# Inventories, Markups, and Real Rigidities in Menu Cost Models*

Oleksiy Kryvtsov
Bank of Canada

Virgiliu Midrigan†
New York University

August 2010

**Abstract**

A growing consensus in New Keynesian macroeconomics is that nominal cost rigidities, rather than countercyclical markups account for the bulk of the real effects of monetary policy shocks. We revisit these conclusions using theory and direct evidence on quantities. We study an economy with nominal rigidities in which goods are storable. Theory predicts that if costs of production are sticky and markups do not vary much in response to, say, expansionary monetary policy, firms react by excessively accumulating inventories in anticipation of future cost increases. In contrast, in the data inventories are fairly constant over the cycle and in response to changes in monetary policy. We show that markups must decline sufficiently in times of a monetary expansion in order to reduce firms' incentive to hold inventories and thus bring the model's inventory predictions in line with the data. Versions of the model consistent with the dynamics of inventories in the data imply that countercyclical markups account for a sizable (50-80%) fraction of the response of real variables to monetary shocks.

**JEL classifications:** E31, F12.
**Keywords:** Inventories, markups, costs.

---

*The views expressed here are those of the authors and do not reflect those of the Bank of Canada. We thank George Alessandria, Ariel Burstein, Mike Dotsey, Mark Getler, Boyan Jovanovic, and Huntley Schaller for useful discussions, as well as seminar participants at the NBER-SITE Price Dynamics Conference at Stanford, Duke Macroeconomics Jamboree, Québec à Montréal, Chicago, NYU, Yale, Federal Reserve Board, the Federal Reserve Banks of Atlanta, Philadelphia, Richmond and St. Louis, as well as at the Bank of Canada.

†Kryvtsov: okryvtsov@bank-banque-canada.ca. Midrigan: virgiliu.midrigan@nyu.edu.

## 1. Introduction

The predictions of New Keynesian sticky price models, widely used for business cycle and policy analysis, are critically determined by the assumptions researchers make about the behavior of costs. If the real marginal cost of production responds strongly to monetary policy shocks, these models predict that such shocks have small and short-lived real effects, as in the work of Chari, Kehoe and McGrattan (2000). In contrast, if the real marginal cost responds slowly to monetary policy shocks, as in the work of Woodford (2002), Christiano, Eichenbaum and Evans (2005) and Dotsey and King (2006), such shocks have much larger and more persistent real effects. Indeed, much of the recent debate about the effects of monetary policy and nominal rigidities is really a debate about costs.

Our paper revisits this debate. We ask, How does the cost of production respond to monetary policy shocks? Since real marginal costs and markups are inversely related, an alternative way to posit our question is, How do markups respond to monetary policy shocks? Are the real effects of monetary policy shocks mostly accounted for by nominal cost rigidities or rather, by countercyclical variation in markups?

We answer this question by studying data on inventories through the lens of a New Keynesian model in which we embed a motive for inventory accumulation. We focus on inventories because theory predicts a tight relationship between prices, costs and inventories, as forcefully argued by Bils and Kahn (2000). If goods are storable, firm prices are determined by the marginal valuation of inventories. In turn, firms produce to the point at which the marginal valuation of inventories is equal to the marginal cost. We exploit these predictions of the theory to show that countercyclical markups account for a sizable fraction of the real effects of monetary policy shocks in versions of the model that replicate the behavior of inventories in the data. Hence, as Bils and Kahn (2000) do, albeit using a different methodology and for monetary-driven business cycles, we find that markups are strongly countercyclical.

Our results stand in sharp contrast to a number of findings in existing work. A growing consensus in New Keynesian macroeconomics is that sticky nominal costs, rather than variable markups, account for the bulk of the response of real activity to monetary policy shocks. Two observations have led researchers to this conclusion. First, studies of micro-price data find that input costs change infrequently and do not react well to nominal shocks, while prices tend to respond quickly to changes in costs[1]. Second, the observation that prices change frequently in the data (Bils and Klenow (2005) and Klenow and Kryvtsov (2008)), has led researchers to conclude that wage rigidities and stickiness in input costs, rather than nominal price rigidities, must be the dominant source of monetary non-neutrality (see for example Christiano, Eichenbaum and Evans (2005)).

Our paper revisits these conclusions. Although input costs are indeed sticky in the data, the marginal cost of production need not necessarily be sticky as well. Rotemberg and Woodford (1999) illustrate, for example, how inference about marginal costs from data on input prices is sensitive to assumptions about the production technology. Observed factor prices do not fully reflect production costs in the presence of factor adjustment frictions or decreasing returns. Moreover, as Stigler and Kindahl (1970), Barro (1977), and Hall (2006) have argued, factor prices in long-term relationships are not necessarily allocative. Contracts that stipulate quantity constraints, non-linear price schedules or other implicit arrangements may prevent buyers from taking advantage of, say, a monetary expansion, by purchasing inputs at the cheaper real input prices.

Existing New Keynesian models abstract from these considerations. Instead, these models assume that firms can easily alter the scale of production in response to nominal shocks by varying the work-week of capital and labor and by freely hiring labor and intermediate inputs at preset wages and input prices. Indeed, Dotsey and King (2006) use the term "real flexibilities" to characterize the assumptions made in this class of models.

Our starting point is the observation that the debate about *costs* is essentially a debate about *quantities*, that is, about the ability of firms to collectively increase the scale of production during expansions. Our approach is therefore to use data on quantities, and in particular data on the stock of inventories, in order to learn about costs. Theory predicts that if the nominal costs of production are indeed sticky and markups do not vary much, firms would take advantage of the stickiness in costs and rapidly accumulate inventories after a, say, expansionary monetary shock when costs are low and expected to increase. In contrast, in the data inventories react slowly to such shocks. Hence, costs of production must be fairly sensitive to monetary policy shocks for the model to replicate

[1]See the work of Goldberg and Hellerstein (2008), Nakamura and Zerom (2010), Eichenbaum, Jaimovich and Rebelo (2010), Gopinath and Itskhoki (2010). See also Nekarda and Ramey (2009) who argue that markups are, in fact, procyclical.

the sluggish dynamics of inventories in the data. We thus find an important role for variation in markups in accounting for the behavior of consumer prices.

We begin our analysis by reviewing several well-known facts about inventories[2]. In the data, inventories are procyclical, but much less volatile then sales. The aggregate US stock of inventories increases by about 0.16% for every 1% increase in sales during a business cycle expansion. We reach a similar conclusion when conditioning fluctuations on identified measures of monetary policy shocks. In response to an expansionary monetary policy shock, the stock of inventories increases by about 0.33% for every 1% increase in sales. Hence, the aggregate stock of inventories is relatively sticky and the aggregate inventory-sales ratio is countercyclical.

We then build a model in which nominal prices and wages change infrequently and firms hold inventories. We conduct our baseline analysis using a model in which inventories arise due to a precautionary stockout-avoidance motive. We then extend the model to allow for non-convexities in the form of fixed ordering costs that give rise to $(S, s)$ rules for inventory adjustment and show that our results are robust to this modification.

We use our model economy to study how the response of inventories to monetary policy shocks depends on the assumptions we make about the nature of costs. A key prediction of the model is that the stock of inventories firms hold is very sensitive to changes in production costs and somewhat less sensitive to changes in markups. This feature, shared by the models with and without non-convexities, is an outcome of the fact that in the model, as in the data, the cost of carrying inventories is fairly low. The low cost of carrying inventories makes it easy for firms to substitute intertemporally by producing and storing goods when production costs are relatively low and drawing down the stock of inventories when production costs are relatively high.

We briefly summarize our findings. We first study a version of our model with nominal wage stickiness and no price rigidities, that is, in which markups are constant. We show that this model accounts extremely poorly for the dynamics of inventories in the data. This is true regardless of whether labor is the only factor of production and hence the marginal cost is proportional to the nominal wage, or whether we introduce capital or firm-level decreasing returns that render marginal costs more volatile than wages. In all these variations of the model inventories increase much more

[2]See, for example, Ramey and West (1999) and Bils and Khan (2000).

strongly in response to an expansionary monetary shock then they do in the data. The reason is that production costs are expected to increase after a monetary expansion as more and more unions reset their nominal wages, thus making it optimal for firms to accumulate inventories in anticipation of future cost increases.

We then introduce nominal price rigidities in addition to nominal wage stickiness. Price rigidities are important for our analysis since they imply that markups decline during booms and thus reduce the firms' incentives to hold inventories. We show that this version of the model can indeed account for the dynamics of inventories in the data, as long as production costs are sufficiently responsive to monetary shocks, due to sufficiently strong diminishing returns to labor. Production costs must be sufficiently responsive to monetary shocks in order to reduce the intertemporal substitution motive. Moreover, when costs are volatile, price rigidities generate strongly countercyclical markups and further reduce the incentive to hold inventories. Overall, we find that versions of our model that account for the dynamics of inventories in the data imply that countercyclical variation in markups accounts for 50-80% of the response of real variables to monetary policy shocks. This stands in sharp contrast to the findings of Christiano, Eichenbaum and Evans (2005) who estimate parameters values that imply that markups play essentially no role in accounting for the real effects of monetary shocks.

Our work is related to a number of recent papers that study the behavior of inventories, costs and markups over the business cycle. Our starting point is the observation of Bils and Kahn (2000) that inventories are closely linked to markups and costs. The main difference between our work and that of Bils and Kahn is that they use data on input prices directly, together with a partial equilibrium model of inventories, in order to measure marginal costs. They find that the growth rate of marginal costs is acyclical and hence the intertemporal substitution motive is weak in the data. They therefore conclude that markups must be countercyclical for the model to account for the fact that the inventory-sales ratio is countercyclical in the data.

Khan and Thomas (2007) have recently argued that a countercyclical inventory-sales ratio is not necessarily evidence of countercyclical markups. They study the dynamics of inventories in a general equilibrium model driven by technology shocks. Such a model accounts well for the behavior of inventories in the data, despite the fact that markups are constant in their economy. Khan and Thomas (2007) show that general equilibrium considerations, and in particular capital accumulation, are critical to this result. Diminishing returns to labor slows the response of marginal costs to a technology shock and hence the incentive for inventory accumulation[3].

As Khan and Thomas (2007) do, we explicitly study the dynamics of inventories in a general equilibrium setting and find an important role for diminishing returns to variable factors in accounting for the inventory facts. While their focus is on technology shocks, ours is on monetary shocks in an economy with nominal rigidities. We find that in such an economy countercyclical markups play an important role: absent markup variation the model's predictions are grossly at odds with the data. The difference in our results stems from the special nature of monetary shocks in driving fluctuations in output. Unlike technology shocks, monetary shocks only affect real activity if nominal prices do not change immediately with changes in monetary policy. Since prices are equal to a markup times costs, monetary shocks only affect output if either i) markups vary or ii) nominal costs are sticky and do not react immediately to changes in monetary policy. Hence, if markups are constant, monetary policy shocks can only generate real effects if nominal costs are sticky. This, however, gives rise to strong variability in inventories due to intertemporal substitution in production which is at odds with the data.

Also related to our analysis is a paper by Jung and Yun (2005) who, as we do, study a sticky price model with inventories. They find that high rates of depreciation and/or convex costs of deviating from a target inventory-sales ratio are necessary to reconcile the model's predictions with the data[4]. These two features shut down the intertemporal substitution motive that is at the heart of our analysis. We argue below that such features are at odds with the micro data, since inventory-sales ratios are, in fact, very volatile at the firm level. In contrast, we present evidence that firm-level decreasing returns of the type that allow our model to match the aggregate behavior of inventories also allow the model to match salient feature of the micro-data regarding the comovement of prices, orders and inventories.

Finally, our work is closely related to the quantitative studies of Klenow and Willis (2006) and Burstein and Hellwig (2007) who also measure the strength of real rigidities using theory and

[3]See also Wen (2008) who studies a stockout-avoidance model of inventories and also finds that a real business cycle model accounts well for the inventory facts.

[4]See also Chang, Hornstein and Sarte (2007) who study the responses to a productivity shock in a sticky price model with inventories.

micro-price data. These researchers focus on an alternative type of real rigidity[5], in the form of strategic complementarities in price setting, and find weak evidence of such complementarities.

## 2. Data

In this section we review several salient facts regarding the cyclical behavior of inventories. These facts are well-known from earlier work[6]. We discuss them briefly for completeness, as they are central to our quantitative analysis below.

We use data from the Bureau of Economic Analysis (NIPA) on monthly final sales, inventories, and inventory-sales ratios for the Manufacturing and Trade sectors from January 1967 to December 2009.[7] These two sectors of the economy account for most (85%) of the U.S. inventory stock; the rest of the stock is in mining, utilities, and construction.

All series are real. Our measure of sales are real final domestic sales. Production is defined as the sum of final sales and the change in the end-of-period inventory stock. We construct the inventory-sales ratio as the ratio of the end-of-period inventory stock to final sales in that period. When reporting unconditional business cycle moments, we HP filter all series with a smoothing parameter equal to 14400. Below we also use a measure of identified monetary policy shocks to report statistics conditional on monetary policy disturbances.

Panel A of Figure 1 presents the time-series of sales and the inventory-sales ratio for Manufacturing and Trade. The Figure shows that the two series are strongly negatively correlated and are almost equally volatile. Every recession is associated with a decline in sales and a similarly-sized increase in the inventory-sales ratio. Likewise, every expansion is associated with an increase in sales and a decline in the inventory-sales ratio of a similar magnitude.

Table 1 quantifies what is evident in the Figure. Panel A reports unconditional statistics for these series. We focus on the series for the entire Manufacturing and Trade sector and briefly discuss the Retail sector to gauge the robustness of these facts.

Notice in the first column of Panel A that the correlation between the inventory-sales ratio

[5]See Ball and Romer (1990).

[6]Ramey and West (1999), Bils and Kahn (2000).

[7]The Bureau of Economic Analysis uses an inventory valuation adjustment to revalue inventory holdings (reported by various companies using potentially different accounting methods) to replacement cost. These adjustments are based on surveys that report the accounting valuation used in an industry and from information on how long goods are held in inventories. See Ribarsky (2004).

and sales for the entire Manufacturing and Trade sector is equal to -0.82. The standard deviation of the inventory-sales ratio is almost as large (1.03 times larger) as the standard deviation of sales. Consequently, the elasticity of the inventory-sales ratio with respect to sales is equal to -0.84.[8] In other words, for every 1% increase in sales at business cycle frequencies, the inventory-to-sales ratio declines by about 0.84%. The stock of inventories is thus fairly constant over the cycle, increasing by only 0.16% ( = -0.84+1) for every 1% increase in sales.

This feature of the data may seem to contradict the well-known fact that inventory investment is strongly procyclical and accounts for a sizable proportion of the volatility of GDP.[9] There is, in fact, no contradiction, since inventory investment is small relative to the entire stock of inventories: monthly inventory investment is equal to 0.22% of the inventory stock in Manufacturing and Trade and 0.29% of the inventory stock in Retail. To see this, we also report the facts on inventory investment. We find it useful to do so by exploiting the following accounting identity:

$$Y_t = S_t + \Delta I_t$$

where $Y_t$ is production, $S_t$ are sales and $\Delta I_t$ is inventory investment. For a measure of how volatile inventory investment is, we compare the standard deviation of production to that of sales (both expressed as log-deviations from an HP trend). Notice in Table 1 that production and sales are strongly correlated and that production is 1.12 times more volatile than sales. We will use this fact, in addition to the facts on the stock of inventories, in order to evaluate the model.

The other columns of Table 1 present several additional robustness checks. We note that the facts above hold if we focus separately on the Retail sector: the elasticity of inventories to sales is equal to 0.24 and production is 1.14 as volatile as sales[10]. These facts also hold conditional on measures of monetary policy shocks. To see this we project the data series on current and 36 lags of Christiano, Eichenbaum and Evans (2002) measures of monetary policy shocks and recompute

[8]This elasticity is defined as the product of the correlation and the ratio of the standard deviations, or equivalently, as the slope coefficient in a regression of log inventory-sales ratio on log sales.

[9]See, e.g. Ramey and West (1999).

[10]Iachoviello, Schiantarelli and Schuh (2007) report that the inventory-sales ratio in Retail is acyclical as they find a low correlation between the inventory-sales ratio in Retail and aggregate GDP. Their results are consistent with ours, since at the monthly frequency aggregate GDP is imperfectly correlated with Retail sales (the correlation of 0.10) and reinforce our conclusion that the stock of inventories is fairly constant over the cycle.

these statistics. We report the resulting series in Panel B of Figure 1. Although monetary shocks account for a small fraction of the business cycle (the standard deviation of these series is about half as large when conditioning on measures of monetary shocks), the main pattern is evident in this Panel as well. In particular, we again find that the inventory-sales ratio is countercyclical. As Table 1 shows, the elasticity of inventories to sales is now equal to 0.33 (0.17 for Retail) and production is 1.10 times more volatile than sales (1.15 in Retail). Thus, in response to an expansionary monetary policy shock, both sales and inventory investment increase, but inventory investment increases much less than sales, and so the inventory-sales ratio declines[11].

The evidence in this Section is robust to the detrending method, the level of aggregation and stage-of-fabrication of inventories. We also checked the robustness of these facts using data from the NBER manufacturing productivity database and measures of monetary shocks from Romer and Romer (2007). See Kryvtsov and Midrigan (2009) for details.

## 3. Model

We study a monetary economy populated by a large number of infinitely lived households, a continuum of monopolistically competitive firms that produce differentiated intermediate goods, a continuum of perfectly competitive firms that produce a final good, and a government. In each period $t$ the commodities are differentiated varieties of labor services, a final labor service, money, a continuum of intermediate goods indexed by $i \in [0, 1]$, and a final good. The final good is used for consumption and investment. In each period $t$, this economy experiences on of infinitely many events $s_{t.}$ We denote by $s^t = (s_0, \ldots, s_t)$ the history (or state) of events up through and including period $t$. The probability density, as of period 0, of any particular history $s^t$ is $\pi(s^t)$. The initial realization $s_0$ is given.

In the model, we have aggregate shocks to the money supply and idiosyncratic demand shocks. We describe the idiosyncratic shocks below. In terms of the money supply shocks, we assume, throughout most of the paper, that the supply of money follows a random-walk process of the form

$$\log M(s^t) = \log M(s^{t-1}) + \log \mu(s^t), \tag{1}$$

[11]See Jung and Yun (2005) who provide similar evidence.

where $\log \mu\left(s^t\right)$ is money growth, a normally distributed i.i.d. random variable with mean 0 and standard deviation $\sigma_\mu$. We consider alternative specifications of monetary policy in a robustness section below.

**A. Households**

Households consume, trade bonds, and work. They also own the capital stock and rent it to intermediate goods producers. We assume frictions in the labor market in the form of sticky wages. In particular, we assume that households are organized in monopolistically competitive unions, indexed by $j$. Each union supplies a differentiated variety of labor services, $l_j\left(s^t\right)$, that aggregates into a final labor service, $l\left(s^t\right)$ according to

$$l\left(s^t\right) = \left(\int l_j\left(s^t\right)^{\frac{\vartheta-1}{\vartheta}} dj\right)^{\frac{\vartheta}{\vartheta-1}}$$

where $\vartheta$ is the elasticity of substitution across different types of labor services. Each union sets its wage $W_j\left(s^t\right)$ and therefore faces demand for its services given by

$$l_j\left(s^t\right) = \left(\frac{W_j\left(s^t\right)}{W\left(s^t\right)}\right)^{-\vartheta} l\left(s^t\right) \tag{2}$$

where $l\left(s^t\right)$ is the amount of labor hired by firms, and $W\left(s^t\right)$ is the aggregate wage rate:

$$W(s^t) = \left(\int W_j\left(s^t\right)^{1-\vartheta} dj\right)^{\frac{1}{1-\vartheta}}.$$

In this economy the markets for state-contingent money claims are complete. We represent the asset structure by having complete, state-contingent, one-period nominal bonds. Let $B_j\left(s^{t+1}\right)$ denote the consumer's holdings of such a bond purchased in period $t$ and state $s^t$ with payoffs contingent on a particular state $s^{t+1}$ at date $t+1$. One unit of this bond pays one unit of money at date $t+1$ if the particular state $s^{t+1}$ occurs and 0 otherwise. Let $Q\left(s^{t+1}|s^t\right)$ denote the price of this bond in period $t$ and state $s^t$. Clearly, $Q\left(s^{t+1}|s^t\right) = \frac{Q\left(s^{t+1}\right)}{Q\left(s^t\right)}$ where $Q\left(s^t\right)$ is the date 0 price of a security that pays one unit if history $s^t$ is realized.

The problem of union $j$ is to choose its members' money holdings $M_j\left(s^t\right)$, consumption

$c_j\left(s^t\right)$, investment $x_j\left(s^t\right)$, state-contingent bonds $B_j\left(s^{t+1}\right)$, as well as a wage $W_j\left(s^t\right)$, to maximize the household's utility:

$$\sum_{t=0}^{\infty}\int_{s^t}\beta^t\pi\left(s^t\right)\left[u\left(c_j\left(s^t\right)\right)-v\left(l_j\left(s^t\right)\right)\right]ds^t \tag{3}$$

subject to the budget constraint

$$P\left(s^t\right)\left[x_j\left(s^t\right)+\frac{\xi}{2}\left(\frac{x_j\left(s^t\right)}{k_j(s^{t-1})}-\delta\right)^2 k_j(s^{t-1})\right]+\int_{s^{t+1}}Q\left(s^{t+1}|s^t\right)B_j\left(s^{t+1}\right)ds^{t+1}+M_j(s^t)$$

$$\leqslant M_j\left(s^{t-1}\right)-P\left(s^{t-1}\right)c_j\left(s^{t-1}\right)+W\left(s^t\right)l_j\left(s^t\right)+\Pi_j\left(s^t\right)+B_j\left(s^t\right)+R\left(s^t\right)k_j\left(s^t\right),$$

a cash-in-advance constraint,

$$P\left(s^t\right)c_j\left(s^t\right)\leq M_j(s^t),$$

and subject to the demand for labor given by (2) as well as subject to the frictions on wage setting. We assume that utility is separable between consumption and leisure.

Here $P(s^t)$ is the price of the final good, $x_j\left(s^t\right)=k_j\left(s^t\right)-(1-\delta)\,k_j\left(s^{t-1}\right)$ is investment, $W(s^t)$ is the nominal wage, $\Pi_j(s^t)$ are firm dividends, and $R\left(s^t\right)$ is the rental rate of capital. Investment is subject to capital adjustment costs, the size of which is governed by $\xi$. The budget constraint says that the household's beginning-of-period balances are equal to unspent money from the previous period, $M_j\left(s^{t-1}\right)-P\left(s^{t-1}\right)c_j\left(s^{t-1}\right)$, labor income, dividends, as well as returns from asset market activity and from rental of the capital stock to firms. The household divides these balances into money holdings, $M_j\left(s^t\right)$, finances investment spending, as well as purchases of state-contingent bonds.

We assume Calvo-type frictions on wage setting. The probability that any given union is allowed to reset its wage at date $t$ is constant and equal to $1-\lambda_w$. A measure $\lambda_w$ of the unions leave their nominal wages unchanged. We choose the initial bond holdings of unions so that each union has the same present discounted value of income. Even though unions differ in the wages they set and hence the amount of labor they supply, the presence of a complete set of securities and the separability between consumption and leisure implies that they make identical consumption and investment choices in equilibrium. Since these decision rules are well-understood, we simply note that the bond prices satisfy

$$Q\left(s^{t+1}|s^{t}\right)=\beta\pi\left(s^{t+1}|s^{t}\right)\frac{u_{c}\left(c\left(s^{t+1}\right)\right)}{u_{c}\left(c\left(s^{t}\right)\right)}\frac{P\left(s^{t}\right)}{P\left(s^{t+1}\right)}$$

where $\pi\left(s^{t+1}|s^{t}\right)$ is the conditional probability of $s^{t+1}$ given $s^{t}$ and we have dropped the $j$ subscript. Similarly, the date 0 prices satisfy:

$$Q\left(s^{t}\right)=\beta^{t}\pi\left(s^{t}\right)\frac{u_{c}\left(c\left(s^{t}\right)\right)}{P\left(s^{t}\right)}.$$

**B. Final good producers**

The final good sector consists of a unit mass of identical and perfectly competitive firms. The final good is produced by combining the goods produced by intermediate goods firms (we refer to these goods as *varieties*) according to:

$$q\left(s^{t}\right)=\left(\int_{0}^{1}v_{i}(s^{t})^{\frac{1}{\theta}}q_{i}(s^{t})^{\frac{\theta-1}{\theta}}di\right)^{\frac{\theta}{\theta-1}}$$

where $q_i(s^t)$ is the amount of variety $i$ purchases by a final good firm, $v_i(s^t)$ is a variety-specific shock and $\theta$ is the elasticity of substitution across varieties. For simplicity we assume that $v_i\left(s^t\right)$ is an iid log-normal random variable.

In this economy, intermediate good firms sell out of their existing stock of inventories, $z_i(s^t)$. We describe the evolution of a firm's stock of inventories below. Given the price and inventory adjustment frictions we assume, this stock of inventories will occasionally be insufficient to meet all demand and intermediate good firms will *stockout*. In such a case, we assume a rationing rule under which all final good firms are allowed to purchase an equal share of that intermediate good's stock of inventories. Since the mass of final good firms is equal to 1, $z_i\left(s^t\right)$ is both the amount of inventories the intermediate good firm has available for sale, as well as the amount of inventories that any particular final good firm can purchase.

The problem of a firm in the final good's sector is therefore:

$$\max_{q_i(s^t)} P\left(s^t\right) q\left(s^t\right) - \int_0^1 P_i\left(s^t\right) q_i\left(s^t\right) di,$$

subject to the inventory constraint

$$\text{s.t. } q_i\left(s^t\right) \leqslant z_i\left(s^t\right) \ \forall i$$

and the final good production technology. Cost minimization by the final good firms implies the following demand for each variety:

$$q_i\left(s^t\right) = v_i(s^t)\left(\frac{P_i\left(s^t\right) + \mu_i\left(s^t\right)}{P\left(s^t\right)}\right)^{-\theta} q\left(s^t\right)$$

where $\mu_i\left(s^t\right)$ is the multiplier on the inventory constraint. Notice here that the shocks $v_i(s^t)$, act as a demand shock for an intermediate goods firm. We will thus refer to such shocks as *demand* shocks. Perfect competition implies that the price of the final good, $P\left(s^t\right)$, is equal to

$$P\left(s^t\right) = \left[\int_0^1 v_i(s^t)\left[P_i\left(s^t\right) + \mu_i\left(s^t\right)\right]^{1-\theta} di\right]^{\frac{1}{1-\theta}}.$$

Also note that if $\mu_i\left(s^t\right) > 0$ so that the inventory constraint binds, then it satisfies:

$$P_i\left(s^t\right) + \mu_i\left(s^t\right) = \left(\frac{z_i\left(s^t\right)}{v_i(s^t) P\left(s^t\right)^{\theta} q\left(s^t\right)}\right)^{\frac{1}{\theta}}.$$

The left hand side of this expression is the price that a firm that stocks out would have chosen absent price adjustment frictions. Since such a firm faces an inelastic demand curve, it would like to increase its price to the point at which final good firms demand exactly all of its stock of inventories. Together with the inventory frictions we describe below, price adjustment frictions give rise to stockouts in the equilibrium of this economy since they prevent firms from increasing their prices.

**C. Intermediate goods firms**

The intermediate good firms are monopolistically competitive. Any given such firm sells a single variety $i$. Such a firm rents capital from consumers, hires labor and produces the intermediate good. It then sells the good to final good firms. The critical assumption we make is that the firm makes the decision of how much to produce, $q_i\left(s^t\right)$, prior to learning the value of $v_i\left(s^t\right)$, the demand shock. This assumption introduces a precautionary motive for holding inventories, the stockout-avoidance motive.

We assume a production function

$$y_i\left(s^t\right)=\left(l_i\left(s^t\right)^{\alpha}k_i\left(s^t\right)^{1-\alpha}\right)^{\gamma},$$

where $y_i\left(s^t\right)$ is output, $k_i\left(s^t\right)$ is the amount of capital firm $i$ rents and $l_i$ is the amount of labor it hires, while $\gamma\leq 1$ determines the degree of returns to scale. Letting $R\left(s^t\right)$ and $W\left(s^t\right)$ denote the rental rate of capital and the aggregate nominal wage rate, respectively, this production function implies that the minimum cost of producing $y_i\left(s^t\right)$ units of the intermediate good is given by $\Omega\left(s^t\right)y_i\left(s^t\right)^{\frac{1}{\gamma}}$, where

$$\Omega\left(s^t\right)=\chi W\left(s^t\right)^{\alpha}R\left(s^t\right)^{1-\alpha},$$

and $\chi$ is a constant.

Intermediate good firms face two frictions. First, they must choose how much to produce, $y_i\left(s^t\right)$, and the price to set, $P_i\left(s^t\right)$, prior to learning their demand shock, $v_i\left(s^t\right)$. Second, they change prices infrequently, in a Calvo fashion. An exogenously chosen faction $1-\lambda_p$ of firms are allowed to reset their nominal prices in any given period; the remaining $\lambda_p$ of firms leave their prices unchanged.

Let $m_i\left(s^{t-1}\right)$ denote the stock of inventories firm $i$ has at the beginning of date $t$. If the firm produces $y_i\left(s^t\right)$ additional units, the amount it has available for sale is equal to $z_i\left(s^t\right)=m_i\left(s^{t-1}\right)+y_i\left(s^t\right)$. Recall that, given a price $P_i\left(s^t\right)$ and stock $z_i\left(s^t\right)$, the firm's sales are equal to:

$$q_i\left(s^t\right)=\min\left(v_i\left(s^t\right)\left(\frac{P_i\left(s^t\right)}{P\left(s^t\right)}\right)^{-\theta}q\left(s^t\right),\ z_i\left(s^t\right)\right) \tag{4}$$

The firm's problem is therefore to choose $p_i\left(s^t\right)$ and $z_i\left(s^t\right) \geqslant m_i\left(s^{t-1}\right)$, so as to maximize its objective given by

$$\max_{p_i(s^t),\ z_i(s^t)\geqslant m_i(s^{t-1})} \sum_{t=0}^{\infty} \int_{s^t} Q\left(s^t\right)\left[P_i\left(s^t\right) q_i\left(s^t\right) - \Omega\left(s^t\right)\left[z_i\left(s^t\right) - m_i\left(s^{t-1}\right)\right]^{\frac{1}{\gamma}}\right] ds^t$$

where recall $Q\left(s^t\right)$ is the date 0 price of one unit of currency to be delivered in state $s^t$ and $m_i\left(s_0\right)$ is given. The constraints are the demand function in (4), the restriction that $z_i\left(s^t\right)$ and $P_i\left(s^t\right)$ are not measurable with respect to $v_i\left(s^t\right)$, as well as the constraint that $P_i\left(s^t\right) = P_i\left(s^{t-1}\right)$ in the absence of a price adjustment opportunity, as well as the law of motion for inventories :

$$m_i\left(s^t\right) = \left(1 - \delta_z\right)\left(z_i\left(s^t\right) - q_i\left(s^t\right)\right)$$

where $\delta_z$ is the rate at which inventories depreciate.

**D. Equilibrium**

Consider now this economy's market-clearing conditions and the definition of *equilibrium*. The market-clearing conditions on labor are:

$$\left(\int l_j\left(s^t\right)^{\frac{\vartheta-1}{\vartheta}} dj\right)^{\frac{\vartheta}{\vartheta-1}} = l(s^t)$$

and

$$l\left(s^t\right) = \int_i l_i(s^t) di$$

The first expression is the production function for producing final labor services $l\left(s^t\right)$ out of the differentiated services supplied by each union. The second expression says that the total amount of the final labor service must be equal to the amount of labor hired by each intermediate goods firm.

Similarly, the market clearing conditions for the final good are:

$$q\left(s^t\right) = \left(\int_0^1 v_i(s^t)^{\frac{1}{\theta}} q_i(s^t)^{\frac{\theta-1}{\theta}} di\right)^{\frac{\theta}{\theta-1}}$$

and

$$\int_0^1 \left( c_j\left(s^t\right) + x_j\left(s^t\right) + \frac{\xi}{2}\left(\frac{x_j\left(s^t\right)}{k_j(s^{t-1})} - \delta\right)^2 k_j(s^{t-1}) \right) dj = q\left(s^t\right)$$

The first expression is the final good production function and the second says that the total consumption and investment of the different households must sum up to the total amount of the final good produced. Since all households make identical consumption and investment decisions, we can write the resource constraint for final goods as:

$$c\left(s^t\right) + x\left(s^t\right) + \frac{\xi}{2}\left(\frac{x\left(s^t\right)}{k(s^{t-1})} - \delta\right)^2 k(s^{t-1}) = q\left(s^t\right)$$

Next, the market-clearing condition on bonds is $B(s^t) = 0$ and the cash-in-advance constraint requires $P\left(s^t\right) c\left(s^t\right) = M\left(s^t\right)$. Finally, the market clearing condition for capital is

$$\int_i k_i\left(s^t\right) di = k\left(s^{t-1}\right)$$

An *equilibrium* for this economy is a collection of allocations for households $c(s^t)$, $M(s^t)$, $B(s^{t+1})$, $k\left(s^t\right)$, $x\left(s^t\right)$, $l_j\left(s^t\right)$ and $W_j(s^t)$; prices and allocations for firms $p_i(s^t)$, $q_i\left(s^t\right)$, $y_i(s^t)$, $l_i\left(s^t\right)$, $k_i\left(s^t\right)$, $z_i\left(s^t\right)$; and aggregate prices $W(s^t)$, $P(s^t)$, $R\left(s^t\right)$ and $Q(s^{t+1}|s^t)$, all of which satisfy the following conditions: (*i*) the consumer allocations solve the consumers' problem; (*ii*) the prices and allocations of firms solve their maximization problem; (*iii*) the market-clearing conditions hold; and (*iv*) the money supply process satisfies the specifications above.

### E. Recursive Formulation and Solution Method

We next recast the problem recursively. At the beginning of period $t$, after the realization of the money shock, $\mu\left(s^t\right)$, but before the realization of the demand shocks, $v_i\left(s^t\right)$, the state of an individual firm $i$ is characterized by its price in the preceding period, $P_i(s^{t-1})$ and its inventory stock $m_i\left(s^{t-1}\right)$. It is convenient to normalize all nominal prices and wages by the current money supply. Specifically, let $p_i(s^t) = P_i(s^{t-1})/M(s^t)$ and $\omega(s^t) = \Omega(s^t)/M(s^t)$ and use similar notation for other prices. Let $\bar{p}\left(s^t\right) = \frac{P(s^t)}{M(s^t)}$ denote the normalized aggregate price level. With this normalization, we can write the state of an individual firm $i$ in $s^t$ as $[p_i, m_i]$. Let $\boldsymbol{\lambda}\left(s^t\right)$ denote the measure of firms over these variables. Let $\boldsymbol{w}\left(s^t\right)$ denote the distribution of wages set by unions.

The only source of aggregate uncertainty is the growth rate of the money supply, $\mu\left(s^t\right)$. However, since money growth is iid, $\boldsymbol{s}=(\boldsymbol{\lambda}, \boldsymbol{w})$ fully characterize the aggregate state of this economy. Let $d(p, z, m, \boldsymbol{s})$ denote the expected dividends of the firm that charges a price $p$ and has $z$ units of inventories after production takes place, evaluated at date 0 prices:

$$d(p, z, m, \boldsymbol{s})=\int \frac{u_c}{\bar{p}}\left(p \min \left(v\left(\frac{p}{\bar{p}}\right)^{-\theta} q, z\right)-\omega(z-m)^{\frac{1}{\gamma}}\right) d \Phi(v),$$

where $\Phi(v)$ is the distribution of demand shocks. We can write the firm's problem recursively as follows:

$$\begin{aligned}
V^a(m, \boldsymbol{s}) &= \max_{p', z \geqslant m} d\left(p', z, m, \boldsymbol{s}\right)+\int \int V\left(\tfrac{p'}{\mu'}, m', \boldsymbol{s}'\right) dF\left(\mu'\right) d \Phi(v), \\
V^n(p, m, \boldsymbol{s}) &= \max_{z \geqslant m} d(p, z, m, \boldsymbol{s})+\int \int V\left(\tfrac{p}{\mu'}, m', \boldsymbol{s}'\right) dF\left(\mu'\right) d \Phi(v).
\end{aligned}$$

Here $V^a$ is the value of a firm that is allowed to reset its price, $V^n$ the value of a firm that cannot reset its price and must sell at its old price $p$, and $V=\left(1-\lambda_p\right) V^a+\lambda_p V^n$ is the expected value of a firm prior to learning whether it can adjust its price. We use $F(\mu)$ to denote the distribution of money growth shocks.

The solution to this problem gives decision rules $p'(m, \boldsymbol{s})$ and $z^a(m, \boldsymbol{s})$ for firms that reset their prices and $z^n(p, m, \boldsymbol{s})$ for firms that do not reset prices, which, together with the equilibrium conditions, are used to compute all other equilibrium objects. For example, the aggregate price level is equal to:

$$\bar{p}(\boldsymbol{s})=\left(\int\left[\left(1-\lambda_p\right) p'(m, \boldsymbol{s})^{1-\theta}+\lambda_p p^{1-\theta}\right] d \boldsymbol{\lambda}(p, m)\right)^{\frac{1}{1-\theta}}$$

Since $\boldsymbol{\lambda}$ and $\boldsymbol{w}$ are large objects, we approximate the solution to this problem using the Krusell-Smith approach. That is, we postulate that aggregate prices and quantities are a function of a small number of moments of the distribution $\boldsymbol{\lambda}$.and $\boldsymbol{w}$**.** For wages, we follow the standard approach in the New Keynesian literature and simply log-linearize the union's wage decisions. Hence the past aggregate wage, $\bar{w}_{t-1}$, is the only relevant variable that characterizes the dynamics of wages. We approximate $\boldsymbol{\lambda}$ using the past aggregate price level, $\bar{p}_{t-1}$, as well as the aggregate stock of inventories, $\bar{m}_{t-1}$. We then postulate that all aggregate variables are log-linear functions of $\bar{w}_{t-1}, \bar{p}_{t-1}, \bar{m}_{t-1}$ as well as the aggregate stock of capital, $k_{t-1}$. For example, we postulate the aggregate price level evolves according to:

$$\log \bar{p}_t = \alpha_0 + \alpha_1 (\log \bar{p}_{t-1} - \log \mu_t) + \alpha_2 (\log \bar{w}_{t-1} - \log \mu_t) + \alpha_3 \log \bar{m}_{t-1} + \alpha_4 \log k_{t-1}.$$

Notice that we must detrend the past price and wage level by the growth rate of the money supply so as to express these state variables relative to the current period money supply, $M_t$.

Given a guess for the coefficients on these aggregate laws of motion, we solve the firm's problem (using splines and projection methods to approximate the firm's value functions and decision rules) and union's problems (using, as standard in Calvo-type models, a log-linear approximation around the steady-state), simulate a time-series of this economy and update the guess for the unknown coefficients $\alpha_i$ using OLS regressions. As Krusell and Smith (1998) and Khan and Thomas (2007) do, we found that these few moments provide a very good approximation, in that the mean squared error between the predicted variables and the actual ones is extremely small (less than 0.01% of the time-series variance of these series).

**F. The Workings of the Model**

We next discuss the decision rules in this economy. We begin by studying a version of the model with constant returns at the firm level ($\gamma = 1$). We then characterize the optimal pricing and inventory decisions in the presence of decreasing returns ($\gamma < 1$) and provide some empirical evidence for decreasing returns using micro-level data.

***Firm-level Constant Returns***

To build intuition, assume away the irreversibility constraint $z_i(s^t) \geqslant m_i(s^{t-1})$. This constraint turns out not to bind for most of the experiments we describe here, with the exception of the economy with non-convexities we describe later on. Moreover, assume that prices are flexible, $\lambda_p = 0$. Recall that $v_i(s^t)$, the demand shocks, are iid and log-normal. Let $\Phi$ denote the cdf and

$\sigma_v^2$ the variance of these shocks. Then we can write the firm's expected sales as

$$R\left(P_i\left(s^t\right), z_i\left(s^t\right)\right) = \int_0^\infty \min\left(v\left(\frac{P_i\left(s^t\right)}{P\left(s^t\right)}\right)^{-\theta} q\left(s^t\right), z_i\left(s^t\right)\right) d\Phi\left(v\right) =$$

$$= \left(\frac{P_i\left(s^t\right)}{P\left(s^t\right)}\right)^{-\theta} q\left(s^t\right) \exp\left(\frac{\sigma_v^2}{2}\right) \Phi\left(\log v_i^*\left(s^t\right) - \sigma_v^2\right) + z_i\left(s^t\right)\left(1 - \Phi\left(\log v_i^*\left(s^t\right)\right)\right) \ , \tag{5}$$

where

$$v_i^*\left(s^t\right) = \frac{z_i\left(s^t\right)}{\left(\frac{P_i(s^t)}{P(s^t)}\right)^{-\theta} q\left(s^t\right)}$$

is the highest value of the productivity shock for which the firm does not stockout. To understand expression (5), notice that the first term is the expected value of sales in those states in which the firm does not stockout, while the second term is the amount of inventories the firm has, $z_i\left(s^t\right)$, times the probability of a stockout. Clearly, $R_z = \left(1 - \Phi\left(\log v_i^*\left(s^t\right)\right)\right) > 0$ : an increase in its stock of inventories allows the firm to sell in those states in which it would otherwise stockout[12].

With constant returns and no irreversibility, the value of a firm is linear in the stock of inventories it has inherited from the previous period: an unsold unit of inventories at date $t$ depreciates to $(1-\delta_z)$ units next period and saves the firm production costs equal to $(1-\delta_z)\, Q\left(s^{t+1}|s^t\right)\Omega\left(s^{t+1}\right)$ evaluated at date $t$ prices. Let

$$\Omega'\left(s^t\right) = (1-\delta_z)\int_{s^{t+1}} Q\left(s^{t+1}|s^t\right)\Omega\left(s^{t+1}\right) ds^{t+1}$$

denote the expected value of these savings. The problem of the firm thus reduces to:

$$\max_{P_i(s^t),\, z_i(s^t)} \left(P_i\left(s^t\right) - \Omega'\left(s^t\right)\right) R\left(P_i\left(s^t\right), z_i\left(s^t\right)\right) - \left(\Omega\left(s^t\right) - \Omega'\left(s^t\right)\right) z_i\left(s^t\right),$$

where, recall, $\Omega(s^t)$ is the marginal cost of production.

To understand this expression, notice that the choice of prices is similar to that in the standard problem of a monopolist, except that $R\left(P_i\left(s^t\right), z_i\left(s^t\right)\right)$ is the demand function and $\Omega'\left(s^t\right)$

[12] To derive this expression, notice that $z$ enters (5) in three places, but two of these terms cancel out.

is the marginal valuation of the goods the firm sells. The choice of inventories, $z_i\left(s^t\right)$, is also straightforward: on one hand a higher $z_i\left(s^t\right)$ increases expected sales, but the firm expects to lose $\left(\Omega\left(s^t\right)-\Omega'\left(s^t\right)\right)z_i\left(s^t\right)$ in inventory carrying costs.

The firm's optimal price is then a markup over its shadow valuation of inventories:

$$P_i\left(s^t\right)=\frac{\varepsilon_i\left(s^t\right)}{\varepsilon_i\left(s^t\right)-1}\Omega'\left(s^t\right).$$

Here $\varepsilon_i\left(s^t\right)$ is the price elasticity of expected sales and is equal to $\theta$ (the elasticity of substitution across varieties) times the share of sales in the states in which the firm does not stockout:

$$\varepsilon_i\left(s^t\right)=\theta\times\frac{\exp\left(\frac{\sigma_v^2}{2}\right)\Phi\left(\log v_i^*\left(s^t\right)-\sigma_v^2\right)}{\exp\left(\frac{\sigma_v^2}{2}\right)\Phi\left(\log v_i^*\left(s^t\right)-\sigma_v^2\right)+v_i^*\left(s^t\right)\left(1-\Phi\left(\log v_i^*\left(s^t\right)\right)\right)}.$$

We next turn to the inventory accumulation decision. The choice of $z_i\left(s^t\right)$ satisfies

$$1-\Phi\left(\log v_i^*\left(s^t\right)\right)=\frac{1-r_i\left(s^t\right)}{P_i\left(s^t\right)/\Omega\left(s^t\right)-r_i\left(s^t\right)}. \tag{6}$$

The left-hand side of this expression is the probability that the firm stocks out. As in Bils and Kahn (2000), the firm chooses a higher stock of inventories (a lower stockout probability) the higher the markup $P_i\left(s^t\right)/\Omega\left(s^t\right)$, and the higher the return to holding inventories, $r_i\left(s^t\right)$, where

$$r_i\left(s^t\right)=\frac{\Omega'\left(s^t\right)}{\Omega\left(s^t\right)}=\left(1-\delta_z\right)\int_{s^{t+1}}Q\left(s^{t+1}|s^t\right)\frac{\Omega\left(s^{t+1}\right)}{\Omega\left(s^t\right)}ds^{t+1}. \tag{7}$$

Stockouts are especially costly for firms that have higher markups since the profit lost by failing to make a sale is greater. Similarly, a higher return to holding inventories (conversely, a lower carrying cost) makes it optimal to increase the stock of inventories available for sale.

One important implication of the model is that inventories are much more sensitive to changes in the return to holding inventories, rather than changes in markups. To see this, we find it useful to log-linearize (6) around the steady-state:

$$\bar{v}\hat{v}_i^*\left(s^t\right) = \frac{1}{\bar{\Phi}\left[\bar{b} - \beta\left(1-\delta_z\right)\right]}\left[\left(1-\bar{\Phi}\right)\bar{b}\left[\hat{P}_i\left(s^t\right) - \hat{\Omega}\left(s^t\right)\right] + \beta\left(1-\delta_z\right)\bar{\Phi}\hat{r}_i\left(s^t\right)\right],$$

where hats denote log-deviations from the steady state, $\bar{b}$ is the steady-state markup, $\bar{v}$ is the pre-sale steady-state inventory-sales ratio, and $1-\bar{\Phi}$ is the steady-state probability of a stockout.

If the stockout probability and markups are low, as in the data, and $\left(1-\bar{\Phi}\right)\bar{b} \approx 0$, then inventories are relatively insensitive to markups. In contrast, as long as the cost of carrying inventories (as determined by $\delta_z$) is sufficiently small, inventories are much more sensitive to fluctuations in the return to holding inventories. Intuitively, if the cost of carrying inventories is sufficiently low, firms find it optimal to intertemporally substitute production in order to react to expected changes in the marginal cost of production and/or changes in the interest rate[13]. Hence, the dynamics of inventories is closely related to the dynamics of costs but also influenced by the behavior of markups. In the next section we exploit this key feature of the model to draw implications for the dynamics of costs in response to monetary policy shocks.

So far we have discussed the model's implications for $v_i^*\left(s^t\right)$. This object, on its own, is not useful to evaluate the model empirically as we do not directly observe it in the data. Notice however that there is a monotonic relationship between the aggregate inventory-to-sale ratio and $v_i^*\left(s^t\right)$. In particular, integrating the distribution of demand shocks, and noting that all firms make the same $v_i^*\left(s^t\right)$ choices, it follows that the end-of-period inventory-sales ratio, which we do observe in the data, is equal to:

$$IS\left(s^t\right) = \frac{v^*\left(s^t\right)\Phi\left(\log v^*\left(s^t\right)\right) - \exp\left(\frac{\sigma_v^2}{2}\right)\Phi\left(\log v^*\left(s^t\right) - \sigma_v^2\right)}{\exp\left(\frac{\sigma_v^2}{2}\right)\Phi\left(\log v^*\left(s^t\right) - \sigma_v^2\right) + v_t^*\left(1-\Phi\left(\log v^*\left(s^t\right)\right)\right)}. \tag{8}$$

[13]See House (2008) who makes a similar argument in the context of a model with investment.

***Firm-level Decreasing Returns***

With decreasing returns, $\gamma < 1$, a firm's marginal cost of producing $y_i\left(s^t\right)$ units of output is increasing in $y_i\left(s^t\right)$. In particular, this cost is now equal to

$$\Omega_i\left(s^t\right) = \Omega\left(s^t\right) y_i\left(s^t\right)^{\frac{1}{\gamma}-1}.$$

Price and production decisions are similar to those in the economy with constant returns, except that now the shadow valuation of inventories depends on how much the firm expects to produce next period:

$$\Omega_i'\left(s^t\right) = (1-\delta_z)\int_{s^{t+1}} Q\left(s^{t+1}|s^t\right)\Omega\left(s^{t+1}\right)\frac{1}{\gamma}\left[z_i\left(s^{t+1}\right) - m_i\left(s^t\right)\right]^{\frac{1}{\gamma}-1} ds^{t+1}.$$

The decisions rules under firm-level decreasing returns differ from those under constant returns along two important dimensions. First, with decreasing returns inventory holdings at the firm level are persistent – the stock of inventories after orders are made, $z_i\left(s^t\right)$, is increasing in the initial stock, $m_i\left(s^{t-1}\right)$. Second, the firm's price is negatively correlated with its initial inventory stock. These two predictions follow from the fact that a firm's marginal cost of production is decreasing in its initial inventory stock: firm-level decreasing returns act much like a convex inventory-adjustment cost.

We illustrate these predictions of the model in Figure 2 in which we contrast the decision rules in an economy with decreasing returns with those in an economy with constant returns. Panel A shows that in the economy with decreasing returns the pre-sale stock, $z_i\left(s^t\right)$, is increasing in the initial stock, $m_i\left(s^{t-1}\right)$. Panel B shows that firms that start with more inventories charge lower prices in the economy with decreasing returns. In contrast, prices and inventories are independent of the initial stock in the economy with constant returns.

***Empirical Evidence on Firm-level Decreasing Returns***

Since decreasing returns play an important role in our analysis below, we briefly present some empirical evidence for these using micro-level data. We find these results of independent interest in light of the increasing role they play in amplifying the real effects of monetary policy shocks in recent studies[14].

We use a dataset of prices, inventories, and orders for a Spanish supermarket used by Aguirregabiria (1999) in his study of markups and inventories in retail firms. See the original study for a detailed description of the data. We use a panel of monthly observations on inventories, prices and orders for 534 products (mostly non-perishable foods and household supplies) sold by the supermarket chain for a period of 29 months from January 1990 to May 1992.

We use the data to establish empirical evidence for the two predictions of the model we discuss above about the relationship between prices, orders and the initial inventory stock. To do so, we estimate the following fixed-effects regressions:

$$\log p_{it} = \alpha_i^p + \beta^p \log m_{it-1} + \varepsilon_{it}^p,$$

$$\log z_{it} = \alpha_i^z + \beta^z \log m_{it-1} + \varepsilon_{it}^z$$

where $m_{it-1}$ is the stock of inventories at the end of period $t-1$ for good $i$, $p_{it}$ is the price for the good at date $t$, and $z_{it}$ is the amount of goods available for sale (the initial stock plus orders). Using the data we estimate an elasticity of prices to inventories, $\beta^p$, equal to -0.023 (the standard error is equal to 0.001) and an elasticity of the pre-sale stock to inventories, $\beta^z$, equal to 0.45 (the standard error is equal to 0.002). The signs of these two elasticities are consistent with the predictions of the theory. We show in our quantitative analysis below that a value of $\gamma$ of about 2/3 best fits these two moments of the data.

We thus conclude that firm-level decreasing returns (or other forms of convex inventory adjustment costs) are necessary to account for the pattern of prices, orders and inventories in the micro data. As we also show below, such decreasing returns also improve the model's ability to account for the behavior of inventories at the aggregate level, especially in an economy with sticky prices.

[14] See for example Altig et. al (2005) and Burstein and Hellwig (2008) who measure the size of decreasing returns using macro and micro-level evidence, respectively.

## G. Parametrization

We next describe how we have chosen parameters to evaluate the model's quantitative implications. We set the length of the period as one month and therefore choose a discount factor of $\beta = .96^{1/12}$. We assume preferences of the form $u(c) - v(n) = \log c - n$. These preferences imply an infinite Frisch labor supply elasticity, and can be interpreted as the outcome of indivisibilities combined with Hansen (1985) and Rogerson (1988) –type lotteries. We focus on these preferences since they imply, in a version of the model without capital and nominal wage rigidities and with constant returns at the firm level, that the marginal cost of production increases one-for-one with the monetary shock[15]. Below we consider the implications of changing the assumptions we make about preferences.

Table 2 reports the parameter values we used in our quantitative analysis. We set the rate at which capital depreciates, $\delta$, equal to 0.01. We set the elasticity of substitution across intermediate goods and varieties of labor, $\theta = \vartheta = 5$, implying a 25% markup, in the range of estimates in existing work. Finally, we assume a frequency of wages changes of once a year, $1 - \lambda_w = 1/12$, consistent with what is typically assumed in existing studies.

We calibrate the inventory parameters, namely, the rate at which inventories depreciate, $\delta_z$, and the volatility of demand shocks, $\sigma_v$, to ensure that the model accounts for two facts about inventories and stockouts in the data. First, as can be seen from the decision rules (6)-(7) above, $\delta_z$ directly affects the frequency of stockouts: a higher cost of carrying inventories make it optimal for firms to stockout more often. Bils (2004) uses micro CPI data from the BLS and reports a frequency of stockouts of 5%.[16] In all of the experiments we consider below we choose $\delta_z$ so that the model generates a 5% frequency of stockouts. Second, $\sigma_v$, the volatility of demand shocks, directly maps into the average inventory-sales ratio in the model, as (8) shows. We thus choose this parameter so as to match an (end-of-period) inventory-sales ratio of 1.4 months, as in the US Manufacturing and Trade sector.

For example, as Panel A. I. of Table 2 shows, in the economy with constant firm-level returns,

[15] This particular parametrization has been widely used in the menu cost literature. See for example Golosov and Lucas (2007).

[16] This statistic excludes observations that are three months or more prior to a product becoming permanently or seasonally unavailable. The overall frequency of stockouts is approximately 8% when focusing on all observations. This number is also consistent with the findings of Aguirregabiria (1999, 2003).

the value of $\sigma_v$ necessary to match these two facts is equal to 0.63, while the rate of depreciation is equal to $\delta_z = 0.91\%$. We show below that such a high volatility of demand is also necessary to reconcile the model with the comovement of prices and quantities in the micro-data. Our estimate of the depreciation rate is in the range of the inventory-carrying costs measured directly in the logistics literature, see for example Richardson (1995).

For the economy with firm-level decreasing returns we must also calibrate the degree of returns to scale, $\gamma$. We do so by requiring the model to match the two elasticities of prices and orders with respect to the initial inventory stock in the Aguirregabiria (1999) data. It turns out that a value of $\gamma$ of 2/3 best fits these two moments of the data. This value of $\gamma$ implies an elasticity of prices to inventories $\beta^p$ = -0.036 and an elasticity of the post-production stock to inventories $\beta^z = 0.36$. Panel A.II. of Table 2 reports the parameter values and moments in this economy with firm-level decreasing returns. A somewhat smaller volatility of demand shocks (0.57 vs. 0.63 earlier) and rate of depreciation (0.75% vs. 0.91% earlier) are required to match the inventory-sales ratio of 1.4 and frequency of stockouts of 0.05 in the data.

## 4. Quantitative Investigation

We use the model to make two related points. First, versions of the model with flexible prices (that imply nearly constant markups) predict a much stronger response of inventories to a monetary policy shock than in the data. Second, models with countercyclical markups (sticky prices) can account for the dynamics of inventories in the data, but only if markups decline (real marginal costs increase) sufficiently in response to an expansionary monetary shock.

To make our first point we study a version of our economy with sticky wages and flexible prices. We then introduce nominal price rigidities and show that if the marginal cost is sufficiently responsive to monetary shocks, the model can indeed account for the dynamics of inventories in the data.

### A. Economy with flexible prices

We start by studying an economy with constant returns to labor. We then allow for decreasing returns to labor, either by assuming decreasing returns to scale at the firm level or by introducing capital in the production function.

***Constant Returns***

We set $\gamma = 1$ and $\alpha = 1$. The marginal cost of production is thus equal to nominal wages:

$$\Omega\left(s^t\right) = W\left(s^t\right).$$

Since we have assumed an infinite Frisch elasticity of labor supply and iid money growth, reset wages are proportional to the money supply and the aggregate wage evolve according to

$$\hat{w}\left(s^t\right) = \lambda_w \hat{w}\left(s^{t-1}\right) - \lambda_w \mu\left(s^t\right),$$

where $\hat{w}\left(s^t\right)$ is the log-deviation of $W\left(s^t\right)/M(s^t)$ from its steady state level.

Figure 3 shows the impulse responses of nominal and real variables in the model to a 1% increase in the money supply. Panel A shows that nominal wages respond gradually to the shock. Since prices are flexible, they track nominal wages closely, although prices decline somewhat relative to wages. This happens because of a decline in the optimal markup induced by inventory accumulation. Panel B shows the response of inventories and sales. We report, as in the data, the response of the real aggregate end-of-period inventory stock defined as:

$$I\left(s^t\right) = \int_0^1 \frac{m_i\left(s^t\right)}{1-\delta_z} di = \int_0^1 \left[z_i\left(s^t\right) - q_i\left(s^t\right)\right] di.$$

Similarly, real sales are computed using:

$$S\left(s^t\right) = \int_0^1 q_i\left(s^t\right) di.$$

Notice that sales rise immediately by about 1% and gradually decline. Moreover, the response of inventories is much greater than that of sales: inventories increase by about 3% on impact and gradually decline. Panel C shows that the reason inventories increase much more than sales is a sharp increase in production. Production is defined as

$$Y\left(s^{t}\right)=\int\left[z_{i}\left(s^{t}\right)-m_{i}\left(s^{t-1}\right)\right]di$$

and is, by definition, equal to sales plus inventory investment:

$$Y\left(s^{t}\right)=\int_{0}^{1}\left[q_{i}\left(s^{t}\right)+\frac{m_{i}\left(s^{t}\right)}{1-\delta_{z}}-m_{i}\left(s^{t-1}\right)\right]di=S\left(s^{t}\right)+I\left(s^{t}\right)-\left(1-\delta_{z}\right)I\left(s^{t-1}\right)$$

Since production increases by about 5% in response to the monetary shock and sales by only 1%, the excess production contributes to the large increase in the stock of inventories.

Table 3 summarizes our findings. In Panel A we report two sets of statistics. The first set are measures of the real effects of monetary shocks which summarize the impulse response of aggregate consumption, $c\left(s^{t}\right)$, to a monetary shock. The first row shows that the average consumption response in the first 2 years after the shock, i.e., the area under the impulse response function in Panel D of Figure 3, is equal to 0.44%. The maximum consumption response is equal to 1.01%. Finally, the half-life of consumption, our measure of the persistence of the real effects, is equal to 8 months.

The second set of statistics we report are those that characterize the behavior of inventories, sales and production. To compute these statistics, we HP-filter these series, as in the data, with a smoothing parameter of 14400. We then contrast the model's predictions with those in the data for which we focus on the Manufacturing and Trade sector, the series conditional on money shocks.

The model does very poorly in accounting for the behavior of inventories in the data. It predicts a strongly procyclical inventory-sales ratio (the correlation with sales is 1 vs. -0.70 in the data) and that the inventory-sales ratio is much more volatile than in the data. The elasticity of the inventory-sales ratio to sales is equal to 2.09 in the model (-0.67 in the data), thus implying that the stock of inventories increases by 3.09% (0.33% in the data) for every 1% increase in sales. The model's counterfactual implications for the stock of inventories imply counterfactual implications for the behavior of inventory investment. The model predicts that production is 3.14 times more volatile than sales (1.10 in the data).

The reason the stock of inventories is very sensitive to monetary shocks in the model is the intertemporal substitution effect. The return to holding inventories is equal to

$$r_i\left(s^t\right) = (1-\delta_z)\int \frac{1}{r\left(s^t\right)} \frac{W\left(s^{t+1}\right)}{W\left(s^t\right)} \pi\left(s^{t+1}|s^t\right) ds^{t+1}$$

where $r\left(s^t\right)$ is the nominal risk-free rate. Since we have assumed that preferences are log-linear and money growth is iid, the nominal interest rate is equal to the expected growth rate of the money supply,

$$r\left(s^t\right) = \frac{1}{\beta}\int \frac{P\left(s^{t+1}\right)c\left(s^{t+1}\right)}{P\left(s^t\right)c\left(s^t\right)} \pi\left(s^{t+1}|s^t\right) ds^{t+1} = \frac{1}{\beta}\int \frac{M\left(s^{t+1}\right)}{M\left(s^t\right)} \pi\left(s^{t+1}|s^t\right) ds^{t+1}$$

and is therefore constant. As a result the return to holding inventories increases after an increase in the growth rate of the money supply, since the nominal interest rate is constant and the cost of production (here the nominal wage) is expected to increase.

We thus conclude that this version of the model generates real effects of monetary shocks for the wrong reasons, by implying a sluggish response of costs to monetary shocks and making it optimal for firms to take advantage of the lower costs by investing in inventories much more than they do in the data.

***Firm-level decreasing returns***

We next assume firm-level decreasing returns and set $\gamma = 2/3$. Decreasing returns imply a slower response of inventories to a monetary shock, since the marginal cost of production increases with the amount produced, thus preventing firms from taking advantage of the sticky wages. Decreasing returns thus act as an adjustment cost that slow down the response of inventories to monetary shocks.

In Table 3 (Panel B) we report the aggregate implications of this version of the model. Clearly, adding decreasing returns diminishes the real effects of money since they make marginal costs more responsive to monetary shocks. The average consumption response to a 1% monetary expansion is equal to 0.27%, thus only 0.61 as large as the average response in the economy with constant returns (0.44%). The maximum consumption response is even more diminished (0.44% vs. 1.01% earlier). The table also shows that the model's implications regarding the response of inventories to monetary shocks improves somewhat, but that decreasing returns alone aren't capable of reproducing the sluggish response of inventories in the data. In particular, the elasticity of inventories to sales is now equal to 2.21. This is smaller than in the economy with constant returns (3.09), but nevertheless much greater than in the data (0.33). Similarly, production is somewhat less volatile (1.71 times more volatile then sales) than in the economy with constant returns (3.14), but much more volatile than in the data (1.10).

***Decreasing Returns at the Aggregate Level***

We next assume decreasing returns (to labor) at the aggregate level by introducing capital as a factor of production. We now have $\gamma = 1$ (no firm-level decreasing returns) and $\alpha = \frac{2}{3}$, implying a capital share of 1/3. Capital accumulation is subject to adjustment costs, the size of which is chosen so that the model implies a relative variability of investment to consumption equal to 4, as in the data. In this economy the marginal cost of production is equal across firms and given by:

$$\Omega\left(s^t\right) = \chi W\left(s^t\right)^{\alpha} R\left(s^t\right)^{1-\alpha}$$

The household's preference for smooth consumption imply that the rental rate of capital increases with an expansionary monetary shock due to the increased demand for capital. Hence, the marginal cost increases more strongly than in the economy with labor only.

Panel C of Table 3 shows that the real effects of monetary shocks are similar now to those in the economy with decreasing returns at the firm level. The Table also shows that adding capital once again bridges the gap between the dynamics of inventories in the model and in the data: this version of the model predicts an elasticity of inventories to sales of 1.42 and a relative volatility of production to sales of 1.43. Both of these are much greater than in the data, but smaller than in the economy with constant returns and in the economy with decreasing returns at the firm level.

Why are inventories more sluggish in the economy with capital than in the economy with firm-level decreasing returns? The reason is that capital accumulation generates more persistence in the marginal cost of production since investment in capital lowers its rental rate in future periods. Since the return to holding inventories is proportional to the expected change in costs, capital accumulation imparts sluggishness in the return to holding inventories and therefore in the inventory stock. To see this, panel D. of Table 3 also reports statistics for an economy with capital in which the stock of capital is fixed. Notice that both the consumption and inventory responses in this version of the model are virtually indistinguishable from those in the economy with firm-level decreasing returns in Panel B.

Panel E reports an alternative extreme experiment in which we assume away capital adjustment costs altogether. In this economy the behavior of inventories is much more in line with the data. The inventory-sales ratio is countercyclical: its correlation with sales is -0.88. Moreover, the elasticity of inventories to sales is now equal to 0.66 (0.33 in the data) and production is only 1.17 times more volatile than sales (1.10 in the data). This improved fit with regards to inventories comes however, at the cost of the model's implications for investment variability. In this version of the model investment is 23 times more volatile than consumption, thus substantially more volatile than in the data. Moreover, the real interest rate now increases during a monetary expansion, in contrast to the data in which real interest rates persistently decline following an expansionary monetary shock[17]. Since the real interest rate is (in addition to the expected change in the real marginal cost) one of the two components that directly affects the cost of carrying inventories, we find this version of the model without capital adjustment costs an unsatisfactory one. We thus conclude that models with constant markups and sticky wages cannot account for the response of inventories to monetary shocks in the data.

**B. Economy with Sticky Prices**

We next assume that prices as well as wages are sticky. Consistent with the evidence in Klenow and Kryvtsov (2008) and Nakamura and Steinsson (2008a) who report that prices change on average once about every 6-10 months, we assume a frequency of price changes of $1 - \lambda_p = \frac{1}{8}$. (Since sticky prices do not change much the inventory-accumulation decisions of firms, the inventory moments are unaffected, and so we keep all other parameter values unchanged. See Panel B of Table 2 for the parameter values and targets in this version of the model). We show that this version of the model can indeed account for the behavior of inventories, but only in the presence of sufficiently large decreasing returns to labor that make the marginal cost of production responsive to monetary shocks.

[17]See, for example, Christiano, Eichenbaum and Evans (2005).

***Constant Returns***

Panel A. of Table 4 reports statistics from the economy with constant returns ($\gamma = \alpha = 1$). Now that prices are sticky, the real effects of money are somewhat greater than in the economy with flexible prices: the average response of consumption is 0.60, thus about 1.35 greater than with flexible prices (0.44%). As for inventories, these are, as earlier, strongly procyclical and very volatile, with an elasticity of inventories to sales equal to 2.68, only slightly lower than in the economy with flexible prices (3.09). Thus sticky prices, on their own, do not improve much the model's ability to account for the inventory facts.

This result is driven by two features of the model. First, the optimal inventory-sales ratio is not very sensitive to variation in markups, as we have shown earlier, and much more sensitive to variation in the return to carrying inventories. Second, when wages are sticky and there are no decreasing returns, sticky prices do not greatly reduce markups: even though prices are sticky, costs are sticky as well.

To see that markups are not very countercyclical here, even though prices are sticky, we conduct the following decomposition of the real response of monetary shocks to a) nominal cost rigidities and b) markup variation. Recall that a cash-in-advance constraint holds in our model:

$$\ln(c_t) = \ln(M_t) - \ln(P_t) = \underbrace{[\ln(M_t) - \ln(\Omega_t)]}_{\textit{cost term}} + \underbrace{[\ln(\Omega_t) - \ln(P_t)]}_{\textit{markup term}}$$

The response of consumption is thus equal to the sum of two terms: one that captures the extent to which costs, $\Omega_t$, decline relative to the money stock (cost term), and another that captures the extent to which prices decline relative to costs (markup term). We report, in Table 4, the average response of the second term (the markup decline) relative to the average response of consumption in order to measure the fraction of the real effects accounted for by countercyclical markups. In terms of the impulse responses of Figure 3, this ratio is equal to the area between the price and cost impulse responses relative to the area between the money supply and price responses. As the row labeled 'markup contribution' in Table 4 shows, markups account for only one-third of the real effects of money in this economy. Since consumption increases by about 0.6% on average in the first 2 years following the monetary shock, this implies that markups decline by an average of only about 0.2%.

***Firm-level Decreasing Returns***

We next introduce decreasing returns at the firm-level, by assuming $\gamma = 2/3$. Figure 4 reports the impulse responses to a monetary shock in this economy. Panel A shows that although the nominal wage is sticky, the average marginal cost of production, $\bar{\Omega}\left(s^t\right) = \int \Omega_i\left(s^t\right) di$, increases sharply after the money shock. This decreases the incentive to invest in inventories since the expected growth in marginal cost is much lower than under constant returns. Panel B shows that now inventories gradually rise after the shock and increase much less than sales do. Moreover, since inventory investment is low, production is only slightly more volatile than sales. Overall, these impulse responses are much more in line with the data.

We report the quantitative predictions of the model in Panel B of Table 4. As in the data, the inventory-sales ratio is countercyclical (the correlation is -0.80 vs. -0.70 in the data). The elasticity of inventories to sales is only slightly greater than in the data (0.51 vs. 0.33 in the data), and production is only 1.14 times more volatile than sales (1.10 in the data).

The fit of the model improves for two reasons. First, marginal costs are more responsive to monetary shocks thereby reducing the intertemporal substitution motive. Second, now that costs are more volatile, sticky prices generate a greater decline in the markups of the firms that do not reset their prices. This drop in markups reduces the incentive to hold inventories and lower the inventory-sales ratio.

The decomposition of the real effects of money shows that in this version of the model countercyclical markups play a more important role: 65% of the average response of consumption to a monetary shock is accounted for by a decrease in markups, calculated as the ratio of the aggregate price level to the average marginal cost, $P(s^t)/\bar{\Omega}\left(s^t\right)$. This is about twice greater than in the economy with constant returns. We show below that variations of the model that do a better job at accounting for the facts on interest rates (in our model nominal interest rates are constant whereas in the data they decline following a monetary expansion), predict an even more important role for markups.

Thus, contrary to what Khan and Thomas (2007) find for *technology* shocks, for the model to account for the response of inventories to *monetary* shocks, countercyclical markups must play an important role. The difference stems from the special nature of monetary shocks. Unlike technology shocks, which shift the production possibilities frontier, monetary shocks can only affect output if either markups adjust or if nominal costs are sticky. The latter induces strong intertemporal substitution in production and investment in inventories, and is thus at odds with the data.

***Decreasing Returns at the Aggregate Level***

We have also considered variations of our economy with sticky prices and wages in which the decreasing returns are at the aggregate level. We do so by increasing the share of capital from 0 to 1/3 and again setting $\gamma = 1$. As above for the case of flexible wages, the predictions of the model do not depend much on whether the decreasing returns are at the firm or at the aggregate level. As Panels C and D of Table 4 show, both the economy with capital adjustment costs and the one with a fixed aggregate stock of capital predict countercyclical inventory-sales ratios and an elasticity of inventories to sales of 0.11 and 0.54, thus in the neighborhood of the 0.33 elasticity in the data. Similarly, in both economies production is only 1.07 (1.20) more volatile than sales (1.10 in the data). The drop in markups accounts for about half of the increase in consumption due to a monetary shock.

## 5. Measuring the Response of Markups

We have shown above that variations of the model with strongly countercyclical markups do a much better job of accounting for the inventory facts than economies with no or little variation in markups. We next attempt to measure precisely the extent to which markups must decline in the aftermath of a monetary expansion in order for the model to account for the response of inventories in the data. We do so by calibrating the degree of decreasing returns necessary to account exactly for the elasticity of inventories to sales in the data. For simplicity, we focus on the version of the model with a fixed stock of capital at the aggregate level. We pin down the share of this fixed factor by matching the elasticity of inventories to sales of 0.33 in the data and then back out the contribution of markups to the total real effects of monetary shocks. We conduct this experiment using our Benchmark economy with Calvo sticky prices and wages and then consider several additional perturbations of the model to gauge the robustness of our results[18].

[18]See also Kryvtsov and Midrigan (2010) who conduct a number of additional robustness experiments in a Smets-Wouters (2007) - type economy with inventories.

**A. Benchmark model**

Panel A. of Table 5 reports the results of this experiment for our Benchmark model with sticky prices and wages. There are two columns in this panel. The first, labeled "Constant Returns," presents results from the economy with no decreasing returns at either the firm or aggregate level (i.e., no capital). The second, labeled "Decreasing Returns," is the economy with a fixed stock of capital at the aggregate level. We choose the share of the fixed factor to match exactly the 0.33 elasticity of inventories to sales in the data.

The table shows that the share of the fixed factor that matches the elasticity of inventories to sales in the data is equal to 0.38. Recall that we interpret this number as simply a measure of how important decreasing returns and other forms of adjustment costs are, and hence a measure of how volatile marginal costs are over the cycle. We do not interpret this number literally as an estimate of the share of capital in production.

With such a share of the fixed factor the model accounts well for the variability of inventory investment: production is 1.13 times more volatile than sales (1.10 times more volatile in the data). Also notice that the average response of consumption is 0.65 as large as in the economy with constant returns (0.39% vs 0.60%). Finally, our decomposition of the consumption response shows that a decline in markups accounts for almost a half (53%) of the overall increase in consumption after the monetary shock. We argue next that this number under-estimates the importance of countercyclical markups since the model fails to account for the behavior of interest rates, one of the two key components that determine the returns to holding inventories.

One may argue that our results critically depend on the assumptions we have made about the size of the inventory carrying costs, as captured by $\delta_z$, the rate at which inventories depreciate. Although the rate of depreciation indeed is a key parameter in our model and the key determinant of the strength of the intertemporal substitution motive, reasonable perturbations of $\delta_z$, in the range of the inventory carrying costs reported in the logistics literature (see Richardson (1995)), do not alter our conclusions much. To see this, we have also conducted an experiment by doubling the rate at which inventories depreciate to $\delta_z = 1.8\%$. With such a parametrization, we found that the model with constant returns produces an elasticity of inventories to sales equal to 2.53, only slightly smaller than in our baseline analysis.

**B. Taylor Rule**

Our Benchmark economy counterfactually predicts that the nominal interest rate is constant following a monetary policy expansion since the nominal interest rate is equal to the expected growth rate of the money supply which is iid. We next modify our assumptions regarding monetary policy and assume that it follows a Taylor-type interest rate rule. We follow Smets and Wouters (2007) and assume that the monetary authority chooses its instrument so as to ensure that the nominal interest rate evolves according to:

$$\begin{aligned} r\left(s^t\right) &= c_r + \rho_i r(s^{t-1}) + (1-\rho_i)\left[r_1 \Delta \log P\left(s^t\right) + r_2 \log y\left(s^t\right)\right] + r_3 \Delta \log y\left(s^t\right) + \varepsilon_{it} \\ \varepsilon_{it} &= \rho_\varepsilon \varepsilon_{it-1} + \epsilon_{it} \end{aligned}$$

where $\Delta \log P\left(s^t\right)$ is inflation, $y\left(s^t\right)$ is output and $\epsilon_{it}$ is a disturbance. Notice that, as is standard in recent studies, we assume interest-rate smoothing, captured by the term $\rho_i$ on the lagged nominal interest rate, as well as that the nominal interest rate reacts to deviations of inflation, output and the output growth rate from their steady-state level. We use the same coefficients in this interest rate rule as in Smets and Wouters (2007) and study the response of our economy to a monetary expansion given by a negative shock $\epsilon_{it}$. With such an interest rate rule, the nominal and real interest rates persistently decline following a monetary policy expansion, as in the data.

Notice in Panel B of Table 5 that we now require a somewhat higher share of the fixed factor (0.43) to match the elasticity of inventories to sales in the data. Intuitively the decline in interest rates makes the return to holding inventories increase even more after a monetary expansion, thereby increasing the incentive to invest in inventories. As a result we need even stronger decreasing returns at the aggregate level to undo the incentive for inventory accumulation. The greater decreasing returns assign an even more important role to countercyclical markups, since costs are now more responsive to a monetary shock. Our markup decomposition shows that 80% of the increase in consumption is accounted for by a decline in markups.

**C. Higher elasticity of intertemporal substitution**

We next assume $\sigma > 1$, and in particular, $\sigma = 1.5$, which is an alternative approach to ensure that the model predicts a decline in nominal interest rates after a monetary expansion, as in the data. Now the nominal interest rate declines following an increase in the growth rate of money supply since

$$r\left(s^{t}\right)=\frac{1}{\beta}\int\frac{P\left(s^{t+1}\right)c\left(s^{t+1}\right)^{\sigma}}{P\left(s^{t}\right)c\left(s^{t}\right)^{\sigma}}\pi\left(s^{t+1}|s^{t}\right)ds^{t+1}=\frac{1}{\beta}\int\mu(s^{t+1})\left(\frac{c\left(s^{t+1}\right)}{c\left(s^{t}\right)}\right)^{\sigma-1}\pi\left(s^{t+1}|s^{t}\right)ds^{t+1}$$

as consumption is highest immediately after the monetary shock and expected to mean-revert in future periods.

Panel C of Table 5 reports the predictions of the model under this parametrization. Once again we find that a greater share of the fixed factor (0.49) than in the Benchmark experiment is necessary to undo the incentive for inventory accumulation and account for the response of inventories in the data. With such a high share the marginal cost responds fairly strongly to the monetary shock and so countercyclical markups once again account for the majority of the real effects of monetary shocks. As in the economy with a Taylor rule, 80% of the average consumption response is accounted for by the decline in markups.

We have also considered an economy with a lower supply elasticity. In particular, Wallenius and Rogerson (2009) estimate macro labor supply elasticities in the range of 2.25–3.0, while we have assumed an infinite labor supply elasticity. It turns out however that the value of the labor supply elasticity does not matter much in our economy since unions face frictions on wage setting – if anything, a lower labor supply elasticity makes wages stickier because of a strategic complementarity in wage setting. Hence, when we lower the labor supply elasticity to 2.5, we find very similar results to those above: for the economy to account for the behavior of inventories the share of the fixed factor must be equal to 0.46 and declining markups account for about 60% of the real effects of monetary shocks.

Overall, we conclude that our results are robust to variations in parameters governing preferences. Moreover, versions of our model that more closely match the dynamics of interest rates in the data predicts an even more important role for countercyclical markups in accounting for the real effects of monetary shocks.

**D. Materials**

So far we have assumed that sticky wages account for the sluggish response of costs to a monetary shock. The literature has identified a number of other mechanisms that give rise to similar outcomes, including use of materials (produced inputs) as a factor of production, as well as variable capital and labor utilization (see e.g., Dotsey and King (2006)). We show below that our conclusions are not specific to any particular source of such 'real rigidities'. In particular, we assume next that wages are flexible but rather, materials are a factor of production, alongside with labor and capital. These materials are purchased from final goods producers and, since prices are sticky, are sold at a price that does not fully react to monetary policy shocks.

Specifically, we now modify the production function of intermediate goods firms to:

$$y_i\left(s^t\right) = \left(l_i\left(s^t\right)^{\alpha} k_i\left(s^t\right)^{1-\alpha}\right)^{\gamma} n_i\left(s^t\right)^{1-\gamma}$$

where $n_i\left(s^t\right)$ is the amount of materials employed by the firm. Materials are purchased from final goods firms at a price $P\left(s^t\right)$ and so the unit cost of production is equal to:

$$\Omega\left(s^t\right) = \chi\left[W\left(s^t\right)^{\alpha} R\left(s^t\right)^{1-\alpha}\right]^{\gamma} P\left(s^t\right)^{1-\gamma}.$$

Even though wages are now flexible, the aggregate price level inherits the stickiness of the intermediate goods' prices and so reacts slowly to monetary shocks. Finally, the resource constraint for final goods is modified to:

$$c\left(s^t\right) + x\left(s^t\right) + \frac{\xi}{2}\left(\frac{x\left(s^t\right)}{k(s^{t-1})} - \delta\right)^2 k(s^{t-1}) + \int_0^1 n_i\left(s^t\right) di = q\left(s^t\right)$$

We set the share of intermediate inputs equal to 0.60, consistent with the evidence in Basu (1995). (See also Nakamura and Steinsson (2008)).

Panel D of Table 5 shows that our earlier conclusions are unchanged under this alternative view of costs. We found, for our baseline case of log-linear preferences, that the share of the fixed factor must be equal to 0.62 in order for the model to match the variability of inventories in the data. Under such parametrization the markups once again play a much more important role and account for 59% of the real effects of monetary shocks.

## 6. An economy with non-convexities

We have introduced sticky prices earlier by adopting the assumption of a constant Calvo hazard of price adjustment. Although popular in recent work, this assumption is not entirely satisfactory as firms cannot react to aggregate and idiosyncratic shocks by choosing the timing of their price changes. We argue next that our results are not driven to this particular assumption. We do so by explicitly modeling the source of price stickiness: firms now face fixed menu costs of changing their prices. Each nominal price change entails a fixed costs, $\kappa_p$, of labor services. In addition, as in Golosov and Lucas (2007), we assume that intermediate goods firms are subject to idiosyncratic shocks to their productivity, $a_i\left(s^t\right)$. The latter allow the model to account for the large variability of prices observed at the micro-level.

We also argue that our results do not depend on the exact motive for inventory accumulation. To do so, we next assume that production entails a fixed cost, $\kappa_z$, of labor resources. This is a form of increasing returns that makes it optimal for firms to bunch production in a few periods and use inventories in order to economize on these fixed costs. Ramey (1991) studies such a model with non-convex costs and argues that these are important in manufacturing. Alternatively, one can interpret the intermediate goods firm as being made up of two units, a manufacturing unit, and a distributor. Under this interpretation, as in the work of Khan and Thomas (2007), one can think of $\kappa_z$ as a fixed cost of ordering inventories. Hall and Rust (2000) and Aguirregabiria (1999) find empirical support for this alternative interpretation.

### A. Setup

We assume that intermediate goods firms produce using a technology given by:

$$y_i\left(s^t\right) = \begin{cases} a_i\left(s^t\right)\left(\left(l_i\left(s^t\right) - \kappa_z\right)^{\alpha}, k_i\left(s^t\right)^{1-\alpha}\right)^{\gamma} \text{ if } l_i\left(s^t\right) > \kappa_z \\ 0 \text{ otherwise} \end{cases},$$

where $\kappa_z$ is the fixed production cost and $a_i\left(s^t\right)$ is the idiosyncratic productivity of the firm. The firm's problem thus reduces to

$$\max_{p_i(s^t),\, z_i(s^t)\geqslant m_i(s^{t-1})} \sum_{t=0}^{\infty} \int_{s^t} Q\left(s^t\right) \left[ \begin{array}{c} P_i\left(s^t\right) q_i\left(s^t\right) - \frac{\Omega(s^t)}{a_i(s^t)} \left[z_i\left(s^t\right) - m_i\left(s^{t-1}\right)\right]^{\frac{1}{\gamma}} - \\ \kappa_p W\left(s^t\right)\left(p_i\left(s^t\right) \neq p_i\left(s^{t-1}\right)\right) - \kappa_z W\left(s^t\right)\left(z_i\left(s^t\right) \neq m_i\left(s^{t-1}\right)\right) \end{array} \right] ds^t.$$

where demand is given, as earlier, by (4). In this economy firms hold inventories both to economies on the fixed ordering costs, as well as to avoid stockouts in the states of the world with high demand.

The optimal decision rules in this economy are of the *(S,s)* type: firms choose to produce (order) additional inventories whenever their current stock is sufficiently low and choose to reset their nominal price whenever their current price is too much out of line. This is illustrated in Figure 5 which shows the regions of the $(p, m)$ space for which an intermediate good firms resets its price/orders. The shaded areas in the left of the Figure is the region where the firm pays the fixed production costs and acquires more inventories: it does so whenever its stock is sufficiently low. The unshaded region is the region in which the firm leaves its price and inventory stock unchanged. These decision rules are discussed in more detail in Aguirregabiria (1999) who studies a partial equilibrium version of the model here.

In Panel C of Table 2 we report the parameter values and moments used to parameterize this economy. We follow Gertler and Leahy (2008) and assume a Poisson process for productivity shocks. Midrigan (2010) argues that such an assumption is necessary for the model to account for the dispersion in the size of price changes in the data. We choose the size of the menu cost (1% of steady state revenues), the frequency (1/8 per month) and size (0.05) of productivity shocks to match the frequency and size of price changes in the BLS data.

As earlier, we target an inventory-sales ratio of 1.40 and a frequency of stockouts of 5%. In addition, we choose a fixed ordering cost equal to 2% of the firm's steady-state revenues so that the frequency with which firms order inventories is equal to 0.5 per month. This number is in the range of estimates from earlier work. Aguirregabiria (1999) reports a frequency of orders equal to 0.79 per month in his data from a Spanish supermarket, while Alessandria et. al (2008), who use data from Hall and Rust (2000) for a large steel wholesaler find a frequency of orders of 0.33 to 0.50 per month.

Table 2 shows that the model requires a lower volatility of productivity, shocks, 0.43 (compared to 0.63 earlier), since now a weaker stockout-avoidance motive is required to match the stock of inventories in the data. Such volatility of demand is also consistent with evidence on the comovement of prices and quantities in the data. Burstein and Hellwig (2008) use the Dominick's data and report that the correlation between prices and quantities is equal to -0.22, thus much lower than the -1 number that would prevail absent shocks to demand. In our model this correlation is equal to -0.26, thus very close to the data.

**B. Aggregate Implications**

In Panel E of Table 4 we report the aggregate implication of the economy with non-convexities and constant returns at both the firm and aggregate level. The real effects of monetary shocks are similar to those in the Calvo economy. For example, the average consumption response is 0.59 (compared to 0.60) earlier. As discussed by Midrigan (2008), the economy with fat-tailed productivity shocks produces similar real effects as the Calvo economy because of a much weaker selection effect. Similarly, the markup decomposition shows that the decline in markups accounts for only 31% of the overall drop in consumption.

As for the inventory statistics, these are again similar to those of the economy without non-convexities. Once again the inventory-sales ratio is strongly procyclical (the correlation is 0.70 vs. -0.70 in the data), the elasticity of inventories to sales is much greater than in the data (1.57 vs. 0.33 in the data) and production is much more volatile than sales (2.2 vs. 1.1 in the data).

Notice also that the economy with fixed costs of ordering produces a somewhat less volatile response of inventories to monetary shocks than our Benchmark model. This happens for two reasons. First, the fixed ordering costs act as an adjustment cost that prevent some firms from responding to the increased returns to holding inventories. Second, as Panel C of Table 2 shows, the model also requires a somewhat greater cost of carrying inventories ($\delta_z = 1.25\%$ vs. 0.91% earlier) to account for the inventory moments in the data. A higher rate of depreciation reduces the firm's ability to substitute intertemporally and slows down the response of inventories a bit.

Overall, we conclude again that an economy in which the real effects of monetary shocks are mostly accounted for by stickiness in costs does a poor job at accounting for the inventory facts. The reason for this result is that in both models (with fixed costs of ordering and without), the strength of the intertemporal substitution effect is primarily determined by a single parameter, the cost of carrying inventories $\delta_z$. As long as this cost is low, as in our model and in the data, firms react strongly to changes in the return to carrying inventories regardless of the underlying reason for inventory accumulation[19].

## 7. Conclusions

We embed a motive for inventory accumulation in a standard New Keynesian model with price and wage rigidities. The model predicts a tight relationship between inventories and the dynamics of costs and markups. We use the theory, together with data on inventories, to evaluate the role of cost rigidities and markups in accounting for the real effects of monetary policy shocks. In the data inventories adjust slowly in response to shocks and are much less volatile than sales. Our theory interprets this fact as implying that countercyclical markups account for a sizable fraction of the real effects of monetary shocks.

## References


[1] Aguirregabiria, Victor, 1999. "The Dynamics of Markups and Inventories in Retailing Firms," *The Review of Economic Studies*, 66(2), 275-308.

[2] Aguirregabiria, Victor, 2005. "Retail Stockouts and Manufacturer Brand Competition," mimeo.

[3] Alessandria, George, Joseph Kaboski, and Virgiliu Midrigan, 2008. "Inventories, Lumpy Trade, and Large Devaluations," forthcoming, *American Economic Review.*

[4] Ball, Laurence and David Romer, 1990, "Real Rigidities and the Non-neutrality of Money," *The Review of Economic Studies,* 57(2), 183-203.

[5] Barro, Robert, 1977, "Long-Term Contracting, Sticky Prices, and Monetary Policy," *Journal of Monetary Economics*, 3: 305-316.


[19] Our results are robust to perturbations of $\delta_z$ in the range of those reported in the literature. Only when $\delta_z$ is implausibly high (in excess of 20% per month), do the model's predictions markedly change. See the working version of this paper, Kryvtsov and Midrigan (2009). There we have also shown that costs of deviating from a target inventory-sales ratio, as in Jung and Yun (2005), also have counterfactual implications since such a model predicts too little variation in the inventory-sales ratio of individual firms relative to what we document using the Aguirregabiria (1999) data.

[6] Basu, Susanto, 1995. "Intermediate Goods and Business Cycles: Implications for Productivity and Welfare," *American Economic Review*, 85(3), 512-31.

[7] Bils, Mark, 2004. "Studying Price Markups from Stockout Behavior," mimeo.

[8] Bils, Mark and James A. Kahn, 2000. "What Inventory Behavior Tells Us About Business Cycles," *American Economic Review*, 90(3), 458-481.

[9] Bils, Mark and Peter J. Klenow, 2004. "Some Evidence on the Importance of Sticky Prices," *Journal of Political Economy,* 112, 947-985.

[10] Burstein, Ariel, and Christian Hellwig, 2007. "Prices and Market Shares in a Menu Cost Model," NBER Working Papers 13455, National Bureau of Economic Research, Inc.

[11] Chang, Yongsung, and Andreas Hornstein, and Pierre-Daniel G. Sarte, 2004. "Productivity, employment, and inventories," Working Paper 04-09, Federal Reserve Bank of Richmond.

[12] Chari, V. V. and Patrick J. Kehoe and Ellen R. McGrattan, 2000. "Sticky Price Models of the Business Cycle: Can the Contract Multiplier Solve the Persistence Problem?," *Econometrica*, 68(5), 1151-1180.

[13] Christiano, Lawrence J. and Eichenbaum, Martin and Charles L. Evans, 1999. "Monetary policy shocks: What have we learned and to what end?," Handbook of Macroeconomics, in: J. B. Taylor & M. Woodford (ed.), Handbook of Macroeconomics, edition 1, volume 1, chapter 2, pages 65-148 Elsevier.

[14] Christiano, Lawrence J. and Eichenbaum, Martin and Charles L. Evans, 2005. "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, University of Chicago Press, 113(1), 1-45.

[15] Dotsey, Michael and Robert G. King, 2006. "Pricing, Production, and Persistence," *Journal of the European Economic Association*, 4(5), 893-928.

[16] Eichenbaum, Martin, and Jaimovich, Nir, and Rebelo, Sérgio, 2010. "Reference Prices and Nominal Rigidities," *American Economic Review*, forthcoming.

[17] Gertler, Mark, and John Leahy, 2008. "A Phillips Curve with an Ss Foundation," *Journal of Political Economy*, 116(3), 533-572.

[18] Goldberg, Pinelopi Koujianou, and Rebecca Hellerstein, 2008. "A Framework for Identifying the Sources of Local-Currency Price Stability with an Empirical Application," Working Papers 1161, Princeton University.

[19] Golosov, Mikhail, and Robert E. Lucas Jr., 2007. "Menu Costs and Phillips Curves," *Journal of Political Economy*, 115, 171-199.

[20] Gopinath, Gita, and Oleg Itskhoki, 2010. "Frequency of Price Adjustment and Pass-through," *Quarterly Journal of Economics*, forthcoming.

[21] Hall, Robert, 2005, "Employment Fluctuations with Equilibrium Wage Stickiness," *American Economic Review,* 95(1): 50-65.

[22] Hall, George and John Rust, 2000. "An Empirical Model of Inventory Investment by Durable Commodity Intermediaries," *Carnegie-Rochester Conference Series on Public Policy*, 52, 171-214.

[23] Hansen, Gary D., 1985. "Indivisible labor and the business cycle," *Journal of Monetary Economics*, 16(3), 309-327, November.

[24] House, Christopher, L., 2008. "Fixed Costs and Long-Lived Investments." *NBER Working Paper*, No. W14402.

[25] Iacoviello, Matteo, Fabio Schiantarelli and Scott Schuh, 2007. "Input and Output Inventories in General Equilibrium," Federal Reserve Bank of Boston Working Paper No. 07-16.

[26] Jung Yongseung and Tack Yun, 2005, "Monetary Policy Shocks, Inventory Dynamics, and Price Setting Behavior " mimeo

[27] Kehoe, Patrick J. and Virgiliu Midrigan, 2007. "Sales and the real effects of monetary policy," Working Papers 652, Federal Reserve Bank of Minneapolis.

[28] Khan, Aubhik and Julia Thomas, 2007. "Inventories and the business cycle: An equilibrium analysis of (S,s) policies." *American Economic Review*, 97(4), 1165-88.

[29] Klenow, Peter J. and Oleksiy Kryvtsov, 2008. "State-Dependent or Time-Dependent Pricing: Does It Matter for Recent U.S. Inflation?" *Quarterly Journal of Economics* 123, 863-904.

[30] Klenow, Peter J. and Jon Willis, 2006. "Real Rigidities and Nominal Price Changes," mimeo.

[31] Kryvtsov, Oleksiy, and Virgiliu Midrigan, 2009. "Inventories, Markups, and Real Rigidities in Menu Cost Models," NBER Working Papers 14651, National Bureau of Economic Research, Inc.

[32] Kryvtsov, Oleksiy, and Virgiliu Midrigan, 2010. "Inventories and Real Rigidities in New Keynesian Business Cycle Models," *Journal of Japanese and International Economies*, 24(2), 259-81.

[33] Krusell, Per and Anthony A. Smith, 1998. "Income and Wealth Heterogeneity in the Macroeconomy," *Journal of Political Economy*, 106(5), 867-896.

[34] Midrigan, Virgiliu, 2008. "Menu Costs, Multi-Product Firms and Aggregate Fluctuations," mimeo.

[35] Nakamura, Emi and Jón, Steinsson, 2008a. "Monetary Non-Neutrality in a Multi-Sector Menu Cost Model," NBER Working Papers 14001, National Bureau of Economic Research, Inc.

[36] Nakamura, Emi and Jón, Steinsson, 2008b. "Five Facts About Prices: A Reevaluation of Menu Cost Models," *Quarterly Journal of Economics*, 123(4), 1415-1464, November..

[37] Nakamura, Emi, and Dawit Zerom, 2010. "Accounting for Incomplete Pass-Through," *Review of Economic Studies*, forthcoming.

[38] Nekarda,Christopher J., and Valerie A. Ramey, 2010. "The Cyclical Behavior of the Price-Cost Markup," mimeo.

[39] Ramey, Valerie A, 1991. "Nonconvex Costs and the Behavior of Inventories," *Journal of Political Economy*, 99(2), 306-34, April.

[40] Ramey, Valerie A. and Kenneth D. West, 1999. "Inventories," Handbook of Macroeconomics, in: J. B. Taylor & M. Woodford (ed.), Handbook of Macroeconomics, edition 1, volume 1, chapter 13, pages 863-923

[41] Ribarsky, Jennifer, 2004, "Real Inventories, Sales, and Inventory-Sales Ratios for Manufacturing and Trade, 1997:I-2003:III," *Survey of Current Business*, February 1, 2004.

[42] Richardson, H. 1995. "Control Your Costs then Cut Them," *Transportation and Distribution*, 94-96.

[43] Rogerson, Richard, 1988. "Indivisible labor, lotteries and equilibrium," *Journal of Monetary Economics*, 21(1), 3-16, January.

[44] Rogerson, Richard, and Johanna Wallenius, 2009. "Retirement in a Life Cycle Model of Labor Supply with Home Production," Working Papers wp205, University of Michigan.

[45] Romer, Christina D. and David H. Romer, 2004. "A New Measure of Monetary Shocks: Derivation and Implications," *The American Economic Review*, 94(4), 1055-1084.

[46] Rotemberg, Julio J, 1984. "A Monetary Equilibrium Model with Transactions Costs," *Journal of Political Economy*, 92(1), pages 40-58, February.

[47] Rotemberg, Julio J., and Michael Woodford, 1999. "The cyclical behavior of prices and costs," Handbook of Macroeconomics, in: J. B. Taylor & M. Woodford (ed.), Handbook of Macroeconomics, edition 1, volume 1, chapter 16, pages 1051-1135 Elsevier.

[48] Smets, Frank and Rafael Wouters, 2007. "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach," *The American Economic Review*, 97(3), 586-606.

[49] Stigler, George, and James Kindhal, 1970, "The Behavior of Industrial Prices," Columbia University Press.

[50] Wen, Yi, 2008. "Input and Output Inventory Dynamics," Federal Reserve Bank of St. Louis Working Paper 2008-008A.

[51] Woodford, Michael, 2003, "Interest and Prices: Foundations of a Theory o Monetary Policy," Princeton University Press.

Table 1: Inventory Facts, US NIPA, Jan 1967 - Dec 2009

| | A. Unconditional | | B. Conditional on monetary shocks | |
|---|---|---|---|---|
| | Manufacturing and Trade | Retail | Manufacturing and Trade | Retail |
| $\rho(IS_t, S_t)$ | -0.82 | -0.65 | -0.70 | -0.58 |
| $\sigma(IS_t) / \sigma(S_t)$ | 1.03 | 1.18 | 0.96 | 1.43 |
| elast. $IS_t$ w.r.t. $S_t$ | -0.84 | -0.76 | -0.67 | -0.83 |
| elast. $I_t$ w.r.t. $S_t$ | 0.16 | 0.24 | 0.33 | 0.17 |
| $\rho(Y_t, S_t)$ | 0.98 | 0.89 | 0.98 | 0.83 |
| $\sigma(Y_t) / \sigma(S_t)$ | 1.12 | 1.14 | 1.10 | 1.15 |

Notes: All series are real, at monthly frequency.
The column labeled 'Unconditional' reports statistics for HP (14400)-filtered data
The column labeled "Conditional on monetary shocks" reports statistics computed using data projected on current and 36 lags of Christiano, Eichenbaum, Evans (2002) measures of monetary policy shocks estimated using a VAR for 1960:01-2000:12

## Table 2: Parameterization

| | Data | Model | | | | |
|---|---|---|---|---|---|---|
| | | A. Flexible Prices | | B. Sticky Prices | | C. Non-convexities |
| | | I. Firm-level CRS | II. Firm-level DRS | I. Firm-level CRS | II. Firm-level DRS | |
| **Parameters** | | | | | | |
| $\theta$ | | 5 | 5 | 5 | 5 | 5 |
| $\delta$ | | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 |
| $1-\lambda_w$ | | 1/12 | 1/12 | 1/12 | 1/12 | 1/12 |
| $1-\lambda_p$ | | 1 | 1 | 1 | 1 | - |
| $\sigma_v$ | | 0.63 | 0.57 | 0.63 | 0.57 | 0.43 |
| $\delta_z$ | | 0.0091 | 0.0075 | 0.0091 | 0.0075 | 0.0125 |
| $\gamma$ | | 1 | 0.667 | 1 | 0.667 | - |
| $\alpha$ | | 1 (or 2/3) | 1 | 1 (or 2/3) | 1 | 1 |
| $\rho$ | | - | - | - | - | 0.12 |
| $a_{max}$ | | - | - | - | - | 0.05 |
| $\kappa_{p,}$ rel. SS revenue | | - | - | - | - | 0.01 |
| $\kappa_{z,}$ rel. SS revenue | | - | - | - | - | 0.02 |
| **Targets** | | | | | | |
| I/S ratio | 1.4 | 1.4 | 1.4 | 1.4 | 1.4 | 1.4 |
| Frequency stockouts | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| Elasticity p to s | -0.023 | 0 | -0.036 | 0 | -0.003 | 0 |
| Elasticity z to s | 0.45 | 0 | 0.36 | 0 | 0.33 | 0 |
| Frequency orders | 0.33-0.79 | 1 | 1 | 1 | 1 | 0.50 |

## Table 3: Business Cycle Predictions of Flexible Price Economies

| | **Data** | **Model** | | | | |
|---|---|---|---|---|---|---|
| | | No Capital | | With Capital | | |
| | | A. Constant Returns | B. Firm-level decreasing returns | C. Adjustment costs | D. Fixed Capital | E. No adustment costs |
| | | *Impulse response of consumption to monetary shock* | | | | |
| average response | | 0.44 | 0.27 | 0.28 | 0.30 | 0.26 |
| maximum response | | 1.01 | 0.44 | 0.39 | 0.49 | 0.28 |
| half-life, months | | 8.0 | 15.1 | 23.0 | 14.9 | 123.4 |
| | | *Inventory Statistics* | | | | |
| $\rho(I/S_t, S_t)$ | -0.70 | 1.00 | 0.72 | 0.55 | 0.75 | -0.88 |
| elast. $I/S_t$ to $S_t$ | -0.67 | 2.09 | 1.21 | 0.42 | 1.34 | -0.34 |
| elast. $I_t$ to $S_t$ | 0.33 | 3.09 | 2.21 | 1.42 | 2.34 | 0.66 |
| $\sigma(Y_t) / \sigma(S_t)$ | 1.10 | 3.14 | 1.71 | 1.43 | 1.77 | 1.17 |
| | | *Investment Statistics* | | | | |
| $\sigma(x_t) / \sigma(c_t)$ | 4 | - | - | 4 | 0 | 23.06 |

Note: all variables HP-filtered with smoothing parameter 14400

average output response computed for first 24 months after shock

## Table 4: Business Cycle Predictions of Sticky Price Economies

| | **Data** | **Model** | | | | |
|---|---|---|---|---|---|---|
| | | No Capital | | Capital | | Non-convexities |
| | | A. Constant Returns | B. Firm-level decreasing returns | C. Adjustment Costs | D. Fixed Capital | E. Constant Returns |
| | | *Impulse response of consumption to monetary shock* | | | | |
| average response | | 0.60 | 0.46 | 0.43 | 0.41 | 0.59 |
| maximum response | | 1.02 | 0.93 | 0.89 | 0.90 | 0.99 |
| half-life, months | | 13.2 | 9.8 | 9.7 | 8.3 | 13.3 |
| markup contribution | | 0.33 | 0.65 | 0.46 | 0.47 | 0.31 |
| | | *Inventory Statistics* | | | | |
| $\rho(I/S_t, S_t)$ | -0.70 | 0.93 | -0.81 | -0.95 | -0.66 | 0.70 |
| elast. $I/S_t$ to $S_t$ | -0.67 | 1.68 | -0.50 | -0.89 | -0.46 | 0.57 |
| elast. $I_t$ to $S_t$ | 0.33 | 2.68 | 0.50 | 0.11 | 0.54 | 1.57 |
| $\sigma(M_t) / \sigma(S_t)$ | 1.10 | 2.87 | 1.14 | 1.07 | 1.20 | 2.16 |
| | | *Investment Statistics* | | | | |
| $\sigma(x_t) / \sigma(c_t)$ | 4 | - | - | 4 | 0 | - |

Note: all variables HP-filtered with smoothing parameter 14400

average output response computed for first 24 months after shock

## Table 5: Measuring the Response of Markups

| | Data | A. Benchmark | | B. Taylor rule | | C. Higher IES | | D. Intermediate Inputs | |
|---|---|---|---|---|---|---|---|---|---|
| | | Constant Returns | Decreasing Returns | Constant Returns | Decreasing Returns | Constant Returns | Decreasing Returns | Constant Returns | Decreasing Returns |
| Share fixed factor | | 0 | 0.38 | 0 | 0.43 | 0 | 0.49 | 0 | 0.62 |
| *Impulse response of consumption to monetary shock* | | | | | | | | | |
| average response | | 0.60 | 0.39 | 0.79 | 0.55 | 0.48 | 0.33 | 0.53 | 0.36 |
| markup contribution | | 0.33 | 0.53 | 0.51 | 0.80 | 0.35 | 0.80 | 0.25 | 0.59 |
| *Inventory Statistics* | | | | | | | | | |
| elast. $I_t$ to $S_t$ | 0.33 | 2.68 | 0.33 | 3.07 | 0.33 | 4.39 | 0.33 | 1.80 | 0.33 |
| $\sigma(M_t) / \sigma(S_t)$ | 1.10 | 2.87 | 1.13 | 1.93 | 1.14 | 4.24 | 1.16 | 2.03 | 1.12 |

Columns A–D are under the heading **Model**.

Figure 1: A. Unconditional HP-filtered Series

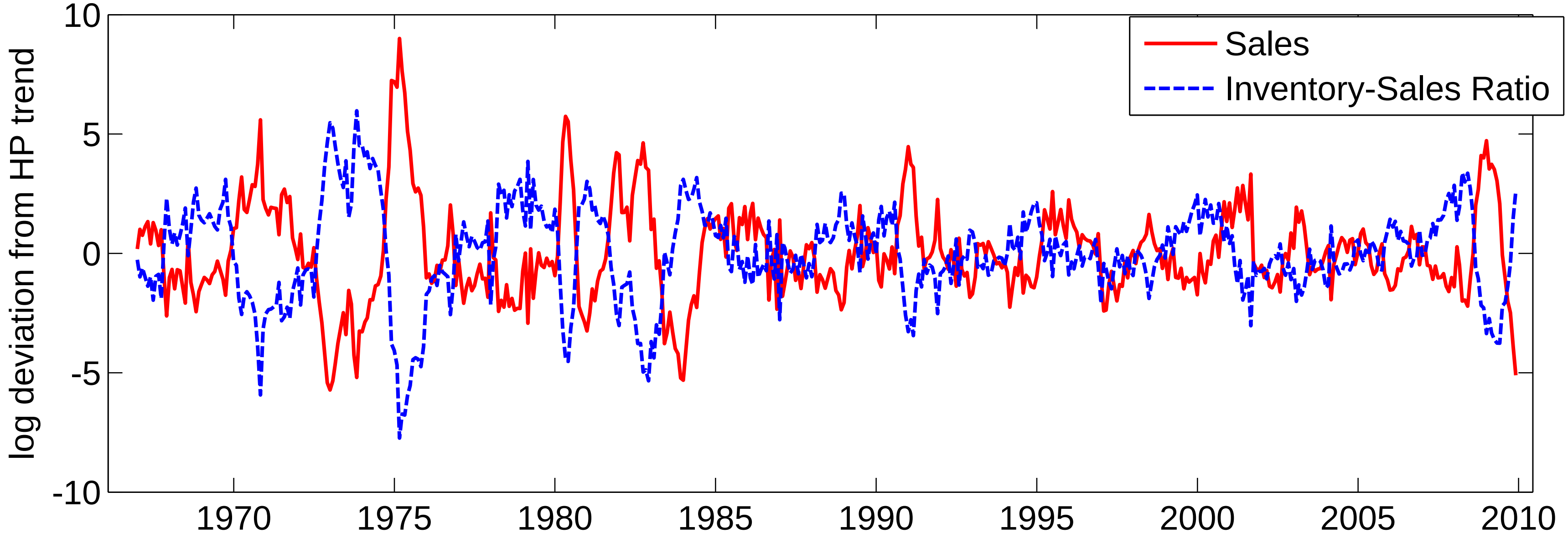


Figure 1: B. Conditional on Monetary Policy Shocks

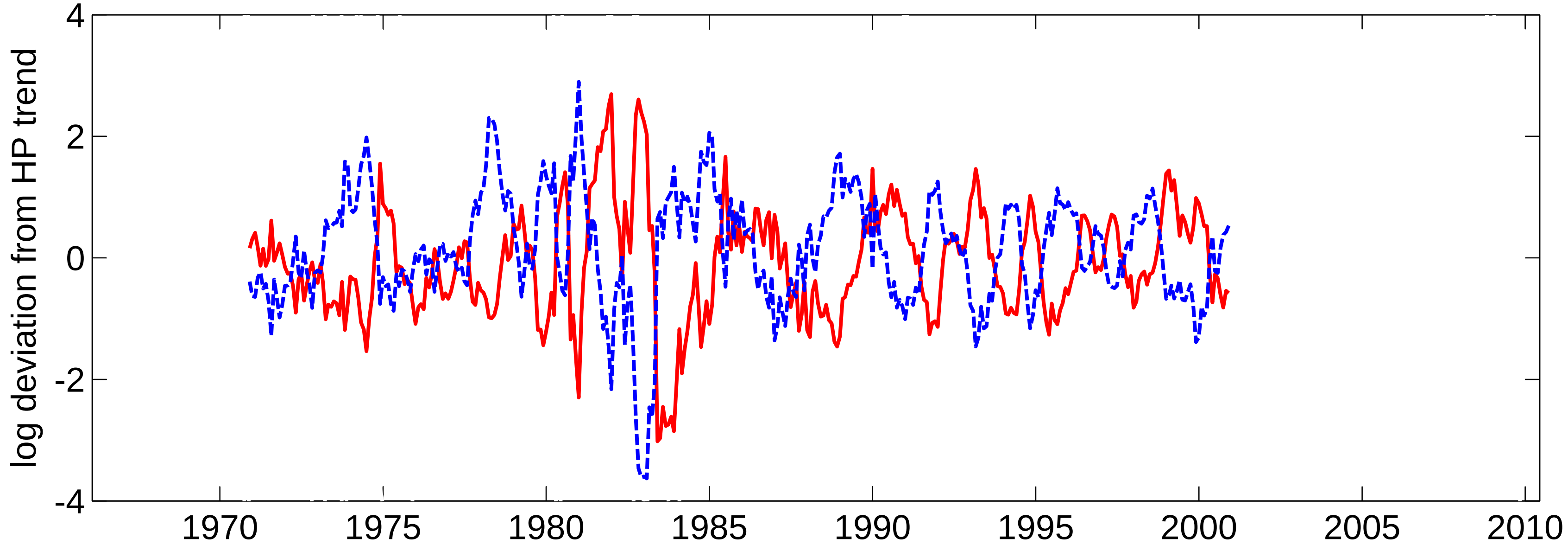

Figure 2: Decision rules in model economies.

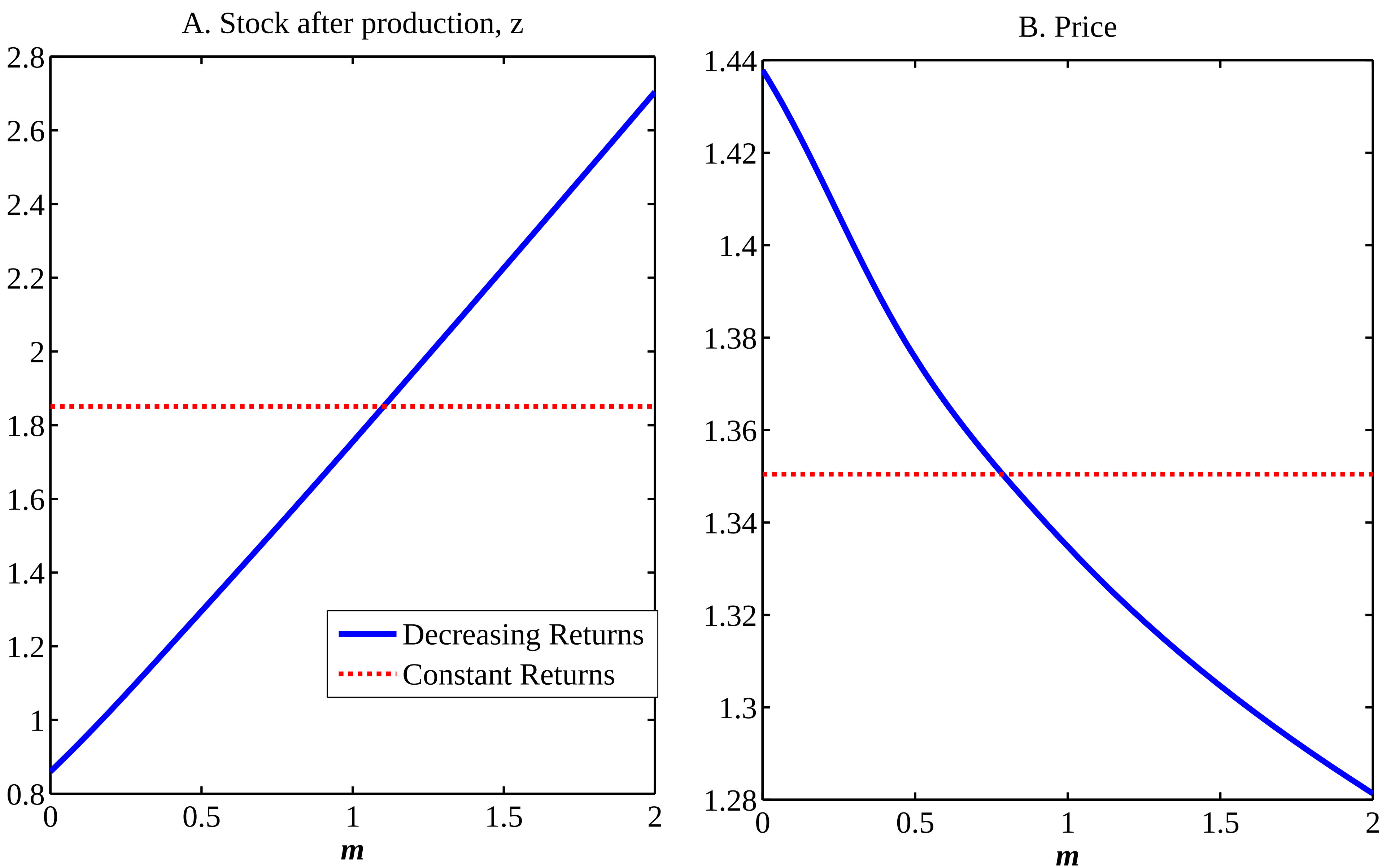

Figure 3: Impulse response to money shock. Flexible prices.

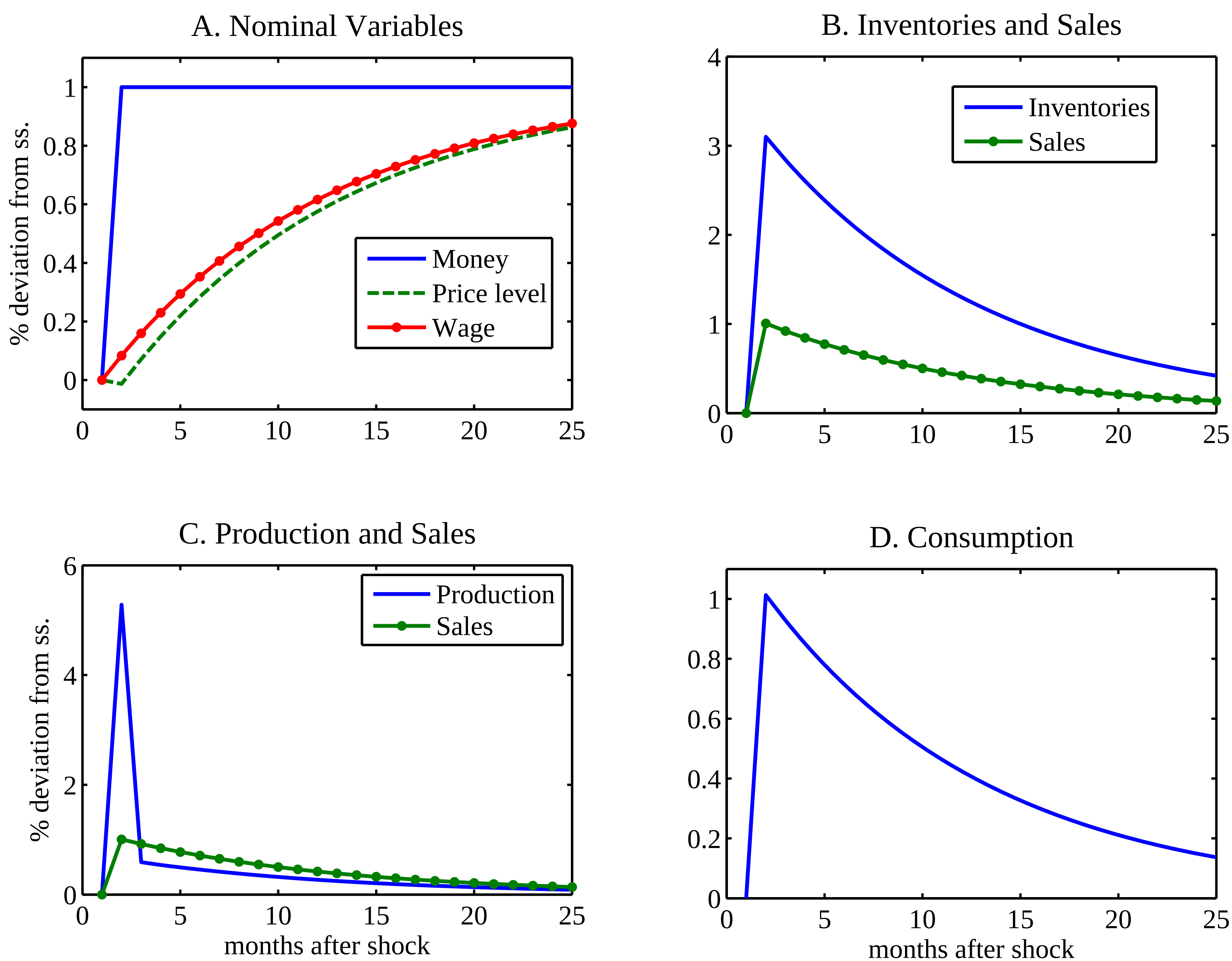

Figure 4: Impulse response to money shock. Sticky prices and decreasing returns.

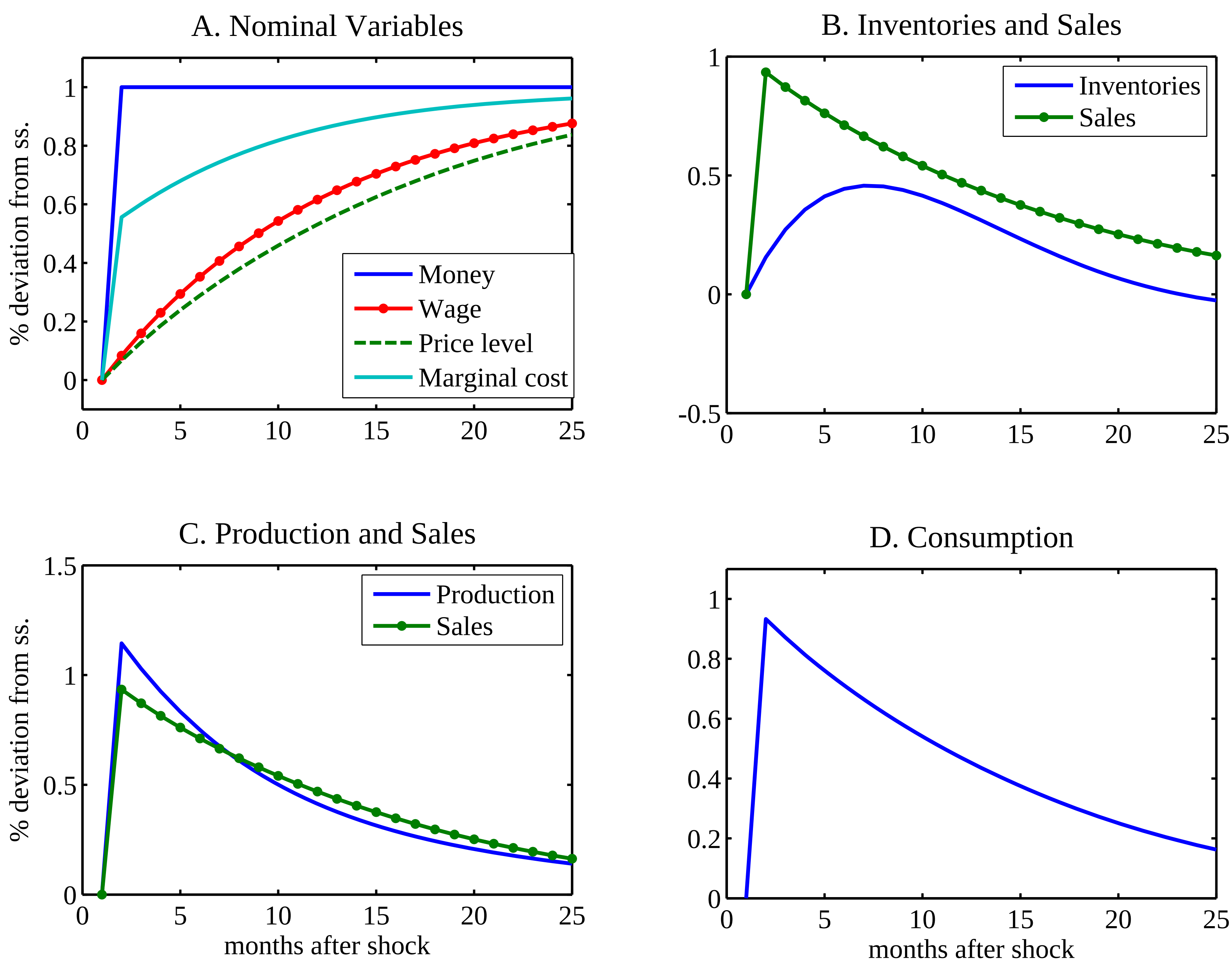

Figure 5: Decision rules. Economy with non-convexities.