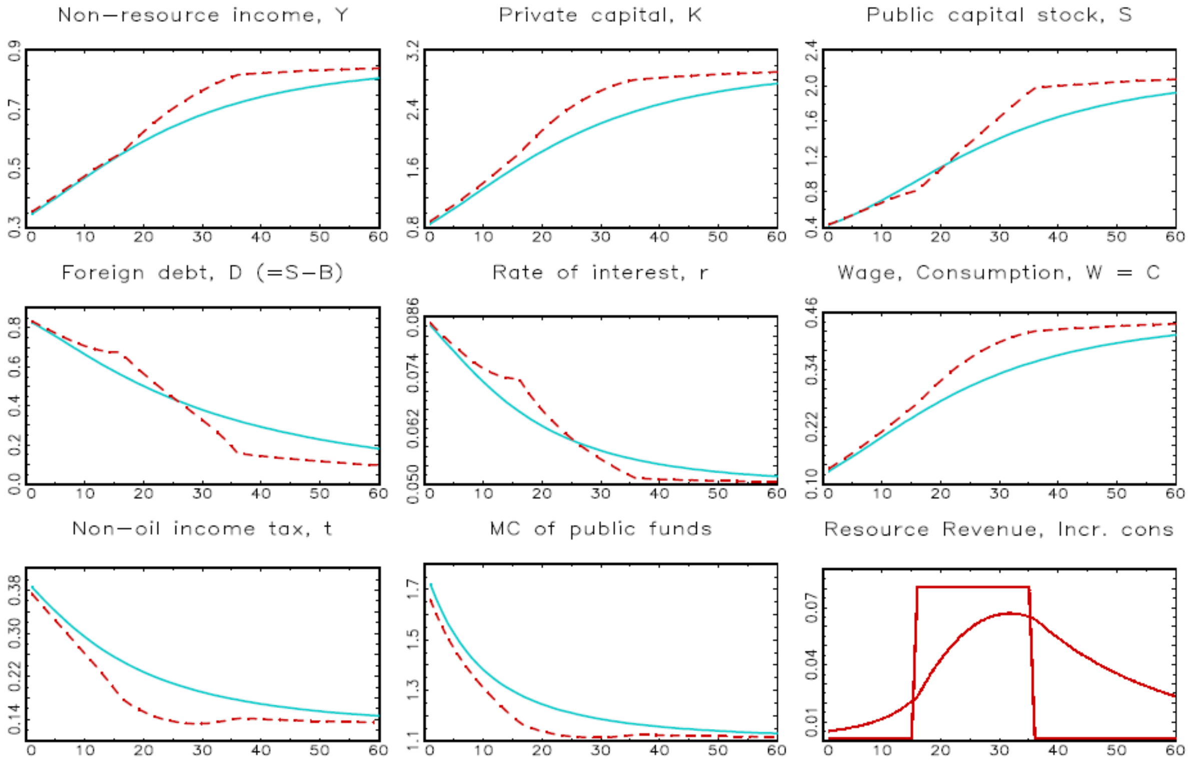
The first-order condition for infrastructure becomes

$$\mathrm{W}_S(S,\tau) = \phi\left[r^* + \Pi(S-B) + (S-B)\Pi'(S-B) + \delta_S - \tau \mathrm{Y}_S\right]. \tag{7.4''}$$

Thus, ϕ greater than unity raises the cost of capital which tends to reduce the optimal level of public infrastructure, although this may be offset as an increase in infrastructure raises income and tax revenue, $\tau \mathrm{Y}_S > 0$.

We once again illustrate the optimal development paths, with and without resource revenue, by numerical example. Figure 5.2 describes the same economy as figure 5.1, but with this restricted set of instruments. Government finance of infrastructure requires distortionary taxation ($\tau > 0$) and hence a shadow premium on public funds ($\phi > 1$). Along the development path without resources there is steady pay back of debt, increasing capital stock and rising income and consumption. This is accompanied by a declining cost of public funds and rate of income tax as accumulation of infrastructure reduces the flow of infrastructure investment (relative to the size of the economy) to be financed. The presence of the distortion means that income and consumption are lower at all dates (including the long run) than they are when lump-sum transfers can be used.

Figure 5.2: Second-best optimal development.



Key: Solid lines without a windfall and dashed lines with an anticipated temporary windfall.

Discovery of the resource revenue causes an immediate decrease in the marginal cost of public funds, as would be expected. There is also an immediate (small) jump in consumption, which is then on an accelerating path. Consumption is equal to the wage rate, so this jump is engineered by a lower income tax rate which attracts private capital and raises income. However, the lower tax requires government borrowing which raises the cost of funds and causes public infrastructure investment to fall below its previous path. It is only once resource revenues flow that the government is able to afford a lower tax rate, higher level of public infrastructure, and sharply falling level of debt. All of these things put income and wages on a rapid growth path.

It is interesting to note that the government could have increased output and wages either by cutting income tax or by increasing the public capital stock. In this example the stock of infrastructure initially falls back as the interest rate rises before it starts to rise steeply. This is a consequence of the fact that a Cobb-Douglas technology (as used in the simulation) implies that neither the tax rate nor the marginal cost of public funds have a direct impact on the optimal stock of infrastructure. Equations (7.1′) and (7.4″) are particularly simple in this case, namely

$$\phi = \frac{1}{1-\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\tau}{1-\tau}\right)} \quad \text{and} \quad \frac{S}{Y} = \frac{\gamma}{r^* + \Pi(D) + D\Pi'(D) + \delta_S} \tag{8}$$

where $0 < \alpha < 1$ denotes the elasticity of output with respect to K and $0 < \gamma < 1$ the elasticity with respect to S. Hence, the optimal public capital/production ratio depends only on parameters and the interest rate and not on the cost of funds or the tax rate.

In summary, inability to raise consumption via lump-sum taxes means that the government must instead increase wages by a combination of lower income tax and higher public infrastructure. The initial response is to lower the tax rate, but at the date when the windfall revenue ceases (t = 35, at the kink) domestic capital stocks (public and private) are approximately 20% higher than they otherwise would have been, indicating increased reliance on the non-resource economy to deliver additional consumption. Comparing the bottom right panels of figures 4.1 and 4.2, it is noteworthy that the resource discovery is associated with a larger increase in consumption at all dates in the second case. The reason is that windfall revenue to government allows it to reduce other distortions that are present in the economy, specifically the rate of income tax.

6. Using public infrastructure to avoid the Ricardian curse

Up to this point government has been the only agent making choices about the intertemporal profile of consumption. In practise there may also be forward-looking private agents who own assets and adjust savings and consumption decisions in response to current and future resource revenues. This raises the possibility that Ricardian consumers may, in some sense, negate the effect of government policy. They fully anticipate their future shares in resource revenues and adjust consumption accordingly, but not necessarily optimally from a social perspective, a ‘Ricardian curse’. Thus, even if the government is seeking to save a substantial share of resource revenues, policy may be undermined by a private consumption boom fuelled by private borrowing, as has happened in some countries.[12] How should government react to this?

To model this, we now assume that households are no longer credit constrained but may own private assets. Aggregate private wealth is denoted by A and can be held in either domestic equity or government bonds which we assume to be perfect substitutes. Thus, the physical assets in the economy, $K + S$, are owned by foreigners (foreign debt F), government (net assets B), and households (wealth A), so $K + S = F + B + A$ (or $K = F - D + A$ where D

$= S - B$ is government debt). Foreign liabilities F thus defined correspond to the excess of public debt over private bond holdings plus net import of capital. We will assume that the interest premium now depends on the asset position of private asset holders as well as that of the government, so $r = r^* + \Pi(F) = r^* + \Pi(K+S - A - B)$.

The production side of the economy is as before, except that we now assume that all investors face the domestic interest r (inclusive of the premium $\Pi(F)$). For simplicity we ignore income taxation, so profit maximisation implies that the marginal product of capital equals the user cost of capital:

$$f_K(K, S) = r^* + \Pi(K+S - A - B) + \delta_K . \tag{9}$$

This implicitly defines $K = \mathrm{K}(S, A+B)$ and correspondingly production $\mathrm{Y}(S, A+B)$, wages $\mathrm{W}(S, A+B)$ and domestic interest rate $\mathrm{r}(S, A+B)$. Capital stock, wages and production are increasing in S and in $A + B$. The interest rate is increasing S and decreasing in $A + B$. For future reference we note that $\mathrm{W}_A + K\mathrm{r}_A = 0$ (see appendix). Effects with respect to S hold with strict equality, but effects with respect to $A + B$ occur only if the economy is highly indebted so $F > \overline{F}$. The responses are intuitive. A higher stock of assets owned by domestic households, A, or the government, B, corresponds to lower foreign liabilities and thus pushes down the premium and the domestic interest rate. Consequently, capital, wage income and output increase. A higher level of public infrastructure boosts the marginal productivity of capital and of labour, hence increases the demand for capital and boosts output. As a result, the domestic interest rate and wage rate rise. For given $A + B$, a higher public infrastructure also increases foreign liabilities and pushes up the domestic rate of interest. Full details of the Cobb-Douglas case are given in the appendix.

Private households have access to domestic capital markets and can smooth their consumption C. They are subject to two government instruments, an asset holding subsidy at rate τ_A and a lump-sum transfer of T_A. Their budget constraint is therefore $\dot{A} = (r + \tau_A)A + W + T_A - C$. The privately optimal growth in consumption is proportional to the gap between the interest rate (inclusive of the asset holding subsidy rate) and households' rate of pure time preference ρ, so that

$$\dot{C} / C = \sigma(r + \tau_A - \rho) . \tag{10}$$

[12] Notably Kazakhstan where public saving has been offset by private borrowing (Esanov and Kuralbeyeva, 2009).

The government borrows and issues debt D (= $S - B$) at rate of interest r. The government budget constraint is thus, $\dot{B} = r(B-S) + N - G - T_A - \tau_A A - \delta_S S$, $B(0) = B_0$. Ricardian equivalence implies that the intertemporal profile of government transfers T_A does not affect private consumption, so we may as well use the consolidated private and public budget constraint:[13]

$$\text{(11)}\quad \dot{A} + \dot{B} = r(A+B) + N + \mathrm{W}(S, A+B) - G - C - (r+\delta_s)S, \quad A(0) + B(0) = A_0 + B_0.$$

This budget constraint, together with $F = K + S - A - B$ and $Y = W + (r + \delta_K)K$ imply that the trade deficit (the excess of public and private spending over production plus windfall revenue) plus interest on foreign liabilities equals the increase in indebtedness of the nation, $\dot{F} = rF + C + G + I_S + I_K - Y - N$, $F(0) = S(0) + K(0) - A_0 - B_0$. The no-Ponzi condition implies that the present discounted value of net exports of goods and services minus windfall revenue exports must cover initial foreign liabilities. These liabilities jump on impact if the government borrows for infrastructure or firms import capital.

Social welfare depends on consumption by households and government, as before, and we maximise with respect to the asset holding subsidy rate τ_A, public consumption G, and public infrastructure S. Notice that transfers or citizen dividends, T_A, do not enter explicitly; Ricardian consumers know the combined budget constraint (11) and hence the implicit value of these payments. Maximisation is subject to (11) and (10) for the state variables A+B and C, with respective co-states and μ and λ. Given that the Pontryagin function is defined by

$$\text{(12)}\quad \begin{aligned} \mathrm{H}(\tau_A, G, S, \lambda, \mu) \equiv \left(\frac{C^{1-1/\sigma} + \psi G^{1-1/\sigma}}{1 - 1/\sigma} \right) + \lambda\sigma\left[\mathrm{r}(S, A+B) + \tau_A - \rho\right]C + \\ \mu\left[\mathrm{r}(S, A+B)(A+B-S) + N + \mathrm{W}(S, A+B) - G - C - \delta_s S\right]. \end{aligned}$$

The first-order conditions are:

$$\text{(12.1)}\quad \mathrm{H}_{\tau_A} = \sigma C \lambda = 0$$

$$\text{(12.2)}\quad \mathrm{H}_G = \psi G^{-1/\sigma} - \mu = 0$$

$$\text{(12.3)}\quad \mathrm{H}_S = \left[\mathrm{W}_S + (A+B-S)\mathrm{r}_S - r - \delta_S\right]\mu + \lambda\sigma C \mathrm{r}_S = 0$$

$$\text{(12.4)}\quad \rho\mu - \dot{\mu} = \mathrm{H}_{A+B} = \left[r + \mathrm{W}_A + (A+B-S)\mathrm{r}_A\right]\mu + \lambda\sigma C \mathrm{r}_A$$

[13] Although noting that the asset holding tax still affects private consumption, as in (10).

(12.5) $$\rho\lambda - \dot{\lambda} = \mathrm{H}_C = C^{-1/\sigma} - \mu + \sigma\left(r + \tau_A - \rho\right)\lambda$$

with the transversality conditions

(12.6) $$\lim_{t\to\infty}\left[\exp(-\rho t)\mu(t)\left(A(t)+B(t)\right)\right] = 0 \text{ and } \lim_{t\to\infty}\left[\exp(-\rho t)\lambda(t)C(t)\right] = 0.$$

6.1. Lump-sum taxes and optimal asset holding subsidy

We first look at the case is in which government can control private consumption growth by using the asset holding subsidy rate τ_A. The first-order condition (12.1) then implies that λ = 0, and hence from (12.2) and (12.5) $\mu = C^{-1/\sigma} = \psi G^{-1/\sigma}$. Consumption paths follow from (12.4) as

(13.1) $$\dot{C}/\sigma C = \dot{G}/\sigma G = \left[r + \mathrm{W}_A + (A+B-S)\mathrm{r}_A - \rho\right] \text{ and } G = \psi^{\sigma} C.$$

The optimal level of public infrastructure is implicitly given in (12.3),

(13.2) $$\mathrm{W}_S = r + \delta_S - \left(A+B-S\right)\mathrm{r}_S,$$

indicating that S increases with the stock of private plus public assets, via an effect on the domestic rate of interest. Analogous to previous cases the optimal path of the economy is described by differential equations for assets, (A+B, equation (11)) and for C (equation (13.1)), with values of G, S, and hence K = K(S, A+B), Y(S, A+B), W(S, A+B), r = r(S, A+B) being computed at each instant. Notice that expressions (13.1) and (13.2) for consumption growth and infrastructure are more complex than the analogous equations in the previous section, (7.4′) and (7.5′), because debt now affects the rate at which the private sector borrows, giving rise to the dependence of K, Y, r and W on (A+B).

Our main focus is on the asset holding subsidy, since it is this that controls the time profile of private consumption. The implied optimal asset holding subsidy rate follows from comparison of (13.1) with (10)

(13.3) $$\tau_A = \left[\mathrm{W}_A + (A+B-S)\mathrm{r}_A\right] = F\mathrm{r}_A \geq 0,$$

where the equality comes from the definition of F and $\mathrm{W}_A + K\mathrm{r}_A = 0$. This is non-negative, since the sign in (13.3) holds with strict inequality for $F > \overline{F} \geq 0$ and equality if debt is below this threshold. The intuition is that an asset holding *subsidy* is required because of a terms-of-trade effect. By saving and raising A, private agents reduce the interest rate premium that the economy has to pay on its foreign debt, an effect that not internalised by individual price-taking asset holders. This interest rate change benefits the economy in aggregate (if $F > 0$), while raising wages ($W_A > 0$) and reducing returns to domestic asset holders ($r_A < 0$). The asset holding subsidy therefore starts relatively high and, without the windfall, falls monotonically to zero. The effect of a resource windfall is to initially increase the asset holding subsidy, followed by a fall in the subsidy rate to below its level absent the resource. This exactly mirrors the path of outstanding debt (as in figures 4.1 or 4.2), since it is this that drives the terms-of-trade effect.

In summary, the asset holding subsidy fully corrects the distortion that arises from households' failure to internalise the adverse effect of their consumption on the interest rate at which the economy borrows. The government funds the optimal level of public consumption and infrastructure and transfers other revenue to households through citizen dividends. Since private agents are Ricardian consumers facing the same cost of capital as the public sector, the timing of transfers is immaterial. However, this requires all consumers to have access to capital markets and any macro-economic impacts of debt on interest rates to be internalised by the asset holding tax such that citizen dividends are optimal.

6.2. Absence of an optimal asset holding subsidy

Time-varying asset holding subsidies may be difficult to implement and are seldom seen in practice. We therefore turn to the case in which this instrument is unavailable and $\tau_A = 0$. Since the return to saving is reduced there is a tendency for households to be on a consumption path that is too flat, involving too much consumption in the early years and too little saving. What is the optimal response to this situation? Since (12.1) no longer applies, necessary conditions are, from (10) with (12.2) – (12.5):

(14.1) $$\dot{C}/C = \sigma(r-\rho)$$

(14.2) $$\frac{\dot{G}}{G} = \sigma\left[r-\rho+\mathrm{W}_A+\mathrm{r}_A\left\{A+B-S+\left(\frac{\sigma\lambda C}{\psi G^{-1/\sigma}}\right)\right\}\right]$$

(14.3) $$\mathrm{W}_s = r+\delta_s-\mathrm{r}_s\left[A+B-S+\left(\frac{\sigma\lambda C}{\psi G^{-1/\sigma}}\right)\right]$$

(14.4) $$\dot{\lambda} = \left[\rho-\sigma(r-\rho)\right]\lambda+\psi G^{-1/\sigma}-C^{-1/\sigma}.$$

The dynamic system (11), (14.1), (14.2) and (14.4) is solved with aggregate assets as a predetermined variable (i.e., $A(0)+B(0)=A_0+B_0$) and private and public consumption as jump variables ($C(0)$ and $G(0)$ free to jump), where at each point of time the interest rate is given by $r(S,A+B)$ and the level of public infrastructure follows from (14.3). Since the government has no access to an asset holding subsidy or lump-sum transfers, it is unable to control the initial level of private consumption[14] and consequently the initial marginal social value of private consumption is free to jump at time zero (i.e. $\lambda(0)$ free). The dynamic system can thus be solved with a standard reverse multiple shooting algorithm. Due to the initial bias towards over-consumption, the social value of a marginal reduction in initial private consumption must be positive. Since C is a forward-looking variable, this implies that its initial co-state must satisfy $\lambda(0) > 0$.[15] We can show that the steady-state value of λ is zero if $\bar{F} = 0$.[16] The government's inability to use the asset holding subsidy thus means that λ is initially positive and then declines to the limiting value of zero. In general, λ need not converge to zero.

The consequences of $\lambda > 0$ are seen by inspection of equations (14.2) and (14.3) for public consumption and infrastructure. From (14.2), $\lambda > 0$ implies slower growth of public consumption G (since $r_A < 0$) and thus a higher value of initial public consumption $G(0)$. And from (14.3), $\lambda > 0$ implies a higher value of S; since $r_S < 0$, the effect is like a lower cost of capital. The only way that the government can, in the absence of the asset holding subsidy, dampen the consumption of Ricardian households is by itself raising spending on public consumption and infrastructure. This spending has a negative income effect on private households, and also causes foreign debt to be larger than it otherwise would be, increasing the interest rate and thereby increasing private saving.

This is illustrated in figure 6.1.[17] The top panel of this figure looks at two cases where there are no resource revenues. The dashed line is if τ_A is set optimally, and the solid line if τ_A is constrained to be zero. In line with the discussion above, public infrastructure is larger – a full 20% larger – if the asset holding subsidy is not available. This has the effect of

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[14] However, the government can by varying G and S control the present value of private consumption $PV(C) = A + B + PV(W+N) - PV(G+(r+\delta_S)S)$, where PV(.) indicates the present value using the market rate of interest. Raising public infrastructure also boosts wages, which offsets the downward adjustment of the private consumption path. Private agents also realise that, if government does not spend the windfall, the windfall is going to accrue to them and thus spend accordingly.

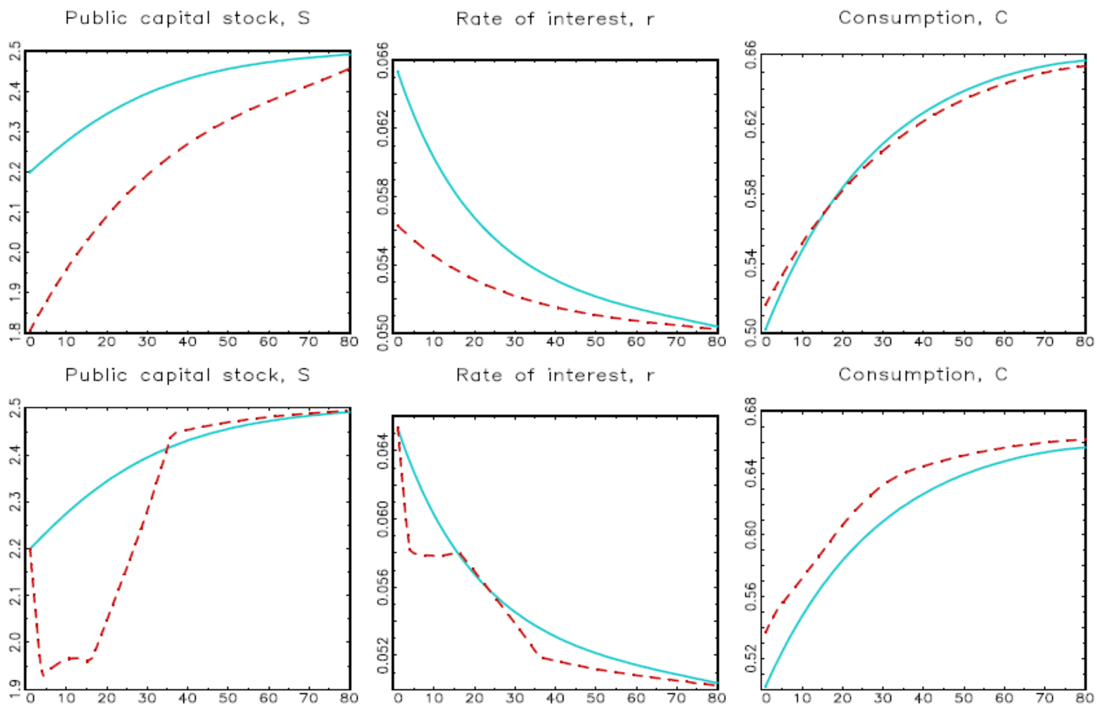
[15] We use the result that at the optimum $\partial U/\partial C(0) = -\lambda(0) < 0$ while $\partial U/\partial[A(0)+B(0)] = \mu(0) > 0$.

[16] In steady state (14.2) gives $W_A + (A+B-S)r_A + \left(\frac{\sigma\lambda C}{\psi G^{-1/\sigma}}\right)r_A = 0$, which implies that $\lambda\Pi' = \Pi'\bar{F}\psi G^{-1/\sigma}/\sigma C.$ Since steady state $\Pi' = 0$ we use a limiting argument; suppose that $\Pi = \Pi(F+\varepsilon)$, $\varepsilon > 0$, and let $\varepsilon \to 0$ to yield steady-state value of $\lambda = 0$ if $\bar{F} = 0$.

[17] Parameters and production function are as in section 4 except that figure 5.1 uses $\Pi = 0.75\rho(F_+)^2$.

increasing indebtedness and hence interest rates. As a consequence the paths of consumption and private capital stock (not illustrated) are very similar in the two cases. Essentially, the government commits to public investment, debt and higher interest rates, in order to prevent over-consumption by the private sector.

Figure 6.1: Counteracting the Ricardian curse



Key: Top panel gives simulations without windfall; dashed lines give case with optimal asset holding subsidy and solid lines without asset holding subsidy.
The bottom panel gives simulations without the asset holding subsidy; dashed lines are with the windfall, and solid lines without.

The bottom panel of figure 6.1 has $\tau_A = 0$, and compares variables with (dashed line) and without (solid line) the anticipated temporary windfall revenue. We see that the effect of the windfall is to cut the stock of public infrastructure. In this example infrastructure investment goes to zero for a short period and the downward-sloping section of the infrastructure stock schedule arises as existing infrastructure depreciates. This perverse effect occurs because the windfall moves the economy closer to development and thereby reduces the magnitude of the distortion that we have built into the system. In particular, resource wealth reduces indebtedness, and so reduces the terms-of-trade effect of private consumption on debt service obligations. Consumption is therefore closer to its first best optimal path and there is less need to control it indirectly through high spending on infrastructure.

Two further points are noteworthy. First, the reduction in infrastructure spending when the resource is discovered occurs because government is assumed to be implementing the second-best optimal policy prior to the windfall; such very high rates of infrastructure investment are not observed in most developing countries. Second, the only reason for private sector over-consumption in this model is the terms-of-trade effect of changing the interest rate. Other distortions affecting the time profile of private consumption (domestic capital market imperfections, high spreads and low returns to private saving) might not be mitigated by the windfall.

7. Concluding remarks

We have established four main results. The first is that a developing economy which is capital scarce with rate of interest greater than the rate of time preference and growing consumption should not follow the prescriptions of the permanent income hypothesis and devote the proceeds of a resource windfall to construction of a SWF. It should instead invest to raise the rate of growth of consumption; in a poor country the gains from reaching the long-run level of consumption sooner outweigh those from raising the long-run level through permanent returns on an SWF.

The second concerns the composition of spending. There should be some immediate increase in consumption (by transfer payments, if these are available), accompanied by investment in a combination of public infrastructure and debt reduction, the latter bringing lower interest rates and higher private investment. If there is a substantial time lag before windfall revenues flow, then the immediate increase in consumption remains optimal, although it has the effect of increasing indebtedness and pushing further into the future the date at which public infrastructure and debt reduction takes place.

Third, if direct transfers to consumers are difficult to implement, then more of the windfall revenue should be devoted to public investment and to tax measures that increase private investment. This is because consumption can be increased only by raising wages in the economy, and higher investment is the means to achieve this.

Finally, the prescription of citizen dividends is optimal only if households have access to capital markets and any tendency to private over- (or under-)consumption can be corrected by a time-varying asset holding subsidy (tax). Access to capital markets means that consumption can be separated from the date at which transfers are made, but private smoothing need not be socially optimal. This can be corrected by a time varying asset holding subsidy. If this instrument is not available, then government will need to correct the over (or under) consumption of Ricardian consumers by other means, such as varying the level of investment in public infrastructure.

These results challenge aspects of the standard advice for handling windfall revenues, for example the recommendation that revenues should be used to build an SWF and, according to some, consumption limited to the interest on this fund (e.g., Barnett and Ossowski, 2003). Developing countries have both an urgent need both for consumption to reduce poverty, and high-return domestic investment opportunities. Our analysis shows how these factors make it optimal to use revenues to grow the domestic economy.

Of course, the analysis abstracts from many important elements that will be the subject of future research. First, if windfall revenue directly impacts creditworthiness, there may be a danger of over-borrowing (e.g. Mansoorian, 1991; Manzano and Rigobon, 2001). Second, the economy may have difficulty in absorbing additional expenditure. At the macro-economic level there may be an appreciation of the real exchange rate and decline of the traded sector (e.g., Corden and Neary, 1982). At the micro-economic level maintaining and raising the efficiency of public expenditure is essential. Third, it is important to allow for endogenous optimal resource depletion and examine how the well-known Hotelling (1931) rule should be modified when the government faces the tough public-finance dilemmas we have highlighted. For example, does it still make sense to have a current-account surplus matching the Hotelling rents? Fourth, the government may be myopic for political reasons or due to competing fractions and the voracity effect (Tornell and Lane, 1999) in which case the government brings forward public spending and postpones taxation. Furthermore, an incumbent, worried about being removed from office by a political rival with preference for a different type of public goods, typically issues too much debt and spends too much on its own pet projects (Alesina and Tabellini, 1990). These political distortions are exacerbated if the incumbent uses the windfall to opportunistically pacify the electorate. On the other hand, governments may prefer to invest in public infrastructure rather than a SWF as the former is more difficult to be raided by future political rivals. Resource-rich countries may also get addicted to high public spending and find it difficult to kick the habit once resource revenues dry up (e.g., Leigh and Olters, 2006; Olters, 2007).

Perhaps most importantly, resource revenues are not only uneven through time, as we have modelled, but also in many cases highly uncertain due to the notorious volatility of commodity prices and uncertainty about future extraction costs. This creates a case for accumulating precautionary buffers in a Sovereign Liquidity Fund to smooth shocks. However, it remains important that, as we have argued in the context of certainty, revenues are used to grow the domestic economy and raise consumption in the short to medium term, and are not simply deposited abroad.

References

Akitobi, B. and T. Stratmann (2008). Fiscal policy and financial markets, Economic Journal, 118, 533, 1971-1985.

Alesina, A. and G. Tabellini (1990). A positive theory of fiscal deficits and government debt, Review of Economic Studies, 57, 3, 403-414.

Aranello, M. (1987). Computing robust standard errors for within-group estimators, Oxford Bulletin of Economics and Statistics, 49, 4, 431-434.

Barnett, S. and R. Ossowski (2003). Operational aspects of fiscal policy in oil-producing countries, in J. Davis, R. Ossowski and A. Fedelino (eds.), Fiscal Policy Formulation and Implementation in Oil-Producing Countries, International Monetary Fund, Washington, D.C.

Barro, R. (1979). On the determination of the public debt, Journal of Political Economy, 87, 5, 940-971.

Basdevant, O. (2008). Are diamonds forever? Using the permanent income hypothesis to analyze Botswana's reliance on diamond revenue, Working Paper No. 08/80, International Monetary Fund, Washington, D.C.

Bjerkholt, O. (2002). Fiscal rule suggestions for economies with non-renewable resources, University of Oslo.

Buiter, W.H. (1984). Saddlepoint problems in continuous time rational expectations models: a general method and some macroeconomic examples, Econometrica, 52, 3, 665-680.

Collier, P., A. Hoeffler and C. Patillo (2001). Flight capital as a portfolio choice, World Bank Economic Review, 15, 1, 55-80.

Collier, P. and A. Hoeffler (2004). Greed and grievance in civil war, Oxford Economic Papers, 56, 563-595.

Collier, P and J.W. Gunning (2005). Asset policies during an oil windfall: some simple analytics, The World Economy, 28, 10, 1401-1415.

Corden, M. and P. Neary (1982). Booming sector and de-industrialization in a small open economy, Economic Journal, 92, 825-848.

Davis, J., R. Ossowski, J. Daniel and S. Barnett (2002). Stabilization and Savings Funds for Non-Renewable Resources: Experience and Fiscal Policy Implications, IMF Occasional Paper No. 205, International Monetary Fund, Washington, D.C.

Esanov and Kuralbeyeva (2009). Managing boom-bust cycle in a resource rich economy: the case of Kazakhstan 2000-2008, mimeo, Oxford.

Fearon, J. (2005). Primary commodity exports and civil war, Journal of Conflict Resolution, 49, 483-507.

Harding, T. and F. van der Ploeg (2009). Is Norway's bird-in-hand stabilization fund prudent enough? Fiscal reactions to hydrocarbon windfalls and graying populations,Working Paper 2830, CESifo, Munich.

Hotelling,H. (1931). The economics of exhaustible resources, Journal of Political Economy, 39, 137-175.

IMF (2007) Guide on Resource Revenue Transparency, IMF, Washington DC

Leigh, D. and J.-P. Olters (2006). Natural resource depletion, habit formation, and sustainable fiscal policy: lessons from Gabon, IMF Working Paper WP/06/193, International Monetary Fund, Washington, D.C.

Mansoorian, A. (1991). Resource discoveries and 'excessive' external borrowing, Economic Journal, 101, 1497-1509.

Mansano, O. and R. Ribogon (2001). Resource curse or debt overhang?, Working Paper 8390, NBER, Cambridge, Mass.

Morshed, A.K.M.M. and S.J. Turnovsky (2004). Sectoral adjustment costs and real exchange rate dynamics in a two-sector dependent economy, Journal of International Economics, 62, 147-177.

Olters, J.P. (2007). Old curses, new approaches? Fiscal benchmarks for oil-producing countries in sub-Saharan Africa, IMF Working Paper WP/07/107, International Monetary Fund, Washington, D.C.

Ploeg, F. van der (2010). Why do many resource-rich countries have negative genuine saving rates? Anticipation of better times or rapacious rent seeking, Resource and Energy Economics, 32, 28-44.

Sachs, J.D. (1981). The current account and macroeconomic adjustment in the 1970s, Brookings Papers on Economic Activity, 1, 201-268.

Sachs, J.D. and A.M. Warner (1997). Natural resource abundance and economic growth, in G. Meier and J. Rauch (eds.), Leading Issues in Economic Development, Oxford University Press, Oxford.

Schmitt-Grohé, S. and M. Uribe (2003). Closing small open economy models, Journals of International Economics, 61, 1, 163-185.

Segura, A. (2006). Management of oil wealth under the permanent income hypothesis: the case of São Tomé and Príncipe, Working Paper No. 06/183, International Monetary Fund, Washington, D.C.

Stock and Watson (2008). Heteroskedasticity-robust errors for fixed effects panel data regression, Econometrica, 76, 1, 155-174.

Tornell, A. and P.R. Lane (1999). The voracity effect, American Economic Review, 89, 1, 22-46.

Turnovsky, S.J. (1997). International Macroeconomic Dynamics, MIT Press, Cambridge, Ma.

Wijnbergen, S. (1984). The Dutch Disease: a disease after all? Economic Journal, 94, 41-55.

Appendix 1: Saddlepath dynamics of capital-scarce economy (section 4)

We establish three propositions, which are used in the discussion of section 4.2.

Proposition 1: With a temporary 'small' windfall of size $N > 0$ from time $\mathrm{T}_0 \geq 0$ to $\mathrm{T}_1 > \mathrm{T}_0$, we have

(A1.1) $$\Delta C(0) + \Delta G(0) = \exp(-\lambda_u \mathrm{T}_0)\left[1 - \exp(-\lambda_u \mathrm{T})\right] N > 0,$$

where $\lambda_u = \frac{1}{2}r^* + \frac{1}{2}\sqrt{r^{*2} + 8\sigma\Pi' Y} > r^* > 0$, T ≡ $T_1 - T_0 > 0$, and Δ indicates deviations from the benchmark trajectories.

Proof: Linearizing equations (3) and (4) around a steady state with zero *N*, that is $F_\infty = \bar{F} = 0, \Pi_\infty = 0$ and $C_\infty = Y/(1+\psi^\sigma)$, yields:[18]

$$\dot{x} = \mathrm{A}x - \begin{pmatrix} \Delta N \\ 0 \end{pmatrix} \text{ with } x \equiv \begin{pmatrix} \Delta F \\ \Delta C \end{pmatrix}, \mathrm{A} \equiv \begin{pmatrix} r^* & 1+\psi^\sigma \\ \Sigma & 0 \end{pmatrix} \text{ and } \Sigma \equiv 2\sigma\Pi' C_\infty > 0.$$

The eigenvalues of the matrix A are:

$$\lambda_s = \frac{1}{2}r^* - \frac{1}{2}\sqrt{r^{*2} + 4\Sigma(1+\psi^\sigma)} < 0 \text{ and } \lambda_u = \frac{1}{2}r^* + \frac{1}{2}\sqrt{r^{*2} + 4\Sigma(1+\psi^\sigma)} > r^* > 0.$$

Since one of the eigenvalues is positive and the other one is negative, the system displays saddlepath dynamics. We solve this system with spectral decomposition of the matrix A (cf., Buiter, 1984):

$$\mathrm{A} \equiv \begin{pmatrix} r^* & 1+\psi^\sigma \\ \Sigma & 0 \end{pmatrix} = \mathrm{N}^{-1}\begin{pmatrix} \lambda_s & 0 \\ 0 & \lambda_u \end{pmatrix}\mathrm{N} \text{ with } \mathrm{N} \equiv \begin{pmatrix} N_{ss} & N_{su} \\ N_{us} & N_{uu} \end{pmatrix},$$

where the columns of the matrix N stack the eigenvectors of the matrix A. The eigenvectors are calculated from the equations:

$$\mathrm{N\,A} = \begin{pmatrix} r^* N_{ss} + \Sigma N_{su} & (1+\psi^\sigma)N_{ss} \\ r^* N_{us} + \Sigma N_{uu} & (1+\psi^\sigma)N_{us} \end{pmatrix} = \begin{pmatrix} \lambda_s N_{ss} & \lambda_s N_{su} \\ \lambda_u N_{us} & \lambda_u N_{uu} \end{pmatrix} = \begin{pmatrix} \lambda_s & 0 \\ 0 & \lambda_u \end{pmatrix}\mathrm{N}.$$

Normalizing such that $N_{uu} = 1$, we obtain $N_{us} = \lambda_u/(1+\psi^\sigma) = \Sigma/(\lambda_u - r^*) > 0$ from equating the two elements in the bottom row of the above matrix equation. Note that the top row gives $N_{ss} = -\Sigma/(1+\psi^\sigma) < 0.$ Defining the vector z ≡ N x, we obtain $z_u(t) = \int_0^\infty \exp(-\lambda_u t) N_{us} \Delta N(t) \mathrm{d}t$ provided we assume that $\lim_{t\to\infty} \exp(-\lambda_u t) z_u(t) = 0$ holds. Restricting the solution to the stable manifold, we obtain $\Delta C(t) = -N_{uu}^{-1} N_{us} \Delta F(t) + N_{uu}^{-1} z_u(t)$ or

$$\Delta C(t) = -N_{us}\Delta F(t) + z_u(t) = \left(\frac{\lambda_u}{1+\psi^\sigma}\right)\left[\int_t^\infty \exp(-\lambda_u(t'-t))\Delta N(t')\mathrm{d}t' - \delta F(t)\right].$$

With the step function $\Delta N(t) = N$, $T_0 < t \le T_1$ and zero at all other instants of time, this equation becomes:

[18] We assume Π′>0 at the steady state. That is not strictly necessary as long as Π′>0 at all points on the adjustment path towards the steady state.

$$\Delta C(t)=\left(\frac{\lambda_u}{1+\psi^{\sigma}}\right)\left[\exp\left(-\lambda_u(\mathrm{T}_0-t)\right)\left(\frac{1-\exp(-\lambda_u\mathrm{T})}{\lambda_u}\right)N-\Delta F(t)\right],\quad 0<t\le \mathrm{T}_0$$

(A.1.2) $$\Delta C(t)=\left(\frac{\lambda_u}{1+\psi^{\sigma}}\right)\left[\left(\frac{1-\exp\left(-\lambda_u(\mathrm{T}_1-t)\right)}{\lambda_u}\right)N-\Delta F(t)\right],\quad \mathrm{T}_0<t<\mathrm{T}_1,$$

$$\Delta C(t)=-\left(\frac{\lambda_u}{1+\psi^{\sigma}}\right)\Delta F(t),\quad t\ge \mathrm{T}_1.$$

Since $\Delta F(0)=0$, it follows that $\Delta C(0)=\exp(-\lambda_u\mathrm{T}_0)\left[\frac{1-\exp(-\lambda_u\mathrm{T})}{1+\psi^{\sigma}}\right]N>0.$ The initial jump in total consumption thus equals (A1.1). Q.E.D.

Proposition 2: With a temporary 'small' windfall of size N starting at time 0 and finishing at t = T_1, we have $\Delta F(t) < 0$ for all $t > 0$, $\Delta\dot{F}(t)<0$ for $0<t<\mathrm{T}_1$ and $\Delta\dot{F}(t)>0$ for $t>\mathrm{T}_1$. We also have:

(A.1.3) $$0<\Delta C(T_1)+\Delta G(T_1)=-\lambda_u\Delta F(\mathrm{T}_1)<\Delta C(0)+\Delta G(0)=\left[1-\exp(-\lambda_u\mathrm{T}_1)\right]N,$$

(A.1.4) $$\Delta F(T_1)=-\left[\frac{1-\exp\left((\lambda_s-\lambda_u)\mathrm{T}_1\right)}{\lambda_u-\lambda_s}\right]N<0.$$

Comparing the outcome in the capital scarce economy with that under the PIH, we have

(A.1.5) $$\Delta C(T_1)+\Delta G(T_1)=\left(\frac{\lambda_u}{\lambda_u-\lambda_s}\right)\left[1-\exp\left((\lambda_s-\lambda_u)\mathrm{T}_1\right)\right]N\ >\left[1-\exp(-r^*\mathrm{T}_1)\right]N.$$

Proof: Setting $\mathrm{T}_0 = 0$ and substituting the last two expressions for $\Delta C(t)$ given in (A.1.2) into $\Delta\dot{F}(t)=r^*\Delta F(t)+\left(1+\psi^{\sigma}\right)\Delta C(t)-\Delta N(t),$ and making use of $r^*-\lambda_u=\lambda_s$ yields:

$$\Delta\dot{F}(t)=-\exp\left(-\lambda_u(\mathrm{T}_1-t)\right)N+\lambda_s\Delta F(t),\quad 0<t<\mathrm{T}_1,$$
$$\Delta\dot{F}(t)=\lambda_s\Delta F(t),\quad t\ge \mathrm{T}_1.$$

Solving these differential equations with the initial condition $\Delta F(0)=0$, we obtain:

(A.1.6) $$\Delta F(t)=-\left[\frac{\exp(\lambda_u t)-\exp(\lambda_s t)}{\lambda_u-\lambda_s}\right]\exp(-\lambda_u\mathrm{T}_1)N<0,\quad 0<t\le \mathrm{T}_1,$$
$$\Delta F(t)=\exp\left(\lambda_s(t-\mathrm{T}_1)\right)\Delta F(\mathrm{T}_1)\to 0 \text{ as } t\to\infty,\quad t>\mathrm{T}_1,$$

where $\Delta F(T_1)$ is as given in (A.1.4). It follows that $\Delta F(t) < 0$ for all $t > 0$. Differentiation of (A.1.6) yields:

(A.1.7) $$\Delta\dot{F}(t)=-\left[\frac{\lambda_u\exp(\lambda_u t)-\lambda_s\exp(\lambda_s t)}{\lambda_u-\lambda_s}\right]\exp(-\lambda_u\mathrm{T}_1)N<0,\quad 0<t\le \mathrm{T}_1,$$
$$\Delta\dot{F}(t)=\lambda_s\exp\left(\lambda_s(t-\mathrm{T}_1)\right)\Delta F(\mathrm{T}_1)>0,\quad t>\mathrm{T}_1,$$

Hence, $\Delta\dot{F}(t) < 0,\ 0 < t < \mathrm{T}_1$ and $\Delta\dot{F}(t) > 0,\ t > \mathrm{T}_1$. Using (A.1.2) from the proof of proposition 1, $\Delta C(T_1) + \Delta G(T_1) = -\lambda_u \Delta F(\mathrm{T}_1)$ and thus using (A1.4):

$$\Delta C(T_1) + \Delta G(T_1) = \left(\frac{\lambda_u}{\lambda_u - \lambda_s}\right)\left[1 - \exp\left((\lambda_s - \lambda_u)\mathrm{T}_1\right)\right] N > 0. \tag{A.1.6}$$

Hence, expression (A.1.3) follows. To establish the inequality sign in (A.1.5), we note that (A.1.6) becomes equal to the outcome under the PIH, $\left[1 - \exp(-r^* \mathrm{T}_1)\right] N$, if $\Sigma^* \equiv 4\Sigma(1 + \psi^\sigma) = 0$. If we differentiate and use $\lambda_u + \lambda_s = r^*$, we find that:

$$\frac{\partial\left[\Delta C(T_1) + \Delta G(T_1)\right]}{\partial \Sigma^*} = \left[\frac{\exp\left((\lambda_s - \lambda_u)\mathrm{T}_1\right) N}{4(\lambda_u - \lambda_s)^3}\right]\left\{-r^*\left[1 - \exp\left((\lambda_s - \lambda_u)\mathrm{T}_1\right)\right] + 2T_1\lambda_u(\lambda_u - \lambda_s)\right\}.$$

To sign the expression in the curly brackets, we note that it equals zero and its derivative with respect to T_1 is positive, $(2\lambda_u - r^*)(\lambda_u - \lambda_s) > 0$, if $\mathrm{T}_1 = 0$. For positive T_1, this derivative is even more positive. Hence, the term in curly brackets is positive for all $\mathrm{T}_1 > 0$ and therefore consumption at $t = \mathrm{T}_1$ is more than under the PIH as indicated in expression (A.1.5). Q.E.D.

Proposition 3: With an anticipated 'small' windfall of size $N > 0$ starting at $t = \mathrm{T}_0$ and finishing at $t = \mathrm{T}_1$, we have

$$\Delta\dot{F}(t) > 0 \text{ for } 0 < t < \mathrm{T}_0, \quad \Delta\dot{F}(t) < 0 \text{ for } \mathrm{T}_0 < t < \mathrm{T}_1 \text{ and } \Delta\dot{F}(t) < 0 \text{ for } t > \mathrm{T}_1.$$

A bigger T_0 implies a bigger reduction in foreign debt at the end of the windfall.

Proof: Upon substitution of (A.1.2) into the differential equation for $\Delta F(t)$, we obtain:

$$\Delta\dot{F}(t) = \exp\left(-\lambda_u(\mathrm{T}_0 - t)\right)\left[1 - \exp(-\lambda_u \mathrm{T})\right] N + \lambda_s \Delta F(t), \quad 0 < t \le \mathrm{T}_0,$$
$$\Delta\dot{F}(t) = \left[1 - \exp\left(-\lambda_u(\mathrm{T}_1 - t)\right)\right] N + \lambda_s \Delta F(t), \quad \mathrm{T}_0 < t < \mathrm{T}_1,$$
$$\Delta\dot{F}(t) = \lambda_s \Delta F(t), \quad t \ge \mathrm{T}_1.$$

Forward integration of this differential equation yields:

$$\Delta F(t) = \exp(\lambda_s t)\left[1 - \exp(-r^* t)\right]\exp(-\lambda_u \mathrm{T})\left[1 - \exp(-\lambda_u \mathrm{T})\right] N / r^* > 0, \quad 0 < t \le \mathrm{T}_0,$$
$$\Delta F(t) = \left[\frac{\exp(\lambda_s t) - 1}{\lambda_s}\right]\exp(-\lambda_u \mathrm{T}_0)\left[1 - \exp(-\lambda_u \mathrm{T})\right] N + \left[\frac{1 - \exp\left(\lambda_s(t - \mathrm{T}_0)\right)}{\lambda_s}\right] N, \quad \mathrm{T}_0 < t \le \mathrm{T}_1, ??$$
$$\Delta F(t) = \exp\left(\lambda_s(t - \mathrm{T}_1)\right)\Delta F(\mathrm{T}_1) \to 0 \text{ as } t \to \infty, \quad t > \mathrm{T}_1.$$

Ahead of the windfall ($t \le \mathrm{T}_0$), the country borrows to make possible an increase in consumption ($\Delta F(t) > 0$). During the windfall ($\mathrm{T}_0 < t \le \mathrm{T}_1$), the country starts to pay back its debt and eventually build up assets. At the end of an anticipated windfall, we have:

$$-\Delta F(\mathrm{T}_1) = -\left(\frac{N}{\lambda_s}\right)\left\{1 - \exp\left(\lambda_s \mathrm{T}_1\right) - \left[1 - \exp(\lambda_s \mathrm{T}_1)\right]\exp(-\lambda_u \mathrm{T}_0)\left[1 - \exp(-\lambda_u \mathrm{T})\right]\right\} > 0\,.$$

Note that

$$\frac{\partial - \Delta F(\mathrm{T}_1)}{\partial \mathrm{T}_0} = -\left(\frac{N}{\lambda_s}\right)\left\{-\lambda_s \exp\left(\lambda_s \mathrm{T}_1\right) + \right.$$

$$\left.\left[\lambda_u \exp(-\lambda_u \mathrm{T}_0) - (\lambda_u - \lambda_s)\exp\left(-(\lambda_u - \lambda_s)\mathrm{T}_0 + \lambda_s \mathrm{T}\right)\right]\left[1 - \exp(-\lambda_u \mathrm{T})\right]\right\}$$

and thus

$$\frac{\partial - \Delta F(\mathrm{T}_1)}{\partial \mathrm{T}_0} = -\left(\frac{N}{\lambda_s}\right)\left\{-\lambda_s \exp\left(\lambda_s \mathrm{T}\right) + \left[\lambda_u - (\lambda_u - \lambda_s)\exp\left(\lambda_s \mathrm{T}\right)\right]\left[1 - \exp(-\lambda_u \mathrm{T})\right]\right\} > 0$$

at $\mathrm{T}_0 = 0$. Hence, the bigger T_0, the more debt is reduced the end of the windfall. After the windfall ($t > \mathrm{T}_1$), the second term in the expression for $\Delta F(t)$ gradually vanishes as $t \to \infty$. Q.E.D.

Appendix 2: Comparative statics of production (section 6)

Profit maximisation implies $f_K(K, S) - \delta_K = r = r^* + \Pi(K + S - A - B)$, this implicitly defining $K(S, A+B)$. Hence, we have $r(S, A+B) = f_K(K(S, A+B), S) - \delta_K$ and $W(S, A+B) = f(K(S, A+B), S) - K(S, A+B)f_K(K(S, A+B), S)$. Comparative statics are; $K_A = \Pi' / (\Pi' - f_{KK})$, $r_A = \Pi' f_{KK} / (\Pi' - f_{KK})$, $W_A = -\Pi' K f_{KK} / (\Pi' - f_{KK})$, so $W_A + K r_A = 0$.

For the Cobb-Douglas case, we obtain $\alpha K^{\alpha-1} S^{\gamma} - \delta_K = r = r^* + \Pi(K + S - A - B)$ and $W = (1-\alpha)K^{\alpha} S^{\gamma}$. Using the implicit function theorem, we establish:

(A.2.1) $$\mathrm{K}_A = \left[\frac{\pi' K}{(1-\alpha)(r+\delta_K) + \pi' K}\right], \quad \mathrm{K}_S = \left[\frac{-\pi' K + \gamma(r+\delta_K)K/S}{(1-\alpha)(r+\delta_K) + \pi' K}\right],$$

(A.2.2) $$\mathrm{r}_A = \left[\frac{-\pi'(1-\alpha)(r+\delta_K)}{(1-\alpha)(r+\delta_K) + \pi' K}\right], \quad \mathrm{r}_S = \frac{\pi'(r+\delta_K)\left(1-\alpha+\gamma K/S\right)}{(1-\alpha)(r+\delta_K) + \pi' K},$$

(A.2.3) $$\mathrm{W}_A = \left[\frac{\pi' \alpha W}{(1-\alpha)(r+\delta_K) + \pi' K}\right] \text{ and } \mathrm{W}_S = \frac{W}{S}\left[\frac{\pi'(\gamma K - \alpha S) + \gamma(r+\delta_K)}{(1-\alpha)(r+\delta_K) + \pi' K}\right].$$

From this, we also find:

(A.2.4) $$Y = \frac{\mathrm{W}(S, A+B)}{1-\alpha} \equiv \mathrm{Y}(S, A+B) \text{ with } \mathrm{Y}_\tau = -\alpha Y\left[\frac{r+\delta_K}{(1-\alpha)(r+\delta_K) + \pi' K}\right] < 0.$$