Evolutionarily Stable Behavior in Winner-Pay Contests

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Abstract

In a winner-pay contest, contestants submit bids to influence the probabilistic award of a prize, the highest bid does not necessarily win, and only the winner pays her bid. This paper considers behavior in such a contest from an evolutionary perspective. I show that a "Tullock" winner-pay contest's unique evolutionarily stable strategy (ESS) differs from its unique Nash equilibrium. For finite populations engaged in such winner-pay contests, ESS behavior entails expenditures in excess of Nash equilibrium levels but never over-dissipation of the prize value. Further, the ESS in such contests globally stabile in that a population playing according to the ESS cannot be invaded by an arbitrary number of mutants using some other strategy. I extend many of the paper's main results to more general contest success functions and to a more general model of evolution.

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1 Introduction

Contests are important non-market forms of allocating valuable prizes or other resources. Contest theory applies to many topics, including rent-seeking and lobbying for political favors, conflict, litigation, R&D competition and patent races, sporting competition, and charitable fundraising.[1] The most-studied forms of contests are all-pay contests in which players compete by making irrecoverable investments of expenditure or effort to influence their probability of winning a prize. The most well-known specifications of contest winning probabilities (also called contest success functions in the literature) are "Tullock" contests where each player's winning probability is her expenditure's share of total expenditure (Tullock, 1980) and all-pay auctions where the player making the largest expenditure wins for certain (Baye et al., 1996).

Yates (2010) introduces into the contest theory literature a winner-pay contest in which contestants submit bids to influence the probabilistic award of a prize, the highest bid does not necessarily win, and only the winner pays her bid. He notes that contests and auctions share the structure of having players compete to influence their probability of winning a prize of some value, but distinguishes between the two mechanisms, reserving the term auction for any competition where the highest bid wins for certain and reserving contest for any competition where the highest bid does not win for certain, due perhaps to circumstances exogenous to a contestant's bid. Like auctions, then, contests can of course differ according to their payment rules as all-pay, winner-pay (first-price), second-price, etc.

This paper provides an evolutionary analysis of the "Tullock" version of the winner-pay contest introduced by Yates (2010). Understanding behavior in winner-pay contests from an evolutionary perspective is important not only because of the many applications of contest theory outlined above. Expenditures often exceed risk-neutral Nash equilibrium levels in experimental all-pay contests.[2] Evolutionary analyses of all-pay contests from Leininger (2003) and Hehenkamp et al. (2004) provide a rationalization for observed over-expenditure

[1] On these applications of contest theory, see Tullock (1980) and Baye et al. (1993) on rent-seeking and lobbying, Hirshleifer (1995) and Garfinkel and Skaperdas (2007) and the references therein on conflict, Farmer and Pecorino (1999) and Baye et al. (2005) on litigation, Baye and Hoppe (2003) on innovation tournaments and patent races, Szymanski (2003) on sporting competition, and Morgan (2000) and Goeree et al. (2005) on the use of contests for charitable fundraising. Nitzan (1994) and Corchón (2007) provide surveys of contest theory more generally.

[2] See, for example, Öncüler and Croson (2005), Hörisch and Kirchkamp (2010), Morgan et al. (2010), and the literature cited in these papers for an overview. Interestingly, even Potters et al. (1998), who correct for design flaws in earlier contest experiments, find evidence of overexpenditure in their "Tullock" contests. To my knowledge, there does not yet existence any experimental evidence on behavior in winner-pay contests, but to the extent that they are very similar in structure to all-pay contests, I conjecture that one would observe bids in excess of Nash equilibrium levels.

(contest bids in excess of Nash equilibrium levels) and over-dissipation (aggregate expenditures in excess of the value of the contest's prize). To the extent that all-pay and winner-pay contests share similar incentives as contests, players' bids may exceed Nash equilibrium levels in practice, and the analysis in this paper provides an evolutionary rationale for such an outcome. Understanding behavior in winner-pay contests from an evolutionary perspective is important also because the incentives present in winner-pay contests emerge, for example, when countries compete to host mega-events like the Olympics or the World Cup, when states compete with tax incentives and other subsidies to attract on-location filming from well-known film studios, or when firms compete for a worker with offers of salary among other perquisites. Yates (2010) also notes that winner-pay contests emerge in inter-state competition for the siting of manufacturing plants, in limited liability contests like Skaperdas and Gan (1995) and Matros and Armanios (2009) when all contest losers are reimbursed their expenditures, in rent-seeking with sunk lobbying effort and bids payable only by the winner as in Haan and Schoonbeek (2003), and in litigation where parties to a dispute delegate litigation to lawyers who are paid only upon the event of winning as in Wärneryd (2000). An evolutionary analysis of a winner-pay contest, then, provides a benchmark for behavior of boundedly rational players in these environments and yields insights into the behavior of players in winner-pay contests that either learn from observing the behavior of more-successful others or who are concerned with relative and not absolute performance.

I derive the unique symmetric Nash equilibrium and unique finite population evolutionarily stable strategy (ESS) in a "Tullock" winner-pay contest and obtain the following results.[3] In the "playing-the-field" model of evolution in which interaction among members of a population is global, the ESS bid exceeds the bid a risk-neutral player would make in Nash equilibrium. This is due to the negative externality that is a hallmark of contests, each player's winning probability is decreasing in rival bids and to the fact that evolution operates on relative fitness in a finite population. In fact, the winner-pay contest ESS is the limiting outcome of symmetric Nash equilibrium behavior as the number of players approaches infinity. While the winner-pay contest ESS features over-expenditure relative to Nash equilibrium, over-dissipation of the prize value cannot be part of ESS behavior. This result is in stark contrast to Hehenkamp et al. (2004), who derive the unique ESS for all-pay "Tullock" contests and find the possibility for prize over-dissipation as part of ESS behavior. Prize over-dissipation is not possible in a winner-pay contest's ESS because an individual bid *is* prize dissipation, so over-dissipation would imply bidding above the prize value, a

[3]An Appendix to the paper shows that many of the main results of the paper generalize either to a more general contest success function than the Tullock (1980) contest success function or to a more general model of evolution among members of a finite population.

population behavior that a mutant bidding below the prize value could successfully invade. The winner-pay contest ESS I derive is robust in the sense that it resists invasion by any finite number of identical mutant strategies. Finally, I derive evolutionarily stable population size in the evolutionary winner-pay contest and show that it increases in the contest's prize value and decreases in the population's subsistence level of fitness and competitiveness of the contest.

This paper contributes to a growing literature on evolutionary approaches to contest theory. This literature divides into two branches, one concerned with the evolution of behavior in contests–direct evolution–and one concerned with the evolution of preferences in contests–the indirect evolution approach pioneered by Güth and Yaari (1992). For direct evolution, Leininger (2003) and Hehenkamp et al. (2004) study evolutionarily stable strategies in all-pay contests and show that they exceed risk-neutral Nash equilibrium levels when evolution occurs in finite populations. Importantly, Hehenkamp et al. (2004) provide an evolutionary rationale for over-dissipation of the prize value, something that I demonstrate is impossible in the ESS in a winner-pay contest, so while the nature of a contest creates similar results for all-pay and winner-pay contests, the difference in payment rules create some divergent results. For indirect evolution, among the papers studying the evolution of preferences in all-pay contests are Wärneryd (2002) on risk attitudes, Eaton and Eswaran (2003) and Leininger (2009) on interdependent preferences, Konrad (2004) and Schmidt (2009) on altruism and envy, Mohlin (2010) on conflict-reducing norms, and Boudreau and Shunda (2010) on the evolution of prize valuation perceptions.

2 Model

In a winner-pay contest, a set of $n \geq 2$ players submit bids $x_i \geq 0$, $i = 1, ..., n$ to influence the probabilistic award of a prize of a common value $v > 0$. Unlike a (winner-pay) auction, the highest bid does *not* necessarily win and unlike a standard all-pay contest, only the winner pays her bid. For player i bidding x_i, payoffs are given by

$$\pi_i(x_1, ..., x_n) := \begin{cases} v - x_i & \text{if player } i \text{ wins} \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

I assume that the prize is allocated by the familiar contest success function from Tullock (1980), so the probability of winning for player i bidding x_i is

$$p_i(x_1, ..., x_n) := \begin{cases} \frac{x_i^r}{x_i^r + \sum_{j \neq i} x_j^r} & \text{if } x_i^r + \sum_{j \neq i} x_j^r \neq 0 \\ \frac{1}{n} & \text{otherwise,} \end{cases} \tag{2}$$

where the exponent $r \in (0, \infty)$ measures the sensitivity of the probability of winning to individual bids.[4] Finally, I assume that players are risk-neutral, so given the bids of all players $j \neq i$, player i's expected payoff (fitness in the evolutionary analysis) is

$$E[\pi_i(x_1, ..., x_n)] := \frac{x_i^r}{x_i^r + \sum_{j \neq i} x_j^r}(v - x_i). \tag{3}$$

3 Results

This section derives results for both Nash equilibrium and evolutionarily stable behavior in the winner-pay contest described by (1) and (2).

3.1 Nash Equilibrium Behavior in Winner-Pay Contests

As a benchmark to the evolutionary analysis below, consider first fully rational Nash equilibrium players. Players simultaneously submit bids to maximize their expected payoffs in (3). All players bidding 0 cannot be a Nash equilibrium because any individual player could win for certain by increasing her bid to some $\varepsilon > 0$. Similarly, bidding in excess of v cannot be part of any Nash equilibrium because such a bid is dominated by a bid of 0. The first-order condition to player i's maximization problem is

$$\frac{r x_i^{r-1} \sum_{j \neq i} x_j^r}{(x_i^r + \sum_{j \neq i} x_j^r)^2}(v - x_i) - \frac{x_i^r}{x_i^r + \sum_{j \neq i} x_j^r} = 0. \tag{4}$$

At a symmetric Nash equilibrium, $x_1 = ... = x_n = x_{NE}$, so (4) becomes

$$\frac{(n-1)r}{n^2 x_{NE}}(v - x_{NE}) - \frac{1}{n} = 0,$$

[4] See Skaperdas (1996) and Kooreman and Schoonbeek (1997) for axiomatizations of this well-known contest success function. The Appendix includes analysis for a more general contest success function and derives results showing that evolutionary stable behavior in a winner-pay contest is generally more aggressive than Nash equilibrium behavior.

with solution

$$x_{NE} = \frac{(n-1)rv}{(n-1)r+n}. \tag{5}$$

Bidding according to (5) is a symmetric Nash equilibrium provided that (5) actually maximizes each player's expected payoffs. The second-order condition to player i's maximization problem is

$$\frac{(r-1)(x_i^r + \sum_{j\neq i} x_j^r) - 2rx_i^r}{x_i^r + \sum_{j\neq i} x_j^r}(v - x_i) - 2x_i < 0. \tag{6}$$

Substituting (5) into the second-order condition and simplifying reveals that it holds if and only if $-nr - n < 0$ and therefore the second-order condition holds at the candidate for symmetric Nash equilibrium bidding in (5) for all $r \in (0, \infty)$. From (3), each player's expected payoffs in equilibrium are

$$\begin{aligned} E[\pi_i(x_{NE}, ..., x_{NE})] &= \frac{1}{n}\left(v - \frac{(n-1)rv}{(n-1)r+n}\right) \\ &= \frac{v}{(n-1)r+n} > 0 \end{aligned}$$

for all $r \in (0, \infty)$.

Proposition 1. *In the unique symmetric pure strategy Nash equilibrium of a winner-pay contest, each player bids an amount*

$$x_{NE} = \frac{(n-1)rv}{(n-1)r+n}.$$

Before moving on to the evolutionary analysis, a few properties of symmetric Nash equilibrium behavior in the winner-pay contest are worth remarking upon. First, equilibrium bidding has expected comparative static properties. Equilibrium bids increase in v since players bid more aggressively for more valuable prizes. Increases in r and n lead to increased equilibrium bids since players respond to a more competitive environment by bidding more competitively. Equilibrium bids increasing in n is in contrast to what one would find in an all-pay contest where players' bids are sunk expenditures and losing is therefore costly; see, e.g., Tullock (1980) and Pérez-Castrillo and Verdier (1992). Because players do not forfeit their bids in the event of losing in a winner-pay contest, losing is not costly and players can respond to increased competition by bidding more aggressively. Second, the limiting behavior of equilibrium bidding behavior is such that

$$\lim_{n\to\infty} x_{NE} \to \frac{rv}{1+r},$$

which I show below to be evolutionarily stable behavior in a "playing-the-field" model of evolution in a *finite* population interacting in a winner-pay contest. Finally, in contrast to the case of an all-pay contest where aggregate expenditure measures prize dissipation, individual expenditure and prize dissipation are identical in a winner-pay contest, so prize dissipation inherits the comparative static properties of individual expenditure.

3.2 Evolutionarily Stable Behavior in Winner-Pay Contests

Members of a finite population of size $n \geq 2$ match to interact in an n-player winner-pay contest described by (1) and (2). Players differ from one another by the strategy they play, that is, by the bids they make in the contest. Since interaction among the population's players is global, the model of evolution I develop in this subsection is often referred to as "playing-the-field." Each time players match, they play the contest once and this determines each strategy's fitness level. The matching to play a winner-pay contest occurs indefinitely, and the population evolves in such a way that strategies earning higher fitness levels proliferate while strategies earning lower fitness levels eventually die off. What behavior emerges as evolutionarily stable in winner-pay contests in finite populations?

I follow the literature on evolutionary behavior in contests and equate evolutionary fitness with the expected payoffs a strategy generates.[5] The fitness of a player i bidding according to x_i is then given by

$$F_i(x_1, ..., x_n) := \frac{x_i^r}{x_i^r + \sum_{j\neq i} x_j^r}(v - x_i). \tag{7}$$

I forgo modeling explicitly the dynamics of evolution and selection and instead apply a static definition of evolutionary stability for finite populations from Schaffer (1988) to characterize evolutionarily stable strategies (ESS) in the population, the stable limit points of evolution and selection. Loosely speaking, an ESS is a strategy that when it predominates the population there exists no other (mutant) strategy that yields higher fitness. In other words, an ESS cannot be successfully invaded by a small number of mutants playing some other strategy when it predominates the population. Without loss of generality, suppose that player 1

[5]This is a reasonable assumption to make if one thinks of this model of evolution as a metaphor for boundedly rational play, learning, and imitation of more successful others in contests.

is a mutant and bids according to $x_1 = x$. Then, from Schaffer (1988), a strategy x_{ESS} is a finite population ESS if and only if

$$x_{ESS} \in \arg\max_x F_1(x, x_{ESS}, ..., x_{ESS}) - F_j(x, x_{ESS}, ..., x_{ESS}), \tag{8}$$

for $j = 2, ..., n$. In other words, a finite population ESS maximizes relative fitness.

All players bidding 0 cannot be an ESS because mutants bidding a small positive amount could successfully invade such a population. Similarly, bidding in excess of v cannot be an ESS because mutants bidding below v could successfully invade such a population. Making use of (7), the first-order condition to the relative fitness maximization problem in (8) is

$$\frac{(n-1)rx^{r-1}x_{ESS}^r}{(x^r+(n-1)x_{ESS}^r)^2}(v-x) - \frac{x^r}{x^r+(n-1)x_{ESS}^r} + \frac{rx^{r-1}x_{ESS}^r}{(x^r+(n-1)x_{ESS}^r)^2}(v-x_{ESS}) = 0. \tag{9}$$

At an ESS, $x = x_{ESS}$, so (9) becomes

$$\frac{r}{nx_{ESS}}(v - x_{ESS}) - \frac{1}{n} = 0,$$

with solution

$$x_{ESS} = \frac{rv}{1+r}. \tag{10}$$

Bidding according to (10) is an ESS provided that (10) actually maximizes relative fitness. The second-order condition to the relative fitness maximization problem is

$$\frac{(n-1)(r-1)x_{ESS}^r - (1+r)x^r}{x^r+(n-1)x_{ESS}^r}(v-x) - 2x + \frac{(n-1)(r-1)x_{ESS}^r - (1+r)x^r}{(n-1)(x^r+(n-1)x_{ESS}^r)}(v-x_{ESS}) < 0. \tag{11}$$

Substituting (10) into the second-order condition and simplifying reveals that it holds if and only if $-nr - n < 0$ and therefore the second-order condition holds at the candidate for ESS bidding in (10) for all $r \in (0, \infty)$. Clearly, relative fitness is 0 at the candidate ESS in (10) and the relative fitness of making a bid of 0 against the candidate ESS played by players $i = 2, ..., n$ is

$$\begin{aligned} F_1(0, x_{ESS}, ..., x_{ESS}) - F_i(0, x_{ESS}, ..., x_{ESS}) &= 0 - \frac{1}{n-1}\left(v - \frac{rv}{1+r}\right) \\ &= -\frac{1}{n-1}\frac{v}{1+r} < 0 \end{aligned}$$

for all $r \in (0, \infty)$.

Proposition 2. *At the unique evolutionarily stable strategy (ESS) of a winner-pay contest among members of a finite population, each player bids an amount*

$$x_{ESS} = \frac{rv}{1+r}.$$

The finite population ESS is the limiting outcome of Nash equilibrium bidding among rational players in a winner-pay contest when $n \to \infty$.

ESS bidding has expected comparative static properties. ESS bids increase in v since more valuable prizes increase the fitness available for the evolution of relatively more aggressive bidding strategies. An increases in r increases the decisiveness of an individual bid for winning, and therefore leads to the evolution of relatively more aggressive bidding strategies. From (7), the individual fitness of an ESS player is

$$\begin{aligned} F_i(x_{ESS}, ..., x_{ESS}) &= \frac{1}{n}\left(v - \frac{rv}{1+r}\right) \\ &= \frac{1}{n}\frac{v}{1+r} \geq 0 \end{aligned} \tag{12}$$

for all $r \in (0, \infty)$, $i = 1, ..., n$. As expected, the fitness of an ESS player increases in v and decreases in n and in r.

Over-dissipation of the prize value does *not* emerge in winner-pay contest ESS for any $r \in (0, \infty)$. This is in sharp contrast to what Hehenkamp et al. (2004) find for ESS in all-pay contests, where over-dissipation of the prize value exists for $r > 1$. In the all-pay contest Hehenkamp et al. (2004) study, over-dissipation is consistent with ESS despite creating negative individual fitness because, starting from a population playing according to the ESS, a mutant bidding less aggressively would in fact experience lower fitness than each member of the ESS population (that is, experience negative relative fitness)and therefore could not successfully invade the ESS population. By contrast, in a winner-pay contest, an individual player's ESS bid *is* prize dissipation. Thus, over-dissipation of the prize in a winner-pay contest is equivalent to an individual bid above the prize value, a strategy that mutants bidding below v could successfully invade because such mutants would earn strictly positive fitness while members of a population over-dissipating the prize continue to experience negative fitness.

While over-dissipation of the prize value is impossible in both Nash equilibrium and ESS behavior, it is the case that ESS behavior involves over-expenditure relative to Nash equilibrium behavior from inspection of (5) and (10).

Proposition 3. *Bidding according to the evolutionarily stable strategy (ESS) in a winner-pay* *contest involves over-expenditure relative to Nash equilibrium bidding behavior since*

$$x_{ESS} = \frac{rv}{1+r} > x_{NE} = \frac{(n-1)rv}{(n-1)r+n}.$$

Since individual bidding is prize dissipation in a winner-pay contest, prize dissipation under ESS bidding exceeds prize dissipation under Nash equilibrium bidding, though in neither case can there exist over-dissipation of the prize value.

Proposition 3 coincides with what Leininger (2003) and Hehenkamp et al. (2004) find when comparing finite population ESS and Nash equilibrium behavior in an all-pay contest. That finite population ESS bidding exceeds Nash equilibrium bidding in a winner-pay contest follows directly from the negative externality in the contest success function in (2). Namely, the hallmark of a contest (all-pay or winner-pay) is that each (active) player's probability of winning the contest decreases in each rival player's effort. Starting from a population bidding in Nash equilibrium as per (5), a mutant bidding (slightly) more aggressively will decrease its own fitness, but will decrease even more the fitness of a member of the population bidding according to the Nash equilibrium. In other words, such a mutant would experience higher relative fitness than a member of the Nash equilibrium population and successfully invade it. Analysis in the Appendix to this paper generalizes this result and shows that ESS bidding is more aggressive than Nash equilibrium bidding for a contest success function more general than (2) and for a more general model of evolution in which players of a finite population of size $N \geq 2$ match at random to interact in n-player winner pay contests, $n \leq N$.

The ESS bidding I derive in Proposition 2 makes use of the definition of a finite population ESS from Schaffer (1988) in which an ESS is evolutionarily stable against the invasion of one mutant. In the interest of robustness, however, one might demand that an ESS be evolutionarily stable against the invasion of a finite number of identical mutants, or at least wish to know the maximum number of identical mutants against which an ESS is evolutionarily stable. The definition of a generalized ESS from Schaffer (1988) accounts for invasion by a finite number of identical mutants as follows: A strategy x_{ESS} is M-stable if m identical mutants playing any strategy $\hat{x} \neq x_{ESS}$ cannot successfully invade (i.e., earns lower relative fitness) a population of $n-m$ players playing according to x_{ESS} with $1 \leq m \leq M$ and $M \leq n-1$. A strategy x_{ESS} is globally stable if it is $(n-1)$-stable.

The finite population ESS bidding I derive in Proposition 2 is highly robust in that it is globally stable.

Proposition 4. *The unique finite population evolutionarily stable strategy*

$$x_{ESS} = \frac{rv}{1+r}$$

for a winner-pay contest is globally stable.

Proof. The proof is by induction. The argument deriving Proposition 2 demonstrates that $x_{ESS} = vr/(1+r)$ is 1-stable. The induction hypothesis is that $x_{ESS} = vr/(1+r)$ is m-stable, so that the relative fitness of one of m identical mutants playing $\hat{x}$ when facing a population playing the ESS is

$$\frac{\hat{x}^r}{m\hat{x}^r + (n-m)\left(\frac{rv}{1+r}\right)^r}(v-\hat{x}) - \frac{\left(\frac{rv}{1+r}\right)^r}{m\hat{x}^r + (n-m)\left(\frac{rv}{1+r}\right)^r}\left(v - \frac{rv}{1+r}\right) < 0 \tag{13}$$

for all $\hat{x} \neq rv/(1+r)$.

To establish that $x_{ESS} = vr/(1+r)$ is $(m+1)$-stable, note that the the relative fitness of one of $m+1$ identical mutants playing $\hat{x}$ when facing a population playing the ESS is

$$\frac{\hat{x}^r}{(m+1)\hat{x}^r + (n-m-1)\left(\frac{rv}{1+r}\right)^r}(v-\hat{x}) - \frac{\left(\frac{rv}{1+r}\right)^r}{(m+1)\hat{x}^r + (n-m-1)\left(\frac{rv}{1+r}\right)^r}\left(v - \frac{rv}{1+r}\right). \tag{14}$$

By the induction hypothesis in (13), the relative fitness of one of these identical $m+1$ mutants is in fact negative for all $\hat{x} \neq rv/(1+r)$ since (14) is simply (13) multiplied by

$$\frac{m\hat{x}^r + (n-m)\left(\frac{rv}{1+r}\right)^r}{(m+1)\hat{x}^r + (n-m-1)\left(\frac{rv}{1+r}\right)^r} > 0.$$

By induction, then, $x_{ESS} = rv/(1+r)$ is globally stable. □

The global stability of ESS bidding in a winner-pay contest complements the result from Hehenkamp et al. (2004) which establishes the global stability of the ESS of an all-pay contest. More generally, it is well-known from Leininger (2006) that the ESS of quasi-submodular symmetric aggregative games are globally stable. The winner-pay contest I study in this paper is a symmetric aggregative game but, unfortunately, the result from Leininger (2006) does not apply in this case because (it can be shown that) players' expected payoffs (fitness) do not exhibit decreasing differences in individual strategy and aggregate everywhere in the strategy space.

Since, from (12), individual fitness of an ESS player in a winner-pay contest is non-negative, it would be interesting to endogenize the population and consider how the population size reacts to changes in the contest's parameters. The evolutionarily stable population size (n_{ESS}) in a winner-pay contest is the population size that, subject to ESS bidding behavior it induces, generates individual fitness no less than some given subsistence level. Let $\overline{F} > 0$ denote the subsistence level of fitness for each player in the population. Populations that generate fitness below $\overline{F}$ will shrink and populations that generate fitness above $\overline{F}$ will grow. Treating population size as a continuous variable, the evolutionarily stable population size then satisfies

$$\frac{1}{n_{ESS}}\frac{v}{1+r} = \overline{F}$$

and is therefore given by

$$n_{ESS} = \frac{1}{\overline{F}}\frac{v}{1+r}, \tag{15}$$

provided that the parameters $\overline{F}$, r, and v are in a configuration such that $n_{ESS} \geq 2$; for example, a sufficiently large v or sufficiently small $\overline{F}$ would ensure a minimally competitive evolutionarily stable population size. Upon inspection of (15), the comparative static properties of evolutionarily stable population size in a winner-pay contest are immediate and as expected.

Proposition 5. *The evolutionarily stable population size (n_{ESS}) in a winner-pay contest decreases in the subsistence level of fitness ($\overline{F}$), decreases in the contest success function's sensitivity to an individual bid (r), and increases in the contest's prize value (v).*

For the case of $r < 1$ in the all-pay contest in Hehenkamp et al. (2004), one obtains results analogous to Proposition 5 for the evolutionarily stable population size defined as above. For the case of $r \in (1, n/(n-1)]$, however, there is over-dissipation of the prize value in the ESS and individual fitness is negative. In this case, then, the evolutionarily stable population size is zero unless one is willing to posit a negative subsistence level of fitness.

4 Conclusion

In this paper, I provide an evolutionary analysis of a winner-pay contest where contestants submit bids to influence the probabilistic award of a prize, the highest bid does not necessarily win, and only the winner pays her bid. As in all-pay contests, evolutionarily stable (ESS) bidding in winner-pay contests exceeds risk-neutral Nash equilibrium levels but, in contrast to the ESS of all-pay contests, can never involve over-dissipation of the contest's prize value. The ESS of winner-pay contests is globally stable in that it resists invasion of an arbitrary number of identical mutants playing according to some other strategy. When endogenizing population size in evolutionary winner-pay contests, I find that the evolutionarily stable population size increases in the contest's prize value and decreases in the population's subsistence level of fitness and competitiveness of the contest. Many of these results continue to hold for either a more general contest success function or a more general model of evolution of a finite population.

Potentially interesting future lines of research include experimental work on winner-pay contests to see whether Nash equilibrium or ESS better explain behavior in this environment. Extending the analysis in this paper to analyze the evolution of multiple populations in asymmetric (in, for example, prize valuation, cost, or contest success function productivity) winner-pay contests could provide further insights into the relationship between Nash equilibrium and ESS behavior in contests and how behavioral differences relate to structural differences among contests. Finally, exploring the relation of ESS to behavior induced by evolutionarily stable preferences as in Leininger (2009) and Boudreau and Shunda (2010) could provide insights into conditions under which these forms of behavior are or are not equivalent in contests that are all-pay, winner-pay, or otherwise.

Appendix

This Appendix contains an analysis which generalizes some results from the body of the paper to a contest success function more general than (2) and for a model of evolution more general than the "playing-the-field" model I develop and study in Section 3 of the paper.

A More General Contest Success Function

The analysis of this section demonstrates the symmetric Nash equilibrium to a winner-pay contest is not evolutionarily stable and that the finite population ESS of a winner-pay contest involves relatively more aggressive bidding than Nash equilibrium behavior. This follows from the negative externality the contest success function generates, a hallmark of contests. Evolutionarily, a player bidding (slightly) more aggressively than Nash equilibrium levels decreases their own fitness but decreases even more the fitness of rival players, resulting in an increase in relative fitness. The analysis below follows Leininger (2003), who obtains an analogous result for all-pay contests.

Suppose a set of $n \geq 2$ players in a winner-pay contest with payoffs described by (1) each face a contest success function $p_i(x_1, ..., x_n)$, $i = 1, ..., n$ with the following properties:

1. *Probability*: For $i = 1, ..., n$, $p_i : \mathbb{R}_n^+ \to [0, 1]$ and $\sum_{i=1}^n p_i = 1$.

2. *Differentiability*: The contest success function is differentiable in own and rival bids with

$$\frac{\partial p_i}{\partial x_i} > 0$$

for $i = 1, ..., n$ and $x_i > 0$, and

$$\frac{\partial p_i}{\partial x_j} < 0$$

for $j = 1, ..., n$, $j \neq i$, and $x_j > 0$.

3. *Anonymity*: For any permutation π of the set of players

$$p_{\pi(i)}(x_1, ..., x_n) = p_i(x_{\pi(1)}, ..., x_{\pi(n)})$$

so that each player's probability of winning depends upon their bid and not upon their identity.

To derive the symmetric Nash equilibrium of this winner-pay contest, each player solves the maximization problem

$$\max_{x_i} p_i(x_{NE}, ..., x_i, x_{NE}, ..., x_{NE})(v - x_i)$$

with first-order condition

$$\frac{\partial p_i}{\partial x_i}(v - x_i) - p_i = 0$$

satisfied at $x_i = x_{NE}$.

On the other hand, an ESS maximizes relative fitness. Suppose then, without loss of generality, that player 1 is a mutant playing according to $x_1 \neq x_{NE}$ against a population playing according to x_{NE}. To find an ESS in this case, evolution solves the relative fitness maximization problem

$$\max_{x_1} p_1(x_1, x_{NE}, ..., x_{NE})(v - x_1) - p_j(x_1, x_{NE}, ..., x_{NE})(v - x_{NE})$$

for $j = 2, ..., n$. Differentiating relative fitness with respect to x_1 and evaluating the differential at $x_1 = x_{NE}$ reveals that

$$\underbrace{\frac{\partial p_1}{\partial x_1}(v - x_1) - p_1}_{=0 \text{ at } x_1 = x_{NE}} \quad \underbrace{-\frac{\partial p_j}{\partial x_1}(v - x_{NE})}_{>0 \text{ because } \frac{\partial p_j}{\partial x_1} < 0 \text{ and } x_{NE} < v} \quad > 0.$$

Proposition 6. *The symmetric interior Nash equilibrium in a winner-pay contest is not* *evolutionarily stable. The evolutionarily stable strategy in a winner-pay contest involves relatively more aggressive bidding than does Nash equilibrium behavior.*

A More General Model of Evolution

In the "playing-the-field" model of evolution I develop and study in Section 3 of this paper, the forces of evolution and selection are at their strongest since interaction among members of the population is global. That is, since the entire population plays itself, it is certain that each member will meet a mutant. Suppose now, instead, that members of a finite population of size $N \geq 2$ match at random to interact in an n-player winner-pay contest described by (1) and (2) with $n \leq N$. Each time players match, they play the contest once and this determines each strategy's fitness level. The random matching in groups of n to play a winner-pay contest occurs indefinitely, and the population evolves in such a way that strategies earning higher fitness levels proliferate while strategies earning lower fitness levels eventually die off.

Suppose, without loss of generality, that player 1 playing according to $x_1 = x$ is a single mutant in the population, the rest of which is playing according to an ESS, $x_{ESS,N}$. Player 1, then, will meet members of the ESS population with certainty and therefore player 1's fitness is given by

$$F_1(x, x_{ESS,N}, ..., x_{ESS,N}) := \frac{x^r}{x^r + (n-1)x_{ESS,N}}(v - x).$$

On the other hand, the probability that a member of the ESS population present in a contest will meet a mutant among the other $n-1$ players in the contest is $(n-1)/(N-1)$ so that the expected fitness of a member of the ESS population is given by

$$\frac{n-1}{N-1}F_j(x, x_{ESS,N}, ..., x_{ESS,N}) + \left(1 - \frac{n-1}{N-1}\right) F_j(x_{ESS,N}, ..., x_{ESS,N}),$$

$j = 2, ..., n$. Ignoring the part of relative fitness that does not depend upon x, a strategy $x_{ESS,N}$ is a finite population ESS if and only if

$$x_{ESS,N} \in \arg\max_x F_1(x, x_{ESS,N}, ..., x_{ESS,N}) - \frac{n-1}{N-1}F_j(x, x_{ESS,N}, ..., x_{ESS,N}),$$

for $j = 2, ..., n$.

Arguments analogous to those in the proof of Proposition 2 derive the following result.

Proposition 7. *At the unique evolutionarily stable strategy among members of a finite pop-*

ulation $N \geq 2$ matching at random to play an n-player winner-pay contest with $n \leq N$, each player bids an amount

$$x_{ESS,N} = \frac{N(n-1)rv}{N(n-1)r + (N-1)n}. \tag{16}$$

Comparison of Nash equlibrium bidding in (5), "playing-the-field" ESS bidding in (10), and ESS bidding in (16) yields the following corollary.

Corollary 1. *Winner-pay contest bidding is weakly most-aggressive in the "playing-the-field" evolutionarily stable strategy and least-aggressive in Nash equilibrium so that*

$$x_{ESS} \geq x_{ESS,N} > x_{NE},$$

with a strict first inequality for $n < N$.

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