

Matching and Inequality in the World Economy*

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January 16, 2009

Abstract

This paper develops tools and techniques to study the impact of exogenous changes in factor supply and factor demand on factor allocation and factor prices in economies with a large number of goods and factors. The main results of our paper characterize sufficient conditions for robust monotone comparative statics predictions in a Roy-like assignment model. These general results are then used to generate new insights about the consequences of globalization.

*We thank Daron Acemoglu, David Autor, Ariel Burstein, Pierre-André Chiappori, Gene Grossman, Gordon Hanson, seminars participants at many institutions, and especially Esteban Rossi-Hansberg for helpful comments and discussions. We have also benefited from stimulating discussions by Pol Antras, Bob Staiger, and Dan Treffer at the NBER Summer Institute and the Princeton IES Summer Workshop. Costinot also thanks Princeton IES for its hospitality.

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1 Introduction

In this paper, we develop tools and techniques to study the impact of exogenous changes in factor supply and factor demand on factor allocation and factor prices in economies with a large number of goods and factors. We then demonstrate how these tools and techniques can be applied to generate new insights about the consequences of globalization.

Understanding the determinants of factor allocation and factor prices in economies with a large number of goods and factors is important for at least two reasons. First, large changes in factor allocation and factor prices do occur at high levels of disaggregation in practice. For instance, a number of authors in the labor and public finance literatures have documented: *(i)* large changes in inequality at the top of the income distribution, Piketty and Saez (2003); *(ii)* divergent trends in inequality in the top and in the bottom halves of the income distribution, Autor, Katz, and Kearney (2008); *(iii)* divergent trends in employment growth of high- and low-wage occupations, Goos and Manning (2007); and *(iv)* changes in both between- and within-group inequality, Juhn, Murphy, and Pierce (1993). Second, even changes occurring at low levels of disaggregation, such as variations in the relative wage of college to non-college graduates, often reflect average changes taken over a large number of imperfectly substitutable factors. To analyze all these phenomena, we need a model with more than two goods and two factors.

Section 2 introduces our theoretical framework. The starting point of our analysis is a Roy-like assignment model in which a continuum of workers, with different skills, are matched to a continuum of intermediate goods or tasks, with different skill intensities. These tasks are then combined into a unique final good using a CES aggregator. All good and labor markets are perfectly competitive.

Section 3 presents tools and techniques to derive robust monotone comparative statics predictions in this environment. We first introduce definitions of skill abundance and skill diversity to conceptualize changes in factor supply, and definitions of skill-biased and extreme-biased technologies to conceptualize changes in factor demand. These definitions do not rely on any functional form assumption, but rely instead on standard concepts in information economics; see e.g. Milgrom (1981). They naturally extend standard notions of relative factor supply and demand from two-good-two-factor models to models with a large number of goods and factors. Using these concepts, we then derive sufficient conditions for various patterns of changes in factor prices—e.g. pervasive changes in inequality—and factor allocation—e.g. job polarization—to occur in a closed economy.

Section 4 uses these general results to shed light on the consequences of globalization. We consider a world economy comprising two countries. Building on our closed economy comparative statics, we first analyze the impact of North-South trade, which we model as trade between countries which differ in: *(i)* skill abundance; or *(ii)* the skill bias of their technologies. When North-South trade is driven by differences in factor endowments, we obtain continuum-by-continuum extensions of the classic two-by-two Heckscher-Ohlin results. In particular, trade integration induces skill downgrading and a pervasive rise in inequality in the North; the converse is true in the South. Perhaps surprisingly, when North-South trade is driven by differences in technological biases, we show that the exact same logic leads to the exact opposite conclusion.

This observation, which arises naturally in the context of our model, implies that predictions regarding the impact of trade integration crucially depend on the correlation between factor endowment and technological differences. In a series of influential papers, see e.g. Acemoglu (1998) and Acemoglu (2003), Acemoglu argues that skill-abundant countries tend to use skill-biased technologies. With this correlation in mind, we should not be surprised if: *(i)* similar countries have different globalization experiences depending on which of these two forces, supply or demand, dominates; and *(ii)* the overall effect of trade liberalization on factor allocation and factor prices tends to be small in practice.

Another benefit of our theoretical framework is that it permits an integrated analysis of North-South and North-North trade. Since we have more than two factors, we can model North-North trade as trade between a more and a less diverse country. As Grossman and Maggi (2000) first emphasized, the notion of diversity—where one country is more diverse than another if it has relatively more workers of extreme skills—is important because it allows us to think about the implications of trade between countries with similar average skill levels. While this accounts for the vast majority of world trade, the standard Heckscher-Ohlin model has nothing to say about its implications for inequality.¹ By contrast, our model predicts that North-North trade integration induces job and wage polarization in the more diverse country. The converse is true in the less diverse country.

Compared to North-South trade, North-North trade may either increase or decrease the relative wage between high- and low-skill workers as well as the relative price of the goods they produce. The consequences of North-North trade are to be found at a higher level of

¹The most common approach to explain North-North trade is the so-called “new” trade theory; see Helpman and Krugman (1985). Its implications for inequality, however, are the same as in the Heckscher-Ohlin model.

disaggregation. When trading partners vary in terms of skill diversity, changes in inequality occur within low- and high-skill workers, respectively. Similarly, North-North trade does not yield a decrease (or increase) in the employment shares of the skill-intensive tasks; instead, it leads to a U-shape (or inverted U-shape) relationship between tasks' employment growth and their skill intensity.

Section 5 presents our final class of comparative statics exercises: the implications of global technological change and offshoring in the world economy. For expositional purposes, we restrict ourselves to the case in which trade is driven only by differences in skill abundance. In this environment, we first show that global skill-biased technological change induces skill downgrading in each country, a pervasive rise in inequality in each country, and an increase in inequality between countries. In addition, we show that our framework also provides sharp predictions regarding the implications of offshoring between a Northern and a Southern country, modeled as the ability of Northern firms to hire Southern workers using the North's superior technology. In our model, offshoring acts like an increase in the size of the Southern country, which makes the world distribution of workers less skill abundant. This, in contrast to the Stolper-Samuelson Theorem, induces skill downgrading and a pervasive rise in inequality in both countries.²

Section 6 discusses two extensions of our basic framework. Our first extension shows that our results can be generalized to the case of an arbitrary, but discrete, number of goods and factors. Unlike standard neoclassical trade models, our comparative static results do not crucially depend on whether or not there are more goods than factors; see e.g. Ethier (1984). Though our approach admittedly is more elegant in the continuum-by-continuum case, none of our results hinge on the dimensionality of our economy. Our second extension shows that our results about changes in inequality over a continuum of skills can also be used to make predictions about between-group inequality (e.g. the skilled-wage premium) and within-group inequality (e.g. the 90 – 10 log hourly wage differential among college graduates) for observationally identical factors. In particular, we demonstrate that North-South trade integration, when driven by factor endowment differences, leads to an increase in between- and within-group inequality in the North, and a decrease in between- and within-group inequality in the South.

Our paper contributes to two distinct literatures. The first one is the assignment litera-

²A related mechanism was first studied by Feenstra and Hanson (1996) and, subsequently, Zhu and Trefler (2005) in an economy with two types of workers, skilled and unskilled. We come back to the relationship between their results and ours in Section 5.

ture; see Sattinger (1993) for an overview. Typical results in this field fall into two broad categories. On the one hand, authors like Becker (1973), Heckman and Honore (1990), Shimer and Smith (2000), Legros and Newman (2002), Legros and Newman (2007), and Costinot (Forthcoming) offer general results focusing on cross-sectional predictions, such as sufficient conditions for positive assortative matching to arise. On the other hand, authors like Teulings (1995), Teulings (2005), Kremer and Maskin (2003), Garicano and Rossi-Hansberg (2006), Antras, Garicano, and Rossi-Hansberg (2006), Gabaix and Landier (2008), Tervio (2008), and Blanchard and Willmann (2008) offer specific comparative statics predictions under strong functional form restrictions on the distribution of skills, worker productivity, and/or the pattern of substitution across goods.

To this literature, our paper offers sufficient conditions for robust monotone comparative statics predictions—without functional form restrictions on the distribution of skills or worker productivity—in a Roy-like assignment model where goods neither have to be perfect substitutes nor perfect complements. These general results are useful because they deepen our understanding of an important class of models in the labor and trade literature, clarifying how relative factor supply and relative factor demand affect factor prices and factor allocations in such environments. Compared to the previous literature, the greater generality of our theory also is useful in that it allows us to offer a unifying perspective on a wide range of phenomena, from technological change to offshoring.

Our paper also contributes to the theory of international trade. There exist many neoclassical trade models analyzing the impact of either trade integration or offshoring on factor allocations and factor prices. These models, however, generally involve either a large number of goods and two factors: Dornbusch, Fischer, and Samuelson (1980), Feenstra and Hanson (1996), and Zhu and Trefler (2005); or two goods and a large number of factors: Grossman and Maggi (2000) and Grossman (2004).³ While extending standard neoclassical trade models to a large number of both goods and factors is, of course, theoretically possible, the predictions derived in such environments, unfortunately, are weak. In a well-known paper, Jones and Scheinkman (1977) shows, for example, that a rise in the price of some good causes an even larger proportional increase in the price of some factor; but depending on the number of goods and factors, it may or may not lead to an even larger proportional decrease

³A related literature to which our paper makes contact investigates the implications of trade integration on inequality in monopolistically competitive environments with heterogeneous firms and labor market imperfections, see e.g. Davis and Harrigan (2007), Amiti and Davis (2008), Helpman, Itskhoki, and Redding (2008), and Sethupathy (2008); or heterogeneous workers and endogenous technology adoption, see e.g. Yeaple (2005).

in the price of some other factor.

By imposing stronger assumptions on the supply-side of our economy, namely those of a Roy-like assignment model, we derive much stronger predictions on the consequences of globalization in economies with an arbitrarily large number of both goods and factors.⁴ In addition to the sharp results that it offers, this new approach enables us to discuss, within a unified framework, phenomena that would otherwise fall outside the scope of standard trade theory, such as pervasive changes in inequality and wage and job polarization.

2 The Closed Economy

2.1 Basic Environment

Endowments. We consider an economy populated by a continuum of workers with skill $s \in \mathbb{R}$. We denote by $V(s) \geq 0$ the inelastic supply of workers with skill s ; and by $S \equiv \{s \in \mathbb{R} | V(s) > 0\}$ the set of skills available in the economy. Throughout this paper, we restrict ourselves to skill distributions such that $S = [\underline{s}, \bar{s}]$, though different V 's may have different supports.

Technology. There is one final good which we use as the numeraire. Producing the final good requires a continuum of intermediate goods or tasks indexed by their skill intensity $\sigma \in \mathbb{R}$. Output of the final good is given by the following CES aggregator⁵

$$Y = \left\{ \int_{\sigma \in \Sigma} B(\sigma) [Y(\sigma)]^{\frac{\varepsilon-1}{\varepsilon}} d\sigma \right\}^{\frac{\varepsilon}{\varepsilon-1}}, \quad (1)$$

where $Y(\sigma) \geq 0$ is the endogenous output of task σ ; $0 \leq \varepsilon < \infty$ is the constant elasticity of substitution across tasks; $B(\sigma) \geq 0$ is an exogenous technological parameter; and $\Sigma \equiv \{\sigma \in \mathbb{R} | B(\sigma) > 0\}$ corresponds to the set of tasks available in the economy. As before, we restrict ourselves to technologies such that $\Sigma = [\underline{\sigma}, \bar{\sigma}]$, though different B 's may have different supports.

⁴Ohnsorge and Treffer (2007) and Costinot (Forthcoming) also use assignment models with many goods and factors in a trade context, but do not derive any comparative statics predictions. In related work, Anderson (2009) offers a neoclassical trade model with a continuum of goods and factors. In his model, however, the allocation of the continuum of factors is exogenously given, and therefore, results are restricted to changes in factor prices.

⁵Only two properties of the CES aggregator are critical for all of our results: (i) constant returns to scale; and (ii) Gorman (1968) separability. Properties (i) and (ii), however, also imply CES, which we therefore assume throughout.

Producing tasks only requires workers. Workers are perfect substitutes in the production of each task, but vary in their productivity, $A(s, \sigma) > 0$. Output of task σ is given by

$$Y(\sigma) = \int_{s \in S} A(s, \sigma) L(s, \sigma) ds, \quad (2)$$

where $L(s, \sigma) \geq 0$ is the endogenous number of workers with skill s performing task σ . We assume that $A(s, \sigma)$ is twice differentiable and strictly log-supermodular:

$$A(s, \sigma) A(s', \sigma') > A(s, \sigma') A(s', \sigma), \text{ for all } s > s' \text{ and } \sigma > \sigma'. \quad (3)$$

Since $A(s, \sigma) > 0$, Property (3) can be rearranged as $\frac{A(s, \sigma)}{A(s, \sigma')} > \frac{A(s', \sigma)}{A(s', \sigma')}$. In other words, high-skill workers have a comparative advantage in tasks with high-skill intensity.

Market structure. All markets are perfectly competitive with all goods being produced by a large number of identical price-taking firms. Total profits for the final good are given by

$$\Pi = \left\{ \int_{\sigma \in \Sigma} B(\sigma) [Y(\sigma)]^{\frac{\varepsilon-1}{\varepsilon}} d\sigma \right\}^{\frac{\varepsilon}{\varepsilon-1}} - \int_{\sigma \in \Sigma} p(\sigma) Y(\sigma) d\sigma, \quad (4)$$

where $p(\sigma) > 0$ is the price of task σ . Similarly, total profits for intermediate good σ are given by

$$\Pi(\sigma) = \int_{s \in S} [p(\sigma) A(s, \sigma) - w(s)] L(s, \sigma) ds, \quad (5)$$

where $w(s) > 0$ is the wage of a worker with skill s . For technical reasons, we also assume that B and V are continuous functions.

2.2 Definition of a Competitive Equilibrium

In a competitive equilibrium, all firms maximize their profits and all markets clear. Profit maximization by final good producers requires

$$Y(\sigma) = I \times [p(\sigma) / B(\sigma)]^{-\varepsilon}, \text{ for all } \sigma \in \Sigma \quad (6)$$

where $I \equiv \left[\int_{s \in S} w(s) V(s) ds \right]$ denotes total income. Since there are constant returns to scale, profit maximization by intermediate good producers requires

$$\begin{aligned} p(\sigma) A(s, \sigma) - w(s) &\leq 0, \text{ for all } s \in S; \\ p(\sigma) A(s, \sigma) - w(s) &= 0, \text{ for all } s \in S \text{ such that } L(s, \sigma) > 0. \end{aligned} \quad (7)$$

Finally, good and labor market clearing require

$$Y(\sigma) = \int_{s \in S} A(s, \sigma) L(s, \sigma) ds, \text{ for all } \sigma \in \Sigma; \quad (8)$$

$$V(s) = \int_{\sigma \in \Sigma} L(s, \sigma) d\sigma, \text{ for all } s \in S. \quad (9)$$

In rest of this paper, we formally define a competitive equilibrium as follows.

Definition 1 *A competitive equilibrium is a set of functions $Y : \Sigma \rightarrow \mathbb{R}^+$, $L : S \times \Sigma \rightarrow \mathbb{R}^+$, $p : \Sigma \rightarrow \mathbb{R}^+$, $w : S \rightarrow \mathbb{R}^+$ such that Conditions (6)-(9) hold.*

2.3 Properties of a Competitive Equilibrium

Given our assumptions on worker productivity, $A(s, \sigma)$, profit-maximization condition (7) imposes strong restrictions on competitive equilibria.

Lemma 1 *In a competitive equilibrium, there exists an increasing bijection $M : S \rightarrow \Sigma$ such that $L(s, \sigma) > 0$ if and only if $M(s) = \sigma$.*

Lemma 1 derives from the fact that: (i) factors of production are perfect substitutes within each task; and (ii) A is strictly log-supermodular. Perfect substitutability, on the one hand, implies the existence of a matching function M summarizing the allocation of workers to tasks. Because of the linearity of the task production function, if a worker of skill s is allocated to task σ , they all are. Log-supermodularity, on the other hand, implies the monotonicity of this matching function. Since high-skill workers are relatively more productive in tasks with high-skill intensity, high- σ firms are willing to bid relatively more for these workers. In a competitive equilibrium, this induces positive assortative matching of high- s workers to high- σ tasks.⁶

The rest of our analysis crucially relies on the following lemma:

Lemma 2 *In a competitive equilibrium, the matching function and wage schedule satisfy:*

$$\frac{dM}{ds} = \frac{A[s, M(s)] V(s)}{I \times \{p[M(s)]/B[M(s)]\}^{-\varepsilon}}, \quad (10)$$

⁶Formally, the log-supermodularity of A is necessary and sufficient for $p(\sigma) A(s, \sigma)$ to satisfy the single crossing property in (s, σ) for all $p(\sigma)$, and therefore, for positive assortative matching to arise for any price schedule.

$$\frac{d \ln w(s)}{ds} = \frac{\partial \ln A[s, M(s)]}{\partial s}, \quad (11)$$

with $M(\underline{s}) = \underline{\sigma}$, $M(\bar{s}) = \bar{\sigma}$, and $p[M(s)] = w(s)/A[s, M(s)]$.

According to Lemma 2, the two key endogenous variables of our model, the matching function, M , and the wage schedule, w , are given by the solution of a system of ordinary differential equations. Equation (10) summarizes how, because of market clearing, factor supply and factor demand determine the matching function. Equation (11) summarizes how, because of profit-maximization, the matching function determines the wage schedule. Once w and M have been computed, Y and p can be computed by simple substitutions using Equations (6) and (7).

3 Comparative Statics in the Closed Economy

Armed with the knowledge that a competitive equilibrium is characterized by Equations (10) and (11), we now investigate how exogenous changes in factor supply, V , and factor demand, B , affect factor allocation and factor prices. In each case, we first determine how exogenous changes in V and B affect the matching function, M . We then consult Equation (11) to draw conclusions about its implications for the wage schedule, w .

3.1 Changes in Factor Supply

3.1.1 Skill Abundance

We first consider a change in factor supply from V to V' such that:

$$V(s)V'(s') \geq V'(s)V(s'), \text{ for all } s \geq s' \quad (12)$$

Property (12) corresponds to the monotone likelihood ratio property; see Milgrom (1981).⁷ It captures the idea that there are relatively more high-skill workers under V than under V' . If $s, s' \in S \cap S'$, Property (12) simply implies $\frac{V(s)}{V(s')} \geq \frac{V'(s)}{V'(s')}$. This is the natural generalization, to a continuum of factors, of the notion of skill abundance in a two-factor model. Property (12), in addition, allows us to consider situations where different sets of skills are available

⁷There exists a close mathematical connection between log-supermodularity and the monotone likelihood ratio property. Formally, if we let $V(s) \equiv \tilde{V}(s, \gamma)$ and $V'(s) \equiv \tilde{V}(s, \gamma')$ with $\gamma \geq \gamma'$, then V and V' satisfies Property (12) if and only if \tilde{V} is log-supermodular.

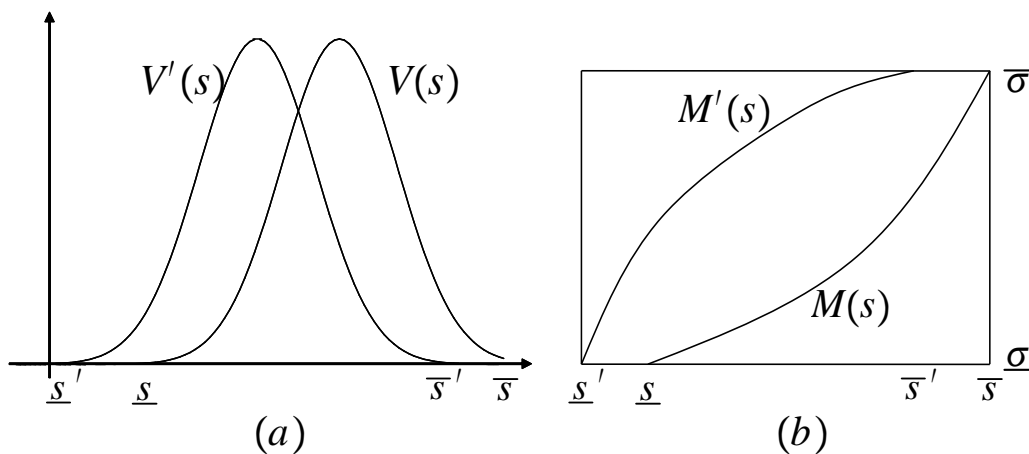


Figure 1: Changes in skill abundance and matching

under V and V' . If $s, s' \notin S \cap S'$, Property (12) implies that $s \in S$ and $s' \in S'$, or equivalently, that S is greater than S' in the strong set order: $\underline{s} \geq \underline{s}'$ and $\bar{s} \geq \bar{s}'$. In other words, the highest-skill workers must be in the economy characterized by V and the lowest-skill workers in the economy characterized by V' . Property (12) is illustrated in Figure 1 (a).

In the rest of this paper, we say that:

Definition 2 V is skill-abundant relative to V' , denoted $V \succeq_a V'$, if Property (12) holds.

We first analyze the impact of a change in skill abundance on matching. Let M and M' be the matching functions associated with V and V' , respectively. Our first result can be stated as follows.

Lemma 3 Suppose $V \succeq_a V'$. Then $M(s) \leq M'(s)$ for all $s \in S \cap S'$.

From a worker standpoint, moving from V to V' implies *task upgrading*: each type of worker performs a task with higher skill intensity under V' . From a task standpoint, this means *skill downgrading*: each task is performed by workers with lower skills under V' . This is illustrated in Figure 1 (b).⁸ At a broad level, the intuition behind Lemma 3 is very simple. As the relative supply of the high-skill workers goes down, market clearing conditions require more tasks to be performed by low-skill workers. So, the M schedule should shift

⁸For expositional purposes, we have chosen to state all our definitions and predictions using weak inequalities. It should be clear, however, that both our definitions and predictions have natural, though slightly more involved, counterparts with strict inequalities.

left. To derive a more precise understanding of Lemma 3, suppose that $M(s) > M'(s)$ for some $s \in S \cap S'$. Then there must be two tasks, $s_1 < s_2$, such that M crosses M' from below at s_1 and from above at s_2 . By our market clearing condition, this can only happen if: (i) the relative supply of s_2 is strictly higher under V' ; and/or (ii) the relative demand for s_2 is strictly lower under V' . On the factor supply side, Property (12) implies $V(s_2)/V(s_1) \geq V'(s_2)/V'(s_1)$, which precisely precludes condition (i). On the factor demand side, Property (3) implies $p[M(s_2)]/p[M(s_1)] \geq p'[M'(s_2)]/p'[M'(s_1)]$, which precisely precludes condition (ii).

Now let us consider the associated impact of a change in skill abundance on wages. Let w and w' be the wage schedules associated with V and V' , respectively, where $V \succeq_a V'$. Combining Lemma 3, Equation (11), and the log-supermodularity of A , we obtain

$$\frac{d \ln w}{ds} = \frac{\partial \ln A[s, M(s)]}{\partial s} \leq \frac{\partial \ln A[s, M'(s)]}{\partial s} = \frac{d \ln w'}{ds}.$$

Integrating the above inequality implies

$$\frac{w(s)}{w(s')} \leq \frac{w'(s)}{w'(s')}, \text{ for all } s > s' \text{ in } S \cap S'. \quad (13)$$

Moving from V to V' leads to a *pervasive rise in inequality*: for any pair of workers, the relative wage of the worker with a higher skill level—who is relatively less abundant under V' —goes up. In our model, a decrease in the relative supply of the high-skill workers triggers a reallocation of all workers towards the skill intensive tasks. Since A is log-supermodular, this increases the marginal return of the high-skill workers relatively more.

3.1.2 Skill Diversity

We now consider the case where V and V' satisfy:

$$(i) V' \succeq_a V, \text{ for all } s < \widehat{s}, \text{ and } (ii) V \succeq_a V', \text{ for all } s \geq \widehat{s}, \text{ with } \widehat{s} \in S'. \quad (14)$$

Property (14) captures the idea that there are relatively more workers with extreme skill levels (either high or low) under V than V' . If different sets of skills are available under V and V' , then Property (14) implies $S' \subseteq S$. Moreover, for any pair of distinct skill levels $s' \leq s < \widehat{s}$ with $s, s' \in S'$, there are relatively more high-skill workers in the economy characterized by V' , $\frac{V'(s)}{V'(s')} \geq \frac{V(s)}{V(s')}$; and for any pair of distinct skill levels $s \geq s' \geq \widehat{s}$ with

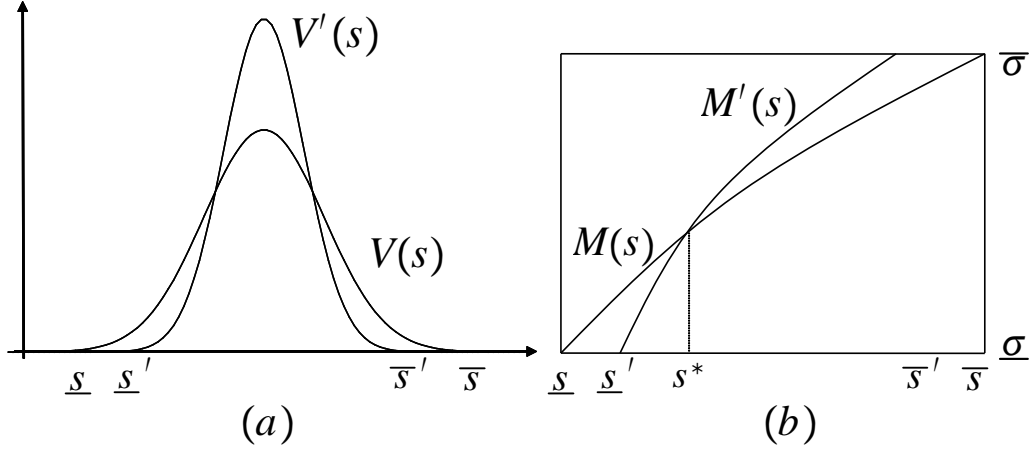


Figure 2: Changes in skill diversity and matching

$s, s' \in S'$, there are relatively more high-skill workers in the economy characterized by V , $\frac{V(s)}{V(s')} \geq \frac{V'(s)}{V'(s')}$. Property (14) is illustrated in Figure 2 (a).

In the rest of this paper, we say that:

Definition 3 V is more diverse than V' , denoted $V \succeq_d V'$, if Property (14) holds.

Definition 3 is a stronger notion of diversity than in Grossman and Maggi (2000) in the sense that we impose likelihood ratio dominance on either side of \hat{s} whereas they only impose first-order stochastic dominance. It also is a weaker notion, however, in the sense that Grossman and Maggi (2000) impose symmetry on V and V' while we do not.

As before, let M and M' be the matching functions associated with V and V' , respectively. Our second result can be stated as follows.

Lemma 4 Suppose $V \succeq_d V'$. Then there exists a skill level $s^* \in S'$ such that $M(s) \geq M'(s)$ for all $s \in [s', s^*]$, and $M(s) \leq M'(s)$ for all $s \in [s^*, \bar{s}']$.

Moving from V to V' implies job polarization: skill upgrading for low skill intensity tasks, $\sigma < \sigma^*$; and skill downgrading for high skill intensity tasks, $\sigma^* < \sigma$, where $\sigma^* \equiv M(s^*) = M'(s^*)$. This is illustrated in Figure 2 (b).⁹

As in the case of a change in skill abundance, the basic intuition behind these two results relies on our market clearing conditions. If $V \succeq_d V'$, the relative supply of high-skill workers

⁹Note that $V \succeq_d V'$ does not guarantee by itself that s^* is in the interior of the support of V' . An example of sufficient conditions that guarantee $s^* \in (s', \bar{s}')$ are $V \succeq_d V'$ and $S' \subset S$.

increases over the range $\underline{s} < s < \widehat{s}$. Thus, more tasks should employ these workers. The converse is true over the range $\widehat{s} < s < \bar{s}$.

Now let us turn to the associated wage schedules, w and w' under the restriction that $V \succeq_d V'$. Combining Lemma 4, Equation (11), and the log-supermodularity of A , we obtain

$$\begin{aligned} \frac{d \ln w}{ds} &\geq \frac{d \ln w'}{ds}, \text{ for all } \underline{s}' < s < s^*; \\ \frac{d \ln w}{ds} &\leq \frac{d \ln w'}{ds}, \text{ for all } s^* < s < \bar{s}'. \end{aligned}$$

Integrating this series of inequalities gives

$$\begin{aligned} \frac{w(s)}{w(s')} &\geq \frac{w'(s)}{w'(s')}, \text{ for all } \underline{s}' \leq s' < s \leq s^*; \\ \frac{w(s)}{w(s')} &\leq \frac{w'(s)}{w'(s')}, \text{ for all } s^* \leq s' < s \leq \bar{s}'. \end{aligned} \tag{15}$$

Within each group of workers—low skill, $s < s^*$, or high skill, $s > s^*$ —changes in skill diversity amount to changes in skill-abundance. For any pair of workers whose abilities are no greater or no less than s^* , the relative wage of the worker whose skill becomes relatively less abundant goes up.

3.2 Changes in Factor Demand

In the previous section, we focused on exogenous changes in factor supply. We now briefly demonstrate how our concepts and techniques can be extended to analyze exogenous changes in factor demand.

3.2.1 Skill-biased technological change

We now consider a shift in the B schedule, from B to B' , such that:

$$B'(\sigma) B(\sigma') \geq B'(\sigma') B(\sigma), \text{ for all } \sigma \geq \sigma' \tag{16}$$

Property (16) captures changes in relative factor demand which are biased towards high-skill workers. Holding prices constant, Equation (6) and Property (16) imply $\frac{Y'(\sigma)}{Y'(\sigma')} \geq \frac{Y(\sigma)}{Y(\sigma')}$ for any pair of tasks $\sigma \geq \sigma'$ in $\Sigma \cap \Sigma'$. In other words, a shift from B to B' increases the relative demand for tasks performed by high-skill workers. Property (16), in addition, allows us to consider situations in which the technologies characterized by B and B' use different sets of tasks. If $\Sigma \neq \Sigma'$, then Property (16) implies Σ' greater than Σ in the strong set order,

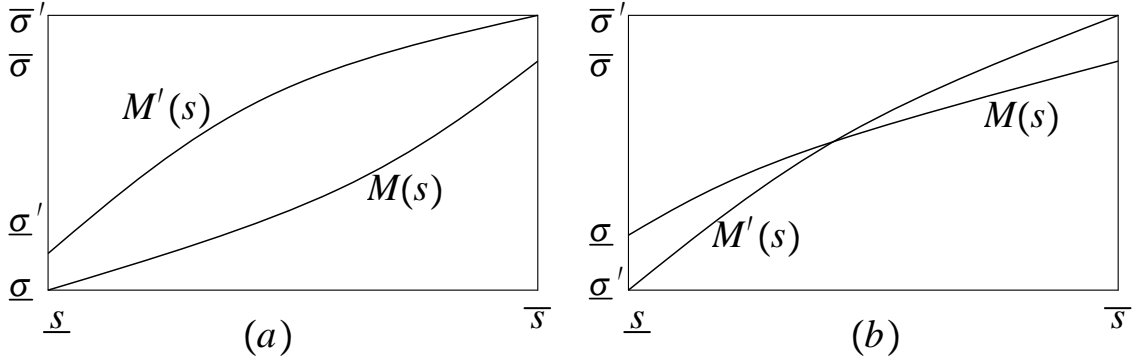


Figure 3: Skill- and extreme-biased technological change and matching

$\underline{\sigma}' \geq \underline{\sigma}$ and $\bar{\sigma}' \geq \bar{\sigma}$. Put simply, the most-skill intensive tasks must be used under B' and the least skill intensive tasks under B .

In the rest of this paper, we say that:

Definition 4 B' is skill-biased relative to B , denoted $B' \succeq_s B$, if Property (16) holds.

Let M and M' denote the matching functions associated with B and B' , respectively. The demand version of the results on changes in factor supply derived in Lemma (3) can be stated as follows.

Lemma 5 Suppose $B' \succeq_s B$. Then $M(s) \leq M'(s)$ for all $s \in S$.

Broadly speaking, if the relative demand for the skill-intensive goods rises, then market clearing conditions require workers to move towards tasks with higher skill intensities in order to maintain equilibrium. This implies skill downgrading at the task level, and task upgrading at the worker level. This is illustrated in Figure 3 (a).

Finally, let w and w' be the wage schedules associated with B and B' , respectively, where $B' \succeq_s B$. Combining Lemma 5, Equation (11), and the log-supermodularity of A , we now obtain

$$\frac{w'_s(s)}{w'(s)} \geq \frac{w_s(s)}{w(s)},$$

which after integration implies

$$\frac{w'(s)}{w'(s')} \geq \frac{w(s)}{w(s')}, \text{ for all } s > s'. \quad (17)$$

Moving from B to B' leads to a pervasive rise in inequality: for any pair of workers, the relative wage of the more skilled worker increases. The mechanism linking the matching function to the wage schedule is the same as in Section 3.1.1. By Lemma 5, an increase in the relative demand for goods with high-skill intensities triggers a reallocation of workers towards such tasks. Given the log-supermodularity of A , this increases the marginal return of high-skill workers relatively more.

3.2.2 Extreme-biased technological change

Finally, we consider a shift in the B schedule, from B to B' , such that:

$$(i) B \succeq_s B' \text{ for all } \sigma < \hat{\sigma}, \text{ and } (ii) B' \succeq_s B \text{ for all } \sigma \geq \hat{\sigma}, \text{ with } \hat{\sigma} \in \Sigma. \quad (18)$$

A shift from B to B' increases the relative demand for tasks with low skill intensities over the range $\sigma < \hat{\sigma}$, and increases the relative demand for tasks with high-skill intensities over the range $\sigma \geq \hat{\sigma}$. Property (18) is reminiscent, for instance, of the impact of computerization, as modeled by Autor, Katz, and Kearney (2006). As in our previous comparative statics exercise, the change in relative factor demand captured by Property (18) may result, among other things, from the introduction of a new set of tasks in the economy, i.e. $\Sigma \subset \Sigma'$.

In the rest of this paper, we say that:

Definition 5 B' is extreme-biased relative to B , denoted $B' \succeq_e B$, if Property (18) holds.

Let M and M' denote the matching functions associated with B and B' , respectively. The demand version of the results on changes in factor supply derived in Lemma 4 can be stated as follows.

Lemma 6 Suppose $B' \succeq_e B$. Then there exists a skill level $s^* \in S$ such that $M(s) \geq M'(s)$ for all $s \in [s, s^*]$, and $M(s) \leq M'(s)$ for all $s \in [s^*, \bar{s}]$.

Moving from B to B' induces workers to reallocate out of intermediate σ tasks and towards extreme σ tasks. We refer to this reallocation as job polarization. This is illustrated in Figure 3(b). As in the case of diversity, relative wages are given by Equation (15). Hence, extreme-biased technological change implies wage polarization as well.

4 The World Economy

In the remainder of this paper we consider a world economy comprising two countries, Home (H) and Foreign (F). Workers are internationally immobile, the unique final good is not traded, and all intermediate goods are freely traded.¹⁰ In each country, we assume that production is as described in Section 2.1 and that factor productivity differences across countries are Hicks-neutral $A_i(s, \sigma) \equiv \gamma_i A(s, \sigma)$ for $i = H, F$, with $\gamma_i > 0$. Hence, cross-country differences in factor endowments, V_H and V_F , and technological biases, B_H and B_F , are the only rationale for trade.¹¹ Throughout this section, we denote by $S_W \equiv S_H \cup S_F$ and $\Sigma_W \equiv \Sigma_H \cup \Sigma_F$ the set of skills and tasks available in the world economy, respectively.

4.1 Free Trade Equilibrium

Before analyzing the consequences of globalization, we characterize a free trade equilibrium. Given our work in Section 2.2, this is a straightforward exercise. A competitive equilibrium in the world economy under free trade is a set of functions $(Y_H, L_H, w_H, Y_F, L_F, w_F, p)$ such that Conditions (6), (7), and (9) hold in both countries, and good markets clear

$$Y_H(\sigma) + Y_F(\sigma) = \int_{s \in S_W} [A_H(s, \sigma) L_H(s, \sigma) + A_F(s, \sigma) L_F(s, \sigma)] ds, \text{ for all } \sigma \in \Sigma_W.$$

Since technological differences at the task level are Hicks-neutral, our model is isomorphic to a model in which tasks are produced using the same technology around the world, but countries' factor supply are given by $\tilde{V}_i \equiv \gamma_i V_i$. Moreover, since factors of production are perfect substitutes within each task, factor price equalization necessarily holds in efficiency units; see Condition (7). Therefore we can focus on the free trade equilibrium that replicates the integrated equilibrium.

Let M_T , w_T , and p_T denote the matching function, the wage expressed in Home units, and the price schedule in the integrated equilibrium, respectively. By Lemma 2, we have

$$\frac{dM_T}{ds} = \frac{A[s, M_T(s)] V(s)}{I_W \times \{p_T[M_T(s)] / B_W[M_T(s)]\}^{-\varepsilon}},$$

¹⁰We briefly discuss the case in which the final good is freely traded at the end of subsection 4.2.

¹¹It should be clear that differences in technological biases are not Ricardian technological differences. In our model, differences in technological biases play very much the same role as differences in preferences in a standard Heckscher-Ohlin model.

$$\frac{d \ln w_T(s)}{ds} = \frac{\partial \ln A[s, M_T(s)]}{\partial s},$$

where $M(\underline{s}_W) = \underline{\sigma}_W$ and $M(\bar{s}_W) = \bar{\sigma}_W$ are the boundary conditions for the matching function; $p_T[M_T(s)] = w_T(s)/\gamma_H A[s, M_T(s)]$ is the price schedule; $B_W[M_T(s)] \equiv \{(I_H/I_W) B_H[M_T(s)]^\varepsilon + (I_F/I_W) B_F[M_T(s)]^\varepsilon\}^{1/\varepsilon}$ characterizes the skill bias of the “world’s technology”; and $I_W \equiv \int_{s \in S_W} w_T(s) [V_H(s) + (\gamma_F/\gamma_H) V_H(s)] ds$ is world income.

4.2 Consequences of North-South Trade

We conceptualize North-South trade as situations where countries differ in either: (i) their skill abundance, $V_H \succeq_a V_F$; or (ii) the skill bias of their technologies, $B_H \succeq_s B_F$.

4.2.1 The Role of Cross-Country Differences in Factor Endowments

To isolate the role of factor supply considerations, we first assume that Home is skill abundant relative to Foreign, $V_H \succeq_a V_F$, but that the final good is produced using the same technology around the world, $B_H = B_F$. In a two-by-two Heckscher-Ohlin model, when the skill-abundant country opens up to trade: (i) the skill intensity of both tasks decreases; (ii) the skill-intensive task expands; and (iii) the skill premium goes up. Conversely, when the unskill-abundant country opens up to trade: (i) the skill intensity of both tasks increases; (ii) the unskill-intensive task expands; and (iii) the skill premium goes down. We now offer continuum-by-continuum extensions of these classic results.¹² Our analysis builds on the following Lemma.

Lemma 7 *Suppose $V_H \succeq_a V_F$. Then $V_W \equiv \gamma_H V_H + \gamma_F V_F$ satisfies $V_H \succeq_a V_W \succeq_a V_F$.*

As in the two-factor model, if Home is skill-abundant relative to Foreign, then Home is skill-abundant relative to the World and the World is skill-abundant relative to Foreign.

We first consider the implications of trade integration on the matching of workers to tasks. Let M_H and M_F be the matching functions at Home and Abroad, respectively, under autarky. By Lemmas 3 and 7, trade integration induces skill downgrading at Home and skill upgrading Abroad:

$$M_H^{-1}(\sigma) \geq M_T^{-1}(\sigma) \geq M_F^{-1}(\sigma) \text{ for all } \sigma \in \Sigma_W. \quad (19)$$

¹²We omit the continuous analogue to the Heckscher-Ohlin Theorem because both Ohnsorge and Treffer (2007) and Costinot (Forthcoming) prove this result with arbitrarily many factors and tasks.

This is the counterpart to Effect (i) in the two-by-two Heckscher-Ohlin model. A direct corollary of Inequality (19) is that for any $\sigma \in \Sigma_W$:

$$\begin{aligned} \int_{M_T^{-1}(\sigma)}^{\bar{s}_H} V_H(s) ds &\geq \int_{M_H^{-1}(\sigma)}^{\bar{s}_H} V_H(s) ds; \\ \int_{\underline{s}_F}^{M_T^{-1}(\sigma)} V_F(s) ds &\geq \int_{\underline{s}_F}^{M_F^{-1}(\sigma)} V_F(s) ds. \end{aligned} \quad (20)$$

According to Inequality (20), the employment share in tasks with high-skill intensities, from σ to $\bar{\sigma}_W$, increases at Home, whereas the employment share in tasks with low skill intensities, from $\underline{\sigma}_W$ to σ , increases Abroad. This is the counterpart to Effect (ii) in the two-by-two Heckscher-Ohlin model.

We now turn to the implications of trade integration on inequality. Let w_H and w_F be the wage schedules at Home and Abroad, respectively, in autarky. As in Section 3.1.1, Inequality (19) and the log-supermodularity of A imply a pervasive rise in inequality in Home and a pervasive fall in inequality Abroad:

$$\begin{aligned} \frac{w_H(s)}{w_H(s')} &\leq \frac{w_T(s)}{w_T(s')}, \text{ for all } s \in S_H; \\ \frac{w_T(s)}{w_T(s')} &\leq \frac{w_F(s)}{w_F(s')}, \text{ for all } s \in S_F. \end{aligned} \quad (21)$$

Inequality (21) is the counterpart to Effect (iii). It captures a strong Stolper-Samuelson effect: *anywhere* in the skill distribution, workers with higher skills get relatively richer in the skill-abundant country under free trade, whereas they get relatively poorer in the other country.

To get a better sense of this effect, denote by $I_i^A(q) \equiv \int_{s_q}^{\bar{s}_i} w_i(s) V_i(s) ds$ and $I_i^T(q) \equiv \int_{s_q}^{\bar{s}_i} w_T(s) (\gamma_i/\gamma_H) V_i(s) ds$ the total earnings of the top ($q \times 100$) % of the skill distribution in country $i = H, F$ under autarky and free trade, respectively. For any $1 > q > q' \geq 0$, Inequality (21) implies

$$\frac{I_H^T(q)}{I_H^T(q')} \geq \frac{I_H^A(q)}{I_H^A(q')}.$$

In other words, changes in inequality are *fractal* in nature: within any truncation of the skill distribution, high-skill workers are getting richer at Home. Similarly, in the Foreign country, we have

$$\frac{I_F^T(q)}{I_F^T(q')} \leq \frac{I_F^A(q)}{I_F^A(q')}.$$

In spite of the large number of goods and factors in this economy, the fundamental forces linking trade integration and inequality remain simple. Because of changes in the relative supply

of skills, trade integration induces skill downgrading in the skill-abundant country. Thus, workers move into tasks with higher skill intensities, which increases the marginal return to skill, and in turn, inequality. Proposition 1 summarizes our results on the consequences of North-South trade when driven by factor-endowment differences.

Proposition 1 *If Home is skill-abundant relative to Foreign, then, all else equal, trade integration induces: (i) skill downgrading at Home and Skill upgrading Abroad; (ii) an increase in the employment share of tasks with high-skill intensities at Home and low-skill intensities Abroad; and (iii) a pervasive rise in inequality at Home and a pervasive fall in inequality Abroad.*

The simple two-by-two Stolper-Samuelson effect, which Proposition 1 (iii) extends to the continuum-by-continuum case, is one of the most tested implications of trade theory. Empirical results, however, are mixed. Finding either direct or indirect support are, for example, O’Rourke and Williamson (1999), Wei and Wu (2001), Menezes-Filho and Muendler (2007), Broda and Romalis (2008), and Michaels (2008); for an extensive list of papers finding violations, see Goldberg and Pavcnik (2007). Goldberg and Pavcnik (2007) provide the following summary of the state of this empirical literature: “Overall, it appears that the particular mechanisms through which globalization affected inequality are country, time, and case specific; that the effects of trade liberalization need to be examined in conjunction with other concurrent policy reforms...” Seen through the lens of our theory, the previous empirical results can be interpreted as follows. For a given country’s globalization experience, cross-country differences in relative factor supply may or may not be the main determinant of changes in inequality. With this in mind, we now turn to the implications of cross-country differences in relative factor demand.

4.2.2 The Role of Cross-Country Differences in Technological Biases

To isolate the role of factor demand considerations, we now assume that countries differ in terms of their final good production functions, $B_H \succeq_s B_F$, but have identical factor supply, $V_H = V_F$. Like in the case of differences in factor supply, our analysis builds on the following Lemma.

Lemma 8 *Suppose $B_H \succeq_s B_F$. Then B_W satisfies $B_H \succeq_s B_W \succeq_s B_F$.*

If Home’s technology is skill biased relative to Foreign’s, then Home’s technology is skill biased relative to the World’s and the World’s technology is skill biased relative to Foreign’s.

We first consider the impact of trade integration on the matching of workers to tasks. Let M_H and M_F be the matching functions at Home and Abroad, respectively, under autarky. By Lemmas 5 and 8, trade integration induces skill upgrading at Home and skill downgrading Abroad:

$$M_H(s) \geq M_T(s) \geq M_F(s), \text{ for all } s \in S_W. \quad (22)$$

Note that if Home and Foreign use different sets of tasks under autarky, $\Sigma_H \neq \Sigma_F$, then trade integration induces workers to move into the production of new tasks. In the Foreign country, the most skilled workers become employed in tasks whose skill intensity is higher than the intensity of any tasks performed under autarky. The converse is true in the Home country, where the least skilled workers become employed in tasks whose skill intensity is lower than the intensity of any tasks performed under autarky.

What happens to the distribution of wages? As in Section 3.2, Inequality (22) and the log-supermodularity of A imply a pervasive fall in inequality in Home and a pervasive rise in inequality Abroad:¹³

$$\frac{w_H(s)}{w_H(s')} \geq \frac{w_T(s)}{w_T(s')} \geq \frac{w_F(s)}{w_F(s')}, \text{ for all } s > s'. \quad (23)$$

To sum up, the consequences of North-South trade driven by demand considerations are the exact opposite of the consequences of North-South trade driven by supply considerations.

Proposition 2 *If Home's technology is skill-biased relative to Foreign's, then, all else equal, trade integration induces: (i) skill upgrading at Home and Skill downgrading Abroad; (ii) an increase in the employment share of tasks with low-skill intensities at Home and high-skill intensities Abroad; and (iii) a pervasive fall in inequality at Home and a pervasive rise in inequality Abroad.*

Propositions 1 and 2 together imply that predictions regarding the impact of globalization crucially depend on the correlation between supply and demand considerations. In a series of influential papers, see e.g. Acemoglu (1998) and Acemoglu (2003), Acemoglu argues that skill-abundant countries tend to use skill-biased technologies. Using our notation, this means that if $V_H \succ_a V_F$, then $B_H \succ_s B_F$. Combining the insights of Propositions 1 and 2, we should therefore not be surprised if: (i) similar countries have different globalization experiences depending on which of these two forces, supply or demand, dominates; and (ii)

¹³Verhoogen (2008) provides a partial equilibrium framework yielding similar predictions, at the firm level, and empirically finds supportive evidence in Mexico.

the *overall* effect of trade liberalization on factor allocation and factor prices tends to be small in practice.

Finally, note that if the final good is freely traded as well, then the consequences of North-South trade integration also depend on whether Home's or Foreign's technology is more efficient. If, for instance, Home's technology is more efficient for all tasks, $B_H(\sigma) > B_F(\sigma)$ for all σ , then North-South trade integration is more likely to increase inequality in both countries.

4.3 Consequences of North-North Trade

To avoid a taxonomic exercise, we focus on the case in which countries only differ in factor supply and conceptualize North-North trade as a situation where Home is more diverse than Foreign, $V_H \succeq_d V_F$. Under this assumption, we demonstrate that the familiar mechanisms at work in North-South trade apply equally well to North-North trade, which allows us, in turn, to generate new results on the consequences of international trade.

Our analysis of North-North trade builds on the following Lemma.

Lemma 9 *Suppose $V_H \succeq_d V_F$. Then $V_W \equiv \gamma_H V_H + \gamma_F V_F$ satisfies $V_H \succeq_d V_W \succeq_d V_F$.*

Consider the Foreign country. By Lemmas 4 and 9, trade integration induces task upgrading for low-skill workers and task downgrading for high-skill workers. Formally, there exists $s_F^* \in [\underline{s}_F, \bar{s}_F]$ such that

$$\begin{aligned} M_F(s) &\leq M_T(s), \text{ for all } s \in [\underline{s}_F, s_F^*]; \\ M_F(s) &\geq M_T(s), \text{ for all } s \in [s_F^*, \bar{s}_F]. \end{aligned} \tag{24}$$

This means skill downgrading for low skill intensity tasks and skill upgrading for high-skill intensity tasks.

The converse is true in the Home country. Namely, there exists $s_H^* \in [\underline{s}_H, \bar{s}_H]$ such that

$$\begin{aligned} M_H(s) &> M_T(s), \text{ for all } s \in [\underline{s}_H, s_H^*]; \\ M_H(s) &< M_T(s), \text{ for all } s \in [s_H^*, \bar{s}_H]. \end{aligned} \tag{25}$$

The differential impact of North-North trade integration on the tasks performed by high- and low-skill workers has stark implications on inequality in the two countries. At Home,

Inequality (24) and the log-supermodularity of A imply

$$\begin{aligned} \frac{w_H^A(s)}{w_H^A(s')} &\geq \frac{w_H(s)}{w_H(s')}, \text{ for all } \underline{s}_H \leq s' < s \leq s_H^*; \\ \frac{w_H^A(s)}{w_H^A(s')} &\leq \frac{w_H(s)}{w_H(s')}, \text{ for all } s_H \leq s' < s' \leq \bar{s}_H. \end{aligned} \quad (26)$$

Moving from autarky to free trade leads to a *polarization* of the wage distribution in the more diverse country. Among the least skilled workers, those with lower skills get relatively richer, whereas the converse is true among the most skilled workers. Similarly, in the less diverse country we have

$$\begin{aligned} \frac{w_F^A(s)}{w_F^A(s')} &< \frac{w_F(s)}{w_F(s')}, \text{ for all } \underline{s}_F \leq s' < s \leq s_F^*; \\ \frac{w_F^A(s)}{w_F^A(s')} &> \frac{w_F(s)}{w_F(s')}, \text{ for all } s_F^* \leq s' < s' \leq \bar{s}_F. \end{aligned} \quad (27)$$

Inequality (27) implies *convergence* Abroad, as the “middle-class” benefits relatively more from free trade. Proposition 3 summarizes our results on the consequences of North-North trade.

Proposition 3 *If Home is more diverse than Foreign, then, all else equal, trade integration induces (i) skill upgrading in tasks with low-skill intensities at Home and high-skill intensities Abroad; (ii) skill downgrading in tasks with high-skill intensities at Home and low-skill intensities Abroad; and (iii) wage polarization at Home and convergence Abroad.*

It is worth emphasizing that, unlike Propositions 1 and 2, Proposition 3 has no clear implications for the *overall* level of inequality. Under North-North trade, the relative wage *between* high- and low-skill workers—as well as the relative price of the goods they produce—may either increase or decrease. The consequences of North-North trade are to be found at a higher level of disaggregation. When trading partners vary in terms of skill diversity, changes in inequality occur *within* low- and high-skill workers, respectively. Similarly, Proposition 3 does not predict a decrease (or increase) in the employment shares of the skill-intensive tasks. According to our theory, North-North trade leads to a U-shape (or inverted U-shape) relationship between tasks’ employment growth and their skill-intensity.¹⁴

¹⁴While we have focused on the consequences of trade integration, our analysis also has natural implications for immigration. Chiquiar and Hanson (2005), for example, document intermediate selection of Mexican migrants into the United States. Within our theoretical framework, such a phenomenon could be interpreted as a change in the Mexican distribution of skills in terms of diversity. Using the same logic as in Proposition 3, our model would then predict that immigration should lead to wage convergence in Mexico.

5 Technological Change in the World Economy

In this section we consider the impact of technological diffusion and skill-biased technological change in the world economy. For expositional purposes, we restrict ourselves to the North-South case where $V_H \succeq_a V_F$, and assume that $\gamma_H \geq \gamma_F$. In other words, the skill-abundant country also is (weakly) more productive in all tasks.

5.1 Global Skill-Biased Technological Change

We first analyze the impact of global skill-biased technological change (SBTC), modelled as a shift from B_W to B'_W such that $B'_W \succeq_s B_W$. We denote M_T and M'_T the matching functions in the integrated equilibrium under B_W and B'_W , respectively, and w_T and w'_T the associated wage schedules. From our previous work in a closed economy, we already know that global SBTC induces skill downgrading/task upgrading in both countries:

$$M_T(s) \leq M'_T(s), \text{ for all } s \in S_W.$$

We also know that this change in matching implies

$$\frac{w'_T(s)}{w'_T(s')} \geq \frac{w_T(s)}{w_T(s')}, \text{ for all } s > s'.$$

which leads to a pervasive rise in inequality within each country. Compared to a closed economy, however, we can further ask how global SBTC affects inequality *between* countries. Let $I_i^{(\prime)} \equiv \int_{s \in S_i} w_T^{(\prime)}(s) (\gamma_i / \gamma_H) V_i(s) ds$ denote total income in country $i = H, F$. Our predictions about the impact of global SBTC on cross-country inequality can be stated as follows.

Lemma 10 *Suppose $V_H \succeq_a V_F$ and $B'_W \succeq_s B_W$. Then total income satisfies $I'_H / I'_F \geq I_H / I_F$.*

According to Lemma 10, an increase in the relative labor demand for skill-intensive tasks worldwide increases inequality between Home and Foreign. The formal argument relies on the fact that log-supermodularity is preserved by multiplication and integration, but the basic intuition is simple: high-skill agents gain relatively more from such a change, and Home has relatively more of them. In our model, within- and between-country inequality tend to go hand in hand: *ceteris paribus*, changes in matching that increase inequality in

both countries also increase inequality across countries. Proposition 4 summarizes our results on the consequences of global SBTC.

Proposition 4 *Global SBTC induces: (i) skill downgrading in each country; (ii) a pervasive rise in inequality in each country; and (iii) an increase in inequality between countries.*

Finally, it is worth pointing out that Proposition 4 also has interesting implications for the consequences of trade liberalization when our CES aggregator is reinterpreted as a utility function. Suppose, for example, that a country's preferences are a function of their aggregate income I , and that wealthier countries have a relative preference for skill-intensive goods: $B_I \succeq_s B_{I'}$ for all $I > I'$. Then, by increasing income in all countries, trade liberalization would lead to a pervasive rise in inequality around the world.

5.2 Offshoring tasks

For our final comparative statics exercise, we analyze the impact of an increase in Foreign workers' productivities from $\gamma_F A(s, \sigma)$ to $\gamma'_F A(s, \sigma)$, where $\gamma'_F > \gamma_F$. A natural way to think about such a technological change is *offshoring*, i.e. the ability of Domestic firms to hire Foreign workers using Home's superior technology.¹⁵ This is the interpretation we adopt in the rest of this subsection.

Our analysis of task offshoring builds on two simple observations. First, as far as the integrated equilibrium is concerned, increasing the productivity of all Foreign workers from γ_F to $\gamma'_F > \gamma_F$ is similar to increasing their supply by γ'_F/γ_F . Second, since Foreign is relatively unskill-abundant, an increase in effective units of Foreign factor supply, from $\gamma_F V_F$ to $\gamma'_F V_F$, makes the World relatively less skill abundant, as we show in the following Lemma.

Lemma 11 *Suppose $V_H \succeq_a V_F$ and $\gamma'_F > \gamma_F$. Then $V_W \equiv \gamma_H V_H + \gamma_F V_F$ and $V'_W \equiv \gamma_H V_H + \gamma'_F V_F$ satisfy $V_W \succeq_a V'_W$.*

To sum up, if domestic firms offshore their production, it is *as if* the World distribution becomes relatively less skill abundant. Therefore, the results of Section 3.1.1 directly imply that

$$M'_T(s) \geq M_T(s) \text{ for all } s \in S_W.$$

where M_T and M'_T are the matching functions in the integrated equilibrium before and after offshoring, respectively. By Lemmas 3 and 11, offshoring induces task upgrading, as the

¹⁵This way of modelling offshoring is in the spirit of Grossman and Rossi-Hansberg (2008).

World's matching function moves closer towards Foreign's matching function under autarky. This implies a pervasive rise in inequality in both countries:

$$\frac{w'_T(s)}{w'_T(s')} \geq \frac{w_T(s)}{w_T(s')}, \text{ for all } s > s'.$$

For any pair of workers in either country, the relative wage of the more skilled worker increases as a result of offshoring. In the integrated equilibrium, offshoring is similar to an increase in the relative size of the Foreign country. As Foreign grows relative to Home, World prices converge to those that hold in Foreign under autarky. Since the wage schedule is steeper Abroad than at Home under autarky, offshoring increases inequality in both countries. Proposition 5 summarizes our results on the consequences of offshoring.

Proposition 5 *Offshoring in the world economy induces: (i) skill downgrading in both countries; and (ii) a pervasive rise in inequality in both countries.*

The previous results are reminiscent of those in Feenstra and Hanson (1996) and, subsequently, Zhu and Trefler (2005). In addition to the fact that they apply to the full distribution of earnings rather than just the skill premium, these results also demonstrate that neither Ricardian technological differences nor a lack of factor-price equalization are necessary to yield these predictions. The key mechanism simply is that offshoring leads to sector upgrading around the world, thereby increasing the marginal return to skill in all countries.

6 Robustness and Extensions

6.1 Number of Goods and Factors

The results derived so far all relied on the assumption that there was a continuum of tasks and a continuum of workers. In neoclassical trade theory, comparative statics predictions on factor allocations and prices typically are very sensitive to assumptions made on the number of goods and factors. The objective of this section is to demonstrate that, by contrast, our results generalize to the case of an arbitrary, but discrete, number of goods and factors. In order to avoid a taxonomic exercise, we focus on a move from V to $V' \preceq_a V$ in the closed economy.

Throughout this section, we assume that there are a discrete number of factors indexed $j = 1, \dots, M$ such that $s_1 < \dots < s_M$, and a discrete number of sectors indexed by $k = 1, \dots, N$

such that $\sigma_1 < \dots < \sigma_N$. The rest of our model is unchanged. In terms of notation, we let $\Sigma(s) \equiv \{\sigma \in \Sigma | L(s, \sigma) > 0\}$ denote the set of tasks employing workers with skills s and $S(\sigma) \equiv \{s \in S | L(s, \sigma) > 0\}$ denote the set of skills employed in task σ , where $L(s, \sigma)$ is the allocation of workers to tasks in a competitive equilibrium. We use similar notation for the assignment functions under V' .

In this environment, we can derive the following counterparts to Lemmas (1) and (3).

Lemma 1 (Discrete) *In a competitive equilibrium, $S(\sigma) \geq S(\sigma')$ in the strong set order for any $\sigma \geq \sigma'$.*

Lemma 3 (Discrete) *Suppose $V \succeq_a V'$. Then $\Sigma(s) \leq \Sigma'(s)$ in the strong set order for all $s \in S \cap S'$.*

This last lemma further implies that if $V \succeq_a V'$, then $\frac{w(s)}{w(s')} \leq \frac{w'(s)}{w'(s')}$, for all $s \geq s'$ in $S \cap S'$. In other words, a move from V to $V' \preceq_a V$ leads to a pervasive rise in inequality, as previously shown in the continuum-by-continuum case. The formal proofs can be found in the Appendix.

6.2 Observable versus Unobservable Skills

Although our theory assumes a continuum of skills, an econometrician is unlikely to observe a continuum of skills in practice. To bring our theory one step closer to data, we now introduce explicitly the distinction between observable and unobservable skills.¹⁶ The objective of this section is to demonstrate how, under reasonable assumptions, our results about changes in inequality over a continuum of unobservable skills can easily be mapped into observable measures of inequality such as: (i) between-group inequality (e.g. the skilled-wage premium); and (ii) within-group inequality (e.g. the 90 – 10 log hourly wage differential among college graduates). For simplicity, we restrict ourselves to the case of North-South trade integration with factor endowment differences: $V_H \succeq_a V_F$ and $B_H = B_F$

Throughout this section, we assume that workers are partitioned into n groups based on some socioeconomic characteristic $e_1 < \dots < e_n$, such as years of education or experience. While firms and workers perfectly observe s , we assume that the econometrician only observes e , but knows the inelastic supply of workers with skill s in group e in country i : $V_i(s, e) \geq 0$.

¹⁶Of course, the analysis of this section has similar implications in the case where the econometrician only observes a coarse measure of task skill intensity, such as ‘‘Occupation’’ or ‘‘Sector’’ of employment.

In particular, the econometrician knows that $V_i(s, e)$ is log-supermodular:

$$V_i(s, e) V_i(s', e') \geq V_i(s, e') V_i(s', e), \text{ for all } s \geq s' \text{ and } e \geq e'. \quad (28)$$

Property (28) captures the idea that, in both countries, high-skill workers are relatively more likely in groups with high levels of education or experience.

Armed with a link between observable and unobservable skills, we may now discuss between- and within-group inequality. For any pair of groups e and e' in country i , we define between-group inequality as the relative average wage between two groups $\bar{w}_i(e) / \bar{w}_i(e') = \int_{s \in S_i} w_i(s) V_i(s, e) ds / \int_{s \in S_i} w_i(s) V_i(s, e') ds$. For any group e in country i , we define within-group inequality as $w_i[s_{90}(e)] / w_i[s_{10}(e)]$, where $s_q(e)$ denotes the skill of the worker at the q^{th} percentile of the wage distribution in education group e .

With the previous notation in hand, we are ready to state the implications of North-South trade integration for between-group and within-group inequality. If $V_H \succeq_a V_F$ and $B_H = B_F$, then:

$$\begin{aligned} (i) \quad & \frac{\bar{w}_H(e)}{\bar{w}_H(e')} \leq \frac{\bar{w}_T(e)}{\bar{w}_T(e')} \leq \frac{\bar{w}_F(e)}{\bar{w}_F(e')} \text{ for all } e \geq e'; \\ (ii) \quad & \frac{w_H[s_{90}(e)]}{w_H[s_{10}(e)]} \leq \frac{w_T[s_{90}(e)]}{w_T[s_{10}(e)]} \leq \frac{w_F[s_{90}(e)]}{w_F[s_{10}(e)]} \text{ for all } e. \end{aligned} \quad (29)$$

Inequality (29) states that North-South trade integration, when driven by factor endowment differences, leads to an increase in between- and within-group inequality at Home, and a decrease in between- and within-group inequality Abroad. Like in Section 4.2.2, North-South trade integration, when driven by technological differences, would lead the exact opposite results. Proposition 6 summarizes the implications of North-South trade driven by factor-endowment differences on between-group and within-group inequality.

Proposition 6 *If Home is skill-abundant relative to Foreign, then, all else equal, trade integration induces an increase in between- and within-group inequality at Home and a decrease in between- and within-group inequality Abroad.*

7 Concluding Remarks

In the assignment literature, comparative statics predictions typically are derived under strong functional form restrictions on the distribution of skills, worker productivity, and/or the pattern of substitution across goods. The first contribution of our paper is to offer sufficient conditions for robust monotone comparative statics predictions—without functional

form restrictions on the distribution of skills or worker productivity—in a Roy-like assignment model where goods neither have to be perfect substitutes nor perfect complements. These general results are useful because they deepen our understanding of an important class of models in the labor and trade literatures, clarifying how relative factor supply and relative factor demand affect factor prices and factor allocations in such environments.

The second contribution of our paper is to show how these general results can be used to derive sharp predictions about the consequences of globalization in economies with an arbitrarily large number of both goods and factors. This new approach enables us to discuss, within a unified framework, phenomena that have been recently documented in the labor and public finance literatures, but would otherwise fall outside the scope of standard trade theory, such as pervasive changes in inequality and wage and job polarization.¹⁷

Finally, while we have emphasized the consequences of globalization, we believe that our general results also have useful applications outside of international trade. As Heckman and Honore (1990) note, “The analysis of choice of geographical location [...], schooling levels [...], occupational choice with endogenous specific human capital [...], choice of industrial sectors [...], and the consequences of these choices for earnings inequality all fall within the general framework of the Roy model.” Accordingly, our tools and techniques can also potentially shed light on each of these choices and their consequences for inequality.

¹⁷Of course, whether or not globalization actually caused such changes is an empirical matter. But to assess empirically whether or not this is the case, we first need a trade model that can “speak” to these phenomena, which our paper provides.

A Proofs (I): The Closed Economy

Proof of Lemma 1. Throughout the proof, we denote $S(\sigma) \equiv \{s \in S \mid L(s, \sigma) > 0\}$ and $\Sigma(s) \equiv \{\sigma \in \Sigma \mid L(s, \sigma) > 0\}$. Clearly $s \in S(\sigma)$ if and only if $\sigma \in \Sigma(s)$. We proceed in 5 steps.

Step 1: $S(\sigma) \neq \emptyset$ for all $\sigma \in \Sigma$ and $\Sigma(s) \neq \emptyset$ for all $s \in S$.

$S(\sigma) \neq \emptyset$ derives from Conditions (6) and (8). $\Sigma(s) \neq \emptyset$ derives from Condition (9).

Step 2: $S(\cdot)$ and $\Sigma(\cdot)$ are weakly increasing in the strong set order.

We first show that $S(\cdot)$ is weakly increasing in the strong set order by contradiction. Suppose there are a pair of tasks $\sigma_0 < \sigma_1$ and a pair of workers $s_0 < s_1$ such that $s_0 \in S(\sigma_1)$ and $s_1 \in S(\sigma_0)$. Condition (7) implies

$$p(\sigma_1) A(s_0, \sigma_1) - w(s_0) = 0; \quad (30)$$

$$p(\sigma_0) A(s_1, \sigma_0) - w(s_1) = 0; \quad (31)$$

$$p(\sigma_0) A(s_0, \sigma_0) - w(s_0) \leq 0; \quad (32)$$

$$p(\sigma_1) A(s_1, \sigma_1) - w(s_1) \leq 0. \quad (33)$$

By Equation (30) and Inequality (32), we have

$$p(\sigma_0) A(s_0, \sigma_0) \leq p(\sigma_1) A(s_0, \sigma_1). \quad (34)$$

By Equations (31) and Inequality (33), we have

$$p(\sigma_1) A(s_1, \sigma_1) \leq p(\sigma_0) A(s_1, \sigma_0). \quad (35)$$

Combining Inequalities (34) and (35), we obtain

$$A(s_0, \sigma_0) A(s_1, \sigma_1) \leq A(s_0, \sigma_1) A(s_1, \sigma_0),$$

which contradicts $A(s, \sigma)$ strictly log-supermodular. Hence, $S(\cdot)$ is weakly increasing in the strong set order. Since $s \in S(\sigma)$ if and only if $\sigma \in \Sigma(s)$, $\Sigma(\cdot)$ must be weakly increasing in the strong set order as well.

Step 3: $S(\sigma)$ is a singleton for all but a countable set of σ .

Let Σ_0 be the subset of tasks σ such that $\mu[S(\sigma)] > 0$, where μ is the Lebesgue measure

over \mathbb{R} . We first show that Σ_0 is a countable set. Choose an arbitrary $\sigma \in \Sigma_0$ and let $\underline{s}(\sigma) \equiv \inf S(\sigma)$ and $\bar{s}(\sigma) \equiv \sup S(\sigma)$. The fact that $\mu[S(\sigma)] > 0$ has strictly positive measure yields $\underline{s}(\sigma) < \bar{s}(\sigma)$. Because $S(\sigma)$ is weakly increasing in σ , we must have $\sum_{\sigma \in \Sigma_0} [\bar{s}(\sigma') - \underline{s}(\sigma')] \leq \bar{s} - \underline{s}$. So for any $\sigma \in \Sigma_0$, there must be $j \in \mathbb{N}$ such that $\bar{s}(\sigma) - \underline{s}(\sigma) \geq (\bar{s} - \underline{s})/j$; and for any $j \in \mathbb{N}$, there must be at most j points $\{\sigma\}$ in Σ_0 for which $\bar{s}(\sigma) - \underline{s}(\sigma) \geq (\bar{s} - \underline{s})/j$. Since the union of countable sets is countable, the two previous observations imply that Σ_0 is a countable set. Now take $\sigma \notin \Sigma_0$. To show that $S(\sigma)$ is a singleton, we proceed by contradiction. If $S(\sigma)$ is not a singleton, then there are $s < s''$ such that $s, s'' \in S(\sigma)$. Using Step 1 and the fact that $\mu[S(\sigma)] = 0$, there also is $s < s' < s''$ such that $s' \in S(\sigma')$ with $\sigma' \neq \sigma$, which contradicts Step 2.

Step 4: $\Sigma(s)$ is a singleton for all but a countable set of s .

Since $\Sigma(s) \neq \emptyset$ and $\Sigma(\cdot)$ is weakly increasing in the strong set order, this follows from the same argument as in Step 3.

Step 5: $S(\sigma)$ is a singleton for all σ .

To obtain a contradiction, suppose that there exists $\sigma \in \Sigma$ for which $S(\sigma)$ is not a singleton. By the same argument as in Step 3, we must have $\mu[S(\sigma)] > 0$. By Step 4, $\Sigma(s) = \{\sigma\}$ for μ -almost all $s \in S(\sigma)$. Hence, Condition (9) implies

$$L(s, \sigma) = V(s) \delta [1 - \mathbb{1}_{S(\sigma)}], \text{ for } \mu\text{-almost all } s \in S(\sigma). \quad (36)$$

where δ is a Dirac delta function. By Step 3 and Condition (9), we must also have $\sigma' \in \Sigma$ for which $S(\sigma') = \{s'\}$ with $s' \in S$ such that

$$L(s', \sigma') \leq V(s') \delta [1 - \mathbb{1}_{S(\sigma')}] \quad (37)$$

Combining Equations (36) and (37) with Conditions (6) and (8), we obtain $\frac{p(\sigma')}{p(\sigma)} = 0$. By the same argument as in Step 2, we must also have $\frac{p(\sigma')}{p(\sigma)} \geq \frac{A(s', \sigma)}{A(s', \sigma')} > 0$. A contradiction.

Steps 2 and 5 imply the existence of a strictly increasing function $M : S \rightarrow \Sigma$ such that $L(s, \sigma) > 0$ if and only if $M(s) = \sigma$. Step 1 requires $M(\underline{s}) = \underline{\sigma}$ and $M(\bar{s}) = \bar{\sigma}$. **QED.** ■

Proof of Lemma 2. We first consider Equation (11). Condition (7) and Lemma 1 imply

$$\begin{aligned} p[M(s)] A[s, M(s)] - w(s) &\geq p[M(s)] A[s + ds, M(s)] - w(s + ds), \\ p[M(s + ds)] A[s + ds, M(s + ds)] - w(s + ds) &\geq p[M(s + ds)] A[s, M(s + ds)] - w(s). \end{aligned}$$

Combining the two previous inequalities

$$\begin{aligned} \frac{p[M(s)] \{A[s+ds, M(s)] - A[s, M(s)]\}}{ds} &\leq \frac{w(s+ds) - w(s)}{ds} \\ &\leq \frac{p[M(s+ds)] \{A[s+ds, M(s+ds)] - A[s, M(s+ds)]\}}{ds}. \end{aligned}$$

Factor market clearing conditions, Equation (9), require w to be continuous. Since $p(\sigma) = w[M^{-1}(\sigma)]/A[M^{-1}(\sigma), \sigma]$, by Condition (7) and Lemma 1, and M^{-1} is continuous, by Lemma 1, p is continuous as well. Taking the limit of the previous chain of inequalities as ds goes to zero, we therefore get

$$w_s(s) = p[M(s)] A_s[s, M(s)]. \quad (38)$$

Since $p[M(s)] = w(s)/A[s, M(s)]$, we can rearrange Equation (38) as

$$\frac{d \ln w(s)}{ds} = \frac{\partial \ln A[s, M(s)]}{\partial s}.$$

This completes the first part of our proof. We now turn to Equation (10). Lemma 1 and Condition (9) imply that, for all $s \in S$,

$$L(s, \sigma) = V(s) \delta[\sigma - M(s)], \quad (39)$$

where δ is a Dirac delta function. Now consider Condition (8). At $\sigma = M(s)$, we have

$$Y[M(s)] = \int_{s' \in S} A[s', M(s)] L[s', M(s)] ds'.$$

Using Equation (39), we can rearrange the previous expression as

$$Y[M(s)] = \int_{s' \in S} A[s', M(s)] V(s') \delta[M(s) - M(s')] ds'.$$

Now set $\sigma' = M(s')$. Since M is a bijection from S to Σ , we have

$$Y[M(s)] = \int_{\sigma' \in \Sigma} A[M^{-1}(\sigma'), M(s)] V[M^{-1}(\sigma')] \delta[M(s) - \sigma'] \frac{1}{M_s(M^{-1}(\sigma'))} d\sigma'.$$

By definition of the Dirac delta function, this simplifies into

$$M_s(s) = \frac{A[s, M(s)] V(s)}{Y[M(s)]}. \quad (40)$$

Combining Equations (40) and (6), we obtain Equation (10). This completes the second part of our proof. $M(\underline{s}) = \underline{\sigma}$ and $M(\bar{s}) = \bar{\sigma}$ derive from the fact M is an increasing bijection from S onto Σ , whereas $p[M(s)] = w(s)/A[s, M(s)]$ derive from Condition (7) and Lemma 1, as previously mentioned. **QED.** ■

Proof of Lemma 3. We proceed by contradiction. Suppose that there exists $s \in S \cap S'$ at which $M(s) > M'(s)$. Since $V \succeq_a V'$, we know that $S \cap S' = [\underline{s}, \bar{s}']$. By Lemma 1, we also know that M and M' are continuous functions such that $M(\underline{s}) = \underline{\sigma} \leq M'(\underline{s})$ and $M'(\bar{s}') = \bar{\sigma} \geq M(\bar{s}')$. So, there must exist $\underline{s} \leq s_1 < s_2 \leq \bar{s}'$ and $\underline{\sigma} \leq \sigma_1 < \sigma_2 \leq \bar{\sigma}$ such that: (i) $M'(s_1) = M(s_1) = \sigma_1$ and $M'(s_2) = M(s_2) = \sigma_2$; (ii) $M'_s(s_1) < M_s(s_1)$ and $M'_s(s_2) > M_s(s_2)$; and (iii) $M(s) > M'(s)$ for all $s \in (s_1, s_2)$. Condition (ii) implies

$$M'_s(s_1)/M'_s(s_2) < M_s(s_1)/M_s(s_2). \quad (41)$$

Combining condition (i) with Inequality (41) and Equation (10), we obtain

$$\frac{V'(s_2)}{V'(s_1)} \left[\frac{p'(\sigma_2)}{p'(\sigma_1)} \right]^\varepsilon > \frac{V(s_2)}{V(s_1)} \left[\frac{p(\sigma_2)}{p(\sigma_1)} \right]^\varepsilon.$$

Using the zero profit condition, this can be rearranged as

$$\frac{V'(s_2)}{V'(s_1)} \left[\frac{w'(s_2)}{w'(s_1)} \right]^\varepsilon > \frac{V(s_2)}{V(s_1)} \left[\frac{w(s_2)}{w(s_1)} \right]^\varepsilon. \quad (42)$$

Inequality (42) requires either $\frac{V'(s_2)}{V'(s_1)} > \frac{V(s_2)}{V(s_1)}$ or $\frac{w'(s_2)}{w'(s_1)} > \frac{w(s_2)}{w(s_1)}$. The former inequality cannot hold because $V \succeq_a V'$, whereas the latter cannot hold because of Equation (11), the log-supermodularity of A , and condition (iii). **QED.** ■

Proof of Lemma 4. We proceed by contradiction. Suppose that there does not exist $s^* \in S \cap S'$ such that $M(s) \geq M'(s)$, for all $s \in [\underline{s}', s^*]$, and $M(s) \leq M'(s)$, for all $s \in [s^*, \bar{s}']$. Since $V \succeq_d V'$, we know that $S \cap S' = [\underline{s}', \bar{s}']$. By Lemma 1, we also know that M and M' are continuous functions such that $M'(\underline{s}') = \underline{\sigma} \leq M(\underline{s}')$ and $M'(\bar{s}') = \bar{\sigma} \geq M(\bar{s}')$. So, there must exist $\underline{s}' \leq s_0 < s_1 < s_2 \leq \bar{s}'$ and $\underline{\sigma} \leq \sigma_0 < \sigma_1 < \sigma_2 \leq \bar{\sigma}$ such that: (i) $M'(s_0) = M(s_0) = \sigma_0$, $M'(s_1) = M(s_1) = \sigma_1$, and $M'(s_2) = M(s_2) = \sigma_2$; (ii)

$M_s(s_0) < M'_s(s_0)$, $M_s(s_1) > M'_s(s_1)$, and $M_s(s_2) < M'_s(s_2)$; and (iii) $M(s) < M'(s)$ for all $s \in (s_0, s_1)$ and $M(s) > M'(s)$ for all $s \in (s_1, s_2)$. At this point, there are two possible cases: $s_1 < \widehat{s}$ and $s_1 \geq \widehat{s}$. If $s_1 < \widehat{s}$, we can follow the same steps as in the proof of Lemma 3 using s_0 and s_1 . Formally, condition (ii) implies

$$M_s(s_0)/M_s(s_1) < M'_s(s_0)/M'_s(s_1). \quad (43)$$

Combining condition (i) with Inequality (43), Equation (10), and the zero profit condition, we obtain

$$\frac{V(s_1)}{V(s_0)} \left[\frac{w(s_1)}{w(s_0)} \right]^\varepsilon > \frac{V'(s_1)}{V'(s_0)} \left[\frac{w'(s_1)}{w'(s_0)} \right]^\varepsilon. \quad (44)$$

Inequality (44) requires either $\frac{V(s_1)}{V(s_0)} > \frac{V'(s_1)}{V'(s_0)}$ or $\frac{w(s_1)}{w(s_0)} > \frac{w'(s_1)}{w'(s_0)}$. The former inequality cannot hold because $V' \succeq_a V$ for all $s < \widehat{s}$, whereas the latter cannot hold because of Equation (11), the log-supermodularity of A , and condition (iii). This completes our proof in the case $s_1 < \widehat{s}$. If $s_1 \geq \widehat{s}$, we can again follow the same steps as in the proof of Lemma 3, but using s_1 and s_2 . The formal argument is identical and omitted. **QED.** ■

Proof of Lemma 5. We proceed by contradiction. Suppose that there exists $s \in [\underline{s}, \bar{s}]$ at which $M(s) > M'(s)$. Since $B' \succeq_s B$, we know that $\underline{\sigma} \leq \underline{\sigma}'$ and $\bar{\sigma} \leq \bar{\sigma}'$. By Lemma 1, we also know that M and M' are continuous functions such that $M(\underline{s}) = \underline{\sigma} \leq \underline{\sigma}' = M'(\underline{s})$ and $M(\bar{s}) = \bar{\sigma} \leq \bar{\sigma}' = M'(\bar{s})$. So, there must exist $\underline{s} \leq s_1 < s_2 \leq \bar{s}$ and $\underline{\sigma}' \leq \sigma_1 < \sigma_2 \leq \bar{\sigma}$ such that: (i) $M'(s_1) = M(s_1) = \sigma_1$ and $M'(s_2) = M(s_2) = \sigma_2$; (ii) $M'_s(s_1) < M_s(s_1)$ and $M'_s(s_2) > M_s(s_2)$; and (iii) $M(s) > M'(s)$ for all $s \in (s_1, s_2)$. Condition (ii) implies

$$M'_s(s_1)/M'_s(s_2) < M_s(s_1)/M_s(s_2). \quad (45)$$

Combining condition (i) with Inequality (45) and Equation (10), we obtain

$$\frac{B'(\sigma_2) p'(\sigma_1)}{B'(\sigma_1) p'(\sigma_2)} < \frac{B(\sigma_2) p(\sigma_1)}{B(\sigma_1) p(\sigma_2)}.$$

Using the zero profit condition, this can be rearranged as

$$\frac{B'(\sigma_2) w'(s_1)}{B'(\sigma_1) w'(s_2)} < \frac{B(\sigma_2) w(s_1)}{B(\sigma_1) w(s_2)}. \quad (46)$$

Inequality (42) requires either $\frac{B'(\sigma_2)}{B'(\sigma_1)} < \frac{B(\sigma_2)}{B(\sigma_1)}$ or $\frac{w'(s_1)}{w'(s_2)} < \frac{w(s_1)}{w(s_2)}$. The former inequality cannot

hold because $B' \succeq_s B$, whereas the latter cannot hold because of Equation (11), the log-supermodularity of A , and condition (iii). **QED.** ■

Proof of Lemma 6. We proceed by contradiction. Suppose that there does not exist $s^* \in S$ such that $M(s) \geq M'(s)$, for all $s \in [\underline{s}, s^*]$, and $M(s) \leq M'(s)$, for all $s \in [s^*, \bar{s}]$. Since $B' \succeq_e B$, we know that $\underline{\sigma}' \leq \underline{\sigma}$ and $\bar{\sigma} \leq \bar{\sigma}'$. By Lemma 1, we also know that M and M' are continuous functions such that $M'(\underline{s}) = \underline{\sigma}' \leq \underline{\sigma} = M(\underline{s})$ and $M(\bar{s}) = \bar{\sigma} \leq \bar{\sigma}' = M'(\bar{s})$. So, there must exist $\underline{s} \leq s_0 < s_1 < s_2 \leq \bar{s}$ and $\underline{\sigma} \leq \sigma_0 < \sigma_1 < \sigma_2 \leq \bar{\sigma}$ such that: (i) $M'(s_0) = M(s_0) = \sigma_0$, $M'(s_1) = M(s_1) = \sigma_1$, and $M'(s_2) = M(s_2) = \sigma_2$; (ii) $M_s(s_0) < M'_s(s_0)$, $M_s(s_1) > M'_s(s_1)$, and $M_s(s_2) < M'_s(s_2)$; and (iii) $M(s) < M'(s)$ for all $s \in (s_0, s_1)$ and $M(s) > M'(s)$ for all $s \in (s_1, s_2)$. At this point, there are two possible cases: $\sigma_1 < \hat{\sigma}$ and $\sigma_1 \geq \hat{\sigma}$. If $\sigma_1 < \hat{\sigma}$, we can follow the same steps as in the proof of Lemma 5 using σ_0 and σ_1 . Formally, condition (ii) implies

$$M_s(s_0)/M_s(s_1) < M'_s(s_0)/M'_s(s_1). \quad (47)$$

Combining condition (i) with Inequality (47), Equation (10), and the zero profit condition, we obtain

$$\frac{B'(\sigma_1)w'(s_0)}{B'(\sigma_0)w'(s_1)} > \frac{B(\sigma_1)w(s_0)}{B(\sigma_0)w(s_1)}. \quad (48)$$

Inequality (48) requires either $\frac{B'(\sigma_1)}{B'(\sigma_0)} > \frac{B(\sigma_1)}{B(\sigma_0)}$ or $\frac{w'(s_0)}{w'(s_1)} > \frac{w(s_0)}{w(s_1)}$. The former inequality cannot hold because $B \succeq_s B'$ for all $\sigma < \hat{\sigma}$, whereas the latter cannot hold because of Equation (11), the log-supermodularity of A , and condition (iii). This completes our proof in the case $\sigma_1 < \hat{\sigma}$. If $\sigma_1 \geq \hat{\sigma}$, we can again follow the same steps as in the proof of Lemma 5, but using σ_1 and σ_2 . The formal argument is identical and omitted. **QED.** ■

B Proofs (II): The World Economy

Proof of Lemma 7. To show that $V_H \succeq_a V_F \iff V_H \succeq_a V_W$, note that

$$\frac{V_H(s)}{V_H(s')} \geq \frac{V_W(s)}{V_W(s')} \iff \frac{V_H(s)}{V_H(s')} \geq \frac{\gamma_H V_H(s) + \gamma_F V_F(s)}{\gamma_H V_H(s') + \gamma_F V_F(s')} \iff \frac{V_H(s)}{V_H(s')} \geq \frac{V_F(s)}{V_F(s')}.$$

The proof that $V_H \succeq_a V_F \iff V_W \succeq_a V_F$ is similar. **QED.** ■

Proof of Lemma 8. To show that $B_H \succeq_s B_F \iff B_H \succeq_s B_W$, note that

$$\begin{aligned} \frac{B_H(\sigma)}{B_H(\sigma')} \geq \frac{B_W(\sigma)}{B_W(\sigma')} &\iff \\ \frac{B_H(\sigma)}{B_H(\sigma')} \geq \left\{ \frac{I'_H [B_H(\sigma)]^\varepsilon + I'_F [B_F(\sigma)]^\varepsilon}{I'_H [B_H(\sigma')]^\varepsilon + I'_F [B_F(\sigma')]^\varepsilon} \right\}^{1/\varepsilon} &\iff \frac{B_H(\sigma)}{B_H(\sigma')} \geq \frac{B_F(\sigma)}{B_F(\sigma')}. \end{aligned}$$

The proof that $B_H \succeq_s B_F \iff B_W \succeq_s B_F$ is similar. **QED.** ■

Proof of Lemma 9. By definition, $V_H \succeq_d V_F$ implies $V_F \succeq_a V_H$, for all $s < \hat{s}$, and $V_H \succeq_a V_F$, for all $s \geq \hat{s}$. Thus, the result follows from Lemma 7 applied separately to $s < \hat{s}$ and $s \geq \hat{s}$. **QED.** ■

Proof of Lemma 10. Define $W(i, j) \equiv \gamma_i \int_{\underline{s}}^{\bar{s}} w(s, j) V(s, i) ds$, where $i = 1$ for Foreign and $i = 2$ for Home; $j = 1$ under B_W and $j = 2$ under B'_W ; $w(s, j)$ is the World wage function for $j = 1, 2$; and $V(s, i) = V_i(s)$. The fact that $V_H \succeq_a V_F$ implies that $V(s, i)$ is log-supermodular. According to Inequality (17), $w(s, j)$ is also log-supermodular. Since log-supermodularity is preserved by multiplication and integration, $W(i, j)$ is log-supermodular; see Karlin and Rinott (1980). This can be rearranged as $\frac{W(2,2)}{W(1,2)} \geq \frac{W(2,1)}{W(1,1)}$, which is equivalent to $\frac{W'_H}{W'_F} \geq \frac{W_H}{W_F}$. **QED.** ■

Proof of Lemma 11. To show that $V_H \succeq_a V_F \iff V_W \succeq_a V'_W$, note that

$$\begin{aligned} \frac{V_W(s)}{V_W(s')} \geq \frac{V'_W(s)}{V'_W(s')} &\iff \\ \frac{\gamma_H V_H(s) + \gamma_F V_F(s)}{\gamma_H V_H(s') + \gamma_F V_F(s')} \geq \frac{\gamma_H V_H(s) + \gamma'_F V_F(s)}{\gamma_H V_H(s') + \gamma'_F V_F(s')} &\iff \frac{V_H(s)}{V_H(s')} \geq \frac{V_F(s)}{V_F(s')}. \end{aligned}$$

QED. ■

C Proofs (III): Discrete Number of Goods and Factors

As mentioned in the main text, we consider the case of a discrete number of factors indexed $j = 1, \dots, M$ such that $s_1 < \dots < s_M$, and a discrete number of sectors indexed by $k = 1, \dots, N$ such that $\sigma_1 < \dots < \sigma_N$. We use the following notation $\Sigma(s) \equiv \{\sigma \in \Sigma | L(s, \sigma) > 0\}$ and $S(\sigma) \equiv \{s \in S | L(s, \sigma) > 0\}$, where $L(s, \sigma)$ is the allocation of workers to sectors in a competitive equilibrium. For any $\sigma \in \Sigma$ and $s \in S$, we let $\lambda(s, \sigma) \equiv L(s, \sigma) / V(s)$. For any $\sigma \in \Sigma$, we let $\underline{s}(\sigma) \equiv \inf S(\sigma)$ and $\bar{s}(\sigma) \equiv \sup S(\sigma)$; and for any $s \in S$, we

let $\underline{\sigma}(s) \equiv \inf \Sigma(s)$ and $\bar{\sigma}(s) \equiv \sup \Sigma(s)$. We use similar notation for the assignment functions under V' . Finally, for any $\sigma \in \Sigma$, we say that $L \succ_{\sigma} L'$ if $\lambda(s, \sigma) \geq \lambda'(s, \sigma)$ for all $s \in S(\sigma) \cup S'(\sigma)$ with strict inequality for at least one s .

C.1 Preliminary results

Throughout this Appendix, we make use of the following results.

Lemma 12 *Suppose that there exist $\sigma > \sigma'$ such that $L \succ_{\sigma} L'$ and $L' \succ_{\sigma'} L$. Then $V \succeq_a V'$ implies $Y(\sigma)/Y'(\sigma) > Y(\sigma')/Y'(\sigma')$.*

Proof. By definition, we know that $Y(\sigma) = \sum_{s \in S(\sigma)} \lambda(s, \sigma) V(s)$. Since $\lambda(s, \sigma) = 0$ for all $s \notin S(\sigma)$, we get $Y(\sigma) = \sum_{s \in S(\sigma) \cup S'(\sigma)} \lambda(s, \sigma) V(s)$. Similarly, we have $Y'(\sigma) = \sum_{s \in S(\sigma) \cup S'(\sigma)} \lambda'(s, \sigma) V'(s)$. Combining the two previous equalities with $L \succ_{\sigma} L'$, we obtain

$$\frac{Y(\sigma)}{Y'(\sigma)} > \frac{\sum_{s \in S(\sigma) \cup S'(\sigma)} \lambda(s, \sigma) V(s)}{\sum_{s \in S(\sigma) \cup S'(\sigma)} \lambda(s, \sigma) V'(s)}. \quad (49)$$

A similar reasoning for σ' implies

$$\frac{Y(\sigma')}{Y'(\sigma')} < \frac{\sum_{s' \in S(\sigma') \cup S'(\sigma')} \lambda(s', \sigma') V(s')}{\sum_{s' \in S(\sigma') \cup S'(\sigma')} \lambda(s', \sigma') V'(s')}. \quad (50)$$

Now note that the following inequality

$$\frac{\sum_{s \in S(\sigma) \cup S'(\sigma)} \lambda(s, \sigma) V(s)}{\sum_{s \in S(\sigma) \cup S'(\sigma)} \lambda(s, \sigma) V'(s)} \geq \frac{\sum_{s' \in S(\sigma') \cup S'(\sigma')} \lambda(s', \sigma') V(s')}{\sum_{s' \in S(\sigma') \cup S'(\sigma')} \lambda(s', \sigma') V'(s')}. \quad (51)$$

can be simplified into

$$\frac{\sum_{s \in S(\sigma)} \lambda(s, \sigma) V(s)}{\sum_{s \in S(\sigma)} \lambda(s, \sigma) V'(s)} \geq \frac{\sum_{s' \in S(\sigma')} \lambda(s', \sigma') V(s')}{\sum_{s' \in S(\sigma')} \lambda(s', \sigma') V'(s')}.$$

which is equivalent to

$$\sum_{s \in S(\sigma)} \sum_{s' \in S(\sigma')} \lambda(s, \sigma) \lambda(s', \sigma') V(s) V'(s') \geq \sum_{s \in S(\sigma)} \sum_{s' \in S(\sigma')} \lambda(s, \sigma) \lambda(s', \sigma') V(s') V'(s).$$

By PAM, we know that $\sigma > \sigma'$ implies $s \geq s'$ for all $s \in S(\sigma)$ and $s' \in S(\sigma')$. In addition, $V \succeq_a V'$ implies $V(s) V'(s') \geq V(s') V'(s)$ for all $s \geq s'$. Hence the previous inequality must hold. Combining Inequalities (49)-(51), we obtain $Y(\sigma)/Y'(\sigma) > Y(\sigma')/Y'(\sigma')$. ■

Lemma 13 *For any $\sigma \in \Sigma$, if $s \in (\underline{s}(\sigma), \bar{s}(\sigma))$, then $\lambda(s, \sigma) = 1$.*

Proof. We proceed by contradiction. If $\lambda(s, \sigma) \neq 1$, then there must be $\sigma' \neq \sigma$ such that $s \in S(\sigma')$. Without loss of generality, suppose that $\sigma' < \sigma$. Profit maximization under $S(\cdot)$ therefore requires

$$p(\sigma)/p(\sigma') = \frac{A(s, \sigma')}{A(s, \sigma)}.$$

Let $\underline{s} = \inf S(\sigma)$. Profit maximization under $S(\cdot)$ also requires

$$p(\sigma)/p(\sigma') \geq \frac{A(\underline{s}(\sigma), \sigma')}{A(\underline{s}(\sigma), \sigma)}.$$

Since A is strictly log-supermodular, $\sigma' < \sigma$ and $\underline{s}(\sigma) < s$ implies

$$\frac{A(\underline{s}(\sigma), \sigma')}{A(\underline{s}(\sigma), \sigma)} > \frac{A(s, \sigma')}{A(s, \sigma)}.$$

A contradiction. ■

Lemma 14 *For any pair sectors $\sigma \geq \sigma'$, if there exist $s \geq s'$ such that $s' \in S(\sigma)$ and $s \in S'(\sigma')$, then: (i) $p'(\sigma')/p'(\sigma) \geq p(\sigma')/p(\sigma)$; and (ii) $Y(\sigma')/Y'(\sigma') \geq Y(\sigma)/Y'(\sigma)$.*

Proof. Profit maximization under $S'(\cdot)$ requires

$$p'(\sigma')/p'(\sigma) \geq \frac{A(s, \sigma)}{A(s, \sigma')}.$$

Similarly, profit maximization under $S(\cdot)$ requires

$$p(\sigma')/p(\sigma) \leq \frac{A(s', \sigma)}{A(s', \sigma')}.$$

Since A is log-supermodular, $\sigma' \leq \sigma$ and $s' \leq s$ implies

$$\frac{A(s, \sigma)}{A(s, \sigma')} \geq \frac{A(s', \sigma)}{A(s', \sigma')}$$

Combining the previous inequalities, we obtain condition (i):

$$p'(\sigma')/p'(\sigma) \geq p(\sigma')/p(\sigma).$$

This last inequality and CES preferences imply

$$Y'(\sigma')/Y'(\sigma) \leq Y(\sigma')/Y(\sigma).$$

which can be rearranged as condition (ii):

$$Y(\sigma')/Y'(\sigma') \geq Y(\sigma)/Y'(\sigma).$$

This completes the proof of Lemma 14. ■

C.2 Skill abundance and matching

We are now ready to derive the counterpart of Lemma 3 in the discrete case.

Theorem 1 *Suppose that $V \succeq_a V'$, then $S'(\sigma) \leq S(\sigma)$ (SSO) for all $\sigma \in \Sigma$.*

Proof We proceed by contradiction. Suppose that there exists $\sigma \in \Sigma$ such that $S'(\sigma) \not\leq S(\sigma)$. So we can define $\sigma_k = \inf \{\sigma \in \Sigma | S'(\sigma) \not\leq S(\sigma)\}$. The rest of the proof is decomposed into 4 Lemmas.

Lemma 15 *There exists $\underline{\sigma}^0 \leq \sigma_k$ such that: (i) either $\bar{s}(\underline{\sigma}^0) < \bar{s}'(\underline{\sigma}^0)$ or $\bar{s}(\underline{\sigma}^0) \leq \bar{s}'(\underline{\sigma}^0)$ and $\lambda'[\bar{s}(\underline{\sigma}^0), \underline{\sigma}^0] > \lambda[\bar{s}(\underline{\sigma}^0), \underline{\sigma}^0]$; (ii) $\underline{s}(\underline{\sigma}^0) \geq \underline{s}'(\underline{\sigma}^0)$; and (iii) $Y(\underline{\sigma}^0)/Y'(\underline{\sigma}^0) \geq Y(\sigma_k)/Y'(\sigma_k)$.*

Proof. There are two possible cases.

Case 1: $\underline{s}'(\sigma_k) \leq \underline{s}(\sigma_k)$. In this case, $S'(\sigma_k) \not\leq S(\sigma_k)$ implies $\bar{s}(\sigma_k) < \bar{s}'(\sigma_k)$. Therefore $\underline{\sigma}^0 \equiv \sigma_k$ trivially satisfies conditions (i)-(iii).

Case 2: $\underline{s}(\sigma_k) < \underline{s}'(\sigma_k)$. In this case, we first show the existence of $\underline{\sigma}^0 < \sigma_k$ such that $\bar{s}(\underline{\sigma}^0) \leq \bar{s}'(\underline{\sigma}^0)$ and $\lambda'[\bar{s}(\underline{\sigma}^0), \underline{\sigma}^0] > \lambda[\bar{s}(\underline{\sigma}^0), \underline{\sigma}^0]$. We proceed in three steps. First note

that $S'(\sigma) \leq S(\sigma)$ for all $\sigma < \sigma_k$. Hence we must have $\underline{\sigma}[\underline{s}(\sigma_k)] \leq \underline{\sigma}'[\underline{s}(\sigma_k)]$. Second note that $\underline{s}(\sigma_k) < \underline{s}'(\sigma_k)$ implies $\bar{\sigma}'[\underline{s}(\sigma_k)] < \sigma_k$. Combining these two observations, we get $\Sigma'[\underline{s}(\sigma_k)] \subset \Sigma[\underline{s}(\sigma_k)]$. Thus there must be $\underline{\sigma}^0 \in \Sigma'[\underline{s}(\sigma_k)]$ such that $\underline{\sigma}^0 < \sigma_k$ and $\lambda'[\underline{s}(\sigma_k), \underline{\sigma}^0] > \lambda[\underline{s}(\sigma_k), \underline{\sigma}^0]$. Third note that PAM implies $\underline{s}(\sigma_k) = \bar{s}(\sigma)$ for all $\sigma \in \Sigma[\underline{s}(\sigma_k)]$. Therefore we have $\underline{s}(\sigma_k) = \bar{s}(\underline{\sigma}^0)$, and in turn, $\lambda'[\bar{s}(\underline{\sigma}^0), \underline{\sigma}^0] > \lambda[\bar{s}(\underline{\sigma}^0), \underline{\sigma}^0]$. Since $\underline{\sigma}^0 \in \Sigma'[\underline{s}(\sigma_k)]$, we further have $\bar{s}'(\underline{\sigma}^0) \geq \underline{s}(\sigma_k) = \bar{s}(\underline{\sigma}^0)$. Hence condition (i) is satisfied. We now turn to conditions (ii) and (iii). Since $S'(\sigma) \leq S(\sigma)$ for all $\sigma < \sigma_k$, $\underline{\sigma}^0 < \sigma_k$ directly implies condition (ii). To show that $Y(\underline{\sigma}^0)/Y'(\underline{\sigma}^0) \geq Y(\sigma_k)/Y'(\sigma_k)$, we use Lemma 14. By construction, we have $\sigma_k \geq \underline{\sigma}^0$, $\underline{s}(\sigma_k) \in S(\sigma_k)$, and $\underline{s}(\sigma_k) \in S'(\underline{\sigma}^0)$. Therefore Lemma 14 directly implies condition (iii). ■

Lemma 16 *There exists $\bar{\sigma}^0 \geq \sigma_k$ such that: (i) either $\underline{s}(\bar{\sigma}^0) < \underline{s}'(\bar{\sigma}^0)$ or $\underline{s}(\bar{\sigma}^0) \leq \underline{s}'(\bar{\sigma}^0)$ and $\lambda[\underline{s}(\bar{\sigma}^0), \bar{\sigma}^0] > \lambda'[\underline{s}(\bar{\sigma}^0), \bar{\sigma}^0]$; and (ii) $Y(\sigma_k)/Y'(\sigma_k) \geq Y(\bar{\sigma}^0)/Y'(\bar{\sigma}^0)$.*

Proof. Since $S'(\sigma_k) \not\leq S(\sigma_k)$, there are two possible cases.

Case 1: $\underline{s}(\sigma_k) < \underline{s}'(\sigma_k)$. In this case, $\bar{\sigma}^0 \equiv \sigma_k$ trivially satisfies conditions (i) and (ii).

Case 2: $\bar{s}(\sigma_k) < \bar{s}'(\sigma_k)$. In this case, PAM implies $\underline{s}(\sigma_{k+1}) \leq \underline{s}'(\sigma_{k+1})$. We now distinguish between two separate subcases.

Case 2-a: $\underline{s}(\sigma_{k+1}) < \underline{s}'(\sigma_{k+1})$. In this subcase, $\bar{\sigma}^0 \equiv \sigma_{k+1}$ trivially satisfies condition (i). To show that $Y(\sigma_k)/Y'(\sigma_k) \geq Y(\bar{\sigma}^0)/Y'(\bar{\sigma}^0)$, we use Lemma 14. By PAM, we have $s = \bar{s}'(\sigma_k) \geq s' = \underline{s}(\sigma_{k+1})$ such that $s' \in S(\sigma_{k+1})$ and $s \in S'(\sigma_k)$. Lemma 14 therefore implies $Y(\sigma_k)/Y'(\sigma_k) \geq Y(\sigma_{k+1})/Y'(\sigma_{k+1})$, and in turn, condition (ii).

Case 2-b: $\underline{s}(\sigma_{k+1}) = \underline{s}'(\sigma_{k+1})$. In this subcase, PAM implies $\bar{s}(\sigma_k) < \underline{s}(\sigma_{k+1})$ and $\bar{s}'(\sigma_k) = \underline{s}'(\sigma_{k+1})$. Hence we have $\underline{\sigma}[\underline{s}(\sigma_{k+1})] > \underline{\sigma}'[\underline{s}(\sigma_{k+1})]$. If $\bar{\sigma}[\underline{s}(\sigma_{k+1})] > \underline{\sigma}'[\underline{s}(\sigma_{k+1})]$, then we can set $\bar{\sigma}^0 \equiv \bar{\sigma}[\underline{s}(\sigma_{k+1})]$, which satisfies $\underline{s}(\bar{\sigma}^0) < \underline{s}'(\bar{\sigma}^0)$, and so, condition (i); and, by Lemma 14, condition (ii) is satisfied as well. If, on the other hand, $\bar{\sigma}[\underline{s}(\sigma_{k+1})] \leq \underline{\sigma}'[\underline{s}(\sigma_{k+1})]$, then we have $\Sigma[\underline{s}(\sigma_{k+1})] \subset \Sigma'[\underline{s}(\sigma_{k+1})]$. Therefore, there must be $\bar{\sigma}^0 \in \Sigma[\underline{s}(\sigma_{k+1})]$ such that $\bar{\sigma}^0 > \sigma_k$ and $\lambda[\underline{s}(\sigma_{k+1}), \bar{\sigma}^0] > \lambda'[\underline{s}(\sigma_{k+1}), \bar{\sigma}^0]$. Finally note that PAM implies $\underline{s}(\sigma_{k+1}) = \underline{s}(\sigma)$ for all $\sigma \in \Sigma[\underline{s}(\sigma_{k+1})]$. Thus we have $\underline{s}(\sigma_{k+1}) = \underline{s}(\bar{\sigma}^0)$, and in turn, $\lambda[\underline{s}(\bar{\sigma}^0), \bar{\sigma}^0] > \lambda'[\underline{s}(\bar{\sigma}^0), \bar{\sigma}^0]$. By construction, we have $\underline{s}(\bar{\sigma}^0) = \underline{s}'(\bar{\sigma}^0) = \underline{s}(\sigma_{k+1})$, and so, condition (i) is satisfied. By Lemma 14, condition (ii) is satisfied as well. ■

Lemma 17 *There exists a sequence $(\underline{s}^n, \underline{\sigma}^n)$ such that: (i) $\underline{s}^n = \underline{s}(\underline{\sigma}^{n-1}) = \underline{s}'(\underline{\sigma}^{n-1})$ with $\lambda(\underline{s}^n, \underline{\sigma}^{n-1}) > \lambda'(\underline{s}^n, \underline{\sigma}^{n-1})$; and (ii) $\underline{\sigma}^n = \underline{\sigma}(\underline{s}^n) = \underline{\sigma}'(\underline{s}^n)$ with $\lambda'(\underline{s}^n, \underline{\sigma}^n) > \lambda(\underline{s}^n, \underline{\sigma}^n)$.*

Proof. To construct a sequence $(\underline{s}^n, \underline{\sigma}^n)$ satisfying conditions (i) and (ii), we iterate the following steps.

Step 1: *There exists \underline{s}^1 such that $\underline{s}^1 = \underline{s}(\underline{\sigma}^0) = \underline{s}'(\underline{\sigma}^0)$ with $\lambda(\underline{s}^1, \underline{\sigma}^0) > \lambda'(\underline{s}^1, \underline{\sigma}^0)$.*

We set $\underline{s}^1 = \underline{s}(\underline{\sigma}^0)$. To show that $\underline{s}(\underline{\sigma}^0) = \underline{s}'(\underline{\sigma}^0)$ and $\lambda[\underline{s}(\underline{\sigma}^0), \underline{\sigma}^0] > \lambda'[\underline{s}(\underline{\sigma}^0), \underline{\sigma}^0]$, we then proceed by contradiction. Suppose that $\underline{s}(\underline{\sigma}^0) \neq \underline{s}'(\underline{\sigma}^0)$ or that $\underline{s}(\underline{\sigma}^0) = \underline{s}'(\underline{\sigma}^0)$ and $\lambda[\underline{s}(\underline{\sigma}^0), \underline{\sigma}^0] \leq \lambda'[\underline{s}(\underline{\sigma}^0), \underline{\sigma}^0]$.

Step 1.1: $L' \succ_{\underline{\sigma}^0} L$. This directly derives from Lemma 13, Lemma 15 conditions (i) and (ii), and the assumption that $\underline{s}(\underline{\sigma}^0) \neq \underline{s}'(\underline{\sigma}^0)$ or $\underline{s}(\underline{\sigma}^0) = \underline{s}'(\underline{\sigma}^0)$ and $\lambda[\underline{s}(\underline{\sigma}^0), \underline{\sigma}^0] \leq \lambda'[\underline{s}(\underline{\sigma}^0), \underline{\sigma}^0]$.

Step 1.2: *There exist $\sigma \geq \sigma_k$ such that $L \succ_{\sigma} L'$ and $Y(\sigma_k)/Y'(\sigma_k) \geq Y(\sigma)/Y'(\sigma)$.* We use the following iterative procedure. If $\bar{s}(\bar{\sigma}^0) > \bar{s}'(\bar{\sigma}^0)$ or $\bar{s}(\bar{\sigma}^0) = \bar{s}'(\bar{\sigma}^0)$ with $\lambda[\bar{s}(\bar{\sigma}^0), \bar{\sigma}^0] \geq \lambda'[\bar{s}(\bar{\sigma}^0), \bar{\sigma}^0]$, then by Lemma 16, we set $\sigma = \bar{\sigma}^0$ and stop. Otherwise, we show that there exists $\bar{\sigma}^1 > \bar{\sigma}^0$ such that: (i) $\underline{s}(\bar{\sigma}^1) < \underline{s}'(\bar{\sigma}^1)$ or $\underline{s}(\bar{\sigma}^1) \leq \underline{s}'(\bar{\sigma}^1)$ and $\lambda[\underline{s}(\bar{\sigma}^1), \bar{\sigma}^1] > \lambda'[\underline{s}(\bar{\sigma}^1), \bar{\sigma}^1]$; and (ii) $Y(\sigma_k)/Y'(\sigma_k) \geq Y(\bar{\sigma}^1)/Y'(\bar{\sigma}^1)$. To do so, we consider two cases separately. If $\bar{s}(\bar{\sigma}^0) < \bar{s}'(\bar{\sigma}^0)$, the same argument as in Lemma 16 case 2 implies the existence of $\bar{\sigma}^1 > \bar{\sigma}^0$ satisfying conditions (i) and (ii). If $\bar{s}(\bar{\sigma}^0) = \bar{s}'(\bar{\sigma}^0)$ and $\lambda[\bar{s}(\bar{\sigma}^0), \bar{\sigma}^0] < \lambda'[\bar{s}(\bar{\sigma}^0), \bar{\sigma}^0]$, we first note that by Lemma 16, we necessarily have $\bar{s}(\bar{\sigma}^0) \neq \underline{s}(\bar{\sigma}^0)$, which implies $\underline{\sigma}[\bar{s}(\bar{\sigma}^0)] = \bar{\sigma}^0$, and in turn, $\underline{\sigma}[\bar{s}(\bar{\sigma}^0)] \geq \underline{\sigma}'[\bar{s}(\bar{\sigma}^0)]$. We can then use the same argument as in Lemma 16 case 2-b to establish the existence of $\bar{\sigma}^1 > \bar{\sigma}^0$ satisfying conditions (i) and (ii). If $\bar{s}(\bar{\sigma}^1) > \bar{s}'(\bar{\sigma}^1)$ or $\bar{s}(\bar{\sigma}^1) = \bar{s}'(\bar{\sigma}^1)$ with $\lambda[\bar{s}(\bar{\sigma}^1), \bar{\sigma}^1] \geq \lambda'[\bar{s}(\bar{\sigma}^1), \bar{\sigma}^1]$, we set $\sigma = \bar{\sigma}^1$ and stop. Otherwise, we show, using the same argument as before, that there exists $\bar{\sigma}^2 > \bar{\sigma}^0$ such that: (i) $\underline{s}(\bar{\sigma}^2) < \underline{s}'(\bar{\sigma}^2)$ or $\underline{s}(\bar{\sigma}^2) \leq \underline{s}'(\bar{\sigma}^2)$ and $\lambda[\underline{s}(\bar{\sigma}^2), \bar{\sigma}^2] > \lambda'[\underline{s}(\bar{\sigma}^2), \bar{\sigma}^2]$; and (ii) $Y(\sigma_k)/Y'(\sigma_k) \geq Y(\bar{\sigma}^2)/Y'(\bar{\sigma}^2)$ etc... Since there only are a finite number of values of σ and since $\lambda[\bar{s}, \bar{\sigma}] \geq \lambda'[\bar{s}, \bar{\sigma}]$ if $\bar{s}(\bar{\sigma}) \neq \underline{s}(\bar{\sigma})$, such an algorithm must converge towards $\sigma \geq \sigma_k$ such that $L \succ_{\sigma} L'$ and $Y(\sigma_k)/Y'(\sigma_k) \geq Y(\sigma)/Y'(\sigma)$.

To conclude the proof of Step 1, we use Lemma 12. Combining Lemma 12 with Step 1.1, Step 1.2, and $V \succeq_a V'$, we get $Y(\underline{\sigma}^0)/Y'(\underline{\sigma}^0) < Y(\sigma)/Y'(\sigma)$. By Lemma 15 and Step 1.2, we have $Y(\underline{\sigma}^0)/Y'(\underline{\sigma}^0) \geq Y(\sigma_k)/Y'(\sigma_k) \geq Y(\sigma)/Y'(\sigma)$. A contradiction.

Step 2: *There exists $\underline{\sigma}^1$ such that $\underline{\sigma}^1 = \underline{\sigma}(\underline{s}^1) = \underline{\sigma}'(\underline{s}^1)$ with $\lambda'(\underline{s}^1, \underline{\sigma}^1) > \lambda(\underline{s}^1, \underline{\sigma}^1)$.*

The formal argument used in Step 2 and all subsequent steps is similar to the one used in Step 1 and is omitted. ■

Lemma 18 *There exists $\bar{n} \geq 1$ such that $\underline{s}^{\bar{n}} = \underline{s}^{\bar{n}+1}$.*

Proof. By construction, we have $\underline{s}^{n+1} \leq \underline{s}^n$ for all n . By assumption, we also have $\underline{s}^n \geq s_1$ for all n . Combining these two observations, there must $\bar{n} \geq 1$ such that $\underline{s}^{\bar{n}} = \underline{s}^{\bar{n}+1}$. ■

(Proof of Theorem 1 continued). To conclude the proof of Theorem 1, note that $\underline{s}^{\bar{n}} = \underline{s}^{\bar{n}+1}$ implies $\underline{\sigma}^{\bar{n}-1} = \underline{\sigma}^{\bar{n}}$. Therefore, we must also have $\lambda(s^{\bar{n}}, \sigma^{\bar{n}-1}) > \lambda'(s^{\bar{n}}, \sigma^{\bar{n}-1})$ and $\lambda'(s^{\bar{n}}, \sigma^{\bar{n}}) > \lambda(s^{\bar{n}}, \sigma^{\bar{n}})$. A contradiction. ■

C.3 Skill abundance and wages

Like in the continuum-by-continuum case, changes in matching caused by changes in skill abundance have strong implications for the distribution of wages.

Theorem 2 *Suppose that $V \succeq_a V'$. Then $w(s)/w(s') \leq w'(s)/w'(s')$ for all $s' \leq s$.*

Proof We first show that $p'(\sigma_{k+1})/p'(\sigma_k) \geq p(\sigma_{k+1})/p(\sigma_k)$ for any pair of adjacent sectors $\sigma_1 \leq \sigma_k < \sigma_{k+1} \leq \sigma_N$. We consider two cases separately.

Case 1: There exist $s \geq s'$ such that $s \in S(\sigma_k)$ and $s' \in S'(\sigma_{k+1})$. In this case, Lemma 14 directly implies $p'(\sigma_{k+1})/p'(\sigma_k) \geq p(\sigma_{k+1})/p(\sigma_k)$.

Case 2: There does not exist $s \geq s'$ such that $s \in S(\sigma_k)$ and $s' \in S'(\sigma_{k+1})$. By Theorem 1, we know that $V \succeq_a V'$ implies $S'(\sigma) \leq S(\sigma)$ (SSO) for all $\sigma \in \Sigma$. We can therefore use the following lemmas.

Lemma 19 *For any pair of adjacent sectors $\sigma_1 \leq \sigma_k < \sigma_{k+1} \leq \sigma_N$, if there does not exist $s \geq s'$ such that $s \in S(\sigma_k)$ and $s' \in S'(\sigma_{k+1})$, then $S'(\sigma) \leq S(\sigma)$ (SSO) for all $\sigma \in \Sigma$ implies: (i) $\sup S(\sigma_k) = \sup S'(\sigma_k) = s_m$; and (ii) $\inf S(\sigma_{k+1}) = \inf S'(\sigma_{k+1}) = s_{m+1}$ for $1 \leq s_m < s_M$.*

Proof. Let $s_{m+1} = \inf S'(\sigma_{k+1})$ for $1 \leq m < M$. If there does not exist $s \geq s'$ such that $s \in S(\sigma_k)$ and $s' \in S'(\sigma_{k+1})$, then $\sup S(\sigma_k) < \inf S'(\sigma_{k+1})$, which can be rearranged as $\sup S(\sigma_k) \leq s_m$. By PAM, we know that $\sup S'(\sigma_k) \geq s_m$. By assumption, we also know that $\sup S(\sigma_k) \geq \sup S'(\sigma_k)$. Combining the last three inequalities, we obtain $\sup S(\sigma_k) = \sup S'(\sigma_k) = s_m$. The argument for property (ii) is similar. On the one hand, $S'(\sigma) \leq S(\sigma)$ (SSO) for all $\sigma \in \Sigma$ implies $s_{m+1} = \inf S'(\sigma_{k+1}) \leq \inf S(\sigma_{k+1})$. On the other hand, PAM and $\sup S(\sigma_k) = s_m$ imply $\inf S(\sigma_{k+1}) \leq s_{m+1}$. Combining the last two inequalities, we obtain $\inf S(\sigma_{k+1}) = \inf S'(\sigma_{k+1}) = s_{m+1}$. This completes the proof of Lemma 19. ■

Lemma 20 *Suppose that σ_k and σ_{k+1} satisfy conditions (i) and (ii) in Lemma 19. Then we must have $L' \succ_{\sigma_k} L$ and $L \succ_{\sigma_{k+1}} L'$.*

Proof. The formal argument is similar to the one used in Lemma 17 and omitted. ■
(Proof of Theorem 2 continued). Since $V \succeq_a V'$, Lemmas 12 and 20 imply

$$Y(\sigma_{k+1})/Y'(\sigma_{k+1}) > Y(\sigma_k)/Y'(\sigma_k).$$

Combining the previous inequality with CES preferences, we obtain

$$p'(\sigma_{k+1})/p'(\sigma_k) \geq p(\sigma_{k+1})/p(\sigma_k).$$

At this point, we have shown that $p'(\sigma_{k+1})/p'(\sigma_k) \geq p(\sigma_{k+1})/p(\sigma_k)$ for any pair of adjacent sectors $\sigma_1 \leq \sigma_k < \sigma_{k+1} \leq \sigma_N$. By transitivity, this implies $p'(\sigma)/p'(\sigma') \geq p(\sigma)/p(\sigma')$ for any pair of sectors $\sigma' \leq \sigma$. To conclude, we note that the previous inequality, PAM, and the zero profit condition imply $w(s)/w(s') \leq w'(s)/w'(s')$ for all $s' \leq s$. **QED.** ■

D Proofs (IV): Observable versus Unobservable Skills

Proof of Proposition 6. We first demonstrate part (i) of Inequality (29) for the Home country. Let $w(s, x) \equiv w_H(s)$ if $x = 1$ and $w(s, x) \equiv w_T(s)$ if $x = 2$. By Inequality (21), $w(s, x)$ is log-supermodular. By assumption, $V_H(s, e)$ is log-supermodular. We know that log-supermodularity is preserved by multiplication and integration; see e.g. Karlin and Rinott (1980). Therefore $\bar{w}(e, x) \equiv \int_{s \in S} w(s, x) V_H(s, e) ds$ must be log-supermodular. This directly implies $\frac{\bar{w}_H(e)}{\bar{w}_H(e')} \leq \frac{\bar{w}_T(e)}{\bar{w}_T(e')}$. The argument in the Foreign country is similar. Part (ii) directly derives from the fact that Inequality (21) holds for all $s \geq s'$. **QED.** ■

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