

# Financial Market Segmentation, Stock Market Volatility and the Role of Monetary Policy

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## **Abstract**

We explore the role of monetary policy in a world of segmented financial markets, where the agents who trade stocks encounter financial income risk although the rest do not. In such an economy, we study how the monetary authority operates when it aims to maximize total welfare. We find that optimal monetary policy has the novel role of sharing the financial market risk traders face, among all agents in the economy. This finding holds for any concave utility function and is not sensitive to the degree of market segmentation. When risk is shared perfectly in this way, consumption is equalized between the two groups, and agents, if given the choice, would be indifferent

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between participating or not in the financial markets. We also explore the implications that this policy has for the volatility of stock prices and of inflation, when compared to the policies of constant money supply, inflation targeting and nominal interest rate pegging. We find that optimal monetary policy is not necessarily associated either with minimal stock price volatility or with minimal inflation volatility.

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JEL Classification: *E44; E52; G12.*

## 1 Introduction

Should the behavior of the stock market be a matter of monetary policy? This widespread concern among central bankers, was addressed by Alan Greenspan in his famous December 1996 talk:

*“But how do we know when irrational exuberance has unduly escalated asset values [...] ? And how do we factor that assessment into monetary policy? [...] But we should not underestimate [...] the complexity of the interactions of asset markets and the economy. Thus, evaluating shifts in balance sheets generally, and in asset prices particularly, must be an integral part of the development of monetary policy.”*

This paper develops a simple model to study some new aspects of the interaction of the monetary authority with the stock market. Specifically, it studies the risk-sharing implications of an optimal, welfare maximizing monetary policy in the presence of a segmented stock market. In addition, this paper addresses the question of whether the optimal policy entails lower fundamentals originated stock price volatility when compared with other, widely used monetary policy rules.

In our simple cash-in-advance model we employ the finding of limited financial market participation, the importance of which is well addressed in the literature (see

[Mankiw and Zeldes \(1991\)](#), [Vissing-Jørgensen \(2002\)](#), [Guiso et al. \(2002\)](#)<sup>1</sup>. The main idea develops as follows: The stock market provides its traders with a risky stream of total dividends, *à la* Lucas tree ([Lucas, 1978](#)), which is shared among them according to the amount of shares they hold. The other group of agents residing in this model economy, the non-traders, does not participate in the financial markets; it has no real income risk but is however, subject to inflation risk. In addition, as is usually assumed in the limited participation literature, capturing the way monetary policy transmits in the economy (starting with [Grossman and Weiss \(1983\)](#), [Rotemberg \(1984\)](#) but following [Lucas \(1990\)](#) in this paper), financial market participants are directly influenced by monetary policy changes, although non-participants are affected only indirectly through price adjustments. Specifically, only the traders receive positive transfers whenever money supply expands, although they get taxed whenever it contracts.

In such an environment, a monetary authority that assigns the population weight to each group of agents can resolve the market failure of limited participation by perfectly sharing the financial risk among traders and non-traders, maximizing in this way total welfare. In particular, optimal monetary policy becomes expansionary whenever dividend income is lower than expected, subsidizing traders with a positive transfer. Such a policy increases the price of the consumption good, dismaying non-traders whose consumption decreases. On the other hand, whenever dividend income is higher than expected, monetary policy contracts, taxes traders and takes away part of their increased income. The consumption good becomes more affordable and as an effect, non-traders realize higher consumption. Then, by operating optimally, monetary policy equalizes the consumption of the two groups. Answering the question asked above, if and how monetary policy should respond to stock market advances,

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<sup>1</sup>For its empirical importance see [Landon-Lane and Occhino \(2008\)](#) and [Mizrach and Occhino \(2008\)](#).

this paper suggests that monetary policy optimally expands in bad times for the financial markets and contracts in good times for the financial markets, because of its risk sharing considerations. This result, as straightforward as it seems, is new in the literature and assigns to monetary policy the novel role of risk-sharing between heterogeneous agents. And it becomes more interesting as the heterogeneity here concerns stock market participation.

Furthermore, we compute the stock price volatility implied by the optimal monetary policy and compare it with the volatility implied by the constant money supply, inflation targeting and interest rate pegging policy rules. In addition, we compute the welfare loss of implementing a stock price volatility minimizing policy instead of the optimal one. It might be interesting to study the answers that this model generates given the recent interest on the issue of how various monetary policy rules affect stock price volatility (e.g., [Bernanke and Gertler \(2000\)](#) and [Bernanke and Gertler \(2001\)](#)). We find that the optimal monetary policy does not necessarily produce lower stock price volatility than the other policy rules and the outcome of the comparison depends on parameter values. The same results hold when we compare the inflation volatility these rules generate, except from the inflation targeting rule which by definition associates with minimum inflation volatility. In addition, the stock price volatility minimizing policy reduces welfare when compared to the optimal policy, and the welfare loss increases with the dividend fluctuations and decreases with the financial market participation rate. Overall, this paper suggests that in the presence of a segmented stock market, there is a new role for monetary policy which is to share financial market risk among all agents in the economy, so they consume the same amount, and is not associated either with minimal stock price volatility or with minimal inflation volatility.

In our paper we take limited participation as given and ignore the interesting

commotions that the participation decision would bring in the model, as Alvarez et al. (2002) and Khan and Thomas (2007) have thoroughly examined. As soon as there exist a positive mass of agents that is discouraged to participate in the financial markets<sup>2</sup>, then financial markets are segmented and our model suggests that monetary policy bears real effects. Under these circumstances, optimal monetary policy shares financial income risk among all agents in the economy, and the optimal monetary policy rule does not depend on the degree of financial market segmentation. Even so we are using an exogenous participation framework, we find that under the optimal monetary policy rule, if agents were given the choice, they would be indifferent between participating in the financial markets or not.

Our work relates to various strands of the literature; in particular it relates to the literature exploring the distributional effects of monetary policy, the financial limited participation literature which explores stock price volatility but ignores monetary policy issues, the empirical literature on the effects of monetary policy on the stock market and vice versa, and the limited participation literature which studies the liquidity effect. Early work on the distributional effects of monetary policy involves models that are not very tractable (Grossman and Weiss, 1983; Rotemberg, 1984), although important attempts to obtain tractability resulted in models that suggest no role for monetary policy (Lucas, 1990; Fuerst, 1992). More recent work has been focusing, similarly to this paper, on the distributional effects of monetary policy<sup>3</sup> and the indications for optimal monetary policy, however for reasons very different than what this work suggests. Monetary policy in Williamson (2005) and Williamson (2006) smoothes across agents the effects of productivity shocks which affect differ-

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<sup>2</sup>because of participation costs, e.g., because of entry costs or lack of information

<sup>3</sup>There is of course large literature exploring the distributional effects of inflation (Doepke and Schneider, 2006; Erosa and Ventura, 2002). In addition, recent work studies monetary policy regimes and the distributional effects that the resulted inflation has (Meh et al., 2008).

ently agents who are better connected with the financial markets than these who are less. Our model focuses on agents heterogeneity in terms of their financial risk holding, and optimal monetary policy smoothes across agents the effects of that risk.

With respect to the financial limited participation literature, Allen and Gale (1994)'s work has been an important motivation for this paper. They assess stock price volatility to be the effect of low participation in financial markets in a model with agents who differ in their liquidity preferences. While the amount of cash available to these agents plays an important role in the model, monetary policy is ignored. In addition, Chien et al. (2007), Guo (2004) and Guvenen and Kuruscu (2006) explore stock price volatility in models with limited participation and idiosyncratic shocks and/or heterogenous preferences. In relation to this literature, we abstract from the issues of endogenous participation, idiosyncratic shocks and heterogenous preferences, to focus on the role of monetary policy in generating stock price volatility.

Concerning the empirical literature, Rigobon and Sack (2001) explore how the federal funds rate reacts to changes in the S&P500 using an improved identification technique. Bernanke and Kuttner (2005) and Rigobon and Sack (2004) study the exact reverse relationship, i.e. the stock market response to monetary policy shocks, and find that stock prices decrease as a response to monetary policy tightening. In addition, Bernanke and Gertler (2000) and Bernanke and Gertler (2001) find that a kind of inflation targeting policy minimizes stock price volatility. These papers are related to this one to the extent that they raise arguments concerning the interplay between the stock market and monetary policy, while the specific methods and models used are unrelated.

More related work is that of Bilbiie (2005) and Andres et al. (2010) who both attempt to specify optimal monetary policy in limited participation New Keynesian models. While Bilbiie (2005) focuses on assigning relative weights on output and

inflation in an interest rate Taylor-type of rule, and [Andres et al. \(2010\)](#) focuses on addressing trade-offs between monetary authority's stabilization goals, we use a very different model to focus on the risk sharing role of optimal monetary policy. In addition, these papers do not address the issue of how stock price volatility is affected by monetary policy. [Challe and Giannitsarou \(2010\)](#) develop a full participation asset pricing New Keynesian model in order to match the empirical documentations of [Bernanke and Kuttner \(2005\)](#) and [Rigobon and Sack \(2004\)](#), i.e., the stock price response to monetary policy shocks. We do not attempt to match the data; we rather explore how various policy assumptions affect the stock market.

Theoretical research has previously utilized the well documented limited participation insight in order to capture the liquidity effect, as in [Alvarez et al. \(2001\)](#), forms of non-neutrality of money as in [Williamson \(2005\)](#) and [Williamson \(2006\)](#) and a positive inflation target as in [Antinolfi et al. \(2001\)](#). While the model we use and the character of the limited participation assumption we employ is very similar to these models, we attempt to answer very different questions.

We proceed as follows: The next section introduces the model economy and studies the competitive equilibrium and asset prices. Section 3 describes the implications and role of the optimal, welfare maximizing monetary policy. Section 4 examines the stock price and inflation volatility for various policy rules and finishes with a subsection discussing the main points. Section 5 concludes.

## 2 The Model Economy

### 2.1 Environment

The model economy consists of the goods market and three asset markets: nominal bond, stock and money market. The bond and stock markets are segmented, so that from a continuum of infinitely lived households of measure one, only  $\lambda \in (0, 1)$  fraction participates in these markets although  $1 - \lambda$  does not. The stock market is introduced in a way similar to [Lucas \(1978\)](#) model. Participating agents receive a share of the stochastic dividend tree according to the amount of stocks they hold. The bonds are introduced for examining the asset pricing of the model and do not affect the agents' behavior.

All agents have identical preferences and seek to maximize their lifetime utility:

$$\sum_{t=0}^{\infty} \beta^t u(c_t^i),$$

where  $\beta$  is the discount factor with  $0 < \beta < 1$ .  $c_t^i$  denotes consumption at time  $t$  for consumer of type  $i$ , with  $i \in \{T, N\}$ , where the two consumer types  $T$  and  $N$  are described below.

The fraction  $1 - \lambda$  of the population which does not participate in the financial markets, or the non-traders ( $i = N$ ) receives every period a fixed real endowment  $y^N$  of the non-storable consumption good. The fraction  $\lambda$  of the population which participates in the financial markets, or the traders ( $i = T$ ), receives every period a fixed real endowment  $y^T$  and a share of the stochastic real total dividend  $\varepsilon_t$ . We assume that there is a firm which receives endowment  $\varepsilon_t$  at period  $t$  and distributes all this amount as dividends to its stock holders. The total dividend  $\varepsilon_t$  is random

and defined as follows:

$$\varepsilon_t = \bar{\varepsilon} + \eta_t. \quad (1)$$

$\eta_t$  is an *iid* shock with mean zero and variance  $\sigma_\varepsilon^2$ , and  $\bar{\varepsilon} > 0$  is the mean of the total dividend shock. At period  $t$  each trader buys  $z_{t+1}$  share of the firm, so  $z_{t+1}\varepsilon_t$  is interpreted as the real dividend each trader receives that period. Consequently, traders have a risky component in their income, while non-traders collect only the fixed endowment  $y^N$ . We define total income in this economy as  $y_t = \varepsilon_t + \lambda y^T + (1 - \lambda)y^N$  and mean income as follows:

$$\bar{y} = \bar{\varepsilon} + \lambda y^T + (1 - \lambda)y^N, \quad (2)$$

In order for total output to be independent from the financial market participation rate  $\lambda$ , we let the total mean income be shared equally among all agents, so that the mean income of the traders equals that of the non-traders, i.e.,  $y^T + \frac{\bar{\varepsilon}}{\lambda} = y^N = \bar{y}$ .

This is a cash-in-advance model, where agents can use only cash when entering the financial and goods markets. Credit is assumed away, introducing the following cash-in-advance constraints:

$$m_t^T + q_t z_t + b_t + \tau_t \geq p_t c_t^T + q_t z_{t+1} + s_t b_{t+1}, \quad (3)$$

for the traders, and

$$m_t^N \geq p_t c_t^N, \quad (4)$$

for the non-traders, where  $p_t$  is the price of the consumption good,  $q_t$  is the price of the share and  $s_t$  is the price of the nominal bond which pays one unit of money next period.

Traders and non-traders enter period  $t$  with available money holdings  $m_t^T$  and  $m_t^N$  respectively. In order to reveal the direct effect that monetary policy has on the financial markets, we let the traders collect a monetary transfer  $\tau_t$  from the monetary authority, although the non-traders do not collect such transfers.<sup>4</sup> Furthermore, agents receive real endowments  $y^T$  and  $y^N$  respectively and traders also receive part of the real total dividend,  $\varepsilon_t$ . In order for money to have value in equilibrium, we assume that no agent consumes her own endowment and dividends, but sells them instead for cash in the goods market. Each household uses the proceeds to buy next period, in the same market, the consumption good from other households.

The financial markets open first (while the goods market remains closed), where at period  $t$  traders can sell the  $b_t$  amount of bonds and  $z_t$  amount of stocks they bought at period  $t - 1$ . Note that  $z_t$  is defined in terms of the amount of titles each trader holds and can be sold at price  $q_t$ , so traders receive  $q_t z_t$  dollars for holding  $z_t$  stocks titles for a period. In contrast, bonds are bought at period  $t - 1$  at the price  $s_{t-1} < 1$  and pay back one unit of money at period  $t$ . Using their money holdings  $m_t^T$ , the money from selling their  $z_t$  stocks, the returns from holding  $b_t$  bonds and the monetary transfer  $\tau_t$ , traders can decide how much new bonds and stocks titles to buy. After the financial markets close, the goods market opens. Households consist of a shopper-seller pair, where the role of the shopper is to get the cash left after the transactions in the financial markets are over, and to buy consumption good from the other agents. The seller gets the real endowment  $y^N$  if she is a non-trader or the real endowment  $y^T$  and part of the real dividends  $\varepsilon_t$  if she is a trader, and sells them for cash. After the operations in the goods market are over, the seller and the shopper meet again, consume the amount of the consumption good the shopper purchased

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<sup>4</sup>This practice is equivalent to open market operations, which affects directly the financial market participants.

and keep the cash the seller received as their money holdings for the next period. Thus, endowments and dividends distributed at period  $t$  do not finance consumption at period  $t$  and do not appear as terms in the cash-in-advance constraints (3) and (4). They are used instead as money holdings for entering period  $t + 1$ .

The budget constraint for the traders is given below:

$$m_t^T + q_t z_t + b_t + \tau_t + p_t z_{t+1} \varepsilon_t + p_t y^T \geq m_{t+1}^T + p_t c_t^T + q_t z_{t+1} + s_t b_{t+1}, \quad (5)$$

where  $d_t = z_{t+1} \varepsilon_t$  are the real dividend payments distributed at period  $t$  (but available to use at  $t + 1$ ).

And similarly, the budget constraint for the non-traders is as follows:

$$m_t^N + p_t y^N \geq m_{t+1}^N + p_t c_t^N. \quad (6)$$

The problem of each household is to maximize their lifetime utility subject to constraints (3) and (5) for the traders and (4) and (6) for the non-traders. Because assets markets operate before the goods market open, holding money after the financial markets close bears positive opportunity cost when the return for bonds or stocks is positive. Only the amount of money required for purchasing the desired amount of consumption good is held and the equilibrium is constructed with binding cash-in-advance constraints. Later we reexamine more carefully the conditions which guarantee that traders' cash-in-advance constraint bind. The budget constraints also bind, as usual. The implications for the budget constraints are:

$$p_t z_{t+1} \varepsilon_t + p_t y^T = m_{t+1}^T, \quad (7)$$

for the traders, and

$$p_t y^N = m_{t+1}^N \quad (8)$$

for the non-traders. The above equations reveal that the cash balances with which the agents begin period  $t + 1$  match the fraction of their wealth that the cash-in-advance constraints prevented them from using at period  $t$ . These are, the proceeds from selling at the goods market the real endowments, and for the case of traders, the real dividends distributed at period  $t$ .

The monetary authority operates by setting the money supply growth each period  $t$ . Whenever money supply increases, the extra cash is distributed as transfers to the traders, although whenever money supply decreases, traders are taxed.<sup>5</sup>

$$\bar{M}_t = \lambda \tau_t + \bar{M}_{t-1},$$

or

$$\bar{M}_t = \bar{M}_{t-1}(1 + \mu_t)$$

where  $\mu_t$  denotes money growth from time  $t - 1$  to time  $t$ .

## 2.2 Competitive Equilibrium and Asset Pricing

This section examines the competitive equilibrium and the asset pricing of the model.

The four market clearing conditions in this economy are as follows:

For the goods market to clear, the total real non-storable consumption good in the economy  $y_t = \varepsilon_t + \lambda y^T + (1 - \lambda)y^N$ , is completely consumed by traders and

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<sup>5</sup>There is nothing in the way we model this authority that prevents us from interpreting it as fiscal authority. However, as we are referring to changes in money supply, we let this authority be the monetary one.

non-traders at every period.

$$\varepsilon_t + \lambda y^T + (1 - \lambda)y^N = \lambda c_t^T + (1 - \lambda)c_t^N.$$

Because of equation (2) that defines mean income, the goods market clearing condition becomes:

$$\bar{y} + \varepsilon_t - \bar{\varepsilon} = \lambda c_t^T + (1 - \lambda)c_t^N. \quad (9)$$

For the stock market to clear, the sum of all shares held across all traders should equal the total stochastic dividend distributed as shares. We assume that all the stochastic total dividend is distributed among the traders:

$$\lambda z_{t+1} = 1 \Rightarrow z_{t+1} = \frac{1}{\lambda}. \quad (10)$$

For the bond market to clear, the sum of all real bonds held across all traders should equal the total supply of them, which is zero:

$$\lambda b_t = 0. \quad (11)$$

For the money market to clear, the total money holdings of traders and non-traders should equal the total amount of money supplied in the economy,  $\bar{M}_t = \bar{M}_{t-1}(1 + \mu_t) = \lambda \tau_t + \bar{M}_{t-1}$ . The money market clearing condition is as follows:

$$\lambda m_{t+1}^T + (1 - \lambda)m_{t+1}^N = \bar{M}_{t-1}(1 + \mu_t), \quad (12)$$

where  $\mu_t$  denotes money growth from time  $t - 1$  to time  $t$ . The extra money supplied at time  $t$  is distributed as transfers to the  $\lambda$  traders.

Substituting the bond, stock and money market clearing conditions in the cash-in-

advance equations (3) and (4) we get the straightforward cash-in-advance restrictions:

$$m_t^T + \tau_t \geq p_t c_t^T,$$

for traders, and

$$m_t^N \geq p_t c_t^N,$$

for non-traders. The above inequalities show that consumption expenditure cannot exceed the monetary resources agents began the period with, plus, for the case of the traders, the monetary transfer. This is true because at any given period, agents do not consume their own endowments and dividend income, but have to sell these resources and finally use them as money balances for the next period.

Furthermore, from the goods market clearing condition (9) and the cash-in-advance constraints (3) and (4) holding with equality, the following condition is implied:

$$p_t Y_t = \lambda q_t(z_t - z_{t+1}) + \bar{M}_t + \lambda(b_t - s_t b_{t+1}).$$

Applying the stock, bond, and money market clearing conditions (10), (11) and (12) in the above equation, we get a version of the quantity equation where velocity equals one and total output is equal to the sum of the deterministic part  $\lambda y^T + (1 - \lambda)y^N = \bar{y} - \bar{\varepsilon}$ , and the stochastic part  $\varepsilon_t$ :

$$p_t = \frac{\bar{M}_t}{\bar{y} + (\varepsilon_t - \bar{\varepsilon})}. \quad (13)$$

While in Alvarez et al. (2001) the instability between prices and money stems from velocity shocks, in this model an important role is played by the stochastic part of output distributed as dividends to the stock market participants. In particular,

an increase in the total real dividend shock the participants receive puts downward pressure on prices while a decrease in the real dividend shock increases the price level.

Equilibrium consumption for the two groups of agents is derived as follows. Combining the non-traders binding cash-in-advance constraint (4) with equation (8) we find:

$$p_{t-1}y^N = p_t c_t^N.$$

Substituting for the consumption good price from equation (13), it turns out that the non-traders consumption is given by the following equation:

$$c_t^N = \frac{p_{t-1}}{p_t} \bar{y} = \bar{y} \frac{\bar{y} + (\varepsilon_t - \bar{\varepsilon})}{\bar{y} + (\varepsilon_{t-1} - \bar{\varepsilon})} \frac{1}{1 + \mu_t}. \quad (14)$$

The above equation together with the market clearing condition for the goods market, given by equation (9), imply that the traders' consumption can be written as follows:

$$\begin{aligned} c_t^T &= \frac{p_{t-1}}{p_t} \left( \bar{y} + \frac{1}{\lambda} (\varepsilon_{t-1} - \bar{\varepsilon}) \right) + \frac{\tau_t}{p_t} = \\ &= \frac{\bar{y} + (\varepsilon_t - \bar{\varepsilon})}{\lambda} \frac{(\varepsilon_{t-1} - \bar{\varepsilon})(1 + \mu_t) + \bar{y}(\lambda + \mu_t)}{(\bar{y} + (\varepsilon_{t-1} - \bar{\varepsilon}))(1 + \mu_t)}. \end{aligned} \quad (15)$$

Note that when there is full participation in the financial markets, consumption for each trader equals total output:

$$c_t^T = \bar{y} + (\varepsilon_t - \bar{\varepsilon}) = y_t,$$

and monetary policy becomes neutral. That is, expansionary monetary policy increases prices but also transfers, which in this case are equally distributed among all agents. These two effects cancel out without affecting consumption. However, in a limited participation economy monetary authority affects prices as before but dis-

tributes transfers only to the traders. In this case, as equations (14) and (15) reveal, monetary policy exhibits distributional effects, controlling the amount consumed by each type of agent. In an expansion, monetary policy creates an inflation tax for all households, but distributes monetary transfers only to the traders; traders' consumption increases and non-traders' consumption decreases. On the other hand, whenever monetary policy contracts, consumption becomes cheaper for both types of agents but only the traders are taxed, so their consumption shrinks although non-traders' consumption increases.

Also, notice how, in the case of limited participation, dividend income affects the consumption of the two groups. The equilibrium consumption equations reveal that an increase in the current total dividends distributed,  $\varepsilon_t$ , implies lower price for the consumption good in period  $t$ , increasing consumption for both traders and non-traders. On the other hand, an increase in total real dividends distributed the previous period,  $\varepsilon_{t-1}$ , decreases the price of the consumption good at period  $t - 1$  and thus the value of the consumption good carried in the form of money balances from period  $t - 1$  to period  $t$ . Assuming that monetary policy does not react to such a shock, that  $1 + \mu_t \geq 0$  and that the dividend shocks are independent across time, consumption in period  $t$  decreases for the non-participants although increases for the participants:

$$\frac{\partial c_t^N}{\partial \varepsilon_{t-1}} = -\frac{(\bar{y} + (\varepsilon_t - \bar{\varepsilon}))\bar{y}}{(1 + \mu_t)(\bar{y} + (\varepsilon_{t-1} - \bar{\varepsilon}))^2} < 0,$$

and

$$\frac{\partial c_t^T}{\partial \varepsilon_{t-1}} = \frac{(\bar{y} + (\varepsilon_t - \bar{\varepsilon}))\bar{y}(1 - \lambda)}{\lambda(1 + \mu_t)(\bar{y} + (\varepsilon_{t-1} - \bar{\varepsilon}))^2} > 0.$$

As explained earlier, the cash-in-advance constraints allow  $\varepsilon_{t-1}$  to be realized in period  $t - 1$  but is available for consumption only in period  $t$ . When  $\varepsilon_{t-1}$  increases, there is an indirect price effect, affecting all agents, and a direct effect affecting only

the traders who actually receive part of the  $\varepsilon_{t-1}$  shock as dividend. Because of the limited participation assumption, the increase in dividends is higher than the decrease in prices and thus the traders are better off. Also note that the increase in traders consumption is  $\frac{1-\lambda}{\lambda}$  times higher than the decrease in non-traders consumption. The increase in traders' consumption depends on the participation rate; the more participating agents there are, the smaller the share of the  $\varepsilon_{t-1}$  shock each of them receives is. Therefore, the increase in traders' consumption is negatively affected by the participation rate.

While the above analysis did not require solving the maximization problem, the study of the asset prices requires such a procedure. In particular the traders' utility maximization problem needs to be solved, subject to the cash-in-advance (3, 4) and budget constraints (5, 6).

It turns out that the price of the bond is determined as follows:

$$\beta E_t \frac{u'(c_{t+1}^T)}{p_{t+1}} = \frac{u'(c_t^T)}{p_t} s_t. \quad (16)$$

Equation (16) describes the pricing of the nominal bond: the utility increments traders expect to receive at period  $t + 1$ , when the bond matures and pays back, equals the foregone utility they suffer from buying the nominal bond at period  $t$ . In addition, this equation reveals the Fisher effect. Defining the nominal interest rate as  $r_t \equiv \frac{1}{s_t} - 1$ , the real rate as  $r_t^r \equiv \frac{p_t}{s_t p_{t+1}} - 1$  and inflation rate as  $\pi_t \equiv \frac{p_{t+1}}{p_t} - 1$ , it turns out that  $r_t = r_t^r \frac{p_{t+1}}{p_t} + \pi_{t+1}$  which gives approximately the Fisher effect.

Note also that for the traders' cash-in-advance constraint to bind, the multiplier associated with the traders' cash-in-advance constraint (3) should be strictly positive, implying that  $s_t < 1$  so the nominal interest rate is strictly positive.

In addition, the first order conditions imply the following expression for the stock

price:

$$\beta E_t \frac{u'(c_{t+1}^T)}{p_{t+1}} (q_{t+1} + p_t \varepsilon_t) = \frac{u'(c_t^T)}{p_t} q_t, \quad (17)$$

which evaluates that the discounted marginal utility expected at period  $t + 1$ , when the dividends are paid and the stock can be traded again, equals the forgone utility at time  $t$  incurred from purchasing the stock.

We will return to the stock price and its volatility later, when we will apply various policy specification in equation (17) and compare across them.

### 3 Optimal Monetary Policy

In this section we study the implications of optimal monetary policy. We consider that the monetary authority acts optimally when aims to maximize total welfare, and we let the instrument of the monetary authority be the money supply growth rate.

We assume that monetary authority assigns equal weight to each agent, no matter in which group an agent belongs to. Then,  $\lambda$  weight is assigned to the group of traders and  $1 - \lambda$  to the group of non-traders. The maximization problem is as follows:

$$Max_{\mu_t} V_t = Max_{\mu_t} \sum_{t=0}^{\infty} \beta^t (\lambda u(c_t^T) + (1 - \lambda)u(c_t^N)).$$

The first order conditions imply the following:

$$\lambda \frac{\partial u(c_t^T)}{\partial c_t^T} \frac{\partial c_t^T}{\partial \mu_t} + (1 - \lambda) \frac{\partial u(c_t^N)}{\partial c_t^N} \frac{\partial c_t^N}{\partial \mu_t} = 0. \quad (18)$$

From equations (14) and (15) which determine consumption in equilibrium, we

can calculate the derivative of consumption with respect to money growth:

$$\frac{\partial c_t^N}{\partial \mu_t} = -\frac{\bar{y} + (\varepsilon_t - \bar{\varepsilon})}{\bar{y} + (\varepsilon_{t-1} - \bar{\varepsilon})} \frac{\bar{y}}{(1 + \mu_t)^2}$$

and

$$\frac{\partial c_t^T}{\partial \mu_t} = \frac{1 - \lambda}{\lambda} \frac{\bar{y} + (\varepsilon_t - \bar{\varepsilon})}{\bar{y} + (\varepsilon_{t-1} - \bar{\varepsilon})} \frac{\bar{y}}{(1 + \mu_t)^2}.$$

Note that non-traders' consumption decreases although traders consumption increases whenever monetary policy expands. This is because expansionary monetary policy increases current prices, affecting negatively all agents; the traders however, receive the monetary transfer and the positive effect is stronger. This is because of the limited participation assumption that prevents monetary authority from directing its transfers to all agents, but shares them instead only among the agents who participate in the financial markets. Also note, as for the case of a change in the previous period stochastic total dividend  $\varepsilon_{t-1}$ , the increase in traders' consumption is  $\frac{1-\lambda}{\lambda}$  times the decrease in non-traders consumption. The increase of traders consumption depends on the participation rate, so that the higher is the participation rate, the smaller is the share of the monetary expansion each of the traders receive.

Substituting the above equations into equation (18) we find the following:

$$\frac{\partial u(c_t^N)}{\partial c_t^N} = \frac{\partial u(c_t^T)}{\partial c_t^T}.$$

Optimal monetary policy will attempt to equate the marginal utility of consumption for the two types of agents. Then for any concave utility specification it turns out that optimal monetary policy equates consumption for the two groups:

$$c_t^T = c_t^N.$$

This implies that optimal money growth is as given below:

$$\mu_t^{opt} = \frac{-(\varepsilon_{t-1} - \bar{\varepsilon})}{\bar{y} + (\varepsilon_{t-1} - \bar{\varepsilon})}$$

or,

$$1 + \mu_t^{opt} = \frac{\bar{y}}{\bar{y} + (\varepsilon_{t-1} - \bar{\varepsilon})}. \quad (19)$$

The above expression reveals a new role for monetary policy, that of sharing the financial income risk among all agents in the economy.<sup>6</sup> Limited participation in the stock market exposes only a fraction of the population to dividend income risk, although all agents are subject to inflation risk. Monetary policy through its distributional effects acquired because of the limited participation assumption, has the ability to undo this market failure<sup>7</sup>, and share the dividend risk among all agents. When the total real dividend is lower than the total mean dividend  $\bar{\varepsilon}$ , optimal monetary policy expands, increasing money supply by the fraction of extra total dividends to total output. This is because whenever traders receive low dividend payments, their consumption decreases. Optimal monetary policy expands and increases traders consumption by distributing higher transfers to them. As monetary policy expands, current prices increase reducing non-traders consumption. On the other hand, whenever the total dividend is higher than the total mean dividend, offering high consumption to traders, then optimal monetary policy contracts, taxing traders, so that prices decrease causing the non-traders' consumption to rise. Also note that because of the cash-in-advance constraint, traders at time  $t$  consume the previous periods' dividend.

Then, optimal policy at time  $t$  reacts to total dividend distributed at time  $t - 1$ .

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<sup>6</sup>Modifying the model so the monetary authority to have intertemporal considerations would not make the Friedman rule optimal. This is because monetary authority has also distributional considerations in this model. This is similarly to Williamson (2005).

<sup>7</sup>We consider the case that limited participation is generated through entry costs, lack of information etc.

As a result, this model suggests that optimal monetary policy responds to advances in the asset markets, however not in the way previous research have considered (). Here the monetary authority responds to real dividend changes in order to redistribute wealth from the privileged financial market participants who were fortunate to encounter good financial markets conditions, to the rest of the population, or, from the rest of the population to the disadvantaged financial market participants who were hit by bad financial markets shocks.

The above result holds for any concave utility function. In addition, it is not sensitive to the fixed endowment assumption. In the case of random mean endowments  $\bar{y}$ , optimal monetary policy would still share the extra risk that participants hold, between the two types of agents and increase in this way total welfare. Also note, that in this model optimal monetary policy reacts to real shocks, without the assumption of price rigidities. This is because of redistribution concerns, similarly to [Williamson \(2005\)](#).

Following optimal monetary policy, the consumption of traders and non traders each period is given below:

$$c_t^{N*} = c_t^{T*} = Y_t = \bar{y} + (\varepsilon_t - \bar{\varepsilon}). \quad (20)$$

We see that optimal monetary policy shares the risk perfectly between the two groups. In addition, although we are using an exogenous participation framework, optimal monetary policy makes agents indifferent between participating in the financial markets or not. Precisely, the non-traders would vote for any policy with  $\mu_t > \mu_t^{opt}$  as such a policy would increase their consumption relative to what they consume under the optimal monetary policy regime. On the contrary, the traders would vote for any policy with  $\mu_t < \mu_t^{opt}$  for the same reason. Optimal monetary policy equates

consumption for the two groups and makes agents indifferent between participating or not in the financial markets.

Finally, note that contrary to [Bilbiie \(2005\)](#), the optimal monetary policy in this model does not depend on the participation rate. This is true because monetary authority has direct effects on the same group of agents that faces the dividends risk, and only on that group. High traders' dividends are offset by contractionary policy and low traders' dividends are enhanced by expansionary policy. Both total dividend and money injections are shared among traders, so that the participation rate does not affect monetary policy's decisions. However, for monetary policy to have any real effect, the limited participation assumption is necessary. Hence, as soon as financial markets are segmented, no matter to what extent, monetary policy can operate in such a way that everybody shares and consumes the total output and becomes indifferent between participating or not in the financial markets.

## 4 Stock Price and Inflation Volatility

### 4.1 Stock Price Volatility

In this section we compute the stock price volatility for four policy rules: optimal, constant money supply, inflation targeting and nominal interest rate pegging. We ask the question whether optimal monetary policy implies lower stock price volatility when compared with the other, frequently cited policy rules, and we find that this is not necessarily the case. In addition, we derive the policy rule associated with minimal stock price volatility and compute the welfare loss of implementing that policy instead of the optimal one. We uncover sizeable welfare losses.

For the analysis that follows we need to specify a utility representation in order

to compute explicitly the stock price and its variance. In this section we use the logarithmic utility function, which is replaced by a general CRRA function in a later section. We first calculate the real stock price,  $\hat{q}_t = \frac{q_t}{p_t}$ , using equation (17). The recursive solution, assuming that there are no bubbles so that the transversality condition holds, is as follows:

$$\hat{q}_t = E_t \sum_{j=1}^{\infty} \beta^j \frac{c_t^T}{c_{t+j}^T} \frac{p_t}{p_{t+j}} \varepsilon_{t+j-1}.$$

That is, we can write the stock price as the expected discounted value of all future dividends. After substituting in, the expressions for prices and trader's consumption given by equations (13) and (15), we find the following expression for the stock price:

$$\begin{aligned} \hat{q}_t &= E_t \sum_{j=1}^{\infty} \beta^j \frac{(\varepsilon_{t-1} - \bar{\varepsilon})(1 + \mu_t) + \bar{y}(\lambda + \mu_t)}{(\bar{y} + (\varepsilon_{t-1} - \bar{\varepsilon}))(1 + \mu_t)} \\ &\quad \frac{(\bar{y} + (\varepsilon_{t+j-1} - \bar{\varepsilon}))(1 + \mu_{t+j})}{(\varepsilon_{t+j-1} - \bar{\varepsilon})(1 + \mu_{t+j}) + \bar{y}(\lambda + \mu_{t+j})} \frac{\varepsilon_{t+j-1}}{\prod_{s=1}^j (1 + \mu_{t+s})}. \end{aligned} \tag{21}$$

In the examples that follow we compute the variance of the stock price for a choice of policy rules. To do that we could either first substitute in equation (21) the policy rule we wish to examine, and then compute the variance of the stock price under that rule, or we could compute the variance of the stock price in (21) and then substitute for  $\mu$ , the policy rule we wish to consider. We are taking the first approach, seeing that in order to calculate the variance of the stock price we first need to linearize around the mean of the total dividend shock. And to do that we need to know how monetary policy reacts to dividend changes. Therefore, substituting first into (21)

the policy rule under consideration and then linearizing around the mean of the total dividend, seems a more appropriate approach to follow.

#### 4.1.1 Optimal Monetary Policy

We compute the stock price under the assumption that monetary policy is conducted optimally:

$$\hat{q}_t^{opt} = E_t \sum_{j=1}^{\infty} \beta^j \frac{\varepsilon_{t+j-1}}{\bar{y}} \prod_{s=1}^j (\bar{y} + (\varepsilon_{t+s-1} - \bar{\varepsilon})).$$

When the shocks hitting total dividends are independently and identically distributed (*iid*), then by linearizing around their mean value,  $\bar{\varepsilon} = \lambda(\bar{y} - y^T)$ , we find that the stock price and its unconditional variance for the optimal monetary policy rule are as follows:

$$\begin{aligned} \hat{q}_t^{opt} &\simeq \frac{\beta\lambda(\bar{y} - y^T)}{1 - \beta} + \frac{\beta}{1 - \beta} \frac{\bar{y}(1 - \beta) + \lambda(\bar{y} - y^T)}{\bar{y}} \eta_t, \\ Var(\hat{q}_t^{opt}) &= \frac{\beta^2 \sigma_\varepsilon^2}{\bar{y}^2(1 - \beta)^2} [\bar{y}(1 - \beta) + \lambda(\bar{y} - y^T)]^2. \end{aligned} \quad (22)$$

where as discussed earlier,  $\sigma_\varepsilon^2$  is the variance of the total dividend and  $\eta_t$  is defined in equation (1).

Higher dividend volatility in (22) translates to higher stock price volatility, since the current dividend shock,  $\eta_t = \varepsilon_t - \bar{\varepsilon}$ , and the stock price are positively correlated. Intuitively, a positive shock in current dividends would make traders want to acquire more stocks. Hence the price of the stock has to increase.

Furthermore, under optimal monetary policy, agents' consumption is not affected by the financial markets' participation rate because it equals aggregate output, as we show in the previous section. However, higher participation increases the mean total dividend, which in turns positively affects the stock price and its variance.

In the following sections we compute the stock price and its variance under various policy rules and compare them to that produced under the optimal monetary policy rule.

#### 4.1.2 Constant Money Supply Policy

We now turn our analysis to the zero money growth policy, which by setting  $\mu_t = 0$  for every  $t$  in equation (21), implies that:

$$\hat{q}_t^{\mu=0} = E_t \sum_{j=1}^{\infty} \beta^j \frac{\lambda \bar{y} + (\varepsilon_{t-1} - \bar{\varepsilon})}{\bar{y} + (\varepsilon_{t-1} - \bar{\varepsilon})} \frac{\bar{y} + (\varepsilon_{t+j-1} - \bar{\varepsilon})}{\lambda \bar{y} + (\varepsilon_{t+j-1} - \bar{\varepsilon})} \varepsilon_{t+j-1}.$$

Assuming *iid* dividend shocks and linearizing around their mean, we get the linearized stock price and its variance:

$$\begin{aligned} \hat{q}_t^{\mu=0} &\simeq \frac{\beta}{1-\beta} \lambda(\bar{y} - y^T) + \frac{\beta}{1-\beta} \frac{(1-\lambda)(\bar{y} - y^T)}{\bar{y}} \eta_{t-1} \\ &\quad + \frac{\beta}{\bar{y}} (\lambda(\bar{y} - y^T) + y^T) \eta_t \\ Var(\hat{q}_t^{\mu=0}) &= \frac{\beta^2 \sigma_\varepsilon^2}{\bar{y}^2 (1-\beta)^2} [(1-\lambda)^2 (\bar{y} - y^T)^2 + (1-\beta)^2 (\lambda(\bar{y} - y^T) + y^T)^2]. \end{aligned} \quad (23)$$

Here we see that higher dividend volatility translates to higher stock price volatility, through its effect on the volatility of consumption and prices. However, the effect of the financial market segmentation on the stock price and its variance is not straightforward. Under the constant money supply rule, consumption depends on the degree of segmentation, since  $c_t^{T,\mu=0} = \frac{p_{t-1}}{p_t} (\bar{y} + \frac{1}{\lambda} (\varepsilon_{t-1} - \bar{\varepsilon}))$ . Higher participation induces a fall in consumption whenever  $\varepsilon_{t-1} - \bar{\varepsilon} = \eta_{t-1} > 0$  as the traders have to share the high dividends with more agents. But this is true for both current and future consumption. That is, whenever the dividend shock affecting consumption in the

next period, i.e  $\eta_t = \varepsilon_t - \bar{\varepsilon}$ , is positive and substantially higher than a positive shock affecting current consumption, i.e  $\eta_{t-1} = \varepsilon_{t-1} - \bar{\varepsilon} > 0$ , then a higher degree of market segmentation decreases future consumption more than the current one, urges traders to buy more assets and hence raises the stock price. The effects of participation on the stock price volatility become ambiguous and depend on the parameter values.

Comparing equations (22) and (23) we see that it is not obvious which of the two policies produces higher stock price volatility. The result would depend again on the parameter values. To illustrate that optimal monetary policy is not necessarily associated with low stock price volatility we use the following example: For  $\bar{y} = 1$ ,  $y^T = 0.9$ ,  $\beta = 0.9$  and  $\sigma_\varepsilon = 0.06$ <sup>8</sup> and leaving the participation rate  $\lambda$  free, we see at Figure (1) that there is a critical value for the participation rate, below which optimal monetary policy produces less volatility than the constant money supply policy, and above which optimal monetary policy generates higher volatility. For convenience, we summarize the parameter values in Table (1).

#### 4.1.3 Inflation Targeting Policy

We now explore the inflation targeting policy and its implication for stock price volatility. Inflation for any monetary policy rule  $\mu_t$  is given below:

$$\pi_t = \frac{p_t - p_{t-1}}{p_{t-1}} = \frac{(1 + \mu_t)(\bar{y} + (\varepsilon_{t-1} - \bar{\varepsilon}))}{\bar{y} + (\varepsilon_t - \bar{\varepsilon})} - 1, \quad (24)$$

and for inflation target  $\pi_t = \bar{\pi}$  the corresponding monetary policy action is:

$$1 + \mu_t^{\bar{\pi}} = (\bar{\pi} + 1) \frac{\bar{y} + (\varepsilon_t - \bar{\varepsilon})}{\bar{y} + (\varepsilon_{t-1} - \bar{\varepsilon})}. \quad (25)$$

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<sup>8</sup>This is between Shiller (1981) estimates of 0.01481 and 0.09828 values, based on two different data sets.

The above equation implies that for  $\bar{\pi} + 1 > 0$ , which is true for positive prices, whenever current dividends are low compared to the previous period, inflationary pressure increases. The monetary authority, aiming at attaining its inflation target, contracts money supply by taxing the traders and reduces inflation back to its target level. Equation (21) for  $\mu_t = \mu_t^{\bar{\pi}}$  becomes:

$$\hat{q}_t^{\mu_t^{\bar{\pi}}} = E_t \sum_{j=1}^{\infty} \beta^j \frac{(\bar{\pi} + 1)(\bar{y} + (\varepsilon_t - \bar{\varepsilon})) - \bar{y}(1 - \lambda)}{(\bar{\pi} + 1)(\bar{y} + (\varepsilon_{t+j} - \bar{\varepsilon})) - \bar{y}(1 - \lambda)} \frac{\varepsilon_{t+j-1}}{(\bar{\pi} + 1)^j}.$$

Linearizing around the mean of the *iid* dividend shocks we get the linearized stock price and its variance for the inflation targeting monetary policy rule:

$$\begin{aligned} \hat{q}_t^{\bar{\pi}} &\simeq \frac{\beta\lambda(\bar{y} - y^T)}{\bar{\pi} + 1 - \beta} + \frac{\beta\eta_t}{(\bar{\pi} + 1 - \beta)(\bar{\pi} + \lambda)(\bar{\pi} + 1)\bar{y}} ((\bar{\pi} + \lambda)\bar{y}(\bar{\pi} + 1 - \beta) + \\ &\quad + \lambda(\bar{y} - y^T)(\pi + 1)^2), \\ Var(\hat{q}_t^{\bar{\pi}}) &= \frac{\beta^2\sigma_{\varepsilon}^2}{(\bar{\pi} + 1 - \beta)^2(\bar{\pi} + \lambda)^2(\bar{\pi} + 1)^2\bar{y}^2} \\ &\quad ((\bar{\pi} + \lambda)\bar{y}(\bar{\pi} + 1 - \beta) + \lambda(\bar{y} - y^T)(\bar{\pi} + 1)^2)^2. \end{aligned}$$

An increase in the variance of the dividend shock increases the volatility of the stock price. The effect of the degree of segmentation is not clear though, as it has ambiguous effects on consumption. For the case of the inflation targeting rule, the consumption of the traders is given below:

$$c_t^{T,\mu=\bar{\pi}} = \frac{1}{\bar{\pi} + 1} (\bar{y} + \frac{1}{\lambda}(\varepsilon_{t-1} - \bar{\varepsilon})) + \frac{(\bar{\pi} + 1)(\bar{y} + (\varepsilon_t - \bar{\varepsilon})) - (\bar{y} + (\varepsilon_{t-1} - \bar{\varepsilon}))}{\lambda(\bar{\pi} + 1)}.$$

Similarly to the constant money supply rule, whenever  $\eta_{t-1} = \varepsilon_{t-1} - \bar{\varepsilon} > 0$ , increased participation decreases current consumption because there are more traders to share

the high dividends. This effect can be seen at the first part of the traders' consumption equation. However, here we have an additional effect through the monetary transfer, which was absent in the constant money supply policy, and shows up at the second part of the consumption equation. Specifically, whenever  $\eta_{t-1} > 0$ , inflation targeting monetary policy contracts. In this case, traders prefer to have many agents in their group, so to share the tax. The total effect of the higher degree of segmentation on the consumption and, as a result, also on the stock price and its variance, depends on the magnitude of the shocks and the specific inflation target the monetary authority sets.

Additionally, depending on the inflation target the monetary authority chooses, we can compare the inflation targeting policy with the optimal and the constant money supply policies. For example, for  $\bar{\pi} = 0$ , the inflation targeting policy implies higher stock price volatility than either of the two alternative policies do. Especially when compared with the optimal policy, the zero inflation targeting policy produces higher stock price volatility because of the limited participation assumption. When there is full participation in the financial markets these two policies generate the same amount of stock price variance. For limited participation however, optimal policy is associated with lower volatility. Hence, in this specific case of zero inflation targeting, optimal monetary policy is indeed associated with lower stock price volatility.

As a second example of inflation targeting policy we consider the policy implied by setting inflation target equal to  $\bar{\pi} = 2\%$ . Using the same parameter values as before, summarized in Table (1), we now compute the stock price variance for the 2% inflation targeting policy. We see at Figure (1) that in this case, choosing the policy that produces the least stock price variability depends crucially on the financial markets participation rate and on the other parameters values.

Using the same parameter values, given in Table (1), and for  $\lambda = 0.35\%$ <sup>9</sup> we see in Figure (2) how stock price volatility varies with the inflation target. It seems that the stock price varies less under the inflation targeting policy when the target is high, implying that an inflation targeting monetary authority which cares about stock price volatility, may be setting higher inflation targets. For these values of the parameters, the optimal policy produces volatility equal to  $Var(\hat{q}_t^{opt}) = 0.0053$  and the constant money supply,  $Var(\hat{q}_t^{\mu=0}) = 0.00378$ . Hence the stock price varies less under the inflation targeting policy when the target is very high, while for lower inflation targets the optimal monetary policy rule and especially the constant money supply policy, generates less volatility than the inflation targeting policy.

#### 4.1.4 Nominal Interest Rate Peg Policy

The last example of policy rule we consider is that of the nominal interest rate peg. Following up with equation (16) and by substituting the traders' equilibrium consumption and the price level, the bond price becomes:

$$s_t = \beta \frac{(\varepsilon_{t-1} - \bar{\varepsilon})(1 + \mu_t) + \bar{y}(\lambda + \mu_t)}{(\bar{y} + (\varepsilon_{t-1} - \bar{\varepsilon}))(1 + \mu_t)} (\bar{y} + (\varepsilon_t - \bar{\varepsilon})) E_t \frac{1}{(\varepsilon_t - \bar{\varepsilon})(1 + \mu_{t+1}) + \bar{y}(\lambda + \mu_{t+1})}.$$

Linearizing the expression inside the expectation around  $\mu_{t+1} = \bar{\mu} = 0$  and solving for the nominal interest rate  $r_t = \frac{1}{s_t} - 1$ , it turns out that the nominal interest rate is:

$$r_t + 1 = \frac{1}{\beta} \frac{(1 + \mu_t)(\bar{y} + (\varepsilon_{t-1} - \bar{\varepsilon}))}{\bar{y} + (\varepsilon_t - \bar{\varepsilon})} \frac{(\lambda \bar{y} + (\varepsilon_t - \bar{\varepsilon}))^2}{(\varepsilon_{t-1} - \bar{\varepsilon})(1 + \mu_t) + \bar{y}(\lambda + \mu_t)} \\ \frac{1}{\lambda \bar{y} + (\varepsilon_t - \bar{\varepsilon}) - (\bar{y} + (\varepsilon_t - \bar{\varepsilon})) E_t(\mu_{t+1})}. \quad (26)$$

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<sup>9</sup>This is approximately the percentage of the US population that Vissing-Jørgensen (2002) classifies as bondholders. Precisely, Vissing-Jørgensen (2002) using the Consumer Expenditure Survey data classifies as stockholders 21.75%, and as bondholders 31.40% of the population interviewed.

Before we explore the stock price implications of the nominal rate peg policy, we first examine the liquidity effect of the model. Differentiating the above expression with respect to current money growth we see that whenever  $E_t(\mu_{t+1}) < \frac{\lambda\bar{y} + (\varepsilon_t - \bar{\varepsilon})}{\bar{y} + (\varepsilon_t - \bar{\varepsilon})}$  then  $\frac{\partial r_t}{\partial \mu_t} < 0$ . Furthermore, this derivative becomes zero when there is full participation in the financial markets, i.e whenever  $\lambda = 1$ , signifying the liquidity effect. In addition, for limited participation and for  $\mu_t + 1 > 0$  it turns out that the nominal interest rate increases with a rise in the expected money supply growth, i.e  $\frac{\partial r_t}{\partial E_t \mu_{t+1}} > 0$ .

To evaluate the stock price volatility we compute the money supply policy implied by pegging the nominal interest rate at the level  $\bar{r}$ , as follows:

$$\mu_t^{\bar{r}} + 1 = \frac{(\bar{r} + 1)\beta(\bar{y} + (\varepsilon_t - \bar{\varepsilon}))\bar{y}(1 - \lambda)}{(\bar{y} + (\varepsilon_{t-1} - \bar{\varepsilon}))[(\bar{r} + 1)\beta(\bar{y} + (\varepsilon_t - \bar{\varepsilon})) - (\lambda\bar{y} + \varepsilon_t - \bar{\varepsilon})]}. \quad (27)$$

The above equation reveals that whenever the dividend shock consumed at the current period is high, i.e.,  $\eta_{t-1} > 0$ , the traders are prompt to buy more assets, the price of the bond rises and the nominal interest rate decreases. The monetary authority which aims at keeping the nominal interest rate at a specific level would then contract, tax the traders, reduce their consumption and bring in this way the nominal interest rate back to its target. In addition, whenever  $\eta_t > 0$  future consumption is expected to rise. Traders tend to buy fewer assets today, forcing the price of the bond to fall and the nominal interest rate to rise. To keep the rate at its target, monetary authorities react by increasing money supply.

The stock price equation (21) for  $\mu = \mu^{\bar{r}}$  becomes:

$$\hat{q}_t^{\bar{r}} = E_t \sum_{j=1}^{\infty} \beta^j \frac{\lambda\bar{y} + (\varepsilon_t - \bar{\varepsilon})}{\bar{y} + (\varepsilon_t - \bar{\varepsilon})} \frac{\bar{y} + (\varepsilon_{t+j} - \bar{\varepsilon})}{\lambda\bar{y} + (\varepsilon_{t+j} - \bar{\varepsilon})} \frac{\varepsilon_{t+j-1}}{((\bar{r} + 1)\beta\bar{y}(1 - \lambda))^j}$$

$$\prod_{s=1}^j \frac{(\bar{y} + (\varepsilon_{t+s-1} - \bar{\varepsilon}))[(\bar{r} + 1)\beta(\bar{y} + (\varepsilon_{t+s} - \bar{\varepsilon})) - (\lambda\bar{y} + (\varepsilon_{t+s} - \bar{\varepsilon}))]}{\bar{y} + (\varepsilon_{t+s} - \bar{\varepsilon})}.$$

Linearizing as usual around  $\bar{\varepsilon} = \lambda(\bar{y} - y^T)$  we find the stock price and its variance for the interest rate pegging policy:

$$\begin{aligned}\hat{q}_t^{\bar{r}} &\simeq \lambda(\bar{y} - y^T) \frac{(\bar{r} + 1)\beta - \lambda}{(\bar{r} + 1)(1 - \lambda) - (\bar{r} + 1)\beta + \lambda} + ((\bar{r} + 1)\beta - \lambda)\eta_t \\ &\quad \frac{(\bar{y} - y^T)(\bar{r} + 1)(1 - \lambda) + \bar{y}((\bar{r} + 1)(1 - \lambda) - (\bar{r} + 1)\beta + \lambda)}{\bar{y}(\bar{r} + 1)(1 - \lambda)((\bar{r} + 1)(1 - \lambda) - (\bar{r} + 1)\beta + \lambda)}\end{aligned}$$

and

$$\begin{aligned}Var(\hat{q}_t^{\bar{r}}) &= \frac{((\bar{r} + 1)\beta - \lambda)^2\sigma_\varepsilon^2}{[(\bar{y}(\bar{r} + 1)(1 - \lambda)((\bar{r} + 1)(1 - \lambda) - (\bar{r} + 1)\beta + \lambda)]^2} \\ &\quad [(\bar{y} - y^T)(\bar{r} + 1)(1 - \lambda) + \bar{y}((\bar{r} + 1)(1 - \lambda) - (\bar{r} + 1)\beta + \lambda)]^2.\end{aligned}\tag{28}$$

The monetary authority' choice of interest rate peg, combined with the parameters values will determine whether or not this policy creates higher stock price volatility than the other policies considered. We first examine the example of pegging the equilibrium rate,  $\bar{r}$ , derived by setting  $\mu = 0$  and  $\varepsilon_t = \bar{\varepsilon}$  in equation (27):

$$\bar{r} + 1 = \frac{1}{\beta}.$$

It turns out that the stock price variance when pegging the equilibrium rate is equal to this produced by targeting zero inflation:

$$Var(\hat{q}_t^{\bar{r}}) = Var(\hat{q}_t^{\bar{r}=0}),$$

which in turn is higher than the stock price variance produced under the optimal monetary policy specification, as we explained in the previous section.

As a second example we consider a monetary authority which pegs a rate of  $\bar{r} + 1 = \frac{2}{\beta}$ . Using for the rest of the parameters the same values as before, summarized in Table (1), we see in Figure (1) that, depending on the parameter values, a different policy produces the least stock price volatility.

In addition, as we did with the inflation targeting policy, we set a value for the degree of financial market participation and see how the stock price variability changes with different nominal rate pegs. We set, as before,  $\lambda = 0.35$  and depict this example in Figure (3). When monetary authority pegs the nominal interest rate at low value, stock price volatility is very high, much higher than what the optimal policy,  $Var(\hat{q}_t^{opt}) = 0.0053$ , or the constant money supply  $Var(\hat{q}_t^{\mu=0}) = 0.00378$ , rules produce. Thus, the exact rate picked by the monetary authority greatly influences the stock price variability generated.

#### 4.1.5 Stock Price Volatility Targeting Policy

We now derive the policy rule that a monetary authority interested in minimizing stock price volatility would implement. We consider a central bank that, as in the case of the optimal rule, reacts to  $\varepsilon_{t-1}$ , the previous period's distributed dividends which become available for consumption in the current period. Specifically, we derive the function  $f(\cdot)$ , given that  $f(\varepsilon_{t-1}) = \mu_t^{Var(q)}$ , where  $\mu_t^{Var(q)}$  is the money growth associated with minimal stock price volatility.

The real stock price equation corresponding to the money supply rule that minimizes stock price volatility is as follows,

$$\hat{q}_t^{Var(q)} = E_t \sum_{j=1}^{\infty} \beta^j \frac{(\varepsilon_{t-1} - \bar{\varepsilon})(1 + \mu_t^{Var(q)}) + \bar{y}(\lambda + \mu_t^{Var(q)})}{(\bar{y} + (\varepsilon_{t-1} - \bar{\varepsilon}))(1 + \mu_t^{Var(q)})}$$

$$\frac{(\bar{y} + (\varepsilon_{t+j-1} - \bar{\varepsilon}))(1 + \mu_{t+j}^{Var(q)})}{(\varepsilon_{t+j-1} - \bar{\varepsilon})(1 + \mu_{t+j}^{Var(q)}) + \bar{y}(\lambda + \mu_{t+j}^{Var(q)})} \frac{\varepsilon_{t+j-1}}{\prod_{s=1}^j (1 + \mu_{t+s}^{Var(q)})},$$

derived by substituting in equation (21) the monetary rule associated with minimum stock price volatility,  $\mu_t = \mu_t^{Var(q)}$ . The objective of the exercise is to specify this policy rule.

Linearizing the above expression around the mean total dividend,  $\bar{\varepsilon} = \lambda(\bar{y} - y^T)$ , we find that the linearized expression for the stock price volatility and the variance of it are respectively:

$$\hat{q}_t^{Var(q)} \simeq \frac{\beta\bar{\varepsilon}}{1 + f(\bar{\varepsilon}) - \beta} + \frac{\beta\bar{\varepsilon}}{1 + f(\bar{\varepsilon}) - \beta} \frac{(1 - \lambda)(1 + f(\bar{\varepsilon}) + \bar{y}f'(\bar{\varepsilon}))}{\bar{y}(1 + f(\bar{\varepsilon}))(\lambda + f(\bar{\varepsilon}))} (\varepsilon_{t-1} - \bar{\varepsilon})$$

and

$$Var(\hat{q}_t^{Var(q)}) = \frac{\beta^2\bar{\varepsilon}^2}{(1 + f(\bar{\varepsilon}) - \beta)^2} \frac{(1 - \lambda)^2(1 + f(\bar{\varepsilon}) + \bar{y}f'(\bar{\varepsilon}))^2}{\bar{y}^2(1 + f(\bar{\varepsilon}))^2(\lambda + f(\bar{\varepsilon}))^2} \sigma_\varepsilon^2.$$

The above expression equals zero whenever the following is true:

$$1 + f(\bar{\varepsilon}) + \bar{y}f'(\bar{\varepsilon}) = 0, \quad (29)$$

from where it turns out that the monetary policy rule that minimizes stock price volatility is as follows:

$$1 + \mu_t^{Var(q)} = \frac{\bar{y} - \varepsilon_{t-1}}{\bar{y}} + c,$$

where  $c$  is a constant, which can be specified by the initial condition 29, as  $c = \frac{\bar{\varepsilon}}{\bar{y}}$ .

Then, the monetary rule that minimizes stock price volatility is as given below:

$$1 + \mu_t^{Var(q)} = \frac{\bar{y} - (\varepsilon_{t-1} - \bar{\varepsilon})}{\bar{y}}, .$$

The above policy turns out to be equivalent to the optimal monetary policy rule in the case of full participation in the financial markets or in the case that this economy is at its steady state, i.e., when the dividend shocks equal their mean. However, there are important differences between the two policies, producing welfare losses, in the case of limited participation in the financial markets and when dividend shocks differ from their mean.

The optimal policy seems to be more expansionary than the stock price volatility minimizing policy. Then, if given the choice between the two policies, financial market participants would prefer the optimal monetary policy, although financial market non participants would prefer the stock price minimizing policy.

We compare the two policies in terms of total welfare:

$$V_t^{Var(q)} - V_t^{opt} = \sum_{t=0}^{\infty} \beta^t (\lambda u(c_t^{T,Var(q)}) + (1-\lambda)u(c_t^{N,Var(q)})) - \sum_{t=0}^{\infty} \beta^t (\lambda u(c_t^{T,opt}) + (1-\lambda)u(c_t^{N,opt})).$$

After substituting for consumption implied by each policy, using equations (15), (14), (20), the logarithmic utility implies the following expression for the welfare loss from implementing stock price volatility minimizing versus the optimal policy rule:

$$V_t^{Var(q)} - V_t^{opt} = \sum_{t=0}^{\infty} \beta^t (\lambda \ln(1 - \frac{(\varepsilon_{t-1} - \bar{\varepsilon})^2}{\lambda(\bar{y})^2}) - \ln(1 - \frac{(\varepsilon_{t-1} - \bar{\varepsilon})^2}{(\bar{y})^2})).$$

The above expression shows that there is no welfare loss when there is full participation in the financial markets or in the case that this economy is at its steady state. However, any other case produces welfare loss. As the expression above indicates, welfare loss is decreasing in the level of financial market participation, as higher degree of participation decreases the financial market participant's consumption under the stock price volatility minimizing policy. In addition, welfare loss increases with

the distance of the dividend shock from its mean, suggesting that optimal monetary policy rule becomes more vital in economies with high dividend fluctuations.

## 4.2 Inflation Volatility

In this section we examine the inflation volatility that the optimal, constant money supply, inflation targeting and interest rate pegging policy rules imply. The general expression for inflation is given by equation (24), in which we substitute the relevant policy rule to get:

$$\pi_t^{opt} = \frac{-(\varepsilon_t - \bar{\varepsilon})}{y + (\varepsilon_t - \bar{\varepsilon})},$$

for the optimal monetary policy rule,

$$\pi_t^{\mu=0} = \frac{-(\varepsilon_t - \varepsilon_{t-1})}{y + (\varepsilon_t - \bar{\varepsilon})},$$

for the constant money supply policy rule,

$$\pi_t^{\bar{\pi}} = 0,$$

for the zero inflation targeting policy rule,

$$\pi_t^{\bar{r}} = \frac{-((\bar{r} + 1)\beta - 1)(\lambda\bar{y} + (\varepsilon_t - \bar{\varepsilon}))}{(\bar{r} + 1)\beta(\bar{y} + (\varepsilon_t - \bar{\varepsilon})) - (\lambda\bar{y} + (\varepsilon_t - \bar{\varepsilon}))},$$

for the nominal interest rate pegging policy rule.

Of course inflation is at its lowest whenever the exact inflation targeting policy is employed. Moreover, comparing the first two equations we see that optimal monetary policy implies lower inflation than the constant money supply policy rule whenever  $\varepsilon_{t-1} > \bar{\varepsilon}$ . That is, every time the total dividend is above average, optimal monetary

policy generates lower inflation than the constant money supply policy. This is true since the constant money supply policy, contrary to the optimal policy, does not react to dividend changes. As a result, all these changes are transferred into prices. When the previous period's total dividend is high, the previous period's prices are low and then, given today's total dividend, inflation is high. On the other hand, optimal policy completely compensates for the effects of the previous period's dividend changes, so there is no effect on inflation. In total, when the previous period's total dividend is high, inflation is higher under the constant money supply policy rule than under the optimal policy rule.

In addition, the nominal interest rate pegging policy generates minimal, equal to zero, inflation if the monetary authority pegs the rate at its equilibrium level,  $\bar{r} + 1 = \frac{1}{\beta}$ . However, if we wish to compare the inflation produced under other pegs, with that produced by the optimal and the constant money supply policy rules, the outcome will depend on the actual choice of rate, how large the dividends shocks are and on the other parameter values.

To compute the inflation variance implied by the policies above, we first linearize the inflation equations around the mean dividend value  $\bar{\varepsilon}$  and then calculate the variance:

$$Var(\pi_t^{opt}) = \frac{\sigma_\varepsilon^2}{\bar{y}^2},$$

for the optimal monetary policy rule,

$$Var(\pi_t^{\mu=0}) = \frac{2\sigma_\varepsilon^2}{\bar{y}^2},$$

for the constant money supply policy rule,

$$Var(\pi_t^{\bar{\pi}}) = 0,$$

for the zero inflation targeting policy rule,

$$Var(\pi_t^{\bar{r}}) = \sigma_{\varepsilon}^2 \frac{((\bar{y} + 1)\beta - 1)^2(\bar{r} + 1)^2\beta^2(1 - \lambda)^2}{((\bar{r} + 1)\beta - \lambda)^4\bar{y}^2},$$

for the nominal interest rate pegging policy rule.

The above analysis implies that inflation variance is minimized under the inflation targeting policy, while the optimal policy is always associated with less inflation volatility than the constant money supply policy. This is again because under the optimal policy the dividend shocks from the previous period are offset, while under constant money supply they are not. These shocks increase the volatility of inflation. On the other hand, the comparison with the interest rate pegging policy rule is not straightforward. When the monetary authority pegs the rate at its equilibrium level  $\bar{r} + 1 = \frac{1}{\beta}$ , then inflation volatility is zero, but for any other choice of rate the result would depend on the parameter values.

Using the same values as in the previous section, given in Table (1), and for  $\lambda = 35\%$  we see in Figure (4) how inflation volatility changes depending on the choice of interest rate peg. For these values of the parameters, given in Table (1), the optimal policy produces volatility equal to  $Var(\pi_t^{opt}) = 0.0036$  while the constant money supply produces volatility equal to  $Var(\pi_t^{\mu=0}) = 0.0072$ . In this case it seems that the interest rate pegging policy, for a rate less than one, is associated with lower inflation volatility compared to the optimal and the constant money supply policies.

### 4.3 Risk Aversion

In this section we use the more general constant relative risk aversion utility function  $u(c_t) = \frac{c_t^{1-a}}{1-a}$  to examine the effects of risk aversion on the stock price and its volatility. Using the stock pricing equation (17) and solving for the recursive form of the real

stock price,  $\hat{q}_t = \frac{q_t}{p_t}$ , assuming as before that the transversality condition holds, we obtain the following expression for the stock price:

$$\hat{q}_t = E_t \sum_{j=1}^{\infty} \beta^j \left( \frac{c_t^T}{c_{t+j}^T} \right)^a \frac{p_t}{p_{t+j}} \varepsilon_{t+j-1},$$

and by substituting in the above expression the price of the consumption good and trader's consumption given by equations (13) and (15), stock price becomes:

$$\begin{aligned} \hat{q}_t &= E_t \sum_{j=1}^{\infty} \beta^j \left( \frac{\bar{y} + (\varepsilon_t - \bar{\varepsilon})}{\bar{y} + (\varepsilon_{t+j} - \bar{\varepsilon})} \frac{(\varepsilon_{t-1} - \bar{\varepsilon})(1 + \mu_t) + \bar{y}(\lambda + \mu_t)}{(\bar{y} + (\varepsilon_{t-1} - \bar{\varepsilon}))(1 + \mu_t)} \right)^a \\ &\quad \left( \frac{(\bar{y} + (\varepsilon_{t+j-1} - \bar{\varepsilon}))(1 + \mu_{t+j})}{(\varepsilon_{t+j-1} - \bar{\varepsilon})(1 + \mu_{t+j}) + \bar{y}(\lambda + \mu_{t+j})} \right)^a \frac{\bar{y} + (\varepsilon_{t+j} - \bar{\varepsilon})}{\bar{y} + (\varepsilon_t - \bar{\varepsilon})} \frac{\varepsilon_{t+j-1}}{\prod_{s=1}^j (1 + \mu_{t+s})}. \end{aligned} \quad (30)$$

Similarly to the previous section, we calculate the linearized version for the price of the stock and its variance for various policy rules.

For the optimal monetary policy, the stock price and its volatility are:

$$\begin{aligned} \hat{q}_t^{opt} &= \frac{\beta \lambda (\bar{y} - y^T)}{1 - \beta} + \frac{\beta}{1 - \beta} \frac{\bar{y}(1 - \beta) + a \lambda (\bar{y} - y^T)}{\bar{y}} \eta_t, \\ Var(\hat{q}_t^{opt}) &= \frac{\beta^2 \sigma_\varepsilon^2}{\bar{y}^2 (1 - \beta)^2} [\bar{y}(1 - \beta) + a \lambda (\bar{y} - y^T)]^2, \end{aligned} \quad (31)$$

where an increase in the risk aversion parameter increases the variance of the stock. This is because for the optimal monetary policy rule, consumption at period  $t + j$  depends only on the shocks  $\eta_{t+j}$ , which in expectation equal their mean, i.e., zero, making the linearized stock price variant only because of the current dividend shocks. Hence, whenever there is a current dividend shock, a more risk averse trader reacts more severely in terms of his willingness to purchase stocks, compared to a less risk

averse trader. This is reflected at the expressions for the stock price and its variance.

For the constant money supply policy, stock price and its volatility are:

$$\begin{aligned}\hat{q}_t^{\mu=0} &\simeq \frac{\beta}{1-\beta} \lambda(\bar{y} - y^T) + a \frac{\beta}{1-\beta} \frac{(1-\lambda)(\bar{y} - y^T)}{\bar{y}} \eta_{t-1} \\ &+ \frac{\beta}{(1-\beta)\bar{y}} (\lambda(\bar{y} - y^T)(2a - 1 - a\beta) + (\bar{y} - a(\bar{y} - y^T)(1 - \beta))) \eta_t, \\ Var(\hat{q}_t^{\mu=0}) &= \frac{\beta^2 \sigma_\varepsilon^2}{\bar{y}^2(1-\beta)^2} [a^2(1-\lambda)^2(\bar{y} - y^T)^2 + \\ &(\lambda(\bar{y} - y^T)(2a - 1 - a\beta) + (\bar{y} - a(\bar{y} - y^T)(1 - \beta)))^2],\end{aligned}\tag{32}$$

where an increase in risk aversion has no clear effect on the stock price variance.

For the inflation targeting policy, stock price and its volatility are:

$$\begin{aligned}\hat{q}_t^{\bar{\pi}} &= \frac{\beta \lambda (\bar{y} - y^T)}{\bar{\pi} + 1 - \beta} + \frac{\beta}{(\bar{\pi} + 1 - \beta)(\bar{\pi} + \lambda)(\bar{\pi} + 1)\bar{y}} ((\bar{\pi} + \lambda)\bar{y}(\bar{\pi} + 1 - \beta) + \\ &+ a\lambda(\bar{y} - y^T)(\pi + 1)^2) \eta_t, \\ Var(\hat{q}_t^{\bar{\pi}}) &= \frac{\beta^2 \sigma_\varepsilon^2}{(\bar{\pi} + 1 - \beta)^2(\bar{\pi} + \lambda)^2(\bar{\pi} + 1)^2\bar{y}^2} \\ &((\bar{\pi} + \lambda)\bar{y}(\bar{\pi} + 1 - \beta) + a\lambda(\bar{y} - y^T)(\bar{\pi} + 1)^2)^2,\end{aligned}\tag{33}$$

where an increase in the risk aversion parameter, increases the variance of the stock price.

We do not compute the stock price volatility for the case of interest rate pegging because this calculation would require ex ante additional assumptions about the correlation of monetary policy's reaction with the dividends shocks. Figure (5) shows how the stock price variance for the optimal monetary policy and the 2% inflation targeting rule changes with the degree of financial markets segmentation. The volatil-

ity of constant money supply policy is much higher to be represented in this graph. A risk aversion parameter of  $a = 2$  is used, while the rest of the parameter values are kept as before, summarized in Table (1). We see, once more, that there is no conclusive evidence concerning which policy minimizes stock price volatility.

## 4.4 Discussion

We conclude this section concerning stock price and inflation volatility by discussing some important points derived from the analysis above. First, it is clear that the optimal monetary policy does not necessarily associate with lower stock price or inflation volatility when compared with the rest of the policy rules considered. The outcome of the comparisons depends on the parameter values.

There are, however, some definite conclusions. Firstly, there is welfare cost from implementing a stock price volatility targeting policy, compared to the optimal policy. The welfare loss increases with higher dividend fluctuations, and decreases with higher financial market participation. Higher financial market participation decreases traders' consumption under the stock price volatility targeting policy, decreasing in this way the welfare cost produced by that policy.

In addition, the optimal monetary policy delivers lower stock price volatility than the inflation targeting policy when the target is set to zero, and lower stock price volatility than the interest rate pegging policy, when the peg is set to its equilibrium level. However, optimal monetary policy generates higher inflation volatility when compared to these two specific policies. Thus, it seems, there is a trade off between inflation volatility and stock price volatility when we compare optimal monetary policy with the policies of zero inflation targeting and equilibrium rate pegging.

In addition, we find that the optimal monetary policy always produces lower in-

flation volatility than the constant money supply policy rule. Also, for the parameter values used, summarized in Table (1), it seems that the nominal interest rate pegging policy gives much less inflation volatility than these two policies. Nevertheless, this result cannot be generalized and depends on the choice of the rate peg and the other parameter values.

Furthermore, we see in Figure (1) that increased participation does not necessarily imply lower stock price volatility. This observation is in contrast to [Allen and Gale \(1994\)](#) who, using a model with individual shocks, argue that high variability in stock prices is encouraged by low stock market participation. When monetary policy actions are taken into account, the effect of the increased financial market participation on stock price volatility may become more complicated.

Moreover, comparing our assessment about the stock price volatility various policies produce to [Bernanke and Gertler \(2001\)](#) results, we argue that limited participation is important in computing stock price volatility, and ignoring this issue might affect the conclusions reached. However, we employ a very different model than the one [Bernanke and Gertler \(2001\)](#) do.

Finally, we explore the variability of the stock prices for a general, constant relative risk aversion utility function, and we observe that there is still no definite answer about which policy produces minimal stock price volatility. The outcome of the comparisons still depends on the parameter values.

In conclusion, it seems that a monetary authority wishing to maximize total welfare does not have the role of minimizing the volatility either of the stock price or of inflation.

## 5 Conclusions

In a segmented financial market model, a novel role arises for the monetary policy: that of sharing the financial market risk the financial market participants face among all agents in the economy, maximizing in this way total welfare. Such a policy does not necessarily imply lower stock price volatility or inflation volatility when compared to other policy rules, indicating that, in our model, minimizing these volatilities is not the focus of optimal monetary policy.

In our setting, optimal monetary policy perfectly shares the dividend risk between traders and non-traders. Whenever dividend income is low, monetary authority distributes monetary transfers to the traders. Such a response increases prices and lowers non-traders' consumption. On the other hand, whenever dividend income is high, monetary policy contracts and taxes traders; as a result, prices decrease and non-traders are benefited. This policy equalizes consumption of the two groups and agents, if given the choice, would be indifferent between participating or not in the financial markets. These results hold for any concave utility function and are not sensitive to the degree of market segmentation.

We also examine what the optimal monetary policy implies for the stock price volatility and inflation volatility and compare these implications with that of others, commonly used monetary policy rules. Specifically, we consider the constant money supply, inflation targeting, nominal interest rate pegging and stock price volatility targeting policy rules and conclude that the optimal monetary policy does not imply lower stock price volatility or inflation volatility than these policies. The outcome of these comparisons depends on the model's parameter values.

In conclusion, this work suggests a new role for the optimal monetary policy, that of sharing financial market risk between agents who encounter this risk and agents

who do not, equalizing in this way their consumption. This policy does not necessarily imply minimal stock price volatility or inflation volatility.

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# Appendix

## Tables

Parameter	Symbol	Value
Mean income	$\bar{y}$	1
Trader's endowment	$y^T$	0.9
Total dividend variation	$\sigma_\varepsilon$	0.06
Discount factor	$\beta$	0.9

Table 1: Parameter Values

## Figures

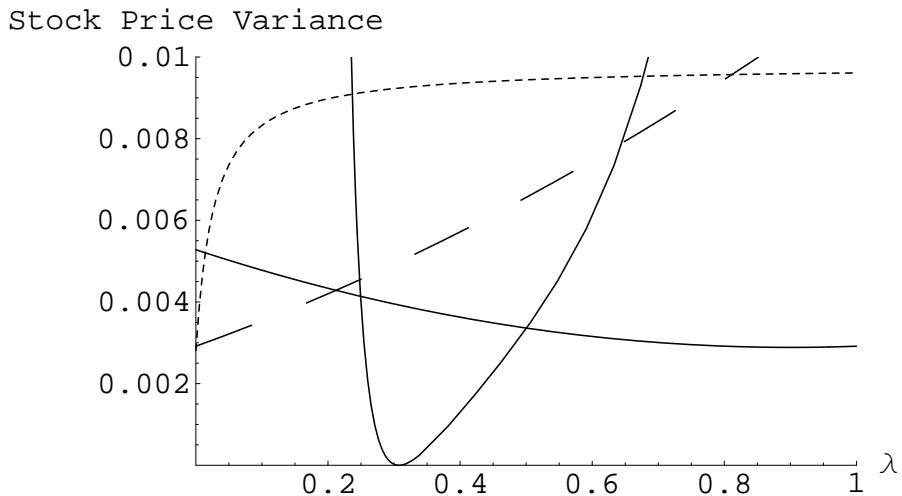


Figure 1: Stock price volatility of optimal (dashed line), constant money supply (downward sloping solid line), 2% inflation targeting (dotted line) and nominal interest rate peg at  $\bar{r} + 1 = \frac{2}{\beta}$  (u -shaped solid line) policy rules, for the parameter values given at Table (1).

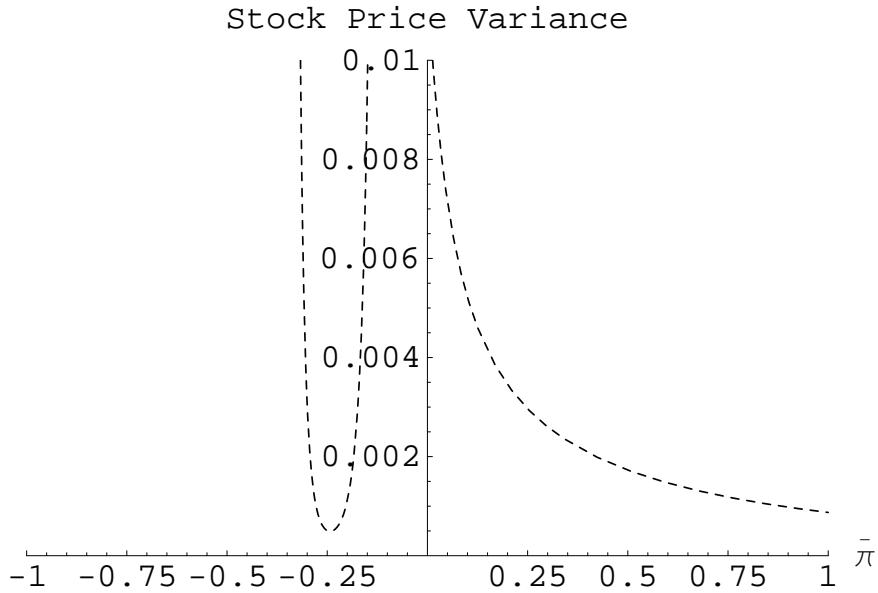


Figure 2: Stock price volatility of the inflation targeting policy, for  $\lambda = 35\%$ , and for the parameter values given in Table (1).

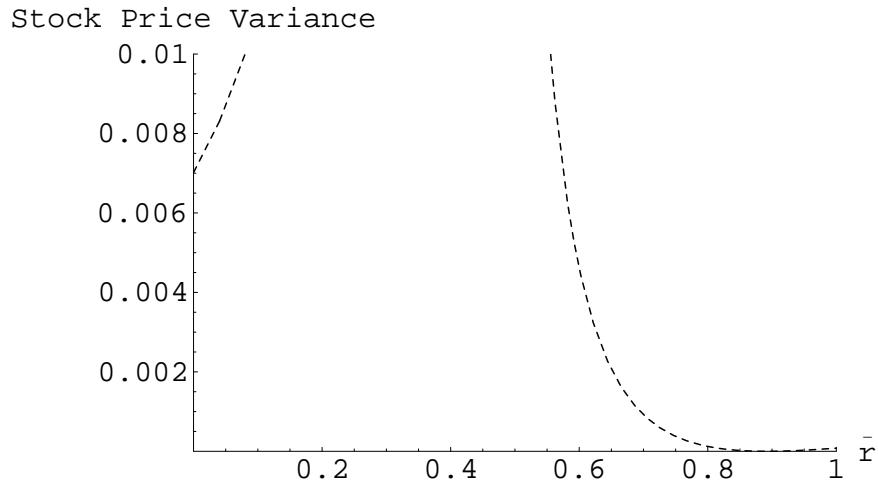


Figure 3: Stock price volatility of the nominal interest rate pegging policy, for  $\lambda = 35\%$ , and for the parameter values given in Table (1).

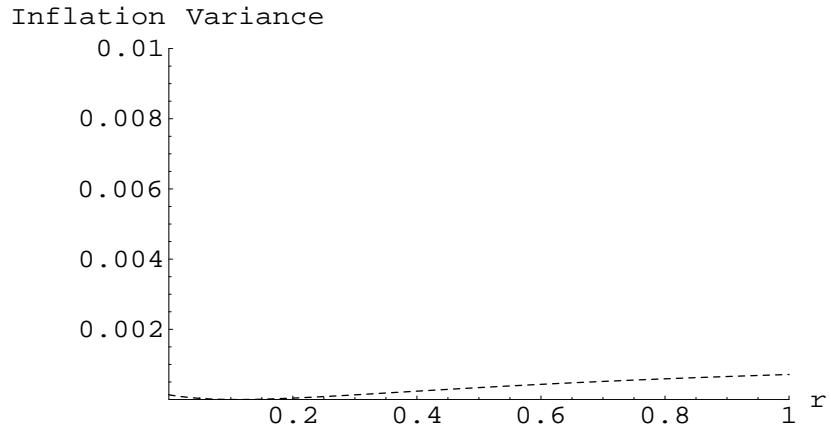


Figure 4: Inflation volatility of the nominal interest rate peg policy for  $\lambda = 35\%$ , and for the parameter values given in Table (1).

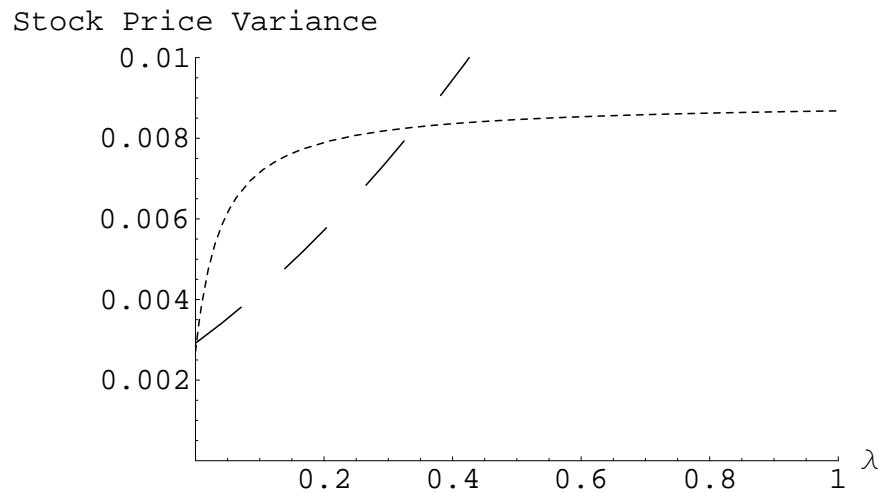


Figure 5: Stock price volatility of optimal (dashed line) and of 2% inflation targeting (dotted line) policy rules, for risk aversion parameter  $a = 2$  and for the parameter values given in Table (1). The constant money supply policy generates much higher volatility to be presented in the graph