Fiscal Risk in a Monetary Union*

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Abstract

Diverse fiscal policies, which are subject to fiscal limits and stochastic shocks, can threaten price stability in a monetary union. The fiscal limits arise due to distortionary taxation and political will. Stochastic shocks are random and could push a fiscally sound policy towards its limit. In equilibrium agents refuse to lend along a path which violates the fiscal limits, creating a crisis. The crisis requires a policy response to restore lending. We focus on two responses, default and policy switching, both of which must be designed to restore fiscal solvency. Under the assumption that agents know the policy response, we show that default poses no risk to price stability, but that policy switching does. We simulate the model to quantify fiscal risk in the European Monetary Union with fiscal variables at end of 2009 values. We find that the probability of a fiscal crisis in Greece and Italy, countries whose debt relative to GDP had strayed far above the 60 percent limit, is positive but small. However, a small increase in debt can lead to a large increase in risk.

Key Words: European Monetary Union, Fiscal Theory of the Price Level, Policy Switching, Default, Financial Crisis

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1 Introduction

What is the threat to a monetary union from diverse fiscal policies in member countries? Understanding this threat is fundamental in the design and implementation of a monetary union. Both the nature and even the existence of the threat have been controversial in the European Monetary Union from its inception. While some have argued that member fiscal policies should be unrestrained, Germany convinced members to impose limits on deficits and debt as a fraction of GDP in the Stability and Growth Pact (SGP) in 1993, without explicitly stating what the threat is or how these limits resolved the threat.

When the Euro was introduced in 2002, markets received it enthusiastically, and later small violations of the SGP limits did not dim the enthusiasm. However, following the worldwide financial crisis and recession, which began in 2007 and accelerated in 2008, several countries experienced large increases in deficits and debts relative to GDP. Markets have responded negatively, with interest rates on government bonds in these countries surging, the Euro depreciating, and some difficulties in debt roll-overs. Articles in the press debate whether the monetary union can hold together. Clearly, market reaction implies that there are threats from fiscal policy to monetary union, and it is important to understand them.

The literature on fiscal sustainability (Bohn 1998, 2007) focuses on the government's intertemporal budget constraint (IBC). A fiscal policy is said to be sustainable if the intertemporal budget constraint is expected to hold. This is also the requirement for fiscal solvency. Bohn showed that a positive response of the primary surplus to debt, both expressed as a fraction of GDP, is sufficient for the IBC to hold and fiscal policy to be solvent. However, he also showed that this restriction on fiscal policy is weak because a country can be fiscally solvent with debt/GDP growing forever, as long as it does not grow faster than the growth-adjusted interest rate. In an equilibrium for which debt/GDP grows forever, the SGP limits on debt/GDP are eventually violated, but these violations have no implications for sustainability and solvency. If governments are always be able to service their debt without resorting to inflation, then markets should not react negatively as debt/GDP grows. But markets have responded negatively in countries for which debt/GDP has spiraled upwards.

World-wide increases in government debt/GDP ratios have sparked interest in a new literature on fiscal limits, where these limits are endogenous to a country's economic and political system and have no relation to the SGP limits or any other limits imposed exogenously on a government. These internal fiscal limits recognize that there is an upper bound to the tax revenue that can be raised because taxes are distortionary; explicitly, there is a top to the Laffer Curve. And the limit on taxation could be politically determined prior to reaching the top of the Laffer Curve. Additionally, there is a minimum below which transfers or government spending on public goods can be reduced. Together these fiscal limits imply an upper bound on the present value of primary surpluses, relative to GDP, that a government can generate, which in turn imply an upper bound on debt/GDP. Therefore, as debt/GDP rises along a path, which is considered sustainable based on an analysis without fiscal limits, markets begin to question fiscal solvency since the primary surplus cannot rise to levels necessary to assure intertemporal budget balance. Davig, Leeper, and Walker (2010a,b), Davig and Leeper (2010), and Cochrane

(2010) have explored the implications of fiscal limits for inflation, and Daniel (2010) explores their implications for an exchange rate crisis. Sims (1997, 1999) considers their implications for price stability in a monetary union, and this paper extends his analysis.

What is the threat imposed by fiscal policy to price stability in a monetary union in the presence of fiscal limits? The answer to this question depends critically on beliefs by agents, the proverbial "anchoring" that policy-makers emphasize. We assume that policy-makers affect agents' beliefs by committing to policy rules, a monetary rule and a surplus rule. The surplus rule yields an equilibrium value for debt/GDP in the long-run, but is subject to shocks over time, examples being the negative shocks created by the financial crisis. The fiscal commitment to the rule is conditional on markets allowing the government to accumulate the debt necessary to carry out the rule. If markets refuse to lend, then the government is forced to respond with an alternative feasible policy. The monetary rule is also conditional. We assume that agents know the conditional responses of both rules. This assumption is counterfactual because agents in fact do not know how governments will respond to a crisis, and much of the market turbulence could be attributed to changing beliefs about the response. However, before we can understand the role of uncertainty regarding policy response, we need to understand the effects of alternative policies in the absence of uncertainty about them. With these assumptions, beliefs are anchored, allowing us to pass over this very important issue, emphasized in Davig, Leeper, and Walker (2010a,b), Davig and Leeper (2010) and Cochrane (2010). Therefore, assuming beliefs are anchored in this way, does the presence of fiscal limits pose a threat to price stability in a monetary union?

The answer depends on the policy response. The first policy response we consider is default, which we define as a reduction in the value of the government's outstanding debt. This yields an alternative model of sovereign default, where the default is due to fiscal limits, not the willingness to pay as in Eaton and Gersovitz (1981), Eaton and Fernandez (1995), and Arellano (2008).[1] The model has several interesting results. First, interest rates in the troubled country rise once default becomes anticipated, reflecting a default-risk premium, and raising the probability of a crisis by accelerating the rate at which debt accumulates. Second, the magnitude of the capital loss on debt due to default is endogenous, determined to restore fiscal solvency, and is always less than one hundred percent. Third, a default crisis occurs when agents refuse to lend. Therefore, the market determines the timing of the crisis, and the crisis occurs before debt actually reaches the fiscal limit. Once debt necessary under the fiscal rule rises above the path passing through the fiscal limit, there is no value for expected future default, and, therefore, for the interest rate, which could both compensate agents for expected default and restore fiscal solvency, implying a refusal to lend. Default restores solvency, but markets remain turbulent with high expectations of additional default.

Under default, there is no effect of the fiscal crisis on the monetary union price level or exchange rate, and the monetary authority can continue its policy of actively fighting inflation, successfully reaching its inflation target even though one country is experiencing default. Therefore, if the monetary authority is concerned solely about the ability to control inflation,[2] it should seek a commitment from each country to default in response

[1] We assume that a government, which is solvent under the fiscal rule to which it has comitted, repays. Therefore, default is due to insolvency. At a more fundamental level, we can view the government as having chosen the parameters of the fiscal rule with knowledge of the implied probability of insolvency, and therefore of default. They choose these parameters over less risky ones based on the cost of default. The cost of default needs to be high to justify high debt. It could be something like reputational loss instead of exclusion from credit markets, which does not have large enough cost.

[2]

to a lending crisis. Germany's demand for an orderly mechanism for debt restructuring in October 2010 could be interpreted as requesting such a commitment.

The second policy response we consider does affect the monetary authority's ability to control inflation and is more akin to work by Davig, Leeper, and Walker (2010a,b), Davig and Leeper (2010), Cochrane (2010), and Sims (1997). In contrast to these models, in which the timing of switching is either endogenously or exogenously chosen by the government, we assume that the timing is determined by the market. Explicitly, once the government can no longer borrow, it switches to an active fiscal policy accompanied by a monetary policy switch to passive. The price level jumps to reduce the real value of government debt, thereby restoring fiscal solvency and allowing the government to borrow again. We show that this crisis and the associated price-level jump will occur before the government actually reaches the fiscal limits. After the switch to passive monetary policy, the monetary authority retains control over expected inflation, but not over actual inflation because the actual price level must experience both positive and negative jumps in response to fiscal shocks to assure intertemporal budget balance.

In the run-up to the crisis, we show that the monetary authority can maintain control over the price level only if it abandons its active inflation-fighting Taylor Rule and allows the interest rate to rise in anticipation of the price level jump which is expected with switching. Alternatively, if the monetary authority maintains its active inflation-fighting stance, it loses control of the price level once a crisis becomes anticipated. And if it

This result ignores wealth transfers away from member countries holding the defaulting country's debt, which could have important financial crisis type effects, creating a recession in countries with negative wealth transfers. This could yield negative fiscal shocks, creating additional potential default crises, i.e. contagion. But, recession and contagion, however undesirable, need not affect the monetary authority's ability to control price.

maintains this stance after fiscal policy switching, then the country can experience a hyperinflation.[3]

Finally, we simulate the model to quantify fiscal risk in the EMU, using the 2009 values for government debt and the primary surplus. Other papers provide estimates of fiscal risk, based on VAR models of debt, but this risk is that of debt relative to GDP reaching an upper bound (Garcia and Rigobon 2004), or beginning to grow (Tanner and Samake 2008), over a particular horizon. Neither of these events need cause a crisis, and both measures miss the non-linear acceleration of debt in the neighborhood of a crisis due to expectations.

This paper is organized as follows. The second section contains a simple open-economy macroeconomic model. The third section considers dynamics leading to a fiscal crisis under alternative responses to the fiscal crisis. The fourth section contains simulations of fiscal risk, and the fifth contains a comparison of the results to the literature. The sixth section concludes.

2 Model

2.1 Overview

In this section, we set up a simple model of a monetary union, which we can use to address fiscal solvency risk. The set-up of the model follows Daniel (2010), with extensions to allow default, a monetary union, and a more general fiscal rule. The model contains four key assumptions. First, international creditors lend to a government only when they expect to receive the market rate of return. Second, there are fiscal limits, specifically

[3] Uribe (2006) shows that a country in this situation can initiate an unanticipated default to reduce government debt and end the hyperinflation.

an upper bound on the present value of primary surpluses relative to output. Third, fiscal policy is subject to stochastic shocks. Together the upper bound and stochastic shocks imply risk on government debt, reflecting the concern by the EMU founders and the reality, that a government's commitment to raise taxes to finance expenditures cannot be totally unconditional. Fourth, we assume that a solvent sovereign always repays.

We fill out the model with enough structure to obtain an equation for the evolution of government debt relative to output. This requires specification of monetary and fiscal policy as well as government budget constraints. We assume that governments can commit to rules, and this commitment anchors expectations. We assume that initially the monetary authority follows a Taylor Rule with an inflation target of zero, and that the fiscal authority follows a rule relating the current primary surplus to past debt. The rule is subject to stochastic shocks, giving fiscal policy risk. The fiscal rule we choose is simple and does not require full specification of a general equilibrium model. However, any rule with fiscal risk could be used to complete the model.

2.2 Credit Markets

We assume that the monetary union consists of N countries. The $j = 1, 2, ..., N$ countries are small enough that they cannot affect the world price level or world interest rate. There is a single good in the world, implying that equilibrium in goods markets requires the law of one price. Normalizing the world price level at unity and assuming no world inflation implies that the equilibrium price level in the monetary union is the exchange rate.

The **first key assumption** is that international creditors are willing to buy and sell country j's government bonds as long as its interest rate, i_{jt}, satisfies interest rate parity.

Interest rate parity is implied by the Euler equations for a representative world agent when the covariance of the country j interest rate with world-agent consumption is zero, or when the world agent is risk neutral. Under the additional assumptions that the world interest rate (i) is constant, interest rate parity can be expressed as

$$\frac{1}{1+i_{jt}} = \left(\frac{1}{1+i}\right) E_t\left[\frac{P_t}{P_{t+1}}\delta_{jt+1}\right], \qquad j = 1, 2, ...N \tag{1}$$

where E_t denotes the expectation conditional on time t information, P_t denotes the price level in the monetary union, and δ_{jt+1} is the fraction of the value of the j country's bond that will be repaid in period $t+1$.

For agents to be willing to lend to the $j'th$ country, its interest rate must be allowed to rise above the world interest rate when there is some possibility of a crisis which will be resolved with either default ($\delta_{jt+1} < 1$) or inflation $\left(\frac{P_t}{P_{t+1}} < 1\right)$. If default is used to resolve fiscal crises, then a country with a positive probability of default in the next period, such that $E_t\delta_{jt+1} < 1$, would have an interest rate which is higher than the rate in member countries for which there is no probability of default. If default is ruled out as a policy response to a crisis, then $\delta_{jt+1} \equiv 1$ $\forall j$, t, and all N member-country interest rates are equal at $1 + i_t$, which is the rate that the union monetary policy sets.

2.3 Monetary Policy

We assume that initially the union monetary authority follows an active policy, targeting inflation with a Taylor rule. Following Davig and Leeper (2010), we express the Taylor rule as

$$\frac{1}{1+i_t} = \frac{1}{1+i}\left(\frac{1}{\pi^*}\right) + \kappa\left(\frac{P_{t-1}}{P_t} - \frac{1}{\pi^*}\right) \quad \kappa > 1, \tag{2}$$

where we set the gross inflation target π^* at unity. Following convention, we assume that the price level can be determined to rule out unstable equilibria.[4] Most results would follow if we assume that the monetary authority has some other way of pegging the price level, as in Sims (1997, 1999).

2.4 Fiscal Policy

2.4.1 Fiscal Limits

The **second key assumption** is that the government faces fiscal limits. Specifically, there is an upper bound on the expected present value of the primary surplus relative to output (s_t) that a government can sustain. The limit is given by

$$E_t \sum_{k=0}^{\infty} s_{t+k} \left(\frac{1}{1+r} \right)^k \leq \frac{(1+r)\,\bar{\varphi}}{r}, \tag{3}$$

where $\bar{\varphi}$ has the interpretation of the value of the surplus at the fiscal limit if the surplus were constant. The assumption of fiscal limits follows a growing literature with papers by Davig and Leeper (2010), Davig, Leeper and Walker (2010a,b), Cochrane (2010), and Sims (1997, 1999). We motivate fiscal limits with the realization that taxes are distortionary such that there is an upper bound on the fraction of income that a government can collect in taxes. Additionally, there is a limit below which government spending cannot fall.

2.4.2 Government Flow Budget Constraint

We assume that member governments issue bonds denominated in the common currency. Assuming that a fraction, η_j, of the union's money supply, M_t, is supported by country $j's$ domestic bonds, a member country's nominal flow government budget constraint is

[4] Sim (1999) and Cochrane (2007) make strong arguements about why this assumption is unreasonable.

given by

$$B_{jt} + \eta_j M_t = \delta_{jt}\left[(1 + i_{jt-1})\,B_{jt-1} + \eta_j M_{t-1}\right] + G_{jt} - \tau_{jt} P_t Y_{jt},$$

where B_{jt} is nominal government bonds held by the public, G_{jt} is nominal government expenditures, Y_{jt} is real output and τ_{jt} is the tax rate on nominal output of country j. Letting small letters denote values relative to output and dropping the j notation to simplify, the values of debt relative to output and the primary surplus relative to output for country j can be expressed respectively as

$$b_t = \frac{1}{P_t Y_t}\left(B_t + \frac{1}{1+i_t}\eta M_t\right),$$
$$s_t = \frac{1}{P_t Y_t}\left(\tau_t P_t Y_t - G_t + \left(\frac{i_t}{1+i_t}\right)\eta M_t\right).$$

Allowing for inflation and default, either of which could be created by a fiscal crisis, the government's flow budget constraint can be expressed in terms of debt and the primary surplus relative to output as[5]

$$b_t = \left(\frac{\delta_t}{1+\pi_t}\right)\left(\frac{1+i_{t-1}}{1+g}\right) b_{t-1} - s_t, \tag{4}$$

where $\pi_t = \frac{P_t}{P_{t-1}} - 1$ is the inflation rate, and $g = \frac{Y_t}{Y_{t-1}} - 1$ is the output growth rate.[6] Imposing interest rate parity from equation (1) and defining γ_t as capital loss on the outstanding stock of debt, such that

$$\gamma_t = \left(1 - \frac{\delta_t}{1+\pi_t}\right)\left(\frac{1+i_{t-1}}{1+g}\right) b_{t-1},$$

the equation for the evolution of debt relative to output can be expressed as

$$b_t = (1+r)\,b_{t-1} - s_t - (\gamma_t - E_{t-1}\gamma_t). \tag{5}$$

[5] This ignores the effect of capital gains or losses on seigniorage revenue $\left(\frac{i_t}{1+i_t}\frac{\eta M_t}{P_t Y_t}\right)$ under the assumption that the fiscal authority can adjust the surplus to offset these.

[6] We assume growth is non-stochastic to simplify the analysis. We could analyze the implications of stochastic growth using a linearized model, but we reserve this for future work.

The growth-adjusted interest rate is denoted by $r = \left(\frac{i-g}{1+g}\right)$, and $(\gamma_t - E_{t-1}\gamma_t)$ is the unexpected capital loss on government debt. Capital loss on debt can occur due to either unexpected inflation or default. Expectations of capital loss raise the interest rate. When the capital loss does not occur, debt accumulates in response to the higher interest rate.

Optimization by the representative agent, together with the assumption that governments do not allow their debt to become negative in the limit,[7] implies a government intertemporal budget constraint (IBC) given by[8]

$$\lim_{T\to\infty} E_t b_{t+T} \left(\frac{1}{1+r}\right)^T = (1+r)\, b_{t-1} - (\gamma_t - E_{t-1}\gamma_t) - E_t \sum_{k=0}^{\infty} s_{t+k} \left(\frac{1}{1+r}\right)^k = 0. \quad (6)$$

The government's IBC is a requirement for fiscal sustainability and solvency.

Together equations (3) and (6) imply a fiscal limit on debt relative to GDP given by

$$b_t \leq \frac{\bar{\varphi}}{r}. \quad (7)$$

Since the model is specified in terms of debt and the primary surplus relative to output, we refer to these variables simply as debt and the surplus when there is no confusion.

2.4.3 Surplus Rule

We assume that the fiscal authority is able to anchor beliefs about future fiscal policy by committing to a surplus rule, under which fiscal limits are expected to be satisfied, at least initially. Satisfaction of fiscal limits requires a stronger response of the surplus to debt than that required by Bohn's sustainability criterion. The response of the surplus to debt must be large enough to make the system in debt and the surplus globally stable,

[7] Sims (1997), Woodford (1997) and Daniel (2001) argue that no country, acting to maximize utility of its own agents, would allow the present-value of its debt to become negative in the limit.
[8] Woodford (1994) derives the constraint as an equilibrium condition for a closed economy.

ruling out an explosive equilibrium. Additionally, the surplus rule must yield a long-run equilibrium which respects the fiscal limits.

The **third key assumption** is that the surplus rule is subject to bounded, zero-mean, stochastic shocks, ν_t. Stochastic shocks represent both truly unanticipated fiscal shocks, as with a war, natural disaster, or the recent financial crisis, as well as fiscal policy responses to the state of the economy. Together, fiscal limits and stochastic shocks imply risk to government debt.

We specify a surplus rule, in which the surplus (s_t) responds to its own lag (s_{t-1}) and a linear combination of the target value of the long-run primary surplus (φ) and debt service (rb_{t-1}) at the growth-adjusted interest rate. The surplus rule for a particular country is given by

$$\begin{aligned} s_t &= (1-\alpha)\, s_{t-1} + \alpha\left[(1-\lambda)\,\varphi + \lambda r b_{t-1}\right] + \nu_t, \qquad (8) \\ \frac{r}{1+r} &< \alpha < 1, \qquad \lambda > 1, \qquad 0 < \varphi \leq \bar{\varphi}, \end{aligned}$$

where $(1-a)$ measures persistence in the surplus, reflecting the desire to smooth the effect of shocks over time, consistent with empirical evidence. The parameter λ determines the responsiveness of the surplus to debt service. The restrictions are imposed to yield a stationary long-run equilibrium which respects the fiscal limit, $\bar{\varphi}$.

The parameters α, λ, and φ are policy choices. We show below that the risk of entering regions where the fiscal limits might bind is affected by these parameter choices. Governments understand this risk, and the parameters they choose $(\alpha,\ \lambda,\ \varphi)$ reflect their risk tolerance, determined in part, by the cost of a fiscal crisis. Empirically, countries do choose surplus rules with risk, and the SGP limits on debt and deficits reflect policy-maker concerns that at least some EMU countries might choose relatively risky rules.

Equations (5) and (8) imply dynamic equations for the surplus and debt

$$s_t = (1-\alpha) s_{t-1} + \alpha (1-\lambda) \varphi + \alpha \lambda r b_{t-1} + \nu_t \tag{9}$$

$$b_t = (1 + r - \alpha \lambda r) b_{t-1} - (1-\alpha) s_{t-1} - \alpha (1-\lambda) \varphi - \nu_t - \gamma_t + E_{t-1} \gamma_t. \tag{10}$$

Letting θ represent eigenvalues, which are assumed to be real and distinct, the characteristic equation for each country is given by

$$(1-\alpha)(1+r) - \theta [2 + r(1-\alpha\lambda) - \alpha] + \theta^2 = 0. \tag{11}$$

Both eigenvalues are less than one under the restriction that $\lambda > 1$, ensuring that the model is globally stable. We refer to such policy as "strongly passive" to reflect the stronger restriction on surplus responsiveness to debt than under passive fiscal policy.

The **fourth key assumption** is that a solvent government always honors its debt. Specifically, when the path of debt and the surplus, implied by the chosen fiscal policy and given in equations (9) and (10), keeps debt from exceeding its fiscal limit, the sovereign honors its debt.

2.5 Dynamics

Governments can face fiscal crises because stochastic shocks could imply a dynamic path along which debt exceeds the fiscal limit. Consider the dynamic behavior of debt and the surplus in a newly-formed monetary union in which each country is committed to strongly passive fiscal policy. Solutions for equations (9) and (10) with $\lambda > 1$ are given in the appendix. It is useful to represent the dynamics of the debt-surplus system using country phase diagrams. We construct the phase diagram for each country by subtracting lagged values of the surplus from equation (9) and lagged values of debt from equation (10) to yield:

$$\Delta s_t = s_t - s_{t-1} = -\alpha s_{t-1} + \alpha (1-\lambda) \varphi + \alpha \lambda r b_{t-1} + \nu_t, \tag{12}$$

$$\Delta b_t = b_t - b_{t-1} = (1-\alpha\lambda) r b_{t-1} - (1-\alpha) s_{t-1} - \alpha (1-\lambda) \varphi - \nu_t - \gamma_t + E_{t-1}\gamma_t. \tag{13}$$

The phase diagram under strongly passive fiscal policy, with $\lambda > 1$ and $\nu_t = \gamma_t - E_{t-1}\gamma_t = 0$, is given in Figure 1. Debt service (rb) is on the vertical axis and the surplus (s) is on the horizontal axis. The fiscal limit on debt service is given by $\bar{\varphi}$ on the vertical axis. The $\Delta s = 0$ and $\Delta b = 0$ schedules intersect at point P with $s_t = \varphi = rb_t$, representing a long-run equilibrium of the globally stable system. If initial debt and the surplus are at point A, then the system is expected to travel along AP, eventually reaching the long-run equilibrium point P.

Taking time $t-1$ expectations of equations (12) and (13) with $\gamma_t - E_{t-1}\gamma_t = 0$, yields the slope of any strongly passive adjustment path as

$$\frac{r\left(E_{t-1}b_t - b_{t-1}\right)}{E_{t-1}s_t - s_{t-1}} = r\left[\frac{rb_{t-1} - s_{t-1}}{\alpha\left(\lambda r b_{t-1} - s_{t-1} + (1-\lambda)\varphi\right) + E_{t-1}\nu_t} - 1\right]. \tag{14}$$

Note that in the range for which the slope of the path is positive, a positive expected future fiscal shock $(E_{t-1}\nu_t)$ reduces the one-period-ahead slope of the adjustment path, such that debt is expected to attain lower values in its approach to a long-run equilibrium. Figure 1 is drawn with expected future fiscal shocks equal to zero.

Over time, actual fiscal shocks (ν_t) could move the system away from its initial passive adjustment path, labelled AP, possibly to an adjustment path like HP. This path violates the government's IBC because it requires that debt be expected to pass through points where it exceeds its fiscal limit. Since the fiscal authority could never service or repay such a large debt, agents could not expect to receive the market rate of return along the path HP, implying that HP cannot be an equilibrium path.

3 Fiscal Solvency Crises

Definition 1 ***Equilibrium**: Given constant values for the world interest rate and world price level, a monetary policy rule (equation 2), a surplus rule (equation 9), a fiscal limit on debt (equation 7), and a policy-response in the event of a fiscal crisis for each country, an equilibrium is a set of time series processes for each country's primary surplus, debt, and capital loss on debt,* $\{b_t, s_t, \gamma_t\}_{t=0}^{\infty}$*, such that each government's flow and intertemporal budget constraints (equations 10 and 6) hold, expectations are rational, the debt for each country does not exceed its fiscal limit, and world agents expect to receive the return on assets determined by interest rate parity (equation 1).*

We consider two possible policy responses to the crisis. In the first, the fiscal authority reduces the value of debt through default and thereafter retains its strongly passive fiscal rule. The monetary authority maintains its active rule. In the second, the fiscal authority switches from strongly passive policy to active, with the monetary authority accommodating with policy designed to minimize systematic inflation. We assume that agents know the policy response to the crisis.[9] Crises are most likely to occur in the region in which debt and the surplus are rising. We restrict initial values to this region.

3.1 Default

Consider the case in which the country responds to a sudden stop of capital by reducing the magnitude of debt through default. Specifically, we **assume that, when a government cannot borrow to continue following the surplus rule, it uses default to reduce debt to the adjustment path that is expected to reach a maximum at** $\hat{\varphi}/r$.

[9] Davig, Leeper, and Walker (2010) show that uncertainty regarding how a shock will ultimately be financed can affect dynamic behavior. Cooper, Kempf, and Peled (2008) show how alternative policy responses can represent multiple equilibria based on agents' beliefs about the policy response.

We are allowing the government to choose a default magnitude larger than necessary to restore solvency,[10] but we are assuming that agents know this choice. The value for $\hat{\varphi}/r$ takes on the interpretation of the maximum politically tolerable value of debt, and it can be reached before taxes are actually at the peak of the Laffer curve or spending and transfers are at some absolute minimums.

We define a boundary locus for debt and show that in equilibrium debt cannot travel above this boundary.

Definition 2 *A **boundary locus** for debt service (rb) is located on the adjustment path tangent to the maximum tolerable value for rb, given by $\hat{\varphi}$. The boundary locus is the portion of this path for which the surplus is rising.*

Figure 1 shows the boundary locus for debt as BLXN. Debt service reaches its maximum tolerable value of $\hat{\varphi}$ at point L, and debt service equals the surplus at point X. Note that the boundary locus is defined with respect to the government's maximum tolerable value of debt, which can be less than its fiscal limit.

Given the government's policy response of reducing the value of debt to the boundary locus, the expectation of one-period-ahead capital loss on government debt is determined by the expectation of the distance between debt along the boundary locus $\left(\hat{b}_t\right)$ and the actual value of debt (b_t). We approximate the value for $\hat{b}_t$, implied by equations (12), (13), and (14), by taking a piecewise linear approximation of this path about s_{t-1} and $\hat{b}_{t-1}$, to yield[11]

$$\hat{b}_t = \hat{b}_{t-1} + \left(\beta_{t-1} - 1\right)\left(s_t - s_{t-1}\right), \tag{15}$$

[10] Solvency requires that debt not travel above $\frac{\bar{\varphi}}{r}$, which is deterimned with taxes at the peak of the Laffer curve and with spending and transfers at minimums.

[11] The path for $\hat{b}_t$ is given by the adjustment path in equation (14) for which b_t has a maximum at $\frac{\hat{\varphi}}{r}$ and has $\gamma_t = E_{t-1}\gamma_t = 0$. We need its value for any given value for s_t. Therefore, we approximate its value at time t using its $t-1$ value $\left(\hat{b}_{t-1}\right)$ together with its $t-1$ slope and the change in the actual surplus $(s_t - s_{t-1})$.

where $\left(\beta_{t-1}-1\right)$ is the slope of the boundary locus BLXN,

$$\beta_{t-1}=\frac{r\hat{b}_{t-1}-s_{t-1}}{\alpha\left(\lambda r\hat{b}_{t-1}-s_{t-1}+(1-\lambda)\,\varphi\right)}, \tag{16}$$

and s_t-s_{t-1} is given by equation (12). The denominator in (16) represents the change in the surplus along the boundary locus and is always positive. Note that $\beta_{t-1}>1$ when debt is rising along BL, $\beta_{t-1}=1$ once debt reaches its maximum level at point L, and $\beta_{t-1}<0$ once the surplus exceeds debt service along the segment XN. We show below that a positive crisis probability requires values of the surplus less than debt service $\left(\beta_{t-1}>0\right)$.

We can compute the distance between $\hat{b}_t$ and b_t, which we label Ω_t, by subtracting equation (10) from equation (15) to yield

$$\Omega_t=\hat{b}_t-b_t=x_t=\mu_{t-1}x_{t-1}+\beta_{t-1}\nu_t+\gamma_t-E_{t-1}\gamma_t, \tag{17}$$

where

$$\mu_{t-1}=1+\frac{r\,(1-\lambda)\,(\varphi-s_{t-1})}{\left(\lambda r\hat{b}_{t-1}-s_{t-1}+(1-\lambda)\,\varphi\right)}>0,$$

and x_{t-1} is the state variable determining the distance, Ω_t, and is given by

$$x_{t-1}=\hat{b}_{t-1}-b_{t-1}. \tag{18}$$

In the default case, the state variable determining the time t distance is equal to the lagged distance.[12] The sign for μ_{t-1} reflects the fact that adjustment paths, conditional on different initial values, do not cross.[13]

[12]The state variable will differ from the lagged difference in the switching case, providing the need for the extra notation.

[13]It also reveals that the measure of the distance is a good approximation of the actual distance only in regions of BLXN for which slope of the boundary locus is not changing too rapidly or for which the slopes of the boundary locus and the adjustment path are both positive. Therefore, we use the approximation only in regions for which $\mu_{t-1}>0$.

We define a shadow value of default, analogous to the shadow value of the exchange rate in generation one currency crisis models (Flood and Garber 1984). Conditional on a crisis in which agents refuse to lend, the shadow value of default represents the reduction in the value of debt needed for the economy to reach the boundary locus. The shadow value can be positive or negative.

Definition 3 *The* ***shadow value*** *of capital loss on debt due to default at time* t, $\tilde{\gamma}_t$, *is defined as the value of* γ_t *for which* $\Omega_t = 0$.

Setting Ω_t to zero in equation (17) yields

$$\tilde{\gamma}_t = E_{t-1}\gamma_t - \left(\mu_{t-1}x_{t-1} + \beta_{t-1}\nu_t\right). \tag{19}$$

Substituting into equation (17) yields an expression for x_t as

$$x_t = \gamma_t - \tilde{\gamma}_t, \tag{20}$$

implying that when default does not occur ($\gamma_t = 0$), the distance and the negative of the shadow value are equivalent.

For $\beta_{t-1} > 0$, we **assume that agents expect default with** $\gamma_t = \tilde{\gamma}_t$ **if** $\tilde{\gamma}_t > 0$**.** We prove that this assumption is consistent with a rational expectations equilibrium below in Proposition 2. Under this assumption, the actual value of capital loss due to default is given by

$$\gamma_t = \max\{\tilde{\gamma}_t, 0\} = \max\left\{E_{t-1}\gamma_t - \left(\mu_{t-1}x_{t-1} + \beta_{t-1}\nu_t\right), 0\right\}, \tag{21}$$

where we have used equation (19) to substitute for $\tilde{\gamma}_t$.

In order to solve for the magnitude of default, γ_t, we must first solve for the expectations of default, $E_{t-1}\gamma_t$.

Proposition 1 *Given a value of the surplus for which* $\beta_{t-1} > 0$*, together with a surplus rule with plans for default when the government cannot borrow, an equilibrium solution for the magnitude of expected default* $(E_{t-1}\gamma_t)$ *exists if and only if* $x_{t-1} \geq 0$.

Corollary 1 *When* $x_{t-1} > 0$*, the probability of a crisis in period* t *is less than one, and when* $x_{t-1} = 0$*, the probability of a crisis in period* t *is one.*

All proofs are contained in the appendix. Intuitively, the Proposition and Corollary imply that creditors can be compensated for expectations of default only if there are some values of the fiscal shock for which there would be no default next period. If default next period would occur for all values of the fiscal shock, then there are no values for actual and rationally-expected one-period-ahead default, which both restore fiscal solvency and provide the market rate of return to international creditors.

We can use the phase diagram in Figure 1 to understand expectations of default and the probability of a crisis. When the system is substantially below its boundary locus BLXN, such that no fiscal shock could send it over, the probability of a crisis next period is zero, implying that expectations of default are zero. The system is governed by the arrows of motion toward long-run equilibrium target values. Once the system reaches the neighborhood of the boundary locus, the probability of a crisis becomes positive and agents begin to expect default. The associated default-risk premium on debt increases the interest rate, causing debt to increase more quickly than shown along illustrated adjustment paths.

Once debt has risen so much that it lies on the boundary locus $(x_{t-1} = 0)$, the probability of a crisis next period is unity, and expectations of default are so high that default next period could be avoided only for the most favorable fiscal shock.[14] Using equation

[14]The probability of the most favorable shock in a continuous distribution is zero.

(21) to solve for the magnitude of default once $x_{t-1} = 0$ yields

$$\gamma_t = E_{t-1}\gamma_t - \beta_{t-1}\nu_t \geq 0.$$

The sign restriction is necessary since default must be greater than or equal to zero for any realization of ν_t, including its upper bound value of $\bar{\nu}$. This yields $E_{t-1}\gamma_t \geq \beta_{t-1}\nu_t$ for all values of υ_t, implying

$$E_{t-1}\gamma_t \geq \beta_{t-1}\bar{\nu}.$$

Therefore, when debt is on the boundary locus, there are multiple equilibria, in which actual and expected default must be positive and can be arbitrarily large. To verify, take the expectation of equation (21), when the probability of default is unity, to yield an identity in the expectation.

A value of $x_{t-1} < 0$ would imply that debt is above the boundary locus even with the most favorable fiscal shock, such that the probability of default is unity. Taking the expectation of equation (21), when the probability of default is unity, yields

$$E_{t-1}\gamma_t = E_{t-1}\gamma_t - \mu_{t-1}x_{t-1},$$

an impossibility. Rationally anticipated default cannot restore fiscal solvency because actual default cannot equal itself plus a gap. Therefore, in equilibrium, the dynamics must bound the system away from positions for which $x_{t-1} < 0$. This criterion determines crisis timing.

Proposition 2 *There is no equilibrium without default in period t if $\tilde{\gamma}_t > 0$. Default given by $\gamma_t = \tilde{\gamma}_t$, restores equilibrium.*

The proof in the appendix shows that if $\tilde{\gamma}_t > 0$ and there is no default, then $x_t < 0$, violating conditions for an equilibrium. Intuitively, in the event of a sudden stop, the country promises default in magnitude sufficient to place the system on the adjustment path toward the maximum tolerable value for debt, thereby restoring fiscal solvency. The sudden stop occurs when $\tilde{\gamma}_t > 0$, and the government responds as promised. Therefore, Proposition 2 validates agents' assumption that the government will default whenever $\tilde{\gamma}_t > 0$.

Corollary 2 *Equilibrium after default requires additional default each period until debt falls below the boundary locus on its approach to the long-run equilibrium value.*

Default places the system on the boundary locus, implying that the expectations of default are large enough that default occurs for any fiscal shock. Post-crisis equilibrium is characterized by repeated default which can be arbitrarily large in magnitude. Therefore, following the crisis, markets remain turbulent for some time. Agents expect additional default, interest rates are high, and additional default is necessary. This pattern does eventually end once the dynamics move the economy toward the long-run equilibrium below the boundary locus or to a region in which surplus is greater than debt service.

Corollary 3 *A government which wants to sustain current fiscal policy as long as possible chooses $\hat{\varphi} = \bar{\varphi}$.*

A larger value for $\hat{\varphi}$ implies a higher boundary locus, implying that the distance between debt along the boundary locus and any initial value is greater. The greater this distance the lower the probability of a crisis.

Proposition 3 *Once the surplus reaches debt service, $\beta_{t-1} \leq 0$, the one-period-ahead probability of a crisis is zero.*

In Figure 1, this is segment XN of the boundary locus. Once $\beta_{t-1} \leq 0$, negative shocks do not send the system above the boundary locus, and positive shocks move the system further below the fiscal limit. Falling debt implies that the system is safe in this region.

To summarize, a crisis country can reduce the magnitude of debt through default to restore fiscal solvency and continue a strongly passive fiscal policy. This policy response has the benefit of not threatening the monetary authority's ability to control the price level. The active Taylor rule is able to keep inflation on target. But default has the cost of implying a sequence of future defaults before the country reaches its long run equilibrium. The crisis country's interest rate will anticipate additional defaults and will remain high even after the initial default.

3.2 Monetary and Fiscal Policy Switching

The second possibility we consider is that a government, which cannot borrow to continue its strongly passive fiscal policy, switches to active with the union monetary authority switching to a passive policy. Before analyzing the switching model, it is necessary to understand equilibrium in a monetary union with one active fiscal policy country, $N-1$ strongly passive fiscal policy countries, and a passive monetary authority which pegs the interest rate.

3.2.1 Active Fiscal Policy in the N'th Country and Strongly Passive in the Others

Consider a monetary union in which fiscal policy is active in the $N\prime th$ country and strongly passive in all others. Passive monetary policy sets the interest rate to be consistent with its inflation target of zero, implying that $E_{t-1}\gamma_t = 0$. The active fiscal authority replaces $(1-\lambda)\,\varphi + \lambda r b_{t-1}$ in the surplus rule, given by equation (9), with a revised target for the long-run surplus.

Definition 4 *Target surplus under switching: Defining $\hat{\varphi}/r$ as the largest tolerable debt, as before, if the path for the post-crisis regime, conditional on current values for debt and* *the surplus, reaches a long-run equilibrium for debt equal to $\tilde{\varphi}/r < \hat{\varphi}/r$, then the revised surplus target is $\tilde{\varphi}$. If not, then the surplus target is $\hat{\varphi}$.*

For $\tilde{\varphi} = \hat{\varphi}$, the active fiscal rule becomes

$$s_t = (1-\alpha)\, s_{t-1} + \alpha\hat{\varphi} + \nu_t. \tag{22}$$

Substituting into equation (5) yields the evolution of debt under active fiscal policy

$$b_t = (1+r)\, b_{t-1} - (1-\alpha)\, s_{t-1} - \alpha\hat{\varphi} - \nu_t - \gamma_t + E_{t-1}\gamma_t. \tag{23}$$

The eigenvalues of the characteristic equation for the active fiscal system are $1+r$ and $1-\alpha$. This is a saddlepath-stable system, in which the government's IBC is not satisfied for positions off the saddlepath, and therefore is not satisfied for any initial value of debt, and hence for any price level. Therefore, the equilibrium value for γ_t is determined to set the coefficient on the explosive root to zero. This is an FTPL model of the price level. Since all other fiscal policies are strongly passive, there is only one unstable root in the system of N countries.

The time paths for the surplus and debt in the active-fiscal-policy country, with $\hat{\varphi}$ as the fiscal target, are given in the appendix. These equations can be used to express the saddlepath relationship between debt and the surplus as

$$\hat{b}_t^{sp} = \left(\frac{1-\alpha}{\alpha+r}\right) s_t + \frac{\alpha\,(1+r)}{(\alpha+r)}\frac{\hat{\varphi}}{r}. \tag{24}$$

We construct the phase diagram for an active-fiscal-policy country by subtracting lagged values of the surplus from equation (22) and lagged values of debt from equation (23), to yield

$$\Delta s_t = s_t - s_{t-1} = -\alpha s_{t-1} + \alpha\hat{\varphi} + \nu_t, \tag{25}$$

$$\Delta b_t = b_t - b_{t-1} = rb_{t-1} - (1-\alpha)\, s_{t-1} - \alpha\hat{\varphi} - \gamma_t + E_{t-1}\gamma_t - \nu_t. \tag{26}$$

The phase diagram under active fiscal policy and with $\nu_t = \gamma_t - E_{t-1}\gamma_t = 0$ is given in Figure 2. The saddlepath has a positive slope and is labeled SP. The larger the value for $\hat{\varphi}$, the higher the saddlepath.

Fiscal shocks, ν_t, move the system away from the saddlepath. To assure that debt does not violate its fiscal limit, there must be one jumping variable to assure that the system is on the saddlepath. Price level jumps create jumps in γ_t. From equation (10), b_t jumps with each jump in γ_t, allowing the system to remain on the saddlepath. The monetary authority loses control over the actual price level because price must respond to offset fiscal shocks. However, it retains control over expected and average inflation. Capital gains and losses on government debt are symmetric, implying that expectations of gains and losses are zero in the active fiscal policy, passive monetary policy regime.[15]

3.2.2 Fiscal Crisis Resolved with Policy Switching

We assume that **when a country cannot borrow to continue its strongly passive fiscal rule, the fiscal authority switches to an active fiscal policy with a fiscal target** $\tilde{\varphi} \leq \hat{\varphi}$, and the monetary authority accommodates to minimize systematic inflation. Monetary accommodation requires abandoning the Taylor Rule and letting the interest rate rise when agents begin to anticipate capital loss, as in equation (1) with $\delta_{jt+1} = 1$. This policy allows the interest rate to be determined by fiscal solvency without creating actual inflation, as with a Taylor Rule. Such a policy is akin to that of fighting exchange rate devaluation (and a price level increase) by allowing the interest rate to rise

[15] Cochrane (2001) shows that the introduction of government bonds allows the monetary authority to trade off some of the jump in the price level for post-crisis expected inflation.

(Daniel 2010). In addition to requiring switching when a country cannot borrow, we allow a country to switch stochastically at any time, in contrast to the default model.

We solve the model by specifying a boundary locus, an algorithm for agent expectations, and a shadow value for capital loss on government debt, as before.

Definition 5 *Conditional on the expectation that a fiscal crisis will be resolved with policy switching, accompanied by a revised target surplus given by Definition (4), and by accommodative monetary policy, a* ***boundary locus*** *for debt service* (rb) *is defined as the piecewise continuous path, given by the saddlepath leading to* $\hat{\varphi}$ *for* $s \leq \hat{\varphi}$ *and by* $rb = \hat{\varphi}$ *for* $s \geq \hat{\varphi}$.

Figure 3 superimposes the saddlepath for an active policy system with $\tilde{\varphi} = \hat{\varphi}$ on the passive policy system for a particular country. The boundary locus for debt is CKM.

For values of $s_t \leq \hat{\varphi}$, equations (22), (23) and (24), can used to express the distance between $\hat{b}_t^{sp}$ and b_t as

$$\Omega_t = \hat{b}_t^{sp} - b_t = \frac{\alpha(1+r)}{\alpha + r}\left(x_{t-1} + \frac{\nu_t}{\alpha}\right) + \gamma_t - E_{t-1}\gamma_t, \tag{27}$$

where x_{t-1} is the state variable determining Ω_t and is given by

$$x_{t-1} = \frac{(1-\alpha)}{\alpha}s_{t-1} - \frac{(r+\alpha-\alpha\lambda r)}{\alpha}b_{t-1} + \frac{\hat{\varphi}}{r} + (1-\lambda)\varphi. \tag{28}$$

Note that, as in the default case, the state variable determining the time t distance receives a $t-1$ subscript since its value is known at time $t-1$. Using equations (22), (23), the state variable evolves as

$$x_t = \frac{(r+\alpha)}{\alpha}(\gamma_t - E_{t-1}\gamma_t) + (1+r)\left(x_{t-1} + \frac{\nu_t}{\alpha}\right) - (\hat{\varphi} - \varphi) - \lambda(\varphi - rb_t). \tag{29}$$

For values of $s_t \geq \hat{\varphi}$, the distance is simply $\frac{\hat{\varphi}}{r} - b_t$, and its evolution is governed by the evolution of b_t. This kink at K in the boundary locus in Figure 3 complicates the analysis, but changes little substantively since few crises occur in this region. To focus on intuition, we relegate analysis in the neighborhood of the kink to the appendix.

Equation (27) shows that for $\nu_t = \gamma_t = E_{t-1}\gamma_t = 0$, a positive value for x_{t-1} implies that b_t is below the boundary locus. However, fiscal shocks (ν_t), expectations of inflation ($E_{t-1}\gamma_t$), and inflation (γ_t) can all affect the position of b_t relative to the boundary locus.

We define a shadow value of capital loss on government debt due to inflation ($\tilde{\gamma}_t$) as in Definition 3. Setting $\Omega_t = 0$, from equation (27), and solving yields

$$\tilde{\gamma}_t = E_{t-1}\gamma_t - \frac{\alpha(1+r)}{\alpha+r}\left(x_{t-1} + \frac{\nu_t}{\alpha}\right). \tag{30}$$

Substituting into equation (29) yields

$$x_t = \frac{(r+\alpha)}{\alpha}(\gamma_t - \tilde{\gamma}_t) - (\hat{\varphi} - \varphi) - \lambda(\varphi - rb_t). \tag{31}$$

Note that, in contrast to the default case, in the absence of actual capital loss, distance is not uniquely determined by the shadow value. This happens because the adjustment path under the original policy crosses the boundary locus.

Assume that agents expect policy switching with $\gamma_t = \tilde{\gamma}_t$ **if** $\tilde{\gamma}_t > 0$. We prove that this assumption is consistent with a rational expectations equilibrium below.[16] If we redefine $\mu_{t-1} = \mu = \frac{\alpha(1+r)}{\alpha+r}$ and $\beta_{t-1} = \beta = \frac{(1+r)}{\alpha+r}$, and let $\tilde{\gamma}_t$ be given by equation (30), then inflation in the crisis period is given by equation (21). Using these redefinitions, Proposition 1 applies directly to the switching case if we replace "plans for default" with "plans for switching." The Proposition determining crisis timing changes.

Proposition 4 *There is no equilibrium without policy-switching in period t if $x_t < 0$ or if $\tilde{\gamma}_t > 0$. Policy switching restores equilibrium.*

[16]In contrast to the default case, under switching, a crisis could occur with $\tilde{\gamma}_t < 0$, as we show below.

The proof is in the appendix. Consider the intuition behind crisis dynamics when the crisis will be resolved with policy-switching. A crisis occurs when the government can no longer borrow to continue with the strongly passive fiscal rule. Assume that debt at time $t-1$ is at point H along path HP in Figure 3. The distance between the debt along the boundary locus CK and the value of debt along the path HP becomes negative. Since this is inconsistent with equilibrium, HP cannot be an equilibrium path. However, the expectation of a regime switch in the future makes point H feasible because the expectation raises the expected present-value surplus to equal the value of outstanding debt.

In the neighborhood of the boundary locus CK, the market begins to anticipate inflation. This anticipation forces the interest rate to increase. Once agents anticipate inflation, the system approaches the boundary locus CK at a faster rate than implied by the adjustment path HP, as shown in Figure 3 by the arrow from point H.

A crisis occurs when agents refuse to lend, and there are three ways in which this can happen. As the strongly passive-fiscal system approaches the boundary locus, a negative fiscal shock ($\nu_t < 0$) could send it over such that $x_t < 0$ and $\tilde{\gamma}_t > 0$. The government's response is to promise policy switching. This implies a regime switch with a price level jump to bring the system to the boundary locus. After the policy switch, the system travels along the boundary locus CK toward the long-run equilibrium at point K.[17]

Alternatively, the system could be in the region in which the slope of the adjustment path becomes very flat and eventually negative, the region labeled LN. A shock could send the system over the saddlepath such that $\tilde{\gamma}_t > 0$, but the dynamics could imply that

[17] Since the probability of devaluation is less than one, when a shock occurs requiring devaluation, its magnitude is greater than expected allowing b to jump downwards.

$x_t > 0$. If agents refuse to lend, then $\gamma_t = \tilde{\gamma}_t$ sets $\Omega_t = 0$, as expected. If agents lend and the government does not switch, then equilibrium expectations of debt devaluation exist since $x_t > 0$, and the policy can continue another period. However, if agents lend and the government switches, then a price level jump sets $\gamma_t = \tilde{\gamma}_t$, yielding an instantaneous capital loss on debt. Since the government could choose to switch, generating an infinitely negative rate of return, agents will refuse to lend in the region for which $\tilde{\gamma}_t > 0$.

Finally, the system could be below the boundary locus in the region for which the slope of the adjustment path exceeds the slope of the boundary locus CK, and the dynamics of the surplus and debt under strongly passive policy could imply that debt next period, in the absence of regime switch, would travel above the boundary locus CK ($x_t < 0$). Agents would not lend into this position since no rationally-expected value for future inflation could place the system on the saddlepath. Regime-switch with no change in the price level allows debt and the surplus to move along a saddlepath below CK, implying a long-run surplus below $\hat{\varphi}$.

Equilibrium after policy switching is characterized by the FTPL. The monetary authority pegs the nominal interest rate. The price level experiences both positive and negative jumps, offsetting fiscal shocks, to keep the system on the saddlepath. On average the jumps are zero, implying that expected inflation and $E_{t-1}\gamma_t$ are both at the monetary's target value of zero.

3.2.3 Alternative monetary policies

It is useful to consider the implications of alternative monetary policy reactions for the behavior of the price level, both in the run-up to the crisis and after the crisis. Suppose that the monetary authority maintains its active inflation-fighting stance with the Taylor Rule in the run-up to the crisis, but switches to interest rate pegging after the crisis. Equations (1) for interest rate parity and (2) for the Taylor Rule can be used to show that if the interest rate must rise as agents anticipate capital loss, then current inflation must rise. In such an equilibrium, fiscal needs determine anticipated inflation, while the Taylor Rule determines actual inflation. Therefore, inflation in the run-up to the crisis would rise, and would change stochastically as expectations of capital loss change. Solution would require a different technique. However, even without full solution, it is obvious that the monetary authority would lose control of both the expected rate of change of prices and of their stochastic behavior due to fiscal shocks. These assumptions would be more similar to Davig, Leeper and Walker (2010), where they show that active monetary policy is not sufficient for the monetary authority to maintain control of inflation even when fiscal policy is strongly passive.

If the monetary authority continues its active stance after the crisis, then hyperinflation is possible. In equilibrium, fiscal needs determine the price level and actual inflation. If the monetary authority reacts actively to an increase in inflation, then the dynamics are explosive. Uribe (2006) shows that a government in such a position could choose to use default to reduce debt, allowing the equilibrium price level and therefore inflation to fall, ending the hyperinflation.

4 Simulations of Crisis Risk

The theoretical model demonstrates that diverse fiscal policies can pose a threat to price stability in a monetary union if there is one country in the monetary union which plans to resolve potential solvency problems with switching. Alternatively, if all commit to default, then there is no threat to price stability, but default has negative economic consequences which are not explicitly the topic of this paper. In this section, we quantify the risk of fiscal crises, created by a failure of private creditors to lend. Explicitly, we ask what are the risks of fiscal crises, both in normal times and following a string of negative shocks like that created by the Great Recession and financial crisis? We consider how a country could modify the parameters of its fiscal rule to reduce risk, and we compare the risks under the alternative policy responses of default and policy switching.

Given parameter values for the N surplus rules, the distribution of ν_t, and the method of crisis resolution, the system can be solved numerically and simulated to quantify the risk of one country in the N-country monetary union encountering a crisis over a given period of time. For the simulations, we use estimates for the parameters of the surplus rule from Daniel and Shiamptanis (2009). They provide group mean estimates of parameters for the surplus rule in real terms using cointegration and error-correction models for a panel of ten EMU countries with annual data over the 1970-2006 period. The baseline parameters we use for the simulations are reported in Table 1.

Table 1: Baseline Parameters

	i	α	λ	g
parameters	0.0422	0.4987	1.3003	0.0262
standard errors	0.0061	0.0717	0.0901	0.0027

We adjust these estimates by the group mean panel estimate of the long-run value of output growth g to provide estimates for the parameters expressed as a fraction of output.[18] These parameters imply a growth-adjusted real interest rate given by $r = \frac{i-g}{1+g} = 0.0156$. For the target value of the long-run primary surplus, φ, we use the value of 0.93% of GDP which implies a target value for long-run debt/GDP of 60%.[19] Under the assumption that fiscal shocks have a normal distribution with mean zero, the panel estimate of their standard error is 1.42% of GDP. We let the upper bound on the fiscal shocks, $\bar{\nu}$, be 4.26% of GDP, which corresponds to three standard deviations. We set the desired maximum value of debt $\frac{\hat{\varphi}}{r}$, which is the effective fiscal limit, to 141% of GDP, larger than any of these countries has experienced within the sample.[20] Additionally, we show that this value for the fiscal limit is consistent with market stability in normal times when debt is low and surpluses are relatively high, and with market volatility when debt rises and surpluses fall.

We use 1,000 replications of a ten-year simulation, under the two fiscal responses, to estimate the probability of a fiscal crisis. In each simulation, initial values of debt/GDP, b_{t-1}, and the primary surplus/GDP, s_{t-1}, are used to set the initial value for the state variable determining the distance, x_{t-1}. Based on x_{t-1}, the critical value for the shock, ν_t^*, defined as the largest value which would create a crisis, and the expectation for capital loss, $E_{t-1}\gamma_t$, are calculated. The dynamic system then receives a fiscal shock, ν_t, from the truncated normal distribution and the value for capital loss, γ_t, is computed. If $\gamma_t = 0$, then next period's surplus and debt are updated using equations (9) and (10), which are

[18]The variables in the paper of Daniel and Shiamptanis (2009) are in real terms, whereas the variables in this paper are expressed as percentages of output. This implies that the α in this paper is $\alpha = \frac{0.5118}{1+g} = 0.4987$.

[19]Since the Daniel and Shiamptanis paper estimates the long-run target value for the surplus as a linear function of output, where the constant is not zero, it provides only bounds for the long-run target value of the surplus relative to output. These bounds contain the Maastricht Treaty limit.

[20]We set the maximum value of the surplus, $\hat{\varphi}$, at 2.2% of GDP which implies an effective fiscal limit for debt, $\frac{\hat{\varphi}}{r}$, of 141%. The largest debt/GDP within the sample was 140% for Belgium in 1993.

then used to update x_t. The process is repeated for ten years. If during the ten-year simulation we have a value of $\gamma_t > 0$ or $x_t < 0$ then there is a crisis and the simulation ends. We repeat the ten-year simulation 1000 times. The probability of a crisis over ten-years is the number of crises divided by 1000, the number of replications.

In normal times, creditors are willing to lend to sovereign governments without risk premia on interest rates. A correctly parameterized model should show the risk of crises to be very low. We proxy normal times as those when countries adhere to the SGP rules, and simulate the model with values for initial debt and the primary surplus equal to the upper bound of the SGP limits. We set debt at 60% of GDP and primary surplus at -2.06% which implies an actual surplus of -3% of GDP. Under the baseline parameter values, fiscal policy is very safe with no crises over ten years in the 1,000 replications. We considered several sensitivity analysis scenarios to raise the risk. These include changing parameter values one at a time by one standard deviations in the risky direction.[21] The probability of a fiscal crisis for a country at the SGP limits is zero under all sensitivity analyses designed to increase risk. Therefore, these results suggests that countries with debt and primary surplus at the SGP limits are perfectly safe over the ten year horizon.

Countries which have experienced a negative string of fiscal shocks have higher debt and lower surpluses than SGP requires. The model implies that these countries have higher risk, and we can use the simulations to quantify this risk. We repeated the simulations for Belgium, France and Germany, using their 2009 values of debt/GDP and primary surplus/GDP,[22] which showed small deviations from the SGP limits. Under the baseline

[21] Experiments included raising i to 0.0483, reducing λ to 1.2102, reducing α to 0.427, and reducing g to 0.0235

[22] The 2009 values for debt/GDP and primary surplus/GDP for Belgium were 101.18% and -2.09%, for France were 84.53% and -5.28%, and for Germany were 77.36% and -0.94%. Source: OECD March 2010.

parameters these three countries are perfectly safe over the ten-year horizon. Additionally, these countries are perfectly safe under all one-standard-deviation parameter changes except for the one-standard deviation reduction in the real interest rate or increase in the growth rate. The higher interest rate increases crisis probability dramatically for Belgium, and moderately for France and Germany.[23]

Next we consider whether high-debt countries like Italy and Greece, which have severely violated the SGP rules, face risk over the next ten years. For Italy (Greece), the 2009 value of debt/GDP was 123.57% (114.88%), and the primary surplus/GDP was -0.60% (-8.17%).[24] Under the baseline parameter values, the probability of crisis for Italy is 2.3% and 3.1%, and for Greece is 1.4% and 2.3%, under default and switching respectively. And sensitivity analyses, designed to increase risk, do increase risk dramatically. Therefore, the simulated model is consistent with stability in sovereign debt markets for EMU countries in normal times, but with increasing risk premia due to the possibility of a fiscal crisis in which agents refuse to lend, once debt has risen and surpluses fallen.

The OECD projects debt to rise in 2011 for many European countries.[25] Therefore, we considered how high debt would have to be for Belgium, France and Germany to begin experiencing risk under baseline parameter values, and how crisis probability changes for Italy and Greece as debt increases from its 2009 value. Figure 4 plots the probability of a fiscal crisis as a function of debt/GDP at baseline values with the primary surplus at its

[23]The increase in the real interest rate raises the probability of a crisis for Belgium to 100% under both policy responses, for France to 8.1% and 10.7%, and for Germany to 0.1% and 0.3% under default and switching, respectively. A reduction in output growth raises the probability of a crisis only for Belgium to 2.3% and 3.1%, under default and switching respectively.

[24]Source: OECD Economic Outlook March 2010.

[25]In our model, this would be due to anticipated future fiscal shocks, based on a larger set of variables than those included in our model, to the persistent reduction in the surplus, and to the increase in the interest rate due to risk.

2009 value. Note that the probability of a crisis is rising in debt at an increasing rate and that this probability reaches unity before debt reaches its effective fiscal limit. Figure 4 shows that crisis probability becomes positive for Belgium, France and Germany once debt exceeds 114%, 111%, and 115% of GDP, respectively. All of these exceed OECD forecasts for 2011[26], implying no risk of a crisis for the countries with moderate SGP deviations at baseline parameter values. However, the OECD forecasts 2011 debt for Italy at 129.7% and for Greece at 130.2%. If debt does reach these levels with surpluses at their 2009 values, then the ten-year risk of a fiscal crisis for Italy rises to 10.4% and 13.4% and for Greece to 69.1% and 94.7%, under default and switching respectively.

Risk depends not only on the values for debt and the surplus, but also on the values for the parameters in the fiscal rule. Figure 5 plots crisis probability as a function of the real growth-adjusted interest rate (r) for five countries, whose debt exceeds SGP limits. The baseline estimate for r is 1.56%, and at this baseline value, neither Belgium, France, nor Germany has risk. However, a small increase in the value for i, well within the 61 basis-point standard deviation, raises r sufficiently for each of these countries to have positive risk. Belgium becomes risky with a value for r 15 basis points higher than the baseline, France for a value of r 35 basis points higher, and Germany for r 56 basis points higher. Figure 5 reveals that risk of a crisis is increasing at an increasing rate in r. Therefore, even though countries with moderate deviations from the SGP limits are safe at baseline parameter estimates, a slight change in the real interest, well within a single standard deviation, can imply significant risk.

[26]OECD forecasts for debt for Belgium, France and Germany are 108.45%, 99.15% and 85.46% of GDP, respectively. (OECD Economic Outlook March 2010)

We consider how the crisis probability changes when the policy parameters α and λ change by one standard deviation in the less-risky direction. A change in the parameters of the fiscal rule is a possible response to an impending crisis; alternatively countries could have different policy parameters. When λ increases to 1.3904, implying that the primary surplus responds more strongly to debt, the probability of a fiscal crisis for Italy falls to 1.4% and 2.2%, and for Greece falls to 0.9% and 1.5%, under default and switching respectively. When α increases to 0.5704, implying less persistence in the primary surplus, the probability of a fiscal crisis for Italy falls to 0.8% and 1.0%, and for Greece falls to 0.1% and 0.3%, under default and switching respectively. The results in Figure 6 show that the probability of fiscal crisis is sensitive to changes in α and λ However, even if countries choose less-risky policy parameters, they cannot eliminate risk for large values of debt/GDP. The risk is due to the fiscal limit and the stochastic shocks to the surplus.

These estimates of risk are based on an effective fiscal limit $\left(\frac{\hat{\varphi}}{r}\right)$ of 141% of GDP. Perhaps risk is lower because Greece and Italy would actually be willing and able to tolerate a higher value for debt. Therefore, we consider how high the effective fiscal limit $\left(\frac{\hat{\varphi}}{r}\right)$ would have to be for Italy and Greece to be safe. Under baseline parameters, Figure 7 shows that crisis probability becomes zero for both countries once $\frac{\hat{\varphi}}{r}$ increases to 148%. Interest rate premia on debt in early 2010 combined with the simulated model implies that the fiscal limit is not this high.

We can use these results to compare crisis risk under the two alternative policy responses, default and switching. We find that the probability of a crisis is always higher under switching. Figure 3 shows that for high values of debt and the surplus, the boundary locus for switching is below the boudary locus for default. Therefore, Germany has another reason to be requesting a commitment to default, and that is, to reduce the probability of a crisis.

The simulations show that crisis probability rises at an increasing rate, either as parameter values change in the risky direction or as debt increases. This is because expectations of debt devaluation rise as the economy approaches the boundary locus. Higher expected debt devaluation increases the interest rate and increases the rate at which debt accumulates. As a country approaches the boundary locus, a slight change in parameters or in debt can create a dramatic change in crisis probability. This illustrates forcefully that a country receiving favorable shocks can substantially reduce and/or eliminate the probability of crisis. It also illustrates the reverse. A country can substantially increase its crisis probability with small changes in debt which push it critically toward the boundary locus. They also indicate that quantitative estimates of risk are highly sensitive to some parameter values.

These simulations of fiscal risk are conditional on fiscal policy following the rule estimated for the panel over the period 1970-2006. The sensitivity analyses allow countries to differ in their parameter values by a single standard deviation. The simulations ignore the likely correlation of fiscal shocks across countries combined with the fact that a fiscal crisis in one country can affect the interest premium in another under policy-switching. This implies that risk is actually higher, and future research is needed to address this.

5 Contributions to the Literature

Consider an alternative fiscal response to those presented in this paper, explicitly the response analyzed by Oviedo and Mendoza (2006). They assume that the government abides by a self-imposed debt limit, which is the upper bound of the value of the primary surplus it can raise. This is the same kind of debt limit Aiyagari (1994) assumes for agents to assure that they repay their debts. Once debt reaches this limit, the government raises the primary surplus to the upper bound and debt ceases to grow. This self-imposed debt limit could in principle be any value of debt at or below the Aiyagari limit, including the sixty percent SGP limit. If a government were able to credibly commit to this policy, then there would be no lending crisis and no threat of either default or inflation. As fiscal shocks send debt toward its limit, interest rates would not rise. Once debt hits the limit, creditors would still be willing to lend. In a growth context, they would actually lend because the limit allows debt to rise at the rate of growth of GDP. Perhaps the SGP limit on debt was supposed to work in this manner with governments themselves not allowing their debt to rise above the limit. However, if the government does not self-impose this limit, the market will allow the government to continue to borrow, albeit at a risk premium.

Sims (1997), Davig and Leeper (2010 a,b), and Davig, Leeper, and Walker (2010) consider fiscal policy in which the passive fiscal authority continues to borrow, but plans to resolve its potential solvency problem by switching to active fiscal policy either exogenously or endogenously. With exogenous switching, agents know that policy could change any time, and with endogenous switching, either the probability of switching is increasing in debt, or switching occurs once debt reaches a government-chosen limit. The models are set up such that switching requires a positive price level jump. The positive probability of switching with a price level jump raises the interest rate in anticipation of a switch. If the probability of switching depends on the level of debt, then interest is higher, the higher is debt. If the monetary authority follows an active Taylor Rule, then the possibility of switching creates inflation even when switching does not occur. Active monetary policy cannot counter this threat.

The model in this paper differs in several ways and yields results which are slightly more positive for the monetary authority. First, we consider the possibility that the crisis government could resolve its potential solvency problems with default. Sims (1999) notes that when government debt is denominated in domestic currency, governments need never default since they can always print money to repay debt. In a monetary union, each individual government gives up the right to print money, and default becomes a realistic resolution for a solvency crisis. This paper shows that if a government could credibly commit to this resolution, then fiscal policy poses no direct threat to price stability in the monetary union.[27] There could be other undesirable effects of default which are not addressed in this paper. However, a commitment to default to restore solvency cedes price level control to the monetary authority. This could justify Germany's demand in October 2010 that any future EMU loans be accompanied by a plan for the orderly restructuring of debt, equivalently a default in our terminology.

In our model, the timing of switching and its implications for a price level jump are different. Switching can occur randomly or be forced in response to a crisis. If it is forced, then switching is the response to a crisis in which agents refuse to lend because additional borrowing would make the government insolvent. This contrasts with other models in which switching is a government decision. In the forced-switching equilibrium, a price level jump on the switching date is usually required to restore solvency and allow

[27] This should be seen as a corroboration of Sims (1999).

the additional borrowing. Therefore, in the neighborhood of forced switching, interest rates rise in anticipation. If the monetary authority maintains its active Taylor Rule, then inflation occurs in the run-up to the crisis, even if the country manages to receive favorable shocks and avoid the crisis.

However, our analysis shows that the monetary authority can change its policy from an active Taylor Rule to one of pegging the price level, as it would with a fixed exchange rate. This policy allows the interest rate to rise with expected inflation. With this change, it does not lose price level control, conditional on the country escaping the crisis. If the crisis occurs, it does lose control with the onset of the crisis, since the post-crisis equilibrium is characterized by the FTPL. Consistent with the FTPL literature, its best policy to avoid a hyperinflation is to switch to interest rate pegging, controlling average and expected inflation, but losing control over actual inflation. Switching could occur without a crisis, but if it does, there is no need for a price level jump to restore solvency because the government is solvent under switching at current prices.[28] Therefore, if unforced switching occurs, then we assume that the fiscal authority chooses a target surplus for the new active fiscal rule which requires no price level jump.[29] With this assumption, if the system is not in the neighborhood of a crisis with forced switching, there is no anticipated inflation and no loss of control of the price level.

We can use the results from this model to consider other possible policy responses by the European Monetary Union to the fiscal crises which began in December 2009 with Greek difficulty in rolling over its debt. The increase in interest rates, the refusal of the

[28]This means that the adjustment path under switching does not violate the upper bound.

[29]This freedom to choose a new target below $\hat{\varphi}$ when the fiscal authority changes its fiscal rule is what allows switching with no price level jump.

private market to lend, and the high levels of debt and deficits relative to GDP, are all consistent with the model's depiction of a fiscal solvency crisis. Therefore, the policy response must resolve the solvency problem.

Official non-concessionary loans do not contribute to solvency. Non-concessionary loans could be accompanied by outside oversight enforcing a fiscal austerity which would be politically impossible in its absence, effectively raising $\hat{\varphi}$ toward $\bar{\varphi}$, or raising short-term tolerable surpluses through increases in the mean of ν_t. Official transfers do contribute to solvency, but carry moral hazard problems and political problems associated with redistribution within the monetary union. The crisis country could succeed from the monetary union and implement switching, or simply create seigniorage, thereby sparing the monetary union from loss of price stability at the cost of breaking it apart. These resolutions, together with that of default, allow the monetary authority to retain price level control. If solvency resolution comes with policy switching and if this is the expected outcome, the monetary authority could lose control over prices in the run-up to the crisis and does lose control after switching.

6 Conclusions

What does this model contribute to our understanding of the threat posed by diverse fiscal policies to price stability in a monetary union? And how can it contribute to understanding and resolving the EMU fiscal crises which began at the end of 2009 with Greek difficulties rolling over debt?

In the presence of fiscal limits and stochastic shocks to the budget surplus, strongly passive fiscal policy is not sufficient to assure that a government will always be able to borrow from the private market to carry out its strongly passive surplus rule. According to the model, even when the government is following a strongly passive fiscal policy, with a long-run value for debt well below the fiscal limit, shocks could send debt and the surplus toward a path which violates the fiscal limits. Agents would not lend along such a path because they cannot expect to receive the market rate of return, creating a lending crisis. The crisis requires a fiscal response because the government cannot continue its strongly passive surplus rule if it cannot borrow. The threat to price stability depends on government plans to resolve the problem that strongly passive fiscal policy does not guarantee solvency.

We consider two responses: maintenance of the strongly passive fiscal policy combined with default to reduce the magnitude of outstanding debt, and policy switching. If all countries in the monetary union commit to resolve a potential solvency crisis with default, then diverse fiscal policies do not threaten the monetary authority's ability to control prices. In contrast, if the policy response to a crisis is fiscal policy switching in even one country, then the monetary authority can lose control of the price level due to a lending crisis in that country. In the run-up to the crisis, the monetary authority has the ability to choose a policy for which it can maintain price-level control. However, after the switch, it loses control over actual inflation, while retaining control over expected and average inflation.

The model makes several additional contributions to the literature. In contrast to other models, crisis timing is market-determined. Once agents refuse to lend, policymakers must respond to restore solvency and lending. Crises occur prior to debt reaching its upper bound. This occurs due to the upward-sloping boundary locus and to the fact that agents might expect that policy-makers have a limited political will and tolerate debt below its actual fiscal limit. When the response is default, the equilibrium magnitude of capital loss on debt is endogenously determined to restore fiscal solvency, and, in contrast to many other models, is never one hundred percent. As in other models, interest rates rise in anticipation of the crisis and accelerate the growth rate of debt, implying that crises develop quickly.

We provide simulations, using estimated parameter values and initial conditions from EMU countries, to determine the probability of a fiscal crisis in the next ten years, under alternative assumptions about the fiscal response to a crisis. We find that a country operating at the upper bound of the SGP Treaty is perfectly safe under the baseline parameter values. Additionally, countries like Belgium, France and Germany with small violations, are also perfectly safe under the baseline parameter values. However, countries like Italy and Greece with high debt, are not safe under the baseline parameter values, and for either a small increase in the real growth-adjusted interest rate or a higher level of debt relative to GDP, risk rises at an increasing rate.

7 Appendix: Default

7.1 Solutions

When fiscal policy is passive and monetary policy active, the time paths for each country's surplus and debt relative to output are

$$\begin{aligned}
s_t &= \varphi + \frac{(\theta_2 - 1 + \alpha)\,\theta_1^t}{(1-\alpha)(\theta_1 - \theta_2)} \left\{ (\alpha - 1)(s_0 - \varphi) + (\theta_1 - 1 + \alpha)\left(b_0 - \frac{\varphi}{r}\right) \right. \\
&\qquad \left. + \sum_{k=1}^{t} \theta_1^{-k} \left[-\theta_1 \nu_k - (\theta_1 - 1 + \alpha)(\gamma_k - E_{k-1}\gamma_k) \right] \right\} \\
&\quad + \frac{(\theta_1 - 1 + \alpha)\,\theta_2^t}{(1-\alpha)(\theta_1 - \theta_2)} \left\{ (1-\alpha)(s_0 - \varphi) - (\theta_2 - 1 + \alpha)\left(b_0 - \frac{\varphi}{r}\right) \right. \\
&\qquad \left. + \sum_{k=1}^{t} \theta_2^{-k} \left[\theta_2 \nu_k + (\theta_2 - 1 + \alpha)(\gamma_k - E_{k-1}\gamma_k) \right] \right\}
\end{aligned} \tag{32}$$

$$\begin{aligned}
b_t &= \frac{\varphi}{r} + \frac{\theta_1^t}{\theta_1 - \theta_2} \left\{ (\alpha - 1)(s_0 - \varphi) + (\theta_1 - 1 + \alpha)\left(b_0 - \frac{\varphi}{r}\right) \right. \\
&\qquad \left. + \sum_{k=1}^{t} \theta_1^{-k} \left[-\theta_1 \nu_k - (\theta_1 - 1 + \alpha)(\gamma_k - E_{k-1}\gamma_k) \right] \right\} \\
&\quad + \frac{\theta_2^t}{\theta_1 - \theta_2} \left\{ (1-\alpha)(s_0 - \varphi) - (\theta_2 - 1 + \alpha)\left(b_0 - \frac{\varphi}{r}\right) \right. \\
&\qquad \left. + \sum_{k=1}^{t} \theta_2^{-k} \left[\theta_2 \nu_k + (\theta_2 - 1 + \alpha)(\gamma_k - E_{k-1}\gamma_k) \right] \right\}
\end{aligned} \tag{33}$$

where $\theta_1 \leq 1$ and $\theta_2 < 1$ are the eigenvalues of the characteristic equation (11). When the country is far from a crisis, $\gamma_t = E_{t-1}\gamma_t = 0$. The values for γ_t and its expectations in the neighborhood of a crisis are endogenized as part of the model's full solution.

7.2 Proofs

7.2.1 Proof of Proposition 1

We prove that there is no value for $E_{t-1}\gamma_t$ when $x_{t-1} < 0$. Assume a value for s_{t-1} for which $\beta_{t-1} > 0$ and $\mu_{t-1} > 0$, and define $f(\nu_t)$ as a bounded, symmetric, mean-zero distribution for ν_t, with bounds $\pm\, \bar{\nu}$. Define ν_t^* as a critical value for ν_t such that

$$\begin{aligned} \gamma_t &> 0 \text{ for } \nu_t < \nu_t^*, \\ \gamma_t &= 0 \text{ for } \nu_t \geq \nu_t^*. \end{aligned}$$

When such a critical value exists, taking the expectation of equation (21) yields

$$E_{t-1}\gamma_t = \int_{-\bar{\nu}}^{\nu_t^*} \gamma_t f(\nu_t)\, d\nu_t = \int_{-\bar{\nu}}^{\nu_t^*} \left[E_{t-1}\gamma_t - \mu_{t-1}x_{t-1} - \beta_{t-1}\nu_t\right] f(\nu_t)\, d\nu_t. \tag{34}$$

Defining $F(\nu_t^*)$ as the cumulative at ν_t^* and collecting terms on the expectation yields

$$\left[1 - F(\nu_t^*)\right] E_{t-1}\gamma_t = -\mu_{t-1}x_{t-1}F(\nu_t^*) - \beta_{t-1}\int_{-\bar{\nu}}^{\nu_t^*} \nu_t f(\nu_t)\, d\nu_t. \tag{35}$$

Substituting into equation (21) yields an implicit expression for γ_t as

$$\left[1 - F(\nu_t^*)\right]\gamma_t = \max\left\{0, -\left[\mu_{t-1}x_{t-1} + \beta_{t-1}\int_{-\bar{\nu}}^{\nu_t^*} \nu_t f(\nu_t)\, d\nu_t + \beta_{t-1}\left[1 - F(\nu_t^*)\right]\nu_t\right]\right\}, \tag{36}$$

where $F(\nu_t^*)$ has the interpretation as the probability of crisis.

To determine the probability of crisis, $F(\nu_t^*)$, and the expectations of default, $E_{t-1}\gamma_t$, first solve for ν_t^*. Define $\chi_t = \int_{-\bar{\nu}}^{\nu_t^*} \nu_t f(\nu_t)\, d\nu_t + \left[1 - F(\nu_t^*)\right]\nu_t^*$. A solution for ν_t^* exists iff there exists a value for ν_t^*, satisfying $-\bar{\nu} \leq \nu_t^* \leq \bar{\nu}$, which sets $\mu_{t-1}x_{t-1} + \beta_{t-1}\chi_t = 0$ such that $\gamma_t = 0$ in equation (36).

Given that $\beta_{t-1} > 0$ and $\mu_{t-1} > 0$, the proof must show that $\chi_t \leq 0$ for all possible values for ν_t^*. Let ν_t^* take on its smallest possible value of $-\bar{\nu}$. Then $\chi_t = -\bar{\nu} < 0$. The derivative of χ_t with respect to ν_t^* is given by $1 - F(\nu_t^*)$. For $\nu_t^* < \bar{\nu}$, the derivative is positive. Therefore, as ν_t^* rises, χ_t rises monotonically. Once ν_t^* takes on its largest possible value, given by $\bar{\nu}$, $1 - F(\bar{\nu}) = 0$, and χ_t takes on its maximum value of zero.

Therefore, $\chi_t \leq 0$ for all feasible values of ν_t^*. Since $\chi_t \leq 0$, a necessary and sufficient condition for $\mu_{t-1}x_{t-1} + \beta_{t-1}\chi_t = 0$ is $x_{t-1} \geq 0$.

When $x_{t-1} \geq 0$, a solution for ν_t^* exists, and the expectations of default are given by the solution of equation (35). When $x_{t-1} < 0$, there is no equilibrium interest which can compensate the lender for expectations of future default and restore fiscal solvency, such that there is no equilibrium without default.

7.2.2 Proof of Corollary 1

When $x_{t-1} > 0$, $\chi_t < 0$, requiring $\nu_t^* < \bar{\nu}$. Therefore, the probability of a crisis, given by $F(\nu_t^*)$, is less than one. When $x_{t-1} = 0$, ν_t^* must set $\chi_t = 0$, implying that $\nu_t^* = \bar{\nu}$. Therefore, the probability of a crisis, given by $F(\bar{\nu})$, is one.

7.2.3 Proof of Proposition 2

Equilibrium in period t requires $x_t \geq 0$. This is because Proposition 1 shows that there can be no rational expectations value for $E_t\gamma_{t+1}$ when $x_t < 0$ under the initial policy mix without default. Therefore, if $x_t < 0$, there is no equilibrium unless the country defaults. Using equation (17) and (19), yields

$$x_t = \mu_{t-1}x_{t-1} + \beta_{t-1}\nu_t - E_{t-1}\gamma_t + \gamma_t = \gamma_t - \tilde{\gamma}_t.$$

Therefore, when $\tilde{\gamma}_t > 0$, $x_t < 0$ unless the country defaults. A positive shadow rate triggers default. Default, with $\gamma_t = \tilde{\gamma}_t$, sets $x_t = 0$, restoring equilibrium by Proposition 1.

7.2.4 Proof of Corollary 2

A default in period t, which brings the system to the boundary locus, implies that $x_t = 0$. When $x_t = 0$, the probability of a crisis in period $t+1$ is unity by Corollary 1, and Proposition 1 yields $E_t \gamma_{t+1} \geq \beta_t \bar{\nu}$. Given a realization for $\nu_{t+1,}$ default occurs in the magnitude to set $x_{t+1} = 0$. The pattern persists until the dynamics imply that debt and the surplus reach a point for which surplus is greater than debt service or until debt falls below BLX.

7.2.5 Proof of Corollary 3

The position of the boundary locus is increasing in $\hat{\varphi}$.

7.2.6 Proof of Proposition 3

The calculations in the proof of Proposition 1 can be used to demonstrate that the expected value of default is zero once $\beta_{t-1} = 0$, implying that the one-period-ahead probability of a crisis is zero. Once $\beta_{t-1} < 0$, negative shocks reduce the distance such that crises do not occur for negative shocks. Positive shocks increase the distance, but since they reduce debt, they cannot send debt above $\frac{\hat{\varphi}}{r}$. Therefore, if the system has not traveled above the boundary locus, as necessary in equilibrium, and has reached a point such that $\beta_{t-1} < 0$, a shock cannot send debt above its fiscal limit from this position. One-period-ahead crisis probability is zero.

8 Appendix: Switching

8.1 Solutions

The time paths for each country's surplus and debt relative to output are

$$s_t = \hat{\varphi} + (1-\alpha)^t \left[s_0 - \hat{\varphi} + \sum_{k=1}^{t} (1-\alpha)^{-k} \nu_k \right],$$

$$b_t = \frac{\hat{\varphi}}{r} + (1-\alpha)^t \left(\frac{1-\alpha}{r+\alpha} \right) \left[s_0 - \hat{\varphi} + \sum_{k=1}^{t} (1-\alpha)^{-k} \nu_k \right]. \tag{37}$$

The requirement that the coefficient on the explosive root be zero implies

$$b_0 - \left(\frac{1-\alpha}{\alpha+r} \right) s_0 + \sum_{k=1}^{t} (1+r)^{-k} \left[E_{k-1}\gamma_k - \gamma_k - \frac{1+r}{\alpha+r} \nu_k \right].$$

8.2 Proof of Proposition 4

Assume first that the system is far enough from the kink in the boundary locus such that it is not relevant.

Proposition 1 with redefinitions shows that there is no equilibrium rational expectations value for $E_t\gamma_{t+1}$ when $x_t < 0$, and there is an equilibrium with $x_t \geq 0$. Therefore, if $x_t < 0$, then there is no equilibrium in the absence of policy switching.

Policy switching restores equilibrium by setting $\Omega_t = 0$. There are three ways in which this can happen, depending on the value for $\tilde{\gamma}_t$. When $\tilde{\gamma}_t > 0$ and $x_t < 0$, a price level jump setting $\gamma_t = \tilde{\gamma}_t$, assures $\Omega_t = 0$, placing the system on the saddlepath.

However, from equation (29), it is possible for $x_t < 0$, when $\tilde{\gamma}_t \leq 0$ since $-(\hat{\varphi} - \varphi) - \lambda(\varphi - rb_t)$ can be negative. In this event, policy switching entails choosing a lower target surplus $\tilde{\varphi} < \hat{\varphi}$, in order to place the system on a lower saddlepath without a price level change. The lower target surplus reduces the distance between debt along the new lower saddlepath and its current value to zero, reducing Ω_t to zero.

Since $-(\hat{\varphi}-\varphi)-\lambda(\varphi-rb_t)$ can also be positive, it is possible for $\tilde{\gamma}_t>0$ and $x_t>0$. If this occurs when $rb_t<\hat{\varphi}$, such that debt is below its desired maximum, then agents could choose to lend, since they there would be an equilibrium with well-defined expectations of inflation. However, if agents lend and switching with a price level jump $\gamma_t=\tilde{\gamma}_t$ occurs, they would experience an instantaneous capital loss to set debt on the boundary locus. Therefore, agents refuse to lend and switching with $\gamma_t=\tilde{\gamma}_t$ occurs.

Now consider modifications necessary in the neighborhood of the flat boundary locus. The distance between the value of debt along the boundary locus and its current value is given by

$$\Omega_t=\begin{cases}\hat{b}_t^{sp}-b_t=\frac{1-\alpha}{\alpha+r}(s_t-\hat{\varphi})+\frac{\hat{\varphi}}{r}-b_t & \text{for } s_t\leq\hat{\varphi}\\ \hat{b}_t^{sp}-b_t=\frac{\hat{\varphi}}{r}-b_t & \text{for } s_t\geq\hat{\varphi}\end{cases}$$

Define

$$y_{t-1}=(1-\alpha)s_{t-1}-r(1-\alpha\lambda)b_{t-1}+\alpha(1-\lambda)\varphi,$$

$$w_{t-1}=\frac{\hat{\varphi}}{r}-b_{t-1}.$$

Using these definitions

$$\Omega_t=\begin{cases}\frac{1+r}{\alpha+r}(y_{t-1}+\alpha w_{t-1}+\nu_t)-E_{t-1}\gamma_t+\gamma_t & \text{for } s_t\leq\hat{\varphi}\\ (y_{t-1}+w_{t-1}+\nu_t)-E_{t-1}\gamma_t+\gamma_t & \text{for } s_t\geq\hat{\varphi}\end{cases}$$

Define ν_t^c as the critical value of the fiscal shock for which the two distances are equal, and equivalently for which $s_t=\hat{\varphi}$. Equating the two values for the distance yields

$$\frac{1+r}{\alpha+r}(y_{t-1}+\alpha w_{t-1})-(y_{t-1}+w_{t-1})=\frac{\alpha-1}{\alpha+r}\nu_t^c.$$

When $\nu_t\leq\nu_t^c$, then $s_t\leq\hat{\varphi}$ and when $\nu_t\geq\nu_t^c$, then $s_t\geq\hat{\varphi}$.

The capital loss on debt is given by

$$\gamma_t = \begin{cases} \max\left\{\left(-\frac{1+r}{\alpha+r}\left(y_{t-1}+\alpha w_{t-1}+\nu_t\right)+E_{t-1}\gamma_t\right),0\right\} & \text{for } \nu_t \le \nu_t^c \\ \max\left\{\left(-\left(y_{t-1}+w_{t-1}+\nu_t\right)+E_{t-1}\gamma_t\right),0\right\} & \text{for } \nu_t \ge \nu_t^c \end{cases} \tag{38}$$

Letting ν_t^* be the critical value below which capital loss occurs and above which it does not, as in Proposition 1, and taking the expectation of the capital loss yields

$$\begin{aligned} E_{t-1}\gamma_t &= \int_{-\bar{\nu}}^{\min\{\nu_t^c,\nu_t^*\}} \left(E_{t-1}\gamma_t - \frac{1+r}{\alpha+r}\left(y_{t-1}+\alpha w_{t-1}+\nu_t\right)\right) f\left(\nu_t\right) d\nu_t \\ &+ \int_{\min\{\nu_t^c,\nu_t^*\}}^{\nu_t^*} \left(E_{t-1}\gamma_t - \left(y_{t-1}+w_{t-1}+\nu_t\right)\right) f\left(\nu_t\right) d\nu_t, \end{aligned}$$

which implies

$$\begin{aligned} \left[1-F\left(\nu_t^*\right)\right] E_{t-1}\gamma_t &= -\int_{-\bar{\nu}}^{\min\{\nu_t^c,\nu_t^*\}} \frac{1+r}{\alpha+r}\left(y_{t-1}+\alpha w_{t-1}+\nu_t\right) f\left(\nu_t\right) d\nu_t \\ &- \int_{\min\{\nu_t^c,\nu_t^*\}}^{\nu_t^*} \left(y_{t-1}+w_{t-1}+\nu_t\right) f\left(\nu_t\right) d\nu_t. \end{aligned}$$

Substituting into equation (38) and letting ξ_t be an indicator function, which equals zero when $\nu_t \le \nu_t^c$ and unity otherwise, yields an implicit expression for γ_t

$$\begin{aligned} \left[1-F\left(\nu_t^*\right)\right]\gamma_t &= -\frac{1+r}{\alpha+r}\left[\left(y_{t-1}+\alpha w_{t-1}\right)+\nu_t\left(1-F\left(\nu_t^*\right)\right)+\int_{-\bar{\nu}}^{\nu_t^*} \nu_t f\left(v_t\right) d\nu_t\right] \\ &+\xi_t\frac{1-\alpha}{\alpha+r}\left[\nu_t\left(1-F\left(\nu_t^*\right)\right)-\nu_t^c\left(1-F\left(\nu_t^c\right)\right)+\int_{\nu_t^c}^{\nu_t^*} \nu_t f\left(v_t\right) d\nu_t\right]. \end{aligned}$$

To determine the probability of a crisis, $F\left(\nu_t^*\right)$, and expectations of debt devaluation, $E_{t-1}\gamma_t$, first solve for ν_t^*. When $\nu_t^* \le \nu_t^c$, this can be done as in Proposition 1 by solving for the value of ν_t^* which satisfies $\left(y_{t-1}+\alpha w_{t-1}\right)+\nu_t^*\left(1-F\left(\nu_t^*\right)\right)+\int_{-\bar{\nu}}^{\nu_t^*} v_t f\left(v_t\right) d\nu_t = 0$. Results are identical to those in Proposition 1.

If not, then the indicator function takes on the value of unity, and the extra term must be included. The term, $\nu_t^*\left(1-F\left(\nu_t^*\right)\right)-\nu_t^c\left(1-F\left(\nu_t^c\right)\right)+\int_{\nu_t^c}^{\nu_t^*} v_t f\left(v_t\right) d\nu_t \ge 0$ for $\nu_t^* \ge \nu_t^c$.

To prove, set $\nu_t^* = \nu_t^c$, and note the term is zero. The derivative with respect to ν_t^* equals $(1 - F(\nu_t^*)) > 0$ for $\nu_t^* < \bar{\nu}$. Therefore, the value of $[1 - F(\nu_t^*)]\,\gamma_t$ is larger for any value for $\nu_t^* > \nu_t^c$, implying that γ_t will be positive for more values of ν_t. This implies a larger ν_t^*.

If there is no value of $\nu_t^* \leq \bar{\nu}$, then no one would have lent into this period, and the crisis would occur in period $t - 1$.

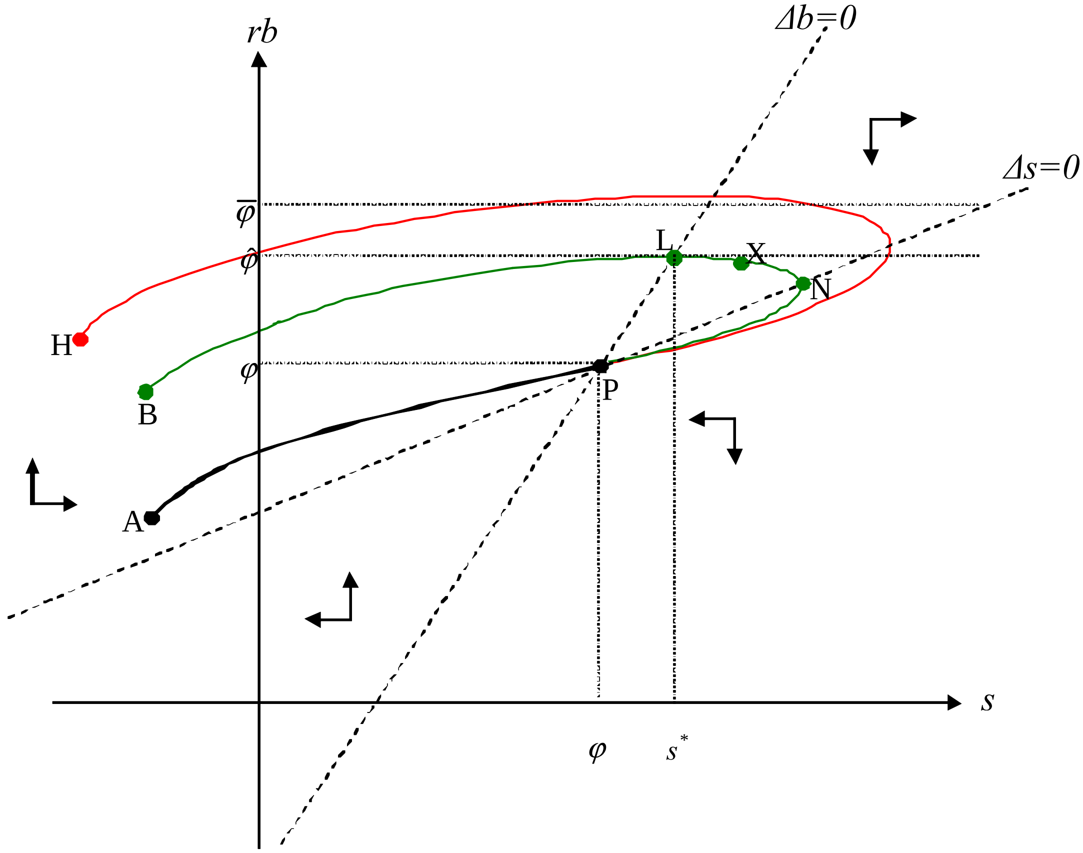

Figure 1: Strongly Passive Fiscal Policy

Note: $s^* = \frac{\hat{\varphi}(1-\alpha\lambda)-\alpha(1-\lambda)\varphi}{1-\alpha}$ is the value of s along the adjustment path BLP at the point L with $rb = \hat{\varphi}$.

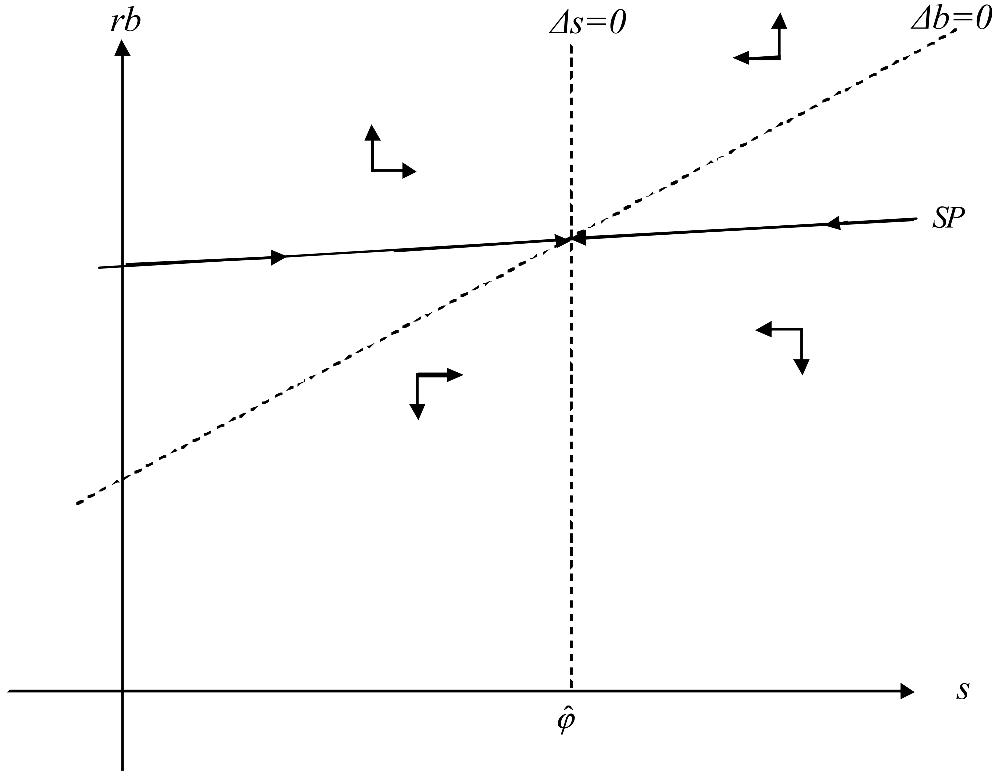

Figure 2: Active Fiscal Policy

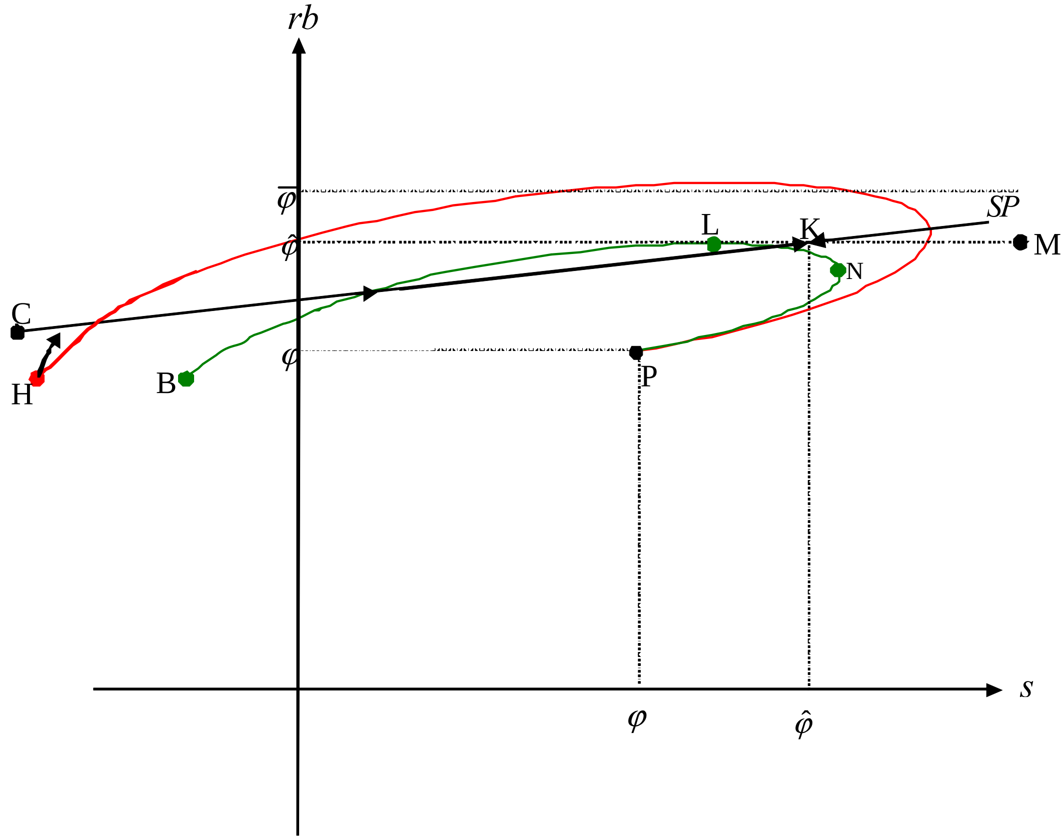

Figure 3: Switching

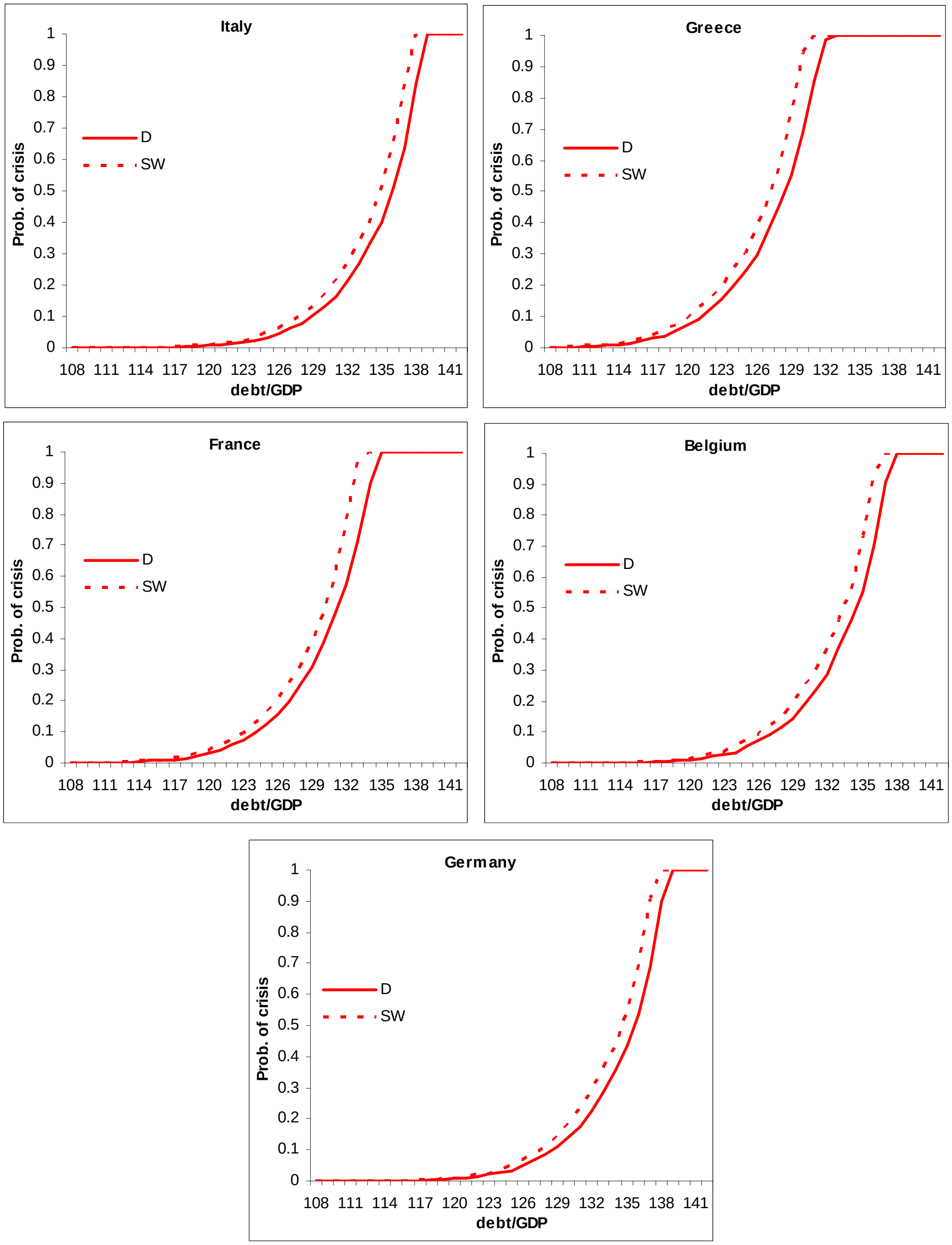

Figure 4: The probability of fiscal crisis as a function of debt/GDP

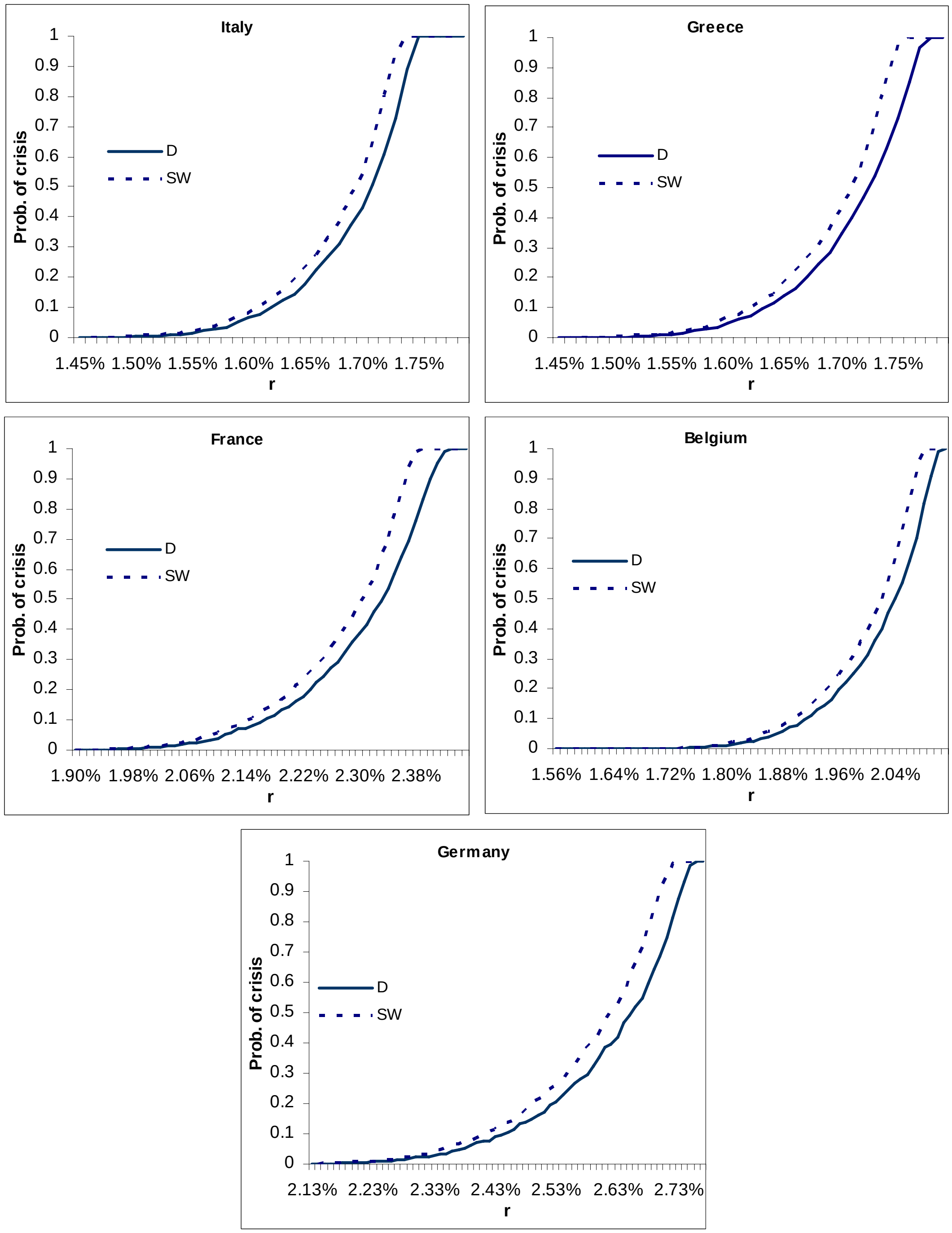

Figure 5: The probability of a crisis as a function of growth-adjusted real interest rate.

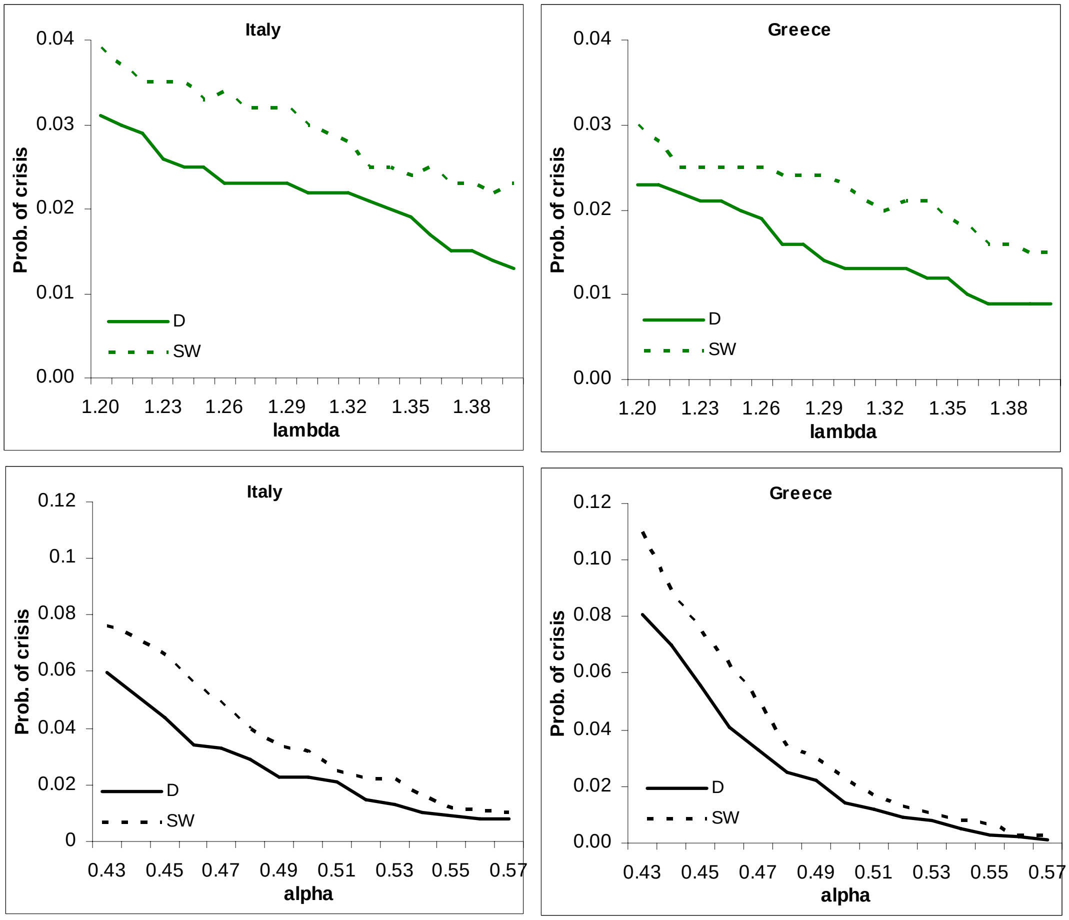

Figure 6: The probability of fiscal crisis as a function of lambda and alpha.

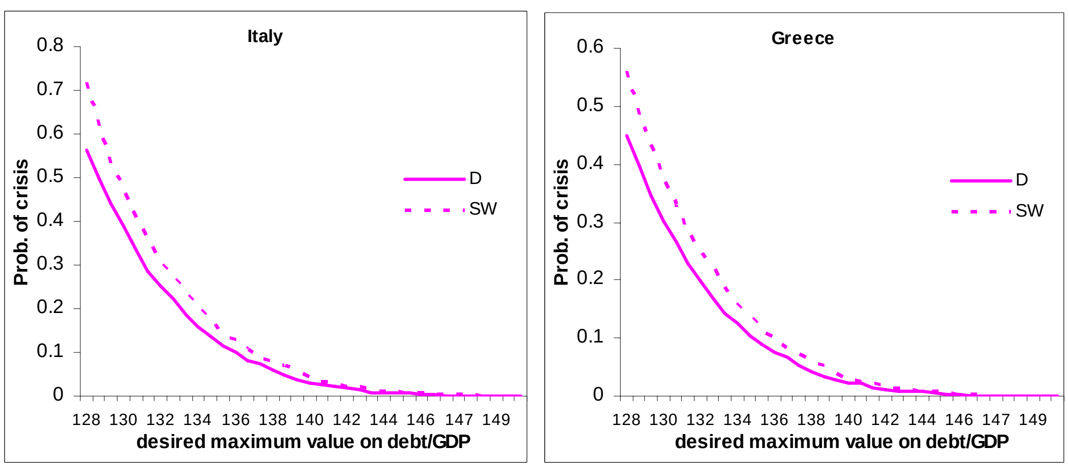

Figure 7: The probability of fiscal crisis as a function of the desired maximum value on debt/GDP.

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