Working Paper Series

The Cyclicality of the User Cost of Labor with Search and Matching

WP 09-12

Marianna Kudlyak
Federal Reserve Bank of Richmond

THE FEDERAL RESERVE BANK OF RICHMOND
RICHMOND ▪ BALTIMORE ▪ CHARLOTTE

The Cyclicality of the User Cost of Labor with Search and Matching*

Marianna Kudlyak[†]

The Federal Reserve Bank of Richmond

Working Paper No. 09-12

Abstract

The user cost of labor captures the hiring wage and the expected effect of the economic conditions at the time of hiring on future wages. In search and matching models, I show that it is the user cost and not the wage that is weighted against the worker's marginal product at the time of hiring; so, the user cost is the allocational variable. I construct its measure in the data and estimate that it is more procyclical than average wages or wages of newly hired workers. I show that the textbook search and matching model cannot simultaneously generate the empirical elasticities of the vacancy-unemployment ratio and of the user cost of labor, irrespectively of the surplus division rule. (JEL: E24, E32, J64, J30.)

Key Words: User Cost of Labor. Cyclicality. Wage Rigidity. Search and Matching.

*This paper is based on the chapters of my doctoral dissertation at the University of Rochester. I am deeply grateful to my adviser, Mark Bils, for his continuous advice during this project. I am also thankful to Arpad Abraham, Mark Aguiar, Paulo Barelli, Andriana Bellou, Yongsung Chang, William Hawkins, Baris Kaymak, Damba Lkhagvasuren, Thomas Lubik, Andriy Norets, Ronni Pavan, Richard Rogerson, Leena Rudanko, and Roman Sysuyev for useful discussions and comments. This paper benefitted from comments from the participants of the Rochester Student Seminar, seminars at the Richmond Fed, the Kansas City Fed, the Cleveland Fed, the University of California Santa Cruz, the University of Montreal, Wilfrid Laurier University, the Kyiv School of Economics, the New York/Philadelphia Workshop on Quantitative Macroeconomics, the System Macroeconomics Meeting in San Francisco and the Econometric Society European Meeting in Barcelona. The views expressed here do not necessarily reflect those of the Federal Reserve Bank of Richmond or the Federal Reserve System. All errors are mine.

[†]Mailing address: Research Department, The Federal Reserve Bank of Richmond, 701 E. Byrd St., Richmond, VA, 23219. E-mail: marianna.kudlyak@rich.frb.org.

I. Introduction

Macroeconomists have long been interested in the cyclicality of the real marginal cost of labor. Using wages as the measure of the cost, the standard conclusion is that there is little (pro)cyclicality, if any. However, the empirical evidence on the cyclicality of individual wages suggests that wages depend on the history of labor market conditions from the start of the job. For example, Beaudry and DiNardo (1991) find evidence that wage depends on the history of unemployment rates from the start of the job. Bils (1985) and subsequent literature find that wages of newly hired workers are more cyclical than wages of workers who do not change jobs.[1] That is, wages may not capture the per period cost of a worker to a firm.

In this paper I argue that the wage is not necessarily the correct measure of the marginal cost of labor. Instead, it is the user cost of labor that is relevant for the hiring decision of a firm. The user cost of labor is analogous to the user cost of capital, which is the difference between the purchase price and the expected price that can be recovered from selling the un-depreciated part at the end of the period. By analogy, the user cost of labor is the difference between the costs of adding a worker starting from the current period and the expected costs of replacing the worker the next period. If the labor market is a spot market, then this difference is the wage. If a worker is contracted for more than one period, then this difference need not be equal to the wage, as economic conditions at the time of hiring may have an impact on the future wage payments within the employment relationship. This impact is captured by the user cost.

I propose a measure of the wage component of the user cost of labor and estimate its cyclicality. I find that it is substantially larger than the cyclicality of individual wages or even the cyclicality of wages of newly hired workers. Then, I show that the standard search and matching model with free entry condition for firms (Pissarides (1985), Mortensen and Pissarides (1994)) cannot simultaneously generate the empirical elasticities of the wage component of the user cost of labor and of the vacancy-unemployment ratio, irrespectively of the surplus division rule at the beginning of the match or the individual wage dynamics

[1] See a recent review in Pissarides (2009).

within employment relationships.

In an economy with search and matching frictions, the user cost of labor can be decomposed into vacancy and wage components. The vacancy component refers to the expected difference in expenses on vacancy creation between the current period and the next. The wage component refers to the expected difference in expenses from starting to pay wages in the current period versus the next period. Future expenses are discounted to take into account the real interest rate and turnover. Notice that the wage component consists of the difference between two present discounted streams of wages. Therefore, in addition to wages at the time of hiring, the wage component of the user cost includes the effects of economic conditions at the time of hiring on future wages. I show that in a search and matching economy with stochastic productivity and exogenous separations, free entry ensures that the worker's current marginal product equals the user cost of labor.

I estimate the cyclicality of the wage component of the user cost using NLSY79 data. Because the user cost is not directly observed in the data, I construct its empirical counterpart based on the behavior of individual wages and turnover. First, I estimate a projection of wages on the two sets of time dummies – a year that indicates the start of the job and a contemporaneous year – and a series of individual- and firm-specific controls. Next, using the estimates of wages and the empirical separation rate, I construct a series for the wage component of the user cost of labor. In the construction, future payments are discounted to take into account anticipated separation rates and the real interest rates. Finally, I project the constructed series of the (log of the) wage component of the user cost of labor on the contemporaneous unemployment rate.

I find that the constructed wage component of the user cost of labor is almost three times as cyclical as the individual wages and also noticeably more cyclical than the wages of newly hired workers. To understand this empirical result, recall the empirical facts on individual wages mentioned above: first, wages of newly hired workers are more cyclical than wages of workers who do not change jobs and, second, wages within employment relationships are smoothed. Consider a firm hiring when the unemployment rate is high. Because the unemployment rate is high, the wage of new hires is low. In addition, the wages in all subsequent periods in the newly formed relationships are relatively lower than wages in the relationships that start under more favorable economic conditions. If the unemployment rate is expected to return to lower levels, then by hiring now as opposed to the subsequent year, a firm 'locks in' a worker to a relatively low stream of wages. In this case, the wage at the time of hiring overstates the wage component of the user cost of labor by the expected difference between the present value of wages to be paid starting from the subsequent year to a worker hired in the subsequent year and the present value of wages to be paid from that time to a worker hired now. Thus, when unemployment is high, the hiring wage is low, but the wage component of the user cost is even lower. This implies that the wage component of the user cost is more responsive to changes in unemployment than the hiring wages.

The estimated -4.5% cyclicality of the wage component of the user cost with respect to unemployment translates into its elasticity with respect to productivity in excess of 1.5. Given the empirical elasticity of the vacancy-unemployment ratio, the stochastic process for productivity and the parameters of the matching function reported in the literature, in the search and matching model, the elasticity of the vacancy component of the user cost is also above 1. However, as explained above, the model's free entry condition shows that productivity equals the sum of the wage and vacancy components, implying that the elasticities of both components cannot be greater than 1. Thus, the search and matching model cannot simultaneously accommodate the empirical elasticity of the vacancy-unemployment ratio and the empirical elasticity of the wage component of the user cost of labor.

This result shows that the solution to the unemployment volatility puzzle noted by Shimer (2005) cannot come from wage setting. That is, Shimer (2005) showed that the standard search and matching model lacks amplification of the productivity shock to generate the empirical volatility of the vacancy-unemployment ratio. A possible amplification mechanism is the rigidity of the statistics from wages that is relevant for the job creation decision of a firm. I show that this mechanism works through making the wage component of the user cost of labor rigid. The estimates of the cyclicality of the wage component of the user cost show that the data lack the required rigidity.

Additionally, to illustrate the allocational role of the wage component of the user cost versus wages, I calculate the cyclicality of the user cost of labor and wages in search and matching models with specific wage settings. I consider two different wage settings that have been widely used in the literature – a model where wages are renegotiated each period by Nash bargaining and models where wages are smoothed due to the implicit contract between a worker and a firm. In the quantitative investigation I hit the economies with the same series of productivity shocks and calculate the cyclicalities from the simulated data. First, I find that, when wages are smoothed within employment relationships, the wage component of the user cost is more cyclical than the wages of newly hired workers, which in turn are more cyclical than the wages of all workers. Second, when the cyclicality of the wage component of the user cost across economies with different wage settings is calibrated to the same target, then the volatility of the vacancy-unemployment ratio in all economies is approximately the same. However, the cyclicality of individual wages, including the cyclicality of wages of newly hired workers, varies significantly depending on the individual wage setting.

The paper is organized as follows. Section 2 discusses the related literature. Section 3 introduces the user cost of labor and presents the main analytical result using the model's free entry condition. Section 4 presents the main empirical results. Section 5 illustrates implications of the estimated elasticity of the wage component of the user cost of labor for the free entry condition using an example. Section 6 presents a quantitative investigation of the cyclicality of the user cost of labor in models with specific wage settings. Section 7 concludes.

II. Related Literature

The main contribution of the paper is to provide an estimate of the cyclicality of the wage component of the user cost of labor in the data. The idea that wage is not allocational for employment if there is history dependence in wages goes back to Barro (1977) and Hall (1980) who argue that what matters to a firm is the value of wages to be paid during the course of a firm-worker relationship. In the words of Hall (1980), "to see what is happening today in the labor market, one should look at the implicit asset prices of labor contracts recently negotiated, not at the average rate of compensation paid to all workers." Barro calls sticky wages just a "façade" of the implication of the long-term labor contracts to short-term macro fluctuations. Kydland and Prescott (1980) note that the weak procyclicality of real wages can suffer from "cyclical measurement bias" because, with implicit contracts, wage payments are not perfectly associated with labor services provided each period. This paper, to my knowledge, is the first attempt to measure the cyclicality of the price of labor taking into account the effect of economic conditions at the time of hiring on future wages.

In addition, this paper contributes to the ongoing discussion on the quantitative behavior of search and matching models. The rigid wage mechanism generated vast interest in the literature and gave rise to developments of the alternative wage setting mechanisms in a search and matching model. Testing the claim for rigidity requires comparing the cyclicality of the wage component of the user cost in the data with the cyclicality implied by the model. Only recently has literature turned to contrasting the wage dynamics in the model with the data (Hagedorn and Manovskii (2008); Rudanko (2009); Pissarides (2009); Haefke, Sonntag, and van Rens (2007)). Although it is acknowledged that the wage is not allocational in the presence of long-term employment relationships, the literature concentrates mostly on individual wage dynamics. The quantitative results in this paper highlight that judging the wage rigidity from the individual wages as opposed to the wage component of the user cost can be misleading.

Contemporaneously, Pissarides (2009) and Haefke, Sonntag and van Rens (2007) examine the dynamics of the wages of newly hired workers in a search and matching model with wages set by Nash bargaining period by period. Under such wage setting, the average wage equals the wage of new hires and equals the wage component of user cost.

In the data, the wages of newly hired workers are more cyclical than the average wage. Thus, the comparison of the dynamics of the wages from the model to the dynamics of the wages of newly hired workers as opposed to the dynamics of the average wage helps to shift away from the rigid wage assumption. However, under period-by-period bargaining, the dynamics of wages from the model should be contrasted with the dynamics of the wage component of the user cost in the data. As I show in the paper, in the data, the latter is noticeably more cyclical than wages of newly hired workers.

The concept of the user cost was introduced by Keynes and clarified in Scott (1953). Later, Jorgenson (1963) applies the term to define the 'shadow' price of capital and Rosen (1969) adopts the term for labor's cost.[2] Despite a long history, the user cost has not been

[2] In Rosen (1969) the user cost of labor refers to the required return to cover the real interest and turnover

employed in the recent literature. While the studies acknowledge that what matters for job creation is the present value of wages, the literature usually proceeds by contrasting models with the average wage in the data. However, the empirical evidence points to the importance of the effect of the economic conditions at the time of hiring for future wages.

III. The User Cost of Labor

In this section, I introduce the user cost of labor in a discrete time search and matching model, and show that the user cost, and not wages, is the key factor in the job creation decision of firms.

A. Environment

The economy is populated by a continuum of infinitely lived profit-maximizing homogeneous firms and a continuum of measure 1 of homogeneous infinitely lived workers. Workers maximize the present discounted value of utility, $u(c)$, with $u'(c) > 0$, $u''(c) \leq 0$. They do not have access to credit markets and cannot save or borrow. Firms and workers discount the future with a common discount factor β, $0 < \beta < 1$.

A firm can choose to remain inactive or to start production. Production requires only labor input. To start production, a firm must enter the labor market and hire a worker. Upon entering the labor market, a firm opens vacancies and searches for a worker. There is free entry; however, a firm must pay a per vacancy cost, c, measured in units of the consumption good. Workers in the economy can be employed or unemployed. An unemployed worker receives a per period unemployment benefit, b, and costlessly searches for a job. Given the number of unemployed workers, u, and the number of vacancies, v, the number of newly created matches in the economy is determined by a matching function, $m(u, v) = Ku^{\alpha}v^{1-\alpha}$, where $\alpha \in [0, 1]$ (Petrongolo and Pissarides (2001)) and K is a positive constant. Given $\theta = \frac{v}{u}$, the labor market tightness, the probability of filling a vacancy for a firm is $q(\theta) = K\theta^{\alpha}$ and the probability of finding a job for an unemployed worker is $\mu(\theta) = K\theta^{1-\alpha}$. While matched, each firm-worker match produces per period output z. The stochastic process for

costs. In addition to Rosen's components, the concept used here encompasses the worker's wage as well as the effect of the economic conditions at the time of hiring on future wages.

z is governed by a stationary first-order Markov process. Workers matched with firms earn wages and cannot search while employed. Generally, one can think of the wage as the result of a surplus division agreement between the firm and the worker, which may or may not entail history dependence in wages. In this section I do not assume a particular surplus division rule either at the beginning or within the employment relationship.

The economy operates according to the following time-line:[3] 1) at the beginning of a period, a firm decides whether to create a job or to stay inactive; if the decision is to create a job, the firm posts vacancies and incurs the vacancy posting cost; also, workers who were unemployed for at least one period costlessly search for jobs; 2) when firms with open vacancies meet unemployed workers, new matches are created; 3) production takes place in both newly created matches and matches that were carried over from the previous period; employed workers receive wages and unemployed workers receive unemployment benefit, b; 4) at the end of a period, a fraction δ of productive matches is randomly selected and exogenously destroyed: the workers who were employed in those matches become unemployed and the firms who operated those matches return to the pool of inactive firms; 5) surviving matches are carried over to the next period.

B. The User Cost of Labor

The only nontrivial economic decision in this environment is a firm's decision of creating a productive match with a worker in the current period versus postponing the creation until the next period. The costs of such a decision are summarized by the user cost of labor: they are all expenses associated with creating and maintaining a match in the current period that can be avoided if the creation is postponed until the next period. Therefore, the user cost includes not the total payments associated with creation of a productive match in the current period, but only the part that is expected to be in excess of what a firm will need to pay the next period.

The user cost of labor can be calculated as the expected present discounted value of the hiring costs and wage payments in a productive match that starts in period t less the expected present discounted value of the costs (hiring costs and wage payments) of replacing

[3] The value functions in the economy are summarized in the appendix.

the worker in period $t + 1$. In the model there are two sources of costs associated with creating a productive match – costs associated with vacancy openings and wage payments to a worker. Thus, the user cost of labor, UC_t, can be decomposed into its two components: the vacancy component, UC_t^V, and the wage component, UC_t^W:

$$UC_t = UC_t^V + UC_t^W.$$

The vacancy component is associated with fixed per period cost on vacancy opening, c, and the probabilities of filling a vacancy in t, q_t, and $t + 1$, q_{t+1}. Suppose, for example, that a worker is always available for hire, i.e. $q_t = q_{t+1} = 1$. Then the vacancy component is $\beta\delta c$, reflecting the real interest rate associated with paying cost c in t instead of delaying it until $t + 1$ and a worker turnover. The turnover cost is due to the possibility of separation in period t, which decreases the expected number of matches surviving until period $t + 1$. However, with search and matching frictions the probability of filling a vacancy may differ between t and $t+1$. Given that to create one match in period t, a firm opens $1/q_t$ vacancies, each at cost c, the vacancy component, UC_t^V, is

$$UC_t^V = \frac{c}{q_t} - \beta(1 - \delta)E_t\frac{c}{q_{t+1}}.$$

The wage component of the user cost of labor is associated with the wage payments to a worker. If wages are renegotiated every period in all matches and do not depend on the history of economic conditions from the start of the job, then the wage component is simply the hiring wage. However, if wages depend on the history of economic conditions from the time a worker is hired, then the wage component should take into account the hiring wage and the effect of the economic conditions at the time of hiring on future wages within the employment relationship. The wage component, UC_t^W, can be calculated as the difference between the expected present discounted value of wages paid to a worker hired in t and the expected present discounted value of wages to be paid to an identical worker hired in $t + 1$:

$$UC_t^W = PDV_t - \beta(1 - \delta)E_tPDV_{t+1}, \tag{1}$$

where $PDV_t = w_{t,t} + \sum_{\tau=t+1}^{\infty}(\beta(1-\delta))^{\tau-t}E_t w_{t,\tau}$.

Substituting expression for PDV_t for (1) yields the following expression for the wage component of the user cost:

$$UC_t^W = w_{t,t} + \sum_{\tau=t+1}^{\infty}(\beta(1-\delta))^{\tau-t}E_t(w_{t,\tau} - w_{t+1,\tau}). \quad (2)$$

The wage component of the user cost at time t consists of two parts: the hiring wage at time t and the expected present discounted value of the differences between wages paid from the next period onward in the employment relationship that starts in period t and the employment relationship that starts in period $t+1$. Unless the second term is 0, the wage component is not equal to the wage at the time of hiring.

Consider the conditions under which the second term in (2) vanishes. An example is the case where the wage is bargained each period and is not history-dependent. Then wages across all matches are equal in every period, and the wage component of the user cost of labor is just the wage at the time of hiring (Nash bargaining in the standard model is an example of such wage setting). The wage component of the user cost will also include only the wage at the time of hiring if the wage is rigid and is not responsive to changes in economic conditions. Finally, if the aggregate shock in the economy is perfectly autocorrelated, then there is no expected change in productivity. In that case, depending on the nature of the wage contract, the expected wages may remain constant due to the unchanged economic conditions.

If wages depend on the history of the economic conditions from the start of the job, the wage component of the user cost of labor does not equal wage. Such history dependence can arise, for example, if workers are risk-averse and cannot save and firms that have access to asset markets provide insurance against fluctuations in productivity. In this case, the wage at the time of hiring is a part of a contractual scheme. Contracts are designed to deliver promised utility to the worker. Firms choose the wage stream to minimize the expected cost of delivery of the promised value. Hence, the wage by itself may not reflect the total wage commitment that the firm takes on at the time of hiring. The wage component of the user cost of labor summarizes the future value of this commitment in present value terms.

It is an empirical question whether wages depend on the history of economic conditions from the start of the job or only on the contemporaneous market conditions. In Section 4, I review the existing empirical evidence that supports for the former.

C. Free Entry and the User Cost of Labor

The key equilibrium condition in a search and matching model is a free entry condition for firms, which implies that the value of vacancy is 0. Given this condition, the following proposition obtains.[4]

Proposition 1. *Given the free entry condition for firms, the marginal productivity of a firm-worker match equals the user cost of labor,* $z_t = UC_t \; \forall t$*:*

(3)

$$z_t = \left(\frac{c}{q(u_t, v_t)} - \beta(1-\delta) E_t \frac{c}{q(u_{t+1}, v_{t+1})} \right) + \left(w_{t,t} + \sum_{\tau=t+1}^{\infty} (\beta(1-\delta))^{\tau-t} E_t (w_{t,\tau} - w_{t+1,\tau}) \right).$$

Equation (3) is intuitive: firms create jobs in period t as long as the marginal benefit from adding a worker exceeds the user cost of labor. With free entry, the firms will enter the labor market until the net benefit is driven to 0. At that point the decision to add a worker exactly equates the current benefit from a worker with the current cost and the present value of the expected future cost resulting from the current decision.

Equation (3) is crucial to understanding the concept of allocational price of labor. Given the dynamics of the wage component of the user cost, the dynamics of individual wages do not have a direct impact on the dynamics of vacancies and unemployment. The dynamics of the wage component of the user cost are what matters for the dynamics of firms' job creation activity.

The result of Proposition 1 illustrates the restrictions imposed by a search and matching model. It allows bringing together the data on unemployment-vacancy ratio and the statistics from wage data that are relevant for the job creation decision. To examine these restrictions, I rewrite equation (3) in terms of elasticities with respect to productivity.

[4]See the appendix for the proof.

First, consider a steady state. Total differentiation of (3) and rearrangement yields:

$$1 = \varepsilon_{UC^V,z}\frac{UC^V}{z} + \varepsilon_{UC^W,z}\left(1 - \frac{UC^V}{z}\right),$$

where $\varepsilon_{UC^V,z}$ and $\varepsilon_{UC^W,z}$ are the elasticities of the vacancy component and the wage component, respectively, with respect to productivity evaluated at steady state values. Then, the vacancy component share in productivity is

$$\frac{UC^V}{z} = \frac{1 - \varepsilon_{UC^W,z}}{\varepsilon_{UC^V,z} - \varepsilon_{UC^W,z}}. \tag{4}$$

In steady state, $UC^V = \frac{c}{K\theta^{-\alpha}}(1-\beta(1-\delta)), UC^V > 0, UC^W > 0$, which yields $\varepsilon_{UC^V,z} = \alpha\varepsilon_{\theta,z}$, where $\varepsilon_{x,z}$ denotes the elasticity of x with respect to productivity, and implies $0 < \frac{UC^V}{z} < 1$. Thus, using $\varepsilon_{UC^V,z} = \alpha\varepsilon_{\theta,z}$, the following must hold from (4):

$$0 < \frac{1 - \varepsilon_{UC^W,z}}{\alpha\varepsilon_{\theta,z} - \varepsilon_{UC^W,z}} < 1. \tag{5}$$

Condition (5) holds if 1) either $\varepsilon_{UC^W,z} < 1 < \alpha\varepsilon_{\theta,z}$, or 2) $\alpha\varepsilon_{\theta,z} < 1 < \varepsilon_{UC^W,z}$. Given the value of the elasticity of the vacancy-unemployment ratio, $\varepsilon_{\theta,z}$, of 7.56 (see, for example, Rudanko (2009), Pissarides (2009)) and a range of values for α that can be found in the literature, $[0.235, 0.72]$, one obtains $\alpha\varepsilon_{\theta,z} > 1$. Thus, for (5) to hold, one should have $\varepsilon_{UC^W,z} < 1$. In Section 4, I provide the estimate of $\varepsilon_{UC^W,z}$.

The analogous argument carries over to the stochastic case.[5] Specifically, assume that z_t follows the $AR(1)$ process in logs with autocorrelation coefficient ρ and normal innovations. Then the elasticity of the vacancy-unemployment ratio with respect to productivity takes the form $\varepsilon_{UC_t^V,z_t} = \alpha\varepsilon_{\theta,z}x_t$,where $x_t > 1$ provided $\rho < \alpha\varepsilon_{\theta,z}$.

Similarly as above, I obtain

$$0 < \frac{1 - \varepsilon_{UC^W,z}}{\alpha\varepsilon_{\theta,z}x_t - \varepsilon_{UC^W,z}} < 1. \tag{6}$$

[5]It can be shown that $\Pr(UC_t^V > 0) > 0.99$, $\Pr(UC_t^W > 0) > 0.99$, which implies $0 < \frac{UC_t^V}{z} < 1$. See the appendix for the details and the derivation of the expression for $\varepsilon_{UC_t^V,z_t}$, given the empirical volatility and autocorrelation of θ.

Condition (A7) holds if 1) either $\varepsilon_{UC^W,z} < 1 < \alpha\varepsilon_{\theta,z}x_t$, or 2) $\alpha\varepsilon_{\theta,z}x_t < 1 < \varepsilon_{UC^W,z}$. Since $x_t > 1$ and $\alpha\varepsilon_{\theta,z} > 1$, for (5) to hold, one should have $\varepsilon_{UC^W,z} < 1$.

Equations (5) and (A7) demonstrate a trade-off between the elasticity of the wage component and the elasticity of the vacancy-unemployment ratio imposed by the free entry condition of the model. Because both the wage component and the vacancy component of the user cost of labor covary positively with productivity, there is a trade-off in the degree of the response of UC_t^W and UC_t^V to changes in productivity. Notice that equations (5) and (A7) are derived without evoking a particular surplus division rule or wage setting. These restrictions allow examining whether the model can potentially generate empirical elasticities $\varepsilon_{\theta,z}$ and $\varepsilon_{UC^W,z}$. Since, as mentioned above, the conventional values for α and empirical estimates of $\varepsilon_{\theta,z}$ deliver the value of $\alpha\varepsilon_{\theta,z}$ that exceeds 1, the answer depends on the value for $\varepsilon_{UC^W,z}$.

In the next section, I find that the estimate of the elasticity of the wage component of the user cost of labor with respect to productivity, $\varepsilon_{UC^W,z}$, is above 1.5. Thus, since both $\alpha\varepsilon_{\theta,z}$ and $\varepsilon_{UC^W,z}$ exceed 1, the model cannot generate both the empirical elasticities of the vacancy-unemployment ratio and the wage component of the user cost of labor. This leads to the conclusion that if the model is to match the volatility of quantities – vacancies and unemployment – and the relevant measure of prices – the wage component of the user cost of labor – then the solution for the unemployment volatility puzzle cannot come from wage-setting. The relevant measure of the price of labor in the data is not rigid.

Additionally, it is worth noting that using the wage component of the user cost of labor as a calibration target as opposed to the wage or the wage of the newly hired workers helps to isolate the quantitative test of the search and matching framework from the issue of the wage setting mechanism. Consider, for example, a search and matching model with Nash bargaining of wages period by period. With such wage setting the wage component of the user cost of labor is identically equal to individual wages and wages of newly hired workers. A researcher faces a menu of three calibration targets when bringing the dynamics of wages from such a model to the data: the dynamics of the individual wages of all workers, the dynamics of wages of newly hired workers, and the dynamics of the wage component of the user cost of labor. The choice would not be crucial for the test if the dynamics of the three statistics were the same. However, this is not the case. As the results in the next section reveal, the wage component of the user cost of labor is more cyclical than the wages of newly hired workers, and wages of newly hired workers are more cyclical than wages of all workers. Calibration of the dynamics of wages from such a model to the dynamics of individual wages in the data is the joint test of the wage setting and search and matching framework and may lead to inaccurate conclusions about the quantitative performance of the search and matching framework. With wage bargaining period by period, the dynamics of wages from the model should be calibrated to dynamics of the wage component of the user cost in the data.

IV. Cyclicality of the Wage Component of the User Cost of Labor in the Data

This section contains the main empirical result of the paper. I construct the empirical counterpart of the wage component of the user cost of labor in the data and measure its cyclicality, which is a proportional response in wages to a one percentage change in the unemployment rate.

A. History Dependence in Wages

Empirical studies of the cyclicality of individual wages provide both direct and indirect evidence for the dependence of wages on the history of the unemployment rates from the start of the job.

Direct evidence of the history dependence in wages is presented in Beaudry and DiNardo (1991). They test whether a contractual wage model is more consistent with the formation of wages than a spot market model. In addition to the contemporaneous unemployment rate in the regressions of real wages on unemployment, they also include the unemployment rate at the start of the job and the minimum unemployment rate since the start of the job. Using PSID data for 1976-84 and CPS data for 1979 and 1983, they find that the effect of the minimum unemployment rate since the start of the job dominates the effects of the other two unemployment rates.

The indirect evidence comes from the differences in the cyclicality of wages of newly hired workers and workers who do not change jobs, i.e., job stayers. Using NLS data, 1966-80, Bils (1985) concludes that there are substantial differences in the cyclicality of wages of workers continuously employed at the same job and those of workers who are newly employed. Numerous studies since, using different data sets, also find that the cyclicality of job changers is substantially higher than that of job stayers (among them Solon, Barsky, and Parker (1994) using the PSID; Shin (1994) using the NLS; and Carneiro, Guimaraes, and Portugal (2009) using matched data). The empirical evidence on the cyclicality of individual wages is summarized in Pissarides (2009). He reports that the general consensus in the literature on the cyclicality of the wages of newly hired workers is -3.0%, while the cyclicality of the wages of job stayers is approximately -1%. This evidence suggests that the wages of newly hired workers are adjusted to reflect the economic conditions at the time of hiring. However within employment relationships, wages are smoothed and respond only weakly to changes in economic conditions.

These empirical findings lead to the following conclusions: (1) wages exhibit dependence on the past history of unemployment, and (2) wages of newly hired workers are more pro-cyclical than wages of workers who remain on the job for some time. In turn, these results imply that wage alone does not summarize the wage commitment a firm takes upon hiring a worker. The relevant measure of a cost of a worker to a firm should take into account both the wage at the time of hiring and the effect of the economic conditions at the time of hiring on future wages.

B. Data

I use the National Longitudinal Survey of Youth (henceforth NLSY), 1978-2004. The survey collects information on work histories of a nationally representative sample of young individuals who were between 14 and 21 years of age in 1979 when the first interview was taken.

I focus on the cross-sectional sample that represents the non-institutionalized civilian population and further restrict my analysis to males. This restriction is typical in other empirical studies of wage cyclicality (see, for example, Beaudry and DiNardo (1991); Shin (1994)). Hence, I work with the following sub-samples, as defined in the NLSY: 1 = cross-sectional white males, 3 = cross-sectional black males, 4 = cross-sectional Hispanic males, 5 = cross-sectional white females, 7 = cross-sectional black females, 8 = cross-sectional Hispanic females. The following sub-samples are not included in the analysis: cross-sectional poor white males (2), cross sectional poor white females (6), all supplemental (9-14) and military sub-samples (15-20).

The data set is suited for the purposes of this study because it separately records wages and other job characteristics for up to five jobs that an individual might hold between two consecutive interviews. By tracking individuals over the years, I can isolate the individual-specific fixed effects. In addition, if a worker simultaneously held more than one job, the NLSY79 kept a separate record for each job, as opposed to PSID data that report the average wage in such cases.

On the other hand, the data contain information on individuals at the early stages of their labor market experience. Because jobs taken at the early stages of an individual's labor experience may be predominantly seasonal or temporary, these job changers may disproportionately affect the wage cyclicality. To alleviate this problem, I restrict the observations included in the wage equation to observations of individuals who started a job at the age of 16 and older, were 20 years old and older at the time of the observation, and reported being out of school. When I use workers' fixed effects in the estimation, the sample is restricted to the workers having more than one observation.

Wage is an hourly pay variable constructed by the NLSY. I deflate wages using the annual CPI index of the year the observation refers to. Unemployment rate is the annual, national, civilian unemployment rate for ages 16+ obtained from the Bureau of Labor Statistics. The contemporaneous unemployment rate is the annual unemployment rate of the calendar year when the respondent reported last working at the job.

C. Estimation Procedure

Given the constant separation rate, δ, and the discount factor, β, the wage component, UC_t^W, is

$$UC_t^W = w_{t,t} + E_t \sum_{\tau=t+1}^{\infty} (\beta(1-\delta))^{\tau-t}(w_{t,\tau} - w_{t+1,\tau}), \tag{7}$$

where w_{t_1,t_2} is a wage in period t_2 at the job that started in t_1.

The cyclicality of the wage component of the user cost of labor is the expected proportional change in the wage component, UC_t^W, in response to a unit change in the unemployment rate, U_t. It can be measured as the projection of $\ln UC_t^W$ on U_t:

$$\gamma = \frac{cov(\ln UC_t^W, U_t)}{var(U_t)}. \tag{8}$$

Let UC_t^{WR} be the realized, ex post value of the wage component, then

$$UC_t^W = E_t(UC_t^{WR}).$$

Given the standard rational expectation argument, the cyclicality of the wage component can be calculated as[6]

$$\gamma = \frac{cov(\ln UC_t^{WR}, U_t)}{var(U_t)}. \tag{9}$$

Now the task is to construct an empirical counterpart of UC_t^{WR} and to estimate the cyclicality in (9). The wage component is not directly observed in the data; hence, I construct an empirical counterpart of UC_t^{WR}, $\widehat{UC_t^{WR}}$, from individual wages and turnover. Calculations of the wage component requires two series of wages for each t in the sample period — a series of wages to be paid to a worker hired in time t and a series of wages to be paid to an identical worker hired the next period. The expression for the wage component assumes infinitely lived

[6]Define a random variable ε_t such that $UC_t^{WR} = UC_t^W \varepsilon_t$, where ε_t is independent of the variables in the information set of a firm in t. Then $cov(\ln UC_t^{WR}, U_t) = cov(\ln UC_t^W, U_t) + cov(\ln \varepsilon_t, U_t)$. Because the information set of a firm in t contains the contemporaneous unemployment rate, U_t, the last term is 0. Then $cov(\ln UC_t^{WR}, U_t) = cov(\ln UC_t^W, U_t)$. This yields expression (9) for the cyclicality of the wage component.

firms and workers; thus the calculations involve infinite sums. I deal with the last issue by truncating the calculations of the sum at different time horizons and checking the sensitivity of the calculated cyclicality indicator to its truncation horizon.

To obtain the series of the wage component, I proceed as follows.

Step 1.

First, I specify the following model for wages in year t of worker j hired in period t_0:

$$\ln w_{j,t_0,t} = c + \sum_{\tau=1}^{T} \gamma_\tau^S D_{\tau,t_0}^S + \sum_{\tau=1}^{T} \gamma_\tau^C D_{\tau,t}^C + \rho t + \Psi X_{j,t} + \alpha_j + \varepsilon_{j,t}, . \tag{10}$$

where D^S and D^C are two sets of time dummy variables that assume values as follows: for the job that starts in t_0 and is observed in t, $D_{\tau,t_0}^S = I(\tau = t_0)$ and $D_{\tau,t}^C = I(\tau = t)$, where $I(\cdot)$ is an indicator function and T is a sample period length in years. The data span the sample period from 1978 to 2004; thus, there are 26 time dummies in each set, excluding the omitted base categories. $X_{j,t}$ is a quadratic in experience; α_j is a worker-specific individual fixed effect and $\varepsilon_{j,t} \sim N(0, \sigma_\varepsilon^2)$. The task here is to obtain the expected wage for each $\{t_0, t\}$ pair in the sample period, conditional on worker characteristics. I estimate equation (10) using OLS regression weighting each observation by sampling weights and controlling for worker-specific fixed effects.

Step 2.

Second, using the coefficient estimates from (10), I calculate the fitted values for wages, $\widehat{w_{t_0,t}}$, for all t_0 and $t : t_0, t = \{1, T\}, t_0 \leq t$:

$$\widehat{w_{t_0,t}} = \exp\left(\widehat{const_w} + \widehat{\rho}\bar{t} + \widehat{\Psi}\overline{X} + \widehat{\gamma_{t_0}^S} + \widehat{\gamma_t^C}\right),$$

where $\bar{t}$ and $\overline{X}$ are sample means. Note that $E_t(\widehat{w_{t_0,t}}) = w_{t_0,t} / \exp\frac{\sigma_\xi^2}{2}$. Assuming that $\sigma_\xi^2 = const$ and $\overline{X}$ is uncorrelated with the contemporaneous unemployment rate, the cyclicality does not depend on the actual values of $\bar{t}$, $\overline{X}$ and σ_ξ^2.

Step 3.

To obtain the series of separation rates, I proceed in two steps: first, I detrend the monthly separation rates; second, I estimate a linear probability model of the detrended monthly separation rates with a set of contemporaneous time dummies as explanatory variables. In the first step, I estimate a linear probability model with the dependant variable taking value 1 if a worker does not work for the same job in the next month and 0 otherwise. The explanatory variables are the quartic in the monthly trend. I subtract the value of a quartic in the trend multiplied by the estimated coefficients from the dependent variable and add the value of a quartic of a trend calculated at the mean multiplied by the estimated coefficients. In the second step, I run the constructed series on a set of contemporaneous time-dummies. Then, using the coefficient estimates on the set of contemporaneous dummies, I obtain fitted projections, $\widehat{\delta_t}$, for all $t : t = \{1, 324\}$.[7] I also obtain the series of the separation rates without detrending. In this case, I estimate the probit regression with the monthly separation rate as a dependent variable and a set of contemporaneous dummies as explanatory variables.

Step 4.

I truncate the horizon in calculating the second component of the realized wage component of the user cost (7), τ_{tr} to 7 years.[8] Truncation of the time horizon for calculation of UC_t^W can be justified by two considerations. First, the discount factor, which includes the turnover rate and the real interest rate, increases. This, in turn, decreases the weight of the terms far in the future. Second, if, for example, the model behind the dependence of wages on the history of unemployment rates is as in Thomas and Worrall (1988) and the unemployment rate follows the mean-reverting process, then wages in the employment relationships that started in different years but that have lasted long enough to experience similar episodes of minimum and maximum unemployment rates will be the same. In that case, the terms in brackets in (7) will be equal to 0 for all τ higher than some high enough τ'.

Finally, I calculate an empirical counterpart of the realized wage component using the constructed series $\widehat{w_{t_1,t_2}}$ and the truncated horizon, τ_{tr}. I set a discount factor, β, at 0.9569

[7] For the robustness check, I have also repeated this procedure with the probit in the first step instead of a linear probability model. See Kudlyak (2007) for more details.

[8] Given the truncation period 7 years and the sample period from 1978 to 2004, the wage component of the user cost of labor can be calculated for 20 years, from 1978 to 1997. This number of observation is typical for the papers on the cyclicality of wages that employ a two-step estimation procedure (Solon, Barsky and Parker (1994), Devereux (2001)). For example, Devereux (2001) reports 22 observations in the second-stage regression.

and annual separation rate $\delta = 0.26$, calculated from the monthly separation rate in Kudlyak (2007). To obtain the cyclicality of the wage component, I regress the logarithm of the constructed realized wage component of the user cost of labor on the unemployment rate and a time trend. The reported cyclicality is the coefficient on the unemployment rate multiplied by 100%.

D. Main Empirical Result

The main results are presented in Table 1. The first row presents the estimates of the cyclicality of the wage component constructed using a constant separation rate. The next rows present the cyclicality of the wage component constructed using the separation rates that depend on the contemporaneous period using the procedure described in Step 3 above for the detrended and non-detrended series of the separation rates, respectively. The cyclicality of the wage component in Table 1 is calculated for the period 1978-1997. The cyclicality of the price of labor calculated using non-detrended series is -4.92%, which implies that as the unemployment rate increases by one percentage point, the constructed price of labor on average decreases by 4.92%.

Table 2 presents the cyclicality results for the constructed wage component truncated at 5, 7 and 9 periods, respectively. For comparison purposes, for all truncation horizons the cyclicality is calculated for 18 periods for which the data on the price of labor in all the cases is available. The last row of Table 2 contains the results for the cyclicality of the price of labor that allows the separation rates to depend on the history of unemployment rates from the start of the job as discussed in the robustness section below. I find that accounting for the separation rate that depends on the history of the unemployment rate from the time of hiring does not change the main empirical results on the cyclicality of the wage component.

For comparison, Table 3 contains the results of the estimated coefficient on the unemployment rate in the wage equation. In the sample of newly hired workers (tenure less than 1 year), the coefficient on the contemporaneous unemployment rate is -3.10%. In the sample of workers who stay at the job for two years and longer, the coefficient on the contemporaneous unemployment rate is 0.29% and is not statistically significant. The results of the estimation indicate that the cyclicality of the wage component is much higher than the cyclicality of individual wages.

The intuition behind the cyclicality of the wage component of the user cost of labor is as follows. Consider a firm that hires a worker toward the end of a recession, when the unemployment rate is high, as opposed to hiring later, when the unemployment rate is expected to return to its lower level. Empirical findings show that wages of newly hired workers are procyclical. Hence, when hiring currently, a firm pays a comparatively lower hiring wage. Once workers are hired, their wages are shielded from the effect of contemporaneous labor market conditions and bear the effect of the past unemployment rates. This argument comes from the empirical findings that wages of newly hired workers are more cyclical than the wages of workers who do not change jobs. Thus, by hiring currently, a firm locks in a worker to a stream of wages that is expected to be lower than the stream of wages to be paid to an identically productive worker hired under more favorable economic conditions. As a result, a per period cost of a worker to a firm in terms of wage payments, the wage component of the user cost of labor, is even lower than the already low hiring wage because the wage component also reflects comparatively low future expected wages. The opposite is true when a worker is hired at the peak of the cycle, when the unemployment rate is low but is expected to rise. Then the wage component is higher than the hiring wage. Thus, the procyclical hiring wage and the lock-in cause the wage component to be more procyclical than the hiring wage.

From the estimation results, I conclude that the cyclicality of the wage component of the user cost of labor is more than -4.5%, which is substantially higher than the cyclicality of individual wages of all workers and also noticeably higher than the cyclicality of wages of newly hired workers.

E. Robustness

Estimation with Time-Varying Separation Rates.—There is empirical evidence that suggest that the unemployment rate at the time of hiring has a positive impact on separation rates. (Bowlus (1995)). To understand the effect the time-varying separation rates might have on the cyclicality of the wage component of the user cost of labor, suppose that the separation rates depend positively on the unemployment rate at the time of hiring.

Then, the workers who are hired when the unemployment rate is high tend to have shorter tenures. Once a worker is separated, a firm must hire a new one to fill the position. But, if the labor market conditions have improved, a new worker is offered a new present discounted value of wages that is expected to be higher than the value paid to the previous employee. Thus, higher separation rates might weaken the lock-in to the initial labor market conditions. To estimate whether this effect is quantitatively important for the cyclicality of the wage component of the user cost, I examine the cyclicality of the wage component that allows the separation rates to depend on the history of unemployment rates from the start of the job.

To incorporate the time varying separation rate,[9] I define the wage component of the user cost of labor taking into account that 1) the probability of separation depends on the period the worker was hired and the contemporaneous period and 2) whenever a worker separates, a firm must rehire a worker to replace the separated one at a new wage agreement or contract. In this context, hiring a worker in t can be thought of as creating a position in period t that will be filled with probability 1 onwards. The wage component of the user cost of labor in period t is the difference between the expected present discounted value of wages paid at the position opened in period t and $t+1$. These two options give the same expected employment levels – one – in all future periods. Therefore, the difference between them gives the implicit price of the services of one worker during the current period. The exact expression for the wage component of the user cost of labor is slightly more complicated than the one given in (2), thus the derivations are delegated to the appendix.

To estimate the cyclicality of the wage component with separation rates that depend on the history, I construct the realized wage component of the user cost of labor using the procedure similar to the one described above. The difference is that to obtain an estimate of the series of monthly separation rates I estimate a linear probability model of the detrended monthly separation rates with two sets of time dummies as explanatory variables: one set of time dummies corresponds to the year the job starts and another set of dummies corresponds to the contemporaneous year. Then, I use monthly fitted projections to obtain annual separation rates. I proceed to estimate the cyclicality as described in the previous

[9] I estimate the response of the separation rates to the unemployment rate at the time of hiring and to the contemporaneous unemployment rate. I find that the unemployment rate at the start of the job has a slight positive impact on the probability of future separation. See Kudlyak (2007) for details.

subsection. As a robustness check, I also construct the wage component of the user cost of labor that allows the separation rate to depend only on the contemporaneous period, δ_t.

The results of the estimation are presented in Table 2. As can be seen from the table, allowing the separation rate to depend on the history of economic conditions from the time of hiring does not change the main result on the cyclicality of the wage component of the user cost. The estimated cyclicality is still larger than -4.5%.

History Dependence in Wages Controlling for Industry.—To check whether the history dependence in wages is driven by a set of industries, I estimate an equation for wages as a function of the past and current unemployment rates and controlling for industry. The results indicate that the magnitude of the dependence of wages on the history does not change if the industry is controlled for.

The estimated equation is similar to the one estimated by Beaudry and DiNardo (1991). In addition to a set of individual controls, I include three measures of unemployment rates: the unemployment rate from the time a worker is hired, the minimum unemployment rate experienced by a worker while on the job, and the contemporaneous unemployment rate. Consistent with the findings in Beaudry and DiNardo (1991), I find that once the effect of the minimum unemployment rate is not restricted to zero, the effect of the contemporaneous unemployment rate decreases substantially. In particular, a one percentage point increase in the unemployment rate experienced during a worker's tenure leads to more than a 3% decrease in wages. The effect of the contemporaneous unemployment rate is close to zero. When, in addition to education, experience, tenure and union variables I add industry dummies, marital status, and region of residence, the coefficients on the minimum, initial, and contemporaneous unemployment rates remain almost unchanged. These results are in Table 4. From the estimation, I conclude that wages are affected by the history of unemployment rates experienced by a worker from the time of hiring. Once the history is considered, the effect of the contemporaneous unemployment rate is comparatively small both statistically and economically, with or without industry dummies.

V. Implications for the Free Entry

Given the semi-elasticity of UC^W with respect to unemployment, $\frac{d\ln UC^W}{du}$, the elasticity of UC^W with respect to productivity is calculated as follows:

$$\frac{d\ln UC^W}{d\ln z} = \frac{d\ln UC^W}{du}\frac{du}{d\ln z},$$

where $\frac{du}{d\ln z}$ is the change in unemployment in response to a percentage change in productivity. Pissarides (2009) provides the following estimates of $\frac{du}{d\ln z}$: -0.34 for the period 1948-2006 and -0.49 for the period 1970-1993. Combining these estimates with the estimates of the semi-elasticity of the wage component of the user cost of labor of -4.5% above, yields the elasticity of the wage component with respect to productivity of 1.530 and 2.205, respectively.

This elasticity and the empirical elasticity of the vacancy-unemployment ratio of 7.56 translates into the elasticity of the wage component and vacancy component of the user cost of labor each in excess of 1. The implications of the elasticity of the wage component of the user cost of labor being greater than 1 for the free entry conditions in the model has been already discussed in Section 3.C. Reiterating that discussion here, the restrictions imposed by the model on the data do not hold.

A. Illustration with Elasticities

To illustrate the findings, I consider the search and matching model as described in Section 3 with two additional assumptions: 1) workers are risk neutral and 2) at the time the match is formed, the surplus between a worker and a firm is divided by a generalized Nash bargaining with constant bargaining shares. Note that there are different wage settings that will consequently deliver different passes of individual wages within employment relationship that encompass this surplus division rule at the beginning of the match. One example of such a wage setting is Nash bargaining period by period in all matches. Another example is a constant wage within the employment relationship.

It can be shown that, given linear utilities for a worker and a firm, models with different wage settings, in which the surplus at the beginning of the match is divided using a constant shares Nash bargaining rule, deliver identically equal the wage component of the user cost of labor and (as is evident from Proposition 1) the same allocations (See also results in Table 9). Thus, to analyze the implications of Proposition 1 for a model with such a surplus division, it is suffice to analyze a model with one of the wage settings with such a surplus division rule at the beginning of a match. A convenient model to analyze is the model with Nash bargaining of wages period by period in all matches, which is widely used in the literature. Thus, in addition to the two additional assumptions above I add the following: 3) the wage is set by Nash bargaining period by period between a worker and a firm with a constant bargaining share of a worker η.

With Nash bargaining period by period, the wage depends only on the contemporaneous economic conditions, $w_{t_1,\tau} = w_{t_2,\tau} = w_\tau$ for all t_1, t_2, τ. Then, the last term in brackets in equation (2) is 0. It implies that with Nash bargaining period by period, $UC_\tau^W = w_\tau$ for all τ. This conveniently allows deriving the closed-form expression for UC_τ^W in the model: first, I derive w_τ and, then, set $UC_\tau^W = w_\tau$. In a steady state the elasticity of the vacancy-unemployment ratio with respect to productivity is

$$\varepsilon_{\theta z} = \frac{1}{1 - b/z} \frac{1 - \beta(1 - \delta - \eta\mu)}{\alpha - \beta(\alpha - \delta\alpha - \eta\mu)} \tag{11}$$

and the elasticity of wages is:

$$\begin{aligned} \varepsilon_{wz} &= \frac{\eta}{\eta(1 - \beta(1 - \delta - \mu)) + (1 - \eta)\frac{b}{z}(1 - \beta(1 - \delta))} \cdot \\ &\quad ((1 - \beta(1 - \delta - \mu)) + \frac{(1 - \eta)(1 - \alpha)\beta\mu(1 - \beta(1 - \delta))}{\alpha - \beta(\alpha - \delta\alpha - \eta\mu)}), \end{aligned} \tag{12}$$

where η is a worker's bargaining power, b is the unemployment benefit, and μ is a steady state value of job finding rate.

It has been discussed that the two parameters are crucial for the volatilities of the vacancy-unemployment ratio and wages: the unemployment benefit and a worker's bargaining power (see, for example, Hornstein, Krusell and Violante (2005)). Without replicating the analysis here, I use equations (11) and (12) to derive the expressions for b/z as a function of $\varepsilon_{\theta z}$ and as a function of ε_{wz}, respectively. I obtain the following equations:

$$\frac{b}{z} = (1 - \frac{1}{\varepsilon_{\theta z}} \frac{1-\beta(1-\delta-\eta\mu)}{\alpha - \beta(\alpha - \delta\alpha - \eta\mu)}). \tag{13}$$

$$\begin{aligned} \frac{b}{z} &= \frac{\eta}{1-\eta} \frac{1}{\varepsilon_{wz}} \frac{1}{1-\beta(1-\delta)} ((1-\beta(1-\delta-\eta\mu))(1-\varepsilon_{wz}) + \\ &\quad \frac{(1-\eta)(1-\alpha)\beta\mu(1-\beta(1-\delta))}{\alpha-\beta(\alpha-\delta\alpha-\eta\mu)}) \text{ if } \frac{b}{z} \neq 0 \end{aligned} \tag{14}$$

Now I can plot two functions of $\frac{b}{z}$: $\frac{b}{z}(\eta|\varepsilon_{\theta z})$ and $\frac{b}{z}(\eta|\varepsilon_{wz})$ given values for $\varepsilon_{\theta z}$ and ε_{wz} and a set of parameters $(\alpha, \beta, \delta, \mu)$. The intersection of the two functions, if one exists, gives pairs of $(\frac{b}{z}, \eta)$ that deliver targeted values of $\varepsilon_{\theta z}$ and ε_{wz}.

I obtain the following parametrization for the quarterly model: $\beta = \frac{1}{1+0.012}$; $\delta = 0.10$; $\mu = 1.35$ (Hornstein, Krusell and Violante (2005), Shimer (2005)). Since literature provides a range of values for the elasticity of the matching function with respect to unemployment, α, I provide results for three different values of α: $a = 0.235$ (Hall (2005)), $a = 0.72$ (Shimer (2005)), and $a = 0.5$ which is the value in the range proposed by Pissarides and Petrongolo (2000). I set $\varepsilon_{\theta z} = 7.56$ (Rudanko (2009), Pissarides (2009)).

It remains to specify the value of ε_{wz}. As shown above, in the model with wage bargaining period by period, wages are equal across all matches in each period. Thus, the average wage equals wages of newly hired workers and equals the wage component of the user cost of labor. However, in the data those three statistics from wages are different. In particular, in the literature on the cyclicality of wages, the wages of workers who stay with employer for some time (job stayers) respond by -1% (up to -1.5%) to one percentage point increase in the unemployment rate, the wages of newly hired workers (job changers) respond by -3%. Rudanko (2009) and Hagedorn and Manovskii (2008) summarize the elasticity of wages of all workers at 0.5 and 0.47, respectively. Pissarides (2009) summarizes the implied elasticity of wages of newly hired workers to be from 1.02 to 1.47. In this paper I find the elasticity of the wage component of the user cost of labor to be from 1.53 to 2.20, based on the cyclicality of -4.5%.

Thus, it is important how the calibration target for ε_{wz} is chosen in the data. Since in the model with Nash bargaining period by period all three responses above are the same, this wage setting cannot be used to describe the behavior of individual wages in the data. To sidestep the question of what the exact wage setting is within employment relationships, one can calibrate ε_{wz} in the model to the elasticity of the wage component of the user cost of labor.

In Figure 1, I plot $\frac{b}{z}(\eta|\varepsilon_{\theta z})$ and $\frac{b}{z}(\eta|\varepsilon_{wz})$, given $\varepsilon_{\theta z} = 7.56$ and $\varepsilon_{wz} = 1.5$. The graphs illustrate two points. First, as stated in the conclusion reached above: given the specified targets for $\varepsilon_{\theta z}$ and ε_{wz} and a set of parameters $(\alpha, \beta, \delta, \mu)$ as described above, two functions $\frac{b}{z}(\eta|\varepsilon_{\theta z} = 7.56)$ and $\frac{b}{z}(\eta|\varepsilon_{wz} = 1.5)$ do not have points in common. Thus, the model cannot generate both the elasticity of the vacancy-unemployment ratio of 7.56 and the elasticity of the wage component of the user cost of labor in excess of 1. Second, given the wage setting, the model can generate the elasticity of the wage component of the user cost of labor equal to 1.5 for only a small set of parameter values. In particular, for $\alpha = 0.72$, there are no admissible values of the pair $(\frac{b}{z}, \eta)$ that can deliver $\varepsilon_{wz} = 1.5$, given the values for (β, δ, μ) (in this case, $\frac{b}{z}$ is negative if $0 < \eta < 1$).

From Figure 1 one can also see that the elasticity of the vacancy-unemployment ratio is very sensitive to the value of $\frac{b}{z}$ and less sensitive to η; and that the empirical value of $\varepsilon_{\theta z}$ requires a high value of $\frac{b}{z}$ (Hornstein, Krusell and Violante (2005), Hagedorn and Manovskii (2008)).

To illustrate conclusions reached in Section 3.C, I plot $\frac{b}{z}(\eta|\varepsilon_{\theta z})$ and $\frac{b}{z}(\eta|\varepsilon_{wz})$ for $\varepsilon_{wz} < 1$. In particular, in Figure 2, I plot $\frac{b}{z}(\eta|\varepsilon_{wz})$ for $\varepsilon_{wz} = 0.5$, which is close to the targets used in Hagedorn and Manovskii (2008) and Rudanko (2009). As can be seen from Figure 2, there exists a pair of $(\frac{b}{z}, \eta)$ that can deliver $\varepsilon_{\theta z} = 7.56$ and $\varepsilon_{wz} = 0.5$. However, as discussed in Section 3.C and above, this calibration assumes a particular wage setting mechanism that lacks support in the data.[10] Focusing on the cyclicality of individual wages might lead to a misleading assessment of the quantitative behavior of the model if the wage setting, which is not a central feature of the model, is specified incorrectly.

[10]See also a review in Pissarides (2009) and a discussion in Martins, Solon and Thomas (2009).

VI. Example: Cyclicality of the User Cost of Labor in Models with Specific Wage Settings

In this section, I examine the cyclicality of the components of the user cost of labor and wages in search and matching models with alternative wage-settings and stochastic productivity.

A. Description of the Models

I consider the environment as described in Section 3 with the assumption that workers are risk averse and an assumption about wage determination. First, I consider models where individual wages depend on the history of economic conditions from the start of the job. Consequently, the wage component of the user cost of labor differs from wage. Second, I consider a model where wages are bargained period by period in all matches. In this case, the wage component of the user cost of labor equals the wage at the time of hiring, which in turn equals the average wage.

In the models with history dependence of wages, wages are the outcome of the implicit self-enforcing contracts between a worker and firm as in Thomas and Worrall (1988). In the models, risk-neutral firms insure risk-averse workers, who do not have access to capital markets, against fluctuations in consumption due to fluctuations in earnings. Three types of contracts are distinguished based on different degrees of commitment: full commitment contracts, contracts with lack of commitment from the worker's side and full commitment from the firm's side, and contracts with lack of commitment from both the worker's and the firm's sides. In the original Thomas and Worrall (1988) environment, workers who renege on the contract are prohibited from entering any contractual arrangements in the future and are bound to trade their labor services at the spot market wage. In the current environment, once unemployed, workers search and enter contractual arrangements as soon as they find a new match. Both firms and workers face search and matching frictions. These frictions influence the value of the outside option through the probability of finding a new match.

Firms open vacancies with associated employment contracts and workers direct their search to the contracts. The vacancies opened with the associated contract σ and the unemployed workers searching for contract σ constitute a labor market with an associated market tightness θ_σ. A contract is a state-contingent sequence of wages that delivers a certain promised value to the worker. Equilibrium contracts are limited to efficient optimal contracts. To ensure a unique contract in equilibrium, I follow Rudanko (2009) and impose the following equilibrium refinement: there does not exist an efficient self-enforcing contract $\sigma'(z)$ and an associated labor market with tightness $\theta_{\sigma'}(z)$ such that the net surpluses from search for a worker and for a firm are at least as much as under $\sigma(z)$ and $\theta_\sigma(z)$ and, for one party, it is strictly more.

Thomas and Worrall (1988) and Rudanko (2009) show that in such an environment, for any history (z^t, z_{t+1}), there exists a $w_{\min}(z_{t+1})$ and $w_{\max}(z_{t+1})$, $w_{\min}(z_{t+1}) \leq w_{\max}(z_{t+1})$, such that the contract wage at $t+1$ is 1) in the contract with full commitment: $w(z^t, z_{t+1}) = w(z^t)$; 2) in the contract with lack of commitment from the worker and full commitment from the firm: $w(z^t, z_{t+1}) = w(z^t)$ if $w_{\min}(z_{t+1}) \leq w(z^t)$ and $w(z^t, z_{t+1}) = w_{\min}(z_{t+1})$ if $w(z^t) < w_{\min}(z_{t+1})$; and 3) in the contract with two-sided lack of commitment: $w(z^t, z_{t+1}) = w_{\max}(z_{t+1})$ if $w(z^t) > w_{\max}(z_{t+1})$, $w(z^t, z_{t+1}) = w(z^t)$ if $w_{\min}(z_{t+1}) \leq w(z^t) \leq w_{\max}(z_{t+1})$, and $w(z^t, z_{t+1}) = w_{\min}(z_{t+1})$ if $w(z^t) < w_{\min}(z_{t+1})$. Thus, whenever possible, the optimal contract offers a constant wage. However, in the contracts with lack of commitment, if the value of the outside option exceeds the value under the contract, the wage is adjusted to prevent reneging.

In addition to the contracting environments above, I also consider a wage setting where wages are determined by bargaining period by period. Every period within employment relationship wages are determined by the following rule:$\frac{W(z)-U(z)}{u'(w(z))} / \ J(z) = \frac{\eta}{1-\eta} \forall z \in Z$,where $W(z)$, $U(z)$ and $J(z)$ are values for an employed worker, an unemployed worker, and a firm with filled vacancy, respectively, and η is a worker's bargaining power.

This condition is well known in the literature: the share of the surplus that an agent obtains from a productive match corresponds to her bargaining power. If workers are risk neutral, then it describes generalized Nash bargaining period by period over total surplus as in the canonical search and matching model (see, for example, Pissarides (1985)). In the appendix, I specify the firm's optimization problem and define the equilibrium in the models described above. Rudanko (2009) provides an excellent treatment of Thomas and Worrall

(1988) contracts in the search and matching model; thus, the reader is relegated to that paper for the details.

B. Quantitative Results I

The parameters of the stochastic process for productivity shocks can be calibrated outside of the models.[11] Then, the only parameter that requires calibration within a model is the cost of posting a vacancy, c, which I calibrate to match the mean monthly job-finding rate, $E(\mu) = 0.45$. The model period is one month. The adopted parameters are reported in Table 5. The discount factor is 0.9960, which corresponds to the annual discount rate of 4.88%. The monthly separation rate is set to 0.034 (Shimer (2005)). I set the bargaining power of workers to equal α to preserve the mathematical equivalence of the competitive search and random search equilibria (Rudanko (2009)).

I obtain corresponding statistics for the models by simulating economies with each of the two different wage settings as follows. First, a vector of aggregate shocks, z, is generated, which is common to the economies. For the panel of 10,000 individuals, an initial employment status is drawn. Then, each period, the separation shock is drawn for each employed individual and his employment status is updated, and for each unemployed individual the job finding shock is drawn and his unemployment status is updated. Given the employment histories, individual wages are generated according to a model-specific wage setting. The first 4,000 periods of the simulated series are discarded; the statistics are based on the series from the last 636 periods. The results that follow are based on the simulations of the economies that have different wage settings but are hit by the exact same sequence of productivity shocks. The cyclicality of the series x is measured as a projection of the logarithm of the series on the unemployment rate, $cov(\ln(x), u)/var(u) * 100$, which is the

[11] To calibrate a stochastic process for productivity, I consider a three-state symmetric Markov process as in Rudanko (2009), $\mathbf{z} = [z_0 - \Delta, z_0, z_0 + \Delta]$, $\Delta > 0$, with the transition matrix (by row): $[\lambda, 1-\lambda, 0; 0.5(1-\lambda), \lambda, 0.5(1-\lambda); 0, 1-\lambda, \lambda]$. The variance of this process, σ_z^2, is $\frac{\Delta^2}{2}$ and the autocorrelation, ρ, is λ. The expected value, $E(z)$, is normalized to 1. The parameters Δ and λ are calibrated to match the standard deviation, 0.02, and autocorrelation, 0.878, of productivity per worker. These empirical targets are obtained from Shimer (2005), Table 1. To find Δ and λ, I draw the initial shock from a stationary distribution of z and, using the initial values for Δ and λ, generate monthly series of length $12T$, where T is the length of the time series in the data in years (from 1951 to 2003); aggregate by summing to obtain quarterly data; calculate the standard deviation and the autocorrelation of the logged quarterly series; and iterate until matching the calibration targets.

semi-elasticity of the series with respect to unemployment.

Table 6 reports the cyclicality of the individual wages of all workers and wages of newly hired workers, and the cyclicality of the components of the user cost of labor assuming log utility function for workers. The cyclicality of individual wages varies across models, with the wages being only mildly procyclical in the implicit contract models and as cyclical as the wage component of the user cost in the period by period bargaining model. Importantly, in models with contracts, the wage component of the user cost of labor is much more procyclical than the wages of newly hired workers. And the wages of newly hired workers are approximately 3 times as cyclical as the wages of all workers.[12] Similar results obtain in Table 7, which contains the results for the CRRA utility function with the coefficient of risk aversion of 3, and Table 11, which contains results for different values of b and different utility functions.

To understand why the cyclicality of the wage component of the user cost in the implicit contract model is higher than the cyclicality of wages at the time of hiring, recall the workings of this wage setting. The implicit contracts offer individual wages that are rigid during the employment relationship to insure workers against fluctuations in consumption. The wages of new hires adjust to reflect the worker's outside option value. Consequently, the wages of newly hired workers are more cyclical than the wages of all workers. For example, when the job finding rate is low, the hiring wage is relatively low. In addition, the wages in all subsequent periods in the employment relationship are relatively lower than the wages in the contracts, initiated under the more favorable economic conditions. The wage component of the user cost takes into account both the lower hiring wage and lower future wage payments. Hence, the wage component of the user cost is more procyclical than the wages of newly hired workers.

C. Quantitative Results II

Note from Table 6, that given $b = 0.70$, the implicit contract model generate a standard deviation of the vacancy-unemployment ratio of approximately 0.0620 and only slightly higher in the model with period by period bargaining, while the empirical counterpart is 0.382

[12]The cyclicality of individual wages in the models with contracts also depends on the number of states of the productivity process. However, qualitatively or quantitatively it does not have an impact on the main results.

(Shimer (2005)). Next, I use b to calibrate the models to match the cyclicality of the wage component of the user cost of labor estimated in Section 4 and check how much volatility of the vacancy-unemployment ratio the models generate.

In Table 8 the cost of opening a vacancy, c, and the consumption of unemployed, b, are calibrated to match the expected value of the job finding rate and the cyclicality of the wage component of the user cost, $cov(\ln(UC^W), u)/var(u) = -0.045$. As can be seen from the table, regardless of the wage setting, the models generate approximately $1/3 - 1/2$ of the empirical volatility of the vacancy-unemployment ratio, 0.382. The strong procyclicality of the wage component of the user cost dampens the response of the job creation to changes in productivity. Alternatively, when the models are calibrated to match the empirical volatility of the vacancy-unemployment ratio, the models generate the wage component of the user cost that is too rigid as compared to its empirical counterpart (See Table 10).

As the results in Table 8 show, once the cyclicality of the wage component of the user cost is calibrated across different models to its empirical counterpart, the economies that are hit by the same sequence of productivity shocks generate very similar dynamics of vacancies and unemployment, regardless of the individual wage setting. In the case where both firms and workers are risk neutral (Table 9), the individual path of wages does not affect the total surplus from job creation, provided the present discounted value of wages at the time of hiring is held constant.[13] In this case, the economies with different wage settings have exactly the same allocations. However, the dynamics of individual wages, including wages of newly hired workers, are determined by the wage setting and differ substantially across economies.

The results demonstrate that, when wages depend on the history from the start of the job, individual wages or wages of newly hired workers are not allocational for employment. With wage smoothing, the dynamics of individual wages are not directly related to the dynamics of the wage component of the user cost. In this case, a weak procyclicality of hiring wages can conceal a substantial procyclicality of the wage component of the user cost.

[13] The implicit contracts do not have a micro-foundation in this context.

VII. Conclusion

This paper contributes to the active debate on the allocational price of labor and its cyclicality. In particular, Pissarides (2009) argues that it is not average wages but wages of newly hired workers that are relevant for the job creation decision of firms. In the paper, I show that it is the wage component of the user cost of labor that is weighted against the marginal revenue product of a worker at the time of hiring. I propose a procedure to estimate its cyclicality and find that it is more procyclical than average wages or even wages of newly hired workers.

The user cost of labor equals the expected present discounted value of the hiring costs and wage payments in a firm-worker match that starts in the current period less the expected present discounted value of the costs of replacing the worker in the subsequent period. In a model with search and matching, the user cost of labor can be decomposed into two components – the vacancy component and the wage component. The wage component of the user cost of labor summarizes the hiring wage as well as the current value of the expected future savings or losses associated with hiring a worker.

With free entry of firms, the marginal productivity of a worker equals the user cost of labor, the sum of the vacancy component and the wage component. This condition allows for testing the quantitative behavior of the search and matching model. The test looks at the model's ability to jointly replicate the elasticities of the vacancy-unemployment ratio and of the wage component of the user cost of labor observed in the data in response to productivity shock. To perform the test requires an estimate of the cyclicality of the wage component of the user cost of labor. In the empirical part of the paper I construct such an estimate, which, to my knowledge, is new in the literature.

I estimate the cyclicality of the wage component of the user cost from the NLSY data. Because it is not directly observed in the data, I construct the wage component of the user cost based on the behavior of individual wages and turnover. I find that a one percentage point increase in unemployment generates more than 4.5% decrease in the wage component of the user cost. This cyclicality is three times higher than the cyclicality of individual wages and also noticeably higher than the cyclicality of wages of newly hired workers.

The cyclicality of the wage component of the user cost of labor translates into elasticity with respect to productivity of above 1.5. Using the free entry condition, I show that the search and matching model cannot simultaneously generate empirical elasticities of the vacancy-unemployment ratio and the wage component of the user cost of labor. This conclusion does not depend on a surplus division rule at the beginning of the match or individual wage dynamics within employment relationships.

In order to examine the cyclicality of the user cost of labor in the search and matching model, I consider economies with different wage settings: 1) implicit contracts and 2) wage bargaining period by period. The simulation results from the models show that in the presence of contractual arrangements, a weak cyclicality of individual wages can conceal a substantial cyclicality of the wage component of the user cost. The results also show that the wage component of the user cost of labor, rather than individual wages or wages of newly hired workers, is allocational for employment. In particular, once the cyclicality of the wage component of the user cost is calibrated to be the same across the models with different wage settings, the models generate very similar volatility of the vacancy-unemployment ratio. However, the cyclicality of individual wages (and the wages of newly hired workers) is different. As discussed above, when the models match the estimated cyclicality of the wage component of the user cost, the generated volatility of the vacancy-unemployment ratio is less than half of its empirical counterpart.

REFERENCES

Beaudry, Paul, and John DiNardo. 1991. "The Effect of Implicit Contracts on the Movement of Wages over the Business Cycle: Evidence from Micro Data." *Journal of Political Economy*, 99(4): 665 – 688.

Bils, Mark. 1985. "Real Wages over the Business Cycle: Evidence from Panel Data." *Journal of Political Economy*, 93(4): 666 – 689.

Bowlus, Audra J.. 1995. "Matching Workers and Jobs: Cyclical Fluctuations in Match Quality." *Journal of Labor Economics*, 13(2): 335-50.

Carneiro, Anabela, Paulo Guimaraes, and Pedro Portugal. 2009. "Real Wages and the Business Cycle: Accounting for Worker and Firm Heterogeneity." Unpublished.

Devereux, Paul. 2001. "The Cyclicality of Real Wages within Employer-Employee Matches." *Industrial and Labor Relations Review,* 54(4): 835 – 850.

Jorgenson, Dale W.. 1963. "Capital Theory and Investment Behavior." *American Economic Review*, 53(2): 247-259.

Haefke, Christian, Marcus Sonntag, and Thijs van Rens. 2007. "Wage Rigidity and Job Creation," Working Paper.

Hagedorn, Marcus, and Iourii Manovskii. 2008. "The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited." *American Economic Review*, 98(4): 1692-1706.

Hall, Robert. 1980. "Employment Fluctuations and Wage Rigidity." Brookings Papers on Economic Activity, Vol. 1980(1), Tenth Anniversary Issue: 91-123.

Hall, Robert. 2005. "Employment Fluctuations with Equilibrium Wage Stickiness." *American Economic Review*, 95(1): 50-65.

Hornstein, Andreas, Per Krusell, and Giovanni L. Violante. 2005. "Unemployment and Vacancy Fluctuations in the Matching Model: Inspecting the Mechanism." *Economic Quarterly*, Federal Reserve Bank of Richmond, Summer: 19-50.

Hosios, Arthur. 1990. "On the Efficiency of Matching and Related Models of Search and Unemployment." *Review of Economic Studies,* 57: 279-298.

Kudlyak, Marianna. 2007. "The Cyclical Price of Labor When Wages Are Smoothed." Unpublished.

Kydland, Finn, and Edward C. Prescott. 1980. "A Competitive Theory of Fluctuations and the Feasibility and Desirability of Stabilization Policy." in S. Fischer, ed., Rational Expectations and Economic Policy, University of Chicago Press: 169-98.

Martins, Pedro, Gary Solon, and Jonathan Thomas. 2009. "Measuring What Employers Really Do about Entry Wages over the Business Cycle." Unpublished.

Mortensen, Dale T., and Christopher A. Pissarides. 1994. "Job Creation and Job Destruction in the Theory of Unemployment." *Review of Economic Studies*, 61(3): 397-415.

Petrongolo, Barbara, and Christopher A. Pissarides. 2001. "Looking into the Black Box: A Survey of the Matching Function." *Journal of Economic Literature*, Vol. XXXIX: 390 - 431.

Pissarides, Christopher A. (1985): "Short-Run Equilibrium Dynamics of Unemployment, Vacancies and Wages," *American Economic Review*, Vol. 75, No.4, pp. 676-690.

Pissarides, Christopher A.. 2009. "The Unemployment Volatility Puzzle: Is Wage Stickiness the Answer?" *Econometrica*, forthcoming.

Rosen, Sherwin. 1969. "On the Interindustry Wage and Hours Structure." *Journal of Political Economy*, 77(2): 249-273.

Rudanko, Leena. 2009. "Labor Market Dynamics under Long Term Wage Contracting." *Journal of Monetary Economics*, 56(2): 170-183.

Scott, A.D.. 1953. "Notes on User Cost." *Economic Journal*, 63(250): 368-384.

Shimer, Robert. 2005. "The Cyclical Behavior of Equilibrium Unemployment and Vacancies." *American Economic Review,* 95(1): 25-49.

Shin, Dongyun. 1994. "Cyclicality of Real Wages among Young Men." *Economics Letters*, 46: 137 – 142.

Solon, Gary, Robert Barsky, and Jonathan A. Parker. 1994. "Measuring the Cyclicality of Real Wages: How Important Is Composition Bias?" *Quarterly Journal of Economics*, CIX(1): 1 – 25.

Thomas, Jonathan, and Tim Worrall. 1988. "Self-Enforcing Wage Contracts." *Review of Economic Studies,* 55(4): 541-553.

Figure 1. Pairs of $(\frac{b}{z}, \eta)$ that generate $\varepsilon_{\theta z} = 7.56$ and $\varepsilon_{wz} = 1.5$

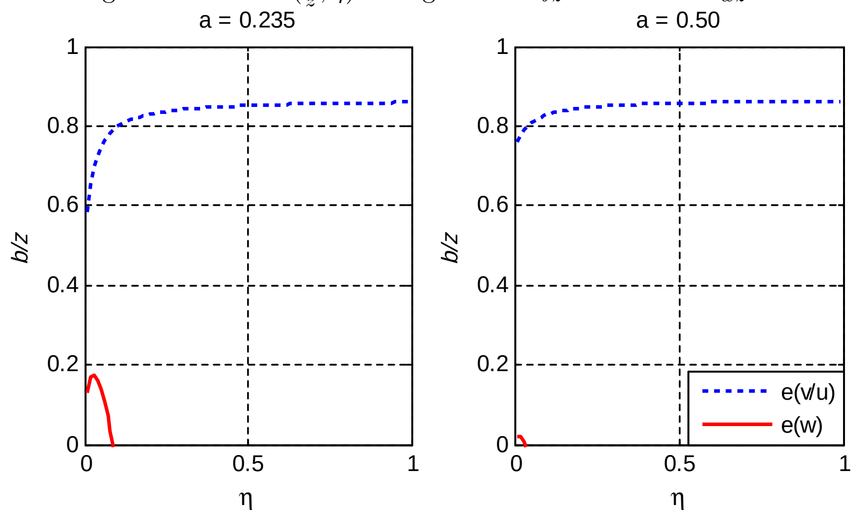

Figure 2. Pairs of $(\frac{b}{z}, \eta)$ that generate $\varepsilon_{\theta z} = 7.56$ and $\varepsilon_{wz} = 0.5$

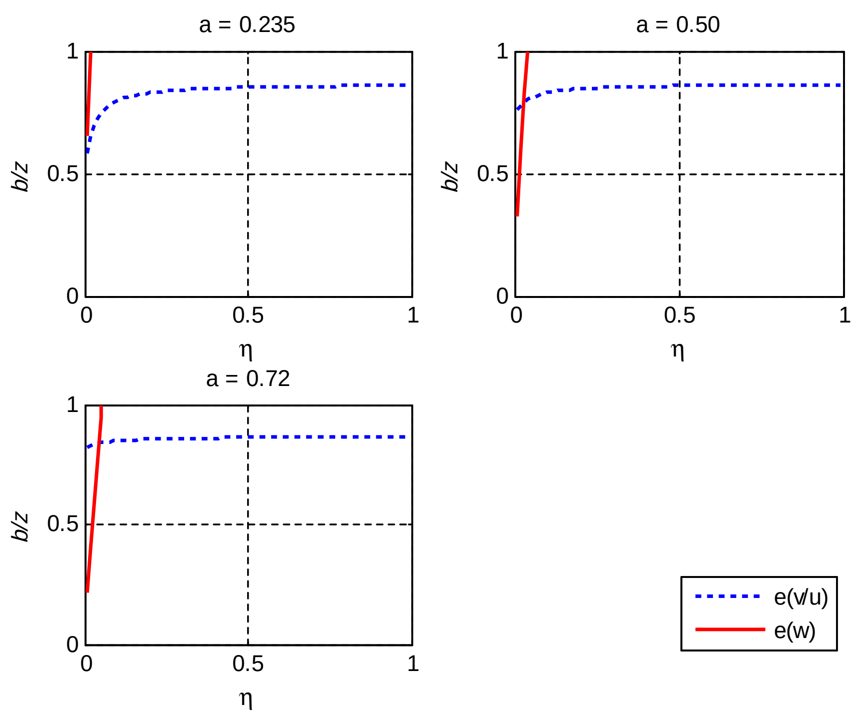

Table 1: Cyclicality of the Wage Component of the User Cost of Labor

	Coefficient on $U_t \cdot 100\%$
UC^W, $\delta_t = const$	-5.29
	(0.97)
UC^W, δ_t	-5.11
	(0.84)
UC^W, δ_t not detrended	-4.92
	(0.81)

Note - The results are from the regression of the natural logarithm of the constructed price of labor on the annual unemployment rate and a time trend (annual). There are 20 observations in each regression - from 1978 to 1997. Time trend is negative and statistically significant. R squared around 0.90. Bootsrapped standard errors in parentheses (1000 replications).

Table 2: Robustness Results

	Coefficient on $U_t \cdot 100\%$		
	$\tau_{tr.} = 5$	$\tau_{tr.} = 7$	$\tau_{tr.} = 9$
UC^W, $\delta_t = const$	-4.87	-5.12	-5.23
	(0.89)	(0.97)	(0.98)
UC^W, δ_t	-4.58	-4.82	-4.95
	(0.77)	(0.88)	(0.92)
UC^W, $\delta_{t_0,t}$	-4.59	-4.83	-4.96
	(0.68)	(0.78)	(0.78)

Note - The results are from the regression of the natural logarithm of the constructed price of labor on the annual unemployment rate and a time trend (annual). There are 18 observations in each regression - from 1978 to 1995. Time trend is negative and statistically significant. R squared around 0.90. Bootsrapped standard errors in parentheses (1000 replications).

Table 3: Wages and the Contemporaneous Unemployment Rate

	All sample	Tenure < 1 y.	Tenure ≥ 2 y.
	1	2	3
U_t	-1.507**	-3.101***	0.292
	(0.705)	(0.716)	(0.733)
Grade	5.710***	4.003***	6.070***
	(0.457)	(0.600)	(1.361)
Experience	0.329***	0.258***	0.376***
	(0.038)	(0.032)	(0.036)
Experience^2	-0.001***	-0.001***	-0.001***
	(0.000)	(0.000)	(0.000)
Tenure	3.642***	1.721	2.114***
	(0.244)	(4.292)	(0.247)
Tenure^2	-0.124***	5.007	-0.077***
	(0.014)	(3.729)	(0.012)
Union	19.565***	20.439***	13.277***
	(1.018)	(1.248)	(1.529)
Union missing	3.288	5.555*	0.602
	(2.095)	(2.770)	(1.213)
Constant	-19.697*	10.729	-29.394
	(9.805)	(7.965)	(20.092)
R^2	0.6324	0.550	0.709
Observations	40850	14576	18546
N of ind	2627	2161	2186

Note - NLSY79, 1978 - 2004, men only, column 1 includes all observations in the sample with the sample restrictions as described in the text. Column 2 includes observations as in column 1 but restricted to the observations with tenure less than 1 year. Column 3 includes observations as in column 1 but restricted to the observations with tenure 2 years and longer. Estimated standard errors in parentheses, clustered by time. The reported coefficients and standard errors are multiplied by 100. P-values: ***p<0.01, ** p<0.05, * p<0.1. Dependent variable: natural logarithm of real hourly wage. All regressions are estimated with fixed effects using sampling weights. Unemployment rate is an annual unemployment rate of the calendar year the wage observation corresponds to.

Table 4: WAGES AND INITIAL, MIN., AND CONTEMPORANEOUS UNEMPLOYMENT RATE, 1978 - 2002

	Industry dummies incl. 1	Industry dummies not incl. 2
U_t	-0.035	-0.085
	(0.954)	(0.992)
U_{t_0}	-1.445***	-1.485***
	(0.368)	(0.395)
$\min U$	-2.821***	-2.987***
	(0.998)	(1.022)
Grade	4.791***	4.750***
	(0.464)	(0.463)
Experience	0.318***	0.343***
	(0.026)	(0.028)
Experience^2	-0.001***	-0.001***
	(0.000)	(0.000)
Tenure	3.464***	3.411***
	(0.360)	(0.336)
Tenure^2	-0.112***	-0.113***
	(0.021)	(0.020)
Union	16.623***	18.872***
	(0.734)	(0.956)
Union missing	3.151**	3.375**
	(1.418)	(1.384)
Constant	18.036*	13.475
	(10.508)	(10.579)
R^2	0.641	0.626
Observations	39132	39132
N of ind.	2623	2623

Note - NLSY79, 1978 - 2002, men only, includes all observation in the sample as described in the text. Dependent variable: natural logarithm of real hourly wage. Standard errors in parentheses. The reported coefficients and standard errors are multiplied by 100. P-values: *** p<0.01, ** p<0.05, * p<0.1. Standard errors are clustered by a contemporaneous year.

Table 5: PARAMETERS

Parameter	Value	Comment
Discount rate, β	.9960	
Separation rate, δ	.0340	Shimer (2005)
Matching function elasticity $(Ku^{\alpha}v^{1-\alpha})$, α	0.5 - 0.7	Petrongolo and Pissarides (2001)
Matching function constant $(Ku^{\alpha}v^{1-\alpha})$, K	0.5	Normalization
Worker's bargaining power, η	α	Hosios (1990), Rudanko (2009)

Table 6: CYCLICALITY OF THE USER COST OF LABOR AND ITS COMPONENTS

Log utility, $\alpha = 0.60$, $b = 0.70$

		Commitment Models			Re-
		Full	1-sided lack of	2-sided lack of	bargain
			Cyclicality		
1	Individual wages (all)	-1.47	-1.47	-1.73	-9.47
2	Individual wages (new hires only)	-4.77	-4.77	-4.99	-9.47
3	Wage component of user cost	-11.15	-11.15	-11.07	-9.47
4	Vacancy component of user cost	-55.06	-55.06	-54.96	-55.14
5	User cost of labor	-11.89	-11.89	-11.82	-10.24
			θ statistics		
6	$\sigma_{\ln(\theta)}$, quarterly	0.0622	0.0622	0.0622	0.0704
			Calibrated parameters		
7	Vacancy creation cost, c	0.2675	0.2675	0.2676	0.2674

Results from simulating the models with risk averse workers (log utility). The vacancy creation cost, c, is calibrated to match $E(\mu(\theta)) = 0.45$. All statistics are calculated from the monthly series unless mentioned otherwise. The cyclicality is calculated as $100cov(\ln(x), u)/var(u)$, which is the semi-elasticity of x with respect to unemployment, u. The corresponding quarterly statistics for the cyclicality of the wage component of the user cost for the models are -11.15, -11.15, -11.08, and -9.47, respectively. $\sigma_{\ln(\theta)}$ is a statistic from quarterly non-HP filtered series. The corresponding statistics from the log-deviations of θ at quarterly frequency from an HP trend with smoothing parameter 10^5 for the models are: 0.0516, 0.0516, 0.0519, 0.0599, respectively.

Table 7: CYCLICALITY OF THE USER COST OF LABOR AND ITS COMPONENTS, CRRA 3 UTILITY

	CRRA $\gamma = 3$ utility, $\alpha = 0.60$, $b = 0.70$				
		Commitment Models			Re-
		Full	1-sided lack of	2-sided lack of	bargain
			Cyclicality		
1	Individual wages (all)	-1.44	-1.44	-1.45	-7.18
2	Individual wages (new hires only)	-4.67	-4.67	-4.68	-7.18
3	Wage component of user cost	-10.92	-10.92	-10.90	-7.18
4	Vacancy component of user cost	-54.98	-54.98	-54.96	-55.23
5	User cost of labor	-11.95	-11.95	-11.93	-8.29
			θ statistics		
6	$\sigma_{\ln(\theta)}$, quarterly	0.0604	0.0604	0.0604	0.0738
			Calibrated parameters		
7	Vacancy creation cost, c	0.3700	0.3700	0.3701	0.3699

Note - Results from simulating the models with risk averse workers (CRRA coefficient $\gamma = 3$). c is calibrated to match $E(\mu(\theta)) = 0.45$. All statistics are calculated from the monthly series unless mentioned otherwise. The cyclicality is calculated as $100cov(\ln(x), u)/var(u)$, which is the semi-elasticity of x with respect to unemployment, u. The corresponding quarterly statistics for the cyclicality of the wage component of the user cost for the models are equal to the ones reported in the table (to the decimal points reported). $\sigma_{\ln(\theta)}$ is a statistic from quarterly non-HP filtered series. The corresponding statistics from the log-deviations of θ at quarterly frequency from an HP trend with smoothing parameter 10^5 are: 0.0514, 0.0514, 0.0514, 0.0738, respectively.

Table 8: Cyclicality of the User Cost of Labor and Its Components and the Volatility of the V-U Ratio

Two calibrated targets: expectation of the job finding rate and the cyclicality of the wage component of the user cost

	Log utility											
	$a = 0.50$				$a = 0.60$				$a = 0.72$			
	Commitment Models			Re-bargain	Commitment Models			Re-bargain	Commitment Models			Re-bargain
	Full	1-sided lack of	2-sided lack of		Full	1-sided lack of	2-sided lack of		Full	1-sided lack of	2-sided lack of	
	Cyclicality											
Wages (all)	-0.59	-0.78	-0.95	-4.50	-0.58	-0.86	-1.56	-4.50	-0.58	-1.13	-2.24	-4.50
Wages (new hires only)	-1.94	-2.18	-2.34	-4.50	-1.94	-2.24	-2.77	-4.50	-1.94	-2.56	-3.51	-4.50
Wage component of UC	-4.50				-4.50				-4.50			
Vacancy component of UC	-37.09	-37.14	-37.07	-37.00	-57.20	-57.29	-57.25	-57.02	-107.64	-107.79	-107.57	-107.10
User cost of labor	-4.90	-4.90	-4.90	-4.94	-4.82	-4.83	-4.83	-4.85	-4.73	-4.73	-4.74	-4.74
	θ statistics											
$\sigma_{\ln(\theta)}$, quarterly	0.1164	0.1163	0.1163	0.1156	0.1475	0.1473	0.1472	0.1468	0.2140	0.2138	0.2136	0.2136
	Calibrated parameters											
Consum.-n of unempl., b	0.8350	0.8335	0.8327	0.8193	0.8710	0.8698	0.8681	0.8616	0.9100	0.9088	0.9075	0.9056
Vacancy creation cost, c	0.1883	0.1901	0.1912	0.2079	0.1041	0.1051	0.1069	0.1123	0.0471	0.0476	0.0486	0.0495

Note - Results from simulating the models with risk averse workers (log utility). c and b are calibrated to match $E(\mu(\theta)) = 0.45$ and the cyclicality of the wage component of the user cost. All statistics are calculated from the monthly series unless mentioned otherwise. The cyclicality is calculated as $100cov(\ln(x), u)/var(u)$. The corresponding quarterly statistics for the cyclicality of the wage component of the user cost for the models are equal to the ones reported in the table (to the decimal points reported). $\sigma_{\ln(\theta)}$ is a statistic from quarterly non-HP filtered series. The corresponding statistics from the log-deviations of θ at quarterly frequency from an HP trend with smoothing parameter 10^5 are: $a = 0.50$: 0.0990, 0.0989, 0.0989, 0.0983; $a = 0.60$: 0.1254, 0.1253, 0.1252, 0.1249; $a = 0.72$: 0.1821, 0.1820, 0.1819, 0.1818.

Table 9: Volatility of the V-U Ratio given the Cyclicality of the User cost and Its Components, $a = 0.60$
Different Utility Functions
Two calibrated targets: expectation of the job finding rate and the cyclicality of the wage component of the user cost

	Linear utility				CRRA 3 utility			
	Commitment Models			Re-	Commitment Models			Re-
	Full	1-sided lack of	2-sided lack of	bargain	Full	1-sided lack of	2-sided lack of	bargain
	Cyclicality							
Wages (all)	-0.58	-0.90	-1.65	-4.50	-0.58	-0.80	-1.36	-4.50
Wages (new hires only)	-1.94	-2.25	-2.85	-4.50	-1.94	-2.21	-2.59	-4.50
Wage component of UC	-4.50				-4.50			
Vacancy component of UC	-57.26				-57.06	-57.32	-57.25	-56.53
User cost of labor	-4.81				-4.87	-4.88	-4.89	-4.99
	θ statistics							
$\sigma_{\ln(\theta)}$, quarterly	0.1480				0.1463	0.1458	0.1453	0.1431
	Calibrated parameters							
Consum.-n of unempl., b	0.8702				0.8715	0.8683	0.8644	0.8381
Vacancy creation cost, c	0.0984				0.1176	0.1211	0.1262	0.1565

Note - Results from simulating the models with workers with utility functions as indicated. c and b are calibrated to match $E(\mu(\theta)) = 0.45$ and the cyclicality of the wage component of the user cost. All statistics are calculated from the monthly series unless mentioned otherwise. The cyclicality is calculated as $100cov(\ln(x), u)/var(u)$. The corresponding quarterly statistics for the cyclicality of the wage component of the user cost for the models are equal to the ones reported in the table (to the decimal points reported). $\sigma_{\ln(\theta)}$ is a statistic from quarterly non-HP filtered series. The corresponding statistics from the log-deviations of θ at quarterly frequency from an HP trend with smoothing parameter 10^5 are: for linear utility 0.1259; the models with CRRA: 0.1244, 0.1240, 0.1236, 0.1217.

Table 10: Cyclicality of the User Cost and Its Components given the Volatility of the V-U Ratio

Different Utility Functions

Two calibrated targets: expectation of the job finding rate and the volatility of the v-u ratio

	$\alpha = 0.60$											
	Linear utility				Logarithmic utility				CRRA 3 utility			
	Commitment Models			Re-	Commitment Models			Re-	Commitment Models			Re-
	Full	1-sided lack of	2-sided lack of	bargain	Full	1-sided lack of	2-sided lack of	bargain	Full	1-sided lack of	2-sided lack of	bargain
	Cyclicality											
Wages (all)	-0.21	-0.51	-1.09	-1.62	-0.21	-0.51	-1.06	-1.62	-0.21	-0.50	-1.01	-1.62
Wages (new hires only)	-0.70	-1.05	-1.39	-1.62	-0.70	-1.04	-1.38	-1.62	-0.70	-1.03	-1.35	-1.62
Wage component of UC	-1.62				-1.62	-1.62	-1.62	-1.62	-1.62	-1.62	-1.61	-1.62
Vacancy component of UC	-76.89				-76.76	-76.80	-76.57	-76.24	-76.47	-76.60	-75.98	-74.98
User cost of labor	-1.75				-1.75	-1.75	-1.75	-1.75	-1.75	-1.75	-1.75	-1.76
	Calibration targets											
$\sigma_{\ln(\theta)}$, quarterly	0.3820											
$E(\mu(\theta))$	0.4500											
	Calibrated parameters											
Consum.-n of unempl., b	0.9434				0.9438	0.9432	0.9428	0.9424	0.9444	0.9427	0.9417	0.9402
Vacancy creation cost, c	0.0415				0.0424	0.0427	0.0432	0.0435	0.0443	0.0454	0.0471	0.0481

Note - Results from simulating the models with workers with different utility functions as indicated. c and b are calibrated to match $E(\mu(\theta)) = 0.45$ and $\sigma_{\ln(\theta)}= 0.0382$. All statistics are calculated from the monthly series unless mentioned otherwise. The cyclicality is calculated as $100cov(\ln(x), u)/var(u)$. The corresponding quarterly statistics for the cyclicality of the wage component of the user cost for the four models and different utility functions are equal with respect to the precision in the tables, except for the log utility, where the statistics are -1.63 for all four models. The statistics, corresponding to $\sigma_{\ln(\theta)}$, from the deviations in logarithms of the vacancy-unemployment ratio at quarterly frequency from an HP trend with smoothing parameter 10^5 for all models are 0.3257.

Table 11: Cyclicality of the user cost of labor and its components. Comparison Across Different Values of the Consumption of the Unemployed

One calibrated target: expectation of the job finding rate

	Log utility, $\alpha = 0.50$											
	$b = 0.40$				$b = 0.70$				$b = 0.90$			
	Commitment Models			Re-bargain	Commitment Models			Re-bargain	Commitment Models			Re-bargain
	Full	1-sided lack of	2-sided lack of		Full	1-sided lack of	2-sided lack of		Full	1-sided lack of	2-sided lack of	
	Cyclicality											
Wages (all)	-2.44	-2.44	-2.44	-13.49	-1.13	-1.13	-1.13	-7.29	-0.33	-0.61	-0.98	-2.42
Wages (new hires only)	-7.78	-7.78	-7.78	-13.49	-3.69	-3.69	-3.69	-7.29	-1.11	-1.39	-1.64	-2.42
Wage component of UC	-18.48	-18.48	-18.48	-13.49	-8.56	-8.56	-8.56	-7.29	-2.56	-2.53	-2.51	-2.42
Vacancy component of UC	-36.37	-36.37	-36.37	-36.39	-36.57	-36.57	-36.57	-36.59	-38.36	-38.43	-38.46	-38.44
User cost of labor	-19.59	-19.59	-19.59	-14.91	-9.25	-9.25	-9.25	-8.01	-2.80	-2.77	-2.75	-2.66
	θ statistics											
$\sigma_{\ln(\theta)}$, quarterly	0.0296	0.0296	0.0296	0.0388	0.0623	0.0623	0.0623	0.0719	0.1991	0.2015	0.2030	0.2093
	Calibrated parameters											
Vacancy creation cost, c	0.9131	0.9131	0.9131	0.9131	0.3688	0.3688	0.3688	0.3687	0.1099	0.1097	0.1100	0.1099

Note - Results from simulating the models with risk averse workers (log utility). c is calibrated to match $E(\mu(\theta)) = 0.45$. All statistics are calculated from the monthly series unless mentioned otherwise. The cyclicality is calculated as $100cov(\ln(x), u)/var(u)$. The corresponding quarterly statistics for the cyclicality of the wage component of the user cost for the models are equal to the ones reported in the table (to the decimal points reported). $\sigma_{\ln(\theta)}$ is a statistic from quarterly non-HP filtered series. The corresponding statistics from the log-deviations of θ at quarterly frequency from an HP trend with smoothing parameter 10^5 are: 0.0251, 0.0251, 0.0251, and 0.0330, respectively, for $b = 0.40$; 0.0530, 0.0530, 0.0530, 0.0611, for $b = 0.70$; 0.1695, 0.1714, 0.1728, 0.1781, for $b = 0.90$.

Appendix to "The Cyclicality of the User Cost of Labor with Search and Matching"

A Derivations and Proofs

A. Value Functions

The values in the economy described in Section 6 can be summarized by the following value functions. Let Ω_t denote a vector of state variables at time t, including the aggregate productivity z_t, and let $\Omega^t \equiv \{\Omega_\tau\}_{\tau=0}^t$. To save on notation, I suppress dependence of the value functions on corresponding histories.

The option value of an inactive firm is assumed to be equal to 0. The value function of a firm with a worker at time t, given that a firm-worker match started at time t_0 is

(A1) $$J_{t_0,t} = z_t - w_{t_0,t} + \beta(1-\delta)E_t J_{t_0,t+1}.$$

The value function of an opened vacancy at t is

(A2) $$V_t = -c + q_t J_{t,t} + \beta(1-q_t)E_t V_{t+1}.$$

The value function of an employed worker at time t, given that a firm-worker match started at time t_0, $W_{t_0,t}$, is

(A3) $$W_{t_0,t} = u(w_{t_0,t}) + \beta E_t\left[(1-\delta)W_{t_0,t+1} + \delta U_{t+1}\right].$$

The value function of an unemployed worker at time t, U_t, is

(A4) $$U_t = u(b) + \beta E_t\left[\mu_{t+1}W_{t_{+1},t+1} + (1-\mu_{t+1})U_{t+1}\right].$$

In this setup, the wage may depend on the history of the labor market conditions from the start of the job. Thus, the wage is indexed by the contemporaneous period and the period a worker is hired.

B. Proof of Proposition 1

Proof.
Consider a value of a firm with a worker at time t given that the productive match starts at time t:

$$J_{t,t} = z_t - w_{t,t} + \beta(1-\delta)E_t J_{t,t+1} = z_t - w_{t,t} + \sum_{\tau=t+1}^{\infty}(\beta(1-\delta))^{\tau-t}E_t(z_\tau - w_{t,\tau}).$$

Then, the expected difference between the value of a firm at time t from the match that starts at time t and the expected present discounted value from the match at time $t+1$ that starts at $t+1$, is

$$\begin{aligned} J_{t,t} - \beta(1-\delta)E_t J_{t+1,t+1} &= z_t - \left[w_{t,t} + \sum_{\tau=t+1}^{\infty}(\beta(1-\delta))^{\tau-t}E_t(w_{t,\tau} - w_{t+1,\tau})\right] = \\ & z_t - UC_t^W. \end{aligned}$$

Substituting the free entry condition, $J_{t,t} = \frac{c}{q(\theta_t)}$, into the left-hand side of the above equation yields

$$\frac{c}{q(\theta_t)} - \beta(1-\delta)E_t\frac{c}{q(\theta_{t+1})} = UC_t^W.$$

Using $UC_t^V = \frac{c}{q(\theta_t)} - \beta(1-\delta)E_t\frac{c}{q(\theta_{t+1})}$, the following equality obtains

$$z_t = UC_t^V + UC_t^W.$$

■

C. $\Pr(UC_t^V > 0)$

I show that $\Pr(UC_t^V > 0) > 0.99$, given the empirical volatility and autocorrelation of z_t.

$UC_t^V > 0$ can be rewritten:

$$\left(\frac{c}{q(\theta(z_t))} - \beta(1-\delta)E_t\frac{c}{q(\theta(z_{t+1}))}\right) > 0$$

or

$$\frac{\theta(z_t)^\alpha}{E_t(\theta(z_{t+1})^\alpha)} > \beta(1-\delta). \tag{A5}$$

Equation (A5) imposes restrictions on the volatility of the stochastic process of $\theta(z_{t+1})$ conditional on $\theta(z_t)$. One can check whether these restrictions hold in the data.

Since $0 < \alpha < 1$, by Jensen's inequality:

$$E_t\left(\theta(z_{t+1})^\alpha\right) \leq \left(E_t\theta(z_{t+1})\right)^\alpha.$$

It implies

$$\frac{\theta(z_t)^\alpha}{E_t(\theta(z_{t+1})^\alpha)} \geq \frac{\theta(z_t)^\alpha}{\left(E_t\theta(z_{t+1})\right)^\alpha}.$$

Thus, to show (A5), it is suffice to show

$$\frac{\theta(z_t)}{\left(E_t\theta(z_{t+1})\right)} > \left(\beta(1-\delta)\right)^{1/\alpha}. \tag{A6}$$

Log-linearization of $\theta(z_{t+1})$ around $\theta(z_t)$ yields

$$\theta(z_{t+1}) \simeq \theta(z_t)(1 + \varepsilon_{\theta(z_t),z_t}\ln\frac{z_{t+1}}{z_t}).$$

Then, (A6) can be rewritten as

$$\frac{1}{(1 + \varepsilon_{\theta(z_t),z_t}E_t\ln\frac{z_{t+1}}{z_t})} > \left(\beta(1-\delta)\right)^{1/\alpha}$$

or, noting that $1 + \varepsilon_{\theta(z_t),z_t}E_t\ln\frac{z_{t+1}}{z_t} > 0$ since $\theta(z_t), \theta(z_{t+1}) > 0$:

$$1 - \left(\beta(1-\delta)\right)^{1/\alpha} > \left(\beta(1-\delta)\right)^{1/\alpha}\varepsilon_{\theta(z_t),z_t}E_t\ln\frac{z_{t+1}}{z_t}. \tag{A7}$$

The stochastic process for z_{t+1} can be specified as

$$\ln z_{t+1} = (1-\rho)\ln\overline{z} + \rho\ln z_t + \iota_{t+1}, \tag{A8}$$

where $\iota_{t+1} \sim N(0, \sigma_\iota^2)$.

Then, inequality (A7) can be rewritten as

$$1 - \left(\beta(1-\delta)\right)^{1/\alpha} > \left(\beta(1-\delta)\right)^{1/\alpha}\varepsilon_{\theta(z_t),z_t}((1-\rho)\ln\overline{z} + \rho\ln z_t) - \left(\beta(1-\delta)\right)^{1/\alpha}\varepsilon_{\theta(z_t),z_t}\ln z_t,$$

which, given $\varepsilon_{\theta(z_t),z_t} > 0$, after simplification yields:

$$\ln\frac{z_t}{\overline{z}} > \frac{\left(\beta(1-\delta)\right)^{1/\alpha} - 1}{\left(\beta(1-\delta)\right)^{1/\alpha}(1-\rho)\varepsilon_{\theta(z_t),z_t}}.$$

Given the stochastic process for z_t,(A8), quarterly values $\beta = 1/(1+0.012)$ and $\delta = 0.01$ (Shimer (2005), Hornstein, Krusell and Violante (2005)), $\rho_z = 0.878$ and $\sigma_z = 0.02$ for quarterly log deviations of z from an HP trend (Shimer (2005)), and a high value of $\alpha = 0.72$ found in the literature, it yields:
(A9)

$$\Pr\left(\ln\frac{z_t}{\bar{z}} > \frac{(\beta(1-\delta))^{1/\alpha}-1}{(\beta(1-\delta))^{1/\alpha}(1-\rho)\varepsilon_{\theta(z_t),z_t}}\right) = \Pr\left(\frac{\ln\frac{z_t}{\bar{z}}}{\sigma_z} > \frac{0.89^{1/\alpha}-1}{0.89^{1/\alpha}(1-\rho)\varepsilon_{\theta(z_t),z_t}\sigma_z}\right) = 1-\Phi\left(\frac{-72.00}{\varepsilon_{\theta(z_t),z_t}}\right),$$

where $\Phi(.)$ is a c.d.f. of the standard normal distribution.

For $\varepsilon_{\theta,z} = 7.56$, the right hand side of (A9) is $1-\Phi(-9.52) >> 0.99$. When the value of $\varepsilon_{\theta(z_t),z_t}$ more than doubles, say, $\varepsilon_{\theta(z_t),z_t} = 20$, then $1-\Phi\left(\frac{-72.00}{\varepsilon_{\theta(z_t),z_t}}\right) = 1-\Phi(-3.6) > 0.99$. Thus, given $\rho_z = 0.878$, $\sigma_z = 0.02$, and $\varepsilon_{\theta,z} = 7.56$, $\Pr(UC_t^V > 0) > 0.99$.■

D. $\Pr(UC_t^W > 0)$

I show that $\Pr(UC_t^W > 0) > 0.99$, given the empirical volatility and autocorrelation of z_t.

$UC_t^W > 0$ can be rewritten as

$$PDV^W(z_t) - \beta(1-\delta)E_t PDV^W(z_{t+1}) > 0.$$

or

$$\frac{PDV^W(z_t)}{E_t PDV^W(z_{t+1})} > \beta(1-\delta). \tag{A10}$$

Log-linearization of $PDV(z_{t+1})$ around z_t yields as

$$PDV^W(z_{t+1}) \simeq PDV^W(z_t)(1+\varepsilon_{PDV^W(z_t),z_t}\ln\frac{z_{t+1}}{z_t}),$$

where $\varepsilon_{PDV^W(z_t),z_t}$ is the elasticity of $PDV^W(z_t)$ at z_t. Note that $1-\varepsilon_{PDV^W(z_t),z_t}(1-\rho)\ln\frac{z_t}{\bar{z}} > 0$ because $PDV^W(z_{t+1}) > 0$, which holds true if all wages are non-negative and at least one is positive.

Equation (A10) can be rewritten:

$$\frac{1}{1-\varepsilon_{PDV^W(z_t),z_t}E_t\ln\frac{z_t}{\bar{z}}} > \beta(1-\delta). \tag{A11}$$

Using the stochastic process for z_t, (A8), inequality (A11) can be rewritten as follows:

$$\ln\frac{z_t}{\bar{z}} > \frac{\beta(1-\delta)-1}{\beta(1-\delta)(1-\rho)\varepsilon_{PDV^W(z_t),z_t}},$$

if $\varepsilon_{PDV^w(z_t),z_t} > 0$, and

$$\ln\frac{z_t}{\bar{z}} < \frac{\beta(1-\delta)-1}{\beta(1-\delta)(1-\rho)\varepsilon_{PDV^W(z_t),z_t}},$$

if $\varepsilon_{PDV^w(z_t),z_t} < 0$.

Given the quarterly parameter values discussed in the appendix above and the stochastic process for z_t, these two cases can be combined as follows:

$$\Pr\left(\ln\frac{z_t}{\bar{z}} > \frac{\beta(1-\delta)-1}{\beta(1-\delta)(1-\rho)|\varepsilon_{PDV^w(z_t),z_t}|}\right) = 1-\Phi\left(\frac{-50.65}{|\varepsilon_{PDV^w(z_t),z_t}|}\right). \tag{A12}$$

To obtain a bound on $\varepsilon_{PDV^w(z_t),z_t}$, consider free entry condition:

$$\frac{c}{\theta^{-\alpha}} = PDV^Z(z_t) - PDV^W(z_t).$$

Differentiating and rearranging yields

$$\alpha\varepsilon_{\theta(z_t),z_t}J(z_t) = \varepsilon_{PDV^Z(z_t),z_t}PDV^Z(z_t) - \varepsilon_{PDV^w(z_t),z_t}PDV^W(z_t),$$

where $J(z_t) \equiv PDV^Z(z_t) - PDV^W(z_t) \geq 0$, given free entry, and $\varepsilon_{PDV^Z(z_t),z_t} > 0$ (see below). Rearranging, it follows:

$$\text{(A13)} \qquad \varepsilon_{PDV^w(z_t),z_t} = \varepsilon_{PDV^Z(z_t),z_t}\frac{PDV^Z(z_t) - \frac{\alpha\varepsilon_{\theta(z_t),z_t}}{\varepsilon_{PDV^Z(z_t),z_t}}J(z_t)}{PDV^Z(z_t) - J(z_t)}.$$

It can be shown that the following holds:

$$\text{(A14)} \qquad \left|\frac{PDV^Z(z_t) - \frac{\alpha\varepsilon_{\theta(z_t),z_t}}{\varepsilon_{PDV^Z(z_t),z_t}}J(z_t)}{PDV^Z(z_t) - J(z_t)}\right| < 1.$$

To see this, note, that if $\varepsilon_{PDV^w(z_t),z_t} > 0$, then $PDV^Z(z_t) - \frac{\alpha\varepsilon_{\theta(z_t),z_t}}{\varepsilon_{PDV^Z(z_t),z_t}}J(z_t) > 0$ because $PDV^Z(z_t) - J(z_t) = PDV^W(z_t) > 0$. Then, equation (A14) can be rewritten: $PDV^Z(z_t) - \frac{\alpha\varepsilon_{\theta(z_t),z_t}}{\varepsilon_{PDV^Z(z_t),z_t}}J(z_t) < PDV^Z(z_t) - J(z_t)$, which holds when $\frac{\alpha\varepsilon_{\theta(z_t),z_t}}{\varepsilon_{PDV^Z(z_t),z_t}} > 1$.

Alternatively, if $\varepsilon_{PDV^w(z_t),z_t} < 0$, then $\frac{PDV^Z(z_t)}{J(z_t)} < \frac{\alpha\varepsilon_{\theta(z_t),z_t}}{\varepsilon_{PDV^Z(z_t),z_t}}$. Then equation (A14) can be rewritten: $-(PDV^Z(z_t) - \frac{\alpha\varepsilon_{\theta(z_t),z_t}}{\varepsilon_{PDV^Z(z_t),z_t}}J(z_t)) < PDV^Z(z_t) - J(z_t)$, which can be rewritten as

$$\text{(A15)} \qquad \frac{\alpha\varepsilon_{\theta(z_t),z_t}}{\varepsilon_{PDV^Z(z_t),z_t}} + 1 < 2\frac{PDV^Z(z_t)}{J(z_t)}.$$

Since $\frac{PDV^Z(z_t)}{J(z_t)} < \frac{\alpha\varepsilon_{\theta(z_t),z_t}}{\varepsilon_{PDV^Z(z_t),z_t}}$, equation (A15) holds if $1 < \frac{\alpha\varepsilon_{\theta(z_t),z_t}}{\varepsilon_{PDV^Z(z_t),z_t}}$.

Thus, $\varepsilon_{PDV^W(z_t),z_t} = \varepsilon_{PDV^Z(z_t),z_t}x_t$,where $|x_t| < 1$ if $1 < \frac{\alpha\varepsilon_{\theta(z_t),z_t}}{\varepsilon_{PDV^Z(z_t),z_t}}$.

Given the stochastic process for z_t, $PDV^Z(z_t)$ can be written:

$$PDV^Z(z_t) = z_t + \sum_{\tau=t+1}(\beta(1-\delta))^{\tau-t}\exp\left((1-\rho)\sum_{k=0}^{\tau-t}\rho^k\ln\overline{z} + \rho^{\tau-t}\ln z_t + \frac{\sigma_i^2}{2}\sum_{k=0}^{\tau-t-1}\rho^k\right).$$

Note the following:

$$\varepsilon_{PDV^Z(z_t),z_t} = \frac{dPDV^Z(z_t), z_t}{dz_t}\frac{z_t}{PDV^Z(z_t), z_t} = \frac{z_t + \sum_{\tau=t+1}(\rho\beta(1-\delta))^{\tau-t}E_t z_\tau}{z_t + \sum_{\tau=t+1}(\beta(1-\delta))^{\tau-t}E_t z_\tau},$$

which delivers $0 < \varepsilon_{PDV^Z(z_t),z_t} < 1$ since $0 < \rho < 1$ and $z_t + \sum_{\tau=t+1}(\beta(1-\delta))^{\tau-t}E_t z_\tau \equiv PDV^Z(z_t) > 0$.

Note that $\alpha\varepsilon_{\theta(z_t),z_t} > 1$ given the values for α and $\varepsilon_{\theta(z_t),z_t}$ as described in Section 3.C. Thus, from $\alpha\varepsilon_{\theta(z_t),z_t} > 1$ and $0 < \varepsilon_{PDV^Z(z_t),z_t} < 1$, it follows that $\frac{\alpha\varepsilon_{\theta(z_t),z_t}}{\varepsilon_{PDV^Z(z_t),z_t}} > 1$. Hence, $|\varepsilon_{PDV^w(z_t),z_t}| = |\varepsilon_{PDV^w(z_t),z_t}x_t| < 1$.

Using $|\varepsilon_{PDV^w(z_t),z_t}| < 1$ in expression (A12) delivers $\Pr\left(UC^W > 0\right) > 0.99$.■

E. Derivation of $\varepsilon_{UC_t^V,z_t} = \alpha\varepsilon_{\theta,z}x_t$

Using (A8), the probability density function for z_{t+1} given z_t is:

$$f(z_{t+1}|z_t) = \frac{1}{z_{t+1}\sigma_{\ln z_{t+1}}\sqrt{2\pi}}\exp(-\frac{\ln(z_{t+1}) - ((1-\rho)\ln\overline{z} + \rho\ln z_t)}{2\sigma^2_{\ln z_{t+1}}}).$$

The elasticity of the vacancy component of the user cost of labor with respect to productivity is:

$$\varepsilon_{UC_t^V,z_t} = \frac{d\left(\frac{c}{K\theta_t^{-\alpha}} - \beta(1-\delta)\int \frac{c}{K\theta_{t+1}^{-\alpha}} f(z_{t+1}|z_t)dz_{t+1}\right)}{dz_t} \frac{z_t}{\frac{c}{K\theta_t^{-\alpha}} - \beta(1-\delta)E_t\frac{c}{K\theta_{t+1}^{-\alpha}}} =$$

$$\frac{\alpha\varepsilon_{\theta,z}\frac{c}{K\theta_t^{-\alpha}} - \rho\beta(1-\delta)\int \frac{c}{K\theta_{t+1}^{-\alpha}} f(z_{t+1}|z_t)dz_{t+1}}{\frac{c}{K\theta_t^{-\alpha}} - \beta(1-\delta)E_t\frac{c}{K\theta_{t+1}^{-\alpha}}} =$$

$$\frac{\alpha\varepsilon_{\theta,z}\left(\frac{c}{K\theta_t^{-\alpha}} - \frac{\rho}{\alpha\varepsilon_{\theta,z}}\beta(1-\delta)E_t\frac{c}{K\theta_{t+1}^{-\alpha}}\right)}{\frac{c}{K\theta_t^{-\alpha}} - \beta(1-\delta)E_t\frac{c}{K\theta_{t+1}^{-\alpha}}} = \alpha\varepsilon_{\theta,z}x_t,$$

where $x_t = \frac{\frac{c}{K\theta_t^{-\alpha}} - \frac{\rho}{\alpha\varepsilon_{\theta,z}}\beta(1-\delta)E_t\frac{c}{K\theta_{t+1}^{-\alpha}}}{\frac{c}{K\theta_t^{-\alpha}} - \beta(1-\delta)E_t\frac{c}{K\theta_{t+1}^{-\alpha}}}$. Since $\frac{c}{K\theta_t^{-\alpha}} - \beta(1-\delta)E_t\frac{c}{K\theta_{t+1}^{-\alpha}} > 0$ (see proof above) and $\rho < \alpha\varepsilon_{\theta,z}$ (for $\rho < 1$, $\varepsilon_{\theta,z} = 7.56$ and $\alpha \in [0.235; 0.72]$), one obtains $x_t > 1$.■

B Empirical Section

A. The Wage Component of the User Cost of Labor with Time-Varying Separation Rates

To define the wage component of the user cost of labor with time-varying separation rates, consider the following thought experiment. A firm hires a worker in period t. Assume that a worker is always available for hire, and the only costs associated with hiring a worker are wage payments. A firm pays according to the wage schedule agreed upon when the worker is hired. Every period, a nonzero probability exists that a worker will exogenously separate from the position. Separation probability, $\delta_{t,\tau}$, may depend on the history of labor market conditions a worker experiences from the time of hiring. After separation, a firm hires a new worker to replace the separated one. A new firm-worker relationship is likely to start with a new wage agreement. In this thought experiment, if a firm hires a worker in some period t, it maintains the number of workers at 1 from that period on by re-hiring in case the worker hired in τ separates. Thus, hiring a worker in t can be thought of as creating a position in period t that will be filled with probability 1 onwards. Then, the expected present discounted value of wages paid to create a position in t onwards is given by

$$PDV_t' = w_{t,t} + E_t[\beta((1-\delta_{t,t})w_{t,t+1} + \delta_{t,t}w_{t+1,t+1}) +$$
$$\beta^2((1-\delta_{t,t})(1-\delta_{t,t+1})w_{t,t+2} + \delta_{t,t}(1-\delta_{t+1,t+1})w_{t+1,t+2} +$$
$$((1-\delta_{t,t})\delta_{t,t+1} + \delta_{t,t}(1-\delta_{t+1,t+1}))w_{t+2,t+2} + ...] =$$

$$w_{t,t} + E_t[\sum_{\tau=t+1}^{\infty}\beta^{\tau-t}\sum_{k=t}^{\tau-1}(\Lambda_{t,k,\tau-1}w_{k+1,\tau})], \quad \text{(B1)}$$

where w_{t_1,t_2} is a wage paid in t_2 to a worker hired in t_1; δ_{t_1,t_2} is a separation rate at the end of t_2 for a worker hired in t_1, conditional that there is no separation between $t_1$and t_2; and $\Lambda_{t,k,\tau}$ is a probability that a separation takes place at the end of period k at the position that a firm opened in t and a new worker is hired in $k+1$ and continues working on that position in τ; and $E_t = E(.|I_t)$ where I_t is the firm's information set at time t. Both wage payments and separation rates are allowed to depend on the history of the labor market conditions from the period a worker is hired.

Equation (B1) states that a worker hired in period t is paid a wage $w_{t,t}$. With probability $1-\delta_{t,t}$ the firm-worker relationship survives until the period $t+1$ and the worker is paid wage $w_{t,t+1}$. With probability $\delta_{t,t}$ the relationship is terminated and the firm hires a new worker at a wage $w_{t+1,t+1}$ to fill the position. By analogy, in period $t+2$ a firm retains a worker hired in period t with probability $(1-\delta_{t,t})(1-\delta_{t,t+1})$ and pays a wage $w_{t,t+2}$. With probability $(1-\delta_{t,t})\delta_{t,t+1}$ that worker is separated and the firm replaces the worker with another at wage $w_{t+2,t+2}$. Also, in period $t+2$ a worker hired in $t+1$ is retained with probability $\delta_{t,t}(1-\delta_{t+1,t+1})$ and receives wage $w_{t+1,t+2}$. In case of separation, with probability $\delta_{t,t}\delta_{t,t+1}$ this worker is replaced with a new one at wage $w_{t+2,t+2}$.

The wage component of the user cost of labor in period t is the difference between the expected present discounted value of wages paid at the position opened in period t and $t+1$:

$$UC_t^W = PDV_t' - \beta E_t PDV_{t+1}',$$

Substituting from (B1), I obtain the following expression for the wage component of the user cost of labor:

$$UC_t^W = w_{t,t} + E_t[\sum_{\tau=t+1}^{\infty} \beta^{\tau-t}(w_{t,\tau}\prod_{k=t}^{\tau-1}(1-\delta_{t,k}) - w_{t+1,\tau}(1-\delta_{t,t})\prod_{k=t+1}^{\tau-1}(1-\delta_{t+1,k}))+$$

$$\sum_{\tau=t+1}^{\infty} \beta^{\tau-t}(\sum_{k=t}^{\tau-1}(\Lambda_{t,k,\tau-1} - (1-\lambda_{t,t})\Lambda_{t+1,k,\tau-1})w_{k,\tau})]. \tag{B2}$$

If separation depends only on the contemporaneous labor market conditions, $\delta_{t_0,t} = \delta_t$ for all t and t_0, then (B2) simplifies to the following expression:

$$UC_t^W = w_{t,t} + E_t \sum_{\tau=t+1}^{\infty} \beta^{\tau-t}(\prod_{k=t}^{\tau-1}(1-\delta_k))(w_{t,\tau} - w_{t+1,\tau}). \tag{B3}$$

If the separation rate is constant, $\delta_{t_0,t} = \delta$, equation (B2) simplifies to

$$UC_t^W = w_{t,t} + E_t \sum_{\tau=t+1}^{\infty} (\beta(1-\delta))^{\tau-t}(w_{t,\tau} - w_{t+1,\tau}).$$

B. Estimation with Time-Varying Separation Rates

To obtain the series of separation rates I proceed in two steps. First, I detrend the monthly separation rates. To do that, I estimate the linear probability model with a dependant variable taking value 1 if a worker does not work for the same job in the next month and 0 otherwise. The explanatory variables are the quartic in the monthly trend. I subtract the value of a quartic in the trend multiplied by the estimated coefficients from the dependent variable and add the value a quartic of a trend calculated at the mean multiplied by the estimated coefficients. In the second step, I estimate a linear probability model of the detrended monthly separation rates with two sets of time dummies as explanatory variables: one set of time dummies corresponds to the year the job starts and another set of dummies corresponds to the contemporaneous year. Then, I use monthly fitted projections to obtain annual separation rates, $\widehat{\delta_{t_1,t_2}^A}$. Annual separation rates are calculated as follows: for all t_1 and t_2 : $t_1, t_2 = \{1978, 2004\}, t_1 < t_2$: $\widehat{\delta_{t_1,t_2}^A} = 1 - \frac{\sum_{\tau^{t1}=1}^{12}\left(\prod_{k^{t2}=1}^{12}(1-\widehat{\delta_{\tau^{t1},k^{t2}}})\right)}{12}$, where $\widehat{\delta_{\tau^{t1},k^{t2}}}$ is a fitted monthly separation rate in a calendar month k of year t_2 at the job that started in a calendar month τ of year t_1. In a similar manner I calculate annual separation rates $\widehat{\delta_{t_1,t_2}^A}$ for $t_1 = t_2$, annualizing monthly separations.

As a robustness check, I also construct the wage component of the user cost of labor as in equation (B3), where the separation rate depends on the contemporaneous period only. In this case, in the empirical model of the separation rates I use only one set of time dummies – the contemporaneous period dummies. Then I use monthly fitted projections to obtain annual separation rates, $\widehat{\delta_t^A}$. For all $t : t = \{1978, 2004\}$: $\widehat{\delta_t^A} = 1 - \prod_{\tau=1}^{12}(1-\widehat{\delta_{\tau^t}})$,where $\widehat{\delta_{\tau^t}}$ is a fitted monthly separation rate in a calendar month τ of year t.

C Quantitative Section

A. Models with Implicit Contracts

The value an employed worker receives in period t from a contract that started in period t_0, $W_\sigma(t_0, z^t)$, is

$$W_\sigma(t_0, z^t) = u(w_\sigma(t_0, z^t)) + E_t \sum_{\tau=t+1}^{\infty} \beta^{\tau-t}(1-\delta)^{\tau-(t+1)} \left[(1-\delta)u(w_\sigma(t_0, \{z^{\tau-1}, z_\tau\})) + \delta U(z_\tau)\right].$$

The value of a newly unemployed worker or a worker who did not find a match in the current period is a sum of the current utility, obtained from consuming an unemployment benefit, b, and the expected discounted value from searching:

$$U(z_t) = u(b) + \beta E_t \left[\mu(\theta_\sigma(\{z^{t+1}, z_t\}))W_\sigma(t+1, \{z^t, z_{t+1}\}) + (1 - \mu(\theta_\sigma(\{z^{t+1}, z_t\})))U(z_{t+1})\right].$$

The value a firm obtains in period t given the aggregate state z_t from a contract σ that started in period t_0 is

$$J_\sigma(t_0, z^t) = z_t - w_\sigma(t_0, z^t) + E_t \sum_{\tau=t+1}^{\infty} (\beta(1-\delta))^{\tau-t}(z_\tau - w_\sigma(t_0, \{z^{\tau-1}, z_\tau\})).$$

Equilibrium contracts are limited to efficient optimal contracts. A contract is efficient if there exists no other contract that offers each party at least as much expected utility and one party strictly more. A contract is optimal if it maximizes the total welfare given the initial promise of a value to one of the parties. An efficient contract cannot be Pareto dominated after any history. Hence, after any history it can be rewritten as a maximization problem. The Pareto frontier is traced by varying the value promised by the contract to the worker and maximizing the value of the firm given the worker's promised value. As in Thomas and Worrall (1988), the history of the productivity realizations from the start of the match can be summarized by the worker's promised value. Given the assumption that z_t follows a first order Markov process, it is sufficient to keep track of the current value of z to determine the expectations. In the presentation that follows the time subscripts are suppressed: z denotes the current value of productivity and z' denotes the value next period.

Let W be the value promised to a worker under the contract. Let $U(z)$ be the value of an unemployed worker given aggregate state z and let $f(z, W, U(z))$ denote a value of a firm from a contract on a Pareto frontier given z, W, $U(z)$, and the evolution of $U(z)$. Then $f(z, W, U(z))$ solves the following dynamic programming optimization problem for all $z \in Z$:

$$f(z, W, U(z)) = \max_{w, \{W(z')\}_{z' \in Z}} z - w + \beta E_z(1-\delta)f(z', W(z'), U(z')) \tag{C1}$$

s. t.

$$W = \varphi(w) + \beta E_z\left[(1-\delta)W(z') + \delta U(z')\right] \tag{C2}$$

$$W(z') \geq U(z') \;\forall\; z' \in Z \tag{C3}$$

$$f(z', W(z'), U(z')) \geq 0 \;\forall\; z' \in Z. \tag{C4}$$

An efficient contract maximizes the value of a firm, f, given the aggregate state, z, the promised value for the worker, W, and the worker's outside option, $U(z)$. The first constraint is a promise-keeping constraint that specifies that a worker gets exactly value W from the contract that pays wage w and promises values $W(z')$ for all states $z' \in Z$ where there is no exogenous separation. The second and third constraints are self-enforcing constraints for the worker and the firm, respectively. By omitting self-enforcing constraints, contracts with different degrees of commitment are obtained: 1) full commitment (by omitting (C3) and (C4)); 2) lack of commitment from the worker's side and full commitment from the firm's side (by omitting

(C4)); and 3) two-sided lack of commitment (when both (C3) and (C4) are present).

I study equilibria of this economy which consist of a contract $\sigma(z)$, value functions for the firm from a contract $\sigma(z)$, f_σ, values promised to the worker at the time of hiring, $W_{h,\sigma}(z)$, values of an unemployed worker, $U(z)$, and a market tightness, $\theta_\sigma(z)$, associated with the contract $\sigma(z)$ for each $z \in Z$, such that

1. (Optimization) Given a vector U, the list of functions $f(z, W_{h,\sigma}(z), U(z))$ solves the dynamic programming problem (C1)-(C4).

2. (Free entry) Firms enter a labor market and post vacancies with the associated contract σ until the value of posting a vacancy is driven to 0:

$$q(\theta_\sigma(z))f(z, W_{h,\sigma}(z), U(z)) = c. \tag{C5}$$

3. The value of an unemployed worker evolves according to the following rule:

$$U(z) = u(b) + \beta E_z \left[\mu(\theta_\sigma(z'))W_{h,\sigma}(z') + (1 - \mu(\theta_\sigma(z')))U(z')\right]. \tag{C6}$$

In addition, I impose the following equilibrium refinement:

4. (Pareto efficiency) There does not exist an efficient self-enforcing contract $\sigma'(z)$ and an associated labor market with tightness $\theta_{\sigma'}(z)$ such that the net surpluses from the search for a worker, $\mu(\theta_{\sigma'}(z))(W_{h,\sigma'}(z) - U(z))$,and for a firm, $-c + q(\theta_\sigma(z))f(z, W_{h,\sigma'}(z), U(z))$, are at least as much as under $\sigma(z)$ and $\theta_\sigma(z)$ and for one party it is strictly more.

This refinement of the set of equilibrium contracts follows Rudanko (2009), who motivates it from the competitive search, in which competitive market-makers specify the set of the efficient self-enforcing contracts that can be posted in the economy. Each contract is offered in a separate market with an associated labor-market tightness, and in equilibrium each market must offer the same surplus from search for firms and the same surplus for workers. Because of competition between market-makers, only markets in which the offered contract is on the Pareto frontier will be opened in equilibrium. Condition 2 combined with Condition 3 determines equilibrium values of the promised value for the worker at the time of hiring, $W_{h,\sigma}(z)$, and an equilibrium value of the market tightness in the market with σ, $\theta_\sigma(z)$.

In this economy unemployment evolves according to the following law, given $u(z_{t_0})$:

$$u(\{z^t, z_{t+1}\}) = u(z^t) + (1 - u(z^t))\delta - \mu(\theta(\{z^t, z_{t+1}\}))u(z^t).$$

The pool of unemployed in the current period consists of unemployed workers from the previous period and those who became unemployed because of the exogenous separations in the previous period, net of the unemployed workers who find jobs in the current period.

Thomas and Worrall (1988) and Rudanko (2006) prove that the optimization problem described above is a concave problem, so the first-order conditions are necessary and sufficient. The first-order conditions for an arbitrary z read:

$$-\lambda_z = -\frac{1}{\varphi'(w)}. \tag{C7}$$

$$-\lambda_z = (1 + \zeta(z'))f_V(z', W(z'), U(z')) + \phi(z') \ \forall z' \in Z, \tag{C8}$$

where λ_z is the Langrange multiplier on the promise-keeping constraint; $\beta\pi(z'|z)\phi(z')$, are Langrange multipliers on the self-enforcing constraints for a worker, and $\beta\pi(z'|z)\zeta(z')$ are Langrange multipliers on self-enforcing constraints for a firm $\forall z' \in Z$. Complimentary slackness conditions: $\lambda_z \geq 0$, $\zeta(z'), \phi(z') \geq 0 \ \forall z'$, and (C3) and (C4). The envelope condition:

$$f_V(z, W(z), U(z)) = -\lambda_z. \tag{C9}$$

Combining the envelope condition, (C9), with the first order conditions gives the following condition, which links the current and next period wage:

$$\frac{1}{\varphi'(w(z,W,U(z)))} = (1+\zeta(z'))\frac{1}{\varphi'(w(z',W(z'),U(z')))} + \phi(z') \ \forall z' \in Z$$

Because of free entry and Pareto optimality, $W_h(z)$ and $\theta(z)$ solve the following maximization problem given $V_u(z)$:[14]

$$\text{(C10)} \qquad \max_{\{\theta(z)\},\{V_h(z)\}} \left\{ \ \mu(\theta(z))(W_h(z) - U(z)) \ \right\}$$
$$\text{s.t. } \ q(\theta(z)f(z, W_h(z), U(z)) = c$$

Combining the first order condition for Pareto optimality problem, (C10), the free entry condition, (C5), the envelope condition, (C9), the first order condition for wages, (C7), and the law of motion for the value of unemployed workers, the following system of equations characterizes the equilibrium objects f, $U(z)$, $W_h(z)$ and $\theta(z)$ $\forall z \in Z$, given the optimal contract.

$$\text{(C11)} \qquad \frac{\alpha}{1-\alpha} f(z, W_h(z), U(z)) = \frac{W_h(z) - U(z)}{\varphi'(w(z, W_h(z), U(z)))}.$$

$$\theta(z) = \left(\frac{c}{f(z, W_h(z), U(z))}\frac{1}{K}\right)^{-\frac{1}{\alpha}}.$$

$$U(z) = u(b) + \beta E_z\left[\mu(\theta_\sigma(z'))W_h(z') + (1-\mu(\theta_\sigma(z')))U(z')\right].$$

B. A Model with Bargaining Period by Period

An equilibrium in the economy with bargaining period by period consists of the set of the value functions for a firm, $J(z)$, (A1), and $V(z)$, (A2), and a worker, $W(z)$, (A3) and $U(z)$, (A4), and a market tightness $\theta(z)$, such that

1. (Free entry) The value of a vacancy is 0:

$$q(\theta(z))J(z) = c.$$

2. (Surplus division) Each period during an employment relationship, the firm and the worker bargain over the match surplus. At the time of bargaining, the outside option value for a worker is the value of unemployment, while the outside option for a firm is 0 (the value of an inactive firm). A matched worker-firm pair divides the total surplus from the match by solving the following maximization problem:

$$\max_{V_e(z)-V_u(z),J_F(z)} \left(W(z) - U(z)\right)^{\eta} J(z)^{1-\eta}$$
$$\text{(C12)} \qquad \text{s.t. } \ \frac{W(z)-U(z)}{u'(w(z))} + J(z) = S(z)$$

where η is a bargaining power of the worker, $u'(w)$ is the marginal utility of income, and $S(z)$ is a total surplus.

3. The value of an unemployed worker evolves according to the following rule:

$$U(z) = u(b) + \beta E_z\left[\mu(\theta(z'))W(z') + (1-\mu(\theta(z')))W(z')\right].$$

■

[14] Rudanko (2009) proves that given fairly mild conditions there is a unique Pareto-efficient contract offered in equilibrium.