Ratings Quality over the Business Cycle*

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Abstract

The reduced accuracy of credit ratings on structured finance products in the boom just preceding the financial crisis has prompted investigation into the business of Credit Rating Agencies (CRAs). While CRAs have long held that their behavior is disciplined by reputational concerns, the value of reputation depends on economic fundamentals that vary over the business cycle. These include income from fees, default probabilities for the securities rated, competition in the labor market for analysts, and expectations about the future. We analyze a dynamic model of ratings where reputation is endogenous and the market environment may vary over time. We find that a CRA is more likely to issue less accurate ratings in boom times than during recessionary periods. Persistence in economic conditions can diminish our results, while mean reversion exacerbates them. Finally, we demonstrate that competition among CRAs yields similar qualitative results.

Keywords: Credit rating agencies, reputation, ratings accuracy

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1 Introduction

The current financial crisis has prompted an examination of the role of credit rating agencies (CRAs). With the rise of structured finance products, the agencies rapidly expanded their ratings business and earned dramatically higher profits (Moody's, for example, tripled its profits between 2002 and 2006). Yet ratings quality seems to have suffered, as the three main agencies increasingly gave top ratings to structured finance products shortly before the financial markets collapsed. This type of behavior has been brought to the public's (and regulators') attention many times, such as during the East Asian Financial Crisis (1997) and the failures of Enron (2001) and Worldcom (2002). Beyond the issue of why the CRAs were off target, these repeated instances raise the question of *when* the CRAs are more likely to be off target.

In this paper, we examine theoretically how the incentives of CRAs to provide quality ratings change in different economic environments, specifically in the booms and recessions of business cycles. Our analysis highlights that both the effective costs of providing high quality ratings and the benefits to the CRA of doing so vary through the business cycle. Specifically, we show that reputation incentives lead naturally to countercyclical ratings quality.

Several moving parts suggest that ratings quality is lower in booms and improves in recessions. First, consider that a CRA's primary expenditure is in skilled human capital. In boom periods, the outside options of current and prospective employees improve substantially, making it more difficult and expensive for a CRA to maintain the same quality of analyst resources.[1] Next, if issues are relatively unlikely to default in boom periods, monitoring a CRA's activities is less effective, and the CRA's returns from investing in ratings quality are likely to be diminished. Furthermore, boom periods are likely to be associated with higher revenues for a CRA, both directly—through higher volume of issues and, perhaps, through higher fees—and indirectly—through advisory and other ancillary services. If a CRA anticipates that boom periods will not continue indefinitely and expects leaner times ahead, it may seek to "milk" its reputation in booms and build its reputation in lean times (when it is relatively cheap to do so).

[1] For example, "At the height of the mortgage boom, companies like Goldman offered million-dollar pay packages to workers like Mr. Yukawa who had been working at much lower pay at the rating agencies, according to several former workers at the agencies. Around the same time that Mr. Yukawa left Fitch, three other analysts in his unit also joined financial companies like Deutsche Bank." This is from "Prosecutors Ask if 8 Banks Duped Rating Agencies," by L. Story, *New York Times*, May 12, 2010.

We formalize these intuitions in a simple model of ratings reputation. We construct an infinite period model where a CRA chooses in each period how much to spend on the accuracy of its ratings by hiring better analysts. The CRA continues to receive fees from issuers as long as it maintains its reputation with investors, who withdraw their business only after an investment with a good rating defaults.

The fundamentals of the economy are characterized by the fees received from the issuer, the probability that an investment will default, labor-market conditions for analysts, and the proportion of investments that are good. In our baseline model, where future shocks are iid draws from a probability distribution, we find unambiguous support for countercyclical ratings quality. We then extend the model to allow for correlation between shocks in different periods. Our findings may be diminished when there is substantial persistence in shocks (positive correlation), but may actually be exacerbated when there is mean-reversion in shocks (negative correlation). We also extend the model to allow for competition between CRAs and demonstrate that similar results hold.

The idea that ratings quality may be countercyclical is consistent with recent empirical work on the market for structured finance products. As a relatively new market for hard-to-evaluate investments, the structured finance market opened up the possibility for accuracy and reputation management by CRAs. Ashcraft, Goldsmith-Pinkham, and Vickery (2010) show that the mortgage-backed security-issuance boom from 2005 to mid-2007 led to ratings quality declines. Griffin and Tang (2009) demonstrate that CRAs made mostly positive adjustments to their models' predictions of credit quality and that the amount adjusted increased substantially from 2003 to 2007. These adjustments were positively related to future downgrades.

Our results are relevant to the current policy debate regarding the role of CRAs. We show that if reputation losses are higher, there are greater incentives to provide accurate ratings. Recent SEC rules promoting full disclosure of ratings history can make it easier for investors to know when a CRA is performing poorly and to punish it. The Dodd-Frank financial reform bill makes CRAs more exposed to liability claims for poor performance. This may give the investors a stick to make punishment credible.

White (2010) highlights the role that regulation has played in enhancing the importance and market power of the three major rating agencies (by granting them a special status and having capital and investment requirements tied to ratings). Given the "protected" position of these agencies,[2] the reputational concerns that discipline CRAs' behavior should be

[2] The Dodd-Frank bill and rulemaking by the SEC will most likely diminish the status of the big three

understood somewhat more broadly than the reduced-form approach taken in our model.[3] The model views the reputational concerns that constrain a CRA's behavior as arising from a reduction in the number of issuers that seek a rating. Although this might appear stark, it may apply well to innovative financial instruments, which have been the focus of public and policy concerns. Indeed, the structured finance market (and the need for ratings) dried up as the crisis hit. The CRAs' reputational incentives may also be viewed as being determined through a regulatory environment that is relatively more or less sympathetic to the CRAs. Lastly, although something similar has not occurred in the recent crisis, the downfall of Arthur Andersen represents a severe punishment to a certification intermediary in a similar business line (auditing).

In the following subsection, we review related theoretical work. In section 6, we formulate the predictions of the model as hypotheses and examine support from recent empirical work.

1.1 Related Theoretical Literature

Mathis, McAndrews, and Rochet (2009) is the closest paper to this one in examining how a CRA's concern for its reputation affects its ratings quality. They present a dynamic model of reputation where a monopolist CRA may mix between lying and truthtelling to build up/exploit its reputation. They focus on whether the equilibrium where the CRA tells the truth in every period exists and demonstrate that truthtelling incentives are weaker when the CRA has more business from rating complex products.[4] Strausz (2005) is similar in structure to Mathis et al. (2009), but examines information intermediaries in general. Our model generalizes their ideas to a richer environment where CRA incentives are linked to a broad set of economic fundamentals that fluctuate and may persist through time. Our paper demonstrates the robustness of these effects to competition and introduces some additional features, such as the connection with labor-market conditions.

Our model also builds on and develops the understanding of firm behavior in business cycles. Several papers analyze how firms maintain collusive behavior through the business cycle, while we analyze incentives to build up or milk reputation. Rotemberg and Saloner (1986) and Dal Bo (2007) consider future states to be iid draws from a known distribution,

CRAs (Standard & Poor's, Moody's, and Fitch).

[3] And in related models of endogenous reputation, such as Mathis, McAndrews and Rochet (2009).

[4] Mathis et al. provide examples of reputation cycles where the CRAís reputational incentives fluctuate, depending on the current level of reputation. These are not linked to economic fundamentals of the business cycle, as they are in our model.

as we do in our main model. Haltiwanger and Harrington (1991) consider a deterministic business cycle. Bagwell and Staiger (1997) and Kandori (1991) add correlation between periods, as we do in the generalization of our model.

In addition to Mathis et al. (2009), there are several other recent theoretical papers on CRAs. Faure-Grimaud, Peyrache and Quesada (2009) look at corporate governance ratings in a market with truthful CRAs and rational investors. They show that issuers may prefer to suppress their ratings if they are too noisy. They also find that competition between rating agencies can result in less information disclosure. Mariano (2008) considers how reputation disciplines a CRA's use of private information when public information is also available. Fulghieri, Strobl and Xia (2010) focus on the effect of unsolicited ratings on CRA and issuer incentives. Bolton, Freixas, and Shapiro (2010) demonstrate that competition among CRAs may reduce welfare due to shopping by issuers. Conflicts of interest for CRAs may be higher when exogenous reputation costs are lower and there are more naïve investors. Skreta and Veldkamp (2009) and Sangiorgi, Sokobin and Spatt (2009) assume that CRAs relay their information truthfully, and they demonstrate how noisier information creates more opportunity for issuers to take advantage of a naive clientele through shopping. In Pagano and Volpin (2009), CRAs also have no conflicts of interest, but can choose ratings to be more or less opaque depending on what the issuer asks for. They show that opacity can enhance liquidity in the primary market but may cause a market freeze in the secondary market.

2 Benchmark Model: Constant Economic Fundamentals

We present a model with a single CRA and many issuers and investors who can interact over an infinite number of discrete periods.[5] As a benchmark, we consider a situation where there is no business cycle and economic fundamentals remain constant across periods.

Each period, an issuer has a new investment. The investment can be good (G) or bad (B). A good investment never defaults and pays out 1. A bad investment defaults with probability p. If it defaults, its payout is zero; otherwise, its payout is 1. The probability that an investment is good is λ. The issuer has no private information about the investment. This implies that the CRA can have a welfare-increasing role of information production by identifying the quality of the investment. Both the issuer and the investors observe the ratings and performance of the investment.

The issuer approaches the CRA at the beginning of the period to evaluate its invest-

[5] Issuers and investors may be long- or short-lived in the model, whereas the CRA is long-lived.

ment. If the CRA gives a good rating, the issuer pays the CRA an amount π. The CRA is not paid for bad ratings. This is a version of the shopping effect described in Bolton, Freixas, and Shapiro (2010) and Skreta and Veldkamp (2009). Mathis, McAndrews, and Rochet (2009) assume that no issue takes place if the rating is bad and that the CRA is not paid in this case, which is equivalent to our approach.

Our focus is on the CRA's ratings policy—i.e., how they monitor or choose the likelihood that their analyses are correct. We model this as a direct cost to the CRA for improving its accuracy. There is no direct conflict of interest, as in Bolton, Freixas, and Shapiro (2010), and we remain agnostic about whether CRAs intentionally produce worse quality ratings. In our model, increasing rating quality is costly, and the CRA maximizes profits given the reality of the business environment.

The cost that the CRA pays for accurate ratings could represent improving analytical models and computing power, performing due diligence on the underlying assets, the staffing resources allocated to ratings, or hiring and retaining better analysts. For the sake of concreteness, we will focus on the employment channel: Hiring better analysts is more costly to the CRA.[6] Investors cannot directly observe the CRA's policy, but must infer it from their equilibrium expectations and from their previous observations of defaults on rated investments.

We model the analyst labor market in a reduced-form manner. In a given period, a CRA pays a wage $w \in [0, \overline{w}]$ to get an analyst of ability $z(w, \gamma) \in [0, 1]$, where γ is a parameter that captures labor-market conditions.[7] When there is no confusion, we suppress the arguments of z. We suppose that it is harder to attract and retain higher-ability analysts, and that it becomes harder at the top end of the wage distribution, meaning that $\frac{\partial z}{\partial w} > 0$ and $\frac{\partial z^2}{\partial w^2} < 0$. We also assume that $\frac{\partial z}{\partial w} \to \infty$ as $w \to 0$, $z(0, \gamma) = 0$, $\frac{\partial z}{\partial w}|_{\overline{w}} = 0$. With respect to the labor-market conditions, we suppose that when γ is larger, the labor market is tighter and it is more difficult to get high-quality workers, so that $\frac{\partial z}{\partial \gamma} < 0$ and $\frac{\partial^2 z}{\partial \gamma \partial w} < 0$.

[6] While there is no empirical work on CRA staffing, internal emails uncovered by the Senate Permanent Subcomittee on Investigations (2010) shed light on the CRAs' staffing situation right before the recent crisis. For example, a Standard & Poor's employee wrote on 10/31/2006: "While I realize that our revenues and client service numbers don't indicate any ill [e]ffects from our severe understaffing situation, I am more concerned than ever that we are on a downward spiral of morale, analytical leadership/quality and client service."

[7] Note that, here, the wage w is the wage per issue rated and, so, lower quality might reflect either a less able analyst or that an analyst (of equal quality) spends less time on a rating. We consider a spot-market for labor. In practice, it is likely that analysts require training and that their skills may improve through time, while employed at the CRA. Here, we ignore such effects.

This implies that a higher wage must be paid in order to maintain quality.

Ability is important for gathering information and figuring out whether the investment is good or bad. All analysts can identify a good investment perfectly ($p(G|G) = 1$). They may, however, make an error about the bad investment with positive probability $1-z$, where $p(B|B) = z$. Therefore, the CRA, through its wage, is choosing its tolerance for mistakes based on both the costs of hiring *and* the incentives for accuracy that are embedded in the dynamics of the model.[8]

These incentives for accuracy arise since we assume that if investors suspect that the CRA is not investing sufficiently in the ratings quality (say $z < \overline{z}$), then they would not purchase the investment product; however, if investors believe that the CRA has hired sufficiently good analysts ($z \geq \overline{z}$), then the rating is of sufficient quality to lead investors to purchase. The cutoff $\overline{z}$ is exogenous here, but it represents the investor's decision to allocate money to this investment as opposed to other opportunities; that is, it could be derived from a participation-constraint or portfolio-allocation problem for the investor. We suppose throughout, that while the CRA maintains its reputation the constraint $z \geq \overline{z}$ does not bind; trivially, if the constraint was violated then investors would not purchase and so issuers would not seek ratings; in this case, the CRA would not be active.

As in any infinitely repeated game, there are many equilibria. We focus on the equilibrium where the CRA is most likely to report honestly or, equivalently, minimize mistakes: i.e., the equilibrium supported by grim-trigger-strategies (see Abreu, 1986). Issuers and investors observe only three states: a good report where the investment returns 1; a bad report; and a good report where the investment defaults. A grim-trigger-strategy here is that investors never purchase an investment rated by a CRA that had previously produced a good report for an investment that subsequently defaulted. This grim outcome is an equilibrium in the continuation game since, if investors do not purchase, then it is optimal for the CRA to set $w = 0$ so that $z < \overline{z}$. Moreover, this equilibrium of the infinitely repeated game has a natural interpretation corresponding to reputation. As in the seminal work of Klein and Leffler (1981), and developed in a wide-ranging literature discussed in Section 4 of Bar-Isaac and Tadelis (2008), the CRA sustains its reputation as long as it is not found to give a good rating to a bad investment, but loses its reputation if it is ever found to do so.

[8] While we have a very simplistic rating structure, we are able to capture the idea that ratings may be inflated—i.e., risky investments receive a stamp of being less risky.

2.1 Analysis

We examine the CRA's problem when economic fundamentals are constant over time. Supposing that the value of maintaining its reputation is V (a value which we characterize below and arises in equilibrium), we can write the CRA's decision as choosing w to maximize

$$V = \pi(\lambda + (1-\lambda)(1-z)) - w + \delta(1-(1-\lambda)(1-z)p)V. \quad (1)$$

This assumes that issuers approach the CRA, which, in turn, supposes that consumers anticipate that the equilibrium $z > \overline{z}$. Under this assumption, in the current period, the CRA pays the wage w and earns the fee whenever it reports a good project, which occurs when the project is good (with probability λ) or when the project is bad, and the employee misreports it (that is, with probability $(1-\lambda)(1-z)$). The probability that the project is bad, the agent misreports, and the project defaults is $(1-\lambda)(1-z)p$; then, in the continuation, no issuer returns to the CRA (anticipating that the CRA would set $w = 0$), and the CRA's continuation value is 0. Otherwise, the CRA earns the continuation value V.

The CRA would choose an optimal wage w^* to satisfy the following first order condition:[9]

$$-\pi(1-\lambda)\frac{\partial z}{\partial w} - 1 + \delta(1-\lambda)\frac{\partial z}{\partial w}pV = 0. \quad (2)$$

Then, the equilibrium requires that the continuation value V is consistent with the CRA's equilibrium decisions, as in (2), and so

$$V = \frac{\pi(\lambda + (1-\lambda)(1-z)) - w}{1-\delta(1-(1-\lambda)(1-z)p)}. \quad (3)$$

We denote the equilibrium wage and value function, defined as the simultaneous solutions to (2) and (3), by w^* and V^* and write z^* to denote $z(w^*, \gamma)$.

Lemma 1 *There is an equilibrium and it is unique; equivalently, the solution to equations (2) and (3), w^* and V^*, exists and is unique.*

Proof. The proof of this lemma and all other omitted proofs appear in the appendix. ■

Given such an equilibrium, we now ask how the CRA's incentives to provide accurate

[9]The first-order approach assumes that the corresponding second-order condition is satisfied. This is ensured by the following assumption on primitives: $\pi\lambda - \frac{1-\delta}{p\delta}\pi > \overline{w}$.

ratings vary with the economic fundamentals.[10] The following proposition characterizes these effects.

Proposition 1 *The CRA's equilibrium choice of investment in ratings quality (w^*) is increasing in the discount factor (δ) and the probability that a bad investment defaults (p) and decreasing if the labor market gets more competitive (γ increases), but is ambiguous in the other parameters: the payment that the CRA receives for a rating (π) and the proportion of good investments (λ).*

Most of these results are intuitive. Consider, first, the effect of the discount rate. The more the CRA values the future, the larger the wage it pays so that fewer mistakes are made and it continues generating rents in the future. Next, turn to labor-market conditions: If the labor market is more competitive, it is more expensive to hire a worker of equal ability, and the CRA's future returns will be lower; both effects lead the CRA to reduce the current wage.

The result on the probability of default is particularly interesting. An increase in the probability of default, p, has two countervailing effects. First, such an increase makes it more likely that the CRA will get caught for misreporting a bad project, and so there is a greater return to hiring a more-able analyst, who is less likely to misreport. However, the more likely it is that the CRA will get caught for misreporting in the future, the less valuable it is to maintain a good reputation, and so there is a greater incentive for the CRA to milk its current reputation and reduce the analyst's wage. We prove that the first effect dominates the second one.

An increase in the fee π that the CRA earns for a good report makes it more tempting to provide good reports today (by reducing w), but also makes it more valuable to survive into the future and continue earning fees—an opposing effect that suggests increasing w. Similarly, countervailing effects lead to ambiguous consequences for changes in the fraction of good investments.

3 Booms and Recessions

We now allow for economic fundamentals to change from period to period. This will vary the incentives of the CRA to produce accurate ratings over time. Specifically, it will lead to

[10] To fully characterize the equilibrium, it remains to verify that, in equilibrium, investors would indeed purchase if the CRA provides a rating. As discussed above, this requires that $z(w^*) > \overline{z}$. If this condition fails, the unique equilibrium is uninteresting: Issuers would not approach the CRA who would set $w = 0$ (or, equivalently, exit the market). Therefore, we suppose that this condition is satisfied.

our main result that ratings quality is countercyclical, i.e. ratings are less accurate in boom times than in recessions. In this section, we assume that the state is independently drawn in each period from the same distribution, though as we show in the following section, qualitative results extend to the case where independence is relaxed.

We parameterize the economic fundamentals by a parameter $s \in [\underline{s}, \bar{s}]$ that represents the state of the economy. Associated with each state are a set of parameters $(\lambda_s, p_s, \pi_s, \gamma_s)$. We order the states such that higher s corresponds to "better" states or booms. Specifically, we suppose that λ_s is non-decreasing in s since the proportion of good projects increases; p_s is non-increasing, reflecting that, in a boom, investments are less likely to default; π_s is non-decreasing in s, which reflects that fees are larger in booms; and γ_s is non-decreasing in s—that is, the labor market gets tighter in booms.[11][12] In each period, the state of the economy s is independently drawn from a continuous pdf $f(.)$, with associated cdf $F(.)$.

We define the expected value for the CRA at state s in equilibrium as V_s, which includes current profits and expected future profits. It is convenient to define $E(V) := \int_{\underline{s}}^{\bar{s}} V_s f(s) ds$. Analogous to the benchmark model, given that state s occurs today, the value function is:

$$V_s = \pi_s(\lambda_s + (1-\lambda_s)(1-z_s)) - w_s + \delta(1-(1-\lambda_s)(1-z_s)p_s)E(V), \tag{4}$$

where, as above, z_s is understood as $z(w_s, \gamma_s)$. In equilibrium, w_s is the optimally chosen wage in state s, given continuation values summarized by $E(V)$; that is, it is the solution to:

$$\arg\max_{w} \pi_s(\lambda_s + (1-\lambda_s)(1-z_s)) - w + \delta(1-(1-\lambda_s)(1-z_s)p_s)E(V), \tag{5}$$

or, equivalently, it is implicitly defined by:[13]

$$\frac{\partial z_s}{\partial w}|_{w_s} = \frac{1}{(1-\lambda_s)(\delta E(V)p_s - \pi_s)}. \tag{6}$$

Equilibrium is characterized by the simultaneous solution of the system of equations (4) and (5) defined at each s. As in the benchmark model, and as shown in Lemma 4 in the

[11] While the asssumptions on the ordering of labor-market conditions and fees are clear, λ_s may actually be decreasing at some point if booms attract lower-quality issuers or investments to get ratings. Similarly, p_s may be increasing at some point. These cases can be understood, given our results.

[12] Note, that while these parameters are correlated with macroeconomic business cycles, these fundamentals are unlikely to be perfectly correlated with traditional business cycle indicators such as aggregrate real growth or aggregate unemployment.

[13] This is under the assumption that the second-order condition is satisfied and that the solution is interior, as in the benchmark model.

appendix, this solution exists and is unique. We denote equilibrium continuation values by V_s^* and the corresponding wages by w_s^*.

We now consider properties of this equilibrium.

Proposition 2 *Investment in ratings quality is lower in boom states than in recession states; that is, w_s^* is decreasing in s.*

Proof. Consider the first-order condition (6). The right-hand side is increasing in λ_s and π_s and decreasing in p_s. It is immediate, therefore, that if γ_s is constant in s, then w_s^* is decreasing in s. Next, recall that γ_s is increasing in s and $\frac{\partial z_s}{\partial w \partial \gamma} < 0$. The conclusion follows immediately since $\frac{\partial^2 z}{\partial w^2} < 0$. ■

This proposition states our main result: Ratings quality is lower in boom states than in recessionary states. The CRA's incentives to exploit its situation depend on how well the economy is doing. The better the economy is doing, the lower is the CRA's ratings accuracy. That is, ratings accuracy decreases with more good investments, lower default probabilities, higher fees, and a tighter labor market.

While some comparative statics in the benchmark model are ambiguous, in our current model, there is no ambiguity. For example, in the benchmark model, increasing fee income increases the current benefit of milking reputation by issuing more positive ratings, but also increases the value of maintaining reputation and earning fee income in the future. Here, the CRA makes the comparison across states without altering future prospects, so only the current incentive to milk reputation arises. Therefore higher fees today mean that the CRA wants to be less accurate to collect them. A similar logic holds with the proportion of good investments. Lower default probabilities imply a lower likelihood of getting caught for reduced accuracy, while a tighter labor market means that hiring good analysts is more costly. All of these point to lower accuracy in boom states.

The assumption that shocks to the economy are independent and identically distributed is critical to this result; tomorrow's state does not depend on today's state. In the next section, we explore how relaxing this assumption affects the results.

For empirical work, it may be interesting to characterize default probabilities for rated products. Note that the probability that a product is rated is given by $\lambda_s + (1 - \lambda_s)(1 - z_s)$ and so the expected probability of default is given by $\frac{(1-\lambda_s)(1-z_s)p_s}{\lambda_s+(1-\lambda_s)(1-z_s)}$. Since this probability is monotonically decreasing in z_s, an increase in fees, π_s, or in the competitiveness of the labor market, γ_s, increases the probability of default for rated products. For the fraction of good projects, λ_s, and the likelihood that a bad project defaults, p_s, there are both direct effects on this default probability and indirect effects through the firm's hiring (w_s, which, in turn, affects z_s). These act in opposite directions, so their overall effect is ambiguous.

4 Correlation across time

To broaden our investigation, it is important to incorporate the fact that the state of the economy today may be linked to the state of the economy tomorrow. We thus extend the model by allowing for correlation of economic fundamentals over time: positive correlation, which implies that a boom is more likely to be followed by a boom, or negative correlation, where a boom is likely to revert to a recession. In order to analyze the effect of correlation, we simplify to suppose that there are only two states $s \in \{R, B\}$, R corresponding to a recessionary period and B to a boom.

Define τ_s as the probability that there is a transition from the current state s to the other state. Note that both τ_B and $1 - \tau_R$ represent the probabilities of moving to a recessionary state in the next period (when starting from the boom and recessionary states, respectively). When $\tau_B = 1 - \tau_R$, each period's state is an independent and identically distributed draw from the same distribution. When $\tau_B < 1 - \tau_R$, there is persistence: it is more likely that a boom state will follow a boom state than a recessionary state, and that a recessionary state will follow a recessionary state. When $\tau_B > 1 - \tau_R$, there is reversion to the mean or negative correlation among states. These transition probabilities are related to the duration of a boom or recession: A higher value of τ_s implies a shorter duration for the state s and a rapid move towards the other state.

Since the CRA is choosing only the current wage, it takes the continuation values as given. As in the benchmark model, we assume that investors anticipate that wages are high enough in each state such that they would purchase after observing a good rating; that is, $z(w_s, \gamma_s) > \overline{z}$ for $s = R$ or B. These conditions can be verified after characterizing the equilibrium wages w_R^* and w_B^*.

We now consider a value function for each state, as in Section 3:

$$V_B = \max_{w_B} \pi_B(\lambda_B + (1 - \lambda_B)(1 - z_B)) - w_B + \delta(1 - (1 - \lambda_B)(1 - z_B)p_B)((1 - \tau_B)V_B + \tau_B V_R)$$
$$V_R = \max_{w_R} \pi_R(\lambda_R + (1 - \lambda_R)(1 - z_R)) - w_R + \delta(1 - (1 - \lambda_R)(1 - z_R)p_R)((1 - \tau_R)V_R + \tau_R V_B) \quad (7)$$

We denote equilibrium values with a star (*). Existence and uniqueness of a solution are not immediate corollaries of our results in Section 3 since those results assume independence of states across time.[14]

Lemma 2 *There exists a unique solution* (V_B^*, V_R^*) *with associated* w_B^* *and* w_R^* *to the system of equations (7).*

We are interested in the difference between accuracy during booms and recessions. We begin by writing the first-order conditions for the decision variables w_B and w_R, respectively:

$$\frac{\partial z}{\partial w}(w_B^*, \gamma_B) = \frac{1}{1-\lambda_H}\frac{1}{\delta p_H((1-\tau_B)V_B^* + \tau_B V_R^*) - \pi_B} \tag{8}$$

$$\frac{\partial z}{\partial w}(w_R^*, \gamma_R) = \frac{1}{1-\lambda_R}\frac{1}{\delta p_R((1-\tau_R)V_R^* + \tau_R V_B^*) - \pi_R} \tag{9}$$

We distinguish between booms and recessions by suggesting that booms involve higher fees ($\pi_B > \pi_R$), a greater proportion of good projects ($\lambda_B > \lambda_R$), lower probabilities of default ($p_B < p_R$) and tighter labor-market competition ($\gamma_B > \gamma_R$). While, it seems natural that the first three effects suggest that it is more valuable to be in a boom than in a recession, so that $V_B^* > V_R^*$, the last force might act in the opposite direction (if it is sufficiently expensive to hire labor in the boom, then the CRA may actually prefer to be in a recession). Considering transition probabilities, if booms are worth more, then transitioning more often to booms from a boom increases the relative value of being in a boom, and transitioning more often from a recession to a recession decreases the relative value of being in a recession.

These intuitions are formalized in the Proposition below:

Proposition 3 *The difference between the value of being in a boom rather than in a recession* $(V_B^* - V_R^*)$*:*

(i) decreases in the probability of default in a boom (p_B) *and the competitiveness of labor-market conditions* (γ_B) *and increases in the proportion of good projects* (λ_B) *and the fee* (π_B)*;*

(ii) increases in the probability of default in a recession (p_R) *and the competitiveness of labor-market conditions* (γ_R) *and decreases in the proportion of good projects* (λ_R) *and the fee* (π_R)*;*

[14] Existence and uniqueness of equilibrium for Section 3 appears as Lemma 4 in the Appendix.

(iii) decreases in the probability of transitioning from a boom to a recession (τ_B) and increases in the probability of transitioning from a boom to a recession (τ_R) if and only if it is more valuable to be in the boom state ($V_B^* > V_R^*$).

In general, the comparison between V_B^* and V_R^* is ambiguous. However, as $V_B^* > V_R^*$ seems to be the interesting (and intuitive) case, we assume this to be true for presentation purposes through the remainder of the paper.[15] Although this is an assumption on endogenous values, it trivially holds where γ_B and γ_R are close enough and we order the other fundamentals according to the business cycle as above.

Assumption A1: The value to a CRA of being in a boom is larger than the value of being in a recession ($V_B^* > V_R^*$)

We now examine how accuracy compares in booms and recessions. We begin by adapting the arguments from the proof of Proposition 2 in the previous section. Define continuation values from the boom and recession states, respectively, as:

$$EV_B^* := (1 - \tau_B)V_B^* + \tau_B V_R^*, \text{ and} \tag{10}$$

$$EV_R^* := (1 - \tau_R)V_R^* + \tau_R V_B^*. \tag{11}$$

As in the proof of Proposition 2, and given the first-order conditions (8) and (9), it follows that $w_B^* \leq w_R^*$ and there is more accuracy in recessions than in booms when:

$$(1 - \lambda_B)(\delta p_B EV_B^* - \pi_B) \leq (1 - \lambda_R)(\delta p_R EV_R^* - \pi_R). \tag{12}$$

As we stated earlier, when $\tau_B = 1 - \tau_R$, each period's state is an iid draw from the same distribution. This implies that the continuation values from a boom and a recession are identical, $EV_B = EV_R$. Here, the intuition and results from our earlier analysis apply, and it is easy to compare accuracy (and the wages paid to analysts) in the boom and recessionary states. We obtain the following result, as a special case of Proposition 2:

Corollary 1 *If states are independent across time ($\tau_B = 1 - \tau_R$), then there is more investment in ratings quality in a recession than in a boom.*

[15] Results on the opposite case ($V_B^* < V_R^*$) can be summarized easily, given the proofs in the appendix.

If booms and recessions do not arise independently of history, then Corollary 1 cannot be applied directly. Condition (12) is not necessarily easy to verify, since the continuation values EV_B and EV_R are endogenously determined. However, given assumption 1, we can state the following:

Proposition 4 *If there is negative correlation between states, then there is more investment in ratings quality in a recession than in a boom.*

This is a direct result of condition (12). Negative correlation or mean reversion implies that the future expected value when in a recession is larger than that in a boom because of the increased likelihood of transitioning to the boom. In the recession, the CRA builds up its reputation so as to reap the benefits of the approaching boom. In the boom, the incentive is to milk reputation since the recession is likely to come soon. This, then, implies that there are more-accurate ratings in a recession than in a boom.

The opposite result—if there is positive correlation, there is higher ratings quality in a boom—does not necessarily hold. Condition (12) is not enough to explore the case of positive correlation. In order to get more insight into the dynamics, we now switch from examining correlation between states to changes in correlation between states (increasing/decreasing the amount of correlation):

Proposition 5 *(i) Decreasing the probability of transitioning from boom to recessionary states (reducing τ_B) increases investment in ratings quality in the boom state (w_B^*) and in the recessionary state (w_R^*)*

(ii) Decreasing the probability of transitioning from recessionary to boom states (reducing τ_R) decreases investment in ratings quality in the boom state (w_B^*) and in the recessionary state (w_R^*)

Decreasing the probability of transitioning from booms to recessions (or, equivalently, increasing the duration of booms) increases ratings quality in both states. In the boom, there is less likelihood that the good times will end soon, meaning that there is less desire to milk reputation. In the recession, the payoff of a transition to a boom increases, meaning that it is a good time to build up reputation. For analogous reasons, increasing the duration of recessions has the reverse effect.

Turning next to changing persistence or mean reversion in both states (specifically, changing the transition probabilities from both states equally), Proposition 5 suggests two contradictory effects. First, decreasing the persistence of a boom state (increasing τ_B) reduces ratings quality in both the boom and the recession, but decreasing the persistence of a recessionary state (increasing τ_R) increases ratings quality. As shown in Proposition 6, either effect can dominate. Intuitively, the effect through the change in τ_B is likely to dominate if the CRA is very often in the boom state (this is likely the case when τ_B is low and τ_R is high), and vice versa.

Proposition 6 *Decreasing the persistence of states (equivalently, increasing mean reversion) equally (increasing τ_B and τ_R by the same amount):*

(i) increases investment in ratings quality in the boom state if and only if $\tau_B - \tau_R > \frac{1}{\delta(1-(1-\lambda_R)(1-z_R)p_R)} - 1$; and

(ii) increases investment in ratings quality in the recessionary state if and only if $\tau_B - \tau_R > 1 - \frac{1}{\delta(1-(1-\lambda_R)(1-z_R)p_R)}$.

Note that $\frac{1}{\delta(1-(1-\lambda_R)(1-z_R)p_R)} - 1 > 0 > 1 - \frac{1}{\delta(1-(1-\lambda_R)(1-z_R)p_R)}$, and so it is never the case that decreasing persistence of states equally can lead to an increase of ratings quality in booms and a decrease of ratings quality in recessions; however, other combinations of outcomes can arise depending on parameters.

In the special case that booms and recessions are of the same duration ($\tau_B = \tau_R$), or sufficiently close, we can obtain a more definitive result:

Corollary 2 *If booms and recessions are of the same duration ($\tau_B = \tau_R$), then decreasing the persistence of states (equivalently, increasing mean reversion) equally (decreasing τ_B and τ_R by the same amount) decreases investment in ratings quality in the boom state (w_B^* decreases) and increases investment in ratings quality in the recessionary state (w_R^* increases).*

Therefore, starting from a benchmark where booms and recessions are of similar duration, increasing persistence diminishes the result that ratings quality is lower in a boom than in a recession. On the other hand, decreasing persistence (or increasing mean reversion) exacerbates the result.

5 Competition

In the main model, we considered a monopoly CRA. Nevertheless, it is important to learn whether the main insights of that model hold when competition is taken into account.

While S&P, Moody's, and Fitch certainly exercise some market power, they also compete for market share. In order to deal with the tractability issue of an infinite period reputation model of competition, we model competition in a very simple fashion, by supposing that the fee that the CRA charges (and/or the volume of issues) depends not only on the state, but also on the extent of competition among CRAs.

Specifically, we assume that there are two CRAs that rate different products (so success rates are independent)[16] and write $\pi_{D,s}$ to denote the fee charged by a duopolist in state s and $\pi_{M,s}$ to denote the fee charged by a monopolist in state s, where $\pi_{M,s} > \pi_{D,s}$. We also revert to the original model where states are continuous and iid draws each period. As before, we consider a grim-trigger strategy equilibrium where investors who observe that an issue with a positive rating from CRA j defaults stop buying investments rated by CRA j.[17]

If one CRA loses the confidence of investors, the market becomes a monopoly. When a CRA acts as a monopolist, the analysis of Section 3 applies. It is straightforward in this case to characterize optimal wages in each state, $w^*_{M,s}$, the continuation value associated with each state, $V^*_{M,s}$, and the expected continuation value, $E(V^*_M)$. These have properties identical to those characterized in Proposition 2.

Using this characterization of the monopoly case, we can, in effect, work backwards to consider duopoly behavior. In particular, we can write down the value for CRA i of being in a duopoly in state s and paying a wage $w_{i,s}$, given that its rival, CRA j, is expected to be paying a wage $w_{j,s}$:

$$V_{i,s} = \begin{array}{c} \pi_{D,s}(\lambda_s + (1-\lambda_s)(1-z_{i,s})) - w_{i,s} \\ +\delta(1-(1-\lambda_s)(1-z_{i,s})p_s)\left[(1-(1-\lambda_s)(1-z_{j,s})p_s)E(V_D) + (1-\lambda_s)(1-z_{j,s})p_s E(V^*_M)\right] \end{array}. \tag{13}$$

This expression is similar to (4). Here, however, the future value for CRA i, if it succeeds in sustaining its reputation, incorporates both the possibility that the rival sustains its

[16] The assumption that CRAs rate different products (and so have independent success rates) is made for ease of presentation. The same forces apply when allowing for correlation in the CRAs success rates, and Lemma 3 and Proposition 7 hold in this richer environment. This analysis is available upon request from the authors.

[17] There is further scope for multiplicity in this environment. For example, consumers may stop trusting all CRAs if one was found to have incorrectly rated an investment. In this case, the analysis would look similar to that of the monopoly case but with lower per-period payoffs. In addition, CRAs may collude; however, this may require the somewhat unreasonable assumption that they observe each other's wage policies.

reputation, so that the CRA continues as a duopolist in the future, and the possibility that the rival firm is found to have assigned a good rating to a bad investment that defaulted, in which case the CRA becomes a monopolist.

In equilibrium, $w^*_{i,s}$ is optimally chosen and so satisfies the first-order condition:

$$-\pi_{D,s}(1-\lambda_s)\frac{\partial z_{i,s}}{\partial w} - 1 + \frac{\partial z_{i,s}}{\partial w}\delta(1-\lambda_s)p_s \begin{bmatrix} (1-(1-\lambda_s)(1-z_{j,s})p_s)E(V^*_D) \\ +(1-\lambda_s)(1-z_{j,s})p_s E(V^*_M) \end{bmatrix} = 0. \quad (14)$$

This allows us to analyze how CRAs react to their expectations of each other's behavior. In this stylized model, part of the reward for a CRA for investing in ratings quality is the possibility that the other CRA falters, and it finds itself a monopolist. Intuitively, if it expects its rival to invest more, then this benefit of investing in ratings quality is diminished, and so the CRA invests less. This intuition is borne out in the following lemma:

Lemma 3 *The CRAs' wage choices are strategic substitutes.*

This lemma demonstrates that if CRA i raises its wages, CRA j would lower its wage in response, and vice-versa. The lemma also ensures that there is a unique symmetric equilibrium.[18] Imposing symmetry, we write the equilibrium wage for this duopoly case as $w^*_{D,s}$ and the CRA's first-order condition (equation 14) as:

$$-\pi_{D,s}(1-\lambda_s)\frac{\partial z_s}{\partial w}|_{w^*_{D,s}} - 1 + \frac{\partial z_s}{\partial w}|_{w^*_{D,s}}\delta(1-\lambda_s)p_s \begin{bmatrix} (1-(1-\lambda_s)(1-z_s)p_s)E(V^*_D) \\ +(1-\lambda_s)(1-z_s)p_s E(V^*_M) \end{bmatrix} = 0. \quad (15)$$

Using this condition, we can now examine the relationship between equilibrium ratings quality and the economic fundamentals. There are now two effects on the incentives of the CRA: the direct effect, as in the monopoly case, and a strategic effect. The direct effect clearly has the same effect on incentives to provide quality ratings as in our monopoly model, analyzed in Proposition 2. A strategic effect arises since the probability of becoming a monopolist rather than a duopolist in the future changes as parameters change.

Since duopoly profits $\pi_{D,s}$ affect only the value of milking a reputation (current profits) and not the value of maintaining it, there is no strategic effect. This means that our previous result that there are lower quality ratings when profits are higher still holds. Other factors

[18] This does not rule out the existence of asymmetric equilibria.

affect both the value of milking and maintaining reputation, so we will have to incorporate the strategic effect. An increase in p, holding all else equal, increases the likelihood that the rival CRA loses its reputation and so increases the likelihood of becoming a monopolist in future, further boosting ratings quality in a recession beyond the direct effects. A fall in λ has a similar effect. However, tighter labor-market conditions (an increase in γ), holding all else constant, reduce the quality of the rival's ratings and, hence, give a CRA an incentive to raise quality in opposition to the direct effect. These intuitions are formalized in the following proposition:

Proposition 7 *There is lower investment in ratings quality in booms than in recessionary states (that is, w_s^* is decreasing in s) when booms and recessions differ in terms of duopoly fees, default rates, and/or the proportion of good investments. However, the effect of labor-market conditions is ambiguous.*

Therefore, countercyclical ratings quality also may be a feature of a competitive ratings market. While competition here changes the value of maintaining a CRA's reputation relative to a market dominated by a monopolist, the economic fundamentals shift incentives in a way mostly similar to that of the monopolist.

A natural next question is whether a monopolist invests more in ratings quality than does a duopolist. This is a question of particular interest, given suggestions by policy-makers and popular commentators that encouraging competition in the credit-rating industry might improve quality. However, previous literature (see in particular, Bar-Isaac, 2005) suggests that there are several forces at play, all acting in opposing directions. In fact, depending on parameters, in our model, ratings quality can be higher under either monopoly or duopoly.[19]

Intuitively, the value of milking a reputation for current returns is higher for a monopolist. This is a force that suggests that the monopolist would produce lower-quality ratings. On the other hand, the value of maintaining reputation to gain future rewards is also higher for a monopolist, suggesting that a monopolist would produce higher-quality ratings. We can see this directly in the first-order conditions that characterize investment

[19] In the context of rating agencies or certifiers, Strausz (2005) argues that a market is likely to be monopolized as it is easier for a monopoly certifier to report honestly and Mariano (2008) also suggests that CRA might have better incentives from reputational concerns as a monopolist than a duopolist.

levels for a monopolist and duopolist:

$$\frac{\partial z_s}{\partial w}|_{w^*_{M,s}} = \frac{1}{1-\lambda_s}\frac{1}{\delta p_s E(V^*_M) - \pi_{M,s}} \tag{16}$$

$$\frac{\partial z_s}{\partial w}|_{w^*_{D,s}} = \frac{1}{1-\lambda_s}\frac{1}{\delta p_s\left[(1-(1-\lambda_s)(1-z_s)p_s)E(V^*_D) + (1-\lambda_s)(1-z_s)p_s E(V^*_M)\right] - \pi_{D,s}} \tag{17}$$

Consider, first, the incentive to milk current reputation: This is trivially higher for a monopolist since $\pi_{M,s} > \pi_{D,s}$. However, the value of maintaining reputation is also higher for a monopolist since $E(V^*_M) > E(V^*_D)$, and so, also, $E(V^*_M) > \begin{bmatrix} (1-(1-\lambda_s)(1-z_s)p_s)E(V^*_D)+ \\ (1-\lambda_s)(1-z_s)p_s E(V^*_M) \end{bmatrix}$. It is easy to find examples where either effect can dominate. For example, suppose that for all s, $\pi_M = 3$, $\pi_D = \frac{3}{2}$, $\lambda = p = \frac{1}{2}$ and $z = \sqrt{w}$. Then, equilibrium wages are higher for a monopolist than for a duopolist when $\delta = 0.95$ ($w^*_M = 0.463$ and $w^*_D = 0.334$), and lower when $\delta = 0.9$ ($w^*_M = 0.106$ and $w^*_D = 0.142$).

The question of whether more competition increases or decreases ratings quality is, thus, an empirical one. Becker and Milbourn (2009) find supporting evidence for competition decreasing ratings quality; they show that increases in market share by Fitch (a proxy for more competition) led to higher ratings and decreased the correlation between bond yields and ratings. Bongaerts, Cremers, and Goetzmann (2009), however, find only a certification role for Fitch in breaking ties between Moody's and Standard and Poors.

6 Empirical Implications

In this section, we examine evidence surrounding testable implications of the model. To examine our hypotheses, we use a set of very recent empirical papers focused on CRAs and ratings quality.

The model shows that ratings quality may be countercyclical. This effect is likely to be exacerbated when economic shocks are negatively correlated and diminished when economic shocks are positively correlated. While we are unable to find direct evidence relating the nature of business cycles to ratings quality, some recent papers document a decrease in ratings quality in the recent boom. Ashcraft, Goldsmith-Pinkham, and Vickery (2010) find that the volume of mortgage-backed security issuance increased dramatically from 2005 to mid-2007, the quality of ratings declined. Specifically, when conditioning on the overall risk of the deal, subordination levels[20] for subprime and Alt-A MBS deals

[20] The subordination level that they use is the fraction of the deal that is junior to the AAA tranche.

decreased over this time period. Furthermore, subsequent ratings downgrades for the 2005 to mid-2007 cohorts were dramatically greater than for previous cohorts. Griffin and Tang (2009) find adjustments by CRAs to their models' predictions of credit quality in the CDO market were positively related to future downgrades. These adjustments were overwhelmingly positive, and the amount adjusted (the width of the AAA tranche) increased sharply from 2003 to 2007 (from six percent to 18.2 percent). The adjustments are not well explained by natural covariates (such as past deals by collateral manager, credit enhancements, and other modeling techniques). Furthermore, 98.6 percent of the AAA tranches of CDOs in their sample failed to meet the CRAs' reported AAA standard (for their sample from 1997 to 2007). They also find that adjustments increased CDO value by, on average, $12.58 million per CDO.

Larger current payoffs should lead to lower ratings quality. He, Qian, and Strahan (2010) find that MBS tranches sold by larger issuers performed significantly worse (market prices decreased) than those sold by small issuers during the boom period of 2004-2006. They define larger by market share in terms of deals. As a robustness check, they also look at market share in terms of dollars and find similar results. Faltin-Traeger (2009) shows that when one CRA rates more deals for an issuer in a half-year period than does another CRA, the first CRA is less likely to be the first to downgrade that issuer's securities in the next half-year.

More-complex investments imply lower ratings quality. Increasing the complexity of investments has two implications for ratings quality. First, it implies more noise regarding the performance of the investment, making it harder to detect whether a CRA can be faulted for poor ratings quality. Second, it implies that CRAs may require more expensive/specialized workers to maintain a given level of quality. Both of these channels decrease the return to investing in ratings quality. Structured finance products are certainly more complex (and the methodology for evaluating them less standardized) than corporate bonds, which provides casual evidence for the recent performance of structured finance ratings. Within the structured finance arena, Ashcraft, Goldsmith-Pinkham, and Vickery (2010) find that the MBS deals that were most likely to underperform were ones with more interest-only loans (because of limited performance history) and lower documentation—i.e., loans that were more opaque or difficult to evaluate.

We also offer two predictions of the model that are testable but have not been examined,

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A smaller fraction means that the AAA tranche is less 'protected' from defaults and, therefore, less costly from the issuer's point of view.

to the best of our knowledge.

1. Ratings quality decisions between CRAs are strategic substitutes. When one CRA chooses to produce better ratings, the other CRAs have incentives to worsen their ratings. Kliger and Sarig (2000) use a natural experiment that seems tailored for testing this hypothesis: Moody's' switch to a finer ratings scale. While their focus is on the informativeness of ratings, it would be interesting to study the strategic aspect of how this affects the quality of Standard and Poor's' ratings.

2. When forecasts of growth/economic conditions are better, ratings quality should be higher. This is because reputation-building is needed for milking in good times, and forecasts should be directly related to CRAs' future payoffs. This is also a prediction of the models of Mathis, McAndrews, and Rochet (2009) and Bolton, Freixas, and Shapiro (2010).

7 Conclusion

In this paper, we analyze how the incentives of CRAs to provide high-quality ratings vary over the business cycle. We define booms as having lower average default probabilities, tighter labor markets, and larger revenue for CRAs than in recessions. When economic shocks are iid, booms have strictly lower quality ratings than do recessions, due to the incentive to milk reputation. These incentives are exacerbated when shocks are negatively correlated (mean reversion) and diminished when shocks are positively correlated. We also put forth a simple model of competition, which demonstrates that countercyclical ratings quality also holds with more than one CRA. Lastly, we find some empirical support for the model and make suggestions for future empirical work.

In order to make our model tractable, we have made several simplifications. First, CRAs can lose their reputation in one fell swoop. It would be interesting to have more continuous changes in reputation. Second, we have not explicitly modeled investors or how CRAs get paid. Digging deeper into competition between CRAs could bring additional insights. Third, it could also prove useful to model the business cycle in a more realistic manner.

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A Proofs

Proof of Lemma 1

Proof. Define the following function:

$$F(w) := (\delta p \frac{\pi(\lambda + (1-\lambda)(1-z)) - w}{1 - \delta(1 - (1-\lambda)(1-z)p)} - \pi)(1-\lambda)\frac{\partial z}{\partial w} - 1. \tag{18}$$

Substituting for V, it is clear that an equilibrium is characterized by $F(w^*) = 0$.

First, note that $F(0) > 0$ since $\frac{\partial z}{\partial w} \to \infty$ as $w \to 0$ and $\delta p \frac{\pi}{1-\delta(1-(1-\lambda)p)} - \pi = \pi \frac{1-\delta(1-p\lambda)}{1-\delta(1-(1-\lambda)p)} > 0$. Further, note that $\frac{\partial z}{\partial w}|_{\overline{w}} = 0$ by assumption, and so $F(\overline{w}) = -1$. Since $F(w)$ is continuous, this is sufficient to prove that there exists a w for which $F(w) = 0$ (i.e., the existence of a solution w^*).

Next, to consider uniqueness, note that any solution, w^*, must satisfy $F(w^*) = 0$. The derivative of $F(w)$ evaluated at w^* is

$$(1-\lambda)\left(\frac{dV^*}{dw}|_{w^*}\delta p \frac{\partial z}{\partial w}|_{w^*} + (\delta V^* p - \pi)\frac{\partial^2 z}{\partial w^2}|_{w^*}\right). \tag{19}$$

Since we assume that the second-order condition of the CRA's maximization problem is negative, the second expression is negative. Next

$$\frac{dV^*}{dw}|_{w^*} = \frac{\begin{bmatrix} (-(1-\lambda)\pi\frac{\partial z}{\partial w}|_{w^*} - 1)[1 - \delta(1-(1-\lambda)(1-z^*)p)] + \\ \delta(1-\lambda)p\frac{\partial z}{\partial w}|_{w^*}[\pi(\lambda + (1-\lambda)(1-z^*) - w^*] \end{bmatrix}}{(1-\delta(1-(1-\lambda)(1-z^*)p))^2}. \tag{20}$$

We focus on the numerator, since the sign of the expression depends only on this part. The numerator can be simplified using (2) and rewriting it as:

$$\delta(1-\lambda)\frac{\partial z}{\partial w}|_{w^*}(-V[1-\delta(1-(1-\lambda)(1-z^*)p)] + p[\pi(\lambda + (1-\lambda)(1-z^*) - w^*]) \tag{21}$$
$$= -\delta(1-\lambda)\frac{\partial z}{\partial w}|_{w^*}[1-\delta(1-(1-\lambda)(1-z^*)p)](1-p)V < 0.$$

Since $F'(w^*) < 0$ for any solution w^*, there can be only one solution, proving uniqueness. ■

Proof of Proposition 1

Proof. The implicit function theorem says (given a variable y) that

$$\frac{dw^*}{dy} = -\frac{\frac{dF}{dy}}{\frac{dF}{dw^*}} \tag{22}$$

where F is defined in (18). From the proof of Lemma 1, $-\frac{dF}{dw^*} > 0$. This implies that the sign of $\frac{dw^*}{dy}$ is the same as that of $\frac{dF}{dy}$. It is also useful to note that the second-order condition of the CRA (which we assume is negative) requires that $\delta pV - \pi > 0$ or, equivalently, $p\delta(\pi\lambda - w) - \pi(1-\delta) > 0$.

First, we examine the effect of δ :

$$\frac{dF}{d\delta} = (1-\lambda)Vp\frac{\partial z}{\partial w} + \delta(1-\lambda)\frac{dV}{d\delta}p\frac{\partial z}{\partial w}$$

This expression is positive since $\frac{\partial z}{\partial w} > 0$ and $\frac{dV}{d\delta} = \frac{\pi(\lambda+(1-\lambda)(1-z))-w}{(1-\delta(1-(1-\lambda)(1-z)p))^2}(1-(1-\lambda)(1-z)p) > 0$.

Second, we examine the effect of p:

$$\begin{aligned} \frac{dF}{dp} &= \delta(1-\lambda)V\frac{\partial z}{\partial w} + \delta(1-\lambda)\frac{dV}{dp}p\frac{\partial z}{\partial w} \\ &= \frac{\delta(1-\delta)(1-\lambda)V}{1-\delta(1-(1-\lambda)(1-z)p)}\frac{\partial z}{\partial w} > 0. \end{aligned} \tag{23}$$

Third, consider labor-market conditions, as summarized by γ:

$$\frac{dF}{d\gamma} = (1-\lambda)(\delta pV - \pi)\frac{\partial^2 z}{\partial w \partial \gamma} + \delta(1-\lambda)\frac{dV}{d\gamma}p\frac{\partial z}{\partial w} < 0 \tag{24}$$

since $\frac{dV}{d\gamma} = \frac{d}{d\gamma}\frac{\pi(\lambda+(1-\lambda)(1-z))-w}{1-\delta(1-(1-\lambda)(1-z)p)} = (1-\lambda)\frac{p\delta(\pi\lambda-w)-\pi(1-\delta)}{(1-\delta(1-(1-\lambda)(1-z)p))^2}\frac{\partial z}{\partial \gamma} < 0$ where the last inequality follows from assuming the CRA's second-order condition is negative (as discussed above).

Next, we examine the effect of π:

$$\frac{dF}{d\pi} = (1-\lambda)\frac{\partial z}{\partial w}(\delta\frac{dV}{d\pi}p - 1) = (1-\lambda)\frac{\partial z}{\partial w}\frac{\delta(1+p\lambda)-1}{1-\delta(1-(1-\lambda)(1-z)p)}, \tag{25}$$

This is negative when $p = 0$, but positive when $\delta = p = 1$, proving that there is no general monotonicity in π.

Finally, consider the effect of λ

$$\frac{dF}{d\lambda} = \frac{\partial z}{\partial w}(\delta p(1-\lambda)\frac{\pi z(1-\delta) + p\delta(\pi - w)(1-z)}{(1-\delta(1-(1-\lambda)(1-z)p))^2} - (\delta pV - \pi)). \tag{26}$$

At $\lambda = 1$, $\frac{dF}{d\lambda} = -\frac{\partial z}{\partial w}(\delta pV - \pi) < 0$ and at $\lambda = 0$ and $p = \delta = 1$, then $\frac{dF}{d\lambda} = \frac{\partial z}{\partial w}(\frac{\pi z(1-z)}{(1-z)^2} + \pi) > 0$.

This proves the results for w^*. To analyze the comparative statics of z^*, since $\frac{\partial z}{\partial w} > 0$ the results with respect to δ, π, λ and p are immediate. The result with respect to γ further requires the properties that $\frac{\partial z}{\partial \gamma} < 0$ and $\frac{\partial^2 z}{\partial \gamma \partial w} > 0$. ■

Lemma 4 *There exists a unique equilibrium in the boom and recessions model.*

Proof. First, we define

$$K(E(V)) := E(V) - \int_{\underline{s}}^{\bar{s}} \{\pi_s(\lambda_s + (1-\lambda_s)(1-z_s^*)) - w_s^* + \delta(1-(1-\lambda_s)(1-z_s^*)p_s)E(V)\} f(s)ds \tag{27}$$

At the equilibrium, integrating equation 4 with respect to s implies $K(E(V^*)) = 0$.

Trivially, $K(0) < 0$. The derivative of $K(E(V))$ with respect to $E(V)$ is (using the envelope condition):

$$1 - \delta \int_{\underline{s}}^{\bar{s}} (1-(1-\lambda_s)(1-z_s^*)p_s) f(s)ds > 0, \tag{28}$$

where the inequality follows since $1 > 1-(1-\lambda_s)(1-z_s^*)p_s$ for all s.

Since $\int_{\underline{s}}^{\bar{s}} \{\pi_s(\lambda_s + (1-\lambda_s)(1-z_s^*)) - w_s^*\} f(s)ds$ is finite, $K(x) > 0$ for x large enough. It follows that there exists a solution to $K(x) = 0$ and it is unique. ■

Proof of Lemma 2

Proof. First, consider existence. Note that $\pi_B(\lambda_B + (1-\lambda_B)(1-z_B)) - w_B$ is bounded from above and $\pi_R(\lambda_R + (1-\lambda_R)(1-z_R)) - w_R$ is bounded from above. Say both are strictly less than A; then, trivially, $V_B < \frac{A}{1-\delta}$ and $V_R < \frac{A}{1-\delta}$. Define two functions from the two equations at marker 7, $V_B(V_R)$ and $V_R(V_B)$. Note that both are increasing and continuous functions, and that both $V_B(0) > 0$ and $V_R(0) > 0$ are positive. Since $V_B(\frac{A}{1-\delta}) < \frac{A}{1-\delta}$ and $V_R(\frac{A}{1-\delta}) < \frac{A}{1-\delta}$, it follows that there must be an odd number of solutions. This is easy to see graphically in the illustrative figure. However, we argue below that $V_B(.)$ and $V_R(.)$ are

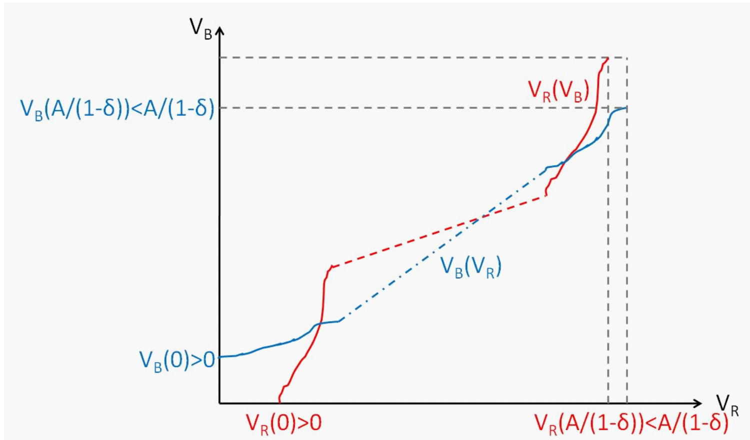

Figure 1: Odd number of solutions

convex and, thereby, show that there cannot be more than two solutions. This will then prove that the solution is unique.

It remains to demonstrate that $V_B(.)$ and $V_R(.)$ are convex. Note, first, that we can consider:

$$w_B^* = \arg\max_w \pi_B(\lambda_B+(1-\lambda_B)(1-z_B))-w+\delta(1-(1-\lambda_B)(1-z_B)p_B)((1-\tau_B)V_B+\tau_B V_R) \tag{29}$$

First, we claim that $\frac{dw_B^*}{dV_R} > 0$. We use the implicit function theorem to do so. Consider the first-order condition of the CRA's maximization problem:[21]

$$-\pi_B \frac{\partial z_B}{\partial w}|_{w_B^*} - 1 + \delta p_B((1-\tau_B)V_B + \tau_B V_R)\frac{\partial z_B}{\partial w}|_{w_B^*} = 0. \tag{30}$$

Taking the derivative of the FOC with respect to V_R and rearranging yields:

$$\frac{dw_B^*}{dV_R} = \frac{\tau_B \delta p_B}{\pi_B - \delta p_B((1-\tau_B)V_B + \tau_B V_R)} \frac{\frac{\partial z_B}{\partial w}}{\frac{\partial^2 z_B}{\partial w^2}} \tag{31}$$

Note that the assumption that the CRA's second-order condition is negative implies that the denominator of the first fraction is negative, and so, since $\frac{\partial^2 z_B}{\partial w^2} < 0$ and $\frac{\partial z_B}{\partial w} > 0$, it follows that $\frac{dw_B^*}{dV_R} > 0$.

[21] It can be shown that the second-order condition is satisfied when λ_B, λ_R and δ are close enough to 1.

Now,

$$\frac{dV_B}{dV_R} = \frac{\partial V_B}{\partial w_B^*}\frac{dw_B^*}{dV_R} + \frac{\partial V_B}{\partial V_R} = \frac{\partial V_B}{\partial V_R} \tag{32}$$

since w_B^* is chosen to maximize V_B (the envelope condition), and so we can write

$$\begin{aligned}\frac{dV_B}{dV_R} &= (\tau_B + (1-\tau_B)\frac{dV_B}{dV_R})\delta(1-(1-\lambda_B)(1-z_B)p_B) \\ &= \frac{\tau_B\delta(1-(1-\lambda_B)(1-z_B)p_B)}{1-(1-\tau_B)\delta(1-(1-\lambda_B)(1-z_B)p_B)} > 0.\end{aligned} \tag{33}$$

Next, to prove convexity, note that

$$\begin{aligned}\frac{d^2V_B}{dV_R^2} &= \frac{d}{dz_B}(\frac{\tau_B\delta(1-(1-\lambda_B)(1-z_B)p_B)}{1-(1-\tau_B)\delta(1-(1-\lambda_B)(1-z_B)p_B)})\frac{\partial z_B}{\partial w}\frac{dw_B^*}{dV_R} \\ &= \frac{(1-\lambda_B)p_B\tau_B\delta}{(1-(1-\tau_B)\delta(1-(1-\lambda_B)(1-z_B)p_B))^2}\frac{\partial z_B}{\partial w}\frac{dw_B^*}{dV_R} > 0.\end{aligned} \tag{34}$$

Analogously, $\frac{d^2V_R}{dV_B^2} > 0$. ■

Proof of Proposition 3

Proof. We start by introducing some additional notation:

$$G_B(p_R, p_B, \gamma_R, \gamma_B, \pi_R, \pi_B, \lambda_R, \lambda_B, \delta) := \begin{array}{c} -V_B + \pi_B(\lambda_B + (1-\lambda_B)(1-z_B)) - w_B \\ +\delta(1-(1-\lambda_B)(1-z_B)p_B)((1-\tau_B)V_B + \tau_B V_R) \end{array} \tag{35}$$

$$G_R(p_R, p_B, \gamma_R, \gamma_B, \pi_R, \pi_B, \lambda_R, \lambda_B, \delta) := \begin{array}{c} -V_R + \pi_R(\lambda_R + (1-\lambda_R)(1-z_R)) - w_R \\ +\delta(1-(1-\lambda_R)(1-z_R)p_R)((1-\tau_R)V_R + \tau_R V_B) \end{array} \tag{36}$$

We suppress the arguments for G_B and G_R and can then rewrite the equations at marker 7 as $G_B = G_R = 0$.

We apply the implicit function theorem, which here implies that

$$\frac{dV_R^*}{da} = -\frac{\det\begin{bmatrix} \frac{\partial G_B}{\partial a} & \frac{\partial G_B}{\partial V_B^*} \\ \frac{\partial G_R}{\partial a} & \frac{\partial G_R}{\partial V_B^*} \end{bmatrix}}{\det\begin{bmatrix} \frac{\partial G_B}{\partial V_R^*} & \frac{\partial G_B}{\partial V_B^*} \\ \frac{\partial G_R}{\partial V_R^*} & \frac{\partial G_R}{\partial V_B^*} \end{bmatrix}} \tag{37}$$

$$\frac{dV_B^*}{da} = -\frac{\det\begin{bmatrix} \frac{\partial G_B}{\partial V_R^*} & \frac{\partial G_B}{\partial a} \\ \frac{\partial G_R}{\partial V_R^*} & \frac{\partial G_R}{\partial a} \end{bmatrix}}{\det\begin{bmatrix} \frac{\partial G_B}{\partial V_R^*} & \frac{\partial G_B}{\partial V_B^*} \\ \frac{\partial G_R}{\partial V_R^*} & \frac{\partial G_R}{\partial V_B^*} \end{bmatrix}} \tag{38}$$

where a is an arbitrary parameter. We begin by analyzing the (common) denominator of both expressions.

As we show in the Lemma below, this determinant is negative. ■

Lemma 5 $\frac{\partial G_B}{\partial V_R^*}\frac{\partial G_R}{\partial V_B^*} - \frac{\partial G_B}{\partial V_B^*}\frac{\partial G_R}{\partial V_R^*}$ *is negative.*

Proof. First, note:

$$\frac{\partial G_B}{\partial V_B^*} = \delta(1-(1-\lambda_B)(1-z_B)p_B)(1-\tau_B) - 1 < 0 \tag{39}$$

$$\frac{\partial G_B}{\partial V_R^*} = \delta(1-(1-\lambda_B)(1-z_B)p_B)\tau_B > 0 \tag{40}$$

$$\frac{\partial G_R}{\partial V_B^*} = \delta(1-(1-\lambda_R)(1-z_R)p_R)\tau_R > 0 \tag{41}$$

$$\frac{\partial G_R}{\partial V_R^*} = \delta(1-(1-\lambda_R)(1-z_R)p_R)(1-\tau_R) - 1 < 0, \tag{42}$$

where we have used the envelope theorem to simplify expressions. This, then, allows us to rewrite

$$\frac{\partial G_B}{\partial V_R^*}\frac{\partial G_R}{\partial V_B^*} - \frac{\partial G_B}{\partial V_B^*}\frac{\partial G_R}{\partial V_R^*} = \delta^2\tau_B\tau_R\alpha_R\alpha_B - (1-\delta\alpha_B(1-\tau_B))(1-\delta\alpha_R(1-\tau_R)), \tag{43}$$

where $\alpha_s := (1-(1-\lambda_s)(1-z_s)p_s)$ and thus $\alpha_s \in (0,1)$

Next, note that

$$\frac{\partial}{\partial \tau_B}\left(\frac{\partial G_B}{\partial V_R^*}\frac{\partial G_R}{\partial V_B^*} - \frac{\partial G_B}{\partial V_B^*}\frac{\partial G_R}{\partial V_R^*}\right) = \delta\alpha_B(\delta\alpha_R - 1) < 0, \tag{44}$$

where the inequality follows since $1 > \alpha_s > 0$.

Finally, note that at $\tau_B = 0$,

$$\frac{\partial G_B}{\partial V_R^*}\frac{\partial G_R}{\partial V_B^*} - \frac{\partial G_B}{\partial V_B^*}\frac{\partial G_R}{\partial V_R^*} = -(1-\delta\alpha_B)(1-\delta\alpha_R(1-\tau_R)) < 0. \tag{45}$$

■

Resumption of Proof of Proposition 3

Proof. Given Lemma 5, we can apply the implicit function theorem and note that $\frac{d}{da}(V_B^* - V_R^*)$ has the same sign as $\det\begin{bmatrix}\frac{\partial G_B}{\partial V_R^*} & \frac{\partial G_B}{\partial a}\\ \frac{\partial G_R}{\partial V_R^*} & \frac{\partial G_R}{\partial a}\end{bmatrix} - \det\begin{bmatrix}\frac{\partial G_B}{\partial a} & \frac{\partial G_B}{\partial V_B^*}\\ \frac{\partial G_R}{\partial a} & \frac{\partial G_R}{\partial V_B^*}\end{bmatrix}$.

We consider several parameters of interest; proofs for other parameters are similar, and so are omitted.

The effect of a change in the probability of default in a boom (p_B)

Consider, first, the comparative static with respect to p_B: $\det\begin{bmatrix}\frac{\partial G_B}{\partial V_R^*} & \frac{\partial G_B}{\partial p_B}\\ \frac{\partial G_R}{\partial V_R^*} & \frac{\partial G_R}{\partial p_B}\end{bmatrix} - \det\begin{bmatrix}\frac{\partial G_B}{\partial p_B} & \frac{\partial G_B}{\partial V_B^*}\\ \frac{\partial G_R}{\partial p_B} & \frac{\partial G_R}{\partial V_B^*}\end{bmatrix} = -\frac{\partial G_B}{\partial p_B}\left(\frac{\partial G_R}{\partial V_R^*} + \frac{\partial G_R}{\partial V_B^*}\right)$, since $\frac{\partial G_R}{\partial p_B} = 0$. Now $\frac{\partial G_B}{\partial p_B} = -\delta(1-\lambda_B)(1-z_B)((1-\tau_B)V_B^* + \tau_B V_R^*) < 0$ and $\frac{\partial G_R}{\partial V_R^*} + \frac{\partial G_R}{\partial V_B^*} = -1 + \delta(1-(1-\lambda_R)(1-z_R)p_R) < 0$.

Consequently, $\frac{d(V_B^* - V_R^*)}{dp_B} < 0$.

The effect of a change in labor-market conditions in a recession (γ_R):

$\det\begin{bmatrix}\frac{\partial G_B}{\partial V_R^*} & \frac{\partial G_B}{\partial \gamma_R}\\ \frac{\partial G_R}{\partial V_R^*} & \frac{\partial G_R}{\partial \gamma_R}\end{bmatrix} - \det\begin{bmatrix}\frac{\partial G_B}{\partial \gamma_R} & \frac{\partial G_B}{\partial V_B^*}\\ \frac{\partial G_R}{\partial \gamma_R} & \frac{\partial G_R}{\partial V_B^*}\end{bmatrix} = \frac{\partial G_R}{\partial \gamma_R}\left(\frac{\partial G_B}{\partial V_R^*} + \frac{\partial G_B}{\partial V_B^*}\right)$, since $\frac{\partial G_B}{\partial \gamma_R} = 0$.

Note that $\frac{\partial G_B}{\partial V_R^*} + \frac{\partial G_B}{\partial V_B^*} = \delta(1-(1-\lambda_B)(1-z_B)p_B) - 1 < 0$ and since $\frac{\partial G_R}{\partial \gamma_R} = (\delta(1-\lambda_R)p_R\,((1-\tau_R)V_R^* + \tau_R V_B^*) - \pi_R(1-\lambda_R))\frac{\partial z}{\partial \gamma_R} < 0$ by the second-order condition and since $\frac{\partial z}{\partial \gamma_R} < 0$. It follows that $\frac{d(V_B - V_R)}{d\gamma_R} > 0$.

The effect of a change in the transition probabilities

i) First, we examine the change with respect to a change in τ_B

$\det\begin{bmatrix}\frac{\partial G_B}{\partial V_R^*} & \frac{\partial G_B}{\partial \tau_B}\\ \frac{\partial G_R}{\partial V_R^*} & \frac{\partial G_R}{\partial \tau_B}\end{bmatrix} - \det\begin{bmatrix}\frac{\partial G_B}{\partial \tau_B} & \frac{\partial G_B}{\partial V_B^*}\\ \frac{\partial G_R}{\partial \tau_B} & \frac{\partial G_R}{\partial V_B^*}\end{bmatrix} = -\frac{\partial G_B}{\partial \tau_B}\left(\frac{\partial G_R}{\partial V_R^*} + \frac{\partial G_R}{\partial V_B^*}\right)$ since $\frac{\partial G_R}{\partial \tau_B} = 0$.

As above, $\frac{\partial G_R}{\partial V_R^*} + \frac{\partial G_R}{\partial V_B^*} < 0$ and $\frac{\partial G_B}{\partial \tau_B} = -\delta(1-(1-\lambda_B)(1-z_B)p_B)(V_B^* - V_R^*)$. It follows that $sign(\frac{\partial G_B}{\partial \tau_B}) = -sign(V_B^* - V_R^*)$. Therefore, $sign\frac{d(V_B^* - V_R^*)}{d\tau_B} = -sign(V_B^* - V_R^*)$.

ii) Second, we examine the change with respect to a change in τ_R

$\det\begin{bmatrix} \frac{\partial G_B}{\partial V_R^*} & \frac{\partial G_B}{\partial \tau_R} \\ \frac{\partial G_R}{\partial V_R^*} & \frac{\partial G_R}{\partial \tau_R} \end{bmatrix} - \det\begin{bmatrix} \frac{\partial G_B}{\partial \tau_R} & \frac{\partial G_B}{\partial V_B^*} \\ \frac{\partial G_R}{\partial \tau_R} & \frac{\partial G_R}{\partial V_B^*} \end{bmatrix} = (\frac{\partial G_B}{\partial V_R^*} + \frac{\partial G_B}{\partial V_B^*})\frac{\partial G_R}{\partial \tau_R}$

As above, $\frac{\partial G_B}{\partial V_B^*} + \frac{\partial G_B}{\partial V_R^*} < 0$. Also, $\frac{\partial G_R}{\partial \tau_R} = \delta(1-(1-\lambda_R)(1-z_R)p_R)(V_B^* - V_R^*)$. It follows that $sign(\frac{\partial G_R}{\partial \tau_R}) = sign(V_B^* - V_R^*)$. Therefore, $sign\frac{d(V_B^* - V_R^*)}{d\tau_R} = -sign(V_B^* - V_R^*)$. ■

Proof of Proposition 5

Proof. First, consider the first-order condition that characterizes w_B^*:

$$-\pi_B(1-\lambda_B)\frac{\partial z_B}{\partial w} - 1 + \delta p_B(1-\lambda_B)\frac{\partial z_B}{\partial w}((1-\tau_B)V_B^* + \tau_B V_R^*) = 0. \tag{46}$$

Taking the total derivative with respect to τ_B, we obtain

$$\begin{aligned} 0 = {} & \delta p_B(1-\lambda_B)\frac{\partial z_B}{\partial w}(V_R^* - V_B^*) + (1-\lambda_B)\frac{\partial^2 z_B}{\partial w^2}\frac{dw_B^*}{d\tau_B}(\delta p_B((1-\tau_B)V_B^* + \tau_B V_R^*) - \pi_B) \\ & + \delta p_B(1-\lambda_B)\frac{\partial z_B}{\partial w}((1-\tau_B)\frac{\partial V_B^*}{\partial \tau_B} + \tau_B\frac{\partial V_R^*}{\partial \tau_B}) \end{aligned} \tag{47}$$

First, note that (using the results from Lemma 5 and the definition of α_s ($s = B, R$) from the same Lemma):

$$\begin{aligned} sign(\frac{\partial V_B^*}{\partial \tau_B}) &= sign(\det\begin{bmatrix} \frac{\partial G_B}{\partial V_R^*} & \frac{\partial G_B}{\partial \tau_B} \\ \frac{\partial G_R}{\partial V_R^*} & \frac{\partial G_R}{\partial \tau_B} \end{bmatrix}) = -sign\frac{\partial G_B}{\partial \tau_B}\frac{\partial G_R}{\partial V_R^*} \\ &= -sign(\delta\alpha_B(1-\delta\alpha_R(1-\tau_R))(V_B^* - V_R^*) = -sign(V_B^* - V_R^*)) \end{aligned}$$

and

$$\begin{aligned} sign(\frac{\partial V_R^*}{\partial \tau_B}) &= sign(\det\begin{bmatrix} \frac{\partial G_B}{\partial \tau_B} & \frac{\partial G_B}{\partial V_B^*} \\ \frac{\partial G_R}{\partial \tau_B} & \frac{\partial G_R}{\partial V_B^*} \end{bmatrix}) = sign(\frac{\partial G_B}{\partial \tau_B}\frac{\partial G_R}{\partial V_B^*}) \\ &= sign(\delta^2\alpha_B\alpha_R\tau_R(V_R^* - V_B^*)) = -sign(V_B^* - V_R^*)). \end{aligned}$$

Now, consider (47): Since $\delta p_B(1-\lambda_B)\frac{\partial z_B}{\partial w} > 0$ and $\frac{\partial^2 z_B}{\partial w^2} < 0$ and $1-\lambda_B > 0$, it follows that $\frac{dw_B^*}{d\tau_B}$ has the same sign as $sign(V_B^* - V_R^*) * sign(\pi_B - \delta p_B((1-\tau)V_B^* + \tau V_R^*))$. Rearranging the FOC as $-\pi_B + \delta p_B((1-\tau)V_B^* + \tau V_R^*) = \frac{1}{(1-\lambda_B)\frac{\partial z_B}{\partial w}}$ and noting that the right-hand side is positive gives the result for w_B^*; that is, $sign(\frac{dw_B^*}{d\tau_B}) = -sign(V_B^* - V_R^*)$.

Analogously, $sign(\frac{dw_R^*}{d\tau_R}) = -sign(V_R^* - V_B^*) = sign(V_B^* - V_R^*)$.

Next, we turn to consider $\frac{dw_B^*}{d\tau_R}$.

Taking the derivative of (46) with respect to τ_R, we obtain:

$$0 = (1-\lambda_B)\frac{\partial^2 z_B}{\partial w^2}\frac{dw_B^*}{d\tau_R}(\delta p_B((1-\tau_B)V_B^*+\tau_B V_R^*)-\pi_B)+\delta p_B(1-\lambda_B)\frac{\partial z_B}{\partial w}((1-\tau_B)\frac{\partial V_B^*}{\partial \tau_R}+\tau_B\frac{\partial V_R^*}{\partial \tau_R}).$$

As above, $(\delta p_B((1-\tau_B)V_B^* + \tau_B V_R^*) - \pi_B) > 0$ and $\frac{\partial^2 z_B}{\partial w^2} < 0$ so that $sign(\frac{dw_B^*}{d\tau_R}) = sign(\delta p_B(1-\lambda_B)\frac{\partial z_B}{\partial w}((1-\tau_B)\frac{\partial V_B^*}{\partial \tau_R} + \tau_B\frac{\partial V_R^*}{\partial \tau_R})) = sign((1-\tau_B)\frac{\partial V_B^*}{\partial \tau_R} + \tau_B\frac{\partial V_R^*}{\partial \tau_R}))$ where the second inequality follows since $\delta p_B(1-\lambda_B)\frac{\partial z_B}{\partial w} > 0$.

Consider

$$\begin{aligned} sign(\frac{\partial V_B^*}{\partial \tau_R}) &= sign(\det\begin{bmatrix} \frac{\partial G_B}{\partial V_R^*} & \frac{\partial G_B}{\partial \tau_R} \\ \frac{\partial G_R}{\partial V_R^*} & \frac{\partial G_R}{\partial \tau_R}\end{bmatrix}) = sign(\frac{\partial G_B}{\partial V_R^*}\frac{\partial G_R}{\partial \tau_R}) \\ &= sign(\delta^2\alpha_B\tau_B\alpha_R(V_B^* - V_R^*)) = sign(V_B^* - V_R^*), \text{ and} \end{aligned} \tag{48}$$

$$\begin{aligned} sign(\frac{\partial V_R^*}{\partial \tau_R}) &= sign(\det\begin{bmatrix} \frac{\partial G_B}{\partial \tau_R} & \frac{\partial G_B}{\partial V_B^*} \\ \frac{\partial G_R}{\partial \tau_R} & \frac{\partial G_R}{\partial V_B^*}\end{bmatrix}) = -sign(\frac{\partial G_B}{\partial V_B^*}\frac{\partial G_R}{\partial \tau_R}) \\ &= sign((1-\delta\alpha_B(1-\tau_B))\delta\alpha_R(V_B^* - V_R^*)) = sign(V_B^* - V_R^*). \end{aligned}$$

This implies $sign(\frac{dw_B^*}{d\tau_R}) = sign(V_B^* - V_R^*)$. Analogously, $sign(\frac{dw_R^*}{d\tau_B}) = -sign(V_B^* - V_R^*)$.

■

Proof of Proposition 6

Proof. Define $\tau_B = \tilde{\tau}_B + \varepsilon$ and $\tau_R = \tilde{\tau}_R + \varepsilon$. We now examine the effect of a change in ε on wages. Taking the derivative of equation (46) with respect to ε yields:

$$\frac{\partial^2 z_B}{\partial w^2}\frac{\partial w_B^*}{\partial \varepsilon}(-\pi_B+\delta p_B((1-\tau_B)V_B^*+\tau_B V_R^*))+\delta p_B\frac{\partial z_B}{\partial w}(V_R^*-V_B^*+(1-\tau_B)\frac{\partial V_B^*}{\partial \varepsilon}+\tau_B\frac{\partial V_R^*}{\partial \varepsilon}) = 0$$

We know that $\frac{\partial^2 z_B}{\partial w^2} < 0$ and $-\pi_B + \delta p_B((1-\tau_B)V_B^* + \tau_B V_R^*) > 0$, so $sign\ (\frac{\partial w_B^*}{\partial \varepsilon}) = sign(V_R^* - V_B^* + (1-\tau_B)\frac{\partial V_B^*}{\partial \varepsilon} + \tau_B\frac{\partial V_R^*}{\partial \varepsilon})$.

We have

$$\frac{\partial V_R^*}{\partial \varepsilon} = -\frac{\det \begin{bmatrix} \frac{\partial G_B}{\partial \varepsilon} & \frac{\partial G_B}{\partial V_B^*} \\ \frac{\partial G_R}{\partial \varepsilon} & \frac{\partial G_R}{\partial V_B^*} \end{bmatrix}}{\frac{\partial G_B}{\partial V_R^*}\frac{\partial G_R}{\partial V_B^*} - \frac{\partial G_B}{\partial V_B^*}\frac{\partial G_R}{\partial V_R^*}} = -\frac{\frac{\partial G_B}{\partial \varepsilon}\frac{\partial G_R}{\partial V_B^*} - \frac{\partial G_B}{\partial V_B^*}\frac{\partial G_R}{\partial \varepsilon}}{\delta^2 \tau_B \tau_R \alpha_R \alpha_B - (1-\delta\alpha_B(1-\tau_B))(1-\delta\alpha_R(1-\tau_R))}$$
$$= -(V_B - V_R)\delta\alpha_R \frac{1-\delta\alpha_B(1-\tau_B+\tau_R)}{\delta^2 \tau_B \tau_R \alpha_R \alpha_B - (1-\delta\alpha_B(1-\tau_B))(1-\delta\alpha_R(1-\tau_R))}$$

and

$$\frac{\partial V_B^*}{\partial \varepsilon} = -\frac{\det \begin{bmatrix} \frac{\partial G_B}{\partial V_R^*} & \frac{\partial G_B}{\partial \varepsilon} \\ \frac{\partial G_R}{\partial V_R^*} & \frac{\partial G_R}{\partial \varepsilon} \end{bmatrix}}{\frac{\partial G_B}{\partial V_R^*}\frac{\partial G_R}{\partial V_B^*} - \frac{\partial G_B}{\partial V_B^*}\frac{\partial G_R}{\partial V_R^*}} = -\frac{\frac{\partial G_R}{\partial \varepsilon}\frac{\partial G_B}{\partial V_R^*} - \frac{\partial G_R}{\partial V_R^*}\frac{\partial G_B}{\partial \varepsilon}}{\delta^2 \tau_B \tau_R \alpha_R \alpha_B - (1-\delta\alpha_B(1-\tau_B))(1-\delta\alpha_R(1-\tau_R))}$$
$$= -(V_B - V_R)\delta\alpha_B \frac{-1+\delta\alpha_R(1-\tau_R+\tau_B)}{\delta^2 \tau_B \tau_R \alpha_R \alpha_B - (1-\delta\alpha_B(1-\tau_B))(1-\delta\alpha_R(1-\tau_R))}$$

Therefore, we have

$$V_R^* - V_B^* + (1-\tau_B)\frac{\partial V_B^*}{\partial \varepsilon} + \tau_B \frac{\partial V_R^*}{\partial \varepsilon}$$
$$= (V_B^* - V_R^*)(-1 - (1-\tau_B)\delta\alpha_B \frac{-1+\delta\alpha_R(1-\tau_R+\tau_B)}{\delta^2 \tau_B \tau_R \alpha_R \alpha_B - (1-\delta\alpha_B(1-\tau_B))(1-\delta\alpha_R(1-\tau_R))}$$
$$-\tau_B \delta\alpha_R \frac{1-\delta\alpha_B(1-\tau_B+\tau_R)}{\delta^2 \tau_B \tau_R \alpha_R \alpha_B - (1-\delta\alpha_B(1-\tau_B))(1-\delta\alpha_R(1-\tau_R))}),$$

so that

$$sign(V_R^* - V_B^* + (1-\tau_B)\frac{\partial V_B^*}{\partial \varepsilon} + \tau_B \frac{\partial V_R^*}{\partial \varepsilon})$$
$$= sign(V_B^* - V_R^*) * sign(-1 - (1-\tau_B)\delta\alpha_B \frac{-1+\delta\alpha_R(1-\tau_R+\tau_B)}{\delta^2 \tau_B \tau_R \alpha_R \alpha_B - (1-\delta\alpha_B(1-\tau_B))(1-\delta\alpha_R(1-\tau_R))}$$
$$-\tau_B \delta\alpha_R \frac{1-\delta\alpha_B(1-\tau_B+\tau_R)}{\delta^2 \tau_B \tau_R \alpha_R \alpha_B - (1-\delta\alpha_B(1-\tau_B))(1-\delta\alpha_R(1-\tau_R))})$$
$$= sign(V_B^* - V_R^*) * sign(\frac{\delta\alpha_R(1+\tau_B-\tau_R)-1}{(1-\delta\alpha_B(1-\tau_B))(1-\delta\alpha_R(1-\tau_R)) - \delta^2 \tau_B \tau_R \alpha_R \alpha_B}).$$

Given Assumption A1, and since by Lemma 5 $(1-\delta\alpha_B(1-\tau_B))(1-\delta\alpha_R(1-\tau_R)) - \delta^2 \tau_B \tau_R \alpha_R \alpha_B > 0$, it follows that $sign(\frac{\partial w_B^*}{\partial \varepsilon}) = sign(\delta\alpha_R(1+\tau_B-\tau_R)-1)$.

We now find $\frac{\partial w_R^*}{\partial \varepsilon}$. The FOC wrt w_R^* is:

$$-\pi_R \frac{\partial z_R}{\partial w} - 1 + \delta p_R \frac{\partial z_R}{\partial w}((1-\tau_R)V_R^* + \tau_R V_B^*) = 0.$$

Taking the derivative of the FOC wrt ε:

$$\frac{\partial^2 z_R}{\partial w^2}\frac{\partial w_R^*}{\partial \varepsilon}(-\pi_R + \delta p_R((1-\tau_R)V_R^* + \tau_R V_B^*)) + \delta p_R \frac{\partial z_R}{\partial w}(V_B^* - V_R^* + (1-\tau_R)\frac{\partial V_R^*}{\partial \varepsilon} + \tau_R \frac{\partial V_B^*}{\partial \varepsilon}) = 0,$$

so $sign \frac{\partial w_R^*}{\partial \varepsilon} = sign(V_B^* - V_R^* + (1-\tau_R)\frac{\partial V_R^*}{\partial \varepsilon} + \tau_R \frac{\partial V_B^*}{\partial \varepsilon})$,where

$$\frac{dV_R^*}{d\varepsilon} = -\frac{\det\begin{bmatrix}\frac{\partial G_B}{\partial \varepsilon} & \frac{\partial G_B}{\partial V_B^*}\\ \frac{\partial G_R}{\partial \varepsilon} & \frac{\partial G_R}{\partial V_B^*}\end{bmatrix}}{\det\begin{bmatrix}\frac{\partial G_B}{\partial V_R^*} & \frac{\partial G_B}{\partial V_B^*}\\ \frac{\partial G_R}{\partial V_R^*} & \frac{\partial G_R}{\partial V_B^*}\end{bmatrix}} \text{ and } \frac{dV_B^*}{d\varepsilon} = -\frac{\det\begin{bmatrix}\frac{\partial G_B}{\partial V_R^*} & \frac{\partial G_B}{\partial \varepsilon}\\ \frac{\partial G_R}{\partial V_R^*} & \frac{\partial G_R}{\partial \varepsilon}\end{bmatrix}}{\det\begin{bmatrix}\frac{\partial G_B}{\partial V_R^*} & \frac{\partial G_B}{\partial V_B^*}\\ \frac{\partial G_R}{\partial V_R^*} & \frac{\partial G_R}{\partial V_B^*}\end{bmatrix}}$$

and so $\frac{\partial V_R^*}{\partial \varepsilon} = -\frac{\delta^2 \alpha_B \alpha_R \tau_R (V_R^* - V_B^*) - (\delta \alpha_B (1-\tau_B) - 1)\delta \alpha_R (V_B^* - V_R^*)}{\delta^2 \tau_B \tau_R \alpha_R \alpha_B - (1-\delta\alpha_B(1-\tau_B))(1-\delta\alpha_R(1-\tau_R))}$ and $\frac{\partial V_B^*}{\partial \varepsilon} = -\frac{\delta \alpha_B \tau_B \delta \alpha_R (V_B^* - V_R^*) - \delta \alpha_B (V_R^* - V_B^*)(\delta \alpha_R (1-\tau_R) - 1)}{\delta^2 \tau_B \tau_R \alpha_R \alpha_B - (1-\delta\alpha_B(1-\tau_B))(1-\delta\alpha_R(1-\tau_R))}$. Therefore, $V_B^* - V_R^* + (1-\tau_R)\frac{\partial V_R^*}{\partial \varepsilon} + \tau_R \frac{\partial V_B^*}{\partial \varepsilon} = (V_B^* - V_R^*)\frac{1-\delta\alpha_B(1-\tau_B+\tau_R)}{(1-\delta\alpha_B(1-\tau_B))(1-\delta\alpha_R(1-\tau_R)) - \delta^2 \tau_B \tau_R \alpha_R \alpha_B}$

Again, the denominator is positive by Lemma 5. ■

Proof of Lemma 3

Proof. We begin by taking the derivative of CRA i's first-order condition with respect to $w_{j,s}$:

$$\begin{array}{c} -\pi_{D,s}(1-\lambda_s)\frac{\partial^2 z_{i,s}}{\partial w^2}\frac{dw_{i,s}}{dw_{j,s}} + \frac{\partial^2 z_{i,s}}{\partial w^2}\frac{dw_{i,s}}{dw_{j,s}}\delta(1-\lambda_s)p_s \begin{bmatrix}(1-(1-\lambda_s)(1-z_{j,s})p_s)E(V_D^*)+\\ (1-\lambda_s)(1-z_{j,s})p_s E(V_M^*)\end{bmatrix} = 0, \\ +\frac{\partial z_{i,s}}{\partial w}\frac{\partial z_{j,s}}{\partial w}\delta(1-\lambda_s)^2 p_s^2 \left[E(V_D^*) - E(V_M^*)\right] \end{array} \tag{49}$$

and, so

$$\frac{dw_{i,s}}{dw_{j,s}} = -\delta(1-\lambda_s)p_s^2 \frac{\frac{\partial z_{i,s}}{\partial w}}{\frac{\partial^2 z_{i,s}}{\partial w^2}} \frac{\frac{\partial z_{j,s}}{\partial w}[E(V_D^*) - E(V_M^*)]}{\delta p_s(1-(1-\lambda_s)(1-z_{j,s})p_s)E(V_D^*) + \delta p_s^2(1-\lambda_s)(1-z_{j,s})E(V_M^*) - \pi_{D,s}}. \tag{50}$$

Since $\frac{\frac{\partial z_{i,s}}{\partial w}\frac{\partial z_{j,s}}{\partial w}}{\frac{\partial^2 z_{i,s}}{\partial w^2}} < 0$ and, as shown in the Lemma below, $E(V_D^*) - E(V_M^*) < 0$, it follows that

$$sign(\frac{dw_{i,s}}{dw_{j,s}}) = -sign(\delta p_s(1-(1-\lambda_s)(1-z_{j,s})p_s)E(V_D^*)+\delta p_s^2(1-\lambda_s)(1-z_{j,s})E(V_M^*)-\pi_{D,s}). \tag{51}$$

Consider the first-order condition of the duopoly (equation 14). Then,

$$-\pi_{D,s}+\delta p_s\left[(1-(1-\lambda_s)(1-z_{j,s})p_s)E(V_D^*)+(1-\lambda_s)(1-z_{j,s})p_sE(V_M^*)\right] = \frac{1}{(1-\lambda_s)\frac{\partial z_{i,s}}{\partial w}} > 0. \tag{52}$$

It follows that $\frac{dw_{i,s}}{dw_{j,s}} < 0$. ■

Lemma 6 $E(V_M^*) > E(V_D^*)$

Proof. Suppose not—i.e., $E(V_D^*) > E(V_M^*)$. Then,

$$V_{i,s} < \pi_{D,s}(\lambda_s + (1-\lambda_s)(1-z_{i.s})) - w_{i,s} + \delta(1-(1-\lambda_s)(1-z_{i,s})p_s)E(V_D^*) \tag{53}$$

for all s. We can then construct a $\tilde{V}_s$ as the optimal continuation value defined by

$$\tilde{V}_s = \pi_{D,s}(\lambda_s + (1-\lambda_s)(1-z_{i.s})) - w_{i,s} + \delta(1-(1-\lambda_s)(1-z_{i,s})p_s)E(\tilde{V}). \tag{54}$$

It is immediate that $E(\tilde{V}) > E(V_D^*)$. The only exogenous parameter that is different in characterizing $\tilde{V}_s$ and $V_{M,s}^*$ is the per-period fee, which is always larger in the monopoly case. It follows that $E(\tilde{V}) < E(V_M^*)$, which is a contradiction. ■

Proof of Proposition 7

Proof. Consider the first-order condition with symmetry imposed in equation 15. Define

$$A := -\pi_{D,s}(1-\lambda_s)\frac{\partial z_s}{\partial w} - 1 + \frac{\partial z_s}{\partial w}\delta(1-\lambda_s)p_s\left[(1-(1-\lambda_s)(1-z_s)p_s)E(V_D^*) + (1-\lambda_s)(1-z_s)p_sE(V_M^*)\right] \tag{55}$$

then

$$\frac{dA}{dw} = \begin{array}{l} -\pi_{D,s}(1-\lambda_s)\frac{\partial^2 z_s}{\partial w^2} + \frac{\partial^2 z_s}{\partial w^2}\delta(1-\lambda_s)p_s(1-(1-\lambda_s)(1-z_s)p_s)E(V_D^*) \\ +(1-\lambda_s)(1-z_s)p_sE(V_M^*)] + \delta(\frac{\partial z_s}{\partial w})^2(1-\lambda_s)^2p_s^2\left[E(V_D^*) - E(V_M^*)\right] \end{array} < 0 \tag{56}$$

where the inequality follows since $-\pi_{D,s}+\delta p_s\left[(1-(1-\lambda_s)(1-z_{j,s})p_s)E(V_D^*) + (1-\lambda_s)(1-z_{j,s})p_sE(V_M^*)\right] > 0$ as in the proof of Lemma **3** above $\frac{\partial^2 z_s}{\partial w^2} < 0$, $\frac{\partial z_s}{\partial w} > 0$ and $E(V_D^*) < E(V_M^*)$ by Lemma 6 above.

It follows by the Implicit Function Theorem that $sign(\frac{dw_s}{dr}) = sign(\frac{dA}{dr})$ for any parameter r. We consider each of our parameters in turn.

First, consider $\pi_{D,s}$:

$$sign(\frac{dw_s}{d\pi_{D,s}}) = sign(-(1-\lambda_s)\frac{\partial z_s}{\partial w}) \tag{57}$$

so $\frac{dw_s}{d\pi_{D,s}} < 0$.

Next,

$$\frac{dA}{dp_s} = \begin{array}{r} \frac{\partial z_s}{\partial w}\delta(1-\lambda_s)\left[(1-(1-\lambda_s)(1-z_s)p_s)E(V_D^*) + (1-\lambda_s)(1-z_s)p_sE(V_M^*)\right] \\ +\frac{\partial z_s}{\partial w}\delta(1-\lambda_s)^2(1-z_s)p_s\left[E(V_M^*) - E(V_D^*)\right] \end{array} > 0 \tag{58}$$

so $\frac{dw_s}{dp_s} > 0$.

Turning, next, to λ,

$$\frac{dA}{d\lambda_s} = \begin{array}{r} \frac{\partial z_s}{\partial w}\{\pi_{D,s} - \delta p_s\left[(1-(1-\lambda_s)(1-z_s)p_s)E(V_D^*) + (1-\lambda_s)(1-z_s)p_sE(V_M^*)\right]\} \\ +\frac{\partial z_s}{\partial w}\delta(1-\lambda_s)p_s^2(1-z_s)\left[E(V_D^*) - E(V_M^*)\right] \end{array} < 0, \tag{59}$$

where, again, the term in the $\{.\}$ is < 0. It follows that $\frac{dw_s}{d\lambda_s} < 0$.

Finally, we turn to consider γ_s,

$$\frac{dA}{d\gamma_s} = \begin{array}{c} (1-\lambda_s)\frac{\partial^2 z_s}{\partial w \partial \gamma_s}\left\{-\pi_{D,s} + \delta p_s\left[(1-(1-\lambda_s)(1-z_s)p_s)E(V_D^*) + (1-\lambda_s)(1-z_s)p_s E(V_M^*)\right]\right\} \\ -\frac{\partial z_s}{\partial w}\frac{\partial z_s}{\partial \gamma_s}\delta(1-\lambda_s)^2 p_s^2\left[E(V_M^*) - E(V_D^*)\right] \end{array}. \tag{60}$$

Note that $\frac{\partial^2 z_s}{\partial w \partial \gamma_s} < 0$, the term in the $\{.\} > 0$ and $\frac{\partial z_s}{\partial \gamma_s} < 0$. Overall ,therefore, $\frac{dw_s}{d\gamma_s}$ is ambiguous. ■