Lumpy Investment and Corporate Tax Policy[*]

Jianjun Miao[†]   Pengfei Wang[‡]

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Abstract

This paper studies the impact of corporate tax policy on the economy in the presence of both convex and nonconvex capital adjustment costs in a dynamic general equilibrium model. We show that corporate tax policy generates both intensive and extensive margin effects via the channel of marginal Q. Its impact is determined largely by the strength of the extensive margin effect, which in turn depends on the cross-sectional distribution of firms. Depending on the initial distribution of firms, the economy displays asymmetric responses to tax changes. We also show that an anticipated decrease in the future corporate income tax rate raises investment and adjustment rate immediately, while an anticipated increase in the future investment tax credit reduces investment and adjustment rate initially. Our general equilibrium analysis demonstrates that a partial equilibrium analysis of tax policy can be quite misleading both quantitatively and qualitatively.

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[†]Department of Economics, Boston University, 270 Bay State Road, Boston, MA 02215. Tel.: 617-353-6675. Email: miaoj@bu.edu. Homepage: http://people.bu.edu/miaoj.

[‡]Department of Economics, Hong Kong University of Science and Technology, Clear Water Bay, Hong Kong. Tel: (+852) 2358 7612. Email: pfwang@ust.hk

1 Introduction

Corporate tax policy is an important instrument to influence firms' capital investment decisions and hence to stimulate the economy. Its transmission channel is through either the user cost of capital according to the neoclassical theory of investment or Tobin's marginal Q according to the q-theory of investment.[1] The neoclassical theory assumes that firms do not face any investment frictions, while the q-theory takes into account of convex capital adjustment costs.

Recent empirical evidence documents that investment at the plant level is lumpy and infrequent, indicating the existence of nonconvex capital adjustment costs.[2] Motivated by this evidence, we address two central questions: (i) How does corporate tax policy affect investment at both the macro- and micro-levels in the presence of nonconvex capital adjustment costs? (ii) Is lumpy investment important quantitatively in determining the impact of tax policy on the economy in the short and long run?

To answer these two questions, one has to overcome two major difficulties. First, in the presence of nonconvex adjustment costs, the standard q-theory widely used in the analysis of tax policy fails in the sense that investment may not be monotonically related to marginal Q (Caballero and Leahy (1996)). Even though a modified q-theory may work, the relationship between investment and marginal Q is nonlinear (Abel and Eberly (1994)). Second, investment at the micro-level is nonlinear, making aggregation difficult. This problem is especially severe in a dynamic general equilibrium context, because one has to deal with the cross-sectional distribution of firms when solving equilibrium prices.

Our solution to these two difficulties is based on the generalized (S,s) approach proposed by Caballero and Engel (1999). The key idea of this approach is to introduce stochastic fixed adjustment costs, which makes aggregation tractable. We combine this approach with the Abel and Eberly (1994) approach by assuming firms face both flow convex and fixed adjustment costs. As a result, at the micro-level, a modified q-theory applies in that investment is a nondecreasing function of marginal Q. In particular, there is a region for the stochastic fixed adjustment costs in which investment is zero, independent of marginal Q. Outside of this region, investment increases with marginal Q.

To resolve the curse of dimensionality issue encountered when numerically solving general equilibrium models with lumpy investment, Khan and Thomas (2003, 2008) and Bachmann et

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[1] See Jorgenson (1963), Hall and Jorgenson (1967), Tobin (1969), and Hayashi (1982). See Abel (1990) for a discussion on the relation between the two theories. See Hassett and Hubbard (1997) for a survey.

[2] See, for example, Domas and Dunne (1998), Caballero, Engel and Haltiwanger (1995), Caballero and Engel (1999), Cooper and Haltiwanger (2006).

al. (2008) use the Krusell and Smith (1998) algorithm, which approximates the distribution of firms by a finite set of moments.[3] Typically, the first moment is sufficient. We deal with the dimensionality problem by making two assumptions as in Miao and Wang (2009): (i) firms' production technology has constant returns to scale, and (ii) fixed capital adjustment costs are proportional to the capital stock.[4] We show that under these two assumptions, we obtain exact aggregation in the sense that the cross-sectional distribution of capital matters only to the extent of its mean. We then characterize equilibrium by a system of nonlinear difference equations, which can be tractably solved numerically. This system also makes the analysis and intuition transparent.

We abstract from aggregate uncertainty and extend Miao and Wang's (2009) model by incorporating tax policy. We study the short- and long-run effects of temporary and permanent changes in the corporate tax rate and the investment tax credit (ITC). We also analyze anticipation effects of tax changes. We find that in the presence of fixed adjustment costs, corporate tax policy affects a firm's decision on the timing and size of investment at the micro-level. At the macro-level, it affects the size of investment for each adjusting firm and the number of total adjusting firms or the adjustment rate. Thus, corporate tax policy has both intensive and extensive margin effects. By contrast, it has an intensive margin effect only according to the neoclassical theory or the q-theory of investment.

We numerically show that the extensive margin effect plays an important role in determining the impact of the corporate tax changes. The strength of this effect depends crucially on the cross-sectional distribution of firms. As Miao and Wang (2009) point out, the larger the steady-state elasticity of the adjustment rate with respect to the investment trigger, the larger is the extensive margin effect. In addition, if the distribution is such that most firms have made capital adjustments prior to tax changes, then expansionary tax policy is less effective in stimulating the economy because the additional number of firms that decide to invest is constrained, while contractionary tax policy is more effective. The opposite is true if the distribution is such that most firms have not made capital adjustments prior to tax changes.

We show that the extensive margin effect is much larger in partial equilibrium than in general equilibrium. In a calibrated experiment, we find that the increase in the investment rate in partial equilibrium in response to an unanticipated temporary 10 percentage points decrease in the corporate income tax rate can be twice as large as that in general equilibrium.

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[3]Thomas (2002) and Gourio and Kashyap (2007) apply the numerical method developed by Dotsey, King and Wolman (1999).

[4]Both assumptions seem innocuous. The first assumption is often used in the workhorse growth model. The second assumption has empirical support by the estimate in Cooper and Haltiwanger (2006).

The intuition is that the general equilibrium price movements dampen the increase in firm profitability and marginal Q.

Furthermore, we show that under constant returns-to-scale technology and perfect competition, a temporary change in the corporate tax rate can have a permanent effect on capital accumulation in partial equilibrium, while it has a temporary effect in general equilibrium. In addition, an unanticipated temporary increase in the ITC has a larger short-run stimulative impact than a permanent increase in the ITC, in contrast to Abel's (1982) result in a partial equilibrium model. The preceding results suggest that a partial equilibrium analysis of tax policy can be quite misleading both quantitatively and qualitatively.

Our paper is related to a large literature on the impact of tax policy on investment beginning from Hall and Jorgenson (1967) and Hall (1971). Most studies use a partial equilibrium q-theory framework with convex adjustment costs (e.g., Summers (1981), Abel (1982), Hayashi (1982), and Auerbach (1989)). Instead, we use a general equilibrium deterministic growth model framework as in Chamley (1981), Judd (1985, 1987), Prescott (2002), McGrattan and Prescott (2005), House and Shapiro (2006), and Gourio and Miao (2008, 2009).[5] Unlike these papers, the present paper incorporates heterogeneous firms subject to both convex and nonconvex capital adjustment costs. Miao (2008) also incorporates these costs in a continuous time model, but he focuses on a steady-state analysis.

We extend Judd's (1987) and Auerbach's (1989) results on anticipation effects of tax policy. In particular, an anticipated decrease in the future corporate income tax rate raises investment and adjustment rate immediately, while an anticipated increase in the future ITC reduces investment and adjustment rate initially. In addition, there is a welfare gain if the increase in the ITC is unanticipated rather than anticipated, contrary to the case of a temporary decrease in the corporate income tax rate.

2   The Model

We consider an infinite-horizon economy that consists of a representative household, a continuum of production units with a unit mass, and a government. Time is discrete and indexed by $t = 1, 2, 3, ...$ We identify a production unit with a firm or a plant. Firms are subject to idiosyncratic shocks to fixed capital adjustment costs. To focus on the dynamic effects of permanent and temporary tax changes, we abstract away from aggregate uncertainty and

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[5]See Ljungqvist and Sargent (2004, Chapter 11) for a textbook treatment. See Auerbach and Kotlikoff (1987) for the study of tax policy in the general equilibrium overlapping-generation framework.

long-run economic growth. By a law of large numbers, all aggregate quantities and prices are deterministic over time.

2.1 Firms

All firms have identical production technology that combines labor and capital to produce output. Specifically, if firm j owns capital K_t^j and hires labor N_t^j, it produces output Y_t^j according to the production function:

$$Y_t^j = F\left(K_t^j, N_t^j\right), \tag{1}$$

Assume that F is strictly increasing, strictly concave, continuously differentiable, and satisfies the usual Inada conditions. In addition, it has constant returns to scale.

Firm j may make investment I_t^j to increase its existing capital stock K_t^j. Investment incurs both nonconvex and convex adjustment costs. We follow Uzawa (1969) and Hayashi (1982) and introduce convex adjustment costs into the capital accumulation equation:

$$K_{t+1}^j = (1-\delta)K_t^j + K_t^j \Phi\left(\frac{I_t^j}{K_t^j}\right), \ \ K_1^j \text{ given}, \tag{2}$$

where δ is the depreciation rate and $\Phi\left(\cdot\right)$ is a strictly increasing and strictly concave function.[6] To facilitate analytical solutions, we follow Jermann (1998) and specify the convex adjustment cost function as:

$$\Phi\left(x\right) = \frac{\psi}{1-\theta}x^{1-\theta} + \varsigma, \tag{3}$$

where $\psi > 0$ and $\theta \in (0,1)$. Nonconvex adjustment costs are fixed costs that must be paid if and only if the firm chooses to invest. As in Cooper and Haltiwanger (2006), we measure these costs as a fraction of the firm's capital stock.[7] That is, if firm j makes new investment, then it pays fixed costs $\xi_t^j K_t^j$, which is independent of the amount of investment. As will be clear later, this modelling of fixed costs is important to ensure that firm value is linearly homogenous. Following Caballero and Engel (1999), we assume that ξ_t^j is identically and independently drawn from a distribution with density ϕ over $[0, \xi_{\max}]$ across firms and over time.

Each firm j pays dividends to households who are shareholders of the firm. Dividends are given by:

$$D_t^j = \left(1-\tau_t^k\right)\left(F(K_t^j, N_t^j) - w_t N_t^j\right) + \tau_t^k \delta K_t^j - \left(1-\tau_t^i\right) I_t^j - \xi_t^j K_t^j \mathbf{1}_{I_t^j \neq 0}, \tag{4}$$

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[6]Assuming convex adjustment costs decrease profits directly as in Abel (1982) and Auerbach (1989) will not change our analysis significantly.

[7]There are several ways to model fixed adjustment costs in the literature. Fixed costs may be proportional to the demand shock (Abel and Eberly (1998)), profits (Caballero and Engel (1999) and Cooper and Haltiwanger (2006)), or labor costs (Thomas (2002) and Khan and Thomas (2003, 2008)).

where w_t is the wage rate, τ_t^k is the corporate income tax rate, and τ_t^i is the investment tax credit (ITC). Here, $\mathbf{1}_{I_t^j \neq 0}$ represents the indicator function taking the value 1 if $I_t^j \neq 0$, and zero otherwise. We have assumed that economic depreciation is equal to physical depreciation. Allowing for more complicated depreciation allowance schemes will complicate our analysis without changing our key insights.

After observing its idiosyncratic shock ξ_t^j, firm j's objective in period t is to maximize cum-dividends market value of equity P_t^j:

$$\max \ P_t^j \equiv E_t \sum_{s=0}^{\infty} \beta^s \frac{\Lambda_{t+s}}{\Lambda_t} D_{t+s}^j, \tag{5}$$

subject to (2) and (4). Here, the expectation is taken with respect to the idiosyncratic shock distribution and $\beta^s \Lambda_{t+s}/\Lambda_t$ is the stochastic discount factor between period t and $t+s$. We will show later that Λ_{t+s} is a household's marginal utility in period $t+s$.

2.2   Households

All households are identical and derive utility from the consumption and labor sequences $\{C_t, N_t\}$ according to the time-additive utility function:

$$E\left[\sum_{t=1}^{\infty} \beta^{t-1} U\left(C_t, N_t\right)\right], \tag{6}$$

where $\beta \in (0,1)$ is the discount factor, and U satisfies the usual assumptions. Each household chooses consumption C_t, labor supply N_t, and shareholdings α_{t+1}^j to maximize utility (6) subject to the budget constraint:

$$C_t + \int \alpha_{t+1}^j \left(P_t^j - D_t^j\right) dj = \int \alpha_t^j P_t^j dj + (1 - \tau_t^n) w_t N_t + T_t, \tag{7}$$

where τ_t^n is the labor income tax rate and T_t denotes government transfers (lump-sum taxes) if $T_t > (<)0$. The first-order conditions are given by:

$$\Lambda_t \left(P_t^j - D_t^j\right) = \beta E_t \Lambda_{t+1} P_{t+1}^j, \tag{8}$$
$$U_1\left(C_t, N_t\right) = \Lambda_t, \tag{9}$$
$$-U_2\left(C_t, N_t\right) = \Lambda_t\left(1 - \tau_t^n\right) w_t. \tag{10}$$

Equations (8)-(9) imply that the stock price P_t^j is given by the discounted present value of dividends as in equation (5). In addition, Λ_t is equal to the marginal utility of consumption.

2.3 Government

The government finances government spending G_t by corporate and personal taxes. We assume lump-sum taxes (or transfers) are available so that the government budget is balanced. The government budget is given by:

$$G_t + T_t = \tau_t^k \int \left(F(K_t^j, N_t^j) - w_t N_t^j - \delta K_t^j \right) dj + \tau_t^n w_t N_t - \tau_t^i \int I_t^j dj, \tag{11}$$

where T_t represents lump-sum transfers (taxes) if $T_t > 0$ ($T_t < 0$).

2.4 Competitive Equilibrium

The sequences of quantities $\{I_t^j, N_t^j, K_t^j\}_{t\geq 1}$, $\{C_t, N_t\}_{t\geq 1}$, prices $\{w_t, P_t^j\}_{t\geq 1}$ for $j \in [0, 1]$, and government policy $\{\tau_t^k, \tau_t^n, \tau_t^i, T_t, G_t\}_{t\geq 1}$ constitute a competitive equilibrium if the following conditions are satisfied:

(i) Given prices $\{w_t\}_{t\geq 1}$, $\{I_t^j, N_t^j\}_{t\geq 1}$ solves firm j's problem (5) subject to the law of motion (2).

(ii) Given prices $\left\{w_t, P_t^j\right\}_{t\geq 1}$, $\{C_t, N_t, \alpha_{t+1}^j\}_{t\geq 1}$ maximizes utility in (6) subject to the budget constraint (7).

(iii) Markets clear in that:

$$\alpha_t^j = 1,$$

$$N_t = \int N_t^j dj,$$

$$C_t + \int I_t^j dj + \int \xi_t^j K_t^j \mathbf{1}_{I_t^j \neq 0} dj + G_t = \int F\left(K_t^j, N_t^j\right) dj. \tag{12}$$

(iv) The government budget constraint (11) is satisfied.

3 Equilibrium Properties

We start by analyzing a single firm's optimal investment policy, holding prices fixed. Next, we conduct aggregation and characterize equilibrium aggregate dynamics by a system of nonlinear difference equations. Finally, we analyze steady state.

3.1 Optimal Investment Policy

To simplify problem (5), we first solve a firm's static labor choice decision. Let $n_t^j = N_t^j / K_t^j$. The first-order condition with respect to labor yields:

$$f'\left(n_t^j\right) = w_t, \tag{13}$$

where we define $f\left(\cdot\right) = F\left(1,\cdot\right)$. This equation reveals that all firms choose the same labor-capital ratio in that $n_t^j = n_t$ for all j. We can then derive firm j's operating profits:

$$\max_{N_t^j} F\left(K_t^j, N_t^j\right) - w_t N_t^j = R_t K_t^j, \tag{14}$$

where $R_t = f(n_t) - w_t n_t$ is independent of j. Note that R_t also represents the marginal product of capital because F has constant returns to scale. Let $i_t^j = I_t^j/K_t^j$ denote firm j's investment rate. We can then express dividends in (4) as:

$$D_t^j = \left[\left(1 - \tau_t^k\right) R_t + \tau_t^k \delta - \left(1 - \tau_t^i\right) i_t^j - \xi_t^j \mathbf{1}_{i_t^j \neq 0}\right] K_t^j,$$

and rewrite (2) as

$$K_{t+1}^j = \left[(1 - \delta) + \Phi(i_t^j)\right] K_t^j. \tag{15}$$

The above two equations imply that equity value or firm value are linear in capital K_t^j. We can then write firm value as $V_t^j K_t^j$ and rewrite problem (5) by dynamic programming:

$$V_t^j K_t^j = \max_{i_t^j} \left[\left(1 - \tau_t^k\right) R_t + \tau_t^k \delta - \left(1 - \tau_t^i\right) i_t^j - \xi_t^j \mathbf{1}_{i_t^j \neq 0}\right] K_t^j + E_t \left[\frac{\beta \Lambda_{t+1}}{\Lambda_t} V_{t+1}^j K_{t+1}^j\right], \tag{16}$$

subject to (15). Substituting equation (15) into equation (16) yields:

$$V_t^j = \max_{i_t^j} \ \left(1 - \tau_t^k\right) R_t + \tau_t^k \delta - \left(1 - \tau_t^i\right) i_t^j - \xi_t^j \mathbf{1}_{i_t^j \neq 0} + g(i_t^j) E_t \left[\frac{\beta \Lambda_{t+1}}{\Lambda_t} V_{t+1}^j\right], \tag{17}$$

where we define:

$$g(i_t^j) = 1 - \delta + \Phi(i_t^j). \tag{18}$$

Since V_t^j depends on ξ_t^j, we write it as $V_t^j = V_t\left(\xi_t^j\right)$ for some function V_t and suppress its dependence on other variables. We integrate out ξ_t^j and define:

$$\bar{V}_t = \int_0^{\xi_{\max}} V_t\left(\xi\right) \phi(\xi) d\xi. \tag{19}$$

Because ξ_t^j is iid across both time and firms and there is no aggregate shock, we obtain:

$$E_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} V_{t+1}^j\right] = \frac{\Lambda_{t+1}}{\Lambda_t} \int_0^{\xi_{\max}} V_{t+1}\left(\xi\right) \phi(\xi) d\xi = \frac{\Lambda_{t+1}}{\Lambda_t} \bar{V}_{t+1}, \tag{20}$$

Define marginal Q as the discounted shadow value of a marginal unit of investment:

$$Q_t = \frac{\beta \Lambda_{t+1}}{\Lambda_t} \bar{V}_{t+1}. \tag{21}$$

Because firm value is linearly homogeneous in capital in our model, marginal Q is equal to average Q (Hayashi (1982)). Using (21), we rewrite problem (17) as:

$$V_t\left(\xi_t^j\right) = \max_{i_t^j} \ \left(1 - \tau_t^k\right) R_t + \tau_t^k \delta - \left(1 - \tau_t^i\right) i_t^j - \xi_t^j \mathbf{1}_{i_t^j \neq 0} + g(i_t^j) Q_t. \tag{22}$$

From this problem, we can characterize a firm's optimal investment policy by a generalized (S,s) rule (Caballero and Engel (1999)).

Proposition 1 *Firm j's optimal investment policy is characterized by the (S, s) policy in that there is a unique trigger value $\bar{\xi}_t > 0$ such that the firm invests if and only if $\xi_t^j \leq \bar{\xi}_t \equiv \min\{\xi_t^*, \xi_{\max}\}$, where the cutoff value ξ_t^* satisfies the equation:*

$$\frac{\theta}{1 - \theta} \left(\psi Q_t\right)^{\frac{1}{\theta}} \left(1 - \tau_t^i\right)^{\frac{\theta - 1}{\theta}} = \xi_t^*. \tag{23}$$

The optimal target investment level is given by:

$$i_t^j = \left(\frac{\psi Q_t}{1 - \tau_t^i}\right)^{\frac{1}{\theta}}. \tag{24}$$

Marginal Q satisfies:

$$Q_t = \frac{\beta \Lambda_{t+1}}{\Lambda_t} \left\{ \left(1 - \tau_{t+1}^k\right) R_{t+1} + \tau_{t+1}^k \delta + \left(1 - \delta + \varsigma\right) Q_{t+1} + \int_0^{\bar{\xi}_{t+1}} \left[\xi_{t+1}^* - \xi\right] \phi(\xi) d\xi \right\}. \tag{25}$$

Equation (23) says that, at the value ξ_t^*, the benefit from investment is equal to the fixed cost of investment so that the firm is indifferent. It is possible that ξ_t^* exceeds the upper support of the fixed costs. In this case, we set the investment trigger $\bar{\xi}_t = \xi_{\max}$.

We should emphasize that the investment trigger $\bar{\xi}_t$ depends on the aggregate capital stock and corporate tax policy. Thus, corporate tax policy affects the timing of investment at the micro-level in that it affects the *adjustment hazard*, $\int_0^{\bar{\xi}_t} \phi(\xi) d\xi$. Under the q-theory of investment, corporate tax policy has only an intensive margin effect on investment in the sense that it can affect how much a firm may invest, but cannot affect how how likely a firm may investment.

Equation (24) shows that the optimal investment level is independent of a firm's characteristics such as its capital or idiosyncratic shock. It is positively related to marginal Q if and only if the firm's idiosyncratic fixed cost shock ξ_t^j is lower than the trigger value $\bar{\xi}_t$, conditioned on the aggregate state and tax policy in the economy. When $\xi_t^j > \bar{\xi}_t$, firm j chooses not to invest. This zero investment is unrelated to marginal Q. As a result, optimal investment may not be related to marginal Q in the presence of fixed adjustment costs, a point made by Caballero and

Leahy (1996). But it is still a nondecreasing function of marginal Q. Thus, a modified q-theory still works (Abel and Eberly (1994)).

Equation (25) is an asset-pricing equation which states that the aggregate marginal Q is equal to the present value of marginal product of capital, plus an option value of waiting because of the fixed adjustment costs. When the shock $\xi_t^j > \bar{\xi}_t$, it is not optimal to pay the fixed costs to make investment. Firms will wait to invest until $\xi_t^j \leq \bar{\xi}_t$ and there is an option value of waiting.

We may interpret (25) as the capital demand equation and (24) as the capital supply equation. Define $\tilde{Q}_t = Q_t / \left(1 - \tau_t^i\right)$ as in Abel (1982) and Goolsbee (1998). It represents the tax-adjusted price of capital.

3.2 Aggregation and Equilibrium Characterization

Given the linear homogeneity feature of firm value, we can conduct aggregation tractably. We define aggregate capital $K_t = \int K_t^j dj$, aggregate labor demand $N_t = \int N_t^j dj$, aggregate output $Y_t = \int Y_t^j dj$, and aggregate investment expenditure in consumption units $I_t = \int I_t^j dj$.

Proposition 2 *The aggregate equilibrium sequences $\{Y_t, N_t, C_t, I_t, K_{t+1}, Q_t, \bar{\xi}_t\}_{t \geq 1}$ are characterized by the following system of difference equations:*

$$I_t/K_t = \left(\frac{\psi Q_t}{1 - \tau_t^i}\right)^{\frac{1}{\theta}} \int_0^{\bar{\xi}_t} \phi(\xi) d\xi, \tag{26}$$

$$K_{t+1} = (1 - \delta + \varsigma)K_t + \frac{\psi}{1 - \theta} K_t \left(I_t/K_t\right)^{1-\theta} \left[\int_0^{\bar{\xi}_t} \phi(\xi) d\xi\right]^\theta, \tag{27}$$

$$Y_t = F(K_t, N_t) = G_t + I_t + C_t + K_t \int_0^{\bar{\xi}_t} \xi \phi(\xi) d\xi, \tag{28}$$

$$\frac{-U_2(C_t, N_t)}{U_1(C_t, N_t)} = (1 - \tau_t^n) F_2(K_t, N_t), \tag{29}$$

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\begin{aligned}
Q_t = \ & \frac{\beta U_1(C_{t+1}, N_{t+1})}{U_1(C_t, N_t)} \left[\left(1 - \tau_{t+1}^k\right) F_1(K_{t+1}, N_{t+1}) + \tau_{t+1}^k \delta + (1 - \delta + \varsigma) Q_{t+1}\right. \\
& \left. + \int_0^{\bar{\xi}_{t+1}} \left(\xi_{t+1}^* - \xi\right) \phi(\xi) d\xi\right],
\end{aligned} \tag{30}
$$

where $\bar{\xi}_t$ and ξ_t^* are given in Proposition 1 and K_1 is given.

We derive equations (26) and (27) by aggregating equations (2) and (24), respectively. Equation (26) reveals that the aggregate investment rate is equal to a firm's target investment rate multiplied by the fraction of adjusting firms (or the *adjustment rate*), $\int_0^{\bar{\xi}_t} \xi \phi(\xi) d\xi$. Thus, corporate tax policy has both intensive and extensive margin effects on investment in the presence of fixed adjustment costs. To see the magnitude of these effects, we log-linearize equation (26) to obtain:

$$\hat{I}_t - \hat{K}_t = \underbrace{\frac{1}{\theta}\left(\hat{Q}_t + \frac{\tau^i}{1-\tau^i}\hat{\tau}_t^i\right)}_{\text{intensive}} + \underbrace{\frac{\bar{\xi}\phi(\bar{\xi})}{\int_0^{\bar{\xi}}\phi(\xi)d\xi}\widehat{\bar{\xi}}_t}_{\text{extensive}}, \tag{31}$$

where for any variable X_t, X denotes its steady state value and $\hat{X}_t = (X_t - X)/X$. The first term on the right side equation (31) captures the usual intensive marginal effect in the q-theory of investment without nonconvex adjustment costs. A change in τ_t^k or τ_t^i affects marginal Q as revealed by asset-pricing equation (30), and thus induces firms to decide how much to invest. The second term on the right-side of equation (31) represents the extensive margin effect. A change in τ_t^k or τ_t^i and the resulting change in marginal Q also affect a firm's decision on when to invest. Thus, tax changes affect the adjustment trigger $\bar{\xi}_t$ and hence the number of firms to make investment. The extent of this extensive margin effect is measured by the coefficient of $\widehat{\bar{\xi}}_t$, which is the steady-state elasticity of the adjustment rate with respect to the adjustment trigger $\bar{\xi}$. The larger the elasticity, the larger is the extensive margin effect.

3.3   Steady State

We consider a deterministic steady state in which government policy variables $\left(\tau_t^k, \tau_t^n, \tau_t^i, T_t, G_t\right)$ stay constant over time. In addition, all aggregate equilibrium quantities, prices and the investment trigger are constant over time, while at the firm-level firms still face idiosyncratic fixed costs shocks. The following proposition characterizes the steady-state aggregate variables $\left(Y, C, N, K, I, Q, \bar{\xi}\right)$, where $\bar{\xi}$ is an interior solution.

Proposition 3 *Suppose*

$$0 < \delta - \varsigma < \frac{\psi}{(1-\theta)^\theta \theta^{(1-\theta)}}\left(\frac{\xi_{\max}}{1-\tau^i}\right)^{1-\theta}\int_0^{\xi_{\max}}\phi(\xi)d\xi.$$

Then the steady-state investment trigger $\bar{\xi} \in (0, \xi_{\max})$ is the unique solution to the equation:

$$\delta - \varsigma = \frac{\psi}{(1-\theta)^\theta \theta^{(1-\theta)}}\left(\frac{\bar{\xi}}{1-\tau^i}\right)^{1-\theta}\int_0^{\bar{\xi}}\phi(\xi)d\xi. \tag{32}$$

10

Given this value $\bar{\xi}$, the steady-state value of Q is given by:

$$Q = \frac{1}{\psi} \left(\frac{\bar{\xi} (1 - \theta)}{\theta} \right)^{\theta} \left(1 - \tau^i \right)^{1 - \theta}. \tag{33}$$

The other steady-state values (I, K, C, N) satisfy:

$$\frac{I}{K} = \frac{Q}{1 - \tau^i} (\delta - \varsigma)(1 - \theta), \tag{34}$$

$$F(K, N) = I + C + K \int_0^{\bar{\xi}} \xi \phi(\xi) \, d\xi + G, \tag{35}$$

$$\frac{-U_2(C, N)}{U_1(C, N)} = (1 - \tau^n) F_2(K, N), \tag{36}$$

$$Q = \frac{\beta}{1 - \beta(1 - \delta + \varsigma)} \left\{ \left(1 - \tau^k \right) F_1(K, N) + \tau^k \delta + \int_0^{\bar{\xi}} \left(\bar{\xi} - \xi \right) \phi(\xi) d\xi \right\}. \tag{37}$$

We can derive some interesting results from this proposition. First, equation (32) shows that the steady-state investment trigger is independent of the corporate tax rate τ^k, but decreases with the ITC τ^i. Thus, the steady-state adjustment rate $\int_0^{\bar{\xi}} \phi(\xi) d\xi$ has the same property. Second, it follows from equation (33) that the steady-state marginal Q is independent of τ^k, but decreases with τ^i. We can also show that the tax-adjusted producer price of capital $\tilde{Q} = Q/\left(1 - \tau^i \right)$ increases with τ^i.[8] Third, it is straightforward from (34) to show that the steady-state investment rate is independent of τ^k, but increases with τ^i. The intuition behind the latter result is as follows. An increase in τ^i has negative an extensive margin effect because it reduces the adjustment rate, but has a positive intensive margin effect because it raises $\tilde{Q}$. Overall, the negative extensive margin effect is dominated by the positive intensive margin effect following an increase in τ^i. Except for the predictions regarding the adjustment rate, the preceding results are still valid in a model with convex adjustment costs only or in a standard growth model without adjustment costs.

4 Numerical Results

We evaluate our lumpy investment model quantitatively and compare this model with two benchmark models. The first one is a standard growth model with distortionary taxes, obtained by removing all adjustment costs in the model presented in Section 2. The second one is obtained by removing fixed adjustment costs only. We call this model *partial adjustment*

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[8]See Goolsbee (1998) for empirical evidence.

model. In each of the benchmark models, all firms make identical decisions and thus they give the same aggregate equilibrium allocations as that in a standard representative-agent and a representative-firm growth model. Because we can characterize the equilibria for all three models by systems of nonlinear difference equations, we can solve them numerically using standard methods.[9] To do so, we need first to calibrate the models.

4.1 Baseline Parametrization

For all model economies, we take the Cobb-Douglas production function, $F(K, N) = K^\alpha N^{1-\alpha}$, and the period utility function, $U(C, N) = \log(C) - \varphi N$, where $\varphi > 0$ is a parameter. We fix the length of period to correspond to one year, as in Thomas (2002), and Khan and Thomas (2003, 2008). Annual frequency allows us to use empirical evidence on establishment-level investment in selecting parameters for the fixed adjustment costs.

We first choose parameter values for preferences and technology to ensure that the steady-state of the standard growth model with taxes is consistent with the long-run values of key postwar U.S. aggregates. Specifically, we set the subjective discount factor to $\beta = 0.9615$, so that the implied real annual interest rate is 4%. We choose the value of φ in the utility function so that the steady-state hours are about 1/3 of available time spent in market work. We set the capital share $\alpha = 0.36$, implying a labor share of 0.64, which is close to the labor income share in the NIPA. We take the depreciate rate $\delta = 0.1$, as in the literature on business cycles.

In a steady state, all tax rates are constant over time. By the estimates from McGrattan and Prescott (2005) and Prescott (2002), we set capital tax rate $\tau^k = 0.35$ and labor tax rate $\tau^n = 0.32$. Because the investment tax credit is typically used as a short-run stimulative policy, we set $\tau^i = 0$ in the steady state. We assume that the government spending G_t is constant over time and equal to 20 percent of the steady-state output in the standard growth model. We fix this level of government spending for all experiments below.

It is often argued that convex adjustment costs are not observable directly and hence cannot be calibrated based on average data over the long run. Thus, we impose the two restrictions, $\psi = \delta^\theta$ and $\varsigma = -\theta\delta/(1-\theta)$, so that the partial adjustment model and the standard growth model give identical steady-state allocations. Following Thomas (2002) and Khan and Thomas (2003), we set $\theta = 1/5.98$, implying that the Q-elasticity of the investment rate is 5.98.

We adopt the power function distribution for the idiosyncratic fixed cost shock with density $\phi(\xi) = \eta\xi^{\eta-1}/(\xi_{\max})^\eta$, $\eta > 0$. We need to calibrate two parameters $\xi_{\max}$ and η. We select values such that our model's steady-state cross-sectional statistics match micro-level evidence on the

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[9] We use the Dynare package to solve our models numerically.

investment lumpiness reported by Cooper and Haltiwanger (2006). Cooper and Haltiwanger (2006) find that the inaction rate is 0.081 and the positive spike rate is about 0.186. A positive investment spike is defined as the investment rate exceeding 0.2. For the power function distribution, the steady-state inaction rate is given by $1 - \left(\bar{\xi}/\xi_{\max}\right)^{\eta}$ and the steady-state investment rate is given by equation (34). Here $\bar{\xi}$ is the steady-state trigger value determined in equation (32). Because our model implies that the target investment rate is identical for all firms, our model cannot match the spike rate observed in the data. Therefore, there are many combinations of η and $\xi_{\max}$ to match the inaction rate of 0.081. As baseline values, we follow Khan and Thomas (2003, 2008) and take a uniform distribution ($\eta = 1$). This implies that $\xi_{\max} = 0.0242$. In this case, we can compute that, in the steady state, total fixed adjustment costs account for 1.8 percent of output, 9.0 percent of total investment spending, and 0.9 percent of the aggregate capital stock, which are reasonable according to the estimation by Cooper and Haltiwanger (2006).

We summarize the baseline parameter values in Table 1.

Table 1. Baseline parameter values

β	φ	α	δ	τ^k	τ^n	τ^i	G	θ	$\xi_{\max}$	η
0.9615	2.5843	0.36	0.1	0.35	0.32	0	0.105	1/5.98	0.0242	1

We suppose the economy in period 1 is in the initial steady state with parameter values given in Table 1. We then consider the economy's responses to changes in corporate tax policy by changing sequences of tax rates $\{\tau_t^k\}$ and ITC $\{\tau_t^i\}$. We hold labor income tax rates τ^n constant at the value in Table 1 and allow lump-sum taxes to adjust to balance government budget. We consider four calibrated models denoted by Growth, PA, Lumpy1, and Lumpy 2. The first three models refer to the standard growth, partial adjustment, and lumpy investment models with the parameter values given in Table 1. "Lumpy2" refers to the lumpy investment model with parameter values given in Table 1 except that we set $\eta = 10$ and recalibrate $\xi_{\max}$ to match the inaction rate of 0.081. We find that it is equal to 0.0224. We use the Lumpy2 model to illustrate the importance of the extensive margin effect. Because the steady-state elasticity of the adjustment rate with respect to the investment trigger is equal to η for the power function distribution, the Lumpy2 model should deliver a larger extensive margin effect than the Lumpy1 model by our analysis in Section 3.2. We emphasize that this effect is not necessarily related to the size of total fixed costs because the steady-state ratio of total fixed costs to output in the Lumpy2 model is 1.7 percent, smaller than that in the Lumpy1 model.[10]

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[10]See Miao and Wang (2009) and Khan and Thomas (2008) for further discussions on this point.

This contrasts with the conventional intuition that fixed costs must be large in order for lumpy investment to be important.

4.2 Temporary Changes in the Corporate Tax Rate

We start with the first policy experiment in which the corporate tax rate τ_t^k decreases by 10 percentage points temporarily and this decrease lasts for only 4 periods. After this decrease, τ_t^k reverts to its previous level. Suppose this tax policy is unanticipated, but once it occurs in period 1, agents have perfect foresight about the future tax rates.

Figure 1 presents the transition dynamics for the four models (Growth, PA, Lumpy1 and Lumpy2) following this tax policy. We find these four models display similar transition dynamics. Because the responses in the standard growth model are well known in the literature (e.g., Ljungqvist and Sargent (2004, Chapter 11)), our analysis will focus on the other three models. For these three models, the decrease in τ_t^k in periods 1-4 raises the after-tax marginal product of capital and hence marginal Q immediately by (30), leading to a jump of investment in the initial period. Note that the impact jump of marginal Q is the highest for the PA model, and the lowest for the Lumpy2 model. Marginal Q starts to decrease until period 4 and then gradually rises to its steady-state value because τ_t^k rises to its original level permanently starting in period 5. Consequently, the investment rate follows a similar path by the intensive margin effect (see (31)). The capital stock rises until period 4, and then decreases monotonically to the original steady state level.

We define the after-tax (gross) interest rate between periods t and $t+1$ as:

$$r_t = \frac{U_1(C_t, N_t)}{\beta U_1(C_{t+1}, N_{t+1})} = \frac{C_{t+1}}{\beta C_t}.$$

It is also equal to the after-tax return on capital by equation (30). In response to the temporary decrease in τ_t^k, the after-tax return on capital rises initially and decreases until period 4. After this period, it gradually rises to its original steady-state level. Due to the substitution effect, consumption drops initially and then rises until period 4. After period 4, it decreases to its original steady-state level. Note that the wealth effect is small because the decrease in τ_t^k must be accompanied by an increase in lump-sum taxes that leaves the government budget balanced.

Given our adopted utility function, the wage rate w_t satisfies $\varphi C_t = (1 - \tau^n) w_t$. Thus, wage and consumption must follow identical dynamics, causing the labor hours to follow a pattern opposite to consumption because the marginal product of labor equals the after-tax wage. Output rises in the initial period because labor rises and capital is predetermined. After period 1, output gradually decreases to its steady-state value.

Figure 1 reveals that the short-run impact on investment and output for the PA model is smaller than for the Growth model because of the convex capital adjustment costs. The presence of fixed capital adjustment costs in the lumpy investment model makes the short-run impact of tax changes larger. The reason is that tax policy has an additional extensive margin effect as discussed in Section 3.2. Figure 1 shows that the adjustment rate rises by about 4 percent in period 1 for the Lumpy1 model. The total impact increase in the investment rate for the Lumpy1 model is about 7 percent, implying that the intensive margin effect contributes to about 3 percent of the increase. As discussed in Section 3.2, the larger the steady-state elasticity of the adjustment rate with respect to the investment trigger, the larger is the extensive margin effect. The transition dynamics for the Lumpy2 model displayed in Figure 1 illustrate this point. The adjustment rate rises by about 8.4 percent on impact, which contributes to most of the increase (about 10 percent) in the investment rate.

Despite the large extensive margin effect, the dynamic responses of the tax change for the lumpy investment model are similar to those for the Growth model and the partial adjustment model. The reason is that the general equilibrium price movements smooth aggregate investment dynamics.[11] Figure 2 illustrates this point by presenting transition dynamics for the partial adjustment and lumpy investment models in partial equilibrium. In particular, we fix the interest rate and the wage rate at their steady-state values prior to tax changes through the transition period so that the system of equations, (23), (26), (27), and

$$Q_t = \beta \left\{ \left(1 - \tau_{t+1}^k\right) R_{t+1} + \tau_{t+1}^k \delta + (1 - \delta + \varsigma) Q_{t+1} + \int_0^{\bar{\xi}_{t+1}} \left[\xi_{t+1}^* - \xi\right] \phi(\xi) d\xi \right\}, \quad (38)$$

characterizes the partial equilibrium dynamics of $\left\{I_t, K_{t+1}, Q_t, \bar{\xi}_t\right\}$, where $\bar{\xi}_t = \min\{\xi_t^*, \xi_{\max}\}$. In response to the tax cut, marginal Q rises by about 1.7, 2.2, and 2.5 percent for the PA, Lumpy1, and Lumpy2 models, respectively, much higher than the corresponding values, 0.7, 0.6, and 0.2 percent in general equilibrium. As a result, the increase in the adjustment rate in partial equilibrium is much higher than that in general equilibrium. In particular, the increase in Q in partial equilibrium is so high that all firms decide to make capital adjustments in the first two periods for the Lumpy1 model and in the first three periods for the Lumpy2 model. This large extensive margin effect causes the aggregate investment rate rises by about 21 and 22 percent for the Lumpy1 and Lumpy2 models, respectively. This increase is much larger than the corresponding increase of 7.2 and 10 percent in general equilibrium.

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[11]Thomas (2002) first makes this point and shows that lumpy investment is quantitatively irrelevent for business cycles. Using a different numerical solution method, Khan and Thomas (2003, 2008) confirm her finding. In an analytical framework, Miao and Wang (2009) provide conditions under which Thomas's result can hold true.

We also emphasize that in a partial equilibrium model with competitive firms and constant-returns-to-scale technology, Q can be determined independently of capital. This can be seen from equation (38), in which the marginal product of capital R_t is constant when the wage rate is fixed.[12] A temporary cut in τ_t^k from periods 1-4 induces Q to rise immediately and then decrease monotonically to its steady-state value. The investment rate also follows the same pattern. Because the investment rate never falls below its steady-state level, a temporary change in τ_t^k has a permanent effect on the capital stock, as revealed in Figure 2. This result is in sharp contrast to that in general equilibrium, suggesting that a partial equilibrium analysis of tax policy can be quite misleading.

We next turn to the case in which the temporary decrease in τ_t^k is anticipated initially to be effective in period 5 and lasts for 4 periods. Figure 3 presents the transition dynamics. The future tax cut raises the future after-tax marginal product of capital, *ceteris paribus*. Because marginal Q reflects the present value of the future after-tax marginal product of capital by (30), it rises immediately. Thus, anticipating the tax cut in the future, adjusting firms react by raising investment immediately. In addition, the adjustment rate also rises immediately in the two lumpy investment models. The investment rate and the adjustment rate continue to rise until period 4. The household reacts by reducing consumption and raising labor supply immediately. From periods 1 to 4, consumption and hours gradually rise. Output also rises from periods 1 to 4. Starting from period 5, the economy's responses are similar to those in the case of unexpected tax change presented in Figure 1.

4.3 Permanent Changes in the Corporate Tax Rate

Figure 4 presents transition dynamics following an unanticipated permanent 10 percentage points cut in the corporate tax rate τ_t^k. The economy's immediate response in this case is qualitatively similar to that in the case of an unanticipated temporary corporate tax cut via the same channel of marginal Q. However, the short-run impact is larger because the response of Q is larger. In the long run, the economy moves to a new steady state after a permanent tax cut, while the steady state does not change after a temporary tax cut. In addition, the transition paths following an unanticipated permanent tax cut are monotonic, rather than non-monotonic.

We emphasize that the impact of tax policy depends on the initial cross-sectional distribution of firms. In our baseline calibration, 91.9 percent of firms have made capital adjustments

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[12]Using equation (14), we can show that $R_t = \alpha \left((1-\alpha)/w_t \right)^{\frac{1-\alpha}{\alpha}}$. It is constant over time when wage w_t is fixed at the initial steady-state value prior to tax changes.

in the initial steady state. This high adjustment rate leaves less room for more firms to make adjustments in response to a decrease in τ^k. Figure 4 reveals that the investment trigger hits the upper support of the fixed cost distribution from periods 1 to 4 for the Lumpy2 model, implying that the extensive margin effect is constrained. If the initial adjustment rate is small, more firms will respond to a capital tax cut, making the extensive margin effect large. To illustrate this point, we conduct an experiment with the initial steady-state adjustment rate equal to 0.2.[13] Figure 5 plots the economy's responses to an unanticipated permanent 10 percentage points decrease in τ^k. We find that both the short- and long-run effects in this case are much larger than the case in Figure 4. The main reason is due to the larger extensive margin effect. In particular, the adjustment rate rises by 10 and 20 percent on impact for the Lumpy1 and Lumpy2 models, respectively.

While the initial high adjustment rate constrains the effectiveness of an expansionary tax policy, it makes a contractionary tax policy more effective. To illustrative this point, Figure 6 presents the economy's response to an unanticipated permanent 10 percentage points increase in τ^k. This figure reveals that for Lumpy2 model, the adjustment rate decreases by about 18 percent immediately, causing the aggregate investment rate to fall by about 20 percent immediately. Comparing Figure 6 with Figure 4, we find that the responses to the tax increase and decrease are roughly symmetric for the standard growth and partial adjustment models, but are highly asymmetric for the lumpy investment model.

When the permanent tax cut is initially anticipated to be enacted in the future, marginal Q rises immediately for the same reason in the case of the anticipated temporary tax cut discussed in Section 4.2. Thus, both the investment rate and the adjustment rate must rise initially. They continue to rise until the enactment date of the tax cut. After this date, the economy transits to the new steady state as in the case presented in Figure 4. We omit a detailed discussion.

4.4   Temporary Changes in the Investment Tax Credit

Suppose a 10 percent ITC is imposed from periods 1-4 unexpectedly, $\tau_t^i = 0.1$ for $t = 1, ..., 4$. In period 5 this policy is repealed so that $\tau_t^i = 0$ for $t \geq 5$. Figure 7 presents the economy's dynamic responses. By equation (26), the increase in the ITC has a direct positive impact on investment by reducing the tax adjusted price of capital $\tilde{Q}_t = Q_t / \left(1 - \tau_t^i\right)$, *ceteris paribus*. But it also has an indirect negative impact on marginal Q by (30) because the increase in the investment rate raises the capital stock and reduces the marginal product of capital, *ceteris paribus*. Our numerical results show that the positive direct effect dominates, and thus $\tilde{Q}_t$ and

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[13]We recalibrate $\xi_{\max}$ in the Lumpy1 and Lumpy2 models to match this adjustment rate, 0.2.

the investment rate rises on impact. The investment rate then falls until period 4. In period 5, it drops sharply below its steady-state level because the ITC is removed in period 5. After period 5, investment gradually rises to its previous steady-state level. Consumption follows the opposite pattern as resources are devoted to investment.

Even though marginal Q falls immediately, the investment trigger rises on impact, because the increase in the ITC dominates the fall in Q in equation (23). This rise is so large that all firms make adjustments immediately (the investment trigger reaches the upper support of fixed costs). In addition, almost all firms in the Lumpy1 model and all firms in the Lumpy2 model continue to make capital adjustments until period 4. As a result, the changes in the investment rate in these 4 periods are similar in the Lumpy1 and Lumpy2 models. The adjustment rate drops below its steady-state level in period 5 and this drop is larger in the Lumpy2 model than in the Lumpy1 model. After period 5, it gradually rises to its previous steady-state level.

In contrast to the case of changes in the corporate income tax rate, the extensive margin effect accounts for less of the increase in the aggregate investment rate in the two lumpy investment models in response to the increase in the ITC. In addition, the initial rise of the investment rate is smaller in the two lumpy investment models than in the standard growth model. This result is also different from that in the case of corporate income tax cuts. The intuition is that the decrease in marginal Q in the short run reduces firms' incentives to make large investment. Even though the extensive margin effect in the presence of fixed costs raises firms' responses to the increase in the ITC, this effect is not large enough.

Next, we suppose the 10 percent ITC is enacted in period 5 and lasts for 4 periods. This tax policy is anticipated initially. Figure 8 presents the economy's responses to this tax policy. Contrary to the responses presented in Figure 7, investment and output decrease immediately, but consumption increases immediately. The intuition comes from the dynamics of marginal Q characterized in Proposition 2. Anticipating the fall of Q in period 5 due to the enactment of the 10 percent ITC, Q falls immediately at date 1. Because the investment rate is determined by equation (26) and $\tau_t^i = 0$ for $t = 1, ..., 4$, the investment rate must decrease from periods 1 to 4. In period 5, the 10 percent ITC makes the new investment good cheaper, but $\tilde{Q}_5 = Q_5 / \left(1 - \tau_5^i \right)$ actually rises. Thus, the investment rate jumps up in period 5. Starting from period 5, the economy's responses are similar to those in the case of the unanticipated temporary increase in the ITC.

18

4.5 Permanent Changes in the Investment Tax Credit

We finally consider two experiments in which there is a permanent 10 percent ITC. Figure 9 presents the economy's responses when this tax change is unanticipated initially. For all four models, the investment rate rises immediately, but the increase is less than that if the ITC is temporary, as shown in Figure 7. This is in sharp contrast to Abel's (1982) result that a temporary ITC provides a greater stimulus to investment than a permanent ITC, except for a competitive firm with constant returns to scale. The reason is that Abel (1982) uses a partial equilibrium model rather than a general equilibrium model. As we point out in Section 4.2, in partial equilibrium Q can be determined independently of capital for competitive firms with constant-returns-to-scale technology. We can then show that the initial rise of the investment rate is independent of the duration of the ITC. By contrast, in general equilibrium, Q and capital must be jointly determined as shown in Proposition 2. In particular, the interest rate and the wage rate change over time in response to tax changes. As Figures 8 and 10 show, the initial fall in Q is larger in response to the permanent increase in the ITC than in response to the temporary increase in the ITC.

A permanent increase in the ITC changes the economy's steady state. For the four models (Growth, PA, Lumpy1 and Lumpy2), the capital stock, output, consumption, and labor are higher in the new steady state. But the steady-state adjustment rates in the two lumpy investment models are lower than their initial steady-state values as discussed in Section 3.3.

When the permanent increase in the ITC is anticipated to be enacted in some future date, both the adjustment rate and the investment rate fall in the short run by the same intuition discussed for the case of the anticipated temporary increase in the ITC. They then jump up at the enactment date of the increase in the ITC. After this date, the transition paths are similar to those presented in Figure 9. We thus omit a detailed discussion.

4.6 Welfare Effects of Tax Policy

Our general equilibrium model allows us to conduct welfare analysis. To evaluate the welfare effect, it is important to make the distinction between the utility gain in the steady state and the utility gain in the transition path. For temporary tax changes, the economy's steady state remains unchanged. Therefore, there is no welfare effect in the steady state. However, in the transition path following a tax change, the equilibrium consumption and leisure may take values different from their steady-state values. There may be short-run costs and intermediate-run benefits. Thus, the indirect lifetime utility level derived from these equilibrium consumption

and leisure sequences after the tax change may be different from that in the equilibrium before the tax change. In the following analysis, we will focus on the welfare effect during transition rather than in the steady state.

We measure welfare gains following a change in tax policy as the percentage consumption compensation needed to leave the household indifferent between the new tax policy and the old tax policy.[14] Specifically, let the indirect life-time utility level before a tax change be given by:

$$U^b = \sum_{t=1}^{\infty} \beta^{t-1} \left[\log \left(C_t^b \right) - \varphi N_t^b \right],$$

where C_t^b and N_t^b denote the equilibrium consumption and labor in period t before the tax change. Let U^a denote the indirect life-time utility level in the equilibrium after the tax change. The welfare gain Δ is defined by the following equation:

$$U^a = \sum_{t=1}^{\infty} \beta^{t-1} \left[\log \left((1 + \Delta) C_t^b \right) - \varphi N_t^b \right].$$

Solving the above equation yields:

$$\Delta = \exp \left[(1 - \beta) \left(U^b - U^a \right) \right] - 1.$$

We use this formula to compute welfare gains for our previous tax policy experiments. Table 2 presents the results.

Table 2. Welfare gains from the tax changes.

	Growth	PA	Lumpy1	Lumpy2
$\tau_t^k = 0.35 \downarrow 0.25$				
$t = 1, ..., 4$	0.06	0.04	0.06	0.08
$t = 5, ..., 8$	0.12	0.09	0.14	0.18
$t = 1, 2, 3, ...$	1.02	0.93	1.25	1.45
$t = 5, 6, 7, ...$	0.97	0.90	1.20	1.40
$\tau_t^i = 0 \uparrow 0.1$				
$t = 1, ..., 4$	0.55	0.40	0.46	0.51
$t = 5, ..., 8$	0.33	0.31	0.32	0.32
$t = 1, 2, 3, ...$	2.42	2.22	2.33	2.35
$t = 5, 6, 7, ...$	1.85	1.83	1.87	1.83

[14]Chamley (1981) and Judd (1987) study marginal excess burden of factor taxation for small tax changes.

Note: This table presents welfare gains in percentage changes of consumption equivalent for the policy experiments studied in Section 4. The first column gives the time periods when tax rates are changed. Parameter values are given in Table 1 except that $\eta = 10$ and $\xi_{\max} = 0.0224$ in the Lumpy2 model.

Table 2 reveals several interesting results. First, the welfare effects for all four models are similar and generally small. The welfare benefits in the lumpy investment model are generally higher than those in the partial adjustment model. In addition, the distribution of fixed costs matter for the calculation of welfare benefits because this distribution determines the extensive margin effect. As we analyzed earlier, the Lumpy2 model generates a higher extensive margin effect than the Lumpy1 model, causing the welfare gains in the Lumpy2 model to be larger than those in the Lumpy1 model. Second, the welfare gains are larger when the temporary corporate income tax cut is anticipated than when it is unanticipated. But there is no anticipation benefit when the corporate income tax cut is permanent. Third, the welfare benefits from a permanent increase in the ITC are larger than those from a temporary increase in the ITC. In addition, there is a welfare gain if the increase in the ITC is unanticipated rather than anticipated, contrary to the case of the temporary decrease in the corporate income tax rate. The last two results are consistent with Judd's (1987) findings in a continuous-time model without capital adjustment costs. The intuition is that, regardless of the presence of capital adjustment costs, the decrease in the future capital income tax rate raises investment immediately and encourages capital formation early, while the increase in the future ITC reduces investment and capital stock in the short run. In addition, the latter also generates volatile consumption as shown in Figure 8.

5   Conclusions

In this paper, we have studied the impact of corporate tax policy on the economy in the presence of both convex and nonconvex capital adjustment costs in a dynamic general equilibrium model with firm heterogeneity. Our model permits exact aggregation and is analytically tractable. Our main results are as follows. First, corporate tax policy generates both intensive and extensive margin effects via the channel of marginal Q. At the micro-level, it affects a firm's decision on the size and timing of investment. At the macro-level, it affects each adjusting firm's investment size and the number of adjusting firms. Second, the impact of corporate tax policy depends on the cross-sectional distribution of firms. If most firms have not made capital adjustments initially, an expansionary corporate tax policy is more effective, while a

contractionary corporate tax policy is less effective. The opposite result holds true if most firms have made capital adjustments initially. Third, introducing convex adjustment costs in the standard growth model smooths the economy's responses to tax changes. Introducing nonconvex adjustment costs on top of it raises the economy's responses to tax changes and brings the equilibrium outcome closer to that in the standard growth model. The responses can be larger than those in the standard growth model for a permanent change in the corporate tax rate, but smaller for a permanent or temporary change in the ITC.

Forth, a permanent increase in the ITC raises the steady-state tax-adjusted price of capital, but reduces the steady-state adjustment rate. In addition, an unanticipated temporary increase in the ITC has a larger short-run stimulative impact than a permanent increase in the ITC, even for competitive firms with constant-returns-to-scale technology. This finding contrasts with Abel's (1982) result in a partial equilibrium model. Finally, we extend Judd's (1987) and Auerbach's (1989) results on anticipation effects of tax policy to a general equilibrium model with both convex and nonconvex capital adjustment costs. In particular, an anticipated decrease in the future corporate income tax rate raises investment and adjustment rate immediately, while an anticipated increase in the future ITC reduces investment and adjustment rate initially. In addition, there is a welfare gain if the increase in the ITC is unanticipated rather than anticipated, contrary to the case of a temporary decrease in the corporate income tax rate.

Our model may be useful for empirical work and some of our results have novel testable implications. For example, our predictions regarding the impact of corporate tax policy on the adjustment rate can be tested empirically.

Appendix

A   Proofs

Proof of Proposition 1:   From (22), we can show that the target investment level i_t^j satisfies the first-order condition:

$$1 - \tau_t^i = g'\left(i_t^j\right)\frac{\beta\Lambda_{t+1}}{\Lambda_t}\bar{V}_{t+1}. \qquad (A.1)$$

By equations (3), (18) and (21), we can derive equation (24). Using this equation, we define $V_t^a\left(\xi_t^j\right)$ as firm value per unit of capital when the firm chooses to invest. It is given by:

$$
\begin{aligned}
V_t^a\left(\xi_t^j\right) &= \left(1 - \tau_t^k\right)R_t + \tau_t^k\delta - \left(1 - \tau_t^i\right)i_t^j - \xi_t^j + g(i_t^j)Q_t \qquad (A.2)\\
&= \left(1 - \tau_t^k\right)R_t + \tau_t^k\delta + \left(1 - \delta + \varsigma\right)Q_t \\
&\quad + \frac{\theta}{1 - \theta}\left(\psi Q_t\right)^{\frac{1}{\theta}}\left(1 - \tau_t^i\right)^{\frac{\theta-1}{\theta}} - \xi_t^j.
\end{aligned}
$$

Define V_t^n as firm value per unit of capital when the firm chooses not to invest. By (22), (18), and (3), it satisfies:

$$V_t^n = \left(1 - \tau_t^k\right)R_t + \tau_t^k\delta + \left(1 - \delta + \varsigma\right)Q_t, \qquad (A.3)$$

which is independent of ξ_t^j. We can then rewrite problem (22) as:

$$V_t\left(\xi_t^j\right) = \max\left\{V_t^a\left(\xi_t^j\right), V_t^n\right\}. \qquad (A.4)$$

Clearly, there is a unique cutoff value ξ_t^* given in (23) satisfying the condition:

$$V_t^a\left(\xi_t^*\right) = V_t^n, \qquad (A.5)$$

$$V_t^a\left(\xi_t^j\right) > V_t^n \text{ if and only if } \xi_t^j < \xi_t^*. \qquad (A.6)$$

Because the support of ξ_t^j is $[0, \xi_{\max}]$, the investment trigger is given by $\bar{\xi}_t \equiv \min\{\xi_t^*, \xi_{\max}\}$.

We can show that:

$$
\begin{aligned}
\bar{V}_t &= \int_0^{\xi_{\max}} V_t(\xi)\phi(\xi)d\xi \\
&= \int_{\bar{\xi}_t}^{\xi_{\max}} V_t^n\phi(\xi)d\xi + \int_0^{\bar{\xi}_t} V_t^a(\xi)\phi(\xi)d\xi \\
&= V_t^n + \int_0^{\bar{\xi}_t}\left[V_t^a(\xi) - V_t^n\right]\phi(\xi)d\xi.
\end{aligned}
$$

23

We use equations (A.2), (A.3) and (23) to derive

$$V_t^a(\xi) - V_t^n = \frac{\theta}{1-\theta}(\psi Q_t)^{\frac{1}{\theta}}\left(1-\tau_t^i\right)^{\frac{\theta-1}{\theta}} - \xi \qquad (A.7)$$
$$= \xi_t^* - \xi.$$

Using the above two equations, (A.3), and (21), we obtain (25). Q.E.D.

Proof of Proposition 2: From (13), we deduce that all firms choose the same labor-capital ratio n_t. We thus obtain $N_t = n_t K_t$. We then derive

$$Y_t = \int Y_t^j dj = \int F\left(K_t^j, N_t^j\right) dj = \int F\left(1, n_t^j\right) K_t^j dj$$
$$= F(1, n_t) \int K_t^j dj = F(1, n_t) K_t = F(K_t, N_t),$$

which gives the first equality in equation (28). The second equality in equation (28) follows from a law of large number, the market clearing condition (12), and Proposition 1. We use equation (13) and $n_t^j = n_t$ to show:

$$F_2(K_t, N_t) = (1 - \tau_t^n) w_t. \qquad (A.8)$$

By the constant return to scale property of F, we also have:

$$R_t = F_1(K_t, N_t). \qquad (A.9)$$

We next derive aggregate investment:

$$I_t = \int I_t^j dj = \int i_t^j K_t^j dj = K_t \int_0^{\bar{\xi}_t} \left(\frac{\psi Q_t}{1 - \tau_t^i}\right)^{\frac{1}{\theta}} \phi(\xi) d\xi,$$

where the second equality uses the definition of i_t^j, the third equality uses a law of large numbers and Proposition 1. We thus obtain (26). By definition,

$$K_{t+1} = \int_0^1 \left[(1 - \delta) + \Phi(i_t^j)\right] K_t^j dj.$$

Substituting the optimal investment rule in equation (24) and using equation (26), we obtain (27).

Finally, equation (30) follows from substitution of equations (9) and (A.9) into equation (25). Equation (29) follows from equations (9), (10) and (A.8). Q.E.D.

Proof of Proposition 3: In an interior steady state, $\xi^* = \bar{\xi}$ and equations (26) and (23) imply that:

$$\frac{I}{K} = \left(\frac{\psi Q}{1 - \tau^i}\right)^{\frac{1}{\theta}} \int_0^{\bar{\xi}} \phi(\xi)d\xi, \tag{A.10}$$

$$\bar{\xi} = \frac{\theta}{1 - \theta}(\psi Q)^{\frac{1}{\theta}}\left(1 - \tau^i\right)^{\frac{\theta - 1}{\theta}}. \tag{A.11}$$

From these two equations, we obtain:

$$\frac{I}{K} = \frac{\bar{\xi}}{1 - \tau^i}\frac{1 - \theta}{\theta}\int_0^{\bar{\xi}}\phi(\xi)d\xi. \tag{A.12}$$

In steady state, equation (27) becomes:

$$\delta - \varsigma = \frac{\psi}{1 - \theta}(I/K)^{1 - \theta}\left[\int_0^{\bar{\xi}}\phi(\xi)d\xi\right]^{\theta}. \tag{A.13}$$

Substituting equation (A.12) into the above equation yields equation (32). The expression on the right-hand side of this equation increases with $\bar{\xi}$. The condition in this proposition guarantees a unique interior solution $\bar{\xi} \in (0, \xi_{\max})$ exists.

Equation (33) follows from (A.11). Equations (A.12) and (A.13) imply that:

$$\delta - \varsigma = \frac{\psi}{1 - \theta}\frac{I}{K}\left(\frac{\xi^*(1 - \theta)}{(1 - \tau^i)\theta}\right)^{-\theta}. \tag{A.14}$$

From this equation and equation (33), we obtain (34). The other equations in the proposition follow from the steady-state versions of equations (29)-(30). Q.E.D.

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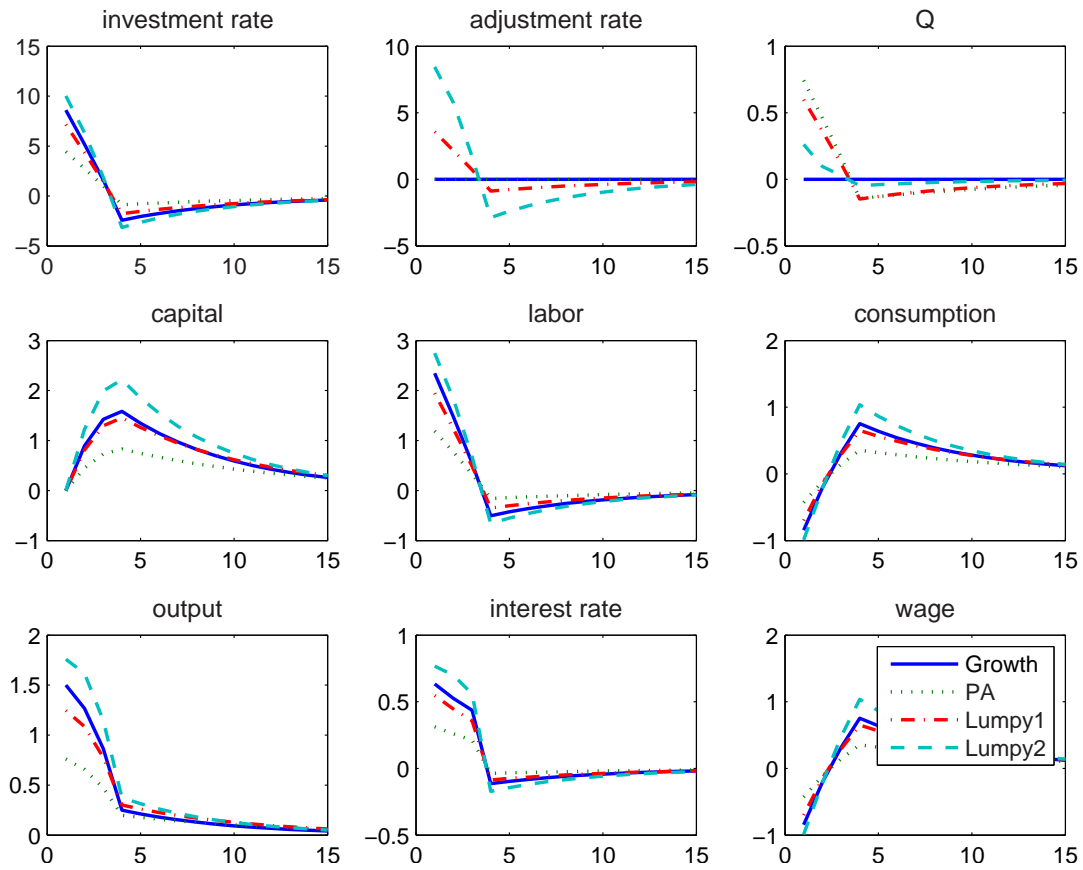
Figure 1: **Response to unanticipated temporary decrease in** τ^k**.** The 10 percentage points tax cut lasts from periods 1 to 4. In each panel, the vertical axis measures the percentage deviation from the initial steady state and the horizontal axis measures time.
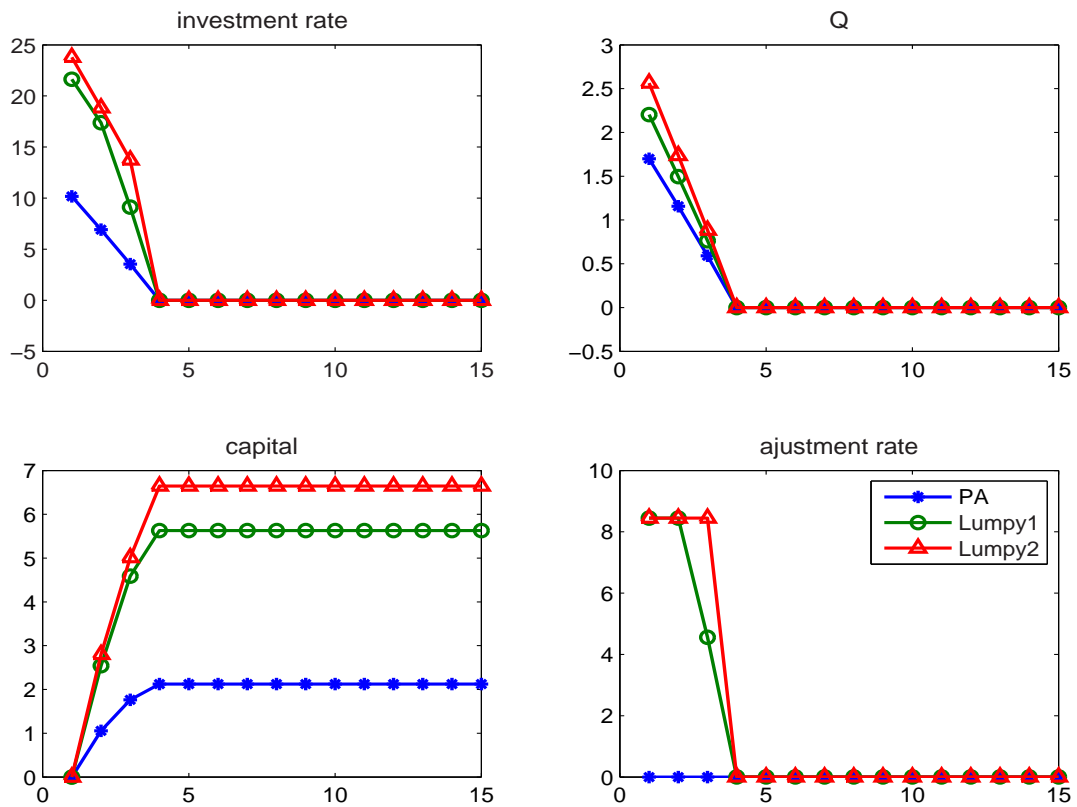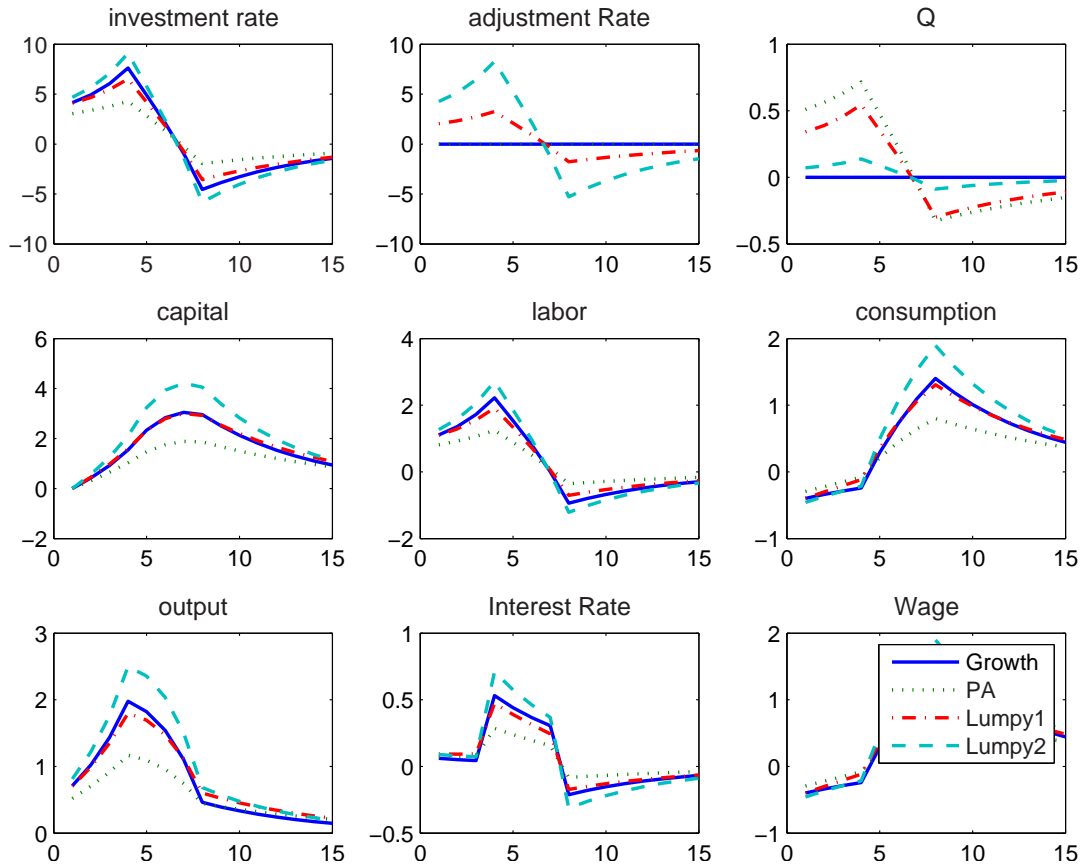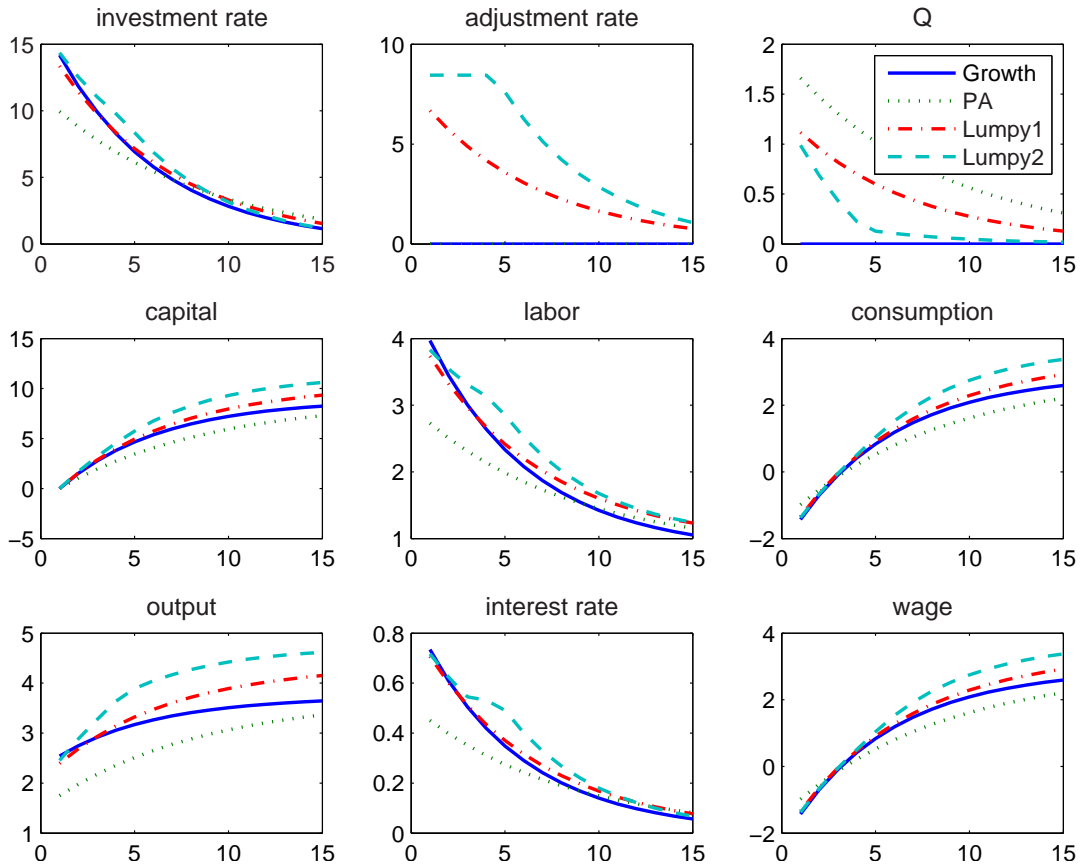
Figure 2: **Response to unanticipated temporary decrease in τ^k in partial equilibrium.**
The 10 percentage points tax cut lasts from periods 1 to 4. In each panel, the vertical axis measures the percentage deviation from the initial steady state and the horizontal axis measures time.
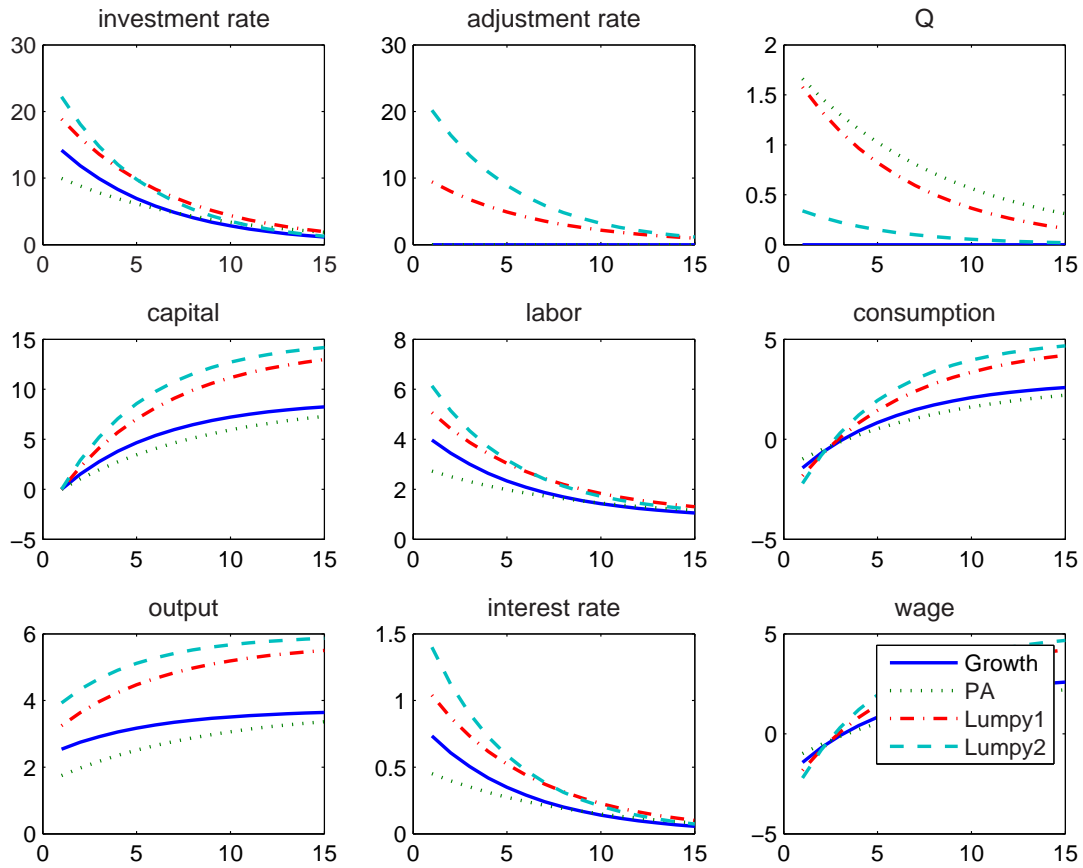
Figure 3: **Response to anticipated temporary decrease in τ^k.** The 10 percentage points tax cut lasts from periods 5 to 8. In each panel, the vertical axis measures the percentage deviation from the initial steady state and the horizontal axis measures time.

Figure 4: **Response to unanticipated permanent decrease in τ^k.** The 10 percentage points tax cut lasts forever starting from period 1. In each panel, the vertical axis measures the percentage deviation from the initial steady state and the horizontal axis measures time.
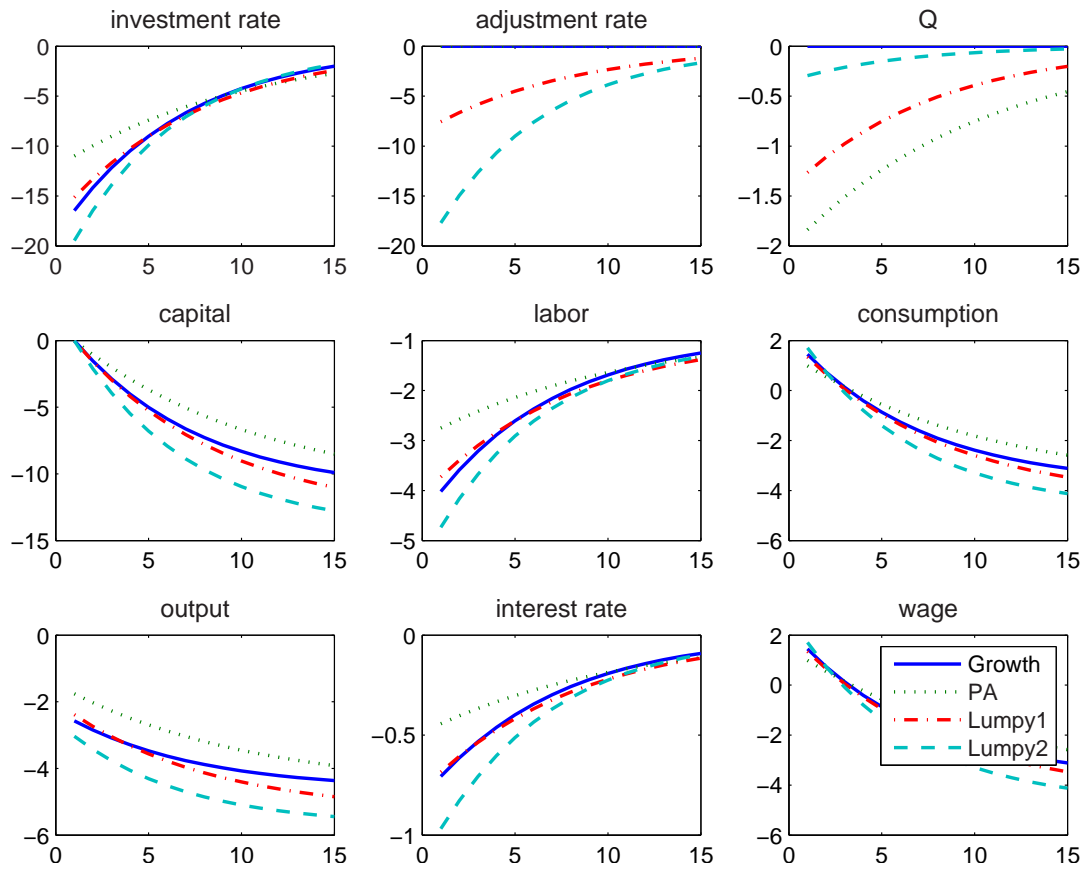
Figure 5: **Response to unanticipated permanent decrease in τ^k when the initial adjustment rate is 0.2.** The 10 percentage points tax cut lasts forever starting from period 1. In each panel, the vertical axis measures the percentage deviation from the initial steady state and the horizontal axis measures time.
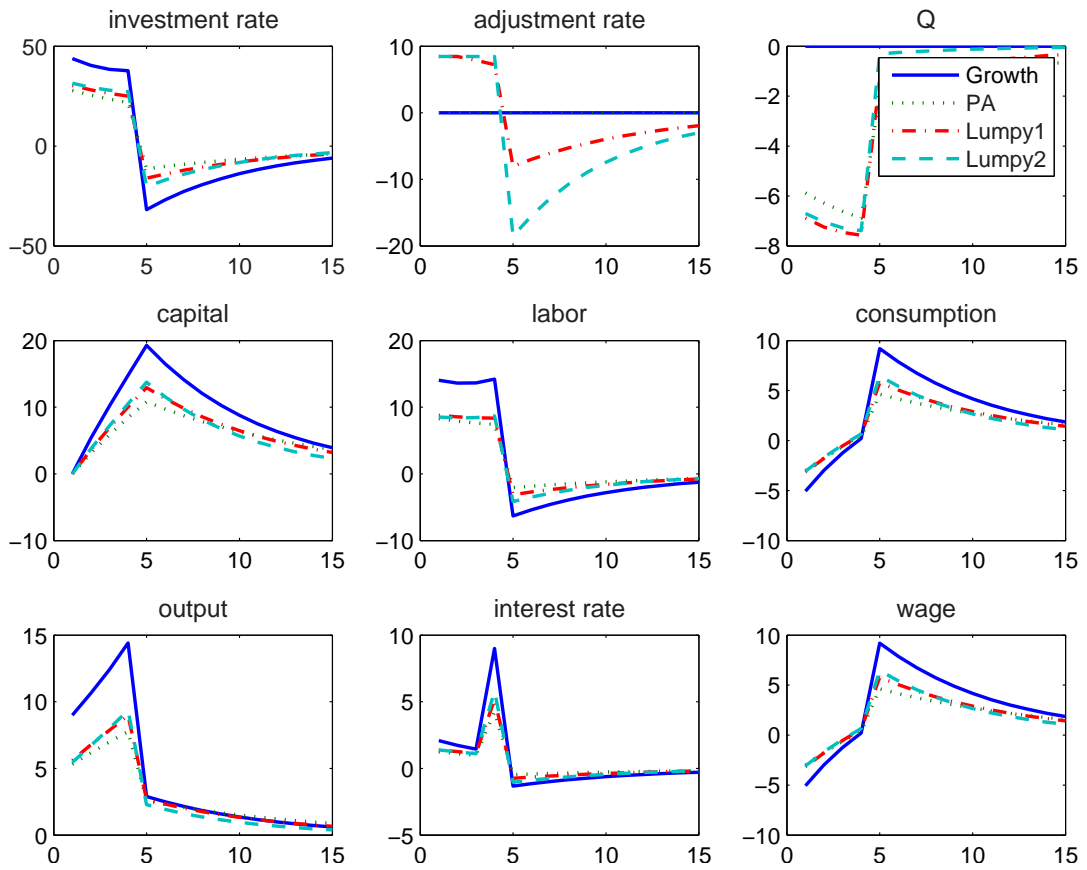
Figure 6: **Response to unanticipated permanent increase in τ^k.** The 10 percentage points tax increase lasts forever starting from period 1. In each panel, the vertical axis measures the percentage deviation from the initial steady state and the horizontal axis measures time.

34

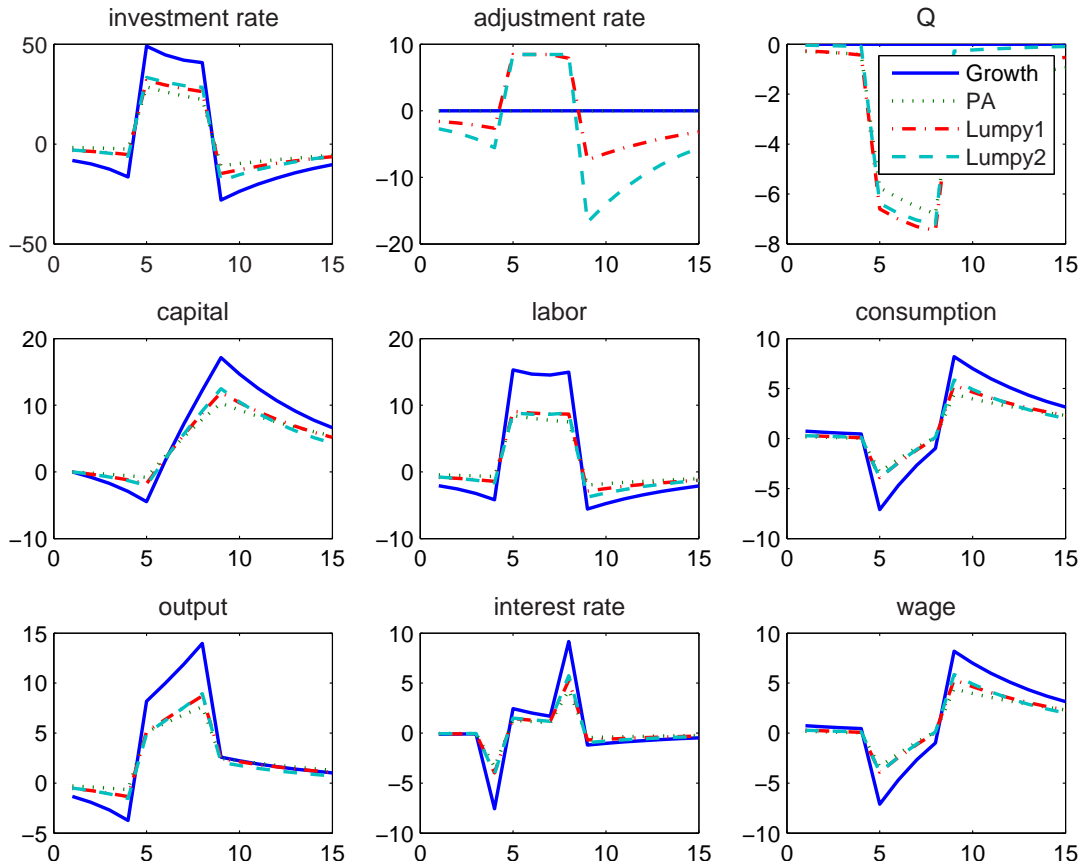Figure 7: **Response to unanticipated temporary increase in τ^i.** The 10 percentage points tax increase lasts from periods 1 to 4. In each panel, the vertical axis measures the percentage deviation from the initial steady state and the horizontal axis measures time.

Figure 8: **Response to anticipated temporary increase in** τ^i**.** The 10 percentage points tax increase lasts from periods 5 to 8. In each panel, the vertical axis measures the percentage deviation from the initial steady state and the horizontal axis measures time.
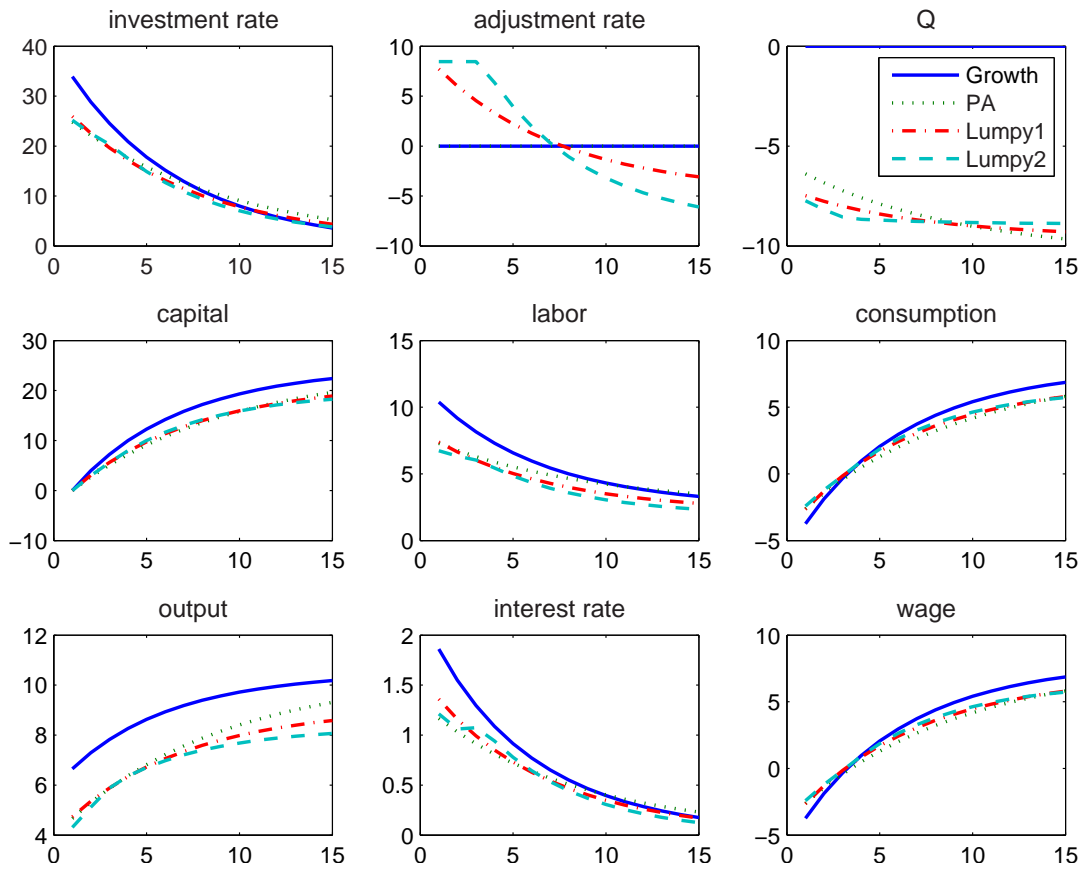
Figure 9: **Response to unanticipated permanent increase in τ^i.** The 10 percentage points tax increase lasts forever starting from period 1. In each panel, the vertical axis measures the percentage deviation from the initial steady state and the horizontal axis measures time.