

# State-Dependent Intellectual Property Rights Policy\*

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## Abstract

What form of intellectual property rights (IPR) policy contributes to economic growth? Should a company with a large technology lead receive the same IPR protection as a company with a more limited advantage? Should followers be able to license the products of technology leaders? We develop a general equilibrium framework to investigate these questions. The economy consists of many industries and firms engaged in cumulative (step-by-step) innovation. IPR policy regulates whether followers in an industry can copy the technology of the leader, whether they can license their innovations, and what types of patent infringement fees they are expected to pay. With full patent protection, followers can catch up to the leader in their industry only by making the same innovation(s) themselves.

We prove the existence of a steady-state equilibrium in a baseline environment and characterize some of its properties. We then quantitatively investigate the implications of different types of IPR policy on the equilibrium growth rate and welfare. The two major results of this exercise are as follows. We find that full patent protection is not optimal (welfare maximizing); instead, optimal policy involves *state-dependent* IPR protection, providing *greater* protection to technology leaders that are further ahead than those that are close to their followers. This form of the optimal policy is a result of a form of “*trickle-down*” effect, which implies that providing greater protection to firms that are further ahead of their followers than a certain threshold increases the R&D incentives also for all technology leaders that are less advanced than this threshold.

**Keywords:** competition, economic growth, endogenous growth, industry structure, innovation, intellectual property rights, licensing, patents, research and development, trickle-down.

**JEL classification:** O31, O34, O41, L16.

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# 1 Introduction

How should the intellectual property rights of a company be protected? Should a firm with a large technology lead receive the same intellectual property rights (IPR) protection as a company with a more limited technological lead, or should IPR policy be coupled with antitrust and used to limit the monopoly power of technology leaders? Despite broad consensus that innovation is central to the long-run performance of an economy, there is relatively little work on the design of innovation policy in general or on the implications of different types of IPR policies in particular. This paper is a first step in this direction. We construct a dynamic general equilibrium model and use it to theoretically and quantitatively investigate the form of optimal IPR policy and the answers to the above questions.

Our framework builds on and extends the step-by-step innovation models of Aghion, Harris and Vickers (1997) and Aghion, Harris, Howitt and Vickers (2001), where a number of (typically two) firms engage in price competition within an industry and undertake R&D in order to improve their production technology. The technology gap between the firms determines the extent of the monopoly power of the leader, and hence the price markups and profits. The purpose of R&D by the follower is to catch up and surpass the leader (as in standard Schumpeterian models of innovation, e.g., Reinganum, 1981, 1985, Aghion and Howitt, 1992, Grossman and Helpman, 1991), while the purpose of R&D by the leader is to escape the competition of the follower and increase its markup and profits. We extend existing models in several directions. Most importantly, we allow for *state-dependent* patent/IPR protection policy, meaning a policy that makes the extent of patent or intellectual property rights protection conditional on the technology gap between different firms in the industry. As in racing-type models in general (e.g., Harris and Vickers, 1985, 1987, Budd, Harris and Vickers, 1993), a large gap between the leader and the follower discourages R&D by both. Consequently, overall R&D and technological progress are greater when the technology gap between the leader and the follower is relatively small.<sup>1</sup> One may then expect that full patent protection may be suboptimal in a world of step-by-step competition. Indeed, it may be conjectured that by stochastically or deterministically allowing the follower to use the technology of the industry leader, particularly in industries where there is a large technology gap between leaders and followers, public policy can and should reduce these gaps, ensure a more competitive environment and also raise the amount of innovative activity.<sup>2</sup>

This reasoning ignores potential incentive and general equilibrium effects of such a policy, however. The anticipation of an IPR policy that implicitly punishes technologically advanced

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<sup>1</sup>Aghion, Bloom, Blundell, Griffith and Howitt (2005) provide empirical evidence that there is greater R&D in British industries where there is a smaller technological gap between firms. See O'Donoghue, Scotchmer and Thisse (1998) for a discussion of how patent life may come to an end because of related innovations.

<sup>2</sup>This is conjectured, for example, in Aghion, Harris, Howitt and Vickers (2001, p. 481).

firms may discourage R&D designed to open such a gap in the first place. Consequently, IPR policy that provides less protection to technologically more advanced firms creates a trade-off. On the one hand, it increases R&D via the *composition effect* outlined in the previous paragraph; on the other, it reduces R&D via a standard *disincentive effect*. A major contribution of our paper is to explore this trade-off, and show that the disincentive effect is always more powerful than the composition effect. In particular, we will show that the *opposite* of the above conjecture is always true in our model in the sense that optimal IPR policy should provide *greater* (not less) protection to technologically more advanced leaders. Underlying this result is what we refer to as the *trickle-down of incentives*: providing relatively low protection to firms with limited leads and greater protection to those that have greater leads not only improves the incentives of firms that are technologically advanced, but also encourages R&D by those that have limited leads because of the prospect of reaching levels of technology gaps associated with greater protection.

Several recent high-profile cases highlight the importance of the issues discussed here for public policy.<sup>3</sup> For example, a recent ruling of the European Commission impelled Microsoft to share secret information about its operating system and products with other software companies (New York Times, December 22, 2004). Similar issues were also central to the US Department of Justice (DOJ) case against Microsoft, which started on May 18th, 1998 and ultimately resulted in a ruling against Microsoft. Figures 1 and 2 show the evolution of R&D by Microsoft and by other top 10 publicly traded R&D investors in the IT sector relative to the sector average before and after the start of the DOJ case against Microsoft.<sup>4</sup>

[FIGURE 1 & 2 HERE]

The relative R&D spending by Microsoft and other industry leaders, which had been steadily and in fact exponentially increasing since the mid-80s, appear to decline after the DOJ action. While one might expect R&D by Microsoft to be discouraged, the decline by other top companies is surprising since they partly benefited from the weakening of, and the restrictions imposed on, Microsoft. Although this decline might be caused by other factors, one interpretation in line with the trickle-down effects we emphasize in this paper is that the DOJ action may be interpreted as adversely affecting firms that build larger technological leads over their rivals, thus discouraging their R&D. Even if this interpretation is correct, it does

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<sup>3</sup>In addition to the Microsoft case, the issue of technological lead has been central in the Department of Justice investigations of Intel (New York Times, May 11, 2009) and the debates about Google's market share (New York Times, February 21, 2009).

<sup>4</sup>All data are from COMPUSTAT. Top 10 firms is determined by the highest 10 R&D investors (except Microsoft) in 1995. The patterns shown in Figures 1 and 2 are very similar if we use in that top 10 investors in 2000 or 1990, or if we benchmark it to the median of the industry rather than the mean. Top 10 investors in Figure 2 are: CA Inc, Continuum Inc, Intergraph Corp, Sterling Software Inc, Oracle Inc, Adobe Inc, Symantec Corp, Electronic Arts Inc, Sybase Inc, Intuit Inc.

not establish that the DOJ action discouraged aggregate innovation in the industry, since it may have induced powerful composition effects by other firms. The purpose of our analysis is to investigate these issues theoretically and quantitatively.

The trade-off identified above may lead to conclusions that depend on several features of the technology of R&D. Nonetheless, our results suggest that this is not the case (at least for the range of possibilities we have investigated). We first start with a partial equilibrium model, which, under some simplifying assumptions, allows an explicit characterization of the trickle-down effect. In particular, we show that in contrast to the standard disincentive effects of uniform relaxation of IPR policy, state-dependent relaxation that provides *greater* protection to technologically more advanced firms creates a *positive incentive effect*. This is because when a particular state for the technology leader (say being  $n^*$  steps ahead of the follower) becomes more profitable, this increases the incentives to perform R&D not only for leaders that are  $n^* - 1$  steps ahead, but for *all* leaders with a lead of size  $n \leq n^* - 1$ . It is this trickle-down effect that generates the positive incentive effect and makes state-dependent IPR, with greater protection for firms that are technologically more advanced than their rivals, preferable to uniform IPR. It is also notable that the ability to choose state-dependent IPR policy typically makes full enforcement of IPR suboptimal.

In the general equilibrium analysis, we relax several assumptions imposed in the partial equilibrium model. We first provide a characterization of the structure of stationary equilibria under the assumption that there is *quick catch-up*, that is, assuming that a follower's ability to catch up with the technology leader is independent of the gap between them. In this quick catch-up scenario, the only relevant dimension of IPR policy is the effective length of patents at different technology gaps. Even though the quick catch-up scenario may be less realistic than situations in which catch-up by followers is more gradual, it enables both continuity with the previous literature (which focuses exclusively on this regime) and a more complete theoretical analysis. Using this scenario, we characterize the optimal (welfare-maximizing) structure of IPR policy. The same effects as in the partial equilibrium analysis make state-dependent relaxation of IPR optimal. In our quantitative analysis, we find that optimal state-dependent IPR policy can increase the growth rate of the economy from about 1.86% to 2.08%, and does so with *fewer* workers employed in the R&D sector (because R&D workers are reallocated towards firms where their efforts directly lead to productivity growth). In contrast, uniform relaxation of IPR policy reduces both welfare and growth. These patterns are quite robust across a variety of different parameter values.

We next study the same set of questions under a *slow catch-up* scenario, where followers gradually close the gap between themselves and technology leaders. The slow catch-up regime is interesting for several reasons. First, it is arguably a better approximation to several industries

where a series of product or process innovations would be necessary to catch up with a leader. Second, one might be concerned that the quick catch-up regime minimizes composition effects and thus our results on the form of optimal IPR policy may not be robust in this alternative environment. Third and most important, the slow catch-up regime enables us to introduce different types of R&D efforts and different dimensions of IPR policy. In particular, in this regime, we allow firms to undertake frontier as well as catch-up R&D. With frontier R&D, they can build on the technology leader’s knowledge base and, if successful, immediately surpass the leader. However, in this case, they may be liable for a *patent infringement fee*. Introducing this realistic dimension of IPR policy, as well as the effective length of patents, is another contribution of our paper. In addition, we also allow followers to *license* the innovation of the technology leader by paying a prespecified license fee.<sup>5</sup> Thus, all in all, in this case we allow for three different dimensions of IPR policy. We study each of these three policies separately and then together. In all cases, we find that optimal IPR policy is state dependent and provides *greater protection* to technologically more advanced firms. In most cases, optimal IPR policy also increases growth by a similar magnitude to that in the quick catch-up regime (though in some cases, it increases welfare but not necessarily growth). Our analysis also shows that both licensing and the possibility of frontier R&D (subject to infringement fees) contributes to growth and welfare. Finally, we show that the results in this case are robust to a variety of different parameter choices.

Our paper is a contribution both to the IPR protection and the endogenous growth literatures. Previous work in industrial organization and in growth theory emphasizes that ex-post monopoly rents and thus patents are central for generating the ex-ante investments in R&D and technological progress, even though monopoly power also creates distortions (e.g., Arrow, 1962, Reinganum, 1981, Tirole, 1988, Romer, 1990, Grossman and Helpman, 1991, Aghion and Howitt, 1992, Green and Scotchmer, 1995, Scotchmer, 1999, Gallini and Scotchmer, 2002, O’Donoghue and Zweimuller, 2004).<sup>6</sup> Much of the literature discusses the trade-off between these two forces to determine the optimal length and breadth of patents. For example, Klemperer (1990) and Gilbert and Shapiro (1990) show that optimal patents should have a long duration in order to provide inducement to R&D, but a narrow breadth so as to limit monopoly distortions. A number of other papers, for example, Gallini (1992) and Gallini and Scotchmer (2002), reach opposite conclusions.

Another branch of the literature, including the seminal paper by Scotchmer (1999) and

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<sup>5</sup>In other words, this is a form of “compulsory licensing” whereby the license fee is determined by IPR policy. We also show that voluntary licensing agreements would not achieve the same results, so our analysis establishes a potential need for compulsory licensing policy. Previous work emphasizing importance of compulsory licensing includes Tandon (1982), Gilbert and Shapiro (1990), and Kremer (2002). See Moser and Voena (2009) for a recent empirical investigation.

<sup>6</sup>Boldrin and Levine (2001) or Quah (2003) argue that patent systems are not necessary for innovation.

the recent interesting papers by Llobet, Hopenhayn and Mitchell (2006) and Hopenhayn and Mitchell (2001), adopts a mechanism design approach to the determination of the optimal patent and intellectual property rights protection system. For example, Scotchmer (1999) derives the patent renewal system as an optimal mechanism in an environment where the cost and value of different projects are unobserved and the main problem is to decide which projects should go ahead. Llobet, Hopenhayn and Mitchell (2006) consider optimal patent policy in the context of a model of sequential innovation with heterogeneous quality and private information. They show that allowing for a choice from a menu of patents will be optimal in this context. To the best of our knowledge, no other paper in the literature has considered state-dependent IPR policy or developed the general equilibrium framework for IPR policy analysis. As a first attempt, we only look at state-dependent patent length, license fees and infringement fees (though similar ideas can be applied to an investigation of the gains from making the breadth of patent awards state-dependent).

Our paper is most closely related to and extends the results of Aghion, Harris and Vickers (1997) and Aghion, Harris, Howitt and Vickers (2001) on endogenous growth with step-by-step innovation.<sup>7</sup> Although our model builds on these papers, it also differs from them in a number of significant ways. First and most importantly, we introduce state-dependent IPR policy. Second, we also introduce and analyze the slow catch-up regime, and in this context, we allow for compulsory licensing and for leapfrogging, which makes the followers directly contribute to the economic growth. We provide a full quantitative analysis of state-dependent IPR policy under these different scenarios. Third, our economy is a full general equilibrium model with competition between production and R&D for scarce labor.<sup>8</sup> Finally, we provide a general existence result and number of analytical results for the general model (with or without IPR policy), while previous literature has focused on the special cases where innovations are either “drastic” (so that the leader never undertakes R&D) or very small, and has not provided existence or general characterization results for steady-state equilibria.

Lastly, our results are also related to the literature on tournaments and races, for example, Fudenberg, Gilbert, Stiglitz and Tirole (1983), Harris and Vickers (1985, 1987), Choi (1991), Budd, Harris and Vickers (1993), Taylor (1995), Fullerton and McAfee (1999), Baye and Hoppe (2003), and Moscarini and Squintani (2004). This literature considers the impact of endogenous or exogenous prizes on effort in tournaments, races or R&D contests. In terms of this literature,

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<sup>7</sup>Segal and Whinston (2005) analyze the impact of anti-trust policy on economic growth in a related model of step-by-step innovation.

<sup>8</sup>This general equilibrium aspect is introduced to be able to close the model economy without unrealistic assumptions and makes our economy more comparable to other growth models (Aghion, Harris, Howitt and Vickers, 2001, assume a perfectly elastic supply of labor). We show that the presence of general equilibrium interactions does not significantly complicate the analysis and it is still possible to characterize the steady-state equilibrium.

state-dependent IPR policy can be thought of as “state-dependent handicapping” of different players (where the state variable is the gap between the two players in a dynamic tournament). To the best of our knowledge, these types of schemes have not been considered in this literature.

The rest of the paper is organized as follows. Section 2 introduces the partial equilibrium model and analytically demonstrates the trickle-down effect. Section 3 presents the basic general equilibrium environment under the assumption that successful innovation by followers will close the entire gap with technology leaders in one step. Section 4 proves the existence of a steady-state equilibrium and characterizes some of its key properties under both uniform and state-dependent IPR policy. Section 5 defines the social welfare objective and outlines our quantitative methods. Section 6 characterizes the structure of optimal IPR policy in the quick catch-up regime quantitatively. Section 7 extends the model to allow for slow catch-up, compulsory license fees and leapfrogging, and quantitatively characterizes the structure of optimal IPR policy under different combinations of these policies. Section 8 concludes, while the Appendix contains the proofs of all the results stated in the text.

## 2 Partial Equilibrium

We first illustrate the main economic force in this paper, the trickle-down effect, using a partial equilibrium model. Consider the following infinite horizon, step-by-step R&D race between two competing firms in continuous time. Each firm maximizes the expected net present discounted value of “net profits,” defined as operating profit minus R&D cost,

$$\mathbb{E}_t \int_t^\infty \exp(-r(s-t)) [\pi_i(s) - \Phi_i(s)] ds,$$

where  $\mathbb{E}_t$  denotes expectation at time  $t$ ,  $r > 0$  is the interest rate,  $\pi_i(t)$  is the instantaneous operating profit flow and  $\Phi_i(t)$  represents the R&D cost of firm  $i$  at time  $t$ . In this game, firm  $i \in \{1, 2\}$  invests in R&D to advance its position relative to its rival  $i' \neq i$ . Suppose that the positions of both firms in this race can be characterized by integer values on the real line, and denote the distance of firm  $i$  from its rival at time  $t$  by  $n_i(t)$ . In the partial equilibrium model, we simplify the analysis by following Aghion, Harris, Howitt and Vickers (2001) and Aghion, Bloom, Blundell, Griffith and Howitt (2005) in assuming that the maximum technology gap between a leader and a follower is 2; this assumption is relaxed in the full general equilibrium model analyzed in the rest of the paper. For now it simplifies the analysis by ensuring that the relative position of firm  $i$  can take five possible values,  $n_i(t) \in \mathbf{N}_I \equiv \{-2, -1, 0, 1, 2\}$ . Let us denote the absolute gap between the firms by  $n(t) \equiv \max\{n_i(t), n_{-i}(t)\}$ , and suppress the time subscripts to simplify notation.

The payoffs in this game are assumed to be stationary and only a function of the relative distance between the firms, thus represented by  $\boldsymbol{\pi} : \mathbf{N}_I \rightarrow \mathbb{R}_+$  (see equation (20) in Section 3).

Here  $\pi_{n_i} \geq 0$  is simply the instantaneous payoff that firm  $i$  obtains when its distance from its competitor is  $n_i$  at time  $t$  and assumed to be a strictly increasing function of  $n_i$ . To advance its relative position, firm  $i$  invests in R&D, which determines the Poisson rate of arrival of innovation,  $x_i \in \mathbb{R}_+$ . Let us also assume that the cost of R&D is linear in the arrival rate of innovation, i.e.,  $\Phi(x_i) = \phi x_i$ , with  $\phi > 0$  (again see below for more general formulations). Each successful innovation is patented and advances firm  $i$ 's state (relative position) by one step, so that following a successful innovation by firm  $i$  at time  $t$  we have:  $n_i(t+) = n_i(t) + 1$  (where  $n_i(t+)$  stands for  $n_i$  immediately following time  $t$ ).

IPR policy governs the expected length of a patent. For simplicity, we model patent length by assuming that it terminates at a Poisson rate. Crucially for our focus, IPR policy is state dependent, and we represent it by the function:  $\eta : \mathbf{N}_{\mathbf{I}} \rightarrow \mathbb{R}_+$ . Here  $\eta(n) \equiv \eta_n < \infty$  is the flow rate at which the patent terminates (patent protection is removed) for a technology leader that is  $n$  steps ahead. When  $\eta_n = 0$ , this implies that there is full protection at technology gap  $n$ , in the sense that patent protection will never be removed. In contrast,  $\eta_n \rightarrow \infty$  implies that patent protection is removed immediately once technology gap  $n$  is reached. When the patent protection is removed, the firm that is behind copies the technology of its competitor and both firms end up *neck-and-neck*, i.e.,  $n = 0$ .

Finally, we take the interest rate  $r$  as exogenous and assume that it satisfies  $r < (\pi_{-1} - \pi_{-2})/4\phi$ . This assumption ensures positive R&D by each firm when  $\eta_n = 0$ . Throughout we will focus on (stationary) Markov Perfect Equilibria (MPE), where strategies (R&D decisions) are only functions of the payoff-relevant state, which is  $n \in \mathbf{N}_{\mathbf{I}}$ . A more formal definition of the MPE in the general equilibrium environment is given below.

The MPE can be characterized by writing the value functions of each firm as a function of the state  $n \in \mathbf{N}_{\mathbf{I}}$ . These value functions are given by the following recursions:

$$rv_2 = \pi_2 + x_{-2}[v_1 - v_2] + \eta_2[v_0 - v_2], \quad (1)$$

$$rv_1 = \max_{x_1 \geq 0} \{ \pi_1 - \phi x_1 + x_1[v_2 - v_1] + x_{-1}[v_0 - v_1] + \eta_1[v_0 - v_1] \}, \quad (2)$$

$$rv_0 = \max_{x_0 \geq 0} \{ \pi_0 - \phi x_0 + x_0[v_1 - v_0] + \tilde{x}_0[v_{-1} - v_0] \}, \quad (3)$$

$$rv_{-1} = \max_{x_{-1} \geq 0} \{ \pi_{-1} - \phi x_{-1} + x_{-1}[v_0 - v_{-1}] + x_1[v_{-2} - v_{-1}] + \eta_1[v_0 - v_{-1}] \}, \quad (4)$$

$$rv_{-2} = \max_{x_{-2} \geq 0} \{ \pi_{-2} - \phi x_{-2} + x_{-2}[v_{-1} - v_{-2}] + \eta_2[v_0 - v_{-2}] \}. \quad (5)$$

In all equations, the first term represents current profits. In equations (2)-(5), the second term subtracts R&D costs from current profits, the third term represents the fact that the firm will successfully innovate at the flow rate  $x_n$  and increase its position by one step. The fourth term incorporates the change in value due to an innovation by the rival firm. In equations (1) and (2) the last term is the change in value for the leader due to patent expiration, which takes place at the rate  $\eta_n$ , while in (4) and (5) is the change in value for the follower. Finally, equation (3)



has the same interpretation except that now  $n = 0$  and the two firms are neck-and-neck and thus there is no IPR policy (and the flow rate of innovation of the other firm is denoted by  $\tilde{x}_0$ , and naturally, in a symmetric equilibrium, we will have  $x_0 = \tilde{x}_0$ ). Note also that in equations (1) and (5), we used the fact that a two-step ahead firm does not undertake any R&D since it has already achieved the maximum feasible lead.

We will now characterize the MPE under two different policy environments: uniform and state-dependent IPR policy.

**Uniform IPR Policy.** Uniform IPR policy corresponds to the case where  $\eta_n = \eta < \infty$ . For  $\eta$  sufficiently small, optimal R&D decisions in equations (2)-(5) can be solved out as (see the Appendix):<sup>9</sup>

$$\begin{aligned} x_{-2}^* &= -4\eta + \frac{\pi_2 - \pi_{-2}}{\phi} - 4r, \quad x_{-1}^* = -3\eta + \frac{\pi_1 - \pi_{-2}}{\phi} - 3r, \\ x_0^* &= -2\eta + \frac{\pi_0 - \pi_{-2}}{\phi} - 2r, \quad \text{and } x_1^* = -\eta + \frac{\pi_{-1} - \pi_{-2}}{\phi} - r. \end{aligned}$$

Inspection of these expressions immediately establishes the following result:

**Proposition 1** *Under uniform IPR policy regime, any relaxation of IPR policy (away from  $\eta = 0$ ) creates a “disincentive effect” and reduces all R&D levels.*

**State-dependent IPR Policy.** We next consider state-dependent policy where the patent protection of a technology leader depends on the technology gap,  $n$ . When both  $\eta_1$  and  $\eta_2$  are sufficiently small, optimal R&D decisions are now given as (see the Appendix):

$$\begin{aligned} x_{-2}^* &= -4\eta_2 + \frac{\pi_2 - \pi_{-2}}{\phi} - 4r, \quad x_{-1}^* = -\eta_1 - 2\eta_2 + \frac{\pi_1 - \pi_{-2}}{\phi} - 3r, \\ x_0^* &= -2\eta_2 + \frac{\pi_0 - \pi_{-2}}{\phi} - 2r, \quad \text{and } x_1^* = \eta_1 - 2\eta_2 + \frac{\pi_{-1} - \pi_{-2}}{\phi} - r. \end{aligned}$$

Inspection of these expressions shows that, in contrast to the uniform IPR case, relaxing patent protection can increase the R&D effort of the one-step leader,  $x_1^*$ . In particular, this can be accomplished by providing a lower protection in the current state (higher  $\eta_1$ ) and/or a higher protection upon a successful innovation (lower  $\eta_2$ ).

**Proposition 2** *Under state-dependent IPR policy regime, relaxing IPR policy (away from  $\eta_n = 0$ ) by weakening current protection (i.e., increasing  $\eta_1$ ) creates a “positive incentive effect” and increases  $x_1^*$ .*

Whether optimal IPR policy will involve  $\eta_1 > 0$  and/or  $\eta_2 > 0$  now depends on the social returns from  $x_n$ 's. For example, if  $x_1$  is socially more beneficial than  $x_{-1}$ ,  $\eta_1 > 0$  will always

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<sup>9</sup>The results in Propositions 1 and 2 do not depend on  $\eta$  being small, but the exact expressions provided here depend on this assumption. Otherwise, some of the equilibrium flow rates would be equal to zero (even though  $r < (\pi_{-1} - \pi_{-2})/4\phi$ ). We maintain the assumption that  $\eta$  (or  $\eta_1$  and  $\eta_2$ ) is small to simplify the expressions.

be preferred. In the context of our general equilibrium model, this will always be the case. Proposition 2 provides a preview of these results.

### 3 General Equilibrium Model

We now describe our baseline dynamic general equilibrium model. To maximize continuity with the previous literature and to provide the sharpest theoretical characterization results, we start with the “quick catch-up” regime, where one innovation by a follower is sufficient to close the gap with the technology leader in the industry. The characterization of the equilibrium in this environment under the different policy regimes is presented in the next section. Alternative assumptions on the form of catch-up are investigated in Section 7.

#### 3.1 Preferences and Technology

Consider the following continuous time economy with a unique final good. The economy is populated by a continuum of 1 individuals, each with 1 unit of labor endowment, which they supply inelastically. Preferences at time  $t$  are given by

$$\mathbb{E}_t \int_t^\infty \exp(-\rho(s-t)) \log C(s) ds, \quad (6)$$

where  $\mathbb{E}_t$  denotes expectations at time  $t$ ,  $\rho > 0$  is the discount rate and  $C(t)$  is consumption at date  $t$ . The logarithmic preferences in (6) facilitate the analysis, since they imply a simple relationship between the interest rate, growth rate and the discount rate (see (7) below).

Let  $Y(t)$  be the total production of the final good at time  $t$ . We assume that the economy is closed and the final good is used only for consumption (i.e., there is no investment), so that  $C(t) = Y(t)$ . The standard Euler equation from (6) then implies that

$$g(t) \equiv \frac{\dot{C}(t)}{C(t)} = \frac{\dot{Y}(t)}{Y(t)} = r(t) - \rho, \quad (7)$$

where this equation defines  $g(t)$  as the growth rate of consumption and thus output, and  $r(t)$  is the interest rate at date  $t$ .

The final good  $Y$  is produced using a continuum 1 of intermediate goods according to the Cobb-Douglas production function

$$\ln Y(t) = \int_0^1 \ln y(j, t) dj, \quad (8)$$

where  $y(j, t)$  is the output of  $j$ th intermediate at time  $t$ . Throughout, we take the price of the final good as the numeraire and denote the price of intermediate  $j$  at time  $t$  by  $p(j, t)$ . We also assume that there is free entry into the final good production sector. These assumptions,

together with the Cobb-Douglas production function (8), imply that the final good sector has the following demand for intermediates

$$y(j, t) = \frac{Y(t)}{p(j, t)}, \quad \forall j \in [0, 1]. \quad (9)$$

Intermediate  $j \in [0, 1]$  comes in two different *varieties*, each produced by one of two infinitely-lived firms. We assume that these two varieties are perfect substitutes and these firms compete a la Bertrand.<sup>10</sup> Firm  $i = 1$  or  $2$  in industry  $j$  has the following technology

$$y(j, t) = q_i(j, t) l_i(j, t) \quad (10)$$

where  $l_i(j, t)$  is the employment level of the firm and  $q_i(j, t)$  is its level of technology at time  $t$ . Each consumer in the economy holds a balanced portfolio of the shares of all firms. Consequently, the objective function of each firm is to maximize expected profits.

The production function for intermediate goods, (10), implies that the marginal cost of producing intermediate  $j$  for firm  $i$  at time  $t$  is

$$MC_i(j, t) = \frac{w(t)}{q_i(j, t)} \quad (11)$$

where  $w(t)$  is the wage rate in the economy at time  $t$ .

When this causes no confusion, we denote the *technology leader* in each industry by  $i$  and the follower by  $-i$ , so that we have:

$$q_i(j, t) \geq q_{-i}(j, t).$$

Bertrand competition between the two firms implies that all intermediates will be supplied by the leader at the “limit” price:<sup>11</sup>

$$p_i(j, t) = \frac{w(t)}{q_{-i}(j, t)}. \quad (12)$$

Equation (9) then implies the following demand for intermediates:

$$y(j, t) = \frac{q_{-i}(j, t)}{w(t)} Y(t). \quad (13)$$

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<sup>10</sup> A more general case would involve these two varieties being imperfect substitutes, for example, with the output of intermediate  $j$  produced as

$$y(j, t) = \left[ \varphi y_1(j, t)^{\frac{\sigma-1}{\sigma}} + (1-\varphi) y_2(j, t)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}},$$

with  $\sigma > 1$ . The model analyzed in the text corresponds to the limiting case where  $\sigma \rightarrow \infty$ . Our results can be easily extended to this more general case with any  $\sigma > 1$ , but at the cost of additional notation. We therefore prefer to focus on the case where the two varieties are perfect substitutes. It is nonetheless useful to bear this formulation with imperfect substitutes in mind, since it facilitates the interpretation of “distinct” innovations by the two firms (when the follower engages in “catch-up” R&D).

<sup>11</sup> If the leader were to charge a higher price, then the market would be captured by the follower earning positive profits. A lower price can always be increased while making sure that all final good producers still prefer the intermediate supplied by the leader  $i$  rather than that by the follower  $-i$ , even if the latter were supplied at marginal cost. Since the monopoly price with the unit elastic demand curve is infinite, the leader always gains by increasing its price, making the price given in (12) the unique equilibrium price.

### 3.2 Technology, R&D and IPR Policy under Quick Catch-up

R&D by the leader or the follower stochastically leads to innovation. We assume that when the leader innovates, its technology improves by a factor  $\lambda > 1$ .

The follower, on the other hand, can undertake R&D to catch up with the frontier technology. We will call this type of R&D as *catch-up R&D* (this is the contrast with *frontier R&D* introduced in Section 7, which will allow the follower to leapfrog the leader). Catch-up R&D can be thought of R&D to discover an *alternative* way of performing the same task as the current leading-edge technology. Because this innovation applies to the follower's variant of the product (recall footnote 10) and results from its own R&D efforts, we assume that it does not constitute infringement on the patent of the leader (see Section 7.4 below for an analysis of infringement fees).

R&D by the leader and follower may have different costs and success probabilities. We simplify the analysis by assuming that both types of R&D have the same costs and the same probability of success. In particular, in all cases, we assume that innovations follow a controlled Poisson process, with the arrival rate determined by R&D investments. Each firm (in every industry) has access to the following R&D technology:

$$x_i(j, t) = F(h_i(j, t)), \quad (14)$$

where  $x_i(j, t)$  is the flow rate of innovation at time  $t$  and  $h_i(j, t)$  is the number of workers hired by firm  $i$  in industry  $j$  to work in the R&D process at  $t$ . This specification implies that within a time interval of  $\Delta t$ , the probability of innovation for this firm is  $x_i(j, t) \Delta t + o(\Delta t)$ .

We assume that  $F$  is twice continuously differentiable and satisfies  $F'(\cdot) > 0$ ,  $F''(\cdot) < 0$ ,  $F'(0) < \infty$  and that there exists  $\bar{h} \in (0, \infty)$  such that  $F'(h) = 0$  for all  $h \geq \bar{h}$ . The assumption that  $F'(0) < \infty$  implies that there is no Inada condition when  $h_i(j, t) = 0$ . The last assumption, on the other hand, ensures that there is an upper bound on the flow rate of innovation (which is not essential but simplifies the proofs). Recalling that the wage rate for labor is  $w(t)$ , the cost for R&D is therefore  $w(t) G(x_i(j, t))$  where

$$G(x_i(j, t)) \equiv F^{-1}(x_i(j, t)), \quad (15)$$

and the assumptions on  $F$  immediately imply that  $G$  is twice continuously differentiable and satisfies  $G'(\cdot) > 0$ ,  $G''(\cdot) > 0$ ,  $G'(0) > 0$  and  $\lim_{x \rightarrow \bar{x}} G'(x) = \infty$ , where

$$\bar{x} \equiv F(\bar{h}) \quad (16)$$

is the maximal flow rate of innovation (with  $\bar{h}$  defined above).

We next describe the evolution of technologies within each industry. Suppose that leader  $i$  in industry  $j$  at time  $t$  has a technology level of

$$q_i(j, t) = \lambda^{n_{ij}(t)}, \quad (17)$$

and that the follower  $-i$ 's technology at time  $t$  is

$$q_{-i}(j, t) = \lambda^{n_{-ij}(t)}, \quad (18)$$

where  $n_{ij}(t) \geq n_{-ij}(t)$  and  $n_{ij}(t), n_{-ij}(t) \in \mathbb{Z}_+$  denote the technology rungs of the leader and the follower in industry  $j$ . We refer to  $n_j(t) \equiv n_{ij}(t) - n_{-ij}(t)$  as the *technology gap* in industry  $j$ . If the leader undertakes an innovation within a time interval of  $\Delta t$ , then its technology increases to  $q_i(j, t + \Delta t) = \lambda^{n_{ijt}+1}$  and the technology gap rises to  $n_j(t + \Delta t) = n_j(t) + 1$  (the probability of two or more innovations within the interval  $\Delta t$  will be  $o(\Delta t)$ , where  $o(\Delta t)$  represents terms that satisfy  $\lim_{\Delta t \rightarrow 0} o(\Delta t) / \Delta t$ ).

When the follower is successful in catch-up R&D within the interval  $\Delta t$ , then its technology improves to

$$q_{-i}(j, t + \Delta t) = \lambda^{n_{ijt}},$$

under *quick catch-up* assumption since the follower is able to catch-up with the leader immediately no matter how big the technology gap is. In this case, the technology gap variable becomes  $n_{jt+\Delta t} = 0$  upon a successful innovation by the follower. In section 7, we will replace this assumption with *slow catch-up* where the follower proceed only by 1 step when it innovates.

In addition to catching up with the technology frontier with their own R&D, followers can also copy the technology frontier because IPR policy is such that some patents expire. In particular, we assume that patents expire at some policy-determined Poisson rate  $\eta$ , and after expiration, followers can costlessly copy the frontier technology, jumping to  $q_{-i}(j, t + \Delta t) = \lambda^{n_{ijt}}$ .<sup>12</sup> As in the partial equilibrium model in Section 2, IPR policy governs the length of the patent and we allow it to be state dependent, so it is represented by the following function:

$$\boldsymbol{\eta} : \mathbb{N} \rightarrow \mathbb{R}_+$$

Here  $\eta(n) \equiv \eta_n < \infty$  is the flow rate at which the patent protection is removed from a technology leader that is  $n$  steps ahead of the follower. When  $\eta_n = 0$ , this implies that there is full protection at technology gap  $n$ , in the sense that patent protection will never be removed. In contrast,  $\eta_n \rightarrow \infty$  implies that patent protection is removed immediately once technology gap  $n$  is reached. Our formulation imposes that  $\boldsymbol{\eta} \equiv \{\eta_1, \eta_2, \dots\}$  is time-invariant. Given this

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<sup>12</sup>Alternative modeling assumptions on IPR policy, such as a fixed patent length of  $T > 0$  from the time of innovation, are not tractable, since they lead to value functions that take the form of delayed differential equations.

specification, we can now write the law of motion of the technology gap in industry  $j$  as follows:

$$n_j(t + \Delta t) = \begin{cases} n_j(t) + 1 & \text{with probability } x_i(j, t) \Delta t + o(\Delta t) \\ 0 & \text{with probability } (x_{-i}(j, t) + \eta_{n_j(t)}) \Delta t + o(\Delta t) \\ n_j(t) & \text{with probability } 1 - (x_i(j, t) + x_{-i}(j, t) + \eta_{n_j(t)}) \Delta t - o(\Delta t) \end{cases} \quad (19)$$

Here  $o(\Delta t)$  again represents second-order terms, in particular, the probabilities of more than one innovations within an interval of length  $\Delta t$ . The terms  $x_i(j, t)$  and  $x_{-i}(j, t)$  are the flow rates of innovation by the leader and the follower; and  $\eta_{n_j(t)}$  is the flow rate at which the follower is allowed to copy the technology of a leader that is  $n_j(t)$  steps ahead. Intuitively, the technology gap in industry  $j$  increases from  $n_j(t)$  to  $n_j(t) + 1$  if the leader is successful. The firms become “neck-and-neck” when the follower comes up with an alternative technology to that of the leader (flow rate  $x_{-i}(j, t)$ ) or the patent expires at the flow rate  $\eta_{n_j}$ .

### 3.3 Profits

We next write the instantaneous “operating” profits for the leader (i.e., the profits exclusive of R&D expenditures). Profits of leader  $i$  in industry  $j$  at time  $t$  are

$$\begin{aligned} \Pi_i(j, t) &= [p_i(j, t) - MC_i(j, t)] y_i(j, t) \\ &= \left( \frac{w(t)}{q_{-i}(j, t)} - \frac{w(t)}{q_i(j, t)} \right) \frac{Y(t)}{p_i(j, t)} \\ &= \left( 1 - \lambda^{-n_j(t)} \right) Y(t) \end{aligned} \quad (20)$$

where  $n_j(t) \equiv n_{ij}(t) - n_{-ij}(t)$  is the technology gap in industry  $j$  at time  $t$ . The first line simply uses the definition of operating profits as price minus marginal cost times quantity sold. The second line uses the fact that the equilibrium limit price of firm  $i$  is  $p_i(j, t) = w(t) / q_{-i}(j, t)$  as given by (12), and the final equality uses the definitions of  $q_i(j, t)$  and  $q_{-i}(j, t)$  from (17) and (18). The expression in (20) also implies that there will be zero profits in neck-and-neck industries, i.e., in those with  $n_j(t) = 0$ . Also clearly, followers always make zero profits, since they have no sales.

The Cobb-Douglas aggregate production function in (8) is responsible for the form of the profits (20), since it implies that profits only depend on the technology gap of the industry and aggregate output. This will simplify the analysis below by making the technology gap in each industry the only industry-specific payoff-relevant state variable.

The objective function of each firm is to maximize the net present discounted value of “net profits” (operating profits minus R&D expenditures). In doing this, each firm will take the

sequence of interest rates,  $[r(t)]_{t \geq 0}$ , the sequence of aggregate output levels,  $[Y(t)]_{t \geq 0}$ , the sequence of wages,  $[w(t)]_{t \geq 0}$ , the R&D decisions of all other firms and policies as given.

### 3.4 Equilibrium

Let  $\boldsymbol{\mu}(t) \equiv \{\mu_n(t)\}_{n=0}^{\infty}$  denote the distribution of industries over different technology gaps, with  $\sum_{n=0}^{\infty} \mu_n(t) = 1$ . For example,  $\mu_0(t)$  denotes the fraction of industries in which the firms are neck-and-neck at time  $t$ . Throughout, we focus on Markov Perfect Equilibria (MPE), where strategies are only functions of the payoff-relevant state variables.<sup>13</sup> This allows us to drop the dependence on industry  $j$ , thus we refer to R&D decisions by  $x_n$  for the technology leader that is  $n$  steps ahead and by  $x_{-n}$  for a follower that is  $n$  steps behind. Let us denote the list of decisions by the leader and the follower with technology gap  $n$  at time  $t$  by  $\xi_n(t) \equiv \langle x_n(t), p_i(j, t), y_i(j, t) \rangle$  and  $\xi_{-n}(t) \equiv \langle x_{-n}(t) \rangle$ .<sup>14</sup> Throughout,  $\boldsymbol{\xi}$  will indicate the whole sequence of decisions at every state, so that  $\boldsymbol{\xi}(t) \equiv \{\xi_n(t)\}_{n=-\infty}^{\infty}$ . We define an allocation as follows:

**Definition 1 (Allocation)** *Let  $\boldsymbol{\eta}$  be the IPR policy sequence. Then an allocation is a sequence of decisions for a leader that is  $n = 0, 1, 2, \dots$  step ahead,  $[\xi_n(t)]_{t \geq 0}$ , a sequence of R&D decisions for a follower that is  $n = 1, 2, \dots$  step behind,  $[\xi_{-n}(t)]_{t \geq 0}$ , a sequence of wage rates  $[w(t)]_{t \geq 0}$ , and a sequence of industry distributions over technology gaps  $[\boldsymbol{\mu}(t)]_{t \geq 0}$ .*

For given IPR sequence  $\boldsymbol{\eta}$ , MPE strategies, which are only functions of the payoff-relevant state variables, can be represented as follows

$$\mathbf{x} : \mathbb{Z} \times \mathbb{R}_+^2 \times [0, 1]^\infty \rightarrow \mathbb{R}_+.$$

This mapping represents the R&D decision of a firm (both when it is the follower and when it is the leader in an industry) as a function of the technology gap,  $n \in \mathbb{Z}$ , the aggregate level of output and the wage,  $(Y, w) \in \mathbb{R}_+^2$ , and R&D decision of the other firm in the industry,  $\tilde{\mathbf{x}} \in [0, 1]^\infty$ . Consequently, we have the following definition of equilibrium:

**Definition 2 (Equilibrium)** *Given an IPR policy sequence  $\boldsymbol{\eta}$ , a Markov Perfect Equilibrium is given by a sequence  $[\boldsymbol{\xi}^*(t), w^*(t), Y^*(t)]_{t \geq 0}$  such that (i)  $[p_i^*(j, t)]_{t \geq 0}$  and  $[y_i^*(j, t)]_{t \geq 0}$  implied by  $[\boldsymbol{\xi}^*(t)]_{t \geq 0}$  satisfy (12) and (13); (ii) R&D policy  $[\mathbf{x}^*(t)]_{t \geq 0}$  is a best response to itself,*

<sup>13</sup>MPE is a natural equilibrium concept in this context, since it does not allow for implicit collusive agreements between the follower and the leader. While such collusive agreements may be likely when there are only two firms in the industry, in most industries there are many more firms and also many potential entrants, making collusion more difficult. Throughout, we assume that there are only two firms to keep the model tractable.

<sup>14</sup>The price and output decisions,  $p_i(j, t)$  and  $y_i(j, t)$ , depend not only on the technology gap, aggregate output and the wage rate, but also on the exact technology rung of the leader,  $n_{ij}(t)$ . With a slight abuse of notation, throughout we suppress this dependence, since their product  $p_i(j, t) y_i(j, t)$  and the resulting profits for the firm, (20), are independent of  $n_{ij}(t)$ , and consequently, only the technology gap,  $n_j(t)$ , matters for profits, R&D, aggregate output and economic growth.

i.e.,  $[\mathbf{x}^*(t)]_{t \geq 0}$  maximizes the expected profits of firms taking aggregate output  $[Y^*(t)]_{t \geq 0}$ , wages  $[w^*(t)]_{t \geq 0}$ , government policy  $\boldsymbol{\eta}$  and the R&D policies of other firms  $[\mathbf{x}^*(t)]_{t \geq 0}$  as given; (iii) aggregate output  $[Y^*(t)]_{t \geq 0}$  is given by (8); and (iv) the labor market clears at all times given the wage sequence  $[w^*(t)]_{t \geq 0}$ .

### 3.5 The Labor Market

Since only the technology leader produces, labor demand in industry  $j$  with technology gap  $n_j(t) = n$  can be expressed as

$$l_n(t) = \frac{\lambda^{-n} Y(t)}{w(t)} \quad \text{for } n \in \mathbb{Z}_+. \quad (21)$$

In addition, there is demand for labor coming for R&D from both followers and leaders in all industries. Using (14) and the definition of the  $G$  function, we can express industry demands for R&D labor as

$$h_n(t) = G(x_n(t)) + G(x_{-n}(t)) \quad \text{for } n \in \mathbb{Z}_+, \quad (22)$$

where  $G(x_n(t))$  and  $G(x_{-n}(t))$  refer to the demand of the leader and the follower in an industry with a technology gap of  $n$ . Note that in this expression,  $x_{-n}(t)$  refers to the R&D effort of a follower that is  $n$  steps behind.

The labor market clearing condition can then be expressed as:

$$1 \geq \sum_{n=0}^{\infty} \mu_n(t) \left[ \frac{1}{\omega(t) \lambda^n} + G(x_n(t)) + G(x_{-n}(t)) \right], \quad (23)$$

and  $\omega(t) \geq 0$ , with complementary slackness, where

$$\omega(t) \equiv \frac{w(t)}{Y(t)} \quad (24)$$

is the labor share at time  $t$ . The labor market clearing condition, (23), uses the fact that total supply is equal to 1, and demand cannot exceed this amount. If demand falls short of 1, then the wage rate,  $w(t)$ , and thus the labor share,  $\omega(t)$ , have to be equal to zero (though this will never be the case in equilibrium). The right-hand side of (23) consists of the demand for production (the terms with  $\omega$  in the denominator), the demand for R&D workers from the neck-and-neck industries ( $2G(x_0(t))$  when  $n = 0$ ) and the demand for R&D workers coming from leaders and followers in other industries ( $G(x_n(t)) + G(x_{-n}(t))$  when  $n > 0$ ).

Defining the index of aggregate quality in this economy by the aggregate of the qualities of the leaders in the different industries, i.e.,

$$\ln Q(t) \equiv \int_0^1 \ln q_i(j, t) dj, \quad (25)$$



the equilibrium wage can be written as:<sup>15</sup>

$$w(t) = Q(t) \lambda^{-\sum_{n=0}^{\infty} n\mu_n(t)}. \quad (26)$$

### 3.6 Steady State and the Value Functions under Quick Catch-up

Let us now focus on steady-state (Markov Perfect) equilibria, where the distribution of industries  $\boldsymbol{\mu}(t) \equiv \{\mu_n(t)\}_{n=0}^{\infty}$  is stationary,  $\omega(t)$  defined in (24) and  $g$ , the growth rate of the economy, are constant over time. We will establish the existence of such an equilibrium and characterize a number of its properties. If the economy is in steady state at time  $t = 0$ , then by definition, we have  $Y^*(t) = Y_0 e^{g^* t}$  and  $w^*(t) = w_0 e^{g^* t}$ , where  $g^*$  is the steady-state growth rate. These two equations also imply that  $\omega(t) = \omega^*$  for all  $t \geq 0$ . Throughout, we assume that the parameters are such that the steady-state growth rate  $g^*$  is positive but not large enough to violate the transversality conditions. This implies that net present values of each firm at all points in time will be finite. This enables us to write the maximization problem of a leader that is  $n > 0$  steps ahead recursively.

First note that given an optimal policy  $\hat{x}$  for a firm, the net present discounted value of a leader that is  $n$  steps ahead at time  $t$  can be written as:

$$V_n(t) = \mathbb{E}_t \int_t^{\infty} \exp(-r(s-t)) [\Pi(s) - w(s) G(\hat{x}(s))] ds$$

where  $\Pi(s)$  is the operating profit at time  $s \geq t$  and  $w(s) G(\hat{x}(s))$  denotes the R&D expenditure at time  $s \geq t$ . All variables are stochastic and depend on the evolution of the technology gap within the industry.

Next taking as given the equilibrium R&D policy of other firms,  $x_{-n}^*(t)$ , the equilibrium interest and wage rates,  $r^*(t)$  and  $w^*(t)$ , and equilibrium profits  $\{\Pi_n^*(t)\}_{n=1}^{\infty}$  (as a function of equilibrium aggregate output), this value can be written as (see the Appendix for the derivation of this equation):<sup>16</sup>

$$r^*(t) V_n(t) - \dot{V}_n(t) = \max_{x_n(t) \geq 0} \left\{ \begin{aligned} & [\Pi_n^*(t) - w^*(t) G(x_n(t))] + x_n(t) [V_{n+1}(t) - V_n(t)] \\ & + (x_{-n}^*(t) + \eta_n) [V_0(t) - V_n(t)] \end{aligned} \right\}, \quad (27)$$

where  $\dot{V}_n(t)$  denotes the derivative of  $V_n(t)$  with respect to time. The first term is current profits minus R&D costs, while the second term captures the fact that the firm will undertake an innovation at the flow rate  $x_n(t)$  and increase its technology lead by one step. The remaining

<sup>15</sup>Note that  $\ln Y(t) = \int_0^1 \ln q_i(j, t) l(j, t) dj = \int_0^1 [\ln q_i(j, t) + \ln \frac{Y(t)}{w(t)} \lambda^{-n_j}] dj$ , where the second equality uses (21). Thus we have  $\ln Y(t) = \int_0^1 [\ln q_i(j, t) + \ln Y(t) - \ln w(t) - n_j \ln \lambda] dj$ . Rearranging and canceling terms, and writing  $\exp \int n_j \ln \lambda dj = \lambda^{-\sum_{n=0}^{\infty} n\mu_n(t)}$ , we obtain (26).

<sup>16</sup>Clearly, this value function could be written for any arbitrary sequence of R&D policies of other firms. We set the R&D policies of other firms to their equilibrium values,  $x_{-n}^*(t)$ , to reduce notation in the main body of the paper.

terms incorporate changes in value due to quick catch-up by the follower (flow rate  $x_{-n}^*(t) + \eta_n$  in the second line).

In steady state, the net present value of a firm that is  $n$  steps ahead,  $V_n(t)$ , will also grow at a constant rate  $g^*$  for all  $n \in \mathbb{Z}_+$ . Let us then define the normalized values as

$$v_n(t) \equiv \frac{V_n(t)}{Y(t)} \quad (28)$$

for all  $n \in \mathbb{Z}$ , which will be independent of time in steady state, i.e.,  $v_n(t) = v_n$ .

Using (28) and the fact that from (7),  $r(t) = g(t) + \rho$ , the recursive form of the steady-state value function (27) can be written as:

$$\rho v_n = \max_{x_n \geq 0} \left\{ (1 - \lambda^{-n}) - \omega^* G(x_n) + x_n [v_{n+1} - v_n] + [x_{-n}^* + \eta_n] [v_0 - v_n] \right\} \text{ for } n \in \mathbb{N}, \quad (29)$$

where  $x_{-n}^*$  is the equilibrium value of R&D by a follower that is  $n$  steps behind, and  $\omega^*$  is the steady-state labor share (while  $x_n$  is now explicitly chosen to maximize  $v_n$ ).

Similarly the value for neck-and-neck firms is

$$\rho v_0 = \max_{x_0 \geq 0} \left\{ -\omega^* G(x_0) + x_0 [v_1 - v_0] + x_0^* [v_{-1} - v_0] \right\}, \quad (30)$$

while the values for followers are given by

$$\rho v_{-n} = \max_{x_{-n} \geq 0} \left\{ -\omega^* G(x_{-n}) + [x_{-n} + \eta_n] [v_0 - v_{-n}] + x_n^* [v_{-n-1} - v_{-n}] \right\} \text{ for } n \in \mathbb{N}. \quad (31)$$

For neck-and-neck firms and followers, there are no instantaneous profits, which is reflected in (30) and (31). In the former case this is because neck-and-neck firms sell at marginal cost, and in the latter case, this is because followers have no sales. These normalized value functions emphasize that, because of growth, the effective discount rate is  $r(t) - g(t) = \rho$  rather than  $r(t)$ .

The maximization problems in (29)-(31) immediately imply that any steady-state equilibrium R&D policies,  $\mathbf{x}^*$ , must satisfy:

$$x_n^* = \max \left\{ G'^{-1} \left( \frac{[v_{n+1} - v_n]}{\omega^*} \right), 0 \right\} \quad (32)$$

$$x_{-n}^* = \max \left\{ G'^{-1} \left( \frac{[v_0 - v_{-n}]}{\omega^*} \right), 0 \right\} \quad (33)$$

$$x_0^* = \max \left\{ G'^{-1} \left( \frac{[v_1 - v_0]}{\omega^*} \right), 0 \right\}, \quad (34)$$

where the normalized value functions, the  $v$ s, are evaluated at the equilibrium, and  $G'^{-1}(\cdot)$  is the inverse of the derivative of the  $G$  function. Since  $G$  is twice continuously differentiable and strictly concave,  $G'^{-1}$  is continuously differentiable and strictly increasing. These equations therefore imply that innovation rates, the  $x_n^*$ s, will increase whenever the incremental value of

moving to the next step is greater and when the cost of R&D, as measured by the normalized wage rate,  $\omega^*$ , is less. Note also that since  $G'(0) > 0$ , these R&D levels can be equal to zero, which is taken care of by the max operator.

The response of innovation rates,  $x_n^*$ , to the increments in values,  $v_{n+1} - v_n$ , is the key economic force in this model. A policy that reduces the patent protection of leaders that are  $n+1$  steps ahead (by increasing  $\eta_{n+1}$ ) will make being  $n+1$  steps ahead less profitable, thus reduce  $v_{n+1} - v_n$  and  $x_n^*$ . This corresponds to the standard *disincentive effect* of relaxing IPR policy. This result corresponds to fact (1) in the toy model. In contrast to existing models, however, here relaxing IPR policy can also create a *positive incentive effect*. Somewhat paradoxically, lower protection for technology leaders that are  $n+1$  steps ahead will tend to reduce  $v_{n+1}$ , thus increasing  $v_{n+2} - v_{n+1}$  and  $x_{n+1}^*$ . This result is very similar to fact (2) in the toy model. We will see this positive incentive effect plays an important role in the form of optimal state-dependent IPR policy. In addition to the incentive effects, relaxing IPR protection may also create a beneficial *composition effect*; this is because, typically,  $\{v_{n+1} - v_n\}_{n=0}^\infty$  is a decreasing sequence, which implies that  $x_{n-1}^*$  is higher than  $x_n^*$  for  $n \geq 1$  (see, e.g., Proposition 4). Weaker patent protection (in the form of shorter patent lengths) will shift more industries into the neck-and-neck state and potentially increase the equilibrium level of R&D in the economy. Finally, weaker patent protection also creates a beneficial “level effect” by influencing equilibrium markups and prices (as shown in equation (12) above) and by reallocating some of the workers engaged in “duplicative” R&D to production. This level effect will also feature in our welfare computations. The optimal level and structure of IPR policy in this economy will be determined by the interplay of these various forces.

Given the equilibrium R&D decisions  $\mathbf{x}^*$ , the steady-state distribution of industries across states  $\boldsymbol{\mu}^*$  has to satisfy the following accounting identities:

$$(x_{n+1}^* + x_{-n-1}^* + \eta_{n+1}) \mu_{n+1}^* = x_n^* \mu_n^* \text{ for } n \in \mathbb{N}, \quad (35)$$

$$(x_1^* + x_{-1}^* + \eta_1) \mu_1^* = 2x_0^* \mu_0^*, \quad (36)$$

$$2x_0^* \mu_0^* = \sum_{n=1}^{\infty} (x_{-n}^* + \eta_n) \mu_n^*. \quad (37)$$

The first expression equates exit from state  $n+1$  (which takes the form of the leader going one more step ahead or the follower catching up the leader) to entry into the state (which takes the form of a leader from state  $n$  making one more innovation). The second equation, (36), performs the same accounting for state 1, taking into account that entry into this state comes from innovation by either of the two firms that are competing neck-and-neck. Finally, equation (37) equates exit from state 0 with entry into this state, which comes from innovation by a follower in any industry with  $n \geq 1$ .

The labor market clearing condition in steady state can then be written as

$$1 \geq \sum_{n=0}^{\infty} \mu_n^* \left[ \frac{1}{\omega^* \lambda^n} + G(x_n^*) + G(x_{-n}^*) \right] \text{ and } \omega^* \geq 0, \quad (38)$$

with complementary slackness.

The next proposition characterizes the steady-state growth rate. As with all the other results in the paper, the proof of this proposition is provided in the Appendix.

**Proposition 3** *Let the steady-state distribution of industries and R&D decisions be given by  $\langle \boldsymbol{\mu}^*, \mathbf{x}^* \rangle$ , then the steady-state growth rate is*

$$g^* = \ln \lambda \left[ 2\mu_0^* x_0^* + \sum_{n=1}^{\infty} \mu_n^* x_n^* \right]. \quad (39)$$

This proposition clarifies that the steady-state growth rate of the economy is determined by two factors: (1) R&D decisions of industries at different levels of technology gap,  $\mathbf{x}^* \equiv \{x_n^*\}_{n=-\infty}^{\infty}$ ; (2) The distribution of industries across different technology gaps,  $\boldsymbol{\mu}^* \equiv \{\mu_n^*\}_{n=0}^{\infty}$ . IPR policy affects these two margins in different directions as illustrated by the discussion above.

## 4 Existence and Characterization of Steady-State Equilibria

We now define a steady-state equilibrium in a more convenient form, which will be used to establish existence and derive some of the properties of the equilibrium.

**Definition 3 (Steady-State Equilibrium)** *Given an IPR policy  $\boldsymbol{\eta}$ , a steady-state equilibrium is a tuple  $\langle \boldsymbol{\mu}^*, \mathbf{v}, \mathbf{x}^*, \omega^*, g^* \rangle$  such that the distribution of industries  $\boldsymbol{\mu}^*$  satisfy (35), (36) and (37), the values  $\mathbf{v} \equiv \{v_n\}_{n=-\infty}^{\infty}$  satisfy (29), (30) and (31), the R&D decision  $\mathbf{x}^*$  is given by (32), (33) and (34), the steady-state labor share  $\omega^*$  satisfies (38) and the steady-state growth rate  $g^*$  is given by (39).*

We next provide a characterization of the steady-state equilibrium, starting first with the case in which there is uniform IPR policy.

### 4.1 Uniform IPR Policy

Let us first focus on the case where IPR policy is uniform, i.e.  $\eta_n = \eta < \infty$  for all  $n \in \mathbb{N}$  and we denote this by  $\boldsymbol{\eta}^{uni}$ . In this case, (31) implies that the problem is identical for all followers, so that  $v_{-n} = v_{-1}$  for  $n \in \mathbb{N}$ . Consequently, (31) can be replaced with the following simpler equation:

$$\rho v_{-1} = \max_{x_{-1} \geq 0} \{-\omega^* G(x_{-1}) + [x_{-1} + \eta] [v_0 - v_{-1}]\}, \quad (40)$$

implying optimal R&D decisions for all followers of the form

$$x_{-1}^* = \max \left\{ G'^{-1} \left( \frac{[v_0 - v_{-1}]}{\omega^*} \right), 0 \right\}. \quad (41)$$

Let us denote the sequence of value functions under uniform IPR as  $\{v_n\}_{n=-1}^{\infty}$ . We next establish the existence of a steady-state equilibrium under uniform IPR and characterize some of its most important properties. Establishing the existence of a steady-state equilibrium in this economy is made complicated by the fact that the equilibrium allocation cannot be represented as a solution to a maximization problem. Instead, as emphasized by Definition 3, each firm maximizes its value taking the R&D decisions of other firms as given; thus an equilibrium corresponds to a set of R&D decisions that are best responses to themselves and a labor share (wage rate)  $\omega^*$  that clears the labor market. Nevertheless, there is sufficient structure in the model to guarantee the existence of a steady-state equilibrium and monotonic behavior of values and R&D decisions.

**Proposition 4** *Consider a uniform IPR policy  $\eta^{uni}$  and suppose that  $G'^{-1}((1 - \lambda^{-1}) / (\rho + \eta)) > 0$ . Then a steady-state equilibrium  $\langle \mu^*, \mathbf{v}, \mathbf{x}^*, \omega^*, g^* \rangle$  exists. Moreover, in any steady-state equilibrium  $\omega^* < 1$ . In addition, if either  $\eta > 0$  or  $x_{-1}^* > 0$ , then  $g^* > 0$ . For any steady-state R&D decisions  $\mathbf{x}^*$ , the steady-state distribution of industries  $\mu^*$  is uniquely determined.*

*In addition, we have the following results:*

- $v_{-1} \leq v_0$  and  $\{v_n\}_{n=0}^{\infty}$  forms a bounded and strictly increasing sequence converging to some  $v_{\infty} \in (0, \infty)$ .
- $x_0^* > x_1^*$ ,  $x_0^* \geq x_{-1}^*$ , and  $x_{n+1}^* \leq x_n^*$  for all  $n \in \mathbb{N}$  with  $x_{n+1}^* < x_n^*$  if  $x_n^* > 0$ . Moreover, provided that  $G'^{-1}((1 - \lambda^{-1}) / (\rho + \eta)) > 0$  and  $x_0^* > x_{-1}^*$ .

**Proof.** See the Appendix. ■

**Remark 1** The condition that  $G'^{-1}((1 - \lambda^{-1}) / (\rho + \eta)) > 0$  ensures that there will be positive R&D in equilibrium. If this condition does not hold, then there exists a trivial steady-state equilibrium in which  $x_n^* = 0$  for all  $n \in \mathbb{Z}_+$ , i.e., an equilibrium in which there is no innovation and thus no growth (this follows from the fact that  $x_0^* \geq x_n^*$  for all  $n \neq 0$ , see the Appendix for more details). Moreover, if  $\eta > 0$ , then this equilibrium would also involve  $\mu_0^* = 1$ , so that in every industry two firms with equal costs compete a la Bertrand and charge price equal to marginal cost, leading to zero aggregate profits and a labor share of output equal to 1. The assumption that  $G'^{-1}((1 - \lambda^{-1}) / (\rho + \eta)) > 0$ , on the other hand, is sufficient to rule out  $\mu_0^* = 1$  and thus  $\omega^* = 1$ . If, in addition, the steady-state equilibrium involves some probability

of catch-up or innovation by the followers, i.e., either  $\eta > 0$  or  $x_{-1}^* > 0$ , then the growth rate is also strictly positive.

In addition to the existence of a steady-state equilibrium with positive growth, Proposition 4 shows that the sequence of values  $\{v_n\}_{n=0}^{\infty}$  is strictly increasing and converges to some  $v_{\infty}$ , and more importantly that  $\mathbf{x}^* \equiv \{x_n^*\}_{n=1}^{\infty}$  is a decreasing sequence, which implies that technology leaders that are further ahead undertake less R&D. Intuitively, the benefits of further R&D are decreasing in the technology gap, since greater values of the technology gap translate into smaller increases in the equilibrium markup (recall (20)). Moreover, the R&D level of neck-and-neck firms,  $x_0^*$ , is greater than both the R&D level of technology leaders that are one step ahead and followers that are one step behind (i.e.,  $x_0^* > x_1^*$  and  $x_0^* \geq x_{-1}^*$ ). This implies that with uniform policy neck-and-neck industries are “most R&D intensive,” while industries with the largest technology gaps are “least R&D intensive”. This is the basis of the conjecture mentioned in the Introduction that reducing protection given to technologically advanced leaders might be useful for increasing R&D by bringing them into the neck-and-neck state.

## 4.2 State-Dependent IPR Policy

We now extend the results from the previous section to the environment with state-dependent IPR policy, though results on monotonicity of values and R&D efforts no longer hold.<sup>17</sup>

**Proposition 5** *Consider the state-dependent IPR policy  $\eta$  and suppose that  $G'^{-1}((1 - \lambda^{-1}) / (\rho + \eta_1)) > 0$ . Then a steady-state equilibrium  $\langle \mu^*, \mathbf{v}, \mathbf{x}^*, \omega^*, g^* \rangle$  exists. Moreover, in any steady-state equilibrium  $\omega^* < 1$ . In addition, if either  $\eta_1 > 0$  or  $x_{-1}^* > 0$ , then  $g^* > 0$ .*

**Proof.** See the Appendix. ■

Unfortunately, it is not possible to determine the optimal (welfare- or growth-maximizing) state-dependent IPR policy analytically. For this reason, in Section 5, we undertake a quantitative investigation of the form and structure of optimal state-dependent IPR policy using plausible parameter values.

## 5 Optimal IPR Policy: Towards A Quantitative Investigation

In the remainder of the paper, we investigate the implications of various different types of IPR policies on R&D, growth and welfare using numerical computations of the steady-state

<sup>17</sup>This is because IPR policies could be very sharply increasing at some technology gap, making a particular state very unattractive for the leader. For example, we could have  $\eta_n = 0$  and  $\eta_{n+1} \rightarrow \infty$ , which would imply that  $v_{n+1} - v_n$  is negative.

equilibrium. Our purpose is not to provide a detailed calibration of the model economy but to highlight the broad quantitative characteristics of the model and its implications for optimal IPR policy under plausible parameter values. We focus on optimal policy, defined as steady-state welfare-maximizing choice of policy (growth-maximizing policies give very similar results and are omitted to save space). In this section, we introduce the measure of steady-state welfare and describe our quantitative methodology. Results are reported in the subsequent sections.

## 5.1 Welfare

Our focus so far has been on steady-state equilibria (mainly because of the very challenging nature of transitional dynamics in this class of models). In our quantitative analysis, we continue to focus on steady states and thus look at steady-state welfare. In a steady-state equilibrium, welfare at time  $t = 0$  can be written as

$$\begin{aligned} \text{Welfare}(0) &= \int_0^\infty e^{-\rho t} \ln \left( Y(0) e^{g^* t} \right) dt \\ &= \frac{\ln Y(0)}{\rho} + \frac{g^*}{\rho^2}, \end{aligned} \tag{42}$$

where the first-line uses the facts that all output is consumed, utility is logarithmic (recall (6)), output and consumption at date  $t = 0$  are given by  $Y(0)$ , and in the steady-state equilibrium output grows at the rate  $g^*$ . The second line simply evaluates the integral. Next, note that

$$\begin{aligned} \ln Y(t) &= \int_0^1 \ln y(j, t) dj \\ &= \int_0^1 \ln \left( \frac{q_{-i}(j, t) Y(t)}{w(t)} \right) dj \\ &= \int_0^1 \ln q_{-i}(j, t) dj - \ln \omega(t) \\ &= \ln Q(t) - \ln \lambda \left( \sum_{n=0}^\infty n \mu_n(t) \right) - \ln \omega(t), \end{aligned} \tag{43}$$

where the first line simply uses the definition in (8), the second line substitutes for  $y(j, t)$  from (13), the third line uses the definition of the labor share  $\omega(t)$ , and the final line uses the definition of  $Q(t)$  from (25) together with the fact that in the steady state  $q_i(j, t) = \lambda^n q_{-i}(j, t)$  in a fraction  $\mu_n(t)$  of industries. The expression in (43) implies that output simply depends on the quality index,  $Q(t)$ , the distribution of technology gaps,  $\boldsymbol{\mu}(t)$  (because this determines markups), and also on the labor share,  $\omega(t)$ . In steady-state equilibrium, the distribution of technology gaps and labor share are constant, while output and the quality index grow at the steady-state rate  $g^*$ . Therefore, for steady-state comparisons of welfare across economies with different policies, it is sufficient to compare two economies with the same level of  $Q(0)$ ,

but with different policies. We can then evaluate steady-state welfare with the distribution of industries given by their steady-state values in the two economies, and output and the quality index growing at the corresponding steady-state growth rates. Expression (43) also makes it clear that only the aggregate quality index  $Q(0)$  needs to be taken to be the same in the different economies. Given  $Q(0)$ , the dispersion of industries in terms of the quality levels has no effect on output or welfare (though, clearly, the distribution of industries in terms of technology gaps between leaders and followers,  $\mu$ , influences the level of markups and output, and thus welfare).

However, note one difficulty with welfare comparisons highlighted by equations (42) and (43); proportional changes in steady-state welfare due to policy changes will depend on the initial level of  $Q(0)$ , which is an arbitrary number. Therefore, proportional changes in welfare are not informative, though this has no effect on ordinal rankings and thus welfare-maximizing policy is well defined and independent of the level of  $Q(0)$ . Equations (42) and (43) also make it clear that changes in steady-state welfare will be the sum of two components: the first is the *growth effect*, given by  $g^*/\rho^2$ , whereas the second is due to changes in  $\ln \lambda (\sum_{n=0}^{\infty} n\mu_n) / \rho - \ln \omega(0)$ . Since changes in the labor share  $\omega(0)$  are largely driven by the distribution of industries, we refer to this as the *distribution effect*. Policies will typically affect both of these quantities. In what follows, we give the welfare rankings of different policies and then report the relative magnitudes of the growth and the distribution effects. This will show that the growth effects will be one or two orders of magnitude greater than the distribution effects and dominate welfare comparisons. So if the reader wishes, he or she may think of the magnitudes of the changes in welfare as given by the proportional changes in growth rates.

## 5.2 Quantitative Methods and Parameter Choices

For our quantitative exercise, we take the annual discount rate as 5%, i.e.,  $\rho_{year} = 0.05$ . In all our computations, we work with the monthly equivalent of this discount rate in order to increase precision, but throughout the tables, we convert all numbers to their annual counterparts to facilitate interpretation.

The theoretical analysis considered a general production function for R&D given by (14). The empirical literature typically assumes a Cobb-Douglas production function. For example, Kortum (1993) considers a function of the form

$$\text{Innovation}(t) = B_0 \exp(\kappa t) (\text{R\&D inputs})^\gamma, \quad (44)$$

where  $B_0$  is a constant and  $\exp(\kappa t)$  is a trend term, which may depend on general technological trends, a drift in technological opportunities, or changes in general equilibrium prices (such as wages of researchers etc.). The advantage of this form is not only its simplicity, but also the



fact that most empirical work estimates a single elasticity for the response of innovation rates to R&D inputs. Consequently, they essentially only give information about the parameter  $\gamma$  in terms of equation (44). A low value of  $\gamma$  implies that the R&D production function is more concave. For example, Kortum (1993) reports that estimates of  $\gamma$  vary between 0.1 and 0.6 (see also Pakes and Griliches, 1980, or Hall, Hausman and Griliches, 1988). For these reasons, throughout, we adopt a R&D production function similar to (44):

$$x = Bh^\gamma \tag{45}$$

where  $B, \gamma > 0$ . In terms of our previous notation, equation (45) implies that  $G(x) = [x/B]^\frac{1}{\gamma} w$ , where  $w$  is the wage rate in the economy (thus in terms of the above function, it is captured by the  $\exp(\kappa t)$  term).<sup>18</sup> Equation (45) does not satisfy the boundary conditions we imposed so far and can be easily modified to do so without affecting any of the results, since in all numerical exercises only a finite number of states are reached.<sup>19</sup> Following the estimates reported in Kortum (1993), we start with a benchmark value of  $\gamma = 0.35$ , and then report sensitivity checks for  $\gamma = 0.1$  and  $\gamma = 0.6$ . The other parameter in (45),  $B$ , is chosen so as to ensure an annual growth rate of approximately 1.9%, i.e.,  $g^* \simeq 0.019$ , in the benchmark economy which features indefinitely-enforced patents. This growth rate together with  $\rho_{year} = 0.05$  also pins down the annual interest rate as  $r_{year} = 0.069$  from equation (7).

We choose the value of  $\lambda$  using a reasoning similar to Stokey (1995). Equation (39) implies that if the expected duration of time between any two consecutive innovations is about 3 years in an industry, then a growth rate of about 1.9% would require  $\lambda = 1.05$ .<sup>20</sup> This value is also consistent with the empirical findings of Bloom, Schankerman and Van Reenen (2005).<sup>21</sup> We take  $\lambda = 1.05$  as the benchmark value. We then check the robustness of the results to  $\lambda = 1.01$

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<sup>18</sup>More specifically, (45) can be alternatively written as

$$\text{Innovation}(t) = Bw(t)^{-\gamma} (\text{R\&D expenditure})^\gamma,$$

thus would be equivalent to (44) as long as the growth of  $w(t)$  can be approximated by constant rate.

<sup>19</sup>For example, we could add a small linear term to the production function for R&D, (45), and also make it flat after some level  $\bar{h}$ . For example, the following generalization of (45),

$$x = \min \{ Bh^\gamma + \varepsilon h; B\bar{h}^\gamma + \varepsilon \bar{h} \}$$

for  $\varepsilon$  small and  $\bar{h}$  large, makes no difference to our simulation results.

<sup>20</sup>In particular, in our benchmark parameterization with full protection without licensing, 24% of industries are in the neck-and-neck state. This implies that improvements in the technological capability of the economy is driven by the R&D efforts of the leaders in 76% of the industries and the R&D efforts of both the leaders and the followers in 24% of the industries. Therefore, the growth equation, (39), implies that  $g \simeq \ln \lambda \times 1.24 \times x$ , where  $x$  denotes the average frequency of innovation in a given industry. A major innovation on average every three years implies a value of  $\lambda \simeq 1.05$ .

<sup>21</sup>The production function for the intermediate good, (10), can be written as  $\log(y(j, t)) = n(j, t) \log(\lambda) + \log(l(j, t))$ , where  $n(j, t)$  is the number of innovations to date in sector  $j$  and represents the “knowledge stock” of this industry. Bloom, Schankerman and Van Reenen (2005) proxy the knowledge stock in an industry by the stock of R&D in that industry and estimate the elasticity of sales with respect to the stock of R&D to be approximately 0.06. In terms of the exercise here, this implies that  $\log(\lambda) = 0.06$ , or that  $\lambda \approx 1.06$ .

and  $\lambda = 1.2$  (expected duration of 8 months and 13 years, respectively). Finally, without loss of generality, we normalize labor supply to 1. This completes the determination of all the parameters in the model except the IPR policy.

As noted above, we begin with the full patent protection regime, i.e.,  $\boldsymbol{\eta} = \{0, 0, \dots\}$ . We then move to a comparison of the optimal (welfare-maximizing) uniform IPR policy  $\boldsymbol{\eta}^{uni}$  to the optimal state-dependent IPR policy. Since it is computationally impossible to calculate the optimal value of each  $\eta_n$ , we limit our investigation to a particular form of state-dependent IPR policy, whereby the same  $\eta$  applies to all industries that have a technology gap of  $n = 5$  or more. In other words, the IPR policy can be represented as:

IPR policy $\rightarrow$	<i>none</i>	$\overbrace{\quad}^{\eta_1}$	$\overbrace{\quad}^{\eta_2}$	$\overbrace{\quad}^{\eta_3}$	$\overbrace{\quad}^{\eta_4}$	$\overbrace{\quad\quad\quad\quad\quad}^{\eta_5}$								
Technology gap: $n \rightarrow$	0	1	2	3	4	5	6	7	8	9	10	11	. . .	$\infty$

We checked and verified that allowing for further flexibility (e.g., allowing  $\eta_5$  and  $\eta_6$  to differ) has little effect on our results.

The numerical methodology we pursue relies on *uniformization* and value function iteration. The details of the uniformization technique are described in the proof of Lemma 1 in the Appendix (for details of value function iteration, see Judd, 1999). In particular, we first take the IPR policy  $\boldsymbol{\eta}$  as given and make an initial guess for the equilibrium labor share  $\omega^*$ . Then for a given  $\omega^*$ , we generate a sequence of values  $\{v_n\}_{n=-\infty}^{\infty}$ , and we derive the optimal R&D policies,  $\{x_n^*\}_{n=-\infty}^{\infty}$  and the steady-state distribution of industries,  $\{\mu_n^*\}_{n=0}^{\infty}$ . After convergence, we compute the growth rate  $g^*$  and welfare, and then check for market clearing in the labor market from equation (23). Depending on whether there is excess demand for or supply of labor,  $\omega^*$  is varied and the numerical procedure is repeated until the entire steady-state equilibrium for a given IPR policy is computed. The process is then repeated for different IPR policies.

In the state-dependent IPR case, the optimal (welfare-maximizing) IPR policy sequences,  $\boldsymbol{\eta}$ , are computed one element at a time, until we find the welfare-maximizing value for that component, for example,  $\eta_1$ . We then move the next component, for example,  $\eta_2$ . Once the welfare-maximizing value of  $\eta_2$  is determined, we go back to optimize over  $\eta_1$  again, and this procedure is repeated recursively until convergence.<sup>22</sup>

## 6 Optimal IPR Policy in the Quick Catch-up Regime

In this section, we start with a quantitative analysis of the baseline model introduced and studied in Sections 3 and 4

<sup>22</sup>After we find a maximizer ( $\boldsymbol{\eta}^*$ ), we also evaluate several random policy combinations around the maximizer to verify the solution.

## 6.1 Full IPR Protection

We start with the benchmark with full protection, which is the case that the existing literature has considered so far (e.g., Aghion, Harris, Howitt and Vickers, 2001). In terms of our model, this corresponds to  $\eta_n = 0$  for all  $n$ . We choose the parameter  $B$  in terms of (45), so that the benchmark economy has an annual growth rate of 1.86%.

[FIGURE 3 & 4 & 5 HERE]

The value function for this benchmark case is shown in Figure 3 (solid line). The value function has decreasing differences for  $n \geq 0$ , which is consistent with the results in Proposition 4, and features a constant level for all followers (since there is no state dependence in the IPR policy). Figure 4 shows the level of R&D efforts for leaders and followers in this benchmark (again solid line). Again consistent with Proposition 4, this figure also shows that the R&D level of a leader declines as the technology gap increases and that the highest level of R&D is for firms that are neck-and-neck (i.e., at the technology gap of  $n = 0$ ). Since there is no state-dependent IPR policy, all followers undertake the same level of R&D effort, which is also shown in the figure.

Figure 5 shows the distribution of industries according to technology gaps (again the solid line refers to the benchmark case). The mode of the distribution is at the technology gap of  $n = 1$ , but there is also a significant concentration of industries at technology gap  $n = 0$ , because innovations by the followers take them to the “neck-and-neck” state.

[TABLE 1 HERE]

The first column of Table 1 also reports the results for this benchmark simulation. As noted above, in each case  $B$  is chosen such that the annual growth rate is equal to 0.0186, which is recorded at the bottom of Table 1 together with the initial consumption and welfare levels according to (42) and (43). The table also shows the R&D levels  $x_0^*$ ,  $x_{-1}^*$  and  $x_1^*$  (0.35, 0.22 and 0.29), the frequencies of industries with technology gaps of 0, 1 and 2. The steady-state value of  $\omega$  is 0.95. Since labor is the only factor of production in the economy,  $\omega^*$  should not be thought of as the labor share in GDP. Instead,  $1 - \omega^*$  measures the share of pure monopoly profits in value added. In the benchmark parameterization, this corresponds to 5% of GDP, which is reasonable.<sup>23</sup> Finally, the table also shows that in this benchmark parameterization 3.2% of the workforce is working as researchers, which is also consistent with US data.<sup>24</sup> These results are encouraging for our simple quantitative exercise, since with very few parameter choices,

<sup>23</sup>Bureau of Economic Analysis (2004) reports that the ratio of before-tax profits to GDP in the US economy in 2001 was 7% and the after-tax ratio was 5%.

<sup>24</sup>According to National Science Foundation (2006), the ratio of scientists and engineers in the US workforce in 2001 is about 4%.

the model generates reasonable numbers, especially for the share of the workforce allocated to research.<sup>25</sup>

## 6.2 Optimal Uniform IPR Protection

For reference, we now characterize optimal uniform IPR policy, that is, we impose that  $\eta_n = \eta$  for all  $n$ , and look for values of  $\eta$  that maximizes the welfare in the economy. Column 2 of Table 1 shows that the welfare-maximizing value of  $\eta$  is not different from zero at the three-digit level. Therefore the results of the full protection case carries over to uniform policy as well. The main reason for this result is the quick catch-up assumption. Recall that the uniform IPR policy discourages innovation, but generates a potential benefit because of the composition effect (bringing more firms into neck-and-neck position). In the quick catch-up regime, firms come into neck-and-neck position at a Poisson rate of 0.22, which results in 35% of sectors being in state 0 and 77% at two-step gap or below. This implies that there are only limited composition gains. In this light, it is not surprising that relaxing the IPR protection uniformly is not beneficial; it generates a significant disincentive effect and little benefit. Therefore, optimal IPR policy is to set full protection,  $\boldsymbol{\eta}^* = 0$ , and thus the value functions, innovation rates and industry distributions under optimal uniform IPR policy are given by the solid lines in Figures 3-5.

## 6.3 Optimal State-Dependent IPR

We next turn to our major question; whether state-dependent IPR makes a significant difference relative to the uniform IPR. In particular, we look for the combination of  $\{\eta_1, \dots, \eta_5\}$  that maximizes the welfare. The new value function, innovation rates and industry distribution are plotted in Figure 3-5 and the numerical results are shown in column 3 of Table 1.

Two features are worth noting. First, the growth rate increases noticeably relative to column 1; it is now 2.04% instead of 1.86%. Secondly and more importantly, we see the key pattern that will be present in all of our quantitative results: optimal state-dependent policy  $\{\eta_1, \dots, \eta_5\}$  provides *greater* protection to technology leaders that are further ahead. In particular, we find that the optimal policy involves  $\eta_1 = 0.71$ ,  $\eta_2 = 0.08$ , and  $\eta_3 = \eta_4 = \eta_5 = 0$ . This corresponds to very little patent protection for firms that are one step ahead of the followers. In particular, since  $\eta_1 = 0.71$  and  $x_{-1}^* = 0.12$ , in this equilibrium firms that are one step behind followers are more than *six times* as likely to catch up with the technology leader because of the expiration of the patent of the leader as they are likely to catch up

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<sup>25</sup>Most endogenous growth models imply that a significantly greater fraction of the labor force should be employed in the research sector and one needs to introduce various additional factors to reduce the profitability of research or to make entry into research more difficult. In the current model, the step-by-step nature of innovation and competition plays this role and generates a plausible allocation of workers between research and production.

because of their own successful R&D. There is then a steep increase in the protection provided to technology leaders that are two steps ahead, and  $\eta_2$  is 1/12th of  $\eta_1$ . Perhaps even more remarkably, after a technology gap of three or more steps, optimal IPR involves full protection, and patents never expire.

This pattern of greater protection for technology leaders that are further ahead may go against a naïve intuition that state-dependent IPR policy should try to boost the growth rate of the economy by bringing the industries with largest technology gaps (where leaders engage in little R&D) into neck-and-neck competition. This composition effect is present, but dominated by another, more powerful force, the *trickle-down* effect. The intuition for the trickle-down effect is as follows: by providing secure patent protection to firms that are three or more steps ahead of their rivals, optimal state-dependent IPR increases the R&D effort of leaders that are one and two steps ahead as well. This is because technology leaders that are only one or two steps ahead now face greater returns to R&D, which will not only increase their profits but also the security of their intellectual property. Mechanically, high levels of  $\eta_1$  and  $\eta_2$  reduce  $v_1$  and  $v_2$ , while high IPR protection for more advanced firms increases  $v_n$  for  $n \geq 3$ , and this increases the R&D incentives of leaders at  $n = 1$  or at  $n = 2$ .<sup>26</sup>

Providing more secure patent protection through less frequent catch-up benefits an  $n$ -step leader more than  $(n + 1)$ -step leader since the preserved profit is higher for a more advanced firm. This results in a steeper value function as illustrated in Figure 3. The slope of the value function is the key determining factor for R&D decisions and this increase in slope reflects itself in overall higher R&D effort by the leaders in Figure 4. It is also notable that state-dependent IPR introduces positive incentive effect while gaining also from the composition. Figure 5 shows that the mode of the new distribution is at  $n = 0$ . The average innovation rate is higher (as reflected on a higher growth rate,  $g = 2.04\%$ ) and the average mark-up is lower ( $C(0)$  increases by 52%). This pattern of greater R&D investments under state-dependent IPR contrasts with uniform IPR, which always reduces R&D of all firms. The possibility that imperfect state-dependent IPR protection can *increase* (rather than reduce) R&D incentives is a novel feature of our approach (and was also shown explicitly in Propositions 1 and 2 in the partial equilibrium model of Section 2).

## 6.4 Robustness

The patterns shown in Figures 3-5 and Table 1 are highly robust. In the working paper version, we repeated this entire exercise for various combinations of values of  $\gamma$  and  $\lambda$  (in particular,

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<sup>26</sup> An alternative intuition, suggested by an anonymous referee, is that when the technology gap is greater, leaders will lose more from a relaxation of IPR. However, this intuition can only be partial, since, as shown in Section 2, state-dependent relaxation of IPR in this form creates a positive incentive effect, which is central to our results (and this is independent of how much technology leaders lose as a result of the relaxation of IPR). As a result, we believe that the trickle-down of incentives is the more correct intuition for our results.

varying  $\gamma$  to  $\gamma = 0.1$  and  $\gamma = 0.6$ , and  $\lambda$  to  $\lambda = 1.01$  and  $\lambda = 1.2$ ). The overall pattern and in fact the quantitative magnitudes are remarkably similar to the baseline reported here. We do not report these robustness checks to save space (they are available upon request); instead, we focus on the results in the slow catch-up regime.

## 7 Optimal IPR Policy in the Slow Catch-up Regime

In this section, we extend our analysis to an environment in which followers close the gap between themselves and technology leaders also step by step. This environment is arguably more realistic for several industries. It also allows us to introduce different types of R&D efforts by followers and to study several different dimensions of IPR policy.

### 7.1 Value Functions

The environment is the same as in Section 3, except that we now assume that successful R&D by followers close is the gap between themselves and the technology leader by one step. We will allow for different types of R&D below. The equivalent expressions for the value functions (29)-(31) in this case are

$$\rho v_n = \max_{x_n \geq 0} \{ (1 - \lambda^{-n}) - \omega^* G(x_n) + x_n [v_{n+1} - v_n] + x_{-n}^* [v_{n-1} - v_n] + \eta_n [v_0 - v_n] \} \text{ for } n \in \mathbb{N}, \quad (46)$$

$$\rho v_0 = \max_{x_0 \geq 0} \{ -\omega^* G(x_0) + x_0 [v_1 - v_0] + x_0^* [v_{-1} - v_0] \}, \quad (47)$$

and

$$\rho v_{-n} = \max_{x_{-n} \geq 0} \{ -\omega^* G(x_{-n}) + x_{-n} [v_{-n+1} - v_{-n}] + x_n^* [v_{-n-1} - v_{-n}] + \eta_n [v_0 - v_{-n}] \} \text{ for } n \in \mathbb{N}. \quad (48)$$

These expressions are intuitive in light of those presented in Section 3, in particular, (29)-(31). The only difference from equations (29)-(31) is that, when a follower innovates, an  $n$ -step leader's value changes from  $v_n$  to  $v_{n-1}$  instead of dropping all the way to  $v_0$ , since this innovation closes the technology gap only by one step. Similarly, in this event, the follower's value changes from  $v_{-n}$  to  $v_{-n+1}$  instead of increasing all the way to  $v_0$ . The rest of the analysis mirrors that in Section 4. In particular, existence of stationary equilibria can be proved using an analogous argument to that provided in the Appendix, but the analogue of the second part of Proposition 4 no longer applies.

### 7.2 Quantitative Results

We next investigate the form of optimal IPR policy in the baseline slow catch-up regime.

### 7.2.1 Full IPR Protection

Under the slow catch-up regime, setting  $\eta_n = 0$ , that is, providing full protection via infinite patent length generates too little catch-up by the followers. Consequently, the steady-state distribution has little mass at or around the “neck-and-neck“ state ( $n = 0$ ). To generate a more plausible distribution with a non-zero share of industries in the neck-and-neck state, we instead impose  $\eta_n = 0.02$ , which implies an expected length of patent protection of 50 years (as the full protection benchmark) under slow catch-up regime.

[TABLE 2 HERE]

The first column of Table 2 reports the results under this scenario. Even with 50 years of protection, the share of industries that are neck-and-neck is only 2%, and the total share of industries that have a gap of less than two steps is only 8%. One implication of this pattern is that a relaxation of IPR policy may now be more powerful because it can affect the composition of industries, reduce the average mark-up in the economy, and perhaps have a large effect on average R&D. Therefore, this is a particularly relevant environment for investigating whether the trickle-down of incentives identified in the previous section is present and robust in different and perhaps more realistic environments.

### 7.2.2 Optimal Uniform IPR Protection

The second column of Table 2 shows optimal uniform IPR policy in this case. Consistent with Proposition 1 in Section 2, relaxing IPR protection creates a powerful disincentive effect. However, it also generates a beneficial composition effect by bringing more and more firms into neck-and-neck competition. For this reason, optimal uniform IPR policy is no longer full protection.

The results in the table show that the optimal policy reduces patent length from  $\eta = 0.02$  (average protection of 50 years) to  $\eta = 0.11$  (average protection of 9 years). This involves a lower innovation rate for technology leaders that are one-step ahead (from 1.1 to 0.15). Similarly average R&D is also reduced and the aggregate growth rate declines from 2.5% to 2.3%. However, because of the increase in the share of neck-and-neck industries (from 2% to 16%) and the increase in the total share of industries that are in the first 3 states (from 8% to 49%), the average mark-up in the economy decreases. This enables a large (19-fold) increase in initial consumption  $C(0)$  (which is the reason why this policy is optimal even though it reduces growth).

### 7.2.3 Optimal State-Dependent IPR

Once again, the most interesting case is when IPR policy is state dependent. In this case, the optimal policy not only benefits from the composition effect, but can do so without sacrificing growth (by exploiting the positive incentive and the trickle-down effects highlighted in Proposition 2 in Section 2).

The optimal state-dependent policies shown in column 3 of Table 2. Under this optimal policy, the share of the first three states increases by an additional 6 percentage point (55%) and the initial consumption further increases relative to the uniform IPR policy by 30%. More interestingly, the innovation rate of a one-step leader now increases from 0.15 to 0.51 (relative to the uniform policy case) and the growth rate increases back to 2.5%. It is noteworthy that these gains are achieved by providing stronger protections to more advanced firms—i.e., exploiting the trickle-down effect. For example, one-step leader is caught up seven times more frequently than a five-step leader due to patent expiration under the optimal state-dependent policy.

### 7.3 Compulsory Licensing

In this subsection, we introduce (compulsory) licensing. Several recent empirical papers suggest that licensing has a significant positive impact on firm innovation (e.g., Moser and Voena, 2009, Almeida and Fernandes, 2008). Consistent with these findings, we model licensing as a way of generating knowledge spillovers to licensee firms. In particular, in addition to independent R&D to proceed one step in the quality ladder, followers can also close all intervening steps by reverse-engineering the current leading-edge technology. But this is only possible by making use of the knowledge generated by the leading-edge technology, and the follower will have to pay a prespecified license fee  $\hat{\zeta}_n(t) \geq 0$  to the leader. The licensing decision of the follower  $-i$  is denoted by  $a_{-i}(j, t) = 1$  ( $a_{-i}(j, t) = 0$  corresponds to independent R&D). Throughout, we allow  $a_{-i}(j, t) \in [0, 1]$  for mathematical convenience. The fees in question are compulsory license fees imposed by policy and are state-dependent, and thus we represent them as:

$$\hat{\zeta}(t) : \mathbb{N} \rightarrow \mathbb{R}_+ \cup \{+\infty\}.$$

Note that  $\hat{\zeta}(t) \equiv \{\hat{\zeta}_1(t), \hat{\zeta}_2(t), \dots\}$  is a function of time. This is natural, since in a growing economy, license fees should not remain constant. As in (28), in what follows we assume that license fees are also scaled up by GDP, so that  $\zeta_n \equiv \hat{\zeta}_n(t)/Y(t)$ , to keep the equilibrium stationary. We discuss voluntary licensing below.



### 7.3.1 Value Functions with Compulsory Licensing

With a similar reasoning to before, relevant value functions in this case can be written as

$$\rho v_n = \max_{x_n \geq 0} \left\{ \begin{array}{l} (1 - \lambda^{-n}) - \omega^* G(x_n) + x_n [v_{n+1} - v_n] \\ + a_{-n}^* x_{-n}^* [v_0 - v_n + \zeta_n] + (1 - a_{-n}^*) x_{-n}^* [v_{n-1} - v_n] + \eta_n [v_0 - v_n] \end{array} \right\} \text{ for } n \in \mathbb{N},$$

where  $a_{-n}^*$  is the equilibrium value of licensing decision by a follower that is  $n$  steps behind, and  $\zeta_n$  is the license fee that it has to pay. The value for neck-and-neck firms remain unchanged while the values for followers becomes

$$\rho v_{-n} = \max_{x_{-n} \geq 0, a_{-n} \in [0,1]} \left\{ \begin{array}{l} -\omega^* G(x_{-n}) + a_{-n} x_{-n} [v_0 - v_{-n} - \zeta_n] \\ + (1 - a_{-n}) x_{-n} [v_{-n+1} - v_{-n}] + x_{-n}^* [v_{-n-1} - v_{-n}] + \eta_n [v_0 - v_{-n}] \end{array} \right\} \text{ for } n \in \mathbb{N},$$

Note that licensing  $a_{-n} \in [0, 1]$  is the new additional decision variable of the follower.

Full IPR protection in this case corresponds to prohibitively high licensing fees, i.e.,  $\zeta_n = \infty$  for all  $n$ , and as in the previous subsection, patent protection has expected duration of 50 years ( $\eta = 0.02$ ). Therefore, the results in this case will be identical to those reported for full protection in the previous subsection (column 1 of Table 2). This is indeed the case; these results are repeated in column 1 of Table 3 for ease of comparison with the remaining results in this table.

[TABLE 3 HERE]

### 7.3.2 Optimal Uniform IPR Protection

Uniform compulsory licensing policy now corresponds to  $\zeta_n = \zeta^* \geq 0$ . The results under the optimal choice of such uniform compulsory licensing policy are reported in the second column of Table 3. This optimal policy involves  $\zeta^* = 1.61$ , which is more than half of the surplus that a three-step follower generates from licensing,  $v_0 - v_{-3} = 2.9$ .

Since this type of licensing allows for more frequent catch-up by followers, a greater share of industries are now in tight competition. In particular, the total share of industries with one or two step gaps goes up to 66% (this number was 8% under full protection). This again corresponds to a powerful composition effect and generates a significant reduction in the average mark-up and a corresponding increase in initial consumption. However, consistent with our previous results, this type of uniform licensing again generates a significant disincentive effect on innovation. In particular, more frequent catch-up implies a shorter durations of positive profits. As a result, innovation incentives are reduced; the innovation rate of a one-step leader is now 0.43 instead of 1.1 and the average growth rate declines from 2.5% to 2.1%.

### 7.3.3 Optimal State-Dependent IPR

As in our previous exercises, the negative incentive effects of uniform relaxations of IPR protection are rectified when policy is state dependent. Optimal state-dependent policy has in

fact qualitatively very similar pattern to those reported above. Most importantly, column 3 of Table 3 shows that optimal state-dependent policy provides greater protection to technology leaders that are more advanced. For example, while a two-step leader receives a license fee of  $\zeta_2^* = 1.5$ , a five-step leader receives more than its double,  $\zeta_5^* = 3.3$ . Given this pattern, the *trickle-down effect* is again at work and generates positive innovation incentives: the innovation rate of a one-step leader increases to  $x_1^* = 0.46$  and the aggregate growth rate goes back to 2.5% from 2.1%. This positive gain is generated without sacrificing the composition effect. Under this policy, 50% of total industries operate with a technology gap less than two and the initial consumption  $C(0)$  is now even higher than under uniform policy (by 40%).

### 7.3.4 Compulsory Versus Bargained License Fees

Our analysis so far has characterized the steady-state equilibrium for a given sequence of license fees  $\zeta$ , implicitly assumed to be determined by IPR policy—i.e., these are compulsory licensing fees for intellectual property that has been patented. This, therefore, corresponds to a world in which once a company patents an innovation, the knowledge embedded in this innovation can be used by its competitors as long as they pay a prespecified license fee.

One may also wish to consider an alternative world in which license fees are determined by bilateral bargaining. To characterize the equilibrium in such a world, one must first conduct exactly the same analysis as we do in this subsection. In other words, one must characterize the equilibrium for a given sequence of license fees, and then taking the license fees agreed by other firms as given, one can consider the bargaining problem between a leader and a follower. In general, there may or may not exist feasible voluntary license fees that the follower and the leader can bargain to (such voluntary agreements may be infeasible even if compulsory licensing is beneficial, since consumers also benefit from licensing).

[FIGURE 6 HERE]

Figure 6 plots the value of licensing to a follower in an industry with an  $n$ -step gap, which is given by  $v_0 - v_{-n+1}$ , and the absolute value of the loss to the leader in the same industry,  $|v_{n-1} - v_0|$ .<sup>27</sup> The overall pattern is that the latter number is unambiguously greater than the former, and this implies that voluntary licensing cannot emerge in this environment. Therefore, compulsory licensing plays a useful role that bilateral licensing agreements between leaders and followers could not achieve, and is thus a useful policy tool. In addition, our analysis shows that compulsory licensing will be useful for welfare precisely when it is state dependent.

<sup>27</sup>Without licensing, the change in follower's value is  $v_{-n+1} - v_{-n}$ . Since licensing takes the follower to  $v_0$ , the change due to licensing is  $v_0 - v_{-n+1}$ . Similar reasoning applies to the leader's loss. All of these values refer to the full protection quantities from Tables 2-5. Note also that Figure 1 has no value for  $n = 0, 1$  since there is no possibility of "beneficial licensing" when the industry is neck-and-neck or has a one-step technology gap.

## 7.4 Leapfrogging and Infringement under Slow Catch-up

Finally, we allow the follower to engage in frontier R&D and “leapfrog” the technology leader. This exercise is useful for two reasons. First, the models analyzed so far do not allow R&D by followers to directly contribute to aggregate growth. One might conjecture that this feature strengthens the *trickle-down* effect. Second, frontier R&D and leapfrogging by followers will allow us to introduce another relevant and important dimension of IPR policy, patent infringement fees.

Followers can now undertake two types of R&D. The first, which is what we have focused on so far, is *catch-up R&D*, corresponding to R&D directed at discovering an *alternative* way of performing the same task as the current leading-edge technology. Catch-up R&D improves the technology of the follower by one step as before. The alternative, *frontier R&D*, involves followers improving the current leading-edge technology. If this type of R&D succeeds, the follower will have improved the leading-edge technology. However, following such an event, the follower will be judged (e.g., by courts) to have infringed the patent of technology leader with probability  $\tau \in (0, 1)$  and will be required to pay a prespecified infringement fee  $\hat{\vartheta}_n \geq 0$  to the leader. The infringement fees are also state dependent and are represented by:

$$\hat{\vartheta}(t) : \mathbb{N} \rightarrow \mathbb{R}_+ \cup \{+\infty\},$$

and we again adopt the normalization  $\vartheta_n \equiv \hat{\vartheta}_n(t)/Y(t)$ , and denote the Poisson arrival rate of innovation by catch-up R&D and frontier R&D by  $x_n^c$  and  $x_n^f$ , respectively. The value of an  $n$ -step leader in this case then takes the following form:

$$\rho v_n = \max_{x_n \geq 0} \left\{ \begin{array}{l} (1 - \lambda^{-n}) - \omega^* G(x_n) + x_n [v_{n+1} - v_n] \\ + x_{-n}^{c*} [v_{n-1} - v_n] + x_{-n}^{f*} [v_{-1} - v_n + \tau \vartheta_n] + \eta_n [v_0 - v_n] \end{array} \right\} \text{ for } n \in \mathbb{N},$$

The main difference from the value functions so far is that the follower has two different arrival rates of innovation. If the follower is successful with frontier R&D, the current leader falls one step behind the follower. However, in this event, with probability  $\tau$ , it receives an infringement fee of  $\vartheta_n$ . With a similar reasoning, the value of an  $n$ -step follower now becomes:

$$\rho v_{-n} = \max_{x_{-n}^c \geq 0, x_{-n}^f \geq 0} \left\{ \begin{array}{l} -\omega^* G(x_{-n}) + x_{-n}^c [v_{-n+1} - v_{-n}] + x_{-n}^f [v_1 - v_{-n} - \tau \vartheta_n] \\ + x_{-n}^{c*} [v_{-n-1} - v_{-n}] + \eta_n [v_0 - v_{-n}] \end{array} \right\} \text{ for } n \in \mathbb{N}.$$

The value of a neck-and-neck firm is unchanged.

Our quantitative analysis now requires an empirical estimate for  $\tau$ . Lanjouw and Schankerman (2001) report that around 10% of the US utility patents are filed for infringement. We therefore set  $\tau = 0.1$ .

Note also that since the followers now improve the aggregate technology frontier through

frontier R&D, the aggregate growth rate becomes

$$g^* = \ln \lambda \left[ 2\mu_0^* x_0^* + \sum_{n=1}^{\infty} \mu_n^* (x_n^* + x_{-n}^{f*}) \right]. \quad (49)$$

Full protection in this case corresponds to infinite patent infringement fees, i.e.,  $\vartheta_n = \infty$ , and given the same parameter choices as before, will be identical to column 1 of Table 2. We repeat these results in column 1 of Table 4 for ease of comparison with the rest of the table.

[TABLE 4 HERE]

#### 7.4.1 Optimal Uniform IPR Protection

In the uniform policy case, we set  $\vartheta_n = \vartheta \geq 0$ . Column 2 of Table 4 shows that the optimal uniform policy in this case is  $\vartheta^* = 14$ . Recall that when a follower undertakes frontier innovation, the probability that it will have to make the payment  $\vartheta^*$  is  $\tau = 0.1$ . Therefore the expected infringement payment is  $\tau \times \vartheta^* = 1.4$  which is more than half of the surplus that a three-step follower generates out of leapfrogging,  $v_1 - v_{-3} = 2.7$ .

Column 2 also shows that under this policy followers undertake more frontier R&D ( $x_{-1}^{*f} = 0.23$ ) than catch-up R&D ( $x_{-1}^{*c} = 0.15$ ). Parallel to the previous uniform policies, the shorter duration of monopoly position resulting from innovation reduces innovation incentives. For example, one-step leaders now innovate at the rate 0.3 instead of 1.1. However, despite this disincentive effect, the growth rate increases slightly because leapfrogging allows followers to directly contribute to aggregate growth, as shown by equation (49).

Column 2 also shows that the share of industries in one-step gap is now much larger,  $\mu_1 = 0.42$ . This is because leapfrogging puts the follower one-step ahead of the previous leader. Thanks to this effect, optimal uniform IPR protection achieves lower average mark-up and higher initial consumption as well as higher growth.

#### 7.4.2 Optimal State-Dependent IPR

State-dependent IPR policy once again exploits the trickle-down effect and creates positive incentive effects on innovation. The form of state-dependent policy is the same as before: technologically more advanced leaders receive more protection in the form of higher infringement fees. While a two-step leader receives  $\vartheta_1^* = 18.1$  in case of infringement, a five-step leader receives more than double of this fee,  $\vartheta_5^* = 43.7$ . In expectation, a three-step follower pays almost 3/4th of the surplus that it generates from leapfrogging ( $\tau \times \vartheta_3^* = 3.1$  versus  $v_1 - v_{-3} = 4.1$ ). As a result of this pattern, state-dependent policy not only generates a greater welfare gain in terms of the initial consumption ( $C(0)$  is now approximately twice the level under the optimal uniform policy), but also by exploiting the trickle-down effect, it increases the equilibrium growth rate by an additional 0.5 percentage point relative to the uniform policy.

## 7.5 Patent Length, Compulsory Licensing and Infringements Fees under Slow Catch-up

In this subsection, we investigate the slow catch-up environment when all three IPR policies are simultaneously present. We do not repeat the value functions to save space.

[TABLE 5 HERE]

Table 5 first shows our benchmark full protection economy in the first column. The second and third columns report the optimal uniform and state-dependent policies with all three types of policies present. The results are very similar to those reported in subsection 7.4 (with only leapfrogging), except that the patent lengths are now set to infinity ( $\eta_n = 0$ ). As a result, the optimal IPR policy in this case involves infinitely long patents with prohibitively high compulsory license fees. The only dimension in which IPR protection is not full is due to moderate infringement fees, which permit followers to undertake frontier R&D and leapfrog technology leaders.

Most importantly for our focus, column 3 again shows the benefits of state-dependent IPR policy. This policy again provides greater protection for technology leaders and exploits the trickle-down effect. As a result, initial consumption is approximately twice the level under uniform IPR and innovation incentives are stronger, increasing the long-run growth rate from 2.5% to 3.3%.

## 7.6 Robustness Checks

Table 6 shows that the patterns documented in Table 5, particularly the gains from state-dependent policy and the major role played by the trickle-down effect, are robust for reasonable changes in parameter values.

[TABLE 6 HERE]

In this table, in each column we change one of the two parameters  $\lambda$  and  $\gamma$  (increasing or reducing  $\lambda$  to 1.2 or 1.01, and increasing or reducing  $\gamma$  to 0.6 or 0.1). In each case, we also change the parameter  $B$  in equation (45) to ensure the growth rate of the benchmark economy with full IPR protection is the same as in our initial baseline economy,  $g^* = 1.86\%$ .

To save space, we only show the results from the optimal state-dependent policies. Table 6 shows that the qualitative patterns in Table 5 are relatively robust. In all cases, optimal state-dependent IPR is shaped by the trickle-down effect. In all of the various parameterizations we have considered (and with different combinations of policies), there is little protection provided to technology leaders that are one-step ahead, but IPR protection grows as the technology gap increases. This is the typical pattern implied by the trickle-down effect. In addition, in all

cases when all three forms of policy are incorporated, optimal IPR policy provides patents of infinite duration and prohibitively high compulsory licensing fees, but deviates from full IPR protection by imposing moderate levels of infringement fees. Most importantly for us, in all cases these infringement fees are state dependent and provide greater protection to technologically more advanced leaders.

## 8 Conclusions

In this paper, we developed a general equilibrium framework to investigate the impact of the extent and form of IPR policy on economic growth and welfare. The two major questions we focused on are whether licensing, which allows followers to build on the leading-edge technology in return of a license fee, has a major impact on the equilibrium growth rate and whether the same degree of patent protection should be given to companies that are further ahead of their competitors as those that are technologically close to their rivals.

In our model economy, firms engage in cumulative (step-by-step) innovation. Leaders innovate in order to widen the technology gap between themselves and the followers, which enables them to charge higher markups. Followers innovate to catch up with or surpass the technology leaders in their industry (by undertaking “frontier R&D”), and can also license the technology of leaders. IPR policy regulates the length of patents, whether licensing is possible and the size of patent infringement fees.

We provided existence and characterization results, and a quantitative analysis of the form of “optimal” (welfare-maximizing) IPR policy. Our results highlight the importance of the trickle-down effect and state-dependent policy. In particular, in several different environments and under different parameter values, we consistently found that state-dependent IPR also leads to a significant improvement in welfare and also typically in the equilibrium growth rate. In our benchmark parameterization, for example, optimal IPR policy increases the growth rate of the economy from 1.86% to 2.04%, and does so while also significantly increasing initial consumption (and in fact reducing the overall amount of resources allocated to the R&D sector). Most interestingly, we found that in all cases, state-dependent policy involves providing greater protection to firms that are further ahead of their rivals than those that are technologically close to their competitors. This result goes against a naïve intuition, which would involve providing less protection to technologically more advanced firms in order to exploit a composition effect (bringing firms that are furthest apart into “neck and neck” competition to both reduce markups and increase R&D which results from tight competition). This naïve intuition is not correct because of the powerful trickle-down effect counteracting it. The trickle-down effect implies that providing greater protection to sufficiently advanced technology leaders not only increases their R&D efforts but also raises the R&D efforts of all technology leaders that are

less advanced than this level. This is because the reward to innovation now includes the greater protection that they will receive once they reach this higher level of technology. Our results suggest that the trickle-down effect is powerful both with and without licensing, and its form and magnitude are relatively insensitive to the exact parameter values used in the quantitative investigation.

Returning to the discussion of the Microsoft and other anti-trust cases discussed in the Introduction, our analysis suggests that in addition to the reasoning based on the composition effect, which may have been important in guiding current policy, the trickle-down effect should also be factored into modern policy analysis. The trickle-down effect implies that reducing the IPR protection of technologically advanced firms may create powerful disincentive effects for all firms, consistent with the pattern shown in Figures 1 and 2. It suggests, instead, that deviations from full IPR protection should take a state-dependent form.

It should be emphasized, however, that our objective in this paper has not been to derive practical policy prescriptions. Our model is simplified in many dimensions and the exact effects of different policies implied by our model exclude a whole host of important factors and ignore potential limitations on the form and complexity of IPR policies. Nevertheless, we believe that our results demonstrate a range of robust and new effects that should be part of the calculus of IPR and competition policy, and should be further investigated both empirically and theoretically in future work.

More generally, the analysis in this paper suggests that a move to a richer menu of IPR policies, in particular, a move towards optimal state-dependent policies, may significantly increase innovation, economic growth and welfare. The results also show that the form of optimal IPR policy may depend on the industry structure (and the technology of catch-up within the industry). The next step in this line of research should be to investigate the robustness of these effects in different models of industry dynamics. It would also be useful to study whether the relationship between the form of optimal IPR policy and industry structure suggested by our analysis also applies when variation in industry structure has other sources (for example, differences in the extent of fixed costs or demand structure causing differential gaps between technology leaders and followers across industries). The most important area for future work is a detailed empirical investigation of the form of optimal IPR policy, using both better estimates of the effects of IPR policy on innovation rates and also structural models that would enable the evaluation of the effects of different policies on equilibrium growth and welfare. We hope that the theoretical framework presented in this paper will be useful in developing models that can be estimated in future work.

## Appendix: Proofs

### Derivation of Optimal R&D Decisions in the Partial Equilibrium Model

Since the costs are linear, optimal R&D decisions imply that in equilibrium,

$$v_{n+1} - v_n = \phi, \text{ for each } n \in \{-2, -1, 0, 1\}. \quad (50)$$

Combining this equation what (1) gives the value of a two-step follower as

$$v_{-2} = \frac{\pi_{-2} + 2\phi\eta_2}{r}.$$

The previous equation, together with (50), implies

$$v_n = v_{-2} + \phi(n+2) = \frac{\pi_{-2} + 2\phi\eta_2}{r} + \phi(n+2) \text{ for } n \in \{-1, 0, 1, 2\}. \quad (51)$$

Now we can use the value of  $v_2$  to solve for  $x_{-2}^*$  from equation (1). Similarly, combining (51) with (2) gives the value of  $x_{-1}^*$ ; (51) with (3) gives  $x_0^*$ . Finally, combining (51) with (4) gives the equilibrium value of  $x_1^*$ . It can be verified that for  $\eta_1$  and  $\eta_2$  sufficiently close to zero, all flow rates are strictly positive. ■

### Derivation of Equation (27)

Fix the equilibrium R&D policies of other firms,  $x_{-n}^*(t)$ , the equilibrium interest and wage rates,  $r^*(t)$  and  $w^*(t)$ , and equilibrium profits  $\{\Pi_n^*(t)\}_{n=1}^\infty$ . Then the value of the firm that is  $n$  steps ahead at time  $t$  can be written as:

$$V_n(t) = \max_{x_n(t)} \{[\Pi_n^*(t) - w^*(t)G(x_n(t))] \Delta t + o(\Delta t) \quad (52)$$

$$+ \exp(-r^*(t + \Delta t) \Delta t) \left[ \begin{array}{l} (x_n(t) \Delta t + o(\Delta t)) V_{n+1}(t + \Delta t) \\ + (\eta_n \Delta t + x_{-n}^*(t) \Delta t + o(\Delta t)) V_0(t + \Delta t) \\ + (1 - x_n(t) \Delta t - \eta_n \Delta t - x_{-n}^*(t) \Delta t - o(\Delta t)) V_n(t + \Delta t) \end{array} \right] \}.$$

The first part of this expression is the flow profits minus R&D expenditures during a time interval of length  $\Delta t$ . The second part is the continuation value after this interval has elapsed.  $V_{n+1}(t)$  and  $V_0(t)$  are defined as net present discounted values for a leader that is  $n+1$  steps ahead and a firm in an industry that is neck-and-neck (i.e.,  $n=0$ ). The second part of the expression uses the fact that in a short time interval  $\Delta t$ , the probability of innovation by the leader is  $x_n(t) \Delta t + o(\Delta t)$ , where  $o(\Delta t)$  again denotes second-order terms. This explains the first line of the continuation value. For the remainder of the continuation value, note that the probability that the follower will catch up with the leader is  $[\eta_n + x_{-n}^*(t)] \Delta t + o(\Delta t)$ . Finally, the last line applies when no R&D effort is successful and patents continue to be enforced, so that the technology gap remains at  $n$  steps. Now, subtract  $V_n(t)$  from both sides, divide everything by  $\Delta t$ , and take the limit as  $\Delta t \rightarrow 0$  to obtain (27). ■

### Proof of Proposition 3

Equations (24) and (26) imply

$$Y(t) = \frac{w(t)}{\omega(t)} = \frac{Q(t) \lambda^{-\sum_{n=0}^\infty n\mu_n^*(t)}}{\omega(t)}.$$



Since  $\omega(t) = \omega^*$  and  $\{\mu_n^*\}_{n=0}^\infty$  are constant in steady state,  $Y(t)$  grows at the same rate as  $Q(t)$ . Therefore,

$$g^* = \lim_{\Delta t \rightarrow 0} \frac{\ln Q(t + \Delta t) - \ln Q(t)}{\Delta t}.$$

Now note the following: during an interval of length  $\Delta t$  (i) in the fraction  $\mu_n^*$  of the industries with technology gap  $n \geq 1$  the leaders innovate at a rate  $x_n^* \Delta t + o(\Delta t)$ ; (ii) in the fraction  $\mu_0^*$  of the industries with technology gap of  $n = 0$ , both firms innovate, so that the total innovation rate is  $2x_0^* \Delta t + o(\Delta t)$ ; and (iii) each innovation increase productivity by a factor  $\lambda$ . Combining these observations, we have

$$\ln Q(t + \Delta t) = \ln Q(t) + \ln \lambda \left[ 2\mu_0^* x_0^* \Delta t + \sum_{n=1}^{\infty} \mu_n^* x_n^* \Delta t + o(\Delta t) \right].$$

Subtracting  $\ln Q(t)$ , dividing by  $\Delta t$  and taking the limit  $\Delta t \rightarrow 0$  gives (39). ■

## Proof of Proposition 4

We prove this proposition in four parts. (1) Existence of a steady-state equilibrium. (2) Properties of the sequence of value functions. (3) Properties of the sequence of R&D decisions. (4) Uniqueness of an invariant distribution given R&D policies.

### Part 1: Existence of a Steady-State Equilibrium.

First, note that each  $x_n$  belongs to a compact interval  $[0, \bar{x}]$ , where  $\bar{x}$  is the maximal flow rate of innovation defined in (16) above. Now fix a labor share  $\tilde{\omega} \in [0, 1]$  and a sequence  $\langle \tilde{\mathbf{x}} \rangle$  of (Markovian) steady-state strategies for all other firms in the economy, and consider the dynamic optimization problem of a single firm. Our first result characterizes this problem and shows that given some  $\mathbf{z} \equiv \langle \tilde{\omega}, \tilde{\mathbf{x}} \rangle$ , the value function of an individual firm is uniquely determined, while its optimal R&D choices are given by a convex-valued correspondence. In what follows, we denote sets and correspondences by uppercase letters and refer to their elements by lowercase letters, e.g.,  $x_n(\mathbf{z}) \in X_n[\mathbf{z}]$ .

**Lemma 1** *Consider a uniform IPR policy  $\eta^{uni}$ , and suppose that the labor share and the R&D policies of all other firms are given by  $\mathbf{z} = \langle \tilde{\omega}, \tilde{\mathbf{x}} \rangle$ . Then the dynamic optimization problem of an individual firm leads to a unique value function  $\mathbf{v}[\mathbf{z}] : \{-1\} \cup \mathbb{Z}_+ \rightarrow \mathbb{R}_+$  and optimal R&D policy  $\hat{\mathbf{X}}[\mathbf{z}] : \{-1\} \cup \mathbb{Z}_+ \rightrightarrows [0, \bar{x}]$  is compact and convex-valued for each  $\mathbf{z} \in \mathbf{Z}$  and upper hemi-continuous in  $\mathbf{z}$  (where  $\mathbf{v}[\mathbf{z}] \equiv \{v_n[\mathbf{z}]\}_{n=-1}^\infty$  and  $\hat{\mathbf{X}}[\mathbf{z}] \equiv \{\hat{X}_n[\mathbf{z}]\}_{n=-1}^\infty$ ).*

**Proof.** Fix  $\mathbf{z} = \langle \tilde{\omega}, \{\tilde{x}_n\}_{n=-1}^\infty \rangle$ , and consider the optimization problem of a representative firm, written recursively as:

$$\begin{aligned} \rho v_n &= \max_{x_n \in [0, \bar{x}]} \left\{ (1 - \lambda^{-n}) - \tilde{\omega} G(x_n) + x_n [v_{n+1} - v_n] + \tilde{x}_{-1} [v_0 - v_n] + \eta [v_0 - v_n] \right\} \text{ for } n \in \mathbb{N} \\ \rho v_0 &= \max_{x_0 \in [0, \bar{x}]} \left\{ -\tilde{\omega} G(x_0) + x_0 [v_1 - v_0] + \tilde{x}_0 [v_{-1} - v_0] \right\} \\ \rho v_{-1} &= \max_{x_{-1} \in [0, \bar{x}]} \left\{ -\tilde{\omega} G(x_0) + x_{-1} [v_0 - v_{-1}] + \eta [v_0 - v_{-1}] \right\}. \end{aligned}$$

We now transform this dynamic optimization problem into a form that can be represented as a contraction mapping using the method of ‘‘uniformization’’ (see, for example, Ross, 1996, Chapter 5). Let  $\tilde{\xi} = \{\tilde{x}_n\}_{n=-1}^\infty$  and  $p_{n,n'}(\xi | \tilde{\xi})$  be the probability that the next state will be  $n'$  starting with state  $n$  when the firm in question chooses policies  $\xi \equiv \{x_n\}_{n=-1}^\infty$  and the R&D policy of other firms is given by  $\tilde{\xi}$ . Using the fact that, because of uniform IPR policy,  $x_{-n} = x_{-1}$  for all  $n \in \mathbb{N}$ , these transition probabilities can be written as:

$p_{-1,0}(\xi   \tilde{\xi}) = \frac{x_{-1} + \eta}{x_n + x_{-1} + \eta}$	$p_{n,0}(\xi   \tilde{\xi}) = \frac{\tilde{x}_{-1} + \eta}{x_n + \tilde{x}_{-1} + \eta}$
$p_{0,-1}(\xi   \tilde{\xi}) = \frac{\tilde{x}_0}{x_0 + \tilde{x}_0}$	$p_{n,n+1}(\xi   \tilde{\xi}) = \frac{x_n}{x_n + \tilde{x}_{-1} + \eta}$
$p_{0,1}(\xi   \tilde{\xi}) = \frac{x_0}{x_0 + \tilde{x}_0}$	

Uniformization involves adding fictitious transitions from a state into itself, which do not change the value of the program, but allow us to represent the optimization problem as a contraction. For this purpose, define the transition rates  $\psi_n$  as

$$\psi_n(\xi | \tilde{\xi}) = \begin{cases} x_n + x_{-1} + \eta & \text{for } n \in \{1, 2, \dots\} \\ x_{-1} + \eta & \text{for } n = -1 \\ 2x_n & \text{for } n = 0 \end{cases}.$$

These transition rates are finite since  $\psi_n(\xi | \tilde{\xi}) \leq \psi \equiv 2\bar{x} + \eta < \infty$  for all  $n$ , where  $\bar{x}$  is the maximal flow rate of innovation defined in (16) in the text (both  $\bar{x}$  and  $\eta$  are finite by assumption).

Now following equation (5.8.3) in Ross (1996), we can use these transition rates and define the new transition probabilities (including the fictitious transitions from a state to itself) as:

$$\tilde{p}_{n,n'}(\xi | \tilde{\xi}) = \begin{cases} \frac{\psi_n(\xi | \tilde{\xi})}{\psi} p_{n,n'}(\xi | \tilde{\xi}) & \text{if } n \neq n' \\ 1 - \frac{\psi_n(\xi | \tilde{\xi})}{\psi} & \text{if } n = n' \end{cases}.$$

This yields equivalent transition probabilities

$\tilde{p}_{-1,-1}(\xi   \tilde{\xi}) = 1 - \frac{x_{-1} + \eta}{2\bar{x} + \eta}$	$\tilde{p}_{-1,0}(\xi   \tilde{\xi}) = \frac{x_{-1} + \eta}{2\bar{x} + \eta}$	$\tilde{p}_{0,1}(\xi   \tilde{\xi}) = \frac{x_0}{2\bar{x} + \eta}$
$\tilde{p}_{0,-1}(\xi   \tilde{\xi}) = \frac{\bar{x}_0}{2\bar{x} + \eta}$	$\tilde{p}_{0,0}(\xi   \tilde{\xi}) = 1 - \frac{x_0 + \bar{x}_0}{2\bar{x} + \eta}$	$\tilde{p}_{n,n+1}(\xi   \tilde{\xi}) = \frac{x_n}{2\bar{x} + \eta}$
$\tilde{p}_{n,0}(\xi   \tilde{\xi}) = \frac{\bar{x}_{-1} + \eta}{2\bar{x} + \eta}$	$\tilde{p}_{n,n}(\xi   \tilde{\xi}) = 1 - \frac{x_n + \bar{x}_{-1} + \eta}{2\bar{x} + \eta}$	

and also defines an effective discount factor  $\beta$  given by

$$\beta \equiv \frac{\psi}{\rho + \psi} = \frac{2\bar{x} + \eta}{\rho + 2\bar{x} + \eta}.$$

Also let the per period return function (profit net of R&D expenditures) be

$$\hat{\Pi}_n(x_n) = \begin{cases} \frac{1 - \lambda^{-n} - \tilde{\omega}G(x_n)}{\rho + 2\bar{x} + \eta} & \text{if } n \geq 1 \\ \frac{-\tilde{\omega}G(x_n)}{\rho + 2\bar{x} + \eta} & \text{otherwise} \end{cases}. \quad (53)$$

Using these transformations, the dynamic optimization problem can be written as:

$$\begin{aligned} v_n &= \max_{x_n} \left\{ \hat{\Pi}_n(x_n) + \beta \sum_{n'} \tilde{p}_{n,n'}(\xi_n | \tilde{\xi}) \tilde{v}_{n'} \right\}, \text{ for all } n \in \mathbb{Z}, \\ &\equiv T\tilde{v}_n, \text{ for all } n \in \mathbb{Z}. \end{aligned} \quad (54)$$

where  $\mathbf{v} \equiv \{v_n\}_{n=-1}^{\infty}$  and the second line defines the operator  $T$ , mapping from the space of functions  $\mathbf{V} \equiv \{\mathbf{v} : \{-1\} \cup \mathbb{Z}_+ \rightarrow \mathbb{R}_+\}$  into itself.  $T$  is clearly a contraction mapping. The innovation rates  $\{\tilde{x}_n\}_{n=-1}^{\infty}$  are upper hemi-continuous therefore  $\tilde{\mathbf{p}} : \{-1\} \cup \mathbb{Z}_+ \times \{-1\} \cup \mathbb{Z}_+ \rightrightarrows [0, 1]$  is upper-hemicontinuous and forms a multivalued stochastic kernel. Then Proposition 2.2 in Blume (1982) implies that  $\tilde{\mathbf{p}}$  has the Feller property. Thus, for given  $\mathbf{z} = \langle \tilde{\omega}, \{\tilde{x}_n\}_{n=-1}^{\infty} \rangle$ ,  $T$  possesses a unique fixed point  $\mathbf{v}^* \equiv \{v_n^*\}_{n=-1}^{\infty}$  (e.g., Stokey, Lucas and Prescott, 1989).

Moreover,  $x_n \in [0, \bar{x}]$ , and  $v_n$  for each  $n = -1, 0, 1, \dots$  given by the right-hand side of (54) is continuous in  $x_n$ , so Berge's Maximum Theorem (Aliprantis and Border, 1999, Theorem 16.31, p. 539) implies that the set of maximizers  $\{\hat{X}_n\}_{n=-1}^{\infty}$  exists, is nonempty and compact-valued for each  $\mathbf{z}$  and is upper hemi-continuous in  $\mathbf{z} = \langle \tilde{\omega}, \{\tilde{x}_n\}_{n=-1}^{\infty} \rangle$ . Moreover, concavity of  $v_n$  in  $x_n$  for each  $n = -1, 0, 1, \dots$  implies that  $\{\hat{X}_n\}_{n=-1}^{\infty}$  is also convex-valued for each  $\mathbf{z}$ , completing the proof. ■

Now let us start with an arbitrary  $\mathbf{z} \equiv \langle \tilde{\omega}, \tilde{\mathbf{x}} \rangle \in \mathbf{Z} \equiv [0, 1] \times [0, \bar{x}]^{\infty}$ . From Lemma 1, this  $\mathbf{z}$  is mapped into optimal R&D decision sets  $\hat{\mathbf{X}}[\mathbf{z}]$ , where  $\hat{x}_n[\mathbf{z}] \in \hat{X}_n[\mathbf{z}]$ . From R&D policies  $\tilde{\mathbf{x}}$ , we calculate

$\boldsymbol{\mu}[\tilde{\mathbf{x}}] \equiv \{\mu_n[\tilde{\mathbf{x}}]\}_{n=0}^{\infty}$  using equations (35), (36) and (37). Then we can rewrite the labor market clearing condition (38) as

$$\begin{aligned}\omega &= \min \left\{ \sum_{n=0}^{\infty} \mu_n \left[ \frac{1}{\lambda^n} + G(\hat{x}_n)\tilde{\omega} + G(\hat{x}_{-n}) \right] \tilde{\omega}; 1 \right\}, \\ &\equiv \varphi(\tilde{\omega}, \tilde{\mathbf{x}})\end{aligned}\tag{55}$$

where due to uniform IPR,  $\hat{x}_{-n} = \hat{x}_{-1}$  for all  $n > 0$ . Next, define the mapping (correspondence)

$$\Phi[\mathbf{z}] \equiv \left( \varphi(\mathbf{z}), \hat{\mathbf{X}}[\mathbf{z}] \right)$$

, which maps  $\mathbf{Z}$  into itself, that is,

$$\Phi: \mathbf{Z} \rightrightarrows \mathbf{Z}.\tag{56}$$

That  $\Phi$  maps  $\mathbf{Z}$  into itself follows since  $\mathbf{z} \in \mathbf{Z}$  consists of  $\tilde{\mathbf{x}} \in [0, \bar{x}]^{\infty}$  and  $\tilde{\omega} \in [0, 1]$ , and the image of  $\mathbf{z}$  under  $\Phi$  consists of  $\hat{\mathbf{x}} \in [0, \bar{x}]^{\infty}$ , and moreover, (55) is clearly in  $[0, 1]$  (since the right-hand side is nonnegative and bounded above by 1). Finally, from Lemma 1,  $\hat{X}_n[\mathbf{z}]$  is compact and convex-valued for each  $\mathbf{z} \in \mathbf{Z}$ , and also upper hemi-continuous in  $\mathbf{z}$ , and  $\varphi$  is continuous. Using this construction, we can establish the existence of a steady-state equilibrium as follows.

We first show that the mapping  $\Phi: \mathbf{Z} \rightrightarrows \mathbf{Z}$  constructed in (56) has a fixed point, and then establish that when  $G'^{-1}((1 - \lambda^{-1}) / (\rho + \eta)) > 0$  this fixed point corresponds to a steady state with  $\omega^* < 1$ . First, it has already been established that  $\Phi$  maps  $\mathbf{Z}$  into itself. We next show that  $\mathbf{Z}$  is compact in the product topology and is a subset of a locally convex Hausdorff space. The first part follows from the fact that  $\mathbf{Z}$  can be written as the Cartesian product of compact subsets,  $\mathbf{Z} = [0, 1] \times \prod_{n=-1}^{\infty} [0, \bar{x}]$ . Then by Tychonoff's Theorem (e.g., Aliprantis and Border, 1999, Theorem 2.57, p. 52; Kelley, 1955, p. 143),  $\mathbf{Z}$  is compact in the product topology. Moreover,  $\mathbf{Z}$  is clearly nonempty and also convex, since for any  $\mathbf{z}, \mathbf{z}' \in \mathbf{Z}$  and  $\lambda \in [0, 1]$ , we have  $\lambda\mathbf{z} + (1 - \lambda)\mathbf{z}' \in \mathbf{Z}$ . Finally, since  $\mathbf{Z}$  is a product of intervals on the real line, it is a subset of a locally convex Hausdorff space (see Aliprantis and Border, 1999, Lemma 5.54, p. 192).

Next,  $\varphi$  is a continuous function from  $\mathbf{Z}$  into  $[0, 1]$  and from Lemma 1,  $\hat{X}_n(\mathbf{z})$  for  $n \in \{-1\} \cup \mathbb{Z}_+$  is upper hemi-continuous in  $\mathbf{z}$ . Consequently,  $\Phi \equiv \langle \varphi[\mathbf{z}], \hat{\mathbf{X}}[\mathbf{z}] \rangle$  has closed graph in  $\mathbf{z}$  in the product topology. Moreover, each one of  $\varphi(\mathbf{z})$  and  $\hat{X}_n(\mathbf{z})$  for  $n = -1, 0, \dots$  is nonempty, compact and convex-valued. Therefore, the image of the mapping  $\Phi$  is nonempty, compact and convex-valued for each  $\mathbf{z} \in \mathbf{Z}$ . The Kakutani-Fan-Glicksberg Fixed Point Theorem implies that if the function  $\Phi$  maps a convex, compact and nonempty subset of a locally convex Hausdorff space into itself and has closed graph and is nonempty, compact and convex-valued  $\mathbf{z}$ , then it possesses a fixed point  $\mathbf{z}^* \in \Phi(\mathbf{z}^*)$  (see Aliprantis and Border, 1999, Theorem 16.50 and Corollary 16.51, p. 549-550). This establishes the existence of a fixed point  $\mathbf{z}^*$  of  $\Phi$ .

To complete the proof, we need to show that the fixed point,  $\mathbf{z}^*$ , corresponds to a steady state equilibrium. First, since  $\hat{x}_n(\omega^*, \{x_n^*\}_{n=-1}^{\infty}) = x_n^*$  for  $n \in \{-1\} \cup \mathbb{Z}_+$ , we have that given a labor share of  $\omega^*$ ,  $\{x_n^*\}_{n=-1}^{\infty}$  constitutes an R&D policy vector that is best response to itself, as required by steady-state equilibrium (Definition 3). Next, we need to prove that the implied labor share  $\omega^*$  leads to labor market clearing. This follows from the fact that the fixed point involves  $\omega^* < 1$ , since in this case (55) will have an interior solution, ensuring labor market clearing. Suppose, to obtain a contradiction, that  $\omega^* = 1$ . Then, as noted in the text, we must have  $\mu_0^* = 1$ . From (35), (36) and (37), this implies  $x_n^* = 0$  for  $n \in \{-1\} \cup \mathbb{Z}_+$ . However, we have shown above that this is not possible when  $G'^{-1}((1 - \lambda^{-1}) / (\rho + \eta)) > 0$ . Consequently, (55) cannot be satisfied at  $\omega^* = 1$ , implying that  $\omega^* < 1$ . When  $\omega^* < 1$ , the labor market clearing condition (38) is satisfied at  $\omega^*$  as an equality, so  $\omega^*$  is an equilibrium given  $\{x_n^*\}_{n=-1}^{\infty}$ , and thus  $\mathbf{z}^* = (\omega^*, \{x_n^*\}_{n=-1}^{\infty})$  is a steady-state equilibrium as desired.

Finally, if  $\eta > 0$ , then (37) implies that  $\mu_0^* > 0$ . Since  $x_0^* > 0$ , equation (39) implies  $g^* > 0$ . Alternatively, if  $x_{-1}^* > 0$ , then  $g^* > 0$  follows from (39). This completes the proof of the existence of a steady-state equilibrium with positive growth.

## Part 2: Properties of the Sequence of Value Functions.

Let  $\{x_n\}_{n=-1}^\infty$  be the R&D decisions of the firm and  $\{v_n\}_{n=-1}^\infty$  be the sequence of values, taking the decisions of other firms and the industry distributions,  $\{x_n^*\}_{n=-1}^\infty$ ,  $\{\mu_n^*\}_{n=-1}^\infty$ ,  $\omega^*$  and  $g$ , as given. By choosing  $x_n = 0$  for all  $n \geq -1$ , the firm guarantees  $v_n \geq 0$  for all  $n \geq -1$ . Moreover, since flow profit satisfy  $\pi_n \leq 1$  for all  $n \geq -1$ ,  $v_n \leq 1/\rho$  for all  $n \geq -1$ , establishing that  $\{v_n\}_{n=-1}^\infty$  is a bounded sequence, with  $v_n \in [0, 1/\rho]$  for all  $n \geq -1$ .

*Proof of  $v_1 > v_0$  :* Suppose, first,  $v_1 \leq v_0$ , then (34) implies  $x_0^* = 0$ , and by the symmetry of the problem in equilibrium (30) implies  $v_0 = v_1 = 0$ . As a result, from (33) we obtain  $x_{-1}^* = 0$ . Equation (29) implies that when  $x_{-1}^* = 0$ ,  $v_1 \geq (1 - \lambda^{-1}) / (\rho + \eta) > 0$ , yielding a contradiction and proving that  $v_1 > v_0$ .  $\square$

*Proof of  $v_{-1} \leq v_0$  :* Suppose, to obtain a contradiction, that  $v_{-1} > v_0$ .

If  $v_1 \leq v_0$ , (33) yields  $x_{-1}^* = 0$ . This implies  $v_{-1} = \eta v_0 / (\rho + \eta)$ , which contradicts  $v_{-1} > v_0$  since  $\eta / (\rho + \eta) < 1$ . Thus we must have  $v_1 > v_0$ . The value function of a neck-and-neck firm can be written as:

$$\begin{aligned} \rho v_0 &= \max_{x_0} \{-\omega^* G(x_0) + x_0 [v_1 - v_0] + x_0^* [v_{-1} - v_0]\}, \\ &\geq \max_{x_0} \{-\omega^* G(x_0) + x_0 [v_1 - v_0]\}, \\ &\geq -\omega^* G(x_{-1}^*) + x_{-1}^* [v_1 - v_0], \\ &\geq -\omega^* G(x_{-1}^*) + x_{-1}^* [v_0 - v_{-1}] + \eta [v_0 - v_{-1}], \\ &= \rho v_{-1}, \end{aligned} \tag{57}$$

which contradicts the hypothesis that  $v_{-1} > v_0$  and establishes the claim.  $\square$

*Proof of  $v_n < v_{n+1}$  :* Suppose, to obtain a contradiction, that  $v_n \geq v_{n+1}$ . Now (32) implies  $x_n^* = 0$ , and (29) becomes

$$\rho v_n = (1 - \lambda^{-n}) + x_{-1}^* [v_0 - v_n] + \eta [v_0 - v_n] \tag{58}$$

Also from (29), the value for state  $n + 1$  satisfies

$$\rho v_{n+1} \geq (1 - \lambda^{-n-1}) + x_{-1}^* [v_0 - v_{n+1}] + \eta [v_0 - v_{n+1}]. \tag{59}$$

Combining the two previous expressions, we obtain

$$\begin{aligned} &(1 - \lambda^{-n}) + x_{-1}^* [v_0 - v_n] + \eta [v_0 - v_n] \\ &\geq 1 - \lambda^{-n-1} + x_{-1}^* [v_0 - v_{n+1}] + \eta [v_0 - v_{n+1}]. \end{aligned}$$

Since  $\lambda^{-n-1} < \lambda^{-n}$ , this implies  $v_n < v_{n+1}$ , contradicting the hypothesis that  $v_n \geq v_{n+1}$ , and establishing the desired result,  $v_n < v_{n+1}$ . Consequently,  $\{v_n\}_{n=-1}^\infty$  is nondecreasing and  $\{v_n\}_{n=0}^\infty$  is (strictly) increasing. Since a nondecreasing sequence in a compact set must converge,  $\{v_n\}_{n=-1}^\infty$  converges to its limit point,  $v_\infty$ , which must be strictly positive, since  $\{v_n\}_{n=0}^\infty$  is strictly increasing and has a nonnegative initial value.  $\square$

The above results combined complete the proof that values form an increasing sequence.  $\blacksquare$

### Part 3: Properties of the Sequence of R&D Decisions.

*Proof of  $x_{n+1}^* < x_n^*$  :* From equation (32),

$$\delta_{n+1} \equiv v_{n+1} - v_n < v_n - v_{n-1} \equiv \delta_n \tag{60}$$

would be sufficient to establish that  $x_{n+1}^* < x_n^*$  whenever  $x_n^* > 0$ . We next show that this is the case.

Let us write:

$$\bar{\rho} v_n = \max_{x_n} \{(1 - \lambda^{-n}) - \omega^* G(x_n) + x_n^* [v_{n+1} - v_n] + x_{-1}^* v_0 + \eta v_0\}, \tag{61}$$

where  $\bar{\rho} \equiv \rho + x_{-1}^* + \eta$ . Since  $x_{n+1}^*$ ,  $x_n^*$  and  $x_{n-1}^*$  are maximizers of the value functions  $v_{n+1}$ ,  $v_n$  and  $v_{n-1}$ , (61) implies:

$$\begin{aligned}\bar{\rho}v_{n+1} &= 1 - \lambda^{-n-1} - \omega^*G(x_{n+1}^*) + x_{n+1}^*[v_{n+2} - v_{n+1}] + x_{-1}^*v_0 + \eta v_0, \\ \bar{\rho}v_n &\geq 1 - \lambda^{-n} - \omega^*G(x_{n+1}^*) + x_{n+1}^*[v_{n+1} - v_n] + x_{-1}^*v_0 + \eta v_0, \\ \bar{\rho}v_n &\geq 1 - \lambda^{-n} - \omega^*G(x_{n-1}^*) + x_{n-1}^*[v_{n+1} - v_n] + x_{-1}^*v_0 + \eta v_0, \\ \bar{\rho}v_{n-1} &= 1 - \lambda^{-n+1} - \omega^*G(x_{n-1}^*) + x_{n-1}^*[v_n - v_{n-1}] + x_{-1}^*v_0 + \eta v_0.\end{aligned}\tag{62}$$

Now taking differences with  $\bar{\rho}v_n$  and using the definitions of  $\delta_n$ s, we obtain

$$\begin{aligned}\bar{\rho}\delta_{n+1} &\leq \lambda^{-n}(1 - \lambda^{-1}) + x_{n+1}^*(\delta_{n+2} - \delta_{n+1}) \\ \bar{\rho}\delta_n &\geq \lambda^{-n+1}(1 - \lambda^{-1}) + x_{n-1}^*(\delta_{n+1} - \delta_n).\end{aligned}$$

Therefore,

$$(\bar{\rho} + x_{n-1}^*)(\delta_{n+1} - \delta_n) \leq -k_n + x_{n+1}^*(\delta_{n+2} - \delta_{n+1}),\tag{63}$$

where

$$k_n \equiv (\lambda - 1)^2 \lambda^{-n-1} > 0.$$

Now to obtain a contradiction, suppose that  $\delta_{n+1} - \delta_n \geq 0$ . From (63), this implies  $\delta_{n+2} - \delta_{n+1} > 0$  since  $k_n$  is strictly positive. Repeating this argument successively, we have that if  $\delta_{n'+1} - \delta_{n'} \geq 0$ , then  $\delta_{n+1} - \delta_n > 0$  for all  $n \geq n'$ . However, we know from Part 2 of the proposition that  $\{v_n\}_{n=0}^\infty$  is strictly increasing and converges to a constant  $v_\infty$ . This implies that  $\delta_n \downarrow 0$ , which contradicts the hypothesis that  $\delta_{n+1} - \delta_n \geq 0$  for all  $n \geq n' \geq 0$ , and establishes that  $x_{n+1}^* \leq x_n^*$ . To see that the inequality is strict when  $x_n^* > 0$ , it suffices to note that we have already established (60), i.e.,  $\delta_{n+1} - \delta_n < 0$ , thus if equation (32) has a positive solution, then we necessarily have  $x_{n+1}^* < x_n^*$ .

We next prove that  $x_0^* \geq x_{-1}^*$  and then show that under the additional condition  $G'^{-1}((1 - \lambda^{-1})/(\rho + \eta)) > 0$ , this inequality is strict.

*Proof of  $x_0^* \geq x_{-1}^*$ :* Equation (30) can be written as

$$\rho v_0 = -\omega^*G(x_0^*) + x_0^*[v_{-1} + v_1 - 2v_0].\tag{64}$$

We have  $v_0 \geq 0$  from Part 2 of the proposition. Suppose  $v_0 > 0$ . Then (64) implies  $x_0^* > 0$  and

$$\begin{aligned}v_{-1} + v_1 - 2v_0 &> 0 \\ v_1 - v_0 &> v_0 - v_{-1}.\end{aligned}\tag{65}$$

This inequality combined with (34) and (41) yields  $x_0^* > x_{-1}^*$ . Suppose next that  $v_0 = 0$ . Inequality (65) now holds as a weak inequality and implies that  $x_0^* \geq x_{-1}^*$ . Moreover, since  $G(\cdot)$  is strictly convex and  $x_0^*$  is given by (34), (64) then implies  $x_0^* = 0$  and thus  $x_{-1}^* = 0$ .  $\square$

We now have the following intermediate lemma.

**Lemma 2** *Suppose that  $G'^{-1}((1 - \lambda^{-1})/(\rho + \eta)) > 0$ , then  $x_0^* > 0$  and  $v_0 > 0$ .*

**Proof.** Suppose, to obtain a contradiction, that  $x_0^* = 0$ . The first part of the proof then implies that  $x_{-1}^* = 0$ . Then (29) implies

$$\rho v_1 \geq 1 - \lambda + \eta[v_0 - v_1].$$

Equation (30) together with  $x_0^* = 0$  gives  $v_0 = 0$ , and hence

$$v_1 - v_0 \geq \frac{1 - \lambda^{-1}}{\rho + \eta}.$$

Combined with this inequality, (34) implies

$$\begin{aligned} x_0^* &\geq \max \left\{ G'^{-1} \left( \frac{1 - \lambda^{-1}}{\omega^* (\rho + \eta)} \right), 0 \right\}, \\ &\geq \max \left\{ G'^{-1} \left( \frac{1 - \lambda^{-1}}{\rho + \eta} \right), 0 \right\}, \end{aligned}$$

where the second inequality follows from the fact that  $\omega^* \leq 1$ . The assumption that  $G'^{-1} \left( (1 - \lambda^{-1}) / (\rho + \eta) \right) > 0$  then implies  $x_0^* > 0$ , thus leading to a contradiction and establishing that  $x_0^* > 0$ . Strict convexity of  $G(\cdot)$  together with  $x_0^* > 0$  then implies  $v_0 > 0$ . ■

*Proof of  $x_0^* > x_{-1}^*$  when  $G'^{-1} \left( (1 - \lambda^{-1}) / (\rho + \eta) \right) > 0$  :* Given Lemma 2,  $G'^{-1} \left( (1 - \lambda^{-1}) / (\rho + \eta) \right) > 0$  implies that  $x_0^* > 0$ . Then (64) implies

$$v_1 - v_0 > v_0 - v_{-1}$$

and as a result  $x_0^* > x_{-1}^*$ . □

*Proof of  $x_0^* > x_1^*$  :* To prove that  $x_0^* > x_1^*$ , let us write the value functions  $v_2$ ,  $v_1$  and  $v_0$  as in (62):

$$\begin{aligned} \bar{\rho}v_2 &= 1 - \lambda^{-2} - \omega^*G(x_2^*) + x_2^*[v_3 - v_2] + x_{-1}^*v_0 + \eta v_0, \\ \bar{\rho}v_1 &\geq 1 - \lambda^{-1} - \omega^*G(x_2^*) + x_2^*[v_2 - v_1] + x_{-1}^*v_0 + \eta v_0, \\ \bar{\rho}v_1 &\geq 1 - \lambda^{-1} - \omega^*G(x_0^*) + x_0^*[v_2 - v_1] + x_{-1}^*v_0 + \eta v_0, \\ \bar{\rho}v_0 &= -\omega^*G(x_0) + x_0^*[v_1 - v_0] + \eta v_0 + x_{-1}^*v_0 + x_0^*[v_{-1} - v_0]. \end{aligned}$$

Now taking differences with  $\bar{\rho}v_n$  and using the definitions of  $\delta_n$ s as in (60), we obtain

$$\begin{aligned} \bar{\rho}\delta_2 &\leq \lambda^{-1}(1 - \lambda^{-1}) + x_2^*(\delta_3 - \delta_2), \\ \bar{\rho}\delta_1 &\geq (1 - \lambda^{-1}) + x_0^*(\delta_2 - \delta_1) + x_{-1}^*[v_0 - v_0] - x_0^*[v_{-1} - v_0], \\ \bar{\rho}\delta_1 &\geq (1 - \lambda^{-1}) + x_0^*(\delta_2 - \delta_1) - x_0^*[v_{-1} - v_0], \\ \bar{\rho}\delta_1 &\geq (1 - \lambda^{-1}) + x_0^*(\delta_2 - \delta_1) - x_0^*[v_{-1} - v_0]. \end{aligned} \tag{66}$$

Next recall from Part 2 that  $v_{-1} - v_0 \leq 0$ . Moreover, the first part of the first part of the proof has established that  $x_{-1}^* - x_0^* \leq 0$ . Therefore  $[x_{-1}^* - x_0^*][v_{-1} - v_0] \geq 0$ , and the last inequality then implies

$$\bar{\rho}\delta_1 \geq (1 - \lambda^{-1}) + x_0^*(\delta_2 - \delta_1).$$

Now combining this inequality with the first inequality of (66), we obtain

$$(\bar{\rho} + x_0^*)(\delta_2 - \delta_1) \leq -(1 - \lambda^{-1})^2 + x_2^*(\delta_3 - \delta_2). \tag{67}$$

Part 2 has already established  $\delta_2 > \delta_3$ , so that the right-hand side is strictly negative, therefore, we must have  $\delta_2 - \delta_1 < 0$ , which implies that  $x_0^* > x_1^*$  and completes the proof. □

The above results together complete the proof of Part 3. ■

#### Part 4: Uniqueness of the Invariant Distribution.

**Lemma 3** *Consider a uniform IPR policy  $\eta^{uni}$  and a corresponding steady-state equilibrium  $\langle \mu^*, \mathbf{v}, \mathbf{x}^*, \omega^*, g^* \rangle$ . Then, there exists  $n^* \in \mathbb{N}$  such that  $x_n^* = 0$  for all  $n \geq n^*$ .*

**Proof.** The first-order condition of the maximization of the value function (29) implies:

$$G'(x_n) \geq \frac{v_{n+1} - v_n}{\omega^*} \text{ and } x_n \geq 0,$$

with complementary slackness.  $G'(0)$  is strictly positive by assumption. If  $(v_{n+1} - v_n)/\omega^* < G'(0)$ , then  $x_n = 0$ . The second part of the proposition implies that  $\{v_n\}_{n=-1}^\infty$  is a convergent and thus a Cauchy sequence, which implies that there exists  $\exists n^* \in \mathbb{N}$  such that  $v_{n+1} - v_n < \omega^* G'(0)$  for all  $n \geq n^*$ . ■

An immediate consequence of Lemma 3, combined with (35) is that  $\mu_n = 0$  for all  $n \geq n^*$  (since there is no innovation in industries with technology gap greater than  $n^*$ ). Thus the law of motion of an industry can be represented by a finite Markov chain. Moreover, because after an innovation by a follower, all industries jump to the neck-and-neck state, this Markov chain is irreducible (and aperiodic), thus converges to a unique steady-state distribution of industries. More formally, there exists  $n^*$  such that  $x_{n^*}^* = 0$  and  $x_n^* = 0$  for all  $n > n^*$ . Combined with the fact  $G'^{-1}((1 - \lambda^{-1}) / (\rho + \eta)) > 0$  and that either  $\eta > 0$  or  $x_{-1}^* > 0$ , this implies that the states  $n > n^*$  are transient and can be ignored. Consequently,  $\{\mu_n^*\}_{n=0}^\infty$  forms a finite and irreducible Markov chain over the states  $n = 0, 1, \dots, n^*$ . To see this, let  $n^* = \min_{n \in \{0, \dots, n^{**}\}} \{n \in \mathbb{N} : v_{n+1} - v_n \leq \omega^* G'(0)\}$ . Such an  $n^*$  exists, since the set  $\{0, \dots, n^{**}\}$  is finite and nonempty because of the assumption that  $G'^{-1}((1 - \lambda^{-1}) / (\rho + \eta)) > 0$ . Then by construction  $x_n^* > 0$  for all  $n < n^*$  and  $x_{n^*}^* = 0$  as desired. Now denoting the probability of being in state  $\tilde{n}$  starting in state  $n$  after  $\tau$  periods by  $P^\tau(n, \tilde{n})$ , we have that  $\lim_{\tau \rightarrow \infty} P^\tau(n, \tilde{n}) = 0$  for all  $\tilde{n} > n^*$  and for all  $n$ . Thus we can focus on the finite Markov chain over the states  $n = 0, 1, \dots, n^*$ , and  $\{\mu_n^*\}_{n=0}^{n^*}$  is the limiting (invariant) distribution of this Markov chain. Given  $\{x_n^*\}_{n=-1}^{n^*}$ ,  $\{\mu_n^*\}_{n=0}^{n^*}$  is uniquely defined. Moreover, the underlying Markov chain is irreducible (since  $x_n^* > 0$  for  $n = 0, 1, \dots, n^* - 1$ , so that all states communicate with  $n = 0$  or  $n = 1$ ). Therefore, by Theorem 11.2 in Stokey, Lucas and Prescott (1989, p. 62) there exists a unique stationary distribution  $\{\mu_n^*\}_{n=0}^\infty$ . ■

## Proof of Proposition 5

We prove this proposition using two crucial lemmas.

**Lemma 4** *Consider the state-dependent IPR policy  $\eta$ , and suppose that  $\langle \mu^*, \mathbf{v}, \mathbf{x}^*, \omega^*, g^* \rangle$  is a steady-state equilibrium. Then there exists a state  $n^* \in \mathbb{N}$  such that  $\mu_n^* = 0$  for all  $n \geq n^*$ .*

**Proof.** There are two cases to consider. First, suppose that  $\{v_n\}_{n \in \mathbb{Z}_+}$  is strictly increasing. Then it follows from the proof of Lemma 3 that there exists a state  $n^* \in \mathbb{N}$  such that  $x_n^* = 0$  for all  $n \geq n^*$ , and as in the proof of Part 4 of Proposition 4, states  $n \geq n^*$  are transient (i.e.,  $\lim_{\tau \rightarrow \infty} P^\tau(n, \tilde{n}) = 0$  for all  $\tilde{n} > n^*$  and for all  $n$ ), so  $\mu_n^* = 0$  for all  $n \geq n^*$ .

Second, in contrast to the first case, suppose that there exists some  $n^{**} \in \mathbb{Z}_+$  such that  $v_{n^{**}} \geq v_{n^{**}+1}$ . Then, let  $n^* = \min_{n \in \{0, \dots, n^{**}\}} \{n \in \mathbb{N} : v_{n+1} - v_n \leq \omega^* G'(0)\}$ , which is again well defined. Then, optimal R&D decision (32) immediately implies that  $x_n^* > 0$  for all states with  $n < n^*$ , and since  $x_{n^*}^* = 0$ , all states  $n > n^*$  are transient and  $\lim_{\tau \rightarrow \infty} P^\tau(n, \tilde{n}) = 0$  for all  $\tilde{n} > n^*$  and for all  $n$ , completing the proof. ■

**Lemma 5** *Consider the state-dependent IPR policy  $\eta$  and suppose that the labor share and the R&D policies of all other firms are given by  $\mathbf{z} = \langle \tilde{\omega}, \tilde{\mathbf{x}} \rangle$ . Then the dynamic optimization problem of an individual firm leads to a unique value function  $\mathbf{v}[\mathbf{z}] : \mathbb{Z} \rightarrow \mathbb{R}_+$  and optimal R&D policy  $\hat{\mathbf{X}}[\mathbf{z}] : \mathbb{Z} \rightrightarrows [0, \bar{x}]$  are compact and convex-valued for each  $\mathbf{z} \in \mathbf{Z}$  and upper hemi-continuous in  $\mathbf{z}$  (where  $\mathbf{v}[\mathbf{z}] \equiv \{v_n[\mathbf{z}]\}_{n=-1}^\infty$ ,  $\hat{\mathbf{X}}[\mathbf{z}] \equiv \{\hat{X}_n[\mathbf{z}]\}_{n=-1}^\infty$ ).*

**Proof.** The proof follows closely that of Lemma 1. In particular, again using uniformization, the maximization problem of an individual firm can be written as a contraction mapping similar to (54) there. The finiteness of the transition probabilities follows, since  $\psi_n(\xi | \tilde{\xi}) \leq \psi \equiv 2\bar{x} + \max_n \{\eta_n\} < \infty$  (this is a consequence of the fact that  $\bar{x}$  defined in (16) is finite and  $\max_n \{\eta_n\}$  is finite, since each  $\eta_n \in \mathbb{R}_+$  and by assumption, there exists  $\bar{n} < \infty$  such that  $\eta_n = \eta_{\bar{n}}$ ). This contraction mapping uniquely determines the value function  $\mathbf{v}[\mathbf{z}] : \mathbb{Z} \rightarrow \mathbb{R}_+$ .

Berge's Maximum Theorem (Aliprantis and Border, 1999, Theorem 16.31, p. 539) again implies that each of  $\hat{X}_n(\mathbf{z})$  for  $n \in \mathbb{Z}$  is upper hemi-continuous in  $\mathbf{z} = \langle \tilde{\omega}, \tilde{\mathbf{x}} \rangle$ , and moreover, since  $v_n$  for  $n \in \mathbb{Z}$

is concave in  $x_n$ , the maximizer of  $\mathbf{v}[\mathbf{z}]$ ,  $\hat{\mathbf{X}} \equiv \left\{ \hat{X}_n \right\}_{n=-\infty}^{\infty}$ , are nonempty, compact and convex-valued. ■

Now using the previous two lemmas, we can establish the existence of a steady-state equilibrium. This part of the proof follows that of Proposition 4 closely. Fix  $\mathbf{z} = \langle \tilde{\omega}, \{\tilde{x}_n\}_{n=-\infty}^{\infty} \rangle$ , and define  $\mathbf{Z} \equiv [0, 1] \times \prod_{n=-\infty}^{\infty} [0, \bar{x}]$ . Again by Tychonoff's Theorem,  $\mathbf{Z}$  is compact in the product topology. Then consider the mapping  $\Phi: \mathbf{Z} \rightrightarrows \mathbf{Z}$  constructed as  $\Phi \equiv \left( \varphi, \hat{\mathbf{X}} \right)$ , where  $\varphi$  is given by (55) and  $\hat{\mathbf{X}}$  is defined in Lemma 5. Clearly  $\Phi$  maps  $\mathbf{Z}$  into itself. Moreover, as in the proof of Proposition 4,  $\mathbf{Z}$  is nonempty, convex, and a subset of a locally convex Hausdorff space. The proof of Lemma 5 then implies that  $\Phi$  has closed graph in the product topology and is nonempty, compact and convex-valued in  $\mathbf{z}$ . Consequently, the Kakutani-Fan-Glicksberg Fixed Point Theorem again applies and implies that  $\Phi$  has a fixed point  $\mathbf{z}^* \in \Phi(\mathbf{z}^*)$ . The argument that the fixed point  $\mathbf{z}^*$  corresponds to a steady-state equilibrium is identical to that in Proposition 4, and follows from the fact that within argument identical to that of Lemma 2,  $G'^{-1} \left( (1 - \lambda^{-1}) / (\rho + \eta_1) \right) > 0$  implies  $x_0^* > 0$ . The result that  $\omega^* < 1$  then follows immediately. Finally, as in the proof of Proposition 4, either  $\eta_1 > 0$  or  $x_{-1}^* > 0$  is sufficient for  $g^* > 0$ . ■



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## Tables

Note: Tables 1-5 give the results of the numerical computations with  $\rho = 0.05$  under three different IPR policy regimes. Tables 1-4 consider a different environment (quick catch-up, slow catch-up, licensing and leapfrogging) at a time, whereas Table 5 combines the latter three environments (slow catch-up, licensing and leapfrogging). Table 6 reports the robustness checks of the state-dependent results of Table 5 with alternative step sizes and R&D elasticity parameters. Depending on the applicability and necessity, Tables 1-6 report the steady-state equilibrium values of the difference in the values  $v_1 - v_{-3}$  and  $v_0 - v_{-3}$ ; the (annual) catch-up and frontier R&D rates of a follower that is one step behind,  $(x_{-1}^c, x_{-1}^f)$ ; the (annual) R&D rate of neck-and-neck competitors,  $x_0^*$ ; the (annual) R&D rate of one-step leader,  $x_1^*$ ; fraction of industries in neck-and-neck competition,  $\mu_0^*$ ; fraction of industries at a technology gap of  $n = 1, 2$ ; the value of “labor share,”  $\omega^*$ ; the ratio of the labor force working in research; log of initial (annual) consumption,  $\ln C(0)$ ; the annual growth rate,  $g^*$ ; and the welfare level according to equation (42). It also reports the welfare-maximizing uniform and state-dependent IPR policies. See text for details.

$\lambda = 1.05, \gamma = 0.35,$ $B = 0.1$	Full IPR	Optimal Uniform IPR	Optimal State-dependent IPR
$\eta_1$	0	0	0.71
$\eta_2$	0	0	0.08
$\eta_3$	0	0	0
$\eta_4$	0	0	0
$\eta_5$	0	0	0
$x_{-1}^*$	0.22	0.22	0.12
$x_0^*$	0.35	0.35	0.25
$x_1^*$	0.29	0.29	0.41
$\mu_0^*$	0.24	0.24	0.46
$\mu_1^*$	0.33	0.33	0.19
$\mu_2^*$	0.20	0.20	0.13
$\omega^*$	0.95	0.95	0.96
Researcher ratio	0.032	0.032	0.028
$\ln C(0)$	33.78	33.78	34.20
$g^*$	0.0186	0.0186	0.0204
Welfare	683.0	683.0	692.1

TABLE 1. OPTIMAL PATENT LENGTH IN QUICK CATCH-UP REGIME

$\lambda = 1.05, \gamma = 0.35$ $B = 0.1, \zeta_n = \infty, \vartheta_n = \infty$	Full IPR	Optimal Uniform IPR	Optimal State-dependent IPR
$\eta_1$	0.02	0.11	0.69
$\eta_2$	0.02	0.11	0.20
$\eta_3$	0.02	0.11	0.14
$\eta_4$	0.02	0.11	0.12
$\eta_5$	0.02	0.11	0.08
$x_{-1}^*$	0.75	0.27	0.17
$x_0^*$	0.99	0.14	0.32
$x_1^*$	1.10	0.15	0.51
$\mu_0^*$	0.02	0.16	0.30
$\mu_1^*$	0.03	0.19	0.15
$\mu_2^*$	0.03	0.14	0.10
$\omega^*$	0.56	0.90	0.90
Researcher ratio	0.150	0.055	0.059
$\ln C(0)$	31.31	34.31	34.57
$g^*$	0.025	0.023	0.025
Welfare	636.3	695.3	701.2

TABLE 2. OPTIMAL PATENT LENGTH IN SLOW CATCH-UP REGIME

$\lambda = 1.05, \gamma = 0.35$ $B = 0.1, \eta_n = 0.02, \vartheta_n = \infty$	Full IPR	Optimal Uniform IPR	Optimal State-dependent IPR
$\zeta_1$	$\infty$	1.61	0
$\zeta_2$	$\infty$	1.61	1.54
$\zeta_3$	$\infty$	1.61	2.45
$\zeta_4$	$\infty$	1.61	2.92
$\zeta_5$	$\infty$	1.61	3.32
$v_0 - v_{-3}$	10.1	2.9	3.2
$x_{-1}^*$	0.75	0.27	0.31
$x_0^*$	0.99	0.39	0.45
$x_1^*$	1.10	0.43	0.46
$\mu_0^*$	0.02	0.21	0.18
$\mu_1^*$	0.03	0.25	0.20
$\mu_2^*$	0.03	0.20	0.12
$\omega^*$	0.56	0.94	0.91
Researcher ratio	0.150	0.043	0.071
$\ln C(0)$	31.31	34.13	34.47
$g^*$	0.025	0.021	0.025
Welfare	636.3	690.9	699.3

TABLE 3. LICENSING IN SLOW CATCH-UP REGIME

$\lambda = 1.05, \gamma = 0.35$ $B = 0.1, \eta_n = 0.02, \zeta_n = \infty$	Full IPR	Optimal Uniform IPR	Optimal State-dependent IPR
$\vartheta_1$	$\infty$	14	0
$\vartheta_2$	$\infty$	14	18.1
$\vartheta_3$	$\infty$	14	31.3
$\vartheta_4$	$\infty$	14	36.6
$\vartheta_5$	$\infty$	14	43.7
$v_1 - v_{-3}$	21.4	2.7	4.1
$x_{-1}^{c*}$	0.75	0.15	0.14
$x_{-1}^{f*}$	0	0.23	0.33
$x_0^*$	0.99	0.30	0.29
$x_1^*$	1.10	0.30	0.39
$\mu_0^*$	0.02	0.14	0.12
$\mu_1^*$	0.03	0.42	0.35
$\mu_2^*$	0.03	0.22	0.17
$\omega^*$	0.56	0.95	0.94
Researcher ratio	0.150	0.028	0.058
$\ln C(0)$	31.31	35.48	36.17
$g^*$	0.025	0.026	0.031
Welfare	636.3	720.0	735.9

TABLE 4. LEAPFROGGING IN SLOW CATCH-UP REGIME

$\lambda = 1.05, \gamma = 0.35$ $B = 0.1$	Full IPR	Optimal Uniform IPR	Optimal State-dependent IPR
$\eta_1$	0.02	0	0
$\eta_2$	0.02	0	0
$\eta_3$	0.02	0	0
$\eta_4$	0.02	0	0
$\eta_5$	0.02	0	0
$\zeta_1$	$\infty$	$\infty$	$\infty$
$\zeta_2$	$\infty$	$\infty$	$\infty$
$\zeta_3$	$\infty$	$\infty$	$\infty$
$\zeta_4$	$\infty$	$\infty$	$\infty$
$\zeta_5$	$\infty$	$\infty$	$\infty$
$\vartheta_1$	$\infty$	16.6	0
$\vartheta_2$	$\infty$	16.6	21.7
$\vartheta_3$	$\infty$	16.6	34.6
$\vartheta_4$	$\infty$	16.6	39.4
$\vartheta_5$	$\infty$	16.6	51.3
$v_1 - v_{-3}$	21.4	3.1	4.6
$v_0 - v_{-3}$	10.1	0.8	2.7
$x_{-1}^{c*}$	0.75	0.16	0.16
$x_{-1}^{f*}$	0	0.23	0.34
$x_0^*$	0.99	0.32	0.29
$x_1^*$	1.10	0.32	0.41
$\mu_0^*$	0.02	0.11	0.36
$\mu_1^*$	0.03	0.41	0.17
$\mu_2^*$	0.03	0.22	0.10
$\omega^*$	0.56	0.94	0.94
Researcher ratio	0.150	0.031	0.065
$\ln C(0)$	31.31	35.62	36.38
$g^*$	0.025	0.027	0.033
Welfare	636.3	723.0	740.5

TABLE 5. ALL THREE POLICIES IN SLOW CATCH-UP REGIME

$\kappa = 0.1$	Optimal State-dependent IPR $\gamma = 0.1$ $B = 0.04$	Optimal State-dependent IPR $\gamma = 0.6$ $B = 0.2$	Optimal State-dependent IPR $\lambda = 1.01$ $B = 0.35$	Optimal State-dependent IPR $\lambda = 1.20$ $B = 0.024$
$\eta_1$	0	0	0	0
$\eta_2$	0	0	0	0
$\eta_3$	0	0	0	0
$\eta_4$	0	0	0	0
$\eta_5$	0	0	0	0
$\zeta_1$	$\infty$	$\infty$	$\infty$	$\infty$
$\zeta_2$	$\infty$	$\infty$	$\infty$	$\infty$
$\zeta_3$	$\infty$	$\infty$	$\infty$	$\infty$
$\zeta_4$	$\infty$	$\infty$	$\infty$	$\infty$
$\zeta_5$	$\infty$	$\infty$	$\infty$	$\infty$
$\vartheta_1$	0	0	3.6	0
$\vartheta_2$	16.7	12.6	5.5	33.4
$\vartheta_3$	35.6	20.0	8.9	82.3
$\vartheta_4$	44.8	23.0	12.3	100.7
$\vartheta_5$	54.0	32.9	15.8	128.7
$v_1 - v_{-3}$	4.1	4.0	1.3	17.5
$v_0 - v_{-3}$	2.6	1.6	0.8	5.2
$x_{-1}^{c*}$	0.23	0.06	0.69	0.02
$x_{-1}^{f*}$	0.28	0.61	0.88	0.10
$x_0^*$	0.27	0.42	0.97	0.10
$x_1^*$	0.29	0.98	1.24	0.10
$\mu_0^*$	0.20	0.01	0.10	0.06
$\mu_1^*$	0.47	0.15	0.26	0.48
$\mu_2^*$	0.19	0.09	0.11	0.22
$\omega^*$	0.95	0.90	0.97	0.11
Researcher ratio	0.011	0.161	0.048	0.087
$\ln C(0)$	36.08	37.54	13.80	118.35
$g^*$	0.027	0.044	0.026	0.035
Welfare	732.4	768.3	286.3	2381.2

TABLE 6. ALL THREE POLICIES IN SLOW CATCH-UP REGIME - ROBUSTNESS CHECKS



# Figures

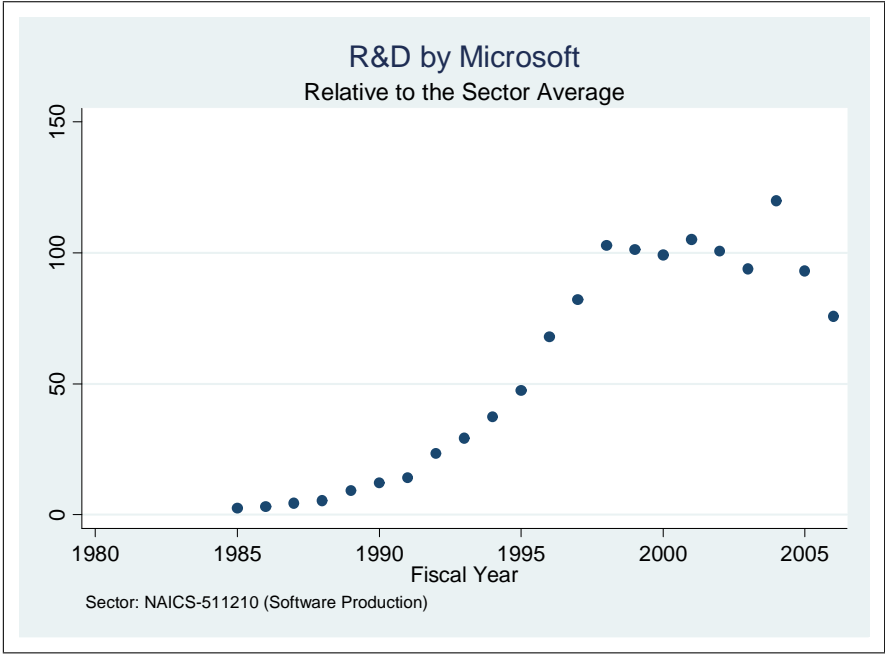


FIGURE 1

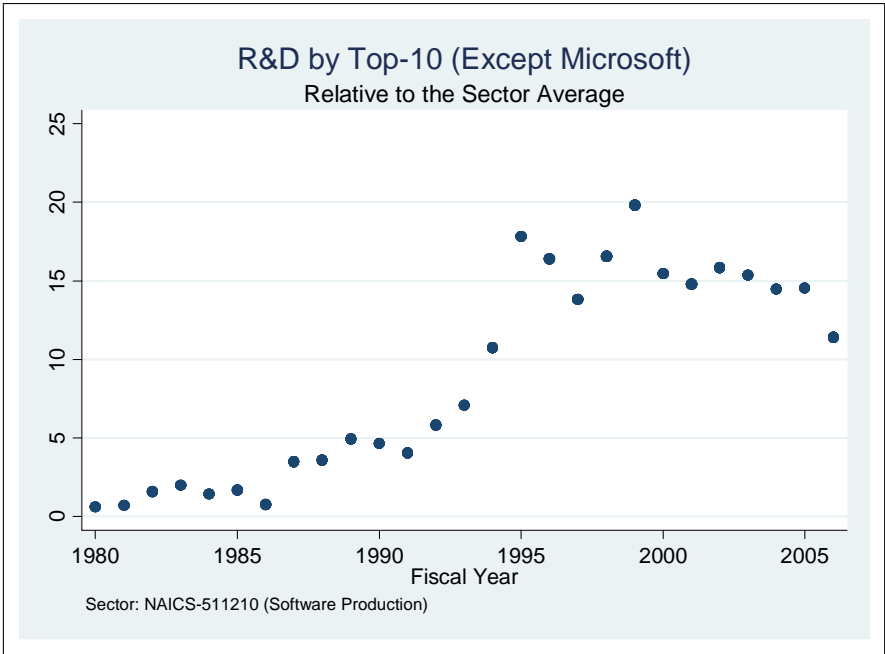


FIGURE 2

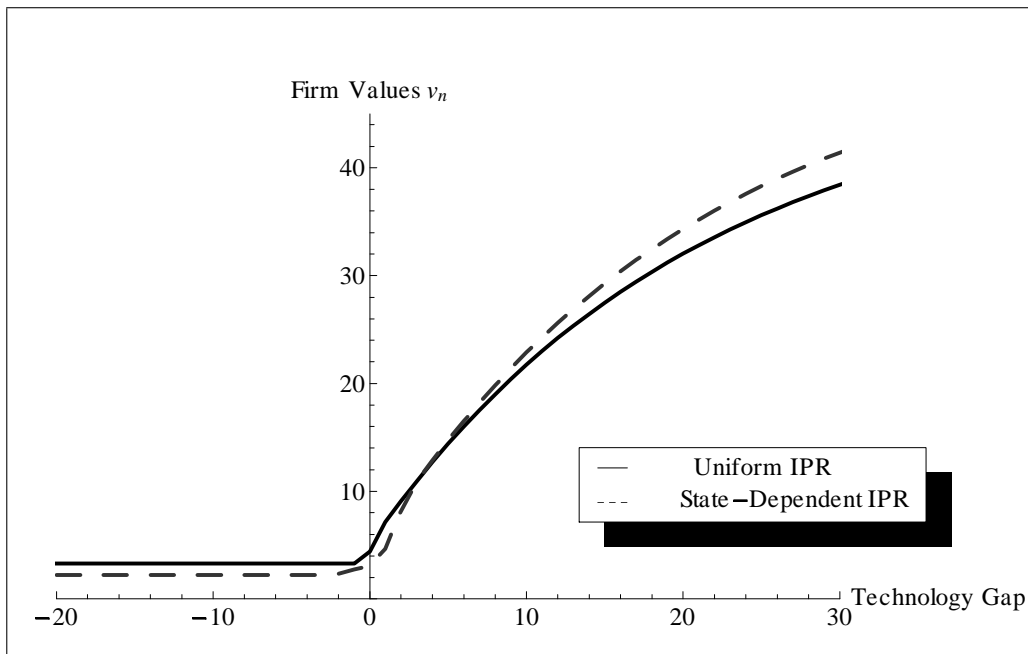


FIGURE 3. VALUE FUNCTIONS

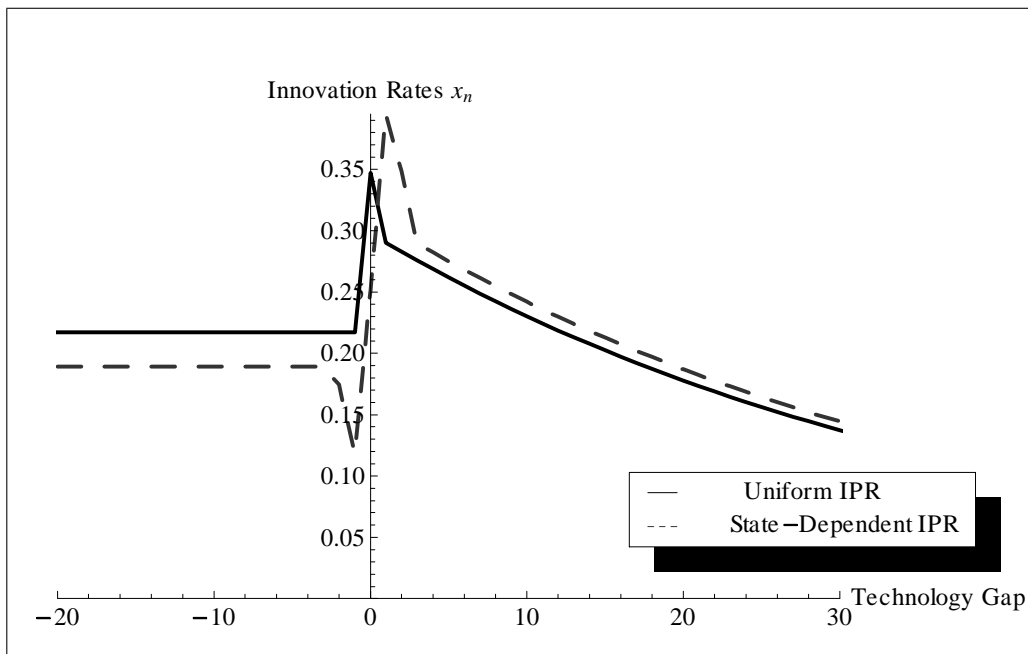


FIGURE 4. R&D EFFORTS

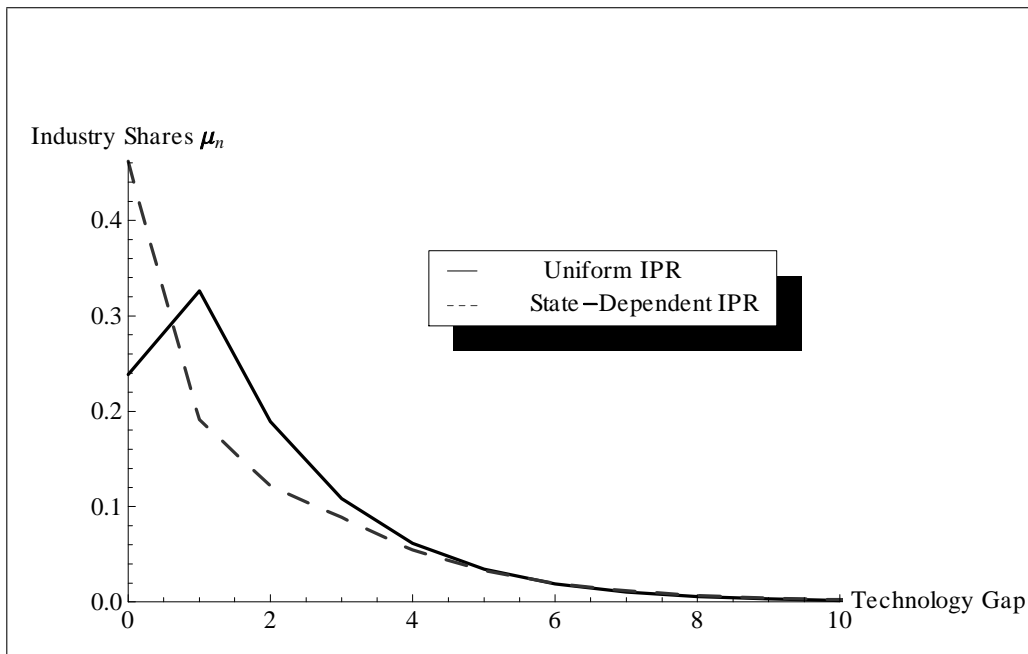


FIGURE 5. INDUSTRY SHARES

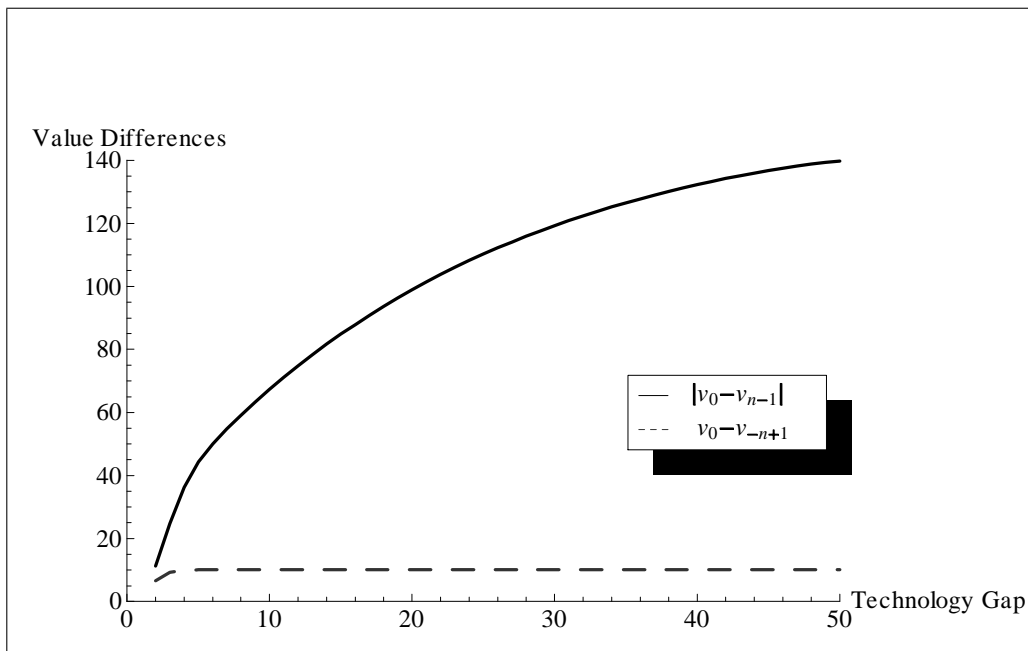


FIGURE 6. VALUE DIFFERENCES UNDER FULL PROTECTION & SLOW CATCH-UP