Median-Unbiased Estimation of Structural Change Models: An Application to PPP

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Abstract

The measurement of persistence in macroeconomic time series has attracted a lot of attention due to its substantial implications for policy making decisions. To measure persistence we use autoregressive models. There are two potential biases in the parameter estimates of autoregressive models: (1) a downward bias which is also referred to as small sample bias (2) an upward bias which is due to neglecting the structural changes in time series. A common exercise to correct for the first bias is to use median-unbiased estimation (Andrews, 1993). We conduct Monte Carlo simulations for a variety of sample sizes and structural breaks and show that inclusion of structural breaks causes a substantial increase in the small sample bias documented in Andrews (1993).

We propose an extension of the median-unbiased technique to account for structural breaks. After establishing how to take care of structural breaks in autoregressive models we apply this extended method on some time series data. One of the most extensively studied time series is the real exchange rate. By accounting for known dates of structural breaks, we estimate the speed of mean reversion of real exchange rates, analyzed previously by Lothian and Taylor (1996) and Taylor (2002). We find that median-unbiased half-lives decrease significantly for these specific examples.

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1 Introduction

The measurement of persistence in macroeconomic time series has attracted a lot of attention due to its substantial implications for policy making decisions. In particular, the speed of adjustment in response to macroeconomic shocks is of crucial importance for a central bank whose policy is oriented towards stability of certain macroeconomic variables. Persistence denotes the effect over time of a shock to a macroeconomic time series. There are many macroeconomic variables for which it is important to know how long it will take to come back to its long run mean when a shock occurs. For example, persistence of real exchange rates, real GDP, real interest rates, and inflation rates is commonly examined.

A common practice in empirical research for measuring persistence is to estimate univariate autoregressive (AR) time series models and calculate the half-life, defined as the number of periods for a unit shock to a time series to decay by 50 percent. If a series is stationary, it will eventually return to its long-run mean and the half-life is finite. If it contains a unit root, the shock will be permanent.[1]

There are two threads of research in the estimation of persistence in macroeconomic time series. First, estimating persistence in time series with AR models is known to produce a downward bias (i.e. half-life estimates are too small) and the bias worsens as the AR coefficient approaches unity. This is often referred to as small sample bias. Andrews (1993) discusses this bias and shows how to construct median-unbiased estimates of the half-life. There are numerous papers that apply this method to macro time series (e.g. Gospodinov (2002), Murray and Papell (2002, 2005a, 2005b), and Rossi (2005)). For example, Murray and Papell (2005a) estimate half-lives of real exchange rates using Lothian and Taylor's (1996) data set. They find that correcting small sample bias raises estimates of half-lives from 5.78 to 6.58 years for the dollar-sterling rate and from 2.73 to 2.94 years for the franc-sterling rate. A potential solution to the problem of small sample bias is to use a long span of data since the bias decreases as the sample size increases. The drawback, however, is that a longer data set has a higher probability of including structural breaks.[2] Neglecting structural changes in the estimation of

[1] If time series has a unit root the whole-life of shock will be infinite. The half-life can be finite but the series still will not be stationary.

[2] For example, Stock and Watson (1996), Garcia and Perron (1996), and Marcellino (2002) illustrate that the

persistence will produce an upward bias, thus half-life estimates are too big.

The second line of research examines persistence in the presence of structural breaks. Hegwood and Papell (1998, 2002) estimate half-lives of real exchange rates and find faster mean reversion after accounting for structural change. For example, the half-life of PPP deviations for the Lothian and Taylor (1996) sterling-dollar rate falls from 5.78 years to 2.32 years. In a nonlinear framework, Lothian and Taylor (2006), after allowing for breaks in the equilibrium exchange rate for the real sterling-dollar exchange rate over a sample period that spans nearly two centuries, compute that its half-life falls from 6.78 years to 3.19 years. Astorga (2007) uses a new data set for the period $1900 - 2005$ to analyze the behavior of real exchange rates in the six largest economies in Latin America. After allowing for trends and structural breaks, he finds that the half-life of the process of the adjusted series ranges from 0.8 to 2.5 years compared to a 1.6 to 7 years for the series that have been detrended only.

In this paper we merge these two strands of research by analyzing median-unbiased estimation in the presence of structural change. Using Monte Carlo simulations for a variety of sample sizes and structural breaks, we show that inclusion of structural breaks causes a substantial increase in the small sample bias documented in Andrews (1993), and the bias increases with both the size and number of breaks. If a model is correctly specified in the sense that structural breaks are fully accounted for, then one might naively expect using Andrews's tables would yield unbiased estimates. We demonstrate, however, that this is not the case. We suggest an extension of median unbiased estimation to include structural breaks, using dummy variables to capture the effect of known structural breaks in the context of Andrews's original setting.

We apply this extended median-unbiased technique on long horizon real exchange rate data to calculate half-lives of purchasing power parity (PPP) deviations. We estimate the speed of mean reversion of the dollar-sterling and franc-sterling real exchange rates analyzed previously by Lothian and Taylor (1996) by accounting for both small sample bias and structural change.[3] We show that if we account for structural breaks while neglecting small sample bias, as in Hegwood and Papell (1998), we understate the persistence. Using exactly median-unbiased

parameters of autoregressive models fitted to economic time series are subject to structural breaks.

[3]We assume the break dates are correct and take them as given from Hegwood and Papell (1998) and Papell and Prodan (2006).

estimation with structural change, we find that half-lives of PPP deviations increase to 2.72 years from 2.32 years for dollar-sterling and to 2.36 years from 2.03 years for franc-sterling. Alternatively, if we only account for small sample bias while neglecting to account for structural breaks, as in Murray and Papell (2005a), we overstate the persistence, as their estimates of persistence were 6.58 years for dollar-sterling and 2.94 years for franc-sterling.

We also analyze several exchange rate series reported in Taylor (2002) and Lopez, Murray, and Papell (2005). Specifically, we analyze real exchange rates for the Netherlands, the UK, and Portugal taking U.S. dollar as the numeraire currency. After correcting the biases with our extended method, we found that half-lives of PPP deviations are much smaller than would be found from autoregressive models that account for neither small sample bias nor structural change. In addition, confidence intervals for these estimates are narrowed considerably. In conclusion, when we incorporate structural breaks in the context of median-unbiased estimation for time series from both data sets we find that the speed of mean reversion increases considerably.

We extend the analysis to examine bias in the context of a time series in which structural change is not accounted for. If a time series does, in fact, have structural breaks but these breaks are ignored, the direction and severity of bias depends on characteristics of structural breaks as well as the size of the autoregressive coefficient. We show that, depending on these two factors, half-life estimators can be either downward or upward biased.

2 Median-Unbiased Estimation with Structural Change

In this section we propose a technique to correct for small sample bias when structural breaks exist in data. We extend the median-unbiased technique of Andrews (1993). In order to assess the effect of structural breaks on the estimation of persistence of time series data, we run a number of Monte Carlo simulations. We consider four different time series models of $Y_t : t = 1, ..., T$. The benchmark model in Andrews (1993) is AR (1) model:

$$Y_t = \mu + \alpha Y_{t-1} + \epsilon_t \quad where\ \alpha \,\epsilon\, [0, 1] \tag{1}$$

The parameter α is the persistence parameter. The innovations ϵ_t are i.i.d. N(0,1). Y_0 is zero. In model 1, if $\alpha \epsilon (0,1)$, Y_t is a strictly stationary and normal AR(1) process. If α=1, Y_t is a normal random walk with drift (i.e, Y_t contains a unit root). We extend this main model to allow for structural breaks. The basic idea is to add dummy variables to the conventional Dickey-Fuller regressions corresponding to known break dates. For our simulations, we take an AR(1) time series and add arbitrary sizes of structural breaks. We add 1 break or 2 breaks at various sizes for comparison reasons. We will concentrate on breaks of one standard deviation (1σ) and three standard deviations (3σ). The other three models we run are the following:

$$Y_t = \mu + \gamma\, DU + \alpha\, Y_{t-1} + \epsilon_t\,, \tag{2}$$

$$Y_t = \mu + \gamma_1\, DU_1 + \gamma_2\, DU_2 + \alpha\, Y_{t-1} + \epsilon_t\,, \tag{3}$$

$$Y_t = \mu + \gamma_1\, DU_1 + \gamma_2\, DU_2 + \alpha\, Y_{t-1} + \epsilon_t \quad \textit{subject to} \quad \gamma_1 + \gamma_2 = 0\,. \tag{4}$$

In model 2, we add one dummy variable, DU, to account for one structural break. $DU = 1$ if $t > tb$ and 0 otherwise. tb is the date of break and it is assumed to be known.

In model 3 and 4, two structural breaks are created to be able to draw conclusions for multiple structural breaks. We add two dummy variables where $DU_i = 1$ if $t > tb_i$ for i=1, 2 and 0 otherwise. tb_i is the date of break i for i=1, 2 and they are assumed to be known. When there are two offsetting breaks in the time series, we run model 4 in which coefficients of two breaks are restricted to sum up to zero. This model is also called the restricted structural change model.

We measure persistence with the half-life. The half life, a nonlinear function of α, is the number of periods for a unit shock to a time series to decay by 50%. It is computed based on α in each model as $ln(0.5)/ln(\alpha)$.

We show that when we include break dummies to take care of the structural breaks, the least squares estimate of α will still be downward biased. To overcome this problem, we correct for median bias. In detail, we create artificial data with structural breaks and run 100,000 Monte Carlo trials and record the median values of the least squares estimates of the autoregressive coefficients for each true value of α. The rest of the bias correction is straight forward. We use this recorded table to correct our OLS estimator of α. For example, if the OLS estimator for α is 0.8, we find the value of "α" such that the median of the least squares estimate is 0.8.

To estimate the speed of adjustment to shocks we use this corrected median-unbiased estimate of α, α_{MU}, in the half-life formula. Since the half-life calculation is a monotonic transformation of α, the median-unbiased estimator and its coverage probabilities of their confidence intervals are preserved.[4] That is the half-life=$ln(0.5)/ln(\alpha_{MU})$ will be median-unbiased since α_{MU} is a median-unbiased estimator for the least squares estimate of α, which is denoted by α_{LS}.

2.1 Small Sample Bias

Let's assume that we have only one break of different sizes right in the middle of the sample. For a sample size of 120, we run 100,000 simulations for each true value of autoregressive unit root, α.

With or without a break we record the median value of the least squares estimate of the autoregressive coefficient for each break and α. Table 1 presents the simulation results for models 1 and 2. An exact replication of Andrews (1993) appears in column (1). If there is no structural break the bias is always negative (downward). As α gets closer to 1 (for persistent series) the bias increases and it is always downward.

We want to learn if structural changes do matter in terms of the small sample bias. We start with the simplest structural change model in which we add one dummy variable to the conventional AR (1) to account for one break. We run model 2 and report the results in columns $2-4$. The bias is still downward as in the benchmark model (column 1), it increases with

[4]There is also another technique called mean-unbiasedness to correct for the small sample bias of least squares estimators. Mean-unbiasedness means that the expected value of an estimator is equal to the true parameter value. Killian (1998) suggested similar corrections based on mean-unbiased estimates of AR parameters. He estimated the bias corrected AR coefficients and confidence intervals for impulse response functions by applying a bootstrap-after-bootstrap method. However even though both methods will work well under AR ($p > 1$), for estimating the half-life for AR (1) mean-unbiasedness technique will not be unbiased under the Half-Life transformation which is a nonlinear transformation.

Also the median-unbiasedness is often more useful than mean-unbiasedness when the parameter space is bounded or when the distributions of estimators are skewed and/or kurtotic (Andrews, 1993).

the size of the autocorrelation and the size of the breaks. In comparison with our benchmark model, the bias increases considerably. For example, for a true autocorrelation of 0.9, the size of bias increases to 0.0494 from 0.0268 with one break of size one standard deviation, thus the bias is almost doubled. When we have a 3σ break, the bias increases even further to 0.0656.

We need to find out how long a macroeconomic time series will take to come back to its long run pattern. To do this, we estimate the speed of convergence by the half-life. To interpret how this bias affects our results economically, let's give a few examples: say that the true persistence parameter α is equal to 0.9 so that the half-life is equal to $ln(0.5)/ln(0.9) = 6.58$ years. For n=120, the speed of convergence is estimated as 5.11 years using our benchmark model. After adding one break of size one standard deviation, we find that the half-life is 4.28 years with a bias of 2.30 years. It is very common that most time series have breaks of size one standard deviation. A bias in this size can create big errors in forecasting and interpreting models. Moreover when the break is large, the researcher would more likely think about structural change and add a break dummy. In this case, since the size of break is bigger the bias gets bigger too. For instance, for a break of size two standard deviations and three standard deviations the half-life falls to 4.11 and 3.83 years. In other words, the bias in half-life can get twice as big as the bias in our benchmark model.

To examine the effect of sample size, we replicate Table 1 with a sample size of 240. The results are presented in Table 2. We find that the bias decreases as sample size gets bigger. For example for a true autocorrelation of 0.9 and with one break of size one standard deviation, the size of bias decreases to 0.0227 from 0.0494 (n=120). In terms of half-life, we estimate 5.29 years instead of 4.28 years meaning getting closer to the true half-life, 6.58 years. Even with larger break of size three standard deviation the half-life increases to 5.00 years. Intuitively, when we increase sample size the small sample bias gets smaller and we can obtain better results.

We proceed by adding more than one break to AR (1), to discuss the small sample bias in the context of multiple structural breaks. Table 3 contains the results for multiple structural breaks with the same and opposite directions. First we run equation 3 where we account for the breaks in an unrestricted model.[5] We don't impose any restrictions for the coefficients of

[5]Bai and Perron (1998) account for the breaks in this way and there is a lot of research which refers to their

breaks. For the same direction, we add two 1σ breaks and again compare it with the one break case (in Table 1). For a true persistence parameter 0.9, the half-life with two breaks is equal to 3.63 years. In other words, adding one more break of the same size decreases the half-life to 3.63 years from 4.28 years where we had one break of size 1σ. The bias is almost twice as big as the bias in our benchmark model. The main point taken here is that the bias increases with the number of breaks.

A key question that arises in the presence of multiple structural breaks is what happens if the two breaks reverse themselves. If two breaks offset each other, the effects of the shocks will dissipate and the series will revert to its long-run mean level. If two breaks are in the same direction, shocks will never wear off. Perron and Qu (2006) show that in small samples, all AR parameters are more efficiently estimated using the restrictions on structural change models.[6] So, we can say that imposing valid restrictions on break dummies increases efficiency for the persistence parameter. Hence, we restrict our model in a way that breaks cancel each other. We add the same number of breaks in the opposite direction as in equation 4. The last column in Table 3 reports the results for two breaks of 1 standard deviation and -1 standard deviation. The corresponding half-life is calculated as 4.35 years with the bias of 2.23 years for a true persistence parameter 0.9. The bias is smaller when it is compared to the unrestricted model where we run equation 3, but it is again very big compared to our benchmark model.

2.2 Structural Breaks

Long horizon time series data has a higher chance of including structural changes. Ignoring these structural changes in the estimation of persistence produces an upward bias.[7] We demonstrate this upward bias using a few examples. We generate artificial data which includes one structural break of 1σ. We run both models 1 and 2 and record the estimated mean values of the least squares estimates of autoregressive coefficient, α. We calculate the bias subtracting α which is estimated by including structural break from the one which is estimated by neglecting

model.

[6]Perron and Qu (2006) also show that in large samples, there is no efficiency gain from imposing valid restrictions as far as the estimates of the break dates are concerned. However efficiency gains occur for the other parameters of the model.

[7]Also Seong et.al (2006) mention the upward bias caused by structural breaks in the trend.

this break. So we compare naive OLS estimation with the estimation where we account for the structural break.

In Table 4, we show that the bias is always positive in column 3. For the first sample, when we don't account for the structural break, the persistence parameter increases to 0.7872 from 0.7500. In other words, the half-life increases to 2.90 years from 2.41 years, with a bias of 0.49 years, thus overstating the persistence. We also conducted the same simulation for a larger sample (n=240) and find that the upward bias decreases with the sample size.[8]

2.3 Misspecified Structural Breaks

In this subsection, we are going to consider the consequences of failing to account for structural breaks. We proceed by discussing other potential biases which might exist due to the misspecification of models. In the presence of structural breaks, if we estimate persistence in the long time series without accounting for the structural breaks, the model will be misspecified. In this case, we run model 1 and calculate the bias by subtracting the true value of α from the estimated median value of OLS coefficient after adding breaks to the artificial data. Table 5 shows the results for one break at different sizes. The main result is that the median bias is not always negative for each true value of α. The bias $(\alpha_1^{'} - \alpha)$ becomes positive after a certain value of α which depends on size of breaks. i.e, neglecting the structural break causes the direction of small sample bias to flip after a certain cut off point, say φ. For example, for n=120 and one break of 3σ, the flipping point is 0.9, meaning the bias is positive for all values of α from 0 to 0.9. Let's assume that the true value of α is 0.5. The autoregressive coefficient would be estimated as 0.8023 because of the positive bias and if we would correct our estimation for median bias naively using Andrews' (1993) tables, it would be increased further to 0.825, making the bias worse. In terms of half-life, instead of 1 year, 3.60 years would be estimated. Therefore, an analysis that doesn't incorporate structural breaks might yield over-estimated half-lives.

When we use a longer data set, there are two basic biases, one downward and one upward, caused by small sampling and neglecting structural breaks, respectively. Until a cut off point, the small sample bias is greater than the bias due to structural breaks, after the cut off point

[8]The results are available upon request.

it is smaller. That's why the combination of these two biases seems to be going to both directions. However there is a range that we can call the "*lucky zone*". For true $\alpha \epsilon [\varphi, 1]$, the bias is smaller than Andrews' (1993) case. So, if the true autoregressive root is in this range for our model and we compute the naive "OLS" model where we do not think about either small sample bias or structural breaks, we are lucky enough to have the smallest bias! Nonetheless, this range is very short and it gets shorter when data has bigger structural breaks. In Table 5, we show that with bigger structural breaks, the upward bias and the cut off point (φ) increase too.

To be able to understand visually how these biases behave, we proceed to draw a decomposition of the opposing bias factors. Figure 1 shows these two biases for one break of 1σ and 3σ. The true value of α is plotted on the horizontal axis and the estimated $\hat{\alpha}$'s for different models are plotted on the vertical axis. The most striking result in these graphs is the increase in the small sample bias when we account for structural breaks as we increase the size of break. The upward bias increases and the range, $[\varphi, 1]$ gets shorter as well.

Table 6 shows where the exact flipping point occurs for a variety of breaks and sample sizes. We demonstrate that if we increase the sample size, the cut off point and the upward bias increases as well (because the model is wrong).[9] [10] To illustrate, for 1 break of size 1 standard deviation, φ rises to 0.83 (n=240) from 0.76 (n=120). [11]

We also checked to see if φ changes with the direction and the number of breaks. In Table 6, part A and part B tell us that the direction of breaks doesn't matter in terms of φ but the number of breaks does.[12] Part C shows that when two breaks are in the same direction, φ increases as much as if they make the impact of one break almost with a total size of individual breaks.

We summarize the results as follows. First, even though median-unbiased estimation has received more attention to correct for small sample bias, it is worse when there is structural change. While this bias increases as we increase the size and the number of breaks, it gets

[9]This particular model is misspecified, as we increase sample size the bias doesn't wear off.

[10]We didn't show the increase in the upward bias when we increase the sample size in Table 6. They are available from the author.

[11]We also set the timing of break at one-third of the sample or two-thirds of the sample to differentiate from the original setting and see if this makes any difference in terms of cut off point. The results didn't change, thus timing of the break point doesn't affect the cut off points.

[12]The same simulations are conducted choosing a break with the opposite direction, the results didn't change.

smaller as we increase the sample size. If a model is correctly specified, allowing for structural changes, then one might naively expect using Andrew's tables would yield unbiased estimates. We show, however, that this is not the case.

Second, neglecting structural breaks produces an upward bias in the estimation of persistence, decreasing as we increase the sample size. However, when there is a structural break and we don't account for it, the small sample bias, can be either downward or upward and it gets worse as we increase the sample size. The direction and severity of bias depends greatly on the characteristics of the structural breaks as well as the size of autoregressive coefficient. Third, there are some times that we don't have to be smart enough but get lucky. However, the problem is that there is no possible way to know if the true autocorrelation is in this lucky range. Therefore, we suggest an extension to the median-unbiased estimation to correct for the small sample bias in the presence of structural breaks.

3 Concrete Example: Application to PPP

In recent macroeconomic literature a popular question asked is how long a macroeconomic time series will take to come back to its long run mean. One of the most extensively studied time series in this literature is the real exchange rate. In this section, we estimate the speed of mean reversion of real exchange rates that are analyzed previously by Lothian and Taylor (1996) and Taylor (2002) by accounting for structural change.

To estimate the speed of reversion back to PPP, we simply run an AR model on real exchange rates. It is well documented in the literature that this estimator suffers from downward small sample bias especially when the real exchange rate is highly persistent.

We use two separate data sets to apply our methodology and make comparisons. The first data set is from Lothian and Taylor (1996). It consists of annual observations of nominal exchange rates for the UK sterling in terms of US dollars $(1791-1990)$ and French francs $(1802-1990)$, and wholesale price indices for US, UK, and France.[13]

The second data set is originally from Taylor (2002) and updated by Lopez, Murray and Papell (2005). It includes annual exchange rates measured as domestic currency units per

[13]For more detail see Lothian and Taylor (1996).

U.S. dollar, and price indices measured as consumer price deflators (or when they are not available, GDP deflators) starting between 1870 and 1892 and ending in 1998. It consists of 16 industrialized countries. Papell and Prodan (2006) also use this data set and classify countries as supporting PPP or TPPP (Trend Purchasing Power Parity). According to their results, we pick countries where there is evidence of structural breaks (where the unit root is rejected) and no trend since we are interested in the convergence to a mean and had no trend in the simulation models. Only 3 countries; the Netherlands, Portugal, and the UK are chosen since they were the ones which are appropriate to analyze.

The real exchange rate with the U.S. dollar (or UK sterling for Lothian and Taylor (1996) data set) as the numeraire currency is calculated as,

$$q_t = e_t + p_t^* - p_t \,, \tag{5}$$

where q_t is the logarithm of the real dollar (or sterling) exchange rate, e_t is the logarithm of the nominal dollar (or sterling) exchange rate, p_t and p_t^* are the logarithms of the domestic CPI and U.S. CPI (or UK sterling CPI) respectively.

3.1 Not Considering Structural Breaks

In this subsection, for comparison reasons, we estimate the speed of mean reversion of real exchange rates using the Median-Unbiased (MU) estimation of Andrews (1993). When the error terms are serially uncorrelated, we run Dickey-Fuller regressions:

$$q_t = c + \alpha\, q_{t-1} + u_t \tag{6}$$

Since the least squares (LS) estimates of the AR parameters are biased downward, we need to correct $\hat{\alpha}_{LS}$ using MU estimation. Then we calculate the half-life as $ln(0.5)/ln(\hat{\alpha}_{MU})$ where $\hat{\alpha}_{MU}$ is the median-unbiased estimator of $\hat{\alpha}_{LS}$. Exactly median-unbiased half-lives are presented in Tables 7 and 8 respectively. When we take into account the serial correlation, we run Augmented Dickey-Fuller (ADF) regressions:

$$q_t = c + \alpha\, q_{t-1} + \sum_{j=1}^{k} \psi_j\, \Delta q_{t-j} + u_t \tag{7}$$

where k is a truncation lag parameter. The lag length is chosen using one of the most recent methods, the general to specific (GS) criterion studied by Hall (1994) and Ng and Perron (1995). Andrews and Chen (1994) extend the MU estimator to the AR(p) case.[14] In AR(p) models with $p > 1$, the median-unbiased estimator of α is no longer exact, but approximate. This is because, the median-unbiased estimator of α depends on the true values of the ψ_j in Equation (7), which are unknown. Andrews and Chen (1994) propose an iterative procedure to obtain approximately median-unbiased estimates of α. Using their technique, approximately median-unbiased half-lives are computed for both data sets. Tables 9 and 10 contain the estimation results for Lothian and Taylor (1996) and Taylor (2002) respectively.

3.2 Considering Structural Breaks

When we consider structural breaks, we estimate the speed of mean reversion of real exchange rates using our extended Median-Unbiased estimation. For both data series, we use the break dates which are estimated by Hegwood and Papell (1998) (for Lothian and Taylor (1996)), and Papell and Prodan (2006) (for Taylor (2002)).[15] We assume that these break dates are correct. When there is no serial correlation we run the equation below:

$$q_t = c + \sum_{i=1}^{nb} \gamma_i\, DU_i + \alpha\, q_{t-1} + u_t \tag{8}$$

Table 7, presents the results for the Lothian and Taylor (1996) which is denominated by sterling real exchange rate. For the U.S., there are two negative breaks at 1863 and 1929 of -0.48 σ and -0.57 σ respectively.[16] For France, there are two structural breaks which are at

[14]See Andrews and Chen (1994) for more explanations about the details of the methodology.

[15]In these papers, 10% trimming is used to avoid finding spurious breaks at the beginning and at the end of samples.

[16]1863 break occurred during the American Civil War and 1929 break is due to Great Depression. Even though the size of breaks are considerably small, they might give an impact of one bigger negative break since they have same direction.

1942 and 1980 of -0.51 σ and 1.01 σ respectively.

Using exactly median-unbiased estimation with structural changes, we find that the half-lives of PPP deviations are 2.72 years for dollar-sterling and 2.36 years for franc-sterling real exchange rates. If we allow for only structural breaks as in Hegwood and Papell (1998), we find the half-lives decrease to 2.32 and 2.03 years for dollar-sterling and franc-sterling understating the persistence. Alternatively, if we only account for small sample bias as in Murray and Papell (2005a), we overstate the persistence as their estimates of half-lives were 6.58 for dollar-sterling and 2.94 years for franc-sterling. We also calculate 95% confidence intervals for the median-unbiased half-lives, since the point estimate of half-life is an incomplete measure of the information. We find a lower bound of 1.76 years and an upper bound of 5.42 years for dollar-sterling. A surprising result is that this interval doesn't include the point estimate, 6.58 years, where we ignore structural breaks. For franc-sterling real exchange rate, 95% confidence intervals for the median-unbiased half-life is [1.53, 4.43]. Table 8 contains the results for the Taylor (2002) which is denominated by dollar real exchange rate. Break dates are taken from Papell and Prodan (2006).[17] The Netherlands has one break in 1970 of 0.82 σ. After correcting the biases with this extended method, the speed of mean reversion of the Netherlands real exchange rate is estimated at 5.19 years. The Portuguese real exchange rate has two breaks in 1916 and 1986 of -0.79 σ and 0.81. For the UK, there are two breaks in 1944 and 1972 of -0.99 σ and 0.80 σ. For both Portugal and the UK, exactly median-unbiased speed of mean reversions are almost doubled when we considered breaks. For all three countries the confidence intervals are narrowed considerably as well.

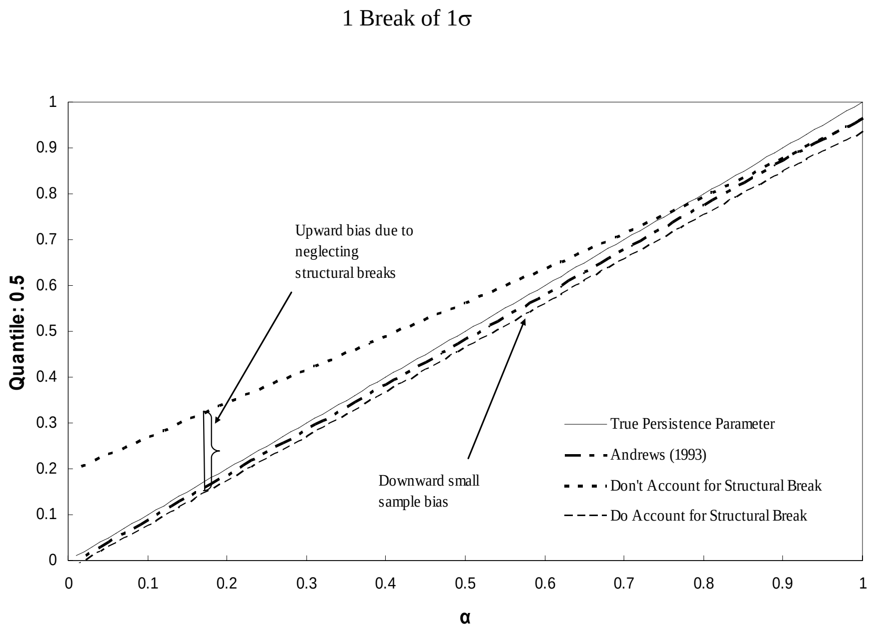
When we consider the serial correlation (AR ($p > 1$)), we run Augmented Dickey-Fuller (ADF) regressions with structural breaks dummies:

$$q_t = c + \sum_{i=1}^{nb} \gamma_i \, DU_i + \alpha \, q_{t-1} + \sum_{j=1}^{k} \psi_j \, \Delta q_{t-j} + u_t \qquad (9)$$

We choose lag length via the general to specific (GS) method. For AR($p > 1$), we extend the approximately median-unbiased estimator of Andrews and Chen (1994). Since shocks do

[17] Papell and Prodan (2006) calculates break dates including the serial correlation. We assume the date of breaks are same with or without serial correlation.

Figure 1: Biases of estimators with artificial data



not decay at a constant rate, we estimate the half-life from the impulse response functions. To estimate impulse response functions we use both α and ψ_j from ADF model.

Andrews and Chen (1994) provide a computationally intensive and iterative method for obtaining approximately median-unbiased estimators of the parameters of the Augmented Dickey-Fuller model $(\alpha, \psi_1, ..., \psi_k)$. Since $\psi_1, ..., \psi_k$ are unknown, they propose a simple iterative procedure that yields an approximately median-unbiased estimator. We extend their procedure in a way to constitute the structural breaks too. First, we compute the least squares estimates of $\alpha, \psi_1, ..., \psi_k$ using ADF regression (equation 9). Second, treating these $\psi_{1LS1}, ..., \psi_{kLS1}$ as though they were true values, we find the value of α such that the least squares estimator has α_{LS} as its median and call this $\alpha_{1,\ AMU}$. Here, when we create the tables for median-unbiased estimator, we create an artificial data whose properties is same with the original data in terms of the structural breaks and the sample size. Then we run 100,000 Monte Carlo trials and record the median values of the least squares estimates of autoregressive coefficient for each true value of α. Third, treating $\alpha_{1,\ AMU}$ as though it was true value of α, we compute a second round of least squares estimators $\psi_{1LS2}, ..., \psi_{kLS2}$ (regressing $q_t - \alpha_{1,\ AMU} * q_{t-1}$ on $DU_1, ..., DU_{nb}$, $\Delta q_{t-1}, .., \Delta q_{t-k}$, and constant). Next, we treat the new $\psi_{1LS2}, ..., \psi_{kLS2}$ as though they were true values, and compute $\alpha_{2,\ AMU}$. We can continue to this procedure either for fixed number of iterations or until convergence, and call this α_{AMU}. In this paper, α_{AMU} is obtained when convergence occurs. The half-life estimation is similar to the AR (1) case. The only difference is that since α_{AMU} will be approximately median-unbiased, half-life calculations will also be approximately median-unbiased.

Table 9 shows extended approximately median-unbiased half-lives for Lothian and Taylor (1996) data. We find that for dollar-sterling real exchange rate the AR (1) specification needs to be augmented by eight lags. The half-life drops to 1.41 years from 4.53 years for dollar-sterling rate. The 95% confidence intervals for the half-life is $[0.97, 1.95]$ years. In other words we are 95% certain that shocks to the dollar-sterling rate decay at a rate between 30% and 51% per year. For France, the lag length is estimated at zero, thus the half-life estimations are same as before.

The extended approximately median-unbiased half-lives for Taylor (2002) data is also calculated in Table 10. For the Netherlands and Portugal, the speed of mean reversion increases more than 50%. Their confidence intervals are narrowed in such a way that they don't cover the point estimates which we calculated without considering structural breaks. Half-life for pound-dollar real exchange rate decreases to 2.80 years from 3.92 years. The 95% confidence intervals for the half-life is $[1.63, 4.15]$ years. So, when we incorporate the structural breaks, the speed of mean reversion increased considerably for this time series data as well. The main reason for this might be that, for these particular examples, structural breaks play an important role when estimating the persistence.

As a result, when there are structural breaks and we don't account for them, the half-life is overestimated, the speed of mean reversion decreases, and the time series becomes more persistent. After considering both serial correlation and structural breaks, median-unbiased half-lives are estimated as even lower than 3 years (except for the Dutch real exchange rate).

4 Conclusions

How can one measure the persistence in long macroeconomic time series involving structural breaks? Previous work can be categorized into two main research areas. The first line of research deals with the small sample bias (downward bias) associated with the parameters of autoregressive models and ended up overestimating the persistence after correcting the bias with the median-unbiased estimation of Andrews (1993). The second line of research examines the persistence in the presence of structural breaks and ended up underestimating the persistence because neglecting structural breaks produces an upward bias in the estimation of persistence. In this paper we unite these two strands of research by analyzing median-unbiased estimation in the presence of structural breaks.

By conducting Monte Carlo simulations for a variety of sample sizes and structural breaks, we draw three main conclusions. First, inclusion of structural breaks causes the small sample bias to increase substantially compared to our benchmark model (Andrews (1993)), and while the bias increases with both the size and number of breaks, it gets smaller with the sample size. Second, the conventional median-unbiased technique is more biased when there are structural breaks and these breaks are ignored. The small sample bias can be either downward or upward and it gets worse as we increase the sample size. Finally, even though in some cases OLS might be the best method to calculate the estimators of AR models, the number of these cases decreases as we increase both the number and size of structural breaks, and sample size. The extended-median-unbiased technique allows us to consider both structural breaks and correction of small sample bias.

As an illustration of the potential usefulness of the technique in applied work we analyze real exchange rates from two different data sets in the context of Purchasing Power Parity. When we use extended exactly median unbiased estimation, the half-life decreases to 2.72 years for dollar-sterling and to 2.36 years for franc-sterling (for Lothian and Taylor (1996) data set). For Taylor (2002)'s data set, the half-lives drop to 5.19, 3.84, and 2.79 years for the Netherlands, Portuguese, and the UK real exchange rates respectively. Corresponding confidence intervals are narrowed considerably as well.

We find that failure to account for structural changes causes us to overestimate the half-lives of PPP deviations. After allowing for both serial correlation and structural breaks, median-unbiased half-lives decrease significantly. We conclude that controlling for structural changes and small sample bias for these data sets result in considerably shorter half-lives.

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Table 1: **Simulation of Models: 1 Break of Sizes 1σ, 2σ, and 3σ with N=120**

	No structural break	Structural breaks		
	Andrews (1993)	1σ	2σ	3σ
α /*Quantile*	0.5	0.5	0.5	0.5
1	0.9639	0.9382	0.9359	0.9312
0.995	0.9581	0.9320	0.9293	0.9248
0.99	0.9537	0.9274	0.9241	0.9190
0.985	0.9491	0.9229	0.9196	0.9141
0.98	0.9452	0.9183	0.9153	0.9091
0.975	0.9414	0.9146	0.9112	0.9048
0.95	0.9197	0.8943	0.8896	0.8821
0.925	0.8970	0.8730	0.8679	0.8589
0.9	0.8732	0.8506	0.8447	0.8344
0.875	0.8496	0.8276	0.8209	0.8103
0.85	0.8253	0.8037	0.7970	0.7854
0.825	0.8008	0.7803	0.7723	0.7600
0.8	0.7767	0.7561	0.7481	0.7351
0.775	0.7522	0.7326	0.7245	0.7107
0.75	0.7281	0.7082	0.6999	0.6854
0.725	0.7037	0.6841	0.6751	0.6603
0.7	0.6791	0.6600	0.6513	0.6362
0.6	0.5812	0.5635	0.5536	0.5387
0.5	0.4826	0.4661	0.4574	0.4429
0.4	0.3843	0.3698	0.3623	0.3494
0.3	0.2860	0.2732	0.2667	0.2568
0.2	0.1886	0.1765	0.1729	0.1660
0.1	0.0903	0.0797	0.0768	0.0748

Notes: For this table we run the following models:

$$Y_t = \mu + \alpha Y_{t-1} + \epsilon_t \,, \quad (1)$$
$$Y_t = \mu + \gamma \, DU + \alpha Y_{t-1} + \epsilon_t \,, \quad (2)$$

where $DU = 1$ if $t > tb$ and 0 otherwise. tb is the date of break.The sample size is 120 and the number of simulations is 100,000. Y_t has 1 break of sizes 1σ, 2σ, and 3σ in the middle of the sample.

Table 2: **Simulation of Models: 1 Break of Sizes 1σ, 2σ, and 3σ with N=240**

	No structural break	Structural breaks		
	Andrews (1993)	1σ	2σ	3σ
α /*Quantile*	0.5	0.5	0.5	0.5
1	0.9819	0.9692	0.9687	0.9674
0.995	0.9764	0.9636	0.9627	0.9612
0.99	0.9725	0.9593	0.9583	0.9569
0.985	0.9684	0.9552	0.9542	0.9524
0.98	0.9641	0.9511	0.9500	0.9481
0.975	0.9597	0.9469	0.9458	0.9437
0.95	0.9363	0.9249	0.9232	0.9203
0.925	0.9119	0.9014	0.8991	0.8956
0.9	0.8873	0.8772	0.8749	0.8704
0.875	0.8630	0.8532	0.8501	0.8454
0.85	0.8380	0.8286	0.8252	0.8197
0.825	0.8136	0.8040	0.8004	0.7947
0.8	0.7888	0.7793	0.7754	0.7691
0.775	0.7638	0.7545	0.7505	0.7437
0.75	0.7391	0.7301	0.7260	0.7187
0.725	0.7146	0.7053	0.7008	0.6937
0.7	0.6894	0.6809	0.6762	0.6684
0.6	0.5905	0.5821	0.5775	0.5694
0.5	0.4915	0.4839	0.4791	0.4714
0.4	0.3924	0.3852	0.3810	0.3746
0.3	0.2932	0.2872	0.2832	0.2781
0.2	0.1946	0.1881	0.1860	0.1831
0.1	0.0948	0.0900	0.0887	0.0868

Notes: For this table we run the following models:

$$Y_t = \mu + \alpha Y_{t-1} + \epsilon_t \,, \quad (1)$$
$$Y_t = \mu + \gamma DU + \alpha Y_{t-1} + \epsilon_t \,, \quad (2)$$

where $DU = 1$ if $t > tb$ and 0 otherwise. tb is the date of break. The sample size is 240 and the number of simulations is 100,000. Y_t has 1 break of sizes 1σ, 2σ, and 3σ in the middle of the sample.

Table 3: **Simulation of Models: 2 Breaks of Sizes (1σ, 1σ) and (1σ, -1σ)**

	No structural break	Structural breaks	
	Andrews (1993)	Unrestricted $(1\sigma, 1\sigma)$	Restricted $(1\sigma, -1\sigma)$
α */Quantile*	0.5	0.5	0.5
1	0.9639	0.9130	0.9558
0.995	0.9581	0.9068	0.9470
0.99	0.9537	0.9017	0.9406
0.985	0.9491	0.8973	0.9352
0.98	0.9452	0.8929	0.9298
0.975	0.9414	0.8885	0.9252
0.95	0.9197	0.8683	0.9009
0.925	0.8970	0.8476	0.8767
0.9	0.8732	0.8262	0.8527
0.875	0.8496	0.8038	0.8287
0.85	0.8253	0.7812	0.8044
0.825	0.8008	0.7584	0.7800
0.8	0.7767	0.7351	0.7551
0.775	0.7522	0.7121	0.7312
0.75	0.7281	0.6883	0.7064
0.725	0.7037	0.6649	0.6826
0.7	0.6791	0.6407	0.6579
0.6	0.5812	0.5454	0.5610
0.5	0.4826	0.4499	0.4639
0.4	0.3843	0.3549	0.3670
0.3	0.2860	0.2600	0.2715
0.2	0.1886	0.1645	0.1760
0.1	0.0903	0.0698	0.0798

Notes: For this table we run the following models:

$$Y_t = \mu + \gamma_1 \, DU_1 + \gamma_2 \, DU_2 + \alpha \, Y_{t-1} + \epsilon_t \, , \quad (3)$$

$$Y_t = \mu + \gamma_1 \, DU_1 + \gamma_2 \, DU_2 + \alpha \, Y_{t-1} + \epsilon_t \, , \quad \textit{subject to} \quad \gamma_1 + \gamma_2 = 0 \, , \quad (4)$$

where $DU_i = 1$ if $t > tb_i$ for i=1, 2 and 0 otherwise. tb is the date of break. The sample size is 120 and the number of simulations is 100,000. Y_t has 2 breaks of sizes (1σ, 1σ) or (1σ, -1σ).

Table 4: **Upward Bias: 1 Break of Size** 1σ

	α		
Sample	(1)	(2)	Bias
1	0.7872	0.7500	0.0372
2	0.6320	0.5588	0.0732
3	0.5581	0.4627	0.0954
4	0.4136	0.2709	0.1427

Notes: For this table we run the following models:

$$Y_t = \mu + \alpha\, Y_{t-1} + \epsilon_t\,, \quad (1)$$
$$Y_t = \mu + \gamma\, DU + \alpha\, Y_{t-1} + \epsilon_t\,, \quad (2)$$

where $DU = 1$ if $t > tb$ and 0 otherwise. tb is the date of break. The sample size is 120 and the number of simulations is 100,000. Y_t has 1 break of 1σ, in the middle of the sample. We record the mean value of the least squares estimates of autoregressive coefficients for each sample. Neglecting structural break will produce an upward bias which is presented in Column 3.

Table 5: **Simulation of Models: 1 Break of Sizes** 1σ**,** 2σ**, and** 3σ

	Bias		
	1σ	2σ	3σ
α	$\alpha^{'} - \alpha$	$\alpha^{'} - \alpha$	$\alpha^{'} - \alpha$
1	−0.0362	−0.0358	−0.0353
0.995	−0.0367	−0.0361	−0.0351
0.99	−0.0364	−0.0353	−0.0339
0.985	−0.0353	−0.0340	−0.0322
0.98	−0.0343	−0.0325	−0.0300
0.975	−0.0331	−0.0311	−0.0278
0.95	−0.0284	−0.0239	−0.0168
0.925	−0.0248	−0.0161	−0.0043
0.9	−0.0216	−0.0080	0.0097
0.875	−0.0182	0.0015	0.0245
0.85	−0.0144	0.0116	0.0404
0.825	−0.0101	0.0225	0.0565
0.8	−0.0061	0.0339	0.0739
0.775	−0.0012	0.0463	0.0915
0.75	0.0036	0.0589	0.1097
0.25	0.0084	0.0720	0.1283
0.7	0.0142	0.0853	0.1468
0.6	0.0377	0.1425	0.2236
0.5	0.0627	0.2017	0.3023
0.4	0.0901	0.2623	0.3809
0.3	0.1174	0.3235	0.4593
0.2	0.1439	0.3828	0.5364
0.1	0.1687	0.4392	0.6113

Notes: For this table we run the following model 1:

$$Y_t = \mu + \alpha Y_{t-1} + \epsilon_t \,,$$

The sample size is 120 and the number of simulations is 100,000. Y_t has 1 break of sizes 1σ, 2σ, and 3σ in the middle of the sample. When there is a structural break and we run model 1, we call the median values of the least squares estimates of autoregressive coefficient for each true value of α as $\alpha^{'}$. Columns 2-4 contain the related biases when we do not account for the structural changes.

Table 6: **The Flipping Point and the Direction of Break**

A) One break

	φ	
Break size	N=120	N=240
0.25	0.20	0.41
0.5	0.55	0.68
1	0.76	0.83
1.5	0.84	0.88
2	0.87	0.91
2.5	0.90	0.93
3	0.91	0.94
3.5	0.92	0.95
4	0.93	0.95

B) Two breaks with opposite signs

$0.25, -0.25$	0.14	0.36
$0.5, -0.5$	0.53	0.66
$1, -1$	0.74	0.82
$1.5, -1.5$	0.82	0.87
$2, -2$	0.86	0.90
$2.5, -2.5$	0.88	0.92
$3, -3$	0.90	0.93
$3.5, -3.5$	0.9	0.94
$4, -4$	0.9	0.94

C) Two breaks with same signs

$0.25, 0.25$	0.46	0.62
$0.5, 0.5$	0.72	0.80
$1, 1$	0.85	0.89
$1.5, 1.5$	0.90	0.93
$2, 2$	0.92	0.94
$2.5, 2.5$	0.94	0.95
$3, 3$	0.95	0.96
$3.5, 3.5$	0.95	0.96
$4, 4$	0.96	0.97

Notes: The number of simulations is 100,000. N is the sample size and φ is the cut off point. Decreasing from $\alpha = \varphi$ to $\alpha = 0$, the bias is upward and increasing (α is the true autoregressive unit root). The same simulations are conducted choosing a break with the opposite direction, the results didn't change.

Table 7: **Exactly Median-Unbiased Half-lives in Dickey-Fuller Regressions: Lothian and Taylor (1996), sterling real exchange rates, 200 years**

Not considering structural breaks:

	n	α_{LS}	HL_{LS}	α_{MU}	95 % CI	HL_{MU}	95 % CI
France-UK	189	0.78	2.75	0.79	[0.7, 0.89]	2.94	[1.94, 5.95]
U.S.-UK	200	0.89	5.77	0.9	[0.835, 0.975]	6.58	[3.84, 27.38]

Considering structural breaks:

	n	α_{LS}	HL_{LS}	α_{MU}	95 % CI	HL_{MU}	95 % CI
France-UK	189	0.71	2.03	0.745	[0.635, 0.855]	2.36	[1.53, 4.43]
U.S.-UK	200	0.74	2.32	0.775	[0.675, 0.88]	2.72	[1.76, 5.42]

Notes: For this table we run the following models:

$$q_t = c + \alpha\, q_{t-1} + u_t\,, \quad (I)$$

$$q_t = c + \sum_{i=1}^{nb} \gamma_i\; DU_i + \alpha\, q_{t-1} + u_t\,, \quad (II)$$

where $DU_i = 1$ if $t > tb_i$ for i=1, 2 and 0 otherwise. tb is the date of break. The break dates are taken as given from Hegwood and Papell (1998). When we don't consider the structural breaks we use model (I) and if we consider the structural breaks we use model (II) to estimate the half-lives. Half-life which is estimated using least squares only is denoted by HL_{LS}. Median-unbiased half-life is denoted by HL_{MU}.

Table 8: **Exactly Median-Unbiased Half-lives in Dickey-Fuller Regressions: Taylor (2002), dollar real exchange rates, 129 years**

Not considering structural breaks:

	n	α_{LS}	HL_{LS}	α_{MU}	$95\,\%\,CI$	HL_{MU}	$95\,\%\,CI$
Netherlands-U.S.	129	0.93	8.92	0.95	[0.875, 1]	13.51	$[5.19, \infty]$
Portugal-U.S.	109	0.88	5.37	0.91	[0.81, 1]	7.35	$[3.29, \infty]$
UK-U.S.	129	0.85	4.23	0.875	[0.775, 0.98]	5.19	[2.72, 34.31]

Considering structural breaks:

	n	α_{LS}	HL_{LS}	α_{MU}	$95\,\%\,CI$	HL_{MU}	$95\,\%\,CI$
Netherlands-U.S.	129	0.84	3.85	0.875	[0.77, 0.995]	5.19	[2.65, 138.28]
Portugal-U.S.	109	0.77	2.63	0.835	[0.7, 0.99]	3.84	[1.94, 68.97]
UK-U.S.	129	0.72	2.14	0.78	[0.65, 0.925]	2.79	[1.61, 8.89]

Notes: For this table we run the following models:

$$q_t = c + \alpha\, q_{t-1} + u_t\,, \quad (I)$$

$$q_t = c + \sum_{i=1}^{nb} \gamma_i\; DU_i + \alpha\, q_{t-1} + u_t\,, \quad (II)$$

where $DU_i = 1$ if $t > tb_i$ for i=1, 2 and 0 otherwise. tb is the date of break. The break dates are taken as given from Papell and Prodan (2006). When we don't consider the structural breaks we use model (I) and if we consider the structural breaks we use model (II) to estimate the half-lives. Half-life which is estimated using least squares only is denoted by HL_{LS}. Median-unbiased half-life is denoted by HL_{MU}.

Table 9: **Approximately Median-Unbiased Half-lives in Augmented Dickey-Fuller Regressions: Lothian and Taylor (1996), sterling real exchange rates, 200 years**

Not considering structural breaks:

	n	k	α_{LS}	HL_{LS}	α_{MU}	$95\,\%CI$	$HL_{IRF,\,MU}$	$95\,\%CI$
France-UK	189	0	0.78	2.75	0.79	[0.7, 0.89]	2.94	[1.94, 5.95]
U.S.-UK	200	5	0.91	7.55	0.94	[0.855, 1]	4.53	[3.02, ∞]

Considering structural breaks:

	n	k	α_{LS}	HL_{LS}	α_{MU}	$95\,\%CI$	$HL_{IRF,\,MU}$	$95\,\%CI$
France-UK	189	0	0.71	2.03	0.745	[0.635, 0.855]	2.36	[1.53, 4.43]
U.S.-UK	200	8	0.54	1.12	0.545	[0.49, 0.605]	1.41	[0.97, 1.95]

Notes: When we account for structural breaks, the lag length, k is estimated after the break dates are taken as given from Hegwood and Papell (1998). K is chosen via the general to specific (GS) criterion studied by Hall (1994) and Ng and Perron (1995). The upper bound for k_{max} is chosen as 8. Equation 9 is run with $k = k_{max}$ and if the last lagged difference is significant then k is set equal to k_{max}, otherwise k is reduced by one and the same process is repeated until the last lagged difference is significant (using a critical value of 1.645). For this table we run the following models:

$$q_t = c + \alpha\, q_{t-1} + \sum_{j=1}^{k} \psi_j\, \Delta q_{t-j} + u_t\,, \quad (I)$$

$$q_t = c + \sum_{i=1}^{nb} \gamma_i\, DU_i + \alpha\, q_{t-1} + \sum_{j=1}^{k} \psi_j\, \Delta q_{t-j} + u_t\,, \quad (II)$$

where $DU_i = 1$ if $t > tb_i$ for i=1, 2 and 0 otherwise. tb is the date of break. When we don't consider the structural breaks we use model (I) and if we consider the structural breaks we use model (II) to estimate the half-lives. Half-life which is estimated using least squares only is denoted by HL_{LS}. Median-unbiased half-life, estimated from impulse functions, is denoted by $HL_{IRF,\,MU}$.

Table 10: **Approximately Median-Unbiased Half-lives in Augmented Dickey-Fuller Regressions: Taylor (2002), dollar real exchange rates, 129 years**

Not considering structural breaks:

	n	k	α_{LS}	HL_{LS}	α_{MU}	95 % CI	$HL_{IRF,\,MU}$	95 % CI
Netherlands-U.S.	129	1	0.90	6.9	0.925	[0.85, 1]	10.33	[4.46, 36.67]
Portugal-U.S.	109	5	0.88	5.55	0.91	[0.81, 1]	8.45	$[2.81, \infty]$
UK-U.S.	129	4	0.85	4.34	0.885	[0.755, 1]	3.92	[3.13, 12.04]

Considering structural breaks:

	n	k	α_{LS}	HL_{LS}	α_{MU}	95 % CI	$HL_{IRF,\,MU}$	95 % CI
Netherlands-U.S.	129	1	0.80	3.02	0.815	[0.735, 0.89]	4.183	[2.89, 6.16]
Portugal-U.S.	109	1	0.74	2.26	0.74	[0.7, 0.78]	2.82	[2.31, 3.44]
UK-U.S.	129	3	0.65	1.61	0.695	[0.56, 0.82]	2.80	[1.63, 4.15]

Notes: For this table we run the following models:

$$q_t = c + \alpha\, q_{t-1} + \sum_{j=1}^{k} \psi_j\, \Delta q_{t-j} + u_t \quad (I)$$

$$q_t = c + \sum_{i=1}^{nb} \gamma_i\, DU_i + \alpha\, q_{t-1} + \sum_{j=1}^{k} \psi_j\, \Delta q_{t-j} + u_t \quad (II)$$

where $DU_i = 1$ if $t > tb_i$ for i=1, 2 and 0 otherwise. tb is the date of break. The truncation lag parameters are taken from Papell and Prodan (2006) along with the break dates. When we don't consider the structural breaks we use model (I) and if we consider the structural breaks we use model (II) to estimate the half-lives. Half-life which is estimated using least squares only is denoted by HL_{LS}. Median-unbiased half-life, estimated from impulse functions, is denoted by $HL_{IRF,\,MU}$.

Figure 2: Biases of estimators with artificial data