International Asset Pricing with Risk Sensitive Rare Events

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Abstract

We propose a frictionless general equilibrium model in which two international consumers with recursive preferences trade two consumption goods and a complete set of date and state contingent securities. Consumption home bias and concern for the temporal distribution of risk generate rich dynamics for international prices and quantities. In our model, exchange rate movements are as volatile as they are in the data. Furthermore, both the volatility of the exchange rate movements and risk-premia are endogenously time varying and history dependent.

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1 Introduction

One of the best established facts in empirical finance is the tendency of volatilities and correlations to vary over time. An extensive literature has documented the following facts: *i)* the volatility of exchange rate movements varies over time (see for example Engle (1982), Gagnon (1993), Thursby and Thursby (1987), and McKenzie (1999) for studies considering different frequencies); *ii)* the correlations of international asset returns are far from being constant (Longin and Solnik (1995) and Cappiello, Engle and Sheppard (2006)); *iii)* consumption has become less risky (Kim and Nelson (1999)); and *iv)* equity risk premia are counter-cyclical (Campbell and Cochrane (1999)).

This paper studies an international asset pricing model, in which time-variation in volatilities and correlations arises endogenously from an optimal *risk-sensitive* trading arrangement. The economy is populated by two agents with recursive preferences, that we shall denote as home and foreign. Consumers are each endowed with the stochastic supply of one country specific good. Endowments are $i.i.d.$ and less than perfectly correlated. Preferences display a bias for the consumption of the domestic good. Trade occurs in frictionless goods markets and in financial markets featuring a complete set of state and date contingent securities. Since the consumers in the two countries enjoy the two goods with different intensities and their endowments are imperfectly correlated, a risk sharing opportunity arises.

Our economy is standard except for the fact that agents have recursive utilities. Our choice to employ recursive preferences is motivated by our interest in the joint dynamics of international quantities and prices. Brandt, Cochrane and Santa-Clara (2006) have shown the impossibility of time-additive preferences to generate both highly volatile and highly cross-country correlated stochastic discount factors. The high volatility is needed to replicate the large equity premia that we observe in a large cross-section of countries (see Hansen and Jagannathan (1991)). The high correlation is needed to account for the degree of volatility of real exchange rate movements that we typically observe between major industrialized countries (see Backus, Foresi and Telmer (1996)). Colacito and Croce (2007) show that a framework featuring Epstein and Zin (1989) preferences and *exogenously* specified consumption dynamics is capable of accounting for both. Lustig and Verdelhan (2007) and Lustig, Roussanov and Verdelhan (2009) point out that large common components and time-varying risk in the dynamics of international stochastic discount factors are needed to account for key features of currency risk premia. Our paper differs from previous work since we let consumption be an *endogenous* variable determined in a general equilibrium model in which agents can choose how much to save. Furthermore, time-varying volatility also arises endogenously.

Efficient allocations can be computed as the solution to a planner's problem for some initial vector of Pareto weights. We document that agents' preferences play a crucial role in the implementation of the optimal risk-sharing scheme. With *time additive* preferences, allocations are *history independent* functions of the endowments. With *recursive preferences* *à* la Epstein and Zin (1989) and Hansen and Sargent (1995), allocations are *history dependent* functions of the endowments. This history-dependence can conveniently be summarized by endogenously time-varying Pareto weights, which alter the conditional distribution of consumption across dates and states of the world. In our model, in fact, consumption growth becomes predictable and its volatility is no longer constant over time, even if endowments are $i.i.d.$ and markets complete.

Since time-varying Pareto weights are a telltale for time-varying distribution of international wealth, our model provides a link between the dynamics of a country's savings and the dynamics of the perceived risk of prices and quantities. First of all, we document that in each country the conditional volatility of the stochastic discount factor varies over time with the domestic level of wealth. This relationship is explained by the very specific attitude toward risk associated with the type of agents considered in this paper. Our agents are willing to give up a conspicuous share of consumption when the supply of the good that they care the most about is abundant, in order to get a marginal increment of consumption during periods of scarce supply. On a history of low realizations to its most preferred good, the affected consumer receives positive transfers from the insurance assets previously accumulated and it can afford to demand an increasing amount of insurance for the future, despite its endowment being currently low. At the same time, the other country's wealth decreases, since a conspicuous share of insurance assets previously purchased expires worthless. This last country finds it increasingly costly to provide more insurance on such a history. At the equilibrium, it is willing to provide insurance only at a price that is increasingly higher than the price that it is willing to pay to be insured. As securities' prices reflect state and date contingent marginal utilities of consumption, a time-varying distribution of prices is equivalent to time-varying volatility of stochastic discount factors and consumption.

For each realization of the endowments there are two effects on utilities. The first effect (income effect) is related to the size of the exogenous supply of the two goods: utilities are monotonically increasing in the level of the endowments. The second effect (substitution effect), instead, is related to the relative supply of the two goods: the risk-sharing mechanism impels that a wealthy country redistributes part of its resources abroad when experiencing a positive shock to the relative supply of its own most preferred good. Focusing on utilities, the second effect mitigates the first one and becomes more intense as the cross-country distribution of savings becomes more spread-out. As a consequence, the international correlation of utilities depends on the time-varying relative intensity of the income effect and the substitution effect. At the equilibrium the cross-country correlation of utilities is indeed an increasing function of the amount of international wealth inequality.

Since risk-sensitive agents are risk-averse in future utility in addition to future consumption, their continuation utilities are a crucial component of their marginal rates of substitutions. This has two consequences in our economy. First of all, the desire to smooth future utilities leads to stochastic discount factors that are more correlated than one period ahead consumption growth rates. Equivalently, international risk sharing, measured as the degree of co-movement of intertemporal marginal rates of substitutions, can be as high as needed to replicate the volatility of exchange rates' fluctuations, despite consumption being modestly correlated across countries. Second, dynamic wealth redistribution lets marginal utilities be more or less correlated depending on the degree of wealth inequality across countries. By no-arbitrage, the time-varying cross-country correlation of the marginal rates of substitution induces time-variation in both exchange rates' volatility and returns' correlations.

In order to make the model closer to the data, we borrow an insight from the rare events literature by postulating a small probability of a large endowment drop in either of the two countries. As shown by Rietz (1988), Barro (2006), Gabaix (2009), and Fahri, Fraiberger, Gabaix, Ranciere and Verdelhan (2009) this ingredient generates the needed volatility of intertemporal marginal rates of substitution. We show that rare events can amplify the time-variation in the market price of risk and other second moments related to asset prices. The convenience of introducing rare events is twofold. First, rare events improve the quantitative performance of our model without affecting the qualitative implications of the optimal risk-sharing scheme. Second, rare disasters allow us to better understand the optimal trading scheme implemented by countries exposed to non-synchronized depressions.

The model delivers a number of qualitative predictions that appear to be consistent with international financial data. Countries with higher international debt tend to have lower consumption volatility. Countries with modest international debt, instead, tend to have higher exchange rate volatility. Finally, countries whose stock market's returns are more correlated show lower currency volatility. We check these predictions focusing on both US data and a wider cross-section of countries. We contribute to the international empirical literature by showing that the model is broadly consistent with the data.

This paper is not the first one featuring endogenously time-varying Pareto weights. Indeed, it is well known that they naturally arise, when markets are not complete. A number of papers have investigated the impact of frictions in international financial markets on international business cycles (see, for example Baxter and Crucini (1995), Heathcote and Perri (2002), Kehoe and Perri (2002), and Kollmann (1996)). In Pavlova and Rigobon (2007), Pareto weights vary over time thanks to exogenous country-specific demand shocks. Chen, Joslin and Tran (2010) show that disagreement about disasters' probabilities can generate a time-varying distribution of wealth in an economy populated by agents with time additive preferences. Our work differs from those just mentioned in at least three major dimensions. First, we obtain rich *endogenous* dynamics in the international distribution of wealth in an environment with *no frictions*. Second, we explicitly address the relevance of the time-variation of wealth for both *quantities* and international *asset prices*. Third, we document that the equilibrium of the economy can be interpreted as one in which agents have a *preference for robustness*. Following this interpretation, we show that disagreement about the transition probabilities across states arises endogenously as a function of the degree of home bias.

Characterizing the dynamics of risk-sensitive allocations is challenging even when preferences are defined over only one consumption good. Lucas and Stokey (1984), Ma (1993) and Kan (1995) provide sufficient conditions for the existence of a recursive representation. Anderson (2005) shows that when preferences are heterogenous it is in general very hard to ensure the stationarity of the equilibrium. Backus, Routledge and Zin (2009) study the dynamics of a two agent economy, in which one agent is infinitely risk averse and the other is risk-neutral with respect to static gambles. In this paper, we document that consumption home bias is both a natural and convenient way of introducing heterogeneity in agents' preferences in an international setting. This produces rich allocations' dynamics in a stationary environment in which no agent eventually receives a negligible amount of wealth.

The rest of this paper is organized as follows. The next section provides the setup of the economy and outlines the solution to the Pareto problem. Section 3 provides a simple two state example, that is used to build on the intuitions of the dynamic economy. Section 4 discusses the generalized setup in which rare events are also part of the state space. Section 5 contains a sensitivity analysis, while section 6 looks at the data, by checking the empirical predictions of the model. Section 7 concludes the paper.

2 The economy

2.1 Endowments, preferences, and markets

There exist two goods, X and Y, whose endowments are stochastic. In each period, there is a realization of a random event s_t. Let $s^t = (s_0, \ldots, s_t)$ denote the history of events up and until time t. Endowments in period t are time-invariant measurable functions of s_t: $(X_t(s^t), Y_t(s^t)) = (X_t(s_t), Y_t(s_t))$. Let $\pi(s_t|s_{t-1})$ be a Markov chain, with given initial realization s_0, so that $\pi(s_0) = 1$. We shall assume that endowments are $i.i.d.$: $\pi(s_t|s_{t-1}) = \pi(s_t)$.

The economy consists of two countries: home (h) and foreign (f), each populated by a representative consumer. The two countries have risk-sensitive preferences defined over domestic and foreign consumption bundles:

$$U_{i,t}(s^t) \;\; = \;\; (1-\delta)\log C_{i,t}(s^t) + \delta\theta \log \sum_{s_{t+1}} \exp\left\{\frac{U_{i,t+1}(s_{t+1}|s^t)}{\theta}\right\} \pi(s_{t+1}|s^t), \quad \forall i \in \{h, f\}$$

where $\theta = \frac{1}{1-\gamma}$ and γ is the coefficient of atemporal risk-aversion. This specification is due to Hansen and Sargent (1995) and is used among others by Tallarini (2000) and Anderson (2005). The main departure from the constant relative risk-aversion case that is often analyzed in the literature lies in the fact that these preferences are non time-additive and they allow agents to be risk-averse in future utility in addition to future consumption.[1] Alternatively they can be interpreted as the special case of Epstein and Zin (1989) preferences in which the intertemporal elasticity of substitution equals 1.

The period utility functions are defined over consumption aggregates of good X and good Y:

$$C_{h,t} = x_{h,t}^{\alpha} y_{h,t}^{1-\alpha} \qquad \text{and} \qquad C_{f,t} = x_{f,t}^{1-\alpha} y_{f,t}^{\alpha} \tag{1}$$

where $x_{i,t}$ and $y_{i,t}$ denote the consumption of good X and good Y in country i at date t. The parameter α captures the degree of bias of the consumption of each representative agent. Specifically, by letting α be larger than $1/2$, we model the assumption of consumption home bias toward good X and good Y for the home and foreign country, respectively.

We let the home country be endowed with good X and the foreign country be endowed with good Y. At each date, trade occurs in a set of claims to one-period ahead state-contingent consumption. There is a complete set of these claims. At each date $t > 0$, the budget constraints of the home and foreign agents are:

$$\begin{aligned} x_{h,t}(s^t) + p_t(s^t) y_{h,t}(s^t) + \sum_{s_{t+1}} q_t(s_{t+1}|s^t) a_{h,t+1}(s_{t+1}, s^t) &\leq X_t(s^t) + a_{h,t}(s^t) \\ x_{f,t}(s^t) + p_t(s^t) y_{f,t}(s^t) - \sum_{s_{t+1}} q_t(s_{t+1}|s^t) a_{h,t+1}(s_{t+1}, s^t) &\leq p_t(s^t) Y_t(s^t) - a_{h,t}(s^t) \end{aligned}$$

where $p_t(s^t)$ denotes the relative price of good Y and good X (the terms of trade), $a_{i,t}(s^t)$ denotes country i's claims to time t consumption of good X, and $q_t(s_{t+1}|s^t)$ gives the price of one unit of time $t+1$ consumption of good X, contingent on the realization s_{t+1} at $t+1$, when the history at t is s^t.

[1]More precisely the CRRA case is nested in this specification as the limiting case in which $\gamma \to 1$.

2.2 Pareto problem

Efficient allocations can be computed as the solution to the planner's problem. We shall decentralize this equilibrium in a later section with the complete set of sequential state and date contingent securities discussed above.

The planner attaches nonnegative Pareto weights $\mu_h = \mu$ and $\mu_f = 1 - \mu$ on the consumers and chooses allocations $\{(x_{i,t}(s^t), y_{i,t}(s^t))\}_{t=0}^{+\infty}$, $i \in \{h, f\}$ to maximize:

$$Q(s_0) = \mu U_{h,0}(s_0) + (1 - \mu) U_{f,0}(s_0)$$

subject to the two economy-wide feasibility constraints:

$$\begin{aligned} x_{h,t}(s^t) + x_{f,t}(s^t) &\leq X_t(s^t) \\ y_{h,t}(s^t) + y_{f,t}(s^t) &\leq Y_t(s^t) \end{aligned}$$

Anderson (2005) suggests a recursive way to characterize this problem in a one good economy. Colacito and Croce (2009) extend this technique to multiple goods economies. The insight is that the solution can be conveniently cast in terms of a time-varying vector of Pareto weights, even though technically there is only a time 0 vector of Pareto weights. Let $\mathcal{M}_t(s^t) = \mu_{h,t}(s^t)/\mu_{f,t}(s^t)$ denote the time t history s^t ratio of Pareto weights. The optimal consumption allocation rule is a sequence of functions, that maps a history s^t into a choice of time t consumption of each good in each country:

$$\begin{aligned} x_{h,t}(s^t) &= \frac{\alpha \mathcal{M}_t(s_t|s^{t-1})}{(1-\alpha) + \alpha \mathcal{M}_t(s_t|s^{t-1})} X_t(s_t), \qquad x_{f,t}(s^t) = \frac{(1-\alpha)}{(1-\alpha) + \alpha \mathcal{M}_t(s_t|s^{t-1})} X_t(s_t) \\ y_{h,t}(s^t) &= \frac{(1-\alpha)\mathcal{M}_t(s_t|s^{t-1})}{\alpha + (1-\alpha)\mathcal{M}_t(s_t|s^{t-1})} Y_t(s_t), \qquad y_{f,t}(s^t) = \frac{\alpha}{\alpha + (1-\alpha)\mathcal{M}_t(s_t|s^{t-1})} Y_t(s_t) \end{aligned}$$

where

$$\mathcal{M}_t(s_t|s^{t-1}) \;=\; \mathcal{M}_{t-1}(s^{t-1}) \frac{\exp\left\{\frac{U_{h,t}(s_t|s^{t-1})}{\theta}\right\}}{\sum_{s_t} \exp\left\{\frac{U_{h,t}(s_t|s^{t-1})}{\theta}\right\} \pi(s_t|s_{t-1})} \Bigg/ \frac{\exp\left\{\frac{U_{f,t}(s_t|s^{t-1})}{\theta}\right\}}{\sum_{s_t} \exp\left\{\frac{U_{f,t}(s_t|s^{t-1})}{\theta}\right\} \pi(s_t|s_{t-1})},$$

$\forall t \geq 1$ and $\mathcal{M}_0(s_0) = \mu/(1-\mu)$. The ratio of the Pareto weights, $\mathcal{M}_t(s^t)$, indexes inequality at time t and summarizes the effects of the history of the exogenous state on consumption allocations. By interpreting the Pareto weights as time varying, consumption allocation rules have the above straightforward representation and the Pareto problem can be written recursively, as discussed in the Appendix.

3 A two states example

3.1 The environment

In order to develop the intuitions of the dynamic economy, we shall focus first on a simplified framework. For each history s_{t-1}, let there be only two equally likely states for time t: $s_t^{HL} = \frac{X_t(s^t)}{Y_t(s^t)} = k$ and $s_t^{LH} = \frac{X_t(s^t)}{Y_t(s^t)} = 1/k$, for some $k > 1$. Accordingly, we shall index all equilibrium prices and quantities with a superscript HL or LH, depending on which of the two $i.i.d.$ states materializes. We shall calibrate the endowments to be equal to 100 in the low supply state and $k = 1.03$. The coefficient of risk aversion γ is set to 25 and the subjective discount factor δ is equal to 0.95 to reflect a yearly decision problem. Home bias is embedded in the parameter $\alpha = 0.98$.

3.2 Characterizing the time-varying Pareto weights

Figure 1 highlights some of the salient features of the dynamics of the Pareto weights. First, *Pareto weights are counter-cyclical*. In periods of large supply of the most preferred good, the domestic Pareto weight *drops*. In periods of scarce supply of the most preferred good, the domestic Pareto weight *rises*. This is an important characteristic of the optimal risk-sharing scheme. It is helpful to think about this mechanism in terms of consumption units. Imagine being at $\mu_t(s^t) = 0.5$. The top two panels of figure 1 suggest that the Pareto weight will drop or rise by an equal amount in the two states of the world. The outcome, however, is quite different when expressed

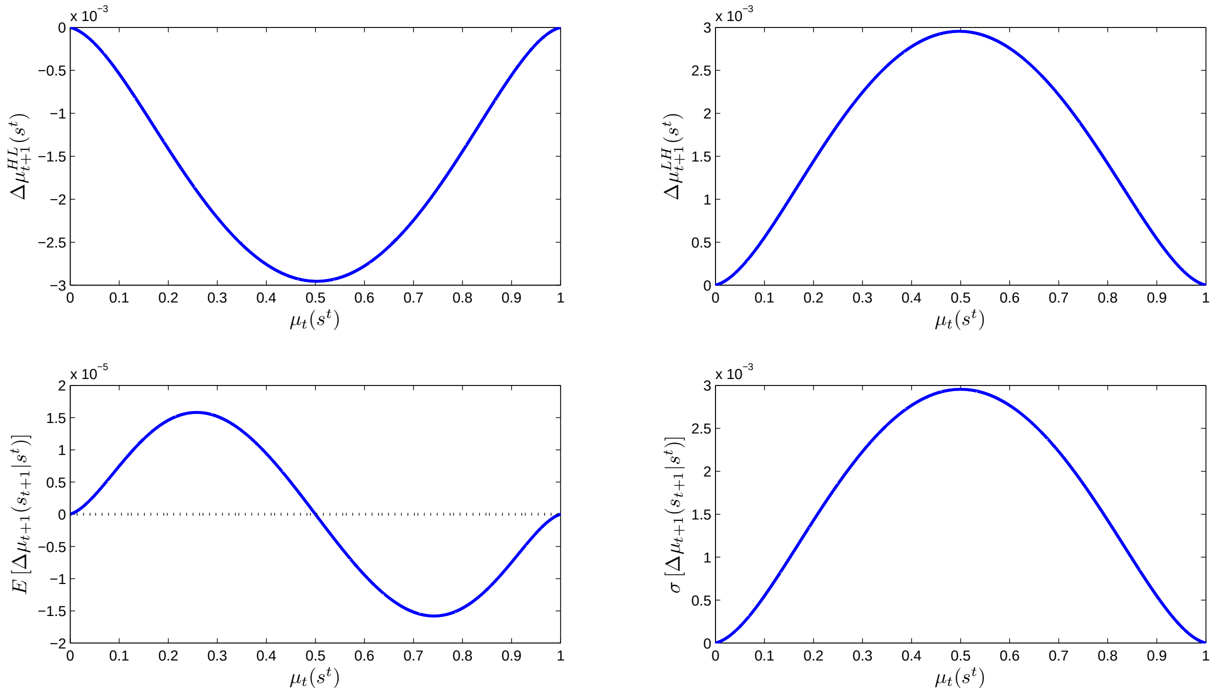

Figure 1: Dynamics of Pareto weights. The top two panels report the change in the home Pareto weight contingent on the two states as a function of the current Pareto weight. According to our notation, $\Delta\mu^{HL}_{t+1}(s^t) := \mu_{t+1}(s^{HL}_{t+1}|s^t) - \mu_t(s^t)$ and $\Delta\mu^{LH}_{t+1}(s^t) := \mu_{t+1}(s^{LH}_{t+1}|s^t) - \mu_t(s^t)$. The bottom two panels report the expected change (left) and the volatility of the change (right) in the home Pareto weight as a function of the current Pareto weight.

in terms of local consumption. Home bias (reflected in the consumption aggregates in (1)) translates into a considerable amount of consumption that must be given up in good times, in order to consume a smaller additional amount during bad times. Equivalently, this economy is populated by agents that are willing to trade-off expected levels for smaller volatilities of their consumption profiles.

Second, *the economy is stationary*. The third panel of figure 1 shows that the home agent is expecting her weight to increase when it is currently low and to decrease when it is currently high. That is, a country that has been hit by a sequence of negative supply shocks and that has therefore been allocated an increasing share of consumption of the two goods via the optimal risk-sharing mechanism, must expect to eventually return those resources to the other agent. Colacito and Croce (2009) show that in this economy no agent eventually receives a negligible amount of wealth forever.

Third, the conditional *volatility* of Pareto weights *changes over time*. It is low when either one of the two agents is close to dying (ie, $\mu_t \approx 0$ or $\mu_t \approx 1$) and it is the largest when the two agents are equally wealthy (ie $\mu_t \approx .5$). This has to do with the concavity of the period utility functions. Specifically, when a consumer has a low Pareto weight, a small increase or decrease in her weight will generate a large shift in her marginal utility. Hence in equilibrium there should be small movements in the current distribution of wealth. The reverse holds for the case in which the amount of wealth is comparable across countries. As we shall see in the remainder of the paper, this time varying property of the volatility of Pareto weights is going to carry over to all other variables in equilibrium.

3.3 Characterizing asset holdings

A competitive equilibrium is characterized by means of an initial distribution of wealth $\{a_{i,0}(s_0)\}$, an allocation $\{x_{i,t}(s^t), y_{i,t}(s^t)\}_{i,s^t}$, asset portfolios $\{a_{i,t+1}(s_{t+1}|s^t)\}_{i,s_{t+1}}$, and pricing kernels $\{q_t(s_{t+1}|s^t)\}_{s_{t+1}}$. In this subsection we focus on the characterization of asset holdings. By virtue of the welfare theorems of economics, we can associate a level of claims to date $t+1$ history s^{t+1} consumption, $a_{i,t+1}(s_{t+1}|s^t)$, to each current Pareto weight, $\mu_t(s^t)$.

The top two panels of figure 2 document that asset holdings in the home country are monotonically increasing in the Pareto weight, regardless of the realization of the endowment shock. The bottom two panels of figure 2 suggest that households will reduce their amount of savings when there is a large supply of the most preferred good, and increase their savings otherwise.[2]

[2]For scaling reasons, the bottom two panels report the monotone transformation $\widetilde{a}_{h,t}(s^t) = \exp\{a_{h,t}(s^t)\} - 1$.

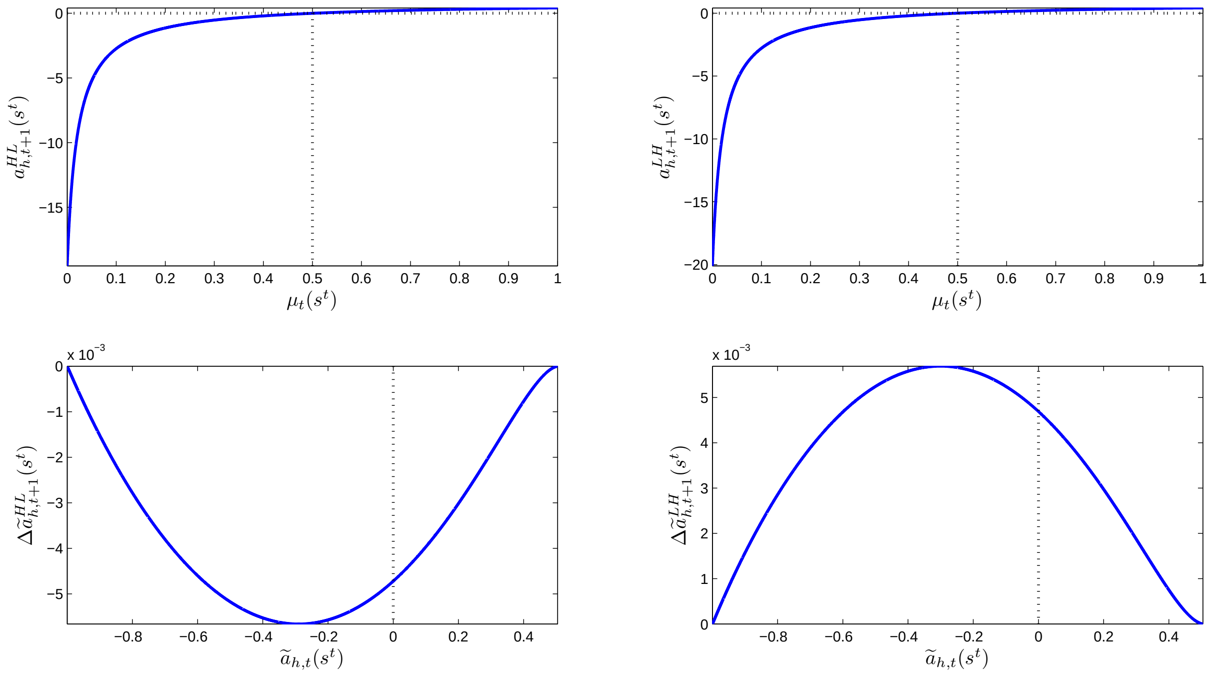

Figure 2: Asset holdings. The top two panels report the home holdings of securities contingent on the two states as a function of the current Pareto weight. According to our notation, $\Delta\widetilde{a}^{HL}_{h,t+1}(s^t) := \widetilde{a}_{h,t+1}(s^{HL}_{t+1}|s^t) - \widetilde{a}_{h,t}(s^t)$ and $\Delta\widetilde{a}^{LH}_{h,t+1}(s^t) := \widetilde{a}_{h,t+1}(s^{LH}_{t+1}|s^t) - \widetilde{a}_{h,t}(s^t)$.The bottom two panels report the change in the holdings of the two state contingent securities as a function of the current asset holdings.

3.4 Characterizing asset prices

For each history s^t, the pricing kernel $q_t(s_{t+1}|s^t)$ attains one of the following two values:

$$\begin{aligned} q^{HL}_t(s^t) &= \delta\left(\frac{x^{HL}_{h,t+1}(s^t)}{x_{h,t}(s^t)}\right)^{-1} \frac{\exp\left\{U^{HL}_{h,t+1}(s^t)/\theta\right\}}{\sum_{s_{t+1}} \exp\left\{U_{h,t+1}(s_{t+1}|s^t)/\theta\right\}\pi(s_{t+1})} \\ q^{LH}_t(s^t) &= \delta\left(\frac{x^{LH}_{h,t+1}(s^t)}{x_{h,t}(s^t)}\right)^{-1} \frac{\exp\left\{U^{LH}_{h,t+1}(s^t)/\theta\right\}}{\sum_{s_{t+1}} \exp\left\{U_{h,t+1}(s_{t+1}|s^t)/\theta\right\}\pi(s_{t+1})} \end{aligned}$$

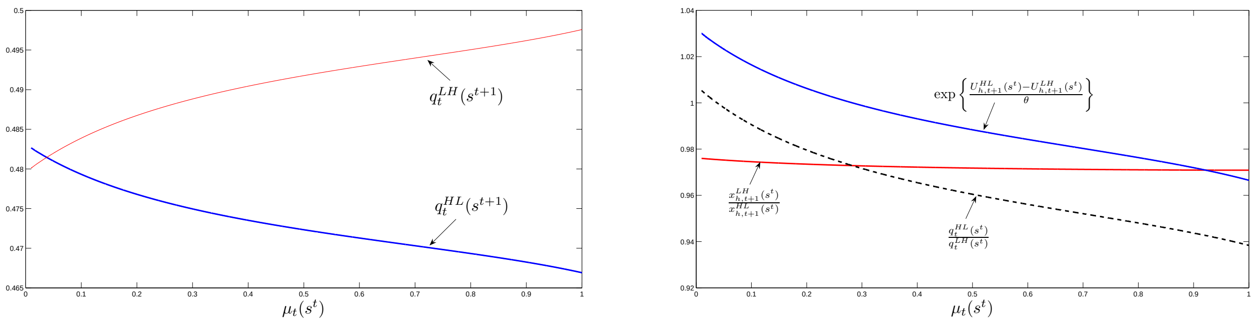

Figure 3: Pricing kernels in the two states model. The left panel shows the prices of the state contingent securities upon realization of either of the two states. The right panel decomposes the ratio of the prices into the ratio of future consumption and the term reflecting continuation utilities.

By taking the ratio of the two prices, we get the following relationship between asset prices, future allocations and continuation utilities:

$$\frac{q_t^{HL}(s^t)}{q_t^{LH}(s^t)} = \frac{x^{LH}_{h,t+1}(s^t)}{x^{HL}_{h,t+1}(s^t)} \exp\left\{\frac{U^{HL}_{h,t+1}(s^t) - U^{LH}_{h,t+1}(s^t)}{\theta}\right\} \quad (2)$$

Figure 3(a) documents that the prices $q_t^{HL}(s^t)$ and $q_t^{LH}(s^t)$ are, respectively, decreasing and increasing in the current Pareto weight attached to the home country. This has an intuitive explanation in light of what reported in the previous subsections. As the home country is hit by a bad endowment shock, her Pareto weight increases (see Figure 1) and her savings get larger and larger (see Figure 2). In a competitive equilibrium, this translates into an increasing price of purchased insurance, $q_t^{LH}(s^t)$, and a decreasing price of provided insurance, $q_t^{HL}(s^t)$.

Figure 3(b) decomposes the ratio of pricing kernels into the share associated to the ratio of current allocations and the share being contributed by relative continuation utilities in the two states of the world. The figure confirms that when agents care about the temporal distribution of risk, the dynamics of prices is mainly determined by future utilities. This has an important implication in our setup. As the gap between security prices widens (see figure 3(a)), so does the gap between the future utilities of the home agent in the two states of the world. Hence, as long as there is dynamics in the international distribution of wealth, the conditional volatility of utility will vary over time. We shall see in the remainder of this paper how this stochastic volatility property carries over to all quantities and prices in the economy.

3.5 Discussion

The analysis of the two states example is instructive, in that it highlights at least one key feature of the specific attitude toward risk of the agents considered in this paper. Upon the realization of a good (relative) endowment draw, our agents are willing to give up current shares of consumption and, consequently, to decrease the expected level of their utilities, in exchange for a decreased volatility of their future utility profiles. This trade-off between the first and second moments of the conditional distribution of the utility can be readily appreciated by taking a second order Taylor expansion of the utility function:[3]

$$U_{i,t}(s^t) \approx (1-\delta)\log C_{i,t}(s^t) + \delta E\left[U_{i,t+1}\left(s_{t+1}|s_t\right)\right] + \frac{\delta}{2\theta} V\left[U_{i,t+1}\left(s_{t+1}|s_t\right)\right] \tag{3}$$

where

$$\begin{aligned} E\left[U_{i,t+1}\left(s_{t+1}|s_t\right)\right] &= \sum_{s_{t+1}} U_{i,t+1}(s_{t+1}|s^t)\pi(s_{t+1}|s^t) \\ V\left[U_{i,t+1}\left(s_{t+1}|s_t\right)\right] &= \sum_{s_{t+1}} \left(U_{i,t+1}(s_{t+1}|s^t) - E\left[U_{i,t+1}\left(s_{t+1}|s_t\right)\right]\right)^2 \pi(s_{t+1}|s^t), \quad \forall i \in \{h, f\} \end{aligned}$$

denote respectively the expectation and the variance of the utility conditional on the current state being s_t.

By ignoring the very last term in (3), the standard time-additive utility case attains. However, with risk-sensitive preferences, agents care not only about the ex-

[3] The expansion is taken about $E\left[U_{i,t+1}\left(s_{t+1}|s_t\right)\right]$. Additionally, the term $\log\left(1+\frac{1}{2\theta^2}V\left[U_{i,t+1}\left(s_{t+1}|s_t\right)\right]\right)$ is approximated as $\frac{1}{2\theta^2}V\left[U_{i,t+1}\left(s_{t+1}|s_t\right)\right]$.

pected future utility level, but also about the conditional distribution of utility. Equivalently, the conditional variance of the utility matters and the preference parameter $\theta = 1/(1-\gamma)$ measures how much agents care about the temporal distribution of risk. Since for any value of risk aversion in excess of 1 the risk-sensitivity parameter θ is negative, agents are willing to trade-off expected utility levels for a lower expected utility volatility.

To further illustrate this point, figure 4 reports the conditional utility volatility against the expected utility obtained in equilibrium over the domain of the home country's Pareto weight. Different curves are associated to different degrees of risk-sensitivity. When $\gamma = 1$, agents have standard time-additive preferences and in equilibrium they will bear the same amount of utility risk, regardless of the expected utility level. For larger γ the situation changes in at least two relevant ways. First, agents are willing to hold a smaller amount of utility risk. This explains the vertical shift of the lines that is associated to increasing the value of γ. Second, agents are willing to substitute a lower expected level for a lower volatility of their utilities. This accounts for the fact that the lines become positively sloped.

Risk-sensitive agents are interested in smoothing future utility in addition to smoothing future consumption. For this reason, time varying conditional volatilities are a natural equilibrium outcome in a risk-sensitive exchange economy.

3.6 Robustness interpretation

Following Barillas, Hansen and Sargent (2009) we can interpret our results in terms of a concern for model misspecification of the two consumers. To this end, we introduce the risk-sensitivity operator

$$\boldsymbol{T}^{\mathbf{1}}(U_{i,t+1}(s_{t+1}|s^t)) = \theta \log \sum_{s_{t+1}} \exp\left\{\frac{U_{i,t+1}(s_{t+1}|s^t)}{\theta}\right\} \pi(s_{t+1}|s^t),$$

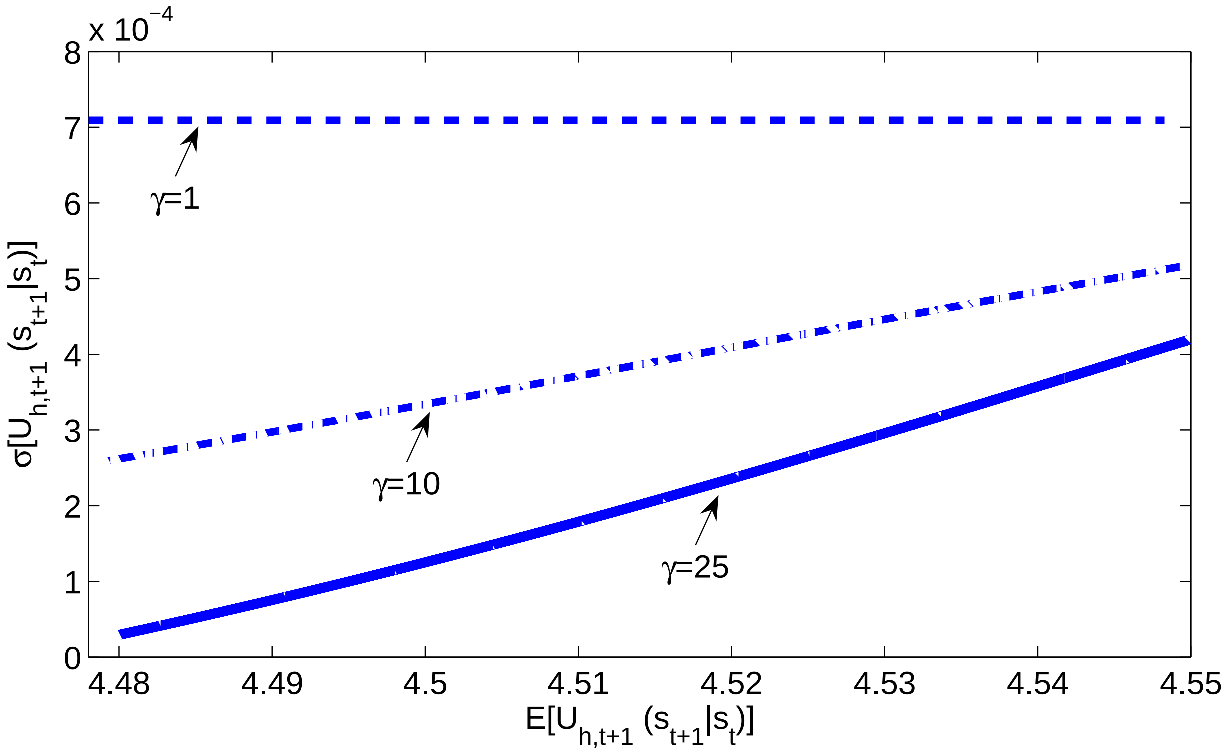

Figure 4: The expected utility-volatility trade-off. The horizontal axis reports the conditional expectation of the utility of the home agent. The vertical axis reports the conditional volatility of the utility of the home agent. The three lines are plotted for increasing values of the risk-aversion coefficient, γ. For comparison, the lines are obtained using utility values over a symmetric interval that includes 95% of the probability distribution for the case of $\gamma = 10$.

which yields the indirect utility function for a problem in which each agent chooses a worst-case distortion to the conditional distribution of the endowments in order to minimize the expected value of the value function plus an entropy penalty. That penalty limits the set of alternative models against which each agent guards. The size of that set is constrained by the parameter θ and it is increasing in θ, with $\theta = -\infty$ signifying the absence of a concern for robustness. Although this recursion is identical to the one that we discussed earlier, the interpretation of the parameter θ is different. In this subsection it measures the degree of concern about model misspecification, while before it is interpreted as a measure of risk aversion.

Following Cogley, Colacito, Sargent and Hansen (2008), the solution to this minimization problem results in an *agent specific* distorted conditional distribution of the

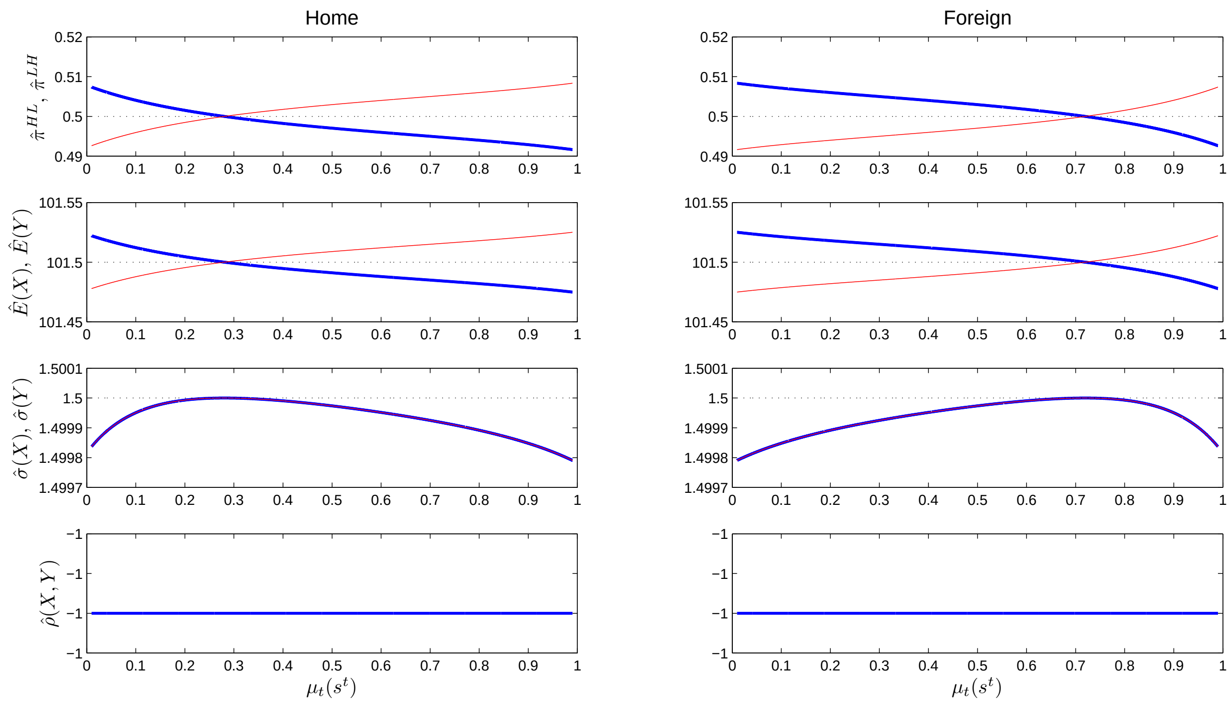

Figure 5: Distorted distributions. The first column refers to the home country, while the second column refers to the foreign country.

endowments' processes:

$$
\begin{aligned}
\widetilde{\pi}_{i,t+1}^{HL}(s^t) &= \pi(s_{t+1}|s^t)\frac{\exp\left\{U_{i,t+1}^{HL}(s^t)/\theta\right\}}{\sum_{s_{t+1}}\exp\left\{U_{i,t+1}(s_{t+1}|s^t)/\theta\right\}\pi(s_{t+1})} \\
\widetilde{\pi}_{i,t+1}^{LH}(s^t) &= \pi(s_{t+1}|s^t)\frac{\exp\left\{U_{i,t+1}^{LH}(s^t)/\theta\right\}}{\sum_{s_{t+1}}\exp\left\{U_{i,t+1}(s_{t+1}|s^t)/\theta\right\}\pi(s_{t+1})}, \quad \forall i \in \{h, f\}
\end{aligned} \tag{4}
$$

The top two panels of figure 5 depict the way in which conditional probabilities are being distorted as a function of $\mu_t(s^t)$. Two things ought to be noticed. First, distorted probabilities are functions of the relative wealth of the two countries. Second, distorted probabilities are country-specific. The explanation has to do with the fact that a robust decision rule is context specific, in the sense that it depends on the preferences and on the details of the stochastic perturbations that concern the two agents. Since agents are heterogeneous in terms of home bias, the distortion is country specific.

Robustness slants probabilities toward the worst case outcomes. The definition of worst case event, however, changes endogenously and in an intuitive fashion. On the one hand, when $\mu_t(s^t)$ is close to unity, the home agent is facing the possibility of being alone in the economy. In this case, she will prefer having an abundant supply of the most preferred good. That is, s_t^{LH} is the worst event. On the other hand, when $\mu_t(s^t)$ is approaching zero, the home agent is facing the prospect of being wiped out of the economy. Via the endogenous risk-sharing scheme, the realization of a high relative supply of local good, s_t^{HL}, would reduce even further the future share of resources. That is s_t^{HL} becomes the worst event.

Distorting conditional probabilities has the effect of distorting conditional moments of the marginal distributions of the endowments. The middle four panels of figure 5 report distorted conditional expectations ($\widehat{E}(X)$ and $\widehat{E}(Y)$) and distorted conditional volatilities ($\widehat{\sigma}(X)$ and $\widehat{\sigma}(Y)$) of the two endowments in the two countries. Distorted conditional expectations are the mirror image of distorted probabilities. The preference for robustness tends to lower the expected supply of the domestic good relative to the other countries' good as a function of the domestic Pareto weight. Conditional second moments are distorted in a such a way that the relative riskiness of the two endowments is unaltered relative to the original probability measure. The last two panels document that the distorted conditional correlations are unaffected by the concern for robustness.

4 Introducing rare events

In this section, we introduce a small probability of a large drop in the endowment of either one of the two goods. Specifically, we assume that there are nine possible states for the joint distribution of the endowments of the two goods. Four of them are equally likely "no-disaster states" in which the endowments of the two goods is either

Table 1: Calibrated Economy

Endowments

X	103	103	100	100	103	100	60	60	60
Y	103	100	103	100	60	60	103	100	60
Probability	0.2375	0.2375	0.2375	0.2375	0.01	0.01	0.01	0.01	0.01

Results

	$\frac{\sigma(\Delta c)}{\sigma(\Delta x)}$	$\frac{\sigma(M)}{E(M)}$	$\sigma(\Delta e)$	$\sigma(\frac{NX}{X})$	$\rho(\Delta c_h, \Delta c_f)$	$\rho(m_h, m_f)$	$E(r_h^c - r_h^f)$	$\frac{E(r_h^c - r_h^f)}{\sigma(r_h^c - r_h^f)}$
Model	0.690	29.224	13.998	2.471	0.372	0.814	1.69	0.19
Data	0.845	-	10.696	2.103	0.245	-	6.120	0.309

Notes - All the statistics are annual and multiplied by 100 (except for correlations and Sharpe ratios). All data source are described in the Appendix. The volatility of Net Exports over Output refers to the US, while all other statistics are computed as an average of major industrialized countries. Preference parameters are calibrated as $\gamma = 25$, $\delta = 0.95$, $\alpha = 0.98$.

100 or 103. Five of them are equally likely "disaster states" in which the supply of at least one of the two goods is 60. We assume that there is a combined 5% probability of a disaster state. These numbers are in line with the closed economy version of Barro (2006). We retain the same preference parameters used in the previous section. Table 1 summarizes the calibration.

This section is organized as follows. First, we are going to outline the differences in the optimal dynamics of the Pareto weights that we obtain in this environment as compared to the simple two states case examined in the previous section. Then, we are going to study the properties of the international stochastic discount factors and their implications for asset prices and exchange rates.

4.1 Pareto weights in the rare event model

Figure 6(a) documents the dynamics of Pareto weights in this environment. The explanation of the cases of unequal supply of the two goods is identical to the discussion reported in the previous section. Notice that in these cases the magnitude of the change in the one period ahead Pareto weight is directly related to the difference in the supply of the two goods. During times of equal endowments' realizations, Pareto weights can be either increasing on decreasing in the current distribution of wealth. This is suggestive of the way in which the risk-sharing scheme is implemented in this economy. A country is willing to lend resources to the country facing a poor realization of its endowment in the expectation of getting part of those resources back as soon as that country fares comparatively better.

Even a small probability of large endowment drops is able to generate large movements in the cross-country distribution of wealth. Figure 6(b) documents this finding. The solid line shows the ergodic distribution of Pareto weights with rare events, while the dashed line is obtained in an economy without disaster events.[4] In both cases, because of our symmetric calibration, the distribution is symmetric around 0.5. The dispersion of the Pareto weights around their mean, however, is significantly greater once rare events are introduced in the economy. When disasters are not perfectly synchronized across countries, indeed, risk-sensitive agents promote a more intense risk-sharing scheme in which Pareto weights or, equivalently, international assets are exposed to relevant adjustments over time.

4.2 Stochastic discount factors in the rare events model

The logarithm of the inter-temporal marginal rates of substitutions through which future uncertain payoffs are being discounted in the two countries are defined as

$$\begin{aligned} m_{i,t+1} &= \log \frac{\partial U_{i,t}/\partial C_{i,t+1}}{\partial U_{i,t}/\partial C_{i,t}} \qquad (5) \\ &= \log \delta - \Delta \log C_{i,t+1} + \frac{U_{i,t+1}}{\theta} - \log \sum_{s^{t+1}} \exp\left\{\frac{U_{i,t+1}}{\theta}\right\} \pi(s_{t+1}), \quad \forall i \in \{h, f\} \end{aligned}$$

[4]We simulate the model over 100 million periods and compute in both cases an histogram with 25 equally distant bins. Since the pictures are constructed on a dicrete grid, the area underneath each curve should not necessarily sum up to one.

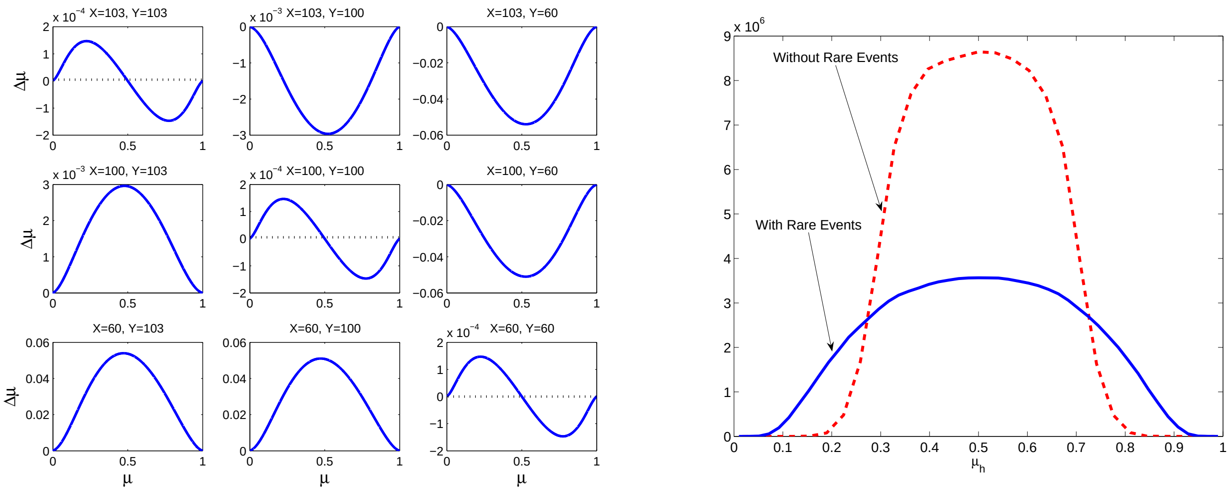

Figure 6: The left panels report the phase diagrams in the model with rare events. Each panel shows the one period ahead change in the home Pareto weight as a function of the current Pareto weight, conditional on each of the nine endowments' realizations. The right panel shows the invariant distribution of Pareto weights in the model with rare events. The results are obtained by simulating the model over 100 million of periods.

where to simplify the notation, we denoted history s^t variables without "(s^t)".[5] We are going to establish four aspects of the stochastic discount factors, concerning their volatilities and cross-country correlations.

4.2.1 The volatility of stochastic discount factors is high

Figure 7 reports the volatility of the home intertemporal marginal rate of substitution as a function of the Pareto weight in the same country. Our stochastic discount factors inherit the high volatility property that has been documented in the rare events literature. This implies that assets' Sharpe ratios are relatively large in this economy. Table 1 reports the equity risk premium and its ratio to the volatility for an asset that pays the consumption bundle as its dividend. The average excess return is about 1.7%, which is roughly 20% of its volatility. [6] In order to assess the success of

[5]We express the stochastic discount factors in units of the respective consumption bundles, because this is the relevant measure to calculate exchange rates in the subsequent sections.

[6]The moderate equity premium produced in this economy is due to an excessively high risk-free rate. Barro (2006) shows that by bringing into the model a default probability on government bonds in the case of a disaster, the model is able to generate a higher equity risk-premium through a reduction

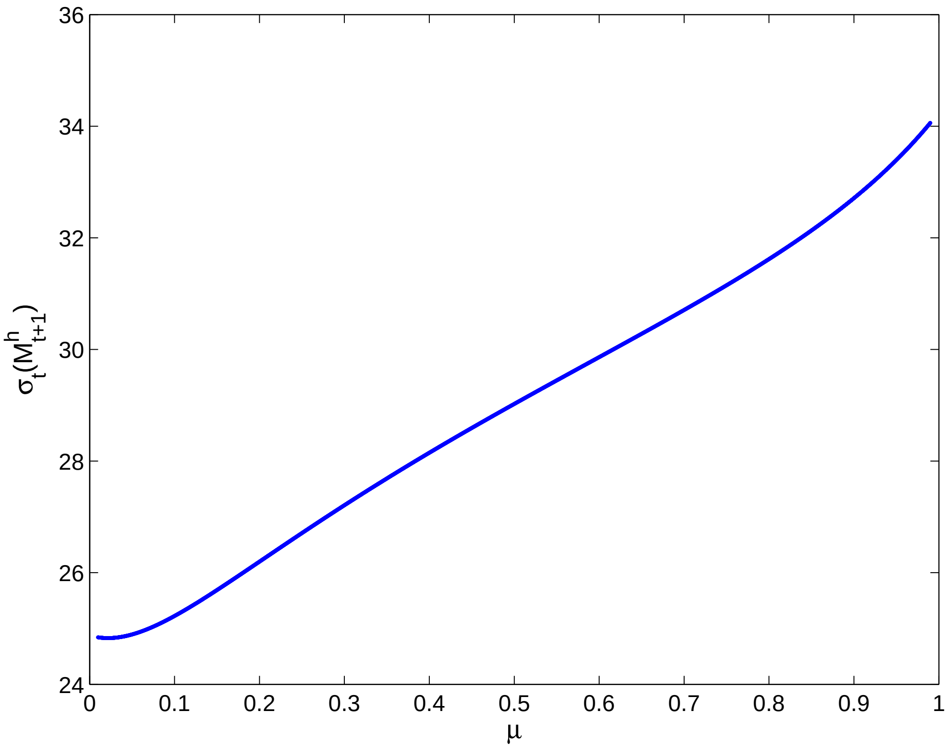

Figure 7: Volatility of the home stochastic discount factor in the rare events model. Conditional volatilities are reported against the current Pareto weight in the home country.

the model in terms of asset pricing, we use its ability of generating sizeable Sharpe ratios as metric.

4.2.2 The volatility of stochastic discount factors varies over time

Figure 7 shows that the volatility of stochastic discount factors is not only high, but also time-varying, ranging from 25% to 34%. The dynamics are driven by the optimal risk sharing scheme. A country that experiences a sequence of positive endowment realizations becomes relatively safer: its market price of risk declines as its Pareto weight decreases. A country that is hit by a sequence of negative shocks is able to smooth out her utility over time at the cost of having to accept an increasing volatility

in the average risk-free rate.

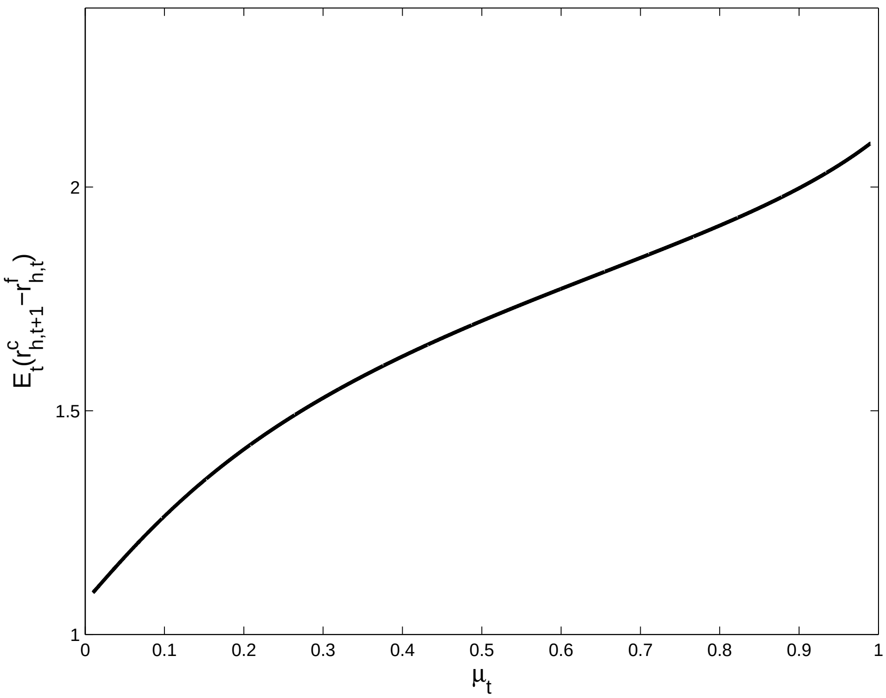

Figure 8: Equity Risk Premium of the home country in the rare events model. Conditional equity premia are reported against the current Pareto weight in the home country.

of consumption: its market price of risk rises as its Pareto weight increases.

Time-varying market prices of risk translate into time varying risk-premia. Figure 8 shows that expected excess returns vary from a little over 1% to a little over 2.1% in this economy. In particular, notice that the one-country versions of our model is associated to $\mu_t = 1$. In the one-country economy, the expected excess return would be constant at 2.1%. In our economy, instead, the excess return is time-varying around an unconditional mean of 1.7%. The 400 basis points difference is a measure of the overall reduction in risk obtained through international trade of goods and securities. In relative terms this difference is very relevant, as it suggests that international trade can reduce equity premia by 20%.

4.2.3 The correlation of stochastic discount factors is high

Brandt et al. (2006) pointed out that with time-additive preferences stochastic discount factors are as correlated as consumption growth rates. This is not necessarily the case with the type of preferences studied in this paper. When agents care about the temporal distribution of risk, they will attempt to equalize their continuation utilities as much as possible, even though consumption profiles are not perfectly aligned. Figure 9 documents that intertemporal marginal rates of substitution are at least twice as correlated as consumption.

This bears an important consequence for real exchange rates. Since markets are complete, the growth rate of the real exchange rate, Δe_{t+1}, is simply the difference of the two log-stochastic discount factors, $m_{f,t+1}$ and $m_{h,t+1}$. Given our symmetric calibration, the unconditional volatility of the exchange rate growth is:

$$\sigma(\Delta e_{t+1}) = \sigma(m_{h,t+1})\sqrt{2(1-\rho(m_{h,t+1}, m_{f,t+1}))}$$

As shown in Table 1, thanks to the high correlation of the stochastic discount factors, $\rho(m_{h,t+1}, m_{f,t+1})$, we are able to reproduce the observed volatility of real exchange rate fluctuations, despite the modest correlation of consumption growth rates across countries. Moreover, we are not exposed to the Brandt et al. (2006) puzzle: in our model the volatility of the exchange rate is consistent with the data even though stochastic discount factors are as volatile as impelled by the Hansen and Jagannathan (1991) bound.

4.2.4 The correlation of stochastic discount factors varies over time

The reallocation of consumption shares that takes place through the optimal risk-sharing scheme makes continuation utilities more or less correlated as a function of the relative wealth of the two countries. Specifically as the wealth spread increases,

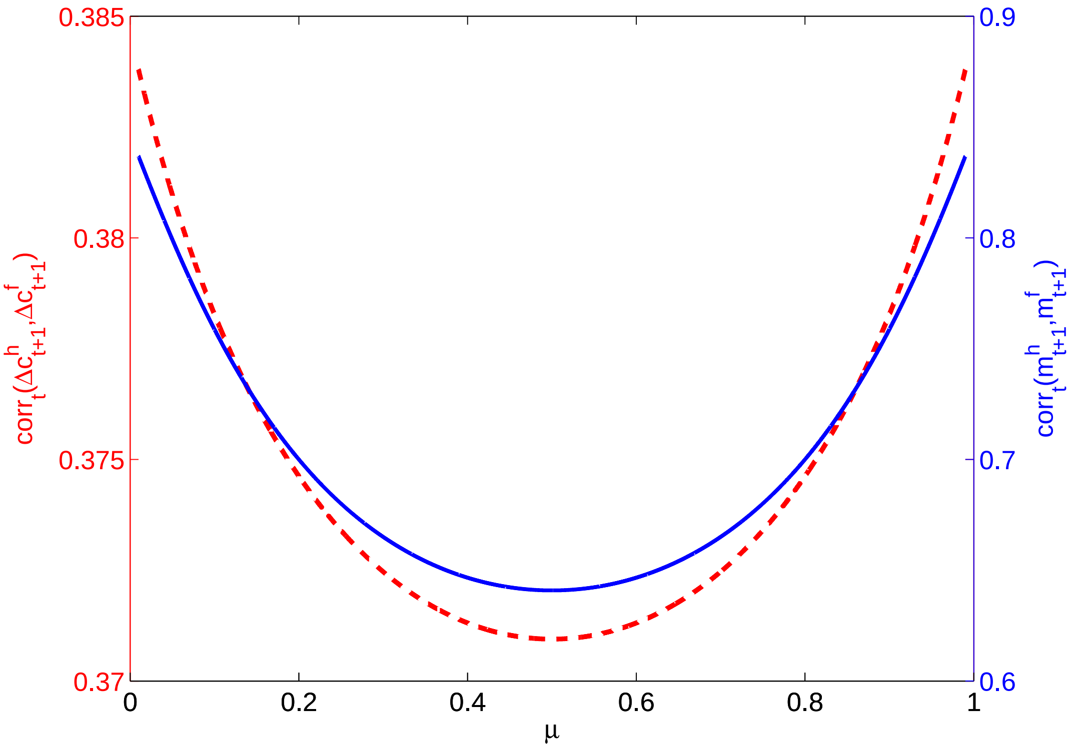

Figure 9: Correlations of growth rates of consumption and marginal utilities across countries. The left scale reports correlations of consumption growth. The right scale reports correlations of marginal rates of substitutions. Both sets of correlations are displayed against the current home Pareto weight.

the concavity of the utility functions is such that a positive endowment shock in the high Pareto weight country leads to an increased utility also in the other country even if it experiences a bad endowment draw. Hence risk sharing is the highest exactly when it is needed the most, that is when either of the two countries is close to dying.

By no arbitrage, the conditional volatility of exchange rate growth is equal to:

$$\sigma_t\left(\Delta e_{t+1}\right) = \sqrt{\sigma_t^2\left(m_{f,t+1}\right) + \sigma_t^2\left(m_{h,t+1}\right) - 2\rho_t\left(m_{h,t+1}, m_{f,t+1}\right)\sigma_t\left(m_{h,t+1}\right)\sigma_t\left(m_{f,t+1}\right)}$$

The combination of time varying volatilities and correlation of stochastic discount factors results in time-varying volatility of exchange rate movements. Figure 10 illustrates this finding. Specifically, it shows that the dynamics of exchange rate's volatility are mainly driven by time-varying conditional correlations. Indeed, by shutting

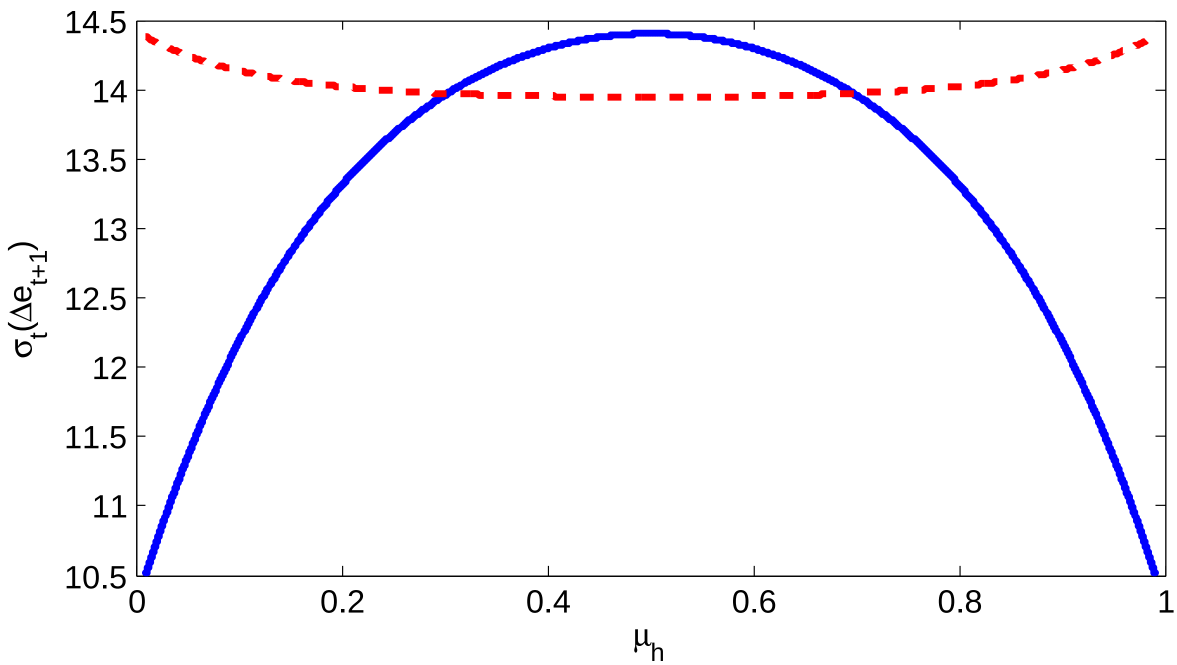

Figure 10: Volatility of the real exchange rate movement in the rare events model. Conditional volatilities are reported against the current Pareto weight in the home country. Dashed line: the correlation of stochastic discount factors is set to its unconditional value. Thick line: the conditional correlation of stochastic discount factors varies as in figure 9.

down this channel and by letting only conditional volatilities of stochastic discount factors change, the resulting volatility of the exchange rate is pretty much flat on the Pareto weight's domain (see dashed line in figure 10). As the conditional correlation varies over time (see thick line in figure 10), so does the volatility of the exchange rate, that can range from as low as 10.5% to as high as 14.5%. Hence, the dynamic risk-sharing arrangement drives the time-varying riskiness of the exchange rate.

5 Sensitivity analysis

In table 2 we report the results of a sensitivity analysis with respect to some of the preference parameters and to the probability distribution across states of the world.

Three aspects seem to emerge.

First, without rare events the model in not able to produce volatile enough stochastic discount factors. This results in equity risk premia that are on average smaller than 0.03% per year and in exchange rates that are too smooth, when compared to the actual data.

Second, introducing a small probability of a joint large endowment drop, allows the model to deliver highly volatile stochastic discount factors and equity premia as large as 3% per year. However, the model's implied exchange rate volatility is still too low due to the extremely high cross-country correlation of marginal rates of substitution. This correlation is mainly driven by the correlation of consumption growth. Under this calibration, in fact, consumption growth rates are highly correlated because of the simultaneous realization of large endowment drops.

Third, spreading the disaster probability to allow for the case of a large endowment drop in one country together with a no-disaster state in the other country, delivers both large equity premia and volatile enough exchange rates' fluctuations.

To summarize, rare events can improve the performance of the model if they are not perfectly correlated across countries.

6 Qualitative implications of the model

In this section, we explore the empirical validity of three qualitative implications of the model. First, the model predicts a negative relationship between the volatility of exchange rate movements, and the international correlation of the returns on the assets that pay consumption as their dividends. This can be readily seen by decomposing returns as

$$R^c_{i,t+1} = \frac{P^c_{i,t+1}/C_{i,t+1} + 1}{P^c_{i,t}/C_{i,t}} \frac{C_{i,t+1}}{C_{i,t}}$$

and noticing that the Euler equation restriction implies a constant price-consumption ratio. Hence the correlation of asset returns is driven by the cross-country correlation

Table 2: Sensitivity Analysis

$\frac{\sigma(\Delta c)}{\sigma(\Delta x)}$	0.682	0.696	0.663	0.707	0.703	0.703	0.690	0.7	0.683	0.686
$\sigma(M)/E(M)$	2.685	3.008	2.131	43.034	15.415	35.464	29.224	36.948	24.630	12.480
$\sigma(\Delta e)$	2.743	1.732	3.458	2.666	2.729	2.884	13.998	10.163	2.632	7.460
$\sigma(NX/X)$	1.196	1.096	1.377	1.255	1.249	1.275	2.471	2.404	15.386	2.050
$corr(\Delta c_h, \Delta c_f)$	0.075	0.646	-0.550	0.987	0.881	0.883	0.372	0.754	0.118	0.282
$corr(m_h, m_f)$	0.693	0.915	0.000	0.999	0.992	0.998	0.814	0.948	0.661	0.858
$E(r_h^c - r_h^f)$	0.030	0.030	0.020	3.430	0.410	1.000	1.690	2.550	1.280	0.350
$\frac{E(r_h^c - r_h^f)}{\sigma(r_h^c - r_h^f)}$	0.020	0.020	0.010	0.300	0.100	0.250	0.190	0.250	0.160	0.280
$(103, 103)$	0.2500	0.4000	0.1000	0.2375	0.2485	0.2488	0.2375	0.1583	0.1583	0.1583
$(103, 100)$	0.2500	0.1000	0.4000	0.2375	0.2485	0.2488	0.2375	0.3167	0.3167	0.3167
$(100, 103)$	0.2500	0.4000	0.1000	0.2375	0.2485	0.2488	0.2375	0.1583	0.1583	0.1583
$(100, 100)$	0.2500	0.1000	0.4000	0.2375	0.2485	0.2488	0.2375	0.3167	0.3167	0.3167
$(103, 60)$	-	-	-	-	-	-	0.0100	0.0050	0.0120	-
$(103, 80)$	-	-	-	-	-	-	-	-	-	0.0120
$(100, 60)$	-	-	-	-	-	-	0.0100	0.0050	0.0120	-
$(100, 80)$	-	-	-	-	-	-	-	-	-	0.0120
$(60, 103)$	-	-	-	-	-	-	0.0100	0.0050	0.0120	-
$(80, 103)$	-	-	-	-	-	-	-	-	-	0.0120
$(60, 100)$	-	-	-	-	-	-	0.0100	0.0050	0.0120	-
$(80, 100)$	-	-	-		-	-	-	-	-	0.0120
$(60, 60)$	-	-	-	0.0500	0.005	0.0050	0.0100	0.0300	0.0020	0.0020
$(80, 80)$	-	-	-	-	-	-	-	-	-	-
γ	25	25	25	25	25	50	25	25	25	25

Notes -

of consumption growth rates. As shown in figures 9 and 10 and as discussed in the previous section, the correlation of consumption growth and exchange rate's volatility are inversely related.

In order to check this empirical prediction, we constructed time series for international stock market correlations and for the volatilities of exchange rate movements, as sample averages of monthly data over ten years overlapping rolling windows. The Appendix discusses the data sources. Throughout this analysis, the United States are the home country, while the foreign country is either a G-7 country or a major US trading partner. The plots in figure 11 show that, for most of the country pairs considered, there is, indeed, the negative relationship suggested by the model.

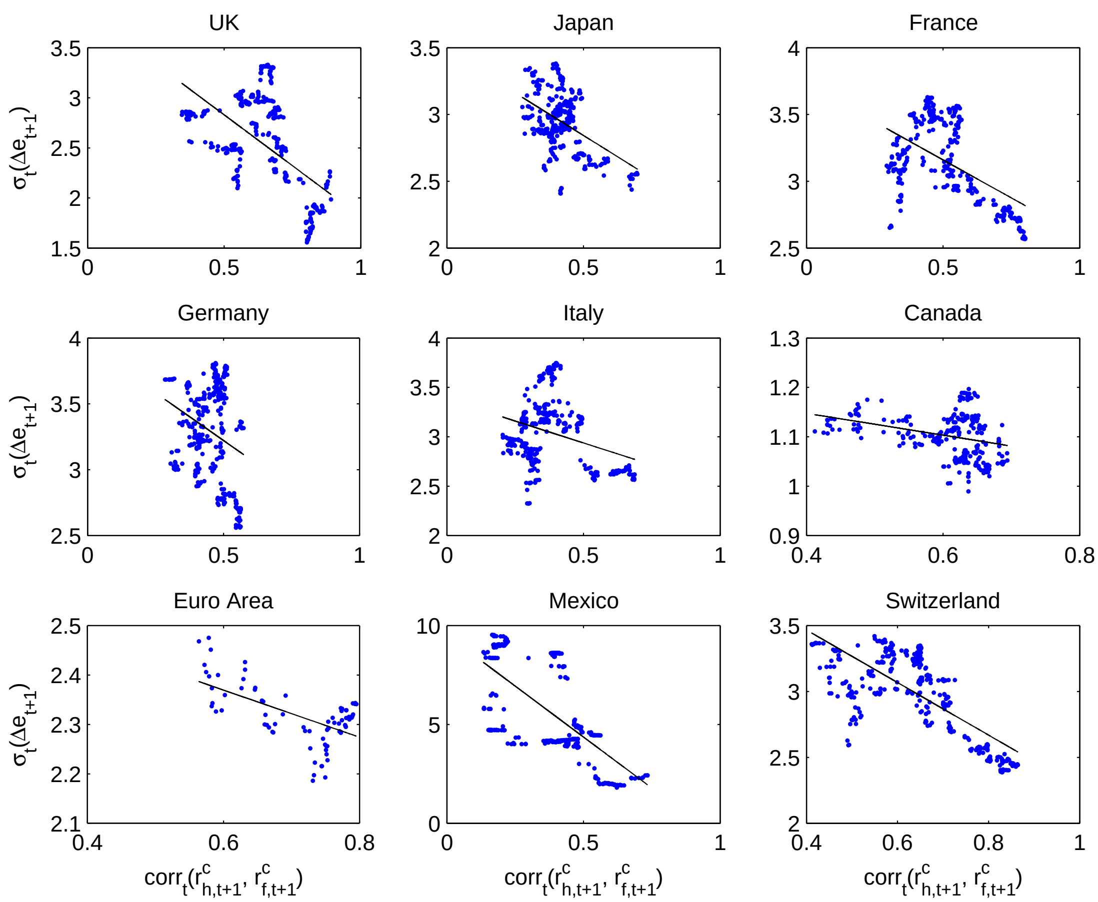

Figure 11: Conditional second moments. This figure shows conditional correlations (horizontal axis) and volatilities of exchange rate movements (vertical axis) in nine countries from 1970 to 2007. Exchange rates are always computed against the US dollar.

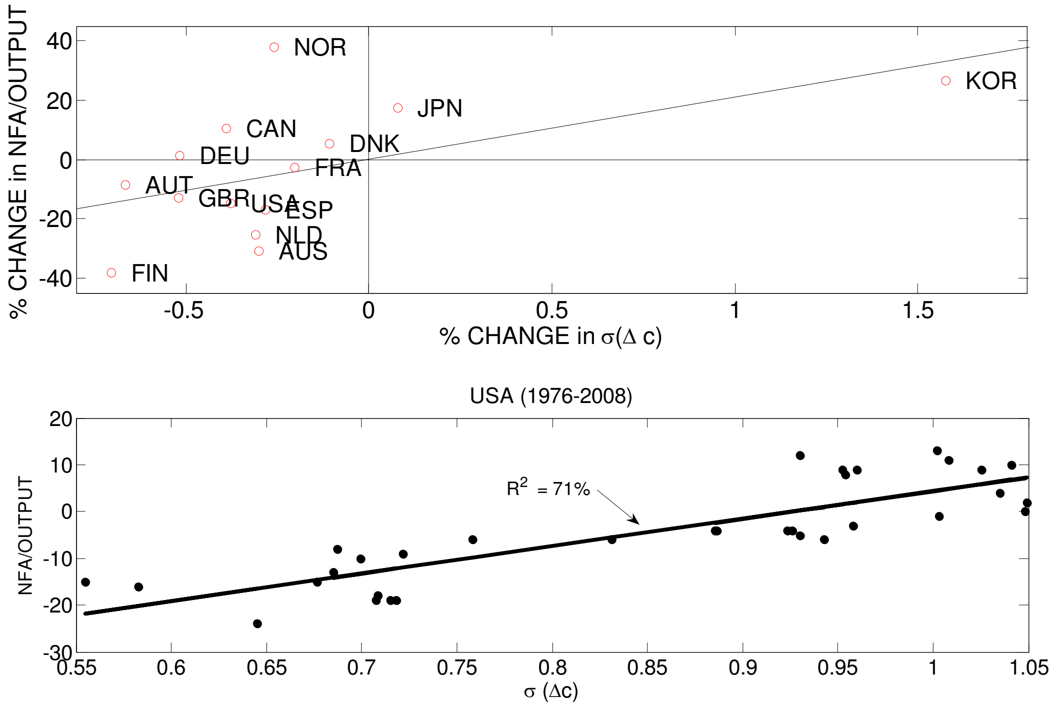

Figure 12: Outstanding level of international savings (Net Foreign Assets) as a percentage of GDP and volatility of consumption growth. In the top panel, we compare statistics computed over the 1970-1985 period and the 1990-2005 period. The bottom panel focuses on US annual data from 1976 to 2008.

The model also suggests that the higher the Pareto weight of a country is, the higher its consumption growth volatility should be. Since there is a one for one mapping between a country's Pareto weight and its outstanding level of international savings (see figure 2), we can check whether savings positively co-vary with consumption growth volatility. Focusing on long periods of time, Fogli and Perri (2009) show that there exists a positive link between output volatility and international net asset position. Using their empirical methodology, we consider a wide cross-section of countries and focus on the change in net foreign assets (NFA) and consumption volatility in the following two sub-samples: 1970-1985 and 1990-2005. In the top panel of figure 12 we show that consumption volatility is a positive function of the net external asset position as well. This novel empirical finding is consistent with the theoretical im-

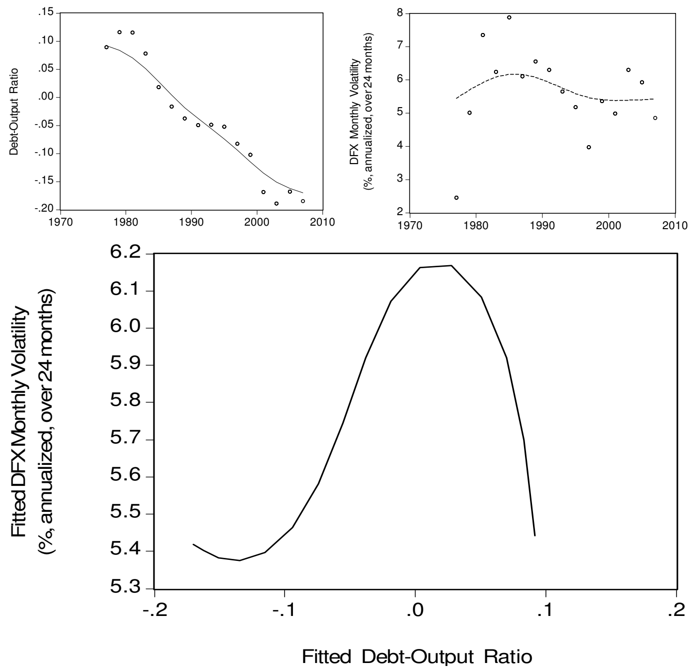

Figure 13: Conditional volatility of exchange rate growth and international debt-output ratio. The two top panels show actual US international debt-output ratio and actual volatility of real exchange rate growth. Circles are used for realized bi-annual averages, the solid lines refer to kernel-based fits. The sample ranges from 1975 to 2007. The bottom panel focuses on kernel-based fits only.

plications of our simple exchange economy. In the bottom panel of figure 12 we focus only on US data and show that the positive connection between international savings and consumption volatility can be identified even at an yearly frequency.[7]

Using the afore mentioned mapping between a country's Pareto weight and its outstanding level of international savings, we can check another interesting condition. According to figure 10, the conditional volatility of the exchange rate should decrease when the debt-output ratio departures from zero. As shown in figure 13, this is consistent with the data. In the top right panel, we focus on the conditional volatility of the US exchange rate growth measured by bi-annual standard deviations.[8] Cir-

[7]Consumption growth volatilities are computed as yearly averages of quarterly volatilities over 15 years overlapping rolling samples.

[8]We focus on the exchange rate of the dollar vs major currencies and compute time-varying stan-

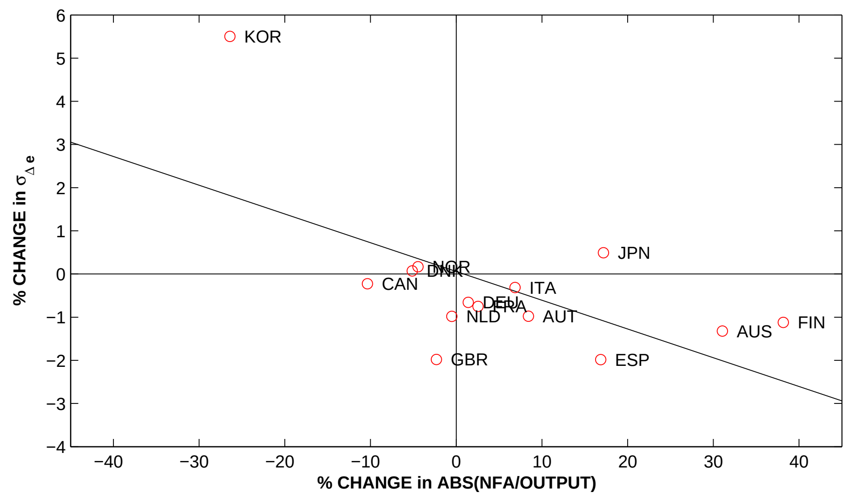

Figure 14: Outstanding level of international savings (Net Foreign Assets–NFA) as a percentage of GDP and volatility of exchange rate growth. We compare statistics computed over the 1970-1985 period and the 1990-2005 period. Real exchange rates are computed against US dollar.

cles refer to actual data, while the solid line is a Kernel-based fit. In order to be consistent, in the top left panel we show bi-annual averages of the US international debt-output ratio (circles) together with a kernel-smoothed fit (solid line). Positive values indicate international surpluses, while negative values refer to international debt. In the bottom panel we plot the fitted conditional volatility of the exchange rate growth as a function of the fitted outstanding US debt-output ratio. Consistently with our theoretical findings, the exchange rate volatility reaches its maximum when the US debt-output ratio is close to zero.

We also check this result in our cross-section of countries. According to figure 10, there is a negative relationship between the absolute value of the debt-output ratio and the volatility of exchange rate movements. Focusing on the same two sub-samples used by Fogli and Perri (2009), we show in figure 14 that this negative relationship can be found across coutries as well.

dard deviations using 24 monthly realizations of the exchange rate growth. Standard deviations are annualized.

7 Concluding remarks

We have proposed a novel model of international risk sharing that is both theoretically challenging and empirically appealing. The theoretical challenge lies in the documented difficulty to obtain interesting dynamics, by ensuring at the same time survivorship of all agents in economies featuring risk-sensitive preferences. The empirical appeal comes from the fact that the dynamic equilibrium displays time-varying second moments of quantities and prices, whose behavior appears to resemble what we observe in the data.

Future developments of this literature should consider the role of investments and specific frictions, whose ability of reproducing a number of stylized facts in the international business cycles literature has been widely documented. A formal estimation of a fully fledged model would be able to shed light into the dynamics of international prices and quantities.

8 Appendix

8.1 The dynamic programming problem

This section describes the iterative algorithm that was used to obtain optimal policy rules. Starting from a social planner's value function Q^k, we considered the following problem:

$$Q^{k+1}(X,Y,\mu) = \max_{\left\{x_i \geq 0, y_i \geq 0, U_i^{j\prime}\right\}} \sum_{i \in \{h,f\}} \mu_i (1-\delta) \log C_i(X,Y,\mu_i) + \delta\theta \log \sum_j \pi^j \exp\left\{U_i^{j\prime}/\theta\right\}$$

subject to

$$\begin{aligned} \min_{\mu'} Q^k(X^j, Y^j, \mu^{j\prime}) - \sum_{i\in\{h,f\}} \mu^{j\prime} U_i^j &\geq 0 \\ x_h + x_f &\leq X \\ y_h + y_f &\leq Y \end{aligned}$$

for each possible endowment pairs $(X^{j\prime}, Y^{j\prime})$ tomorrow. Let $Q^0(X,Y,\mu)$ be an arbitrary finite function which is continuous and convex in μ. Following, Lucas and Stokey (1984) and Kan (1995) the social planner's value function can be computed by iterating on the above starting from Q^0.

8.2 Data sources

Consumption and GDP

These series are the same used by Heathcote and Perri (2002) and Fogli and Perri (2009). The original data series are from OECD.

Exchange rates, stock market returns, and risk-free rates

The data series for exchange rates are from the IMF International Financial Statistics (IFS) over the sample 1974:1–1998:4 for G-7 countries. Stock markets' returns are obtained from IFS, series name Share Prices, also for G-7 countries. Risk-free rates are Treasury Bill rates. Real series are obtained by subtracting CPI inflation from the same source.

International Debt

The data series for US international debt are from the Bureau of Economic Analysis (BEA), year-end international positions, 1976-2008. US debt-output ratio is computed using both nominal annual GDP and debt from the BEA. Data on international debt for all the other countries studied in this paper are from Lane and Milesi-Ferretti (2007).

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