

Bad News for News Shocks? The Expectational-Stability and Comovement Tradeoff in a News-Driven Business Cycle Model

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Abstract

This paper investigates whether adaptive learning and news shocks, operating in tandem, can generate business cycle dynamics that match the data. In doing so, this paper adopts expectational stability (“E-stability”) as a natural criterion for rationality: plausible equilibria should arise from an adaptive learning formulation where agents form forecasts from a correctly specified model whose parameters are updated in real time. In examining the model’s stability properties, I find that the rational expectations equilibrium (REE) is not learnable for calibrated parameter values capable of generating news-driven recessions. In particular, I uncover a tradeoff between E-stability, or learnability, and the model’s ability to produce realistic business cycles: REE associated with parameter values necessary for generating recessions in response to news shocks are not learnable, and parameter values that imply learnable REE cannot generate empirically plausible recessions in response to news shocks. This finding thus has implications on the plausibility of rational expectations solutions in business cycle models driven by news.

Keywords: adaptive learning, E-stability, news-driven business cycle models

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1 Introduction

There has been renewed interest in business cycle models driven by news, partly due to the investment boom in the late 1990s and a key empirical finding by Beaudry and Portier (2006) that an identified news shock uncorrelated with current technology can explain approximately 50% of business cycle fluctuations.¹ The news-driven business cycle literature posits a theory of fluctuations resurrecting an old idea from Arthur Pigou where booms and busts can arise from agents' forecast errors.² In these models, agents receive news about future productivity which can produce aggregate fluctuations without any concurrent change in fundamentals. Beaudry and Portier (2004), Jaimovich and Rebelo (2009), and Lorenzoni (2010) are recent examples. Models with news shocks can generate recessions absent any technological regress, which is both intuitively appealing and consistent with recent recessionary episodes.

While the existing literature has been somewhat successful in producing empirically plausible news-driven cycles, these models make a strong assumption about individual rationality. In particular, these models assume *rational expectations* where agents know the true structure of the economy and optimal forecasts are formed using all available information. Under rational expectations, news-driven cycles emerge in part because agents coordinate on equilibria that depend on news about future productivity. A promising literature has emerged that relaxes the rational expectations assumption in favor of *adaptive learning*, where agents behave as econometricians who formulate forecasting models and update the parameters of their model in real-time.³ Thus a natural question is whether deviating from rational expectations in the form of learning can still lead

¹Using a structural VAR approach, Beaudry and Portier (2006) isolate two disturbances from post-war U.S. data: one that represents innovations in stock prices orthogonal to innovations in TFP, and one that drives long-run movements in TFP. These two disturbances are found to be almost perfectly collinear and induce the same dynamics, thus indicating that stock price movements that are not correlated with current changes in TFP are highly correlated with long-run changes in TFP. The authors interpret their identified shock as representing news about future technological opportunities.

²Pigou (1927): "... a rise in prices, however brought about, by creating some actual and some counterfeit prosperity for business man, is liable to promote an error of optimism, and a fall in prices an error of pessimism, and this mutual stimulation of errors and price movements may continue in a vicious spiral until it is checked by some interference from outside."

³Evans and Honkapohja (2001) provides a comprehensive review and treatment of the adaptive learning approach.

agents to coordinate on a news-driven equilibrium.

The objective of this paper is to investigate the effect of adaptive learning in a news-driven economy. In a general business cycle environment, Eusepi and Preston (2011) establish that learning can improve the model's internal propagation. Rather than an exogenous news process that affects the economy, shifts in expectations through the learning process drive the business cycle with stationary fluctuations around the rational expectations equilibrium (REE). Agents' shifting beliefs over time due to learning about the economy's structure can add additional frictions to the model; thus it is reasonable to suspect that the interplay between news and learning can yield realistic business cycle dynamics.

However, intuition presupposes that the equilibrium in the news-driven model is an appropriate benchmark. This paper follows an extensive literature that judges the REE as plausible if it can be learned by economic agents, endowed with a correctly specified model whose parameters are updated in real time.⁴ Despite the extensive theoretical work on news shock models, there has been little work done examining the stability of these models under learning.⁵ To generate additional insight, this paper focuses on the Jaimovich and Rebelo (2009) business cycle model driven by news about investment-specific productivity and examines its learnability properties.

In assessing whether an equilibrium is stable under learning, the standard approach in the literature assumes that agents know the functional form of the model economy but must learn the true rational expectations equilibrium parametrization. To model adaptive learning, agents are endowed with an econometric forecasting model and must estimate the parameters of that model in real-time using a recursive least squares learning algorithm. While attempting to learn this parametrization, agents' forecasts of future endogenous variables will necessarily differ from rational expectations forecasts. Hence, the crucial question is whether the adaptive learning process

⁴This approach follows Lucas (1986), Bray and Savin (1906), Marcet and Sargent (1989), and Evans and Honkapohja (2001).

⁵The stability of rational expectations under learning in standard real business cycle models has been studied by Evans and Honkapohja (2001), Bullard and Duffy (2004), Carceles-Poveda and Giannitsarou (2007), and Eusepi and Preston (2008).

leads agents toward, or away from the REE. If the learning process converges to the REE, that equilibrium is said to be stable under adaptive learning, or “Expectationally Stable” (E-stable). In this way, stability under learning can provide an important check on the plausibility of rational expectations equilibria.

The central results of this paper can be concisely summarized. When introducing adaptive learning to the Jaimovich and Rebelo (2009) news-driven business cycle model, there is a tradeoff between learnability and the model’s ability to produce realistic business cycles. In particular, REE associated with parameter values necessary for generating recessions in response to news shocks are not learnable, and parameter values that imply learnable REE cannot generate empirically plausible recessions in response to news shocks. This finding thus has implications on the plausibility of rational expectations solutions in business cycle models driven by news. In turn, this paper ties with a previous literature that demonstrated that non-convex real business cycle models yield equilibria that depend on sunspots and are not learnable. In particular, Evans and McGough (2005) and Duffy and Xiao (2005) showed that the set of parameter values consistent with empirically realistic business cycles are inconsistent with E-stable equilibria. Like the real business cycle model with non-convexities, news-driven models depend on strong informational assumptions, and the parameter values that yield stability will not produce empirically plausible economic dynamics.

The organization of this paper proceeds as follows. Section 2 reviews the Jaimovich and Rebelo (2009) environment of a one-sector business cycle model augmented with gradual capital adjustment and news about investment-specific productivity. Section 3 introduces learning and derives stability conditions for the model. The main numerical results of the paper are presented and discussed in Section 4. Finally, Section 5 concludes and offers directions for future research.

2 The Environment

The basic environment is the Jaimovich and Rebelo (2009), henceforth JR, business cycle model augmented with a new class of preferences, variable utilization, and gradual capital adjustment.

These additions to the standard neoclassical growth model can produce empirically recognizable business cycles in response to both contemporaneous shocks and news shocks about future productivity. In this setting, news shocks consist of information that can predict future fundamentals but has no effect on current fundamentals.

2.1 One-Sector Business Cycle Model

The economy consists of identical agents who maximize their expected lifetime utility (U) over consumption (C_t) and labor hours (N_t):

$$U = E_0 \left\{ \sum_{t=0}^{\infty} \frac{\beta^t}{1-\sigma} (C_t - \psi N_t^\theta X_t)^{1-\sigma} - 1 \right\} \quad (1)$$

where

$$X_t = C_t^\gamma X_{t-1}^{1-\gamma} \quad (2)$$

The operator E denotes the expectation conditional on agents' time t information set. $0 < \beta < 1$ is the discount factor; $\theta > 1$ and $\psi > 0$ parameterizes the elasticity of the labor supply; and $\sigma > 0$ represents the curvature of agents' utility function.

The JR model introduces a new class of preferences that parameterizes the short-run wealth effect on the labor supply. These preferences nest two classes of utility functions used in the business cycle literature: King, Plosser, and Rebelo (1988) preferences when $\gamma = 1$ (designed to be consistent with balanced growth) and Greenwood, Hercowitz, and Huffman (1988) preferences when $\gamma = 0$ (designed to shut down the wealth effect on labor).⁶ The variable X_t makes agents' preferences non-separable in consumption and labor hours.

Cobb-Douglas production of output (Y_t) depends on total factor productivity (A_t), capital stock

⁶Using a Bayesian estimation approach, Schmitt-Grohe and Uribe (2007) estimate that $\gamma = 0$, that is a near-zero wealth elasticity of labor supply. These preferences are consistent with steady-state growth so long as $0 < \gamma \leq 1$.

(K_t), capital utilization (u_t), and labor (N_t):

$$Y_t = A_t(u_t K_t)^{1-\alpha} N_t^\alpha \quad (3)$$

Output is used for consumption and investment:

$$Y_t = C_t + \frac{I_t}{z_t} \quad (4)$$

Here z_t represents the productivity of investment goods. Increases in z_t can be interpreted as investment-specific technological progress.

The evolution of capital is governed by the following:

$$K_{t+1} = I_t \left[1 - \phi\left(\frac{I_t}{I_{t-1}}\right) \right] + [1 - \delta(u_t)] K_t \quad (5)$$

Changing investment from its level in the previous period incurs an adjustment cost $\phi(\cdot)$. There are no adjustment costs in steady state, so that $\phi(1) = 0$ and $\phi'(1) = 0$. Capital depreciation is represented by the function $\delta(u_t)$, which is convex in the rate of utilization: $\delta'(u_t) > 0$ and $\delta''(u_t) \geq 0$.

2.1.1 Key Mechanisms of the Model

In general, the main obstacle in business cycle models driven by news is to produce positive comovement among the major macroeconomic variables, such as output, consumption, investment, and labor hours. Broad-based comovement is a key empirical regularity in the data, so it is important that any business cycle model have this feature. Crucially however, producing positive comovement is a difficulty for news-driven models since with only news shocks and no other shocks to aggregate technology, good news about the future produces recessions rather than booms. This difficulty is due to the wealth effect on labor where agents work less in response to good news. Hence when agents learn about future productivity, the key modeling challenge is to get output to

rise in reaction to good news.⁷ To get positive comovement with only news shocks, there has to be additional mechanisms to overwhelm this wealth effect.

The key mechanisms in the JR model that produce empirically plausible business cycles are (i) preferences that minimize the wealth effect on labor, (ii) variable capital utilization, and (iii) capital adjustment costs.⁸ First, the non-standard preferences prevents agents from reducing labor hours in response to good news since their future expected income has increased. Second, capital adjustment costs induce agents to increase their labor supply before the actual productivity increase. Agents build up a large capital stock in anticipation of the productivity increase, and to minimize adjustment costs, they start as early as possible (with no capital adjustment costs, agents would reduce investment after learning about good news, and would wait until the increased productivity actually arrives to increase their investment). Finally, variable capital utilization is introduced to get labor to rise in response to good news.

2.1.2 First Order Conditions

The competitive equilibrium of the JR model can be found by solving the Social Planner's problem: maximize the representative agent's utility subject to the resource constraint (3), accounting identity (4), and capital law of motion (5).

The first order conditions with respect to C_t , X_t , N_t , u_t , K_{t+1} , and I_t are the following, where μ_t , λ_t , and η_t are the Lagrange multipliers associated with the constraints:

$$(C_t - \psi N_t^\theta X_t)^{-\sigma} + \mu_t \gamma C_t^{\gamma-1} X_t^{\gamma-1} = \lambda_t \quad (6)$$

$$(C_t - \psi N_t^\theta X_t)^{-\sigma} \psi N_t^\theta + \mu_t = \beta E_t[\mu_{t+1}(1 - \gamma) C_{t+1}^\gamma X_t^{-\gamma}] \quad (7)$$

$$(C_t - \psi N_t^\theta X_t)^{-\sigma} \theta \psi N_t^{\theta-1} X_t = \lambda_t \alpha A_t (u_t K_t)^{1-\alpha} N_t^{\alpha-1} \quad (8)$$

⁷Standard real business cycle models predict that when agents learn that future productivity will be high, they work less and consume more, leading to decreased investment and hence output. This is due to a wealth effect on labor, since good news makes agents expect higher lifetime income.

⁸For more detail and discussion, see Jaimovich and Rebelo (2009).

$$\lambda_t(1 - \alpha)A_t u_t^{-\alpha} K_t^{1-\alpha} N_t^\alpha = \eta_t \delta'(u_t) K t \quad (9)$$

$$\eta_t = \beta E_t[\lambda_{t+1}(1 - \alpha)A_{t+1} u_{t+1}^{1-\alpha} K_{t+1}^{-\alpha} N_{t+1}^\alpha + \eta_{t+1}[1 - \delta(u_{t+1})]] \quad (10)$$

$$\frac{\lambda_t}{z_t} = \eta_t \left[1 - \phi\left(\frac{I_t}{I_{t-1}}\right) - \phi'\left(\frac{I_t}{I_{t-1}}\right) \frac{I_t}{I_{t-1}} \right] + E_t \left[\beta \eta_{t+1} \phi'\left(\frac{I_{t+1}}{I_t}\right) \left(\frac{I_{t+1}}{I_t}\right)^2 \right] \quad (11)$$

When agents expect productivity A_t or z_t to increase in the future, investment will also increase in the future. With investment adjustment costs, investment will rise immediately, leading to a decrease in $\frac{\eta_t}{\lambda_t}$, the value of capital per consumption unit. This is because agents build up capital gradually over time due to adjustment costs. As $\frac{\eta_t}{\lambda_t}$ falls, capital is less valuable, so that agents must use their existing capital more intensively. This increased utilization raises the marginal product of labor. So long as the wealth effect on the labor supply is sufficiently low (so that $\gamma \rightarrow 0$), N_t will also rise. Hence all three model elements working together can generate comovement between consumption, output, investment, and labor hours in response to good news about the future.

2.1.3 Competitive Equilibrium

Definition. *An equilibrium of the JR model will be a collection of sequences for C_t , X_t , N_t , u_t , K_{t+1} , and I_t that satisfies the economy's resource constraint (3), accounting identity (4), capital law of motion (5), and agents' transversality condition.*

The rational expectations solution to the linearized system, following Blanchard and Kahn (1980) and described in King, Plosser, and Rebelo (2002), will take the form of a stationary vector autoregression and can be used for model simulations and computing business cycle moments. For benchmark parameters presented in Table 1, Jaimovich and Rebelo (2009) show that the one-sector model can produce positive comovement in response to both contemporaneous shocks to A_t or z_t and to news about future values of A_t or z_t . Further, when calibrated with these parameters, the model is able to generate empirically plausible business cycle statistics.

Table 1: **JR Parameter Values**

	Parameter
Discount factor	$\beta = 0.985$
Labor share	$\alpha = 0.64$
Curvature in utility	$\sigma = 1$
Elasticity in labor supply	$\theta = 1.4$
Utility parameter	$\gamma = 0.001$
Adjustment costs	$\phi''(1) = 1.3$
Elasticity of utilization	$\frac{\delta''(\bar{u})\bar{u}}{\delta'(\bar{u})} = 0.15$

2.2 Plausibility of Rational Expectations Equilibrium

To arrive at a solution to the JR model, it is implicitly assumed that expectations are formed according to the rational expectations hypothesis. Under this assumption, expectations are pinned down by the structure of the economy since expectational errors can be solved out as a function of the structural disturbances and hence eliminated from the system. Economic agents are assumed to have knowledge about the parameters of the economy, the correct model, and the distributions of the shocks. Hence by imposing rational expectations, agents are required to know the true data generating process of the entire economy. While this remains the standard approach in macroeconomics, I take an alternative approach in the rest of this paper by relaxing the rational expectations assumption.

3 Adaptive Learning

The literature on adaptive learning studies the plausibility of rational expectations equilibria by insisting on logical consistency between econometricians and private-sector agents. Rather than rational expectations, this literature assumes that agents behave as econometricians who formulate forecasting models and update the parameters of their model in real-time.⁹ Since econometricians and professional forecasters develop economic models that they estimate based on available data and

⁹Sargent (1993) describes adaptive learning as “putting the agents and the econometrician on the same footing.”

update as new data becomes available, it is reasonable to expect that private-sector agents behave similarly. Therefore in this part of the paper, agents are endowed with the same information set as the econometrician or professional forecaster.

To model private agent learning, expectations are formed from a near-rational expectations formation mechanism. Agents are endowed with a linear forecasting model, which is their perceived model of the economy. Although this perceived law of motion (PLM) has a similar structural form as the rational expectations solution of the system, agents do not know the reduced-form coefficients. Instead, they learn these coefficients over time by observing historical data and employing a recursive updating learning algorithm. Agents estimate the parameters of their forecasting model and then use the estimated model to form expectations. This then leads to the actual law of motion (ALM) of the economy. Since the ALM is obtained by inserting agents' expectations into the reduced form model, it is implicitly assumed that under adaptive learning, the Euler equation is the basis for economic decisions.¹⁰ The parameter values from the resulting ALM will therefore be the actual parameter values of the economy and will depend on agents' perceptions.

While attempting to learn about the model economy, agents' forecasts of future endogenous variables will necessarily differ from rational expectations forecasts. Thus, the crucial question is whether the adaptive learning process leads agents toward or away from the rational expectations equilibrium (REE). If the learning process leads agents to the REE, that equilibrium is said to be stable under adaptive learning, or "E-stable." Otherwise, the equilibrium is said to be unstable or unlearnable. In this way, stability under learning can provide an important robustness check on the plausibility of rational expectations equilibria.¹¹

¹⁰An alternative would combine the Euler equation with the expected lifetime budget constraint, as in Eusepi and Preston (2010). The actual law of motion would then depend on infinite-step-ahead expectations. Although "infinite horizon" learning is fully optimal in an anticipated utility framework, this paper adopts the so called "Euler equation" learning to keep the model's reduced-form comparable with JR. However, in many models the E-stability properties are identical under Euler equation and infinite-horizon learning. Evans and Honkapohja (2004) provides a discussion of this issue in the context of a New Keynesian model.

¹¹In fact, stability under learning can be used as an equilibrium selection device in models with multiple equilibria. If there are many equilibria, of which only one is E-stable, the E-stable equilibria is argued to be the more likely equilibrium to be observed in reality.

3.1 Learning Algorithm

The reduced-form of the JR model can be characterized by the following two equations:

$$\widehat{E}_t X_{t+1} = W X_t + R \widehat{E}_t z_{t+1} + Q z_t \quad (12)$$

$$z_t = z_{t-1} + \epsilon_t \quad (13)$$

where $X_t = [x_{t-1}, k_t, i_{t-1}, \mu_t, \lambda_t, \eta_t]'$ represents the vector of endogenous state variables and shadow prices, z_t is an exogenous variable representing investment-specific technological progress, and \widehat{E}_t are (possibly) non-rational expectaionts. All variables are in logarithms. The matrices W , R , and Q are defined in the Appendix. The growth rate of z_t is $\epsilon_t \in \{\epsilon_l, \epsilon_h\}$, which follows a two-point Markov process with transition matrix π :

$$\pi = \begin{bmatrix} p & 1-p \\ 1-q & q \end{bmatrix}$$

Taking expectations and calculating $\widehat{E}_t z_{t+1}$, the reduced-form system (12) and (13) simplifies to a single equation:

$$\widehat{E}_t X_{t+1} = W X_t + (R + Q) z_t + R r_1 + R r_2 \widehat{s}_t \quad (14)$$

where

$$\widehat{s}_t = s_t - 1, s_t \in \{1, 2\}$$

$$r_1 = p\epsilon_l + (1-p)\epsilon_h$$

$$r_2 = ((1-q)\epsilon_l + q\epsilon_h) - (p\epsilon_l + (1-p)\epsilon_h)$$

Equation (14) is the reduced-form representation of the model economy. The nature of the model's determinacy is governed by the reduced-form parameters in equation (14). A model is said to be determinate if it has a unique REE, indeterminate if it has multiple REE, and explosive

otherwise. Determinacy of the JR model requires three eigenvalues to lie outside and three to lie inside the unit circle, so that the dynamical system is a saddle.¹² I find that the model is always determinate.

Further, the reduced form model (14) is assumed to determine X_t whether or not expectations are formed rationally, where the operator \widehat{E}_t denotes agents' potentially non-rational forecasts of the endogenous variables. Under the standard rational expectations assumption, a REE is any non-explosive solution to the expectational difference equation (14). The rational expectations representation of the JR model driven by news will take the following form:

$$X_t = \overline{A}X_{t-1} + \overline{B}z_t + \overline{C}z_{t-1} + \overline{D} + \overline{F}\widehat{s}_t \quad (15)$$

Under adaptive learning, agents know the functional form of the model economy but are initially uninformed as to the correct, rational expectations coefficient values.¹³ Instead, agents' expectations are formed using their Perceived Law of Motion (PLM), which follows the same functional form as the rational expectations equilibrium solution:

$$X_t = AX_{t-1} + Bz_t + Cz_{t-1} + D + F\widehat{s}_t \quad (16)$$

The matrices A, B, C, D, F represent agents' perceived parameter values and may be different from the rational expectations matrices $\overline{A}, \overline{B}, \overline{C}, \overline{D}, \overline{F}$ at each point in time.¹⁴

In lieu of rational expectations, agents form forecasts given their PLM:

$$\widehat{E}_t X_t = X_t = AX_{t-1} + Bz_t + Cz_{t-1} + D + F\widehat{s}_t \quad (17)$$

¹²In general, checking for determinacy amounts to verifying that n_{cs} eigenvalues of W are greater than one in absolute value and n_s eigenvalues of W are less than one in absolute value, where n_{cs} is the number of costate variables and n_s is the number of state variables.

¹³This paper follows the Euler-equation learning approach, where the model under adaptive learning is driven by the same reduced-form equations as under rational expectations. Agents in principle could also be learning the model's microfoundations: in fact Preston (2008) and Eusepi and Preston (2010) show that infinite-horizon learning and subjective expectations can also be modeled to deliver increased business cycle volatility and persistence relative to rational expectations.

¹⁴To simplify notation I have eliminated time subscripts on the learning matrices.

$$\widehat{E}_t X_{t+1} = A[AX_{t-1} + Bz_t + Cz_{t-1} + D + F\widehat{s}_t] + B[z_t + r_1 + r_2\widehat{s}_t] + Cz_t + D + FE_t\widehat{s}_{t+1} \quad (18)$$

Substituting these beliefs into the reduced-form model leads to the Actual Law of Motion (ALM):

$$X_t = T_A X_{t-1} + T_B z_t + T_C z_{t-1} + T_D + T_F \widehat{s}_t \quad (19)$$

The matrices T_A , T_B , T_C , T_D , and T_F depends on agents' beliefs (A, B, C, D, F) and on the parameters of the model (W, R, Q) :

$$T_A = W^{-1}A^2 \quad (20)$$

$$T_B = W^{-1}[(A + I)B + C - (R + Q)] \quad (21)$$

$$T_C = W^{-1}AC \quad (22)$$

$$T_D = W^{-1}[(A + I)D + (B - R)r_1 + F(1 - p)] \quad (23)$$

$$T_F = W^{-1}[AF + (B - R)r_2 + F(2q - 1)] \quad (24)$$

$$(25)$$

The mapping from agents' beliefs to the true data generating process for the economy takes the following form:

$$T(A, B, C, D, F) = (T_A, T_B, T_C, T_D, T_F)' \quad (26)$$

This “T-map” maps the PLM to the ALM, and has a very intuitive interpretation. If agents perceived the economy follows the law of motion (16), with parameters fixed at (A, B, C, D, F) , then their forecasts would be given by (17) and (18). Then the economy's actual law of motion—determined, in part, by these forecasts—would have the same form, but with parameters $T(A, B, C, D, F)$. A REE is simply a fixed point to this “T-map,” as it identifies a situation where agents perceptions match reality.

3.2 E-Stability

If agents' learning process leads to the REE, that equilibrium is said to be stable under adaptive learning, or expectationally stable (E-stable). Otherwise, the REE is said to be expectationally unstable or unlearnable. More precisely, the E-stability principle states that if agents use a recursive least squares learning algorithm, then E-stable rational expectations equilibria are locally stable under learning. The intuition behind this principle is that agents should update their parameter estimates in the direction of forecast errors for reasonable learning rules. In this way, stability under adaptive learning provides a check on the plausibility of rational expectations solutions.

Definition. Let θ_t denote the vector of coefficients in the PLM and $T(\theta_t)$ denote the vector of coefficients in the ALM. To be “E-Stable,” the rational expectations solution must be locally asymptotically stable under the following ordinary differential equation:

$$\frac{d\theta}{d\tau} = T(\theta_t) - \theta_t \quad (27)$$

Verifying stability under learning amounts to checking if this differential equation, evaluated at the REE values for θ_t is locally stable. Conveniently, conditions for local asymptotic stability can be easily computed by examining the eigenvalues of the Jacobian matrix DT . If all eigenvalues of DT have real parts less than one, then the rational expectations equilibrium is E-stable. In this case, adjusting parameters in the direction of the forecast error will lead the parameters toward the rational expectations equilibrium. Evans and Honkapohja (2001) show that the PLM will converge to the ALM with probability one if $T(\theta_t) - \theta_t$ is a stable system and with probability zero if it is unstable.

Following the method from Evans and Honkapohja (2001) results in the following E-stability conditions for the JR model:

$$DT_A = \bar{A}' \otimes W^{-1} + I \otimes W^{-1}\bar{A} \quad (28)$$

$$DT_B = \bar{A} \otimes W^{-1} + I \otimes W^{-1} \quad (29)$$

$$DT_C = W^{-1}\bar{A} \quad (30)$$

$$DT_D = \bar{A} \otimes W^{-1} + I \otimes W^{-1} \quad (31)$$

$$DT_F = I \otimes W^{-1}\bar{A} + (2q - 1)' \otimes W^{-1} \quad (32)$$

Proposition 1. *A rational expectations solution $\bar{A}, \bar{B}, \bar{C}, \bar{D}, \bar{F}$ is E-unstable if any eigenvalues of (i) DT_A , (ii) DT_B , (iii) DT_C , (iv) DT_D , and (v) DT_F have real parts greater than one.*

If the rational expectations solution to the JR model is E-stable, then private-sector agents can learn the equilibrium law of motion for capital, investment, and the economy's shadow prices. If instead the solution is E-unstable, then the learning process will not converge to the true rational expectations equilibrium.

4 Numerical Results

As is standard in the business cycle literature, the multivariable nature of the model precludes analytic results. This section contains the main numerical results of the paper. I consider the learnability properties of the JR model by applying the E-stability result derived in Proposition 1. I find that REE associated with parameter values necessary for generating recessions in response to news shocks are not learnable, and that parameter values that imply learnable REE cannot generate recessions in response to news shocks. Hence there is a tradeoff between E-stability and the model's ability to produce realistic business cycles.

4.1 Case 1: Benchmark JR

For the benchmark calibration in the JR model,¹⁵ I find that the rational expectations solution violates the E-stability conditions derived in Proposition 1. This means the unique REE for the

¹⁵ $\sigma = 1, \theta = 1.4, \beta = 0.985, \alpha = 0.64, \gamma = 0.001, \phi''(1) = 1.3, \frac{\delta''(\bar{u})\bar{u}}{\delta'(\bar{u})} = 0.15$

model driven by news about future technology is not learnable, even when agents adopt a forecasting model that has the same form as the true rational expectations representation. This finding is in contrast to the standard real business cycle model, which has an E-stable rational expectations solution, as shown by Evans and Honkapohja (2001). That the unique, stationary REE for the benchmark JR model is not stable under learning also demonstrates that determinacy need not imply learnability. Since the JR model can reduce to the standard real business cycle model, there must exist parameter regions where the rational expectations solution is E-stable. In what follows, I examine the parameter space outside the benchmark JR calibration.

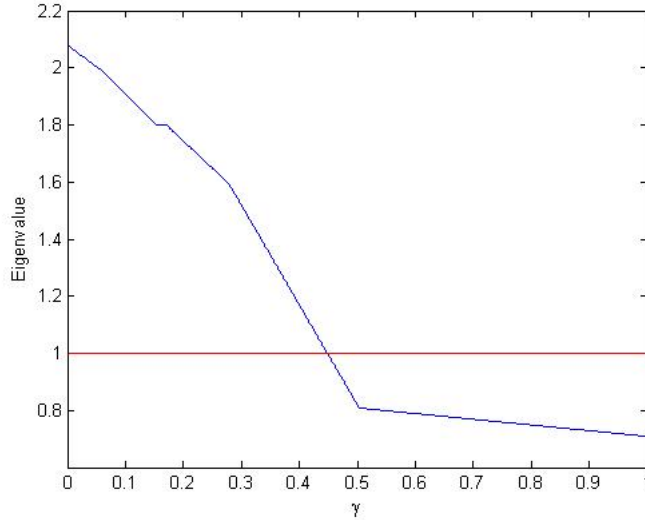
Table 2 presents the constrained parameter space required for generating comovement in response to news about investment-specific productivity as reported in Jaimovich and Rebelo (2009). The most striking result is that the rational expectations solution is unlearnable for $\gamma < 0.4$, which is a necessary constraint for the JR model to produce empirically plausible business cycles. At the limit value of $\gamma = 1$, which corresponds to the standard utility representation of King, Plosser, and Rebelo (1988), I find that the rational expectations solution is learnable since the E-stability conditions in Proposition 1 are satisfied. However in this case, $\gamma = 1$ cannot generate positive comovement between output and labor since the wealth effect makes agents work less in response to good news about the future.

Table 2: **JR Parameter Space**

Description	Benchmark	Constraint
Utility parameter	$\gamma = 0.001$	$\gamma < 0.4$
Adjustment costs	$\phi''(1) = 1.3$	$\phi''(1) > 0.4$
Elasticity of utilization	$\frac{\delta''(\bar{u})\bar{u}}{\delta'(\bar{u})} = 0.15$	$\frac{\delta''(\bar{u})\bar{u}}{\delta'(\bar{u})} < 5$

Based on Proposition 1, the model's E-stability is coming from the Jacobian matrix DT_A . Figure 1 depicts the stability region as a function of the preference parameter γ . This E-stability region is based on the maximum explosive eigenvalue of the Jacobian matrix DT_A as a function of γ . For $0 \leq \gamma \leq 1$, the rational expectations solution is learnable when $\gamma > 0.448$ and unlearnable

Figure 1: E-Stability Region– Case 1



otherwise.

To get a better sense of these results, I next separately consider the stability properties of the three key ingredients of the JR model in more detail: the new class of preferences, variable capital utilization, and investment adjustment costs. The main results for this exercise are presented in the following three cases to get a clearer picture of how each of the model’s three key mechanisms affect E-stability: (a) JR preferences, no variable capital utilization, no investment adjustment costs; (b) KPR preferences, variable capital utilization, no investment adjustment costs; and (c) KPR preferences, no variable capital utilization, investment adjustment costs.

4.2 Case 2: JR Preferences, No Capital Utilization, No Adjustment Costs

I first consider the model with JR preferences ($\gamma = 0.001$), no variable capital utilization ($u_t = 1$, $\delta(u_t) = \delta$, $\delta''(\bar{u})\bar{u}/\delta'(\bar{u}) \rightarrow \infty$), and no investment adjustment costs ($\phi(x) = 0$ for all x , $\phi''(1) = 0$) to investigate the role of the new class of preferences in the JR model. All other parameter values

follow the benchmark JR specification as reported in Table 1. I find that the determinate rational expectations solution is E-unstable, violating Proposition 1. Since the equilibrium corresponding to this calibration is unstable, it cannot be reached by agents following an adaptive learning rule. This result suggests that the special preferences designed to shut down the wealth effect on labor and the non-separability of the utility function is an important factor at influencing the instability result reported in Case 1.

Figure 2 displays the E-stability region of a version of the JR model with no variable utilization and no capital adjustment costs. As in Figure 1, the E-stability region is based on the maximum explosive root of the Jacobian matrix DT_A as a function of γ . The rational expectations solution is E-stable for $\gamma > 0.818$ and unstable otherwise. Compared to the previous case with baseline JR specifications, the model with constant utilization and no adjustment costs requires a larger wealth effect on the labor supply for the E-stability conditions to hold. As $\gamma \rightarrow 1$, the utility function becomes the standard neoclassical specification and hence reduces to the standard optimal growth setting. In this case, the rational expectations equilibrium is stable under adaptive learning, as demonstrated by Evans and Honkapohja (2001).

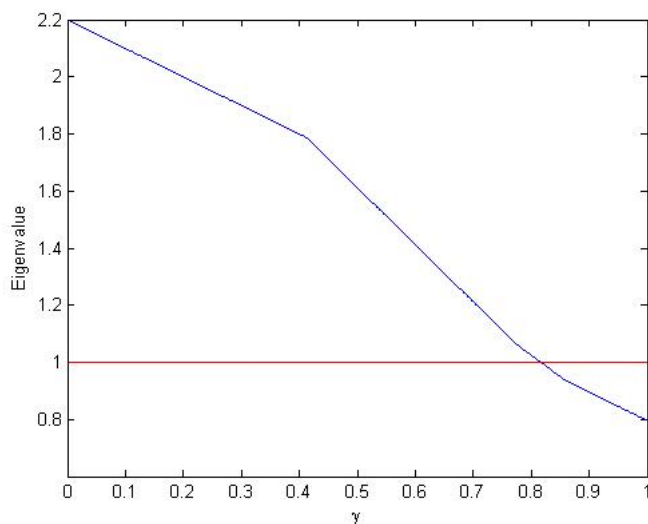
Table 3: **Business Cycle Statistics** (Rational Expectations)

	Data	Case 1	Case 2	Case 3	Case 4
	1947–2009	JR	Preferences	Utilization	Adjustment Costs
$\text{std}(y_t)$	1.66	0.98	0.87	0.99	0.82
$\text{std}(c_t)$	1.39	0.77	0.61	0.74	0.64
$\text{std}(i_t)$	4.91	3.21	2.93	3.27	2.29
$\text{std}(n_t)$	1.62	0.69	0.39	0.68	0.21
$\text{corr}(y_t, y_{t-1})$	0.79	0.87	0.88	0.90	0.87
$\text{corr}(y_t, c_t)$	0.69	0.85	-0.71	-0.65	-0.24
$\text{corr}(y_t, i_t)$	0.90	0.89	0.67	0.81	0.83
$\text{corr}(y_t, n_t)$	0.87	1.00	0.14	-0.21	-0.54

Further, the model under this calibration does not produce realistic business cycles. Column 4 of Table 3 reports simulated business cycle moments for the model.¹⁶ Volatilities of output, con-

¹⁶Table 3 was constructed by generating 5000 model simulations with 300 periods each, for each case. Business

Figure 2: E-Stability Region– Case 2



sumption, investment, and labor hours are all lower than than both the data and the benchmark JR specification. Moreover, this version of the model does not generate comovement among the aggregate macroeconomic variables since the simulated correlations are not all positive. In particular, output and hours are roughly acyclical, which confirms a feature of the JR model that the special preferences alone are not enough to produce the strong procyclicality between output and hours.

Since these results suggest E-stability hinges upon the new class of preferences introduced by the JR model, I investigate this further by considering separately the two extreme versions of the model with $\gamma = 0$ and $\gamma = 1$ respectively, with no capital utilization and no adjustment costs. Preferences with $\gamma = 0$ and $\gamma = 1$ correspond to classes of utility function widely used in the business cycle literature and represent polar cases of constant income effect. The following exercise

cycle statistics are averaged across simulations. All variables are in logarithms and have been detrended using the Hodrick-Prescott filter with a smoothing parameter of 1600. The data in column 2 corresponds to post-war U.S. quarterly data from 1947 to 2009.

can thus clarify the nature of the model's instability by examining the simplified business cycle environment with the other two key mechanisms shut down.

4.2.1 JR Preferences with $\gamma = 0$ and $\gamma = 1$, No Capital Utilization, No Adjustment Costs

The simplified model for $\gamma = 0$ and $\gamma = 1$ will reduce to a system of expectational difference equations in terms of just consumption, capital, and the productivity shock. The reduced-form model for both these cases will be characterized by the following three equations, whether or not expectations are formed rationally:

$$c_t + \eta_1 k_t = \psi_2 \widehat{E}_t c_{t+1} + \psi_1 \widehat{E}_t k_{t+1} + \pi_1 a_t \quad (33)$$

$$k_t = \delta_2 c_{t-1} + \delta_1 k_{t-1} - \pi_2 a_{t-1} \quad (34)$$

$$a_t = \rho a_{t-1} + v_t \quad (35)$$

This system is the general reduced-form representation of the model economy under JR preferences, no capital utilization, and no investment adjustment costs. All variables are in logarithms where the particular coefficients will depend on whether $\gamma = 0$ or $\gamma = 1$. Stacking the model and eliminating expectations by defining a martingale difference sequence $\epsilon_t = c_t - \widehat{E}_{t-1} c_t$ yields:

$$\begin{bmatrix} 1 & \eta_1 & -\pi_1 \\ \delta_2 & \delta_1 & -\pi_2 \\ 0 & 0 & \rho \end{bmatrix} \begin{bmatrix} c_t \\ k_t \\ a_t \end{bmatrix} = \begin{bmatrix} \psi_2 & \psi_1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \left(\begin{bmatrix} c_{t+1} \\ k_{t+1} \\ a_{t+1} \end{bmatrix} - \begin{bmatrix} \epsilon_t \\ 0 \\ v_t \end{bmatrix} \right) \quad (36)$$

In matrix form, the model becomes $H_t x_t = F(x_{t+1} - \phi_{t+1})$, where $x_t \equiv [c_t, k_{t+1}, a_t]'$ and $\phi_t \equiv [\epsilon_t, 0, v_t]'$. For some forecast error ϵ_t , every REE for x_t will satisfy the dynamic system

$$x_t = M x_{t-1} + \phi_t \quad (37)$$

where $M = F^{-1}H$.¹⁷

In what follows, I present agents' perceived law of motion under adaptive learning to derive stability conditions for this simplified environment, written in terms of the reduced-form coefficients.

Under adaptive learning, agents' PLM will take the same form as the REE. When agents know the capital accumulation equation $k_t = \delta_1 k_{t-1} + \delta_2 c_{t-1} - \pi_2 a_{t-1}$, the PLM for consumption will follow:

$$c_t = ac_{t-1} + bk_{t-1} + c + da_{t-1} + ea_t \quad (38)$$

Note that the constant term c is included in the PLM so that agent must learn whether the steady-state value is zero. Given their PLM, agents form expectations of future consumption and capital according to

$$\widehat{E}_t c_{t+1} = a\widehat{E}_t c_t + b\widehat{E}_t k_t + c + da_t + e\epsilon_t \quad (39)$$

$$\widehat{E}_t k_{t+1} = \delta_1 \widehat{E}_t k_t + \delta_2 \widehat{E}_t c_t - \pi_2 a_t \quad (40)$$

Inserting expectations $\widehat{E}_t c_{t+1}$ and $\widehat{E}_t k_{t+1}$ in the reduced form system yields the actual law of motion for consumption:

$$c_t = T_a c_{t-1} + T_b k_{t-1} + T_c + T_d a_{t-1} + T_e a_t \quad (41)$$

The coefficients for the ALM T_a , T_b , T_c , T_d , and T_e depend on agent's beliefs and the reduced-form parameters in the following way:

$$T_a = \psi_1 \delta_2 a + \psi_2 (a^2 + b\delta_2) \quad (42)$$

$$T_b = \psi_1 (\delta_1^2 + \delta_2 b) + \psi_2 (ab + b\delta_1) \quad (43)$$

¹⁷The model's determinacy properties can be evaluated by computing the eigenvalues of the matrix $M = F^{-1}H$ and are governed by the reduced form parameters. When the model is determinate so that the REE is unique, there will be a unique forecast error ϵ_t that satisfies $x_t = Mx_{t-1} + \phi_t$. There will be multiple forecast errors that makes the system $x_t = Mx_{t-1} + \phi_t$ non-explosive if the model is indeterminate. In this latter case there will be multiple REEs.

$$T_c = c[\psi_1\delta_2 + \psi_2(1 + a)] \quad (44)$$

$$T_d = \psi_1(\delta_2d - \delta_1\pi_2) + \psi_2(ad - b\pi_2) \quad (45)$$

$$T_e = \pi_1 + \psi_1\delta_2e + \psi_2[(a + \rho)e + d] \quad (46)$$

As before, the T-map $T(a, b, c, d, e) = (T_a, T_b, T_c, T_d, T_e)'$ provides a mapping from agents' beliefs (PLM) to reality (ALM):

$$(a, b, c, d, e) \rightarrow T(a, b, c, d, e) \quad (47)$$

Denoting the vector of coefficients by $\theta = [a, b, c, d, e]'$, a fixed point of the T-map θ^* is E-stable if the differential equation $\frac{d\theta}{d\tau} = T(\theta) - \theta$ is locally asymptotically stable at θ^* . As in Section 3 and detailed in Evans and Honkapohja (2001), the E-stability Principle says that E-stable representations can be learned by agents following a recursive least squares learning algorithm. Verifying stability under learning requires that eigenvalues of the derivative $T(\theta) - \theta$ evaluated at θ^* have negative real parts. Noting that the subsystems for T_a and T_b decouples, the following derivatives will determine the model's stability properties:

$$DT_{ab} = \begin{bmatrix} \psi_1\delta_2 + 2\psi_2a & \psi_2\delta_2 \\ \psi_2b & \psi_1\delta_2 + \psi_2(a + \delta_1) \end{bmatrix} \quad (48)$$

$$DT_c = \psi_1\delta_2 + \psi_2(1 + a) \quad (49)$$

$$DT_d = \psi_1\delta_2 + \psi_2a \quad (50)$$

$$DT_e = \psi_1\delta_2 + \psi_2(a + \rho) \quad (51)$$

E-stability requires that the derivatives DT_{ab} , DT_c , DT_d , and DT_e have real parts less than one. Denoting $u \equiv \text{trace}(DT_{ab}) = 2\psi_1\delta_2 + \psi_2(3a + \delta_1)$ and $v \equiv \text{det}(DT_{ab}) = (\psi_1\delta_2 + 2\psi_2a)(\psi_1\delta_2 +$

$\psi_2(a + \delta_1) - \psi_2^2 \delta_2 b$ results in the following conditions in terms of the reduced-form parameters that must hold in order for the simplified model's REE to be E-stable:

$$\frac{1}{2} \sqrt{\frac{u}{2} \pm \frac{u^2}{4-v}} < 1 \quad (52)$$

$$\psi_1 \delta_2 + \psi_2(1 + a) < 1 \quad (53)$$

$$\psi_1 \delta_2 + \psi_2 a < 1 \quad (54)$$

$$\psi_1 \delta_2 + \psi_2(a + \rho) < 1 \quad (55)$$

I can now apply the E-stability conditions derived above to the two extreme parameterizations of the simplified model with $\gamma = 0$ and $\gamma = 1$, respectively. JR preferences with $\gamma = 0$ reduces to a standard business cycle model with no wealth effect on the labor supply. In this case, I find that the E-stability conditions do not hold. In particular, the roots of the derivative DT_{ab} are greater than one: since the E-stability condition $\frac{1}{2} \sqrt{\frac{u}{2} \pm \frac{u^2}{4-v}} < 1$ is not satisfied, REE under this parametrization is not learnable by agents adopting an adaptive learning rule. JR preferences with $\gamma = 1$ results in a version of the standard neoclassical growth model, and in contrast with the previous case, I find that the REE following this parametrization is E-stable since the stability conditions all hold.

4.3 Case 3: KPR Preferences, Capital Utilization, No Adjustment Costs

Next I consider the model with standard KPR preferences ($\gamma = 1$), variable capital utilization, and no investment adjustment costs ($\phi(x) = 0$ for all x , $\phi''(1) = 0$) to investigate the role of variable capital utilization on the model's learnability properties. All other parameter values follow the benchmark JR specification. This environment corresponds to the neoclassical growth model augmented with variable utilization and news about future productivity. As before, I solve the model and verify whether the rational expectations solution satisfies Proposition 1. I find that

the model under this calibration is E-stable, as all eigenvalues for each of the Jacobian matrices have real parts less than one. Since this system is stable, agents will learn this equilibrium with probability one.

Table 4: **Business Cycle Statistics** (Adaptive Learning)

	Case 3	Case 4
	Utilization	Adjustment Costs
$\text{std}(y_t)$	1.02	0.83
$\text{std}(c_t)$	0.75	0.64
$\text{std}(i_t)$	3.29	2.33
$\text{std}(n_t)$	0.68	0.23
$\text{corr}(y_t, y_{t-1})$	0.92	0.88
$\text{corr}(y_t, c_t)$	-0.65	-0.24
$\text{corr}(y_t, i_t)$	0.80	0.85
$\text{corr}(y_t, n_t)$	-0.21	-0.52

However under this calibration, the model cannot generate empirically plausible business cycles, as indicated by the simulated moments in column 5 of Table 3. Although the volatilities are similar to the benchmark JR model due to the role of capital utilization as an amplification mechanism, the simulated correlations come in with the wrong signs and magnitudes. As in the previous case, this version of the model does not produce aggregate comovement among the key macroeconomic variables. This is because $\gamma = 1$ cannot generate positive comovement among output and labor since the wealth effect makes agents work less in response to good news about the future.

Although this version of the model is E-stable, introducing adaptive learning has quantitatively small effects on improving the empirical fit. Column 2 of Table 4 reports simulated business cycle moments under a least squares recursive learning rule. Learning only slightly increases business cycle volatility and persistence.¹⁸ Since under this learning scheme only one-period-ahead forecasts matter, agents learn the rational expectations equilibrium quickly so that the overall effect of learning is negligible.

¹⁸This finding that introducing Euler-equation learning in a business cycle environment does little to enhance volatility and persistence is in line with Williams (2003).

4.4 Case 4: KPR Preferences, No Capital Utilization, Adjustment Costs

Finally I consider the model with standard KPR preferences ($\gamma = 1$), no variable capital utilization ($u_t = 1$, $\delta(u_t) = \delta$, $\delta''(\bar{u})\bar{u}/\delta'(\bar{u}) \rightarrow \infty$), and investment adjustment costs to investigate the role of convex adjustment costs on the model's E-stability properties. As before, all other parameter values follow the benchmark JR specification. I find that the rational expectations solution is E-stable, satisfying Proposition 1.

As in the previous case, the model under this specification does not produce realistic business cycles. Column 6 of Table 3 reports simulated business cycle moments. Again, the correlations are not all positive, indicating that all three ingredients of the model together are necessary to generate aggregate comovement. Column 3 of Table 4 reports simulated moments under adaptive learning. As in the previous case, introducing learning only slightly increases business cycle volatility and persistence. The effects are quantitatively small because agents learn the rational expectations quickly. Introducing a different learning scheme, such as infinite-horizon learning as in Preston and Eusepi (2010), may provide additional propagation.

Taken together, these results show that the JR news-driven business cycle model has an unstable rational expectations equilibrium, and that this E-instability result is contingent on the parametrization of the utility function. When the parametrization is chosen such that preferences are close to the classical KPR specification ($\gamma = 1$), as in the standard real business cycle environment, the unique rational expectations solution is E-stable and hence can be learned by private sector agents through an adaptive learning process. Although E-stable, the equilibrium in both Case 3 and Case 4 do not yield plausible business cycles in model simulations. I find that neither case delivers the requisite comovement between the aggregate macroeconomic variables. Table 5 summarizes this stability and comovement tradeoff in the JR model.

Table 5: **E-Stability and Comovement Tradeoff**

Case 1	Case 2	Case 3	Case 4
JR	Preferences	Utilization	Adjustment Costs
E-unstable	E-unstable	E-stable	E-stable
Comovement	No comovement	No comovement	No comovement

4.5 Discussion

News-driven business cycle models, such as the JR model considered in this paper, have a similar flavor as business cycle models with sunspots, where the economy is driven by self-fulfilling shifts in private sector expectations. Sunspots are solutions that depend on an exogenous stochastic process that can effectively act as a coordinating device causing agents to change their expectations in a self-fulfilling manner. Sunspots cause shifts between multiple equilibria, which may thereby produce economic fluctuations. For example, business cycle models with multiple equilibria such as Benhabib and Farmer (1994) can have an indeterminate steady state where there exists sunspot equilibria. However, Evans and McGough (2005) identify a “stability puzzle” with respect to these models and conjectures why indeterminacy almost always implies instability in sunspot-driven business cycle models.

In this paper, I obtain an analogous instability result as Evans and McGough (2005), except that the JR model is always determinate for the parameter region of interest. I conjecture that news functions as a kind of coordinating device for agents that can lead them away from learning the rational expectations solution, much like in the sunspot models. With news however, beliefs are not self-fulfilling. Furthermore, since the JR model exhibits determinacy, the stability results derived in this paper are not sensitive to representations of the PLM, unlike the case with sunspot equilibria.

The news process introduced in this paper and in Jaimovich and Rebelo (2009) represent information about future technological change and does not represent shifts between multiple equilibria as in Evans and McGough (2005) or shifts unrelated to fundamentals. Like sunspots however, news

about future fundamentals may act as a coordinating device that prevents agents from learning the rational expectations equilibrium. Intuitively, this may be interpreted as optimism or pessimism about the economy that is based on good or bad news, rather than random shifts in beliefs. Hence this interpretation differs from the Keynesian “animal spirits” view where fluctuations are driven by random waves of market sentiment and is closer in spirit to Pigou’s conjecture that errors of undue optimism or pessimism in market forecasts can generate business cycle dynamics.

5 Conclusion

This paper examines stability under learning of the rational expectations equilibrium in an augmented business cycle model driven by news about future technology. As the main contribution of this paper, I find that at parameter values necessary for generating recessions in response to bad news, the rational expectations solution to the one-sector JR model is unstable under learning. This finding thus casts doubt on the plausibility of the rational expectations equilibrium of the model. However, this finding does not imply that all models with news shocks have unlearnable equilibria. In fact, a promising direction for future research is to establish E-stability properties in a wider class of business cycle models driven by news. A further challenge will be to construct calibrated versions of news-driven business cycle models that are stable under learning.

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