Precautionary Saving over the Business Cycle*

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Abstract

We study the macroeconomic implications of time-varying precautionary saving in a tractable general equilibrium model with both aggregate and uninsurable idiosyncratic risk. In the model, agents facing incomplete markets and borrowing constraints respond to countercyclical changes in unemployment risk by altering their buffer stock of wealth, with a direct impact on aggregate consumption. In a calibrated version of the model, the response of aggregate consumption to a typical NBER recession is found to be twice as large as that implied by a comparable representative agent economy, and about 30% larger than that implied by a comparable economy with full insurance but wherein a fraction of households permanently faces a binding borrowing constraint.

JEL codes: E20, E21, E32

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1 Introduction

How important are changes in precautionary asset accumulation for the propagation of business cycle shocks? In this paper, we attempt to answer this question by constructing a tractable model of time-varying precautionary saving behaviour driven by countercyclical changes in unemployment risk. Because households are assumed to be imperfectly insured against this risk, they rationaly respond to such changes by altering their buffer stock of precautionary wealth. This in turn amplifies the consumption response to aggregate shocks affecting unemployment.

Our motivation for investigating the role of precautionary saving over the business cycle is based on earlier empirical evidence, which points out to a significant role for the precautionary motive in explaining the accumulation of wealth by individuals and their variations over time. Empirical studies that focus on the cross-sectional dispertion of wealth suggest that households facing higher income risk accumulate more wealth, all else equal (Carroll, 1994; Carroll and Samwick, 1997, 1998; Carroll et al., 2003). This argument has been extended to the time-series dimension by Carroll (1992), Gourinchas and Parker (2001) and Parker and Preston (2005), who argue that changes in precautionary wealth accumulation may substantially amplify consumption fluctuations. We take stock of their results and construct a general equilibrium model in which the strength of the precautionary motive is explicitely related to the extent of unemployment risk, the main source of income fluctuations for most households (at least at business cycle frequencies.)

The present contribution is both methodological and substantitive. From a methodological point of view, we exhibit a class of heterogenous agents model with incomplete markets, borrowing contraints and both aggregate (i.e., productivity) and idiosyncratic (i.e., labour market transition) shocks than can be solved by exact cross-household aggregation and rational expectations. This approach makes it possible to incorporate time-varying precautionary saving into general equilibrium analysis using simple solution methods –including linearisation. We are able to do because our model has two key features. First, it endogenously generates a cross-sectional distribution of wealth with a limited number of states –rather than the large-dimentional heterogeneity implied by most heterogenous-agent models.[1] Sec-

[1] As is well known, the lack of perfect cross-household insurance against individual income risk usually produces a considerable amount of household heterogeneity, because the decision (state) of every household typically depends on the whole history of income shocks that this household has faced (see, e.g., Huggett, 1993; Aiyagari, 1994; Krussel and Smith, 1998). The most popular appraoch to solving such models with

ond, in our model a substantial fraction of the households does not achieve full self-insurance in equilibrium (despite precautionary wealth accumulation), and thus experiences a discontinuous drop in income and consumption when unemployment strikes. This drop being of first-order magnitude, changes in the perceived likelihood that it will occur have a correspondingly first-order impact on the intensitity of the precautionary motive for accumulating assets *ex ante*. Our main theoretical result is the derivation of a (common) Euler equation for employed households that explicitly connnects precautionary wealth accumulation to the risk of experiencing an unemployment spell.

From a substantive point of view, our paper aims at identifying and quantifying the specific role of incomplete insurance and precautionary wealth accumulation, as opposed to mere borrowing constraints, in shaping the behaviour of aggregate consumption during a typical recession (of size equal to the average NBER recession). To this purpose, we use a calibrated version of the model that matches the evidence on the share of "permanent income consumers" and the distribution of wealth in the U.S. economy (in addition with matching other usual quantities). The predicted fall in aggregate consumption during a recession is found to be about twice as large as that implied by a comparable representative agent economy, and about 30% larger than that implied by a comparable full-insurance economy where a fraction agents consume all their income due to binding borrowing constraints. Our analysis thus lends support to the view that borrowing constraints *and* time-varying precautionary saving substantially propagate recessions by amplifying consumption fluctuations.

Our contribution is closely related to the analysis of incomplete-market models with aggregate shocks involving large-dimensional cross-sectional heterogeneities in income and wealth. In their pioneering contribution, Krusell and Smith (1998) computed the time-series properties of a benchmark model with incomplete markets and borrowing constraints and found market incompleteness to moderately raise the unconditional consumption-output correlation, relative to that under frictionless financial markets. We find an increase of similar magnitude in this correlation when we subject our model to the same experiment as theirs –that is, simulating our economy using a joint process for the exogenous state variables that is parameterised to match U.S. post-war data. The specificity of our approach, however, is to extract the *conditional* response of aggregate consumption to a typical recession, rather than computing unconditional moments. We are able to do so because our equilibrium with

aggregate shocks has been to rely on "approximate aggregation", that is, to analyse bounded-rationality equilibria that are approximately self-confirming (Krusell and Smith, 1998).

limited cross-sectional heterogeneity makes it possible to consider continuous changes in exogenous state variables (i.e., productivity and labour market transition rates) and hence to compute standard impulse-response functions in response to such changes. Moreover, we study the consumption response to changes in exogenous state variables conditional on entering an NBER recession, and these changes have larger amplitude than when measured over the entire post-war period. This suggests that while time-variations in precautionary wealth accumulation may not seem to greatly affect the association between consumption and ouput "on average", they may in fact affect it substantially when the economy is hit by a large aggregate shock.

More recently, Krusell et al. (2010) and Nakajima (2010) have considered a version of the incomplete-market model wherein individual transition rates in the labour market are endogenised via search-and-matching frictions. An important difference between our paper and theirs is their modelling of the size and costs of individual labour income uncertainty. Under the calibration of the model proposed by Shimer (2005), the model generates high wage flexibility, little employment volatility, and hence a small effect of aggregate shocks on idiosyncratic unemployment risk. Under the "small firm surplus" calibration of Hagerdorn and Manovskii (2008), employment is highly volatile but unemployment is almost not costly, again giving the households little incentives to hoard assets for precautionary purpose ex ante. We adopt a different approach, which is closer in spirit to Krusell and Smith (1998). More specifically, when running our impulse-response experiments, we treat labour market transition rates as exogenous and construct their typical movements during a recession from the data (by averaging over NBER recessions).[2] This ensures that unemployment fluctuations –and the associated variations in idiosyncratic income risk– have realistic magnitudes. This property, jointly with a substantial income-equivalent loss associated with unemployment spells, implies that households have good reasons to fear such spells and thus to form a significant buffer stock of precautionary wealth while still employed.

The remainder of the paper is organised as follows. The following section introduces the model. It starts by describing households' consumption-saving decisions in the face of idiosyncratic unemployment risk, spells out firms optimality conditions, and finally characterises the equilibrium that results from their interactions. In Section 3, we introduce the parameter restrictions that make our model tractable by endogenously limiting the cross-sectional het-

[2]Even though we treat transition rates as exogenous in our baseline specification, endogenising them via labour search and matching is a straigthforward extension, as we show in the Appendix.

erogeneity in household wealth. In section 4, the model is calibrated and impulse-response functions are drawn and discussed, with particular attention being paid to the response of aggregate consumption to aggregate shocks. Section 6 concludes.

2 The model

The economy is populated by a continuum of households indexed by i and uniformy distributed along the unit interval, as well as a representative firm. All households rent out labour and capital to the firm, which latter produces the unique (all-purpose) good in the economy. Markets are competitive but there are frictions in the financial markets, as we describe further below.

2.1 Households

Every household i is endowed with one unit of labour, which is supplied inelastically to the representative firm if the household is employed. All households are subject to idiosyncratic changes in labour market status between "employment" and "unemployment". Employed households earn the real wage w_t, while unemployed households earn a fixed home production income $\delta > 0$.

We assume that households can be of two types, *impatient* and *patient*, with the former and the latter having subjective discount factors $\beta \in (0, 1)$ and $\beta^p \in (\beta, 1)$, respectively. As will become clear below, patient households will end up holding a large fraction of total wealth in equilibrium, leaving the impatient with only a small fraction to self-insure against unemployment risk.[3] The introduction of patient households in our incomplete-market environment is necessary to generate a realistic wealth dispertion, but the equilibrium with limited cross-sectional wealth heterogeneity that we construct in Section 3 could be studied without them. Impatient households occupy the subinterval $[0, \Omega]$, $\Omega \in [0, 1)$, while patient households cover the complement interval $(\Omega, 1]$. The former are thus in proportion Ω in the economy.

[3] A typical implication of models with heterogenous discount factors and borrowing constraint is that the constraint is binding for all impatient households in equilibrium, so the latter hold zero or negative wealth (e.g., Becker, 1980; Becker and Foias, 1987; Kiyotaki and Moore, 1997; Iacoviello, 2005). In our model, the precautionary motive will cause impatient households to hold a small but positive amout of asset wealth, despite their subjective discount rate being lower than the interest rate.

Idiosyncratic unemployment risk. The unemployment risk faced by individual households is summarised by two probabilities: the probability that a household who is employed at date $t-1$ becomes unemployed at date t (the employment exit probability s_t), and the probability that a household who is unemployed at date $t-1$ stays so in the following period (one minus the job-finding rate, i.e., $1-f_t$). Both s_t and f_t are stochastic and know at date t.[4] The law of motion for employement is:

$$n_t = (1 - n_{t-1}) f_t + (1 - s_t) n_{t-1} \tag{1}$$

One typically thinks of (f_t, s_t) as being ultimately driven by underlying shocks governing the job creation policy of the firms and the natural breakdown of existing employment relashionships. For example, and as we explicitly show in Appendix A, stochastic changes in (f_t, s_t) are the direct outcome of a labour search and matching structure a la Diamond-Mortensen-Pissarides, wherein the two idiosyncratic transition rates are affected by the underlying aggregate productivity shocks. However, we wish to emphasise here that the key market friction leading to precautionary saving behaviour and its variations over time is the inability of some households to perfectly insure against such transitions, a property that does not a priori depend on the specific modelling of the labour market being adopted. For this reason, we take those transition as given in our baseline specification, and will ultimately extract them from the data in the quantitative implementation of the model.

Impatient households. Impatient households maximise their expected life-time utility $E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^i)$, $i \in [0, \Omega]$, where c_t^i is consumption by household i at date t and $u(.)$ a period utility function satisfying $u'(.) > 0$ and $u''(.) \leq 0$. We restrict the set of assets that impatient households have access to in two ways. First, we assume that these households cannot issue assets contingent on their employment status –that is, there is not unemployment insurance scheme, either public or private.[5] Second, we assume they cannot borrow against future

[4] Note that this formulation is fully consistent with labour market flows taking place *within* the period. In this case, the measured period-to-period employment exit probability, s_t, is affected by the job-finding rate f_t, because newly unemployed households at (the end of) date t include those who have become *and* stayed unemployment during period t. See Appendix A for a version of the model with labour search-and-matching that explicitly produces within-period flows into and out of the unemployment pool.

[5] Alternatively, one could interpret the home production parameter δ as the outcome of an unemployment insurance scheme. Our result would be unaltered provided that that the scheme is funded by lump sum taxing the employed.

income.[6] Given these restrictions, together with the fact that the representative firm makes zero profits at all times (see Section 2.1), the only asset that can be used to smooth out idiosyncratic labour income fluctuations are claims to the capital stock. We denote by e_t^i household'i employment status at date t, with $e_t^i = 1$ if the household is employed and zero otherwise. The budget and non-negativity constraints faced by an impatient household i are then:

$$a_t^i + c_t^i = e_t^i w_t + \left(1 - e_t^i\right) \delta + R_t a_{t-1}^i, \tag{2}$$

$$c_t^i,\ a_t^i \geq 0. \tag{3}$$

where a_t^i is household i's holdings of claims to the capital stock of the representative firm by the end of date t. The Euler condition for impatient households is:

$$u'\left(c_t^i\right) = \beta E_t \left(u'\left(c_{t+1}^i\right) R_{t+1}\right) + \varphi_t^i, \tag{4}$$

where φ_t^i is the Lagrange coefficient associated with the borrowing constraint $a_t^i \geq 0$, with $\varphi_t^i > 0$ if the constraint is binding and $\varphi_t^i = 0$ otherwise. Condition (4), together with the given initial asset holdings a_{-1}^i and the terminal condition $\lim_{t\to\infty} \beta^t E_t u'\left(c_t^i\right) = 0$, fully characterise the optimal asset holdings of impatient households.

Patient households. We assume that patient households maximise the lifetime expected utility $E_0 \sum_{t=0}^{\infty} \beta^p u^p\left(c_t^i\right)$, $i \in (\Omega, 1]$, where $\beta^p \in (\beta, 1)$ is their discount factor and $u^p(.)$ their instant utility function, which is assumed to be increasing and stricly concave over $[0, \infty)$. In contrast to impatient households, patient households have complete access to asset markets –including the full set of Arrow-Debreu securities and loan contracts. As noticed by Mertz (1995) and Hall (2009), full insurance implies that these households collectively behave like a large "representative family" in which the family head ensures equal ex post marginal utility of consumption for all its members –despite the fact that the members experience heterogenous employment statuses due to the random process of job creation and destruction. Since consumption is the only argument in the period utility, equal marginal utility implies equal consumption, so the budget constraint of this hypothetical family is:

$$C_t^p + A_t^p = R_t A_{t-1}^p + (1 - \Omega)\left(w_t n_t + (1 - n_t)\delta\right), \tag{5}$$

[6] The assumption of no borrowing follows Bewley (1977), Scheinkman and Weiss (1986), Zeldes (1989), Carrol (1991) and more recently Krusell and Smith (1998) and Heathcote (2005), among many others. As discussed by Heathcote, this condition may overstate actual borrowing limits, but also understate it if some of the agent's assets (e.g., durables) may not readily be liquidated to smooth out shocks to individual income.

where C_t^p (≥ 0) and A_t^p denote the consumption and end-of-period asset holdings of the family (both of which must be divided by $1-\Omega$ to find the per-member analogues). Because patient and impatient households are perfectly symmetric from the point of view of the firm, the family enjoys a share $1-\Omega$ of aggregate labour income, which also includes home production. The Euler equation summarising the optimal asset allocation of the family is given by:

$$u^{p\prime}\left(C_t^p\right) = \beta^p E_t\left(u^{p\prime}\left(C_{t+1}^p\right) R_{t+1}\right). \tag{6}$$

As is now well understood since Becker (1980), Becker and Foias (1987), and more recently Kiyotaki and Moore (1997) and Iacoviello (2005), under heterogenous discount factors and borrowing limits patient households tend to accumulate large quantities of assets at the expense more impatient ones, asymptotically leading the former to hold the entire asset stock. This will not occur in our economy because impatient households have a specific (precautionary) motive for holding wealth.

2.2 Production

The representative firm produces output, Y_t, out of capital K_t and employment n_t according to the production function $Y_t = z_t G\left(K_t, n_t\right)$, where $G\left(.,.\right)$ exhibits positive, decreasing marginal products and constant returns to scale (CRS), and where $\{z_t\}_{t=0}^{\infty}$ is a stochastic aggregate productivity process. Defining $k_t \equiv K_t/n_t$ and $g\left(k_t\right) \equiv G\left(k_t, 1\right)$ gives $Y_t = z_t n_t g\left(k_t\right)$. Capital depreciates at a rate $\mu_t \in [0,1]$. The capital stock is owned by the households and rented out to the firm in every period; as usual, the supply of capital results from date $t-1$ households' asset holding decisions, and the demand for it from date t firm's decision (conditional on z_t), with the price of capital freely adjusting to clear the market.

The optimal demand for capital by the representative firm obeys

$$k_t = g^{\prime -1}\left(\frac{R_t - 1 + \mu_t}{z_t}\right). \tag{7}$$

The quantity of labour used by the representative firm is given by (1). With a competitive labour market, the equilibrium real wage is $w_t = g\left(k_t\right) - k_t g'\left(k_t\right)$. With a noncompetitive labour market structure such as that implied by search frictions, the wage must includes a discount in order to make it worthwhile for the firm to pay for vacancy opending costs (see the Appendix for details).

2.3 Market clearing

Since households are uniformly distributed over $[0,1]$, with a share Ω of impatient households in the economy, clearing of the market for claims to the capital stock is given by:

$$A_{t-1}^{P}+\int_{0}^{\Omega}a_{t-1}^{i}di=n_{t}k_{t}, \tag{8}$$

where the left hand side is total asset holdings by the households at the end of date $t-1$ and the right hand side the demand for capital by the representative firm at date t. On the other hand, clearing of the goods market requires:

$$C_{t}^{P}+\int_{0}^{\Omega}c_{t}^{i}di+I_{t}=z_{t}n_{t}g\left(k_{t}\right)+\left(1-n_{t}\right)\delta, \tag{9}$$

where $\int_{0}^{\Omega}c_{t}^{i}di$ is total consumption by impatient households, $I_{t}=n_{t+1}k_{t+1}-\left(1-\mu_{t}\right)n_{t+1}k_{t+1}$ aggregate investment and $\left(1-n_{t}\right)\delta$ total home production income.

We define an *equilibrium* of this economy as a sequence of households' decision variables $\{C_{t}^{p},c_{t}^{i},A_{t}^{p},a_{t}^{i}\}_{t=0}^{\infty}$, $i\in[0,\Omega]$, firm's capital $\{K_{t}\}_{t=0}^{\infty}$, and aggregate variables $\{f_{t},s_{t},n_{t},w_{t},R_{t}\}_{t=0}^{\infty}$ that satisfy the households' and the firm's optimality conditions (4), (6) and (7), together with the market-clearing conditions (8)-(9), given the forcing sequences $\{z_{t},\mu_{t}\}_{t=0}^{\infty}$ and the initial wealth distribution $\left(A_{-1}^{p},a_{-1}^{i},\right)_{i\in[0,\Omega]}$.

3 A minimal cross-sectional heterogeneity equilibrium

As is well known, the joint assumption of incomplete markets and borrowing constraints generally preclude the reduction of the model's dynamics to a small-scale dynamic system. This is because the asset holding decisions of a particular household depend its accumulated wealth, while the later is determined by the entire history of idiosyncratic income shocks. In consequence, the asymptotic cross-sectional distribution of wealth usually has infinitely many states, and hence infinitely many Euler equations would be necessary to exactly describe the behaviour of the economy (Aiyagari, 1994; Krusell and Smith, 1997). In this paper, we circumvent this issue by making specific assumptions about impatient household's period utility and the tightness of the borrowing constraint, which ensure that the cross-sectional distribution of wealth and implied number of household types are both finite.

3.1 Assumptions and conjectured equilibrium

Let us first assume that the instant utility function for impatient households $u(c)$ is i) continuous, increasing and differentiable over $[0,+\infty)$, ii) strictly concave with local relative risk aversion coefficient $\xi(c) = -cu''(c)/u'(c) > 0$ over $[0,c^*]$, where c^* is an exogenous, positive threshold, and iii) linear with slope $\eta > 0$ over $(c^*,+\infty)$ (see Figure 2). Essentially, this utility function (an extreme form of decreasing relative risk aversion) implies that high-consumption (i.e., relatively wealthy) households do not mind moderate consumption fluctuations –i.e., as long as the implied optimal consumption level says inside $(c^*,+\infty)$– but dislike substantial consumption drops –those that would cause consumption to fall inside $[0,c^*]$. In the equilibrium that we are focusing on, "moderate consumption fluctuations" refer to consumption changes triggered by variations in asset and wage incomes *conditional on the agent remaining employed*; in contrast, "substantial consumption drops" refer to those triggered by the large falls in current income that are associated with a *change in employment status from employed to unemployed.* In other words, we are constructing an equilibrium in which:

$$\forall i \in [0,\Omega],\ e_t^i = 1 \Rightarrow c_t^i > c^*,\ e_t^i = 0 \Rightarrow c_t^i \leq c^*. \tag{10}$$

As we shall see shortly, one implication of this utility function and consumption rankings is that employed households fear unemployment and consequently engage in precautionary saving behaviour *ex ante* in order to limit (but without being able to fully eliminate) the associated shooting up in marginal utility. As a result, their asset holdings will be well defined despite the fact that these agents are locally risk-neutral.

The second feature of the equilibrium that we are constructing is that the borrowing constraint in (3) is binding for all unemployed households (that is, the Lagrange multiplier in (4) is positive), so that their end-of-period asset holdings are zero (rather than negative). In short, the equilibrium that we are constructing has the following property:

$$\forall i \in [0,\Omega],\ e_t^i = 0 \Rightarrow E_t\left(\beta u'\left(c_{t+1}^i\right) R_{t+1}/u'\left(c_t^i\right)\right) < 1 \text{ and } a_t^i = 0. \tag{11}$$

Equations (10)–(11) have direct implications for the optimal asset holdings of employed households. By construction, a household who is employed at date t has asset wealth $a_t^i R_{t+1}$ at the beginning of date $t+1$. If the household falls into unemployment at date $t+1$, it liquidates assets after having collected asset incomes, so the household enjoys consumption

$$c_{t+1}^i = \delta + a_t^i R_{t+1} \tag{12}$$

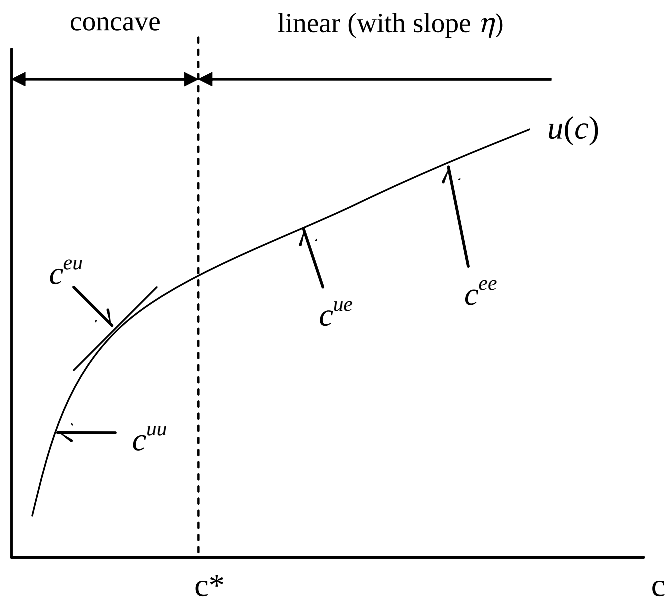

Figure 1: Instant utility function of impatient households

and marginal utility $u'\left(\delta + a^i_{t+1}R_{t+1}\right)$. On the other hand, if this houshold stays employed, it enjoys marginal utility η, as it did in period t (by equation (10)). Therefore, if employed agents' consumption is higher than c^* while unemployed agents consumption is lower than c^*, then the optimal asset holding a^i_t of a typical employed household i must satisfy the following Euler equation:

$$\eta = \beta E_t\left[\left((1 - s_{t+1})\,\eta + s_{t+1}u'\left(\delta + a^i_t R_{t+1}\right)\right)R_{t+1}\right], \tag{13}$$

where marginal utility at date $t+1$ (inside the expectations operator) has been broken into the two possible employment statuses that this household may experience at that date, weighted by their probabilities of occurence. Note that equation (13) uniquely pins down a^i_t as a function of aggregate variables *only*. This in turn implies that asset holdings are symmetric across employed agents –and hence independent of their employment history up to date $t-1$. We may thus write:

$$\forall i \in [0,\Omega]\,,\ e^i_t = 1 \Rightarrow a^i_t = a_t > 0. \tag{14}$$

Equations (11) and (14) show that in this equilibrium the cross-sectional distribution of wealth has two states, so that the economy effectively has exactly *four* types of impatient households –since from (2) the type of an agent depens on both beginning- and end-of-period wealth. We call these types "ij", $i,j = e,u$, where i (j) refers to the household's employment status in the previous (current) date. For example, a "ue" houshold is currently employed but was unemployed in the previous period, and its consumption at date t is c_t^{ue}, and so on. The ranking of consumption level for these households is depicted in Figure 1.

3.2 Cross-sectional implications

We already know that all patient households in our economy share the same Euler equation (thanks to full insurance). In the conjectured equilibrium under consideration, the employed, impatient households also share the same Euler equation (although a different one from that of patient households.) Equations (13)–(14) imply that we may write it as follows:

$$E_t\left(M_{t+1}R_{t+1}\right)=1, \tag{15}$$

where M_{t+1} is the common pricing kerner of employed, impatient households:

$$M_{t+1}=\beta\left[1+s_{t+1}\left(\frac{u'\left(\delta+a_tR_{t+1}\right)-\eta}{\eta}\right)\right]. \tag{16}$$

Equations (15)–(16) clarify the importance of the risk of falling into unemployment for the determination of precautionary asset accumulation. Holding asset returns fixed, an increase in the employment exit probability s_{t+1} tends to raise M_{t+1} (since $\left[u'\left(\delta+a_tR_{t+1}\right)-1\right]/\eta>1$ by (10)), so $u'\left(\delta+a_tR_{t+1}\right)$ must go down (i.e., $c_t^{eu}=\delta+a_tR_{t+1}$ must go up) for the Euler equation to hold. This is achieved by raising date t asset holdings, a_t, in (16). It is important to stress here that time-variations in the probability to become unemployed, s_{t+1}, have a *first-order effect* on precautionary asset accumulation at the individual level, a_t. This is because even without aggregate risk a change in employment status from employment to unemployment at date $t+1$ is associated with a large fall in individual consumption, and hence with a global (rather than local) rise in marginal utility from η to $u'\left(c_{t+1}^{eu}\right)>\eta$ (see Figure 1). The probability s_{t+1} weights these two potential outcomes, so even small changes in s_{t+1} have a sizeable impact on asset holdings and consumption choices.[7]

[7] Linearising (13)–(16), we find an individual asset accumulation rule of the form

$$a_t=a^*+\Gamma_sE_t\left(s_{t+1}-s^*\right)+\Gamma_RE_t\left(R_{t+1}-R^*\right),$$

where stars denote steady state values, a^* is given by (20) below, and Γ_s and Γ_R are constant coefficients. It can be shown that

$$\Gamma_s=\frac{s^*\beta^p\left(\beta^p-\beta\right)u'^{-1}\left(\eta\left(\beta^p/\beta+s^*-1\right)/s^*\right)}{\xi^*\left(\beta^p-\beta+\beta s^*\right)}>0,$$

thereby illustrating the first-order positive effect of a rise in the probability of becoming unemployed on precautionary asset accumulation.

The homogeneity in asset holding behaviour across impatient households imply that we can straightforwardly aggregate or decompose their consumption/saving choices. For example, from (11) and (14), total asset holdings by impatient households is:

$$A_t^I \equiv \int_0^\Omega a_t^i di = \Omega n_t a_t, \tag{17}$$

which can be substituted into the market-clearing condition (8). Similarly, aggregating the budget constraints of impatient households (equation (2)) under (11)–(14), we find their total consumption to be:

$$C_t^I \equiv \int_0^\Omega c_t^i di = \underbrace{\Omega \left(n_t w_t + (1 - n_t)\,\delta + (R_t - 1)\,A_{t-1}^I\right)}_{\text{net income}} - \underbrace{\Omega\Delta\left(n_t a_t\right)}_{\text{change in asset wealth}}, \tag{18}$$

where Δ is the difference operator.

Equation (18) summarises the determinant of total consumption by impatient households in the economy. At date t, their aggregate net income is given by past asset accumulation and current factor payments. The change in their total asset holdings, $\Omega\Delta\left(n_t a_t\right)$, depends on both the change in the number of precautionary savers, Ωn_t, and the assets held by each of them, a_t; in the remainder of the paper we also refer to the former and the latter as the "extensive" and "intensive" asset holding margins, respectively. The former is determined by employment flows is thus beyond the households' control. The latter is their key choice variable and obeys (15)–(16).

3.3 Steady state and existence conditions

We shall work out the conditions for our equilibrium with minimal cross-sectional heterogeneity to exist under the maintained assumption that aggregate shocks have small magnitude. Hence, these conditions will be satisfied in the stochastic equilibrium provided that they are so in the steady state.

Steady state. In the steady state, the real interest rate is determined by the discount rate of the most impatient agents, i.e., $R^* = 1/\beta^p$. From (7) and (1), the steady state levels of employment and capital per employee are:

$$n^* = \frac{f^*}{f^* + s^*},\ k^* = g'^{-1}\left(\frac{1}{\beta^p} - 1 + \mu^*\right). \tag{19}$$

The central variable in our model is the level of asset holdings that employed, impatient households choose to hold as a buffer against unemployment risk. Using (15)–(16) and rearranging, we find their steady state value to be

$$a^* = \beta^p \left[u'^{-1} \left(\frac{\eta \left(\beta^p/\beta + s^* - 1 \right)}{s^*} \right) - \delta \right]. \tag{20}$$

Finally, from (8) and (17), steady state (total) asset holdings by patient households are $A^{p*} = n^* \left(k^* - \Omega a^* \right)$. The other relevant steady state values directly follow.

Existence conditions. The equilibrium described so far requires two sets of conditions to be satisfied. First, the ranking of consumption levels for impatient households in (10) must be satisfied in equilibrium. For this to be the case, the consumption level of eu households, c_t^{eu}, must be lower than that of ue households, c_t^{ue}. From the budget constraint (2) and the asset holdings conditions (11) and (14), we have $c_t^{eu} = \delta + a_{t-1} R_t$ and $c_t^{ue} = w_t - a_t$, so the equilibrium requires

$$\delta + a_{t-1} R_t < w_t - a_t$$

at all dates. Second, the borrowing limit must be effectively binding for all unemployed, impatient households –so that (11) is satisfied. Such a household leaves unemployment in the next period with probability f_{t+1}, and in this case enjoy marginal utility η (by condition (10)); it remains unemployed with complementary probability, in which case it enjoys marginal utility $u'(\delta)$ (by (10) and the budget constraint (2)). Unemployed households can be of two types, uu and eu, and requires both to be borrowing-constrained; however, since $c_t^{uu} = \delta < c_t^{eu} = \delta + a_{t-1} R_t$ (and hence $u'(c_t^{uu}) > u'(c_t^{eu})$), a necessary and sufficient condition for both types to be constrained is:

$$u' \left(\delta + a_{t-1} R_t \right) > \beta E_t \left(\left(f_{t+1} \eta + \left(1 - f_{t+1} \right) u'(\delta) \right) R_{t+1} \right)$$

for all t. Evaluating the latter to conditions at the steady state and noting that $w^* = g(k^*) - k^* g'(k^*)$, the steady state counterpart of the latter two inequalities are:

$$\delta + a^* R^* < g(k^*) - k^* g'(k^*) - a^*, \tag{21}$$

$$u' \left(\delta + a^* R^* \right) > \beta \left(f^* \eta + \left(1 - f^* \right) u'(\delta) \right) R^*, \tag{22}$$

Substituting (19) into (21)–(22) and using the fact that $R^* = 1/\beta^p$, we obtain the following existence proposition.

Proposition 1. A sufficient condition for the minimal heterogeneity equilibrium described above to exist is:

$$u'^{-1} \left[\frac{\eta \left(\beta^p/\beta + s^* - 1 \right)}{s^*} \right] < \min \left[\frac{g(k^*) - k^* g'(k^*) - \delta \beta^p}{1 + \beta^p}, u'^{-1} \left(\frac{\beta \left(f^* \eta + \left(1 - f^* \right) u'(\delta) \right)}{\beta^p} \right) \right],$$

where k^* is given by (19).

The inequality in Proposition 1 can straightforwardly be checked once specific values are assigned to the deep parameters of the model. A we argue next, it is satisfied for plausible such values when we calibrate the model to the U.S. economy. The reason for which it does is as follows. Our limited-heterogeneity equilibrium requires that impatient, unemployed households be borrowing-constrained (i.e., they would like to borrow against future income but are prevented from doing so), while impatient, employed households accumulate sufficiently little wealth in equilibrium (so that this wealth be exhausted within a quarter of unemployment). In the U.S., the quarter-to-quarter probability of leaving unemployment is reasonably high and the replacement ratio reasonably low, leading the unemployed's expected income to be sufficiently larger than current income for these households to be willing to borrow. On the other hand, the distribution of wealth is fairly unequal, leading a large fraction of the population (the impatient in our model) to hold a very small fraction of total wealth.

4 Precautionary asset accumulation and consumption fluctuations

The model developed above implies that some households rationally respond to countercyclical changes in unemployment risk by raising precautionary asset accumulation –and hence by cutting down individual consumption more than they would without the precautionary motive. We now wish to assess the extend of this effect when realistic unemployment shocks are fed into our model economy. For this purpose, we calibrate the model and then compare it to two natural benchmarks. The first benchmark is a model with households heterogeneity and borrowing constraints but in which full unemployment insurance is available. The basic motivation for studying this case is that with full insurance impatient (but constrained) households behave like the "rule-of-thumb" consumers originally proposed by Campbell and Mankiw (1991) to explain the high sensitivity of aggregate consumption to current income. Hence, comparing the economies with and without full unemployment insurance will allow us to isolate the specific role of changes in precautionary wealth accumulation for the dynamics of aggregate consumption –as opposed to the mere presence of binding borrowing constraints. The second benchmark is the representative agent economy, in which neither incomplete markets nor borrowing constraints are posited. The comparision between our model and the representative agent one will provide a measure of the joint roles of borrowing constraints and incomplete markets in magnifiying the consumption response to a typical unemployment shock.

We first describe the baseline calibration of our economy and then show how the two alternative benchmark models (the economy with borrowing constraints but full insurance, and that with a representative agent) can be recovered as special cases of it.

4.1 Baseline model

Preferences and household shares. The first set of parameters to calibrate are preferences parameters. We set the discount factor of impatient households β^p, which governs the steady state real interest rate, to the standard value of 0.99. Iacoviello (2005) discusses the evidence on the cross-sectional distribution of discount factors and accordingly sets that of impatient households, β, to 0.95; we follow him here and refer the reader to his paper for the motivating evidence. The period utility of patient households is of the CRRA form, i.e., $u^p(c) = c^{1-\sigma}/(1-\sigma)$, $\sigma > 0$. We choose a utility function for impatient households, $u(c)$, that is as close as possible as that of patient household –despite the fact that it must include a linear portion to produce our equilibrium with limited cross-sectional heterogeneity. Accordingly, we first assume that $u(c) = c^{1-\sigma}/(1-\sigma)$ for $c \in [0, c^*]$. Regarding the linear part, we proceed as follows. First, we note that the marginal utility of impatient asset holders (i.e., η in (20)) governs desired steady state asset holdings a^*. Now, the great majority of these households are of the ee type, i.e., they were employed in the previous period and are still so in the current period. We thus set η so that the marginal utility of ee households is the same as if their behaviour was governed by the utility function of patient households. That is, we set:

$$\eta = u^{p\prime}(c^{ee}) = (w + a^*(1 - 1/\beta^p))^{-\sigma}.$$

The latter equation indicates that the appropriate value of η depends on a^*. Since a^* also depends on η (by (20)), we jointly solve for the fixed point (a^*, η) using a (rapidly converging) iterative procedure. In our baseline calibration we set $\sigma = 1$. Given the other parameters of the model, we obtain $\eta = 0.49$. The last parameter to calibrate in $u(.)$ is c^*, which we set slightly below c^{ue}, again to minimise the distance between the two period utility functions. Note that while the implied period utility function for impatient households is continuous and concave, it is not differentiable all over $[0, \infty)$; however, it can be made so by "smooth pasting" the two portions of the function in an arbitrarily small neighboorhood of c^*.

We set the share of impatient households, Ω, to 1/2. Campbell and Mankiw (1989) estimate that about one half of U.S. households do not behave as "permanent-income consumers". While in their model those who do not adopt the simple rule-of-thumb consumption rule of consuming their entire income, this clearly should be interpreted as a shortcut for rational consumption behaviour under (implicit) borrowing constraints and buffer stock saving (Mankiw, 2000).[8] More recently, Gali *et al.* (2007) find that a similar portion of such households is necessary to account for the output response to government spending shocks in a New Keynesian model with rigid prices and wages.

Production and income The production function is assumed to be $Y_t = z_t K_t^{\alpha} n_t^{1-\alpha}$, with $\alpha = 1/3$ and $z^* = E(z) = 1$. The depreciation rate is $\mu = 0.025$.

For expositional simplicity we have motivated the income earned while unemployed, δ, as "home production". However, the dynamics of the model is virtually identical if we assume that δ results from the collection unemployment benefits that are paid lump sum by employed workers (who are a lot more numerous than the unemployed.) In the U.S. income replacement rates vary greatly across households. For a typical earner of the mean wage, the OECD indicators report net replacement ratios ranging from 0.06 to 0.75, depending on the length of unemployment, the type of household (numbers of children and wage earners) and income prior to unemployment; we set δ so that the replacement ratio produced by the model (δ/w^*) is 0.40, approximately in the middle of this range and similar to the value used by Shimer (2005).[9]

Our calibration is summarised in Table 2. Note that under our calibration the wealth

[8] Interestingly, Mankiw (2000, p. 121) argues that "the consumption literature on 'buffer stock saving' can be seen as providing a richer description of this rule-of-thumb behavior. Buffer stock savers are individuals who have high discount rates and often face binding borrowing constraints. Their savings might not be exactly zero: they might hold a small buffer stock as a precaution against very bad income shocks". This is precisely what happens in our model.

[9] In an alternative calibration approach, Hadgedorn and Manovskii (2008) propose to incorporate the implicit value of leisure into home production income, and accordingly set the corresponding replacement ratio to 95.5%. However controversial this approach may be (see, e.g., Mortensen and Nagypál, 2007; Costain and Reiter, 2008), it suffices to note here that in this case households would suffer unemployment at very small cost, which would give them little reason to save for precautionary purpose in the first place.

distribution is rather unequal, with the poorest half (our impatient households) holding 0.52% of total wealth. This is a very plausible number (bearing in mind that empirical wealth fractiles are sensitive to the wealth measure being used.) Data from the 2007 Survey of Consumer Finances show that the share of net worth, defined broadly as all assets minus all liabilities, held by the poorest half is 2.5%. However, this net worth concept is not the adequate wealth measure here because it includes net equity in owner-occupied housing, while the primary residence cannot easily be sold to provide for current consumption.[10] Removing net equity housing from the computation of the poorest half's wealth share gives 0.52%, our implied value. Removing some other assets with little fungibility (e.g., vehicules or pension plans) still lowers this share, but the exclusion of some of these items would arguably be more debatable. Given the wealth share of the poorest half, the condition stated in Proposition 1 above holds by a large margin. In particular, households who fall into unemployment without enjoying full insurance are predicted to exhaust their buffer stock of precautionary wealth within a quarter, and hence to live entirely out of home production (i.e., unemployment benefits) thereafter.[11]

[10] See the discussion in Wolff (2007), who constructs "nonhome wealth" fractiles on the ground that owner-occupied housing is essentially illiquid. While a number of households have recently returned their houses against the cancelation of their mortgages, this behaviour is arguably a specific feature of the current crisis rather than the rule.

[11] We have also experimented a version of the model in which impatient households hold a somewhat larger fraction of total wealth and take two quarters, rather than one, to fully liquidate fungible wealth when remaining unemployed. The aggregate dynamics implied by this specification turns out to differ only marginally from our baseline case. This is because, for realistic transition rates in the labour market, the fraction of households remaining unemployed for two quarters in a row is very small. Consequently, these households as a whole hold a vanishingly small share of total wealth and have a equally small impact on aggregate consumption.

Parameters	Symbol	Value	Source or target
Risk aversion	σ	1	Krussel and Smith (1998)
Slope/impatient	η	0.49	See text
Discount factor/patient	β^p	0.99	$R = 1.01$
Discount factor/impatient	β	0.95	Iacovello (2005)
Share of impatient hous.	Ω	0.50	Campbell & Mankiw (1989)
Home production	δ	0.82	$\delta/w^* = 0.4$ (Shimer, 2005)
Capital share	α	0.33	
Depreciation rate	μ	0.025	
Job-finding rate	f^*	0.83	Shimer (2005)
Job separation rate	s^*	0.05	Shimer (2005)
Implied steady state values			
Unemployment rate (%)	$1 - n^*$	5.79	
Wealth share of impatient households (%)	$\theta a^*/k^*$	0.52	

Table 2. Parameters and implied steady state values (baseline model)

4.2 Alternative benchmarks

Borrowing constraint and full insurance. In this economy, the (exogenous) heterogeneity between patient and impatient households as well as the borrowing limit are maintained, but all agents are assumed to enjoy full insurance against individual unemployment shocks. Formally, this economy is one with two representative families (one patient, one impatient) within which equal ex post consumption prevails across family members. Since the impatient family would like to borrow from the patient one, but is prevented from doing so by a (binding) borrowing constraint, its optimal consumption plan is corner and leads current income to be entirely consumed in every period.[12] We thus have $a_t = 0$ and $C_t^I = \Omega\left(n_t w_t + (1 - n_t)\,\delta\right)$.

[12]See Becker (1981) and Becker and Foias (1987). Iacoviello (2005) construct a version of this model structure where (employed) households are able to borrow against future housing wealth, subject to a limited commitment problem a la Kiyotaki and Moore (1997).

Representative agent. In this economy, all agents share the same discount factor, face no borrowing constraint, and fully share the labour income risk generated by individual transitions in the labour market. It is recovered as a special case of our baseline model when the share of impatient households is set to 0 (so that the steady state interest rate is maintained at $R^* = 1/\beta^p$).

Table 3 below compares the three models according to the fraction of households facing a binding borrowing constraint. In our baseline economy with borrowing constraints and incomplete markets, only impatient, unemployed households do, so this share is $\Omega(1-n^*)$. In the borrowing-constrained and full-insurance economy, all impatients households (including employed ones) do, so this share is simply Ω. Finally, the borrowing constraint never binds in the representative agent economy.

Model	Symbol	Value
Baseline	$\Omega(1-n^*)$	2.90
Borrowing constraint and full insurance	Ω	50.0
Representative agent	0	0

Table 3. Fraction of households at the borrowing constraint.

4.3 Impulse-response analysis

To assess the role of precautionary saving in recessions, we analyse impulse response functions after different negative transitory shocks. We focus on the consumption choices of households and hence design our impulse-response experiments so that the environment of the households is clearly identified. Impulse-response functions are drawn by assuming that the economy is initially in the steady state and then faces exogenous joint changes in f_t, s_t and z_t, whose dynamics is constructed to be consistent with a typical NBER recession. To summarise, the paths of f_t and s_t are taken from the data, while the path for z_t is that which equalises the average wage path along a recession and that produced by our baseline model (See Appendix B for details about the way we construct the forcing sequences and the implied consumption responses) As in any impulse-response experiement, households are assumed to have perfect foresight about the evolution of the exogenous variables once a deviation from the steady state occurs.

The first row of Figure 2 displays the paths of the exogenous variables after the start of the recession. The second row show the impact of these forcing processes on the determinants of individual incomes, that is, unemployment, the real interest rate and the real wage. By construction, the job-finding and separation probability paths are identical across the three model variants described above, giving a unique unemployment path. For our given paths of the Solow residual and the transition probabilities, the three model variants produce fairly homogenous responses in the interest rate and wage responses to the shock.

The key observation is that the consumption responses differ markedly across the three specification, as is shown in the last row of Figure 2. The first panel of that row displays the responses of aggregate consumption to the shock in the three economies. Despite small differences in aggregate income, the economy with incomplete markets and precautionary saving predicts a stronger impact effect and a faster adjustment back to the steady state, relative to the two other economies. Comparing troughs, aggregate consumption in the former falls about twice as much as in the representative economy and about 30% more than in the economy with complete markets and borrowing constraints. Note also that the consumption response to the shock is faster, due to the forward-looking dependence of precautionary savings on transition rates in the labour market. The last two panels in decompose the response of aggregate consumption in the three economies in that of impatient versus patient households. Impatient households are responsible for the strength of the consumption drop, notably in the economy with incomplete unemployment insurance.

Given the importance of the reaction of impatient housholds to the shocks for the determination of aggregate consumption, it may be useful to disaggregate their behaviour further into the various types that effectively compose this subgroup. This is done in Figure 3. The first row of Figure 3 focuses on their asset holding behaviour, with the first panel showing their total assets (i.e., $A_t^I = \Omega n_t a_t,$), and the second and third panel decomposing changes in total assets into the intensive (a_t) and extensive (Ωn_t) asset holding margins, respectively. Following the shock, the extensive margin diminishes (due to the fall int the number of precautionary savers), but the intensive margin (i.e., the typical asset holdings of a precautionary saver) rises sufficiently for total assets to rise. The second row of Figure 3 decomposes total consumption by impatient households (i.e., C_t^I) into their components and the relative weight of the four types of impatient households in the economy. It illustrates that the dynamics of C_t^I is primarily driven by ee households, the most numerous type amongst the precautionary savers.

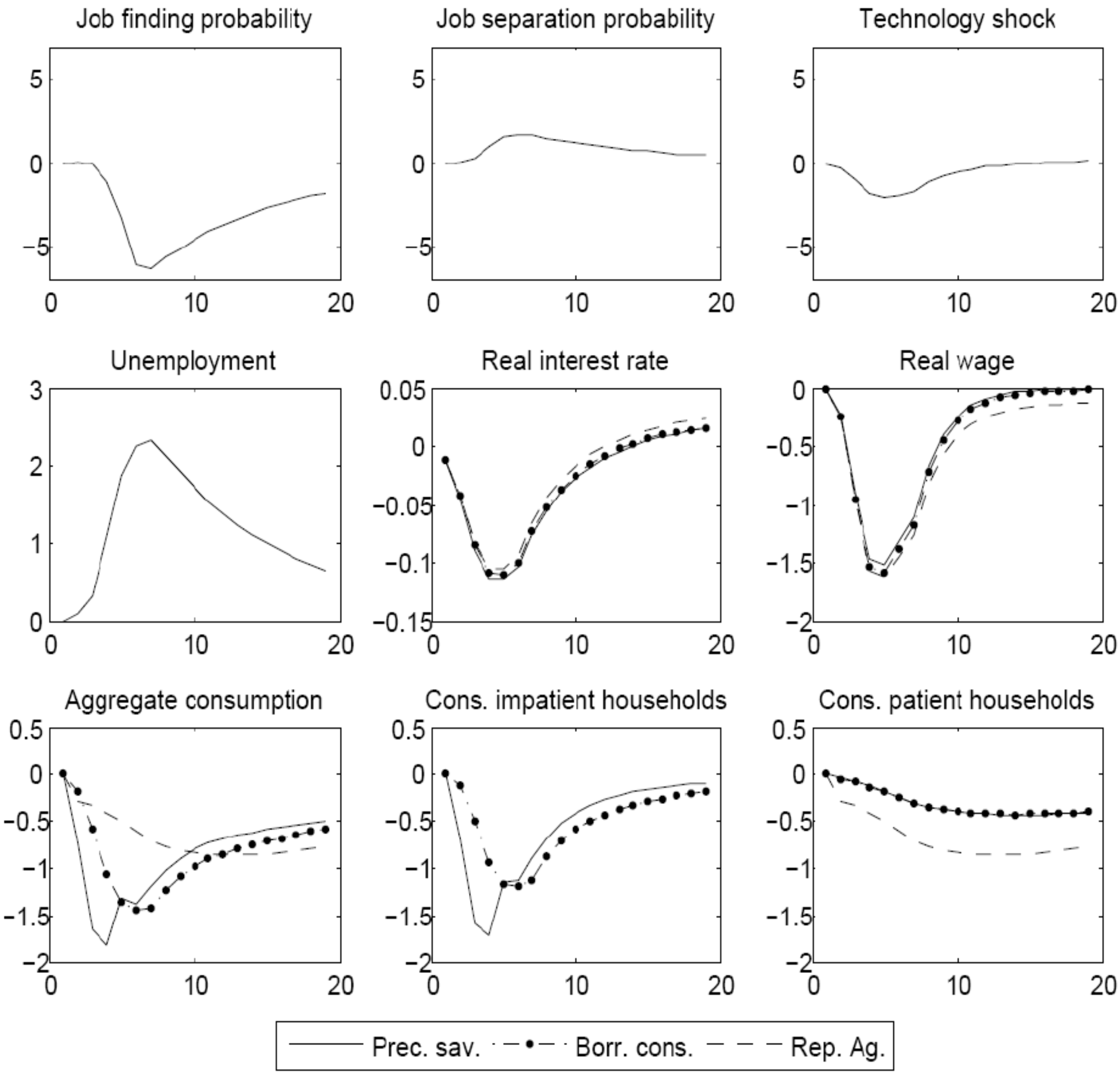

Figure 2: Models' responses to a recessionary shock

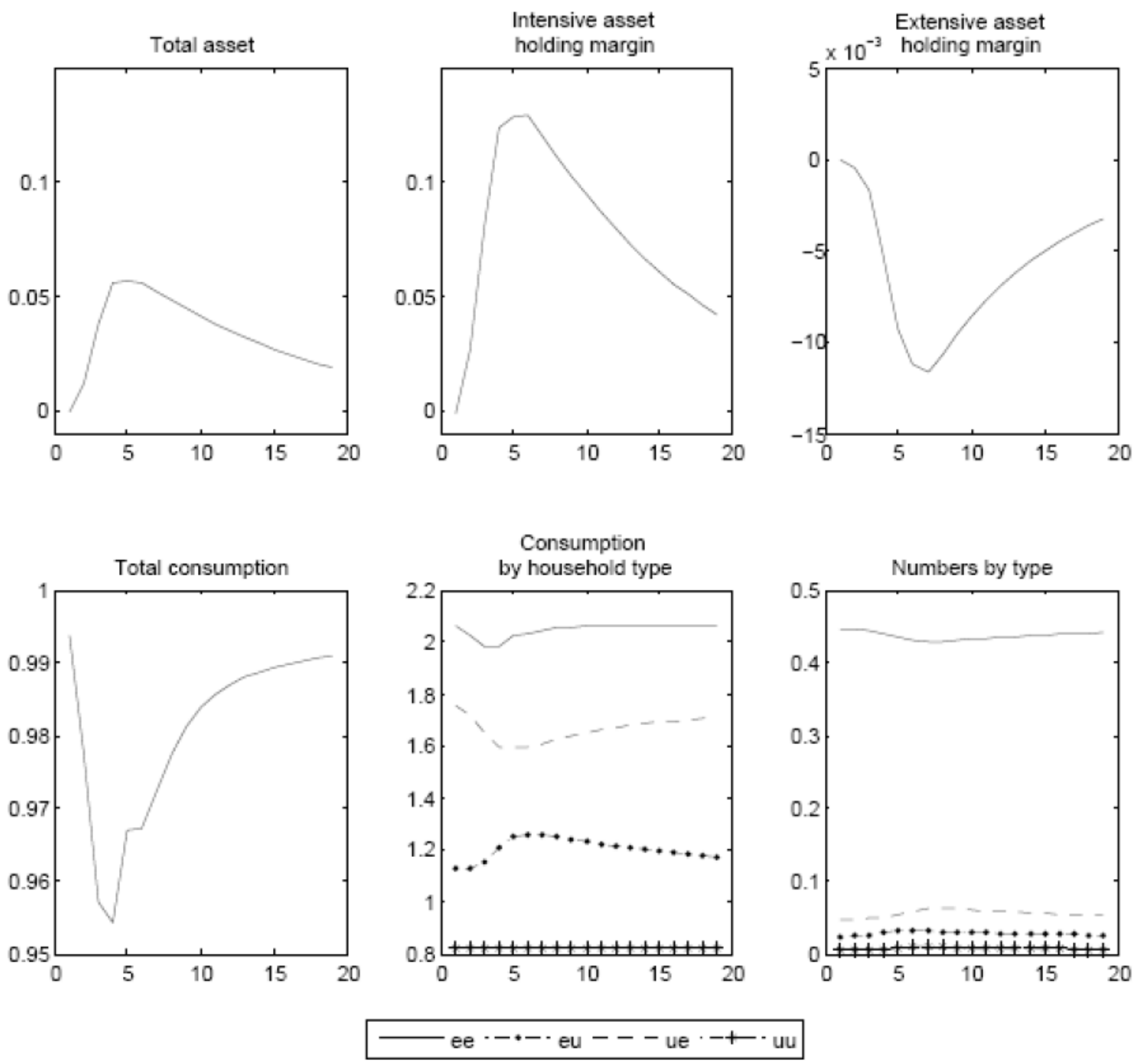

Figure 3: Desaggregated behaviour of impatient households

4.4 Unconditional moments

We now compare the aggregate time series properties of the three models described above. To do so, we simulate these models imposing a joint process for the exogenous state vector $X_t = [f_t, s_t, z_t]'$, where the latter is estimated via the following VAR:[13]

$$X_t = AX_{t-1} + \Omega\varepsilon_t, \tag{23}$$

where A and Ω are 3×3 matrices and $\varepsilon_t = [\varepsilon_t^1, \varepsilon_t^2, \varepsilon_t^3]'$ is i.i.d with distribution $\mathcal{N}(0,1)$. The second moments resulting from this calibration are provided in Table 4. Exactly as in Krusell and Smith (1998), we find small differences between our baseline economy (with borrowing constraint and incomplete markets) and the representative agent model, with the most significant difference lying in the consumption-output correlation. Moreover, our baseline economy and the economy with borrowing constraint but full insurance are virtually indistinguishable using these statics (while they clearly were in the impulse-response experiments carried out above).

Model	Std(Ct)	Std(Yt)	Corr(Yt,Ct)
Baseline	0.022	0.044	0.942
Borrowing constraint and full insurance	0.022	0.044	0.945
Representative agent	0.020	0.046	0.757

Table 4. Aggregate time series properties.

Two main conclusions can be drawn from Table 4. First, as instructive as they may be, unconditional second moments computed in this way may not fully account for the contribution of borrowing constraints and incomplete markets in affecting the aggregate consumption response to a typical recession event. Second, this approach is entirely ineffective at isolating the specific contribution of precautionary savings for the magnification of consumption fluctuations, relative as opposed to the direct contribution of binding borrowing constraints.

4.5 Sensitivity

We now evaluate the sensitivity of results to parameter changes. We focus on the following three parameters: the share of impatient households Ω, the value of home production δ, and

[13] We follow the methodology of Rios-Rull and Santaeulalia-Llopis (2009) to estimate a quarterly process for the technology shock. We estimate a Solow residual on U.S. data for the period 1948Q1-2008Q2. Changing the number of lags in the VAR does not affect the results.

the curvature of the utility function (σ). In each case, we look at the impact of a parameter change on each of the three economies described above, focusing on three summary statics. The first one is the wealth share of impatient households, $\Omega a/k$ (which we provide only for the precautionary saving economy, because it is either equal to 0 or irrelevant in the other two). The second statistics is the maximum fall in aggregate consumption (ΔC) when the same paths for the job-finding rate, the job-separation rate and total factor productivity as in Section 4.3 are imposed. The third statistics is the unconditional consumption-output correlation using the stochastic process estimated in Section 4.4. Table 5 summarises the results.

	Values			Baseline			Borrow. Cons.		Rep. Agent	
	Ω	δ/w	σ	$\frac{\Omega a}{k}(\%)$	ΔC (%)	Corr (Y_t, C_t)	ΔC (%)	Corr (Y_t, C_t)	ΔC (%)	Corr (Y_t, C_t)
1.	0.5	0.4	1	0.52	-1.83	0.94	-1.44	0.94	-0.86	0.75
2.	0.75	0.4	1	0.77	-2.58	0.98	-1.88	0.98	-0.86	0.75
3.	0.5	0.95	1	0.00	-0.71	0.94	-0.71	0.94	-0.47	0.74
4.	0.5	0.4	2	0.82	-1.48	0.96	-1.28	0.96	-0.61	0.77

Table 5. Sensitivity Analysis. All other parameters are as in Table 3.

Row 1 summarises the results obtained under the calibration of Table 3. Row 2 is the same economy with a share of impatient households increased from 50% to 75%. The consumption share of impatient households in the precautionary saving economy increases from 0.52% to 0.77%. This produces a much larger fall in aggregate consumption (-2.58%, instead of -1.83%), because these households' consumption reacts much more strongly to changes in labour market conditions than patient households. The consumption-output correlation also increases, because at the individual level it is higher for impatient households than for patient ones. Similar changes occur in the economy with borrowing constrained and full insurance, although to a less extent. Hence, rising Ω increases the distance between the two model. The statistics for the representative agent economy are not affected, since $\Omega = 0$ by construction.

Row 3 shows the impact of an increase in home production, relative to the baseline calibration (40% of the steady state real wage.).We set it to 0.95% of the real wage, following the suggestion of Hadgedorn and Manovskii (2008). Under this calibration, the borrowing constraint becomes binding for all impatient households (including employed ones) in our baseline economy; their asset holdings is thus 0. The reason for this is that with this value of δ households suffer a very small income loss when they fall into unemployment. As a consequence, the precautionary motive is to weak relative to the desire to borrow that follows their impatience (i.e., the fact that the equilibrium interest rate is lower than their subjective discount rate). This implies that our baseline economy and that with borrowing constraint and full insurance become identical. The fall in consumption is reduced to -0.71% in both economies and the consumption-output is unchanged and equal to 0.94. The fall in consumption is smaller because the consumption of unemployed households is higher, thanks to the high valuie of home production. For the same reason, the consumption fall incurred by the representative agent is also smaller. To summarise, an increase in home production reduces the fall in aggregate consumption in all economies economies, but a very high level of home production does not seem consistent with the saving decision of the poorest half (which is small but positive.)

Row 4 shows the impact of an increase in the curvature of the utility function. In this economy we recomputed the value of η, so that in equilibrium $\eta = (c^{ee})^{-2}$ (see Section 4.1). This causes the wealth share of the poorest half to rise in our baseline model (from 0.52% to 0.82%). The reason for this is the increased desire to smooth consumption by impatient households, which leads to more self-insurance in equilibrium. A implication of this higher buffer stock is a more moderate fall in aggregate consumption after the shock (-1.48%, instead of -1.83%) in our baseline economy. The correlation between consumption and output is increased mildly from 0.94 to 0.96, thanks to the more muted consumption fall. In the borrowing constrained economy, the fall in aggregate consumption is reduced to -1.44%. The reason for this result is that the consumption of patient households falls less, due to their lower elasticity of intertemporal substitution ($1/\sigma$). This is also true in the representative agent economy, wherein the consumption fall is reduced to -0.61%, (againt -0.86% in the calibration of Table 3).

Appendix A. Endogenising transitions rates

This appendix shows how (f_t, s_t) can be determined by the job creation policy of the firm in a labour market with (random) search and matching frictions. We use the same timing convention and form of the employment contract as in Hall (2009), which allows us to be as close as possible to our baseline model with exogenous transition rates.

More specifically, our timing is as follows. At the very beginning of date t, a fraction ρ of existing employment relationships are broken, thereby creating a job seekers pool of size $1-(1-\rho)\,n_{t-1}$ (that is, unemployment at the end of date $t-1$, $1-n_{t-1}$, *plus* the broken relationships at the beginning of date t, ρn_{t-1}). Members of this pool then have a probability f_t to find a job within the same period, and stay unemployed at the end of the period with complementary probability. It follows that the quarter-to-quarter separation rate is $s_t = \rho\,(1-f_t)$.

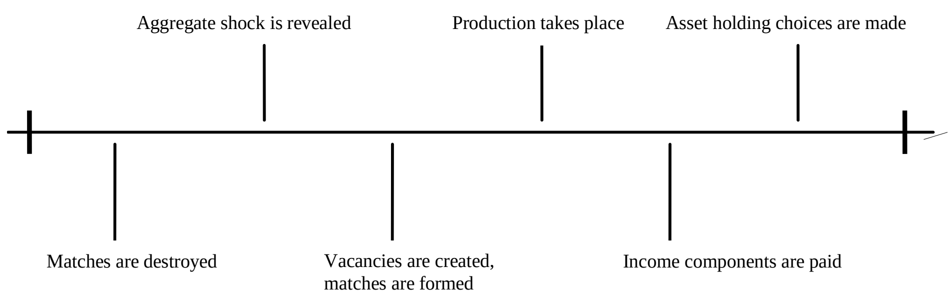

Figure 4: Time line

The job-finding rate, f_t, is determined as follows. Given its knowledge of $1-(1-\rho)\,n_{t-1}$ and z_t, the firm posts v_t vacancies at cost $c>0$ each and a fraction λ_t of which are filled in the current period. Total employment at the end of date t is thus:

$$n_t = (1-\rho)\,n_{t-1} + \lambda_t v_t. \tag{24}$$

The vacancy filling rate λ_t is related to the vacancy opening policy of the firm via the matching technology. The number of matches M_t formed at date t is assumed to depend on both the size of the job seekers' pool and the number of posted vacancies, v_t, according to the function $M_t = M\,(1-(1-\rho)\,n_{t-1}, v_t)$, which is increasing and strictly concave in both arguments and has CRS. Thus, the vacancy-filling rate satisfies $\lambda_t = M_t/v_t = m\,(\theta_t)$, where $\theta_t \equiv v_t/\,(1-(1-\rho)\,n_{t-1})$ is the market tightness ratio, and where the function $m\,(\theta_t) \equiv M\left(\theta_t^{-1}, 1\right)$ is strictly decreasing in θ_t.

Once matched, the households and the firm split the match surplus according to bilateraly efficent dynamic contracts that are negociated at the time of the match and implemented as planned for the duration of the match. Following Hall (2009) and Stevens (2004), we restrict our attention to a simple class of dynamic contracts whereby the firm pays the worker its full marginal product, except at the time of the match when the worker is paid below marginal product. The profit flow extracted by the firm on new matches motivates –and finances– the payment of vacancy opening costs, but existing matches generate no quasi-rents thereafter. Formally, this arrangement is equivalent to a "fee contract" in which any matched worker i enjoys the wage $w_t = z_t G_2(K_t, n_t)$ at any point in time but pays a fixed fee $\psi_t > 0$ to the firm at the time of hiring; that is, the worker is actually paid $\tilde{w}_t = w_t - \psi_t > 0$ during the (one-period) probation time and the full wage thereafter. We let ψ_t respond to the aggregate states, i.e., $\psi_t = \psi(z_t)$, $\psi'(.) > 0$. The representative firm maximises its instantaneous profit flow

$$\Pi_t = z_t G(K_t, n_t) - n_t w_t - (R_t - 1 + \mu) K_t + v_t (\lambda_t \psi_t - c), \tag{25}$$

subject to (24), and taking n_{t-1}, λ_t, R_t, as well as the contract $(\tilde{w}_t, w_t)$, as given. The optimal choice of capital per employee is given by equation (7). On the other hand, from (25) the firm expands vancancy openings until $z_t G_2(K_t, n_t) \lambda_t - \lambda_t w_t + \lambda_t \psi_t - c = 0$. Since $G_2(K_t, n_t) = w_t$ in the class of contracts under consideration, the economywide vacancy-filling rate that results from these openings is:

$$\lambda_t = c/\psi(z_t) \equiv \lambda(z_t), \ \lambda'(.) < 0. \tag{26}$$

From (25)–(26) and the the CRS assumption, the firm makes no pure profits in equilibrium (i.e., old matches generate no profit, while the quasi-rent extracted from new matches is exhausted in the payment of vacancies costs.). From the matching function specified above, the tightness ratio that results from the optimal vacancy policy of the firm is $\theta_t = m^{-1}(\lambda_t)$. Hence, the job-finding rate in this economy is:

$$f_t = \lambda_t . m^{-1}(\lambda_t) \equiv f(z_t), \ f'(.) > 0, \tag{27}$$

and the employment exit probability $s(z_t) = \rho(1 - f(z_t))$. Note that under this structure the firm's problem is static and thus well defined despite the fact that impatient and patient households do not share the same pricing kernel.

Appendix B. Impulse-response experiments

Data. We analyse the behavior of the job finding probability, the job separation probability, the unemployment rate, the real wage, the real interest rate, and real aggregate consumption of non durable goods and services.

We first construct quarterly labour market transition rates over the period 1948Q1-2010Q3 from the Current Population Survey data and following the same procedure as described in Shimer (2007). The cyclical component is extracted using a HP filter with smoothing parameter 10^5. We then average the paths of these rates over NBER recessions.

We construct an average real quarterly wage over the same period by dividing the total the sum of total wages and salary accruals for all employees (taken from the NIPA tables) by the number of nonfarm payrolls provided by Current Employment Statistics survey (BLS). Unemployment level is taken from the CPS. The measure of real consumption is the sum of personal consumption expenditures on non durable goods and services (NIPA), where we use the price index for personal consumption expenditures as a deflator (BLS). All variables are HP-filtered with the same smoothing parameter as that applied to labour market transition rates (10^5).

We then average the paths of labour market transition rates and the real wage over the the 11 post-1948 NBER recessions, starting for the first quarters of each recession.

While there is a relative homogeneity in the behaviour of those variables over the first few quarters, the way they revert to the mean thereafter varies considerably across recessions. The average duration of an NBER recession being of 5 quarters, we only keep the average paths of these variables for the first 5 quarters. Afterwards, we impose that f_t and s_t return to their steady state values according to an AR(1) process of autocorrelation coefficient of 0.9, and that the real wage return to its steady state value according to an AR(1) with autocorrelation coefficient of 0.7. These coefficients approximately correspond the the average reversion speed of those variable across NBER recessions (although, as we argued, there are sizeable variations around this average speed).

Extraction of underlying shocks. We describe here how we choose the paths for f_t, s_t, z_t.to reproduce an average recession. Since in the model the processes for f_t and s_t are exogenous, we their paths to their actual mean values as described above.

The choice of the productivity path $\{z_t\}$ is chosen such that path of the real wage in our baseline model be identical to the wage path described above (average of NBER recessions over the first five quarters and average reversion speed thereafter). The algorithm to find this path is the following.

1) First, define a number of periods, T, such that the economy is back to steady state at date $t_0 + T$ if the aggregate shock hits the economy in period t_0. Since transitions rate are exogenous, so is the number of employed agents n_t, for $t_0 \leq t \leq t + T$.

2) For $t \geq 1$, assume a path for the real interest rate r_t and assume that the real wage follows an exogenously given path

3) From the first order conditions of the firm, z_t is recovered from r_t and w_t, as follows:

$$z_t = \frac{(r_t + \mu)^\alpha}{\alpha^\alpha (1-\alpha)^{1-\alpha} w_t^{-(1-\alpha)}}$$

Note that the previous equality allows iterating on the path of r_t on to obtain the path of z_t as a simple function of prices, r_t and w_t.

4) Moreover, from the first order conditions of the firm, one infers the capital stock in use at date t, K_t^d, from the path of n_t, w_t and z_t, as follows:

$$K_t^d = n_t \left(\frac{w_t}{(1-\alpha) z_t} \right)^{\frac{1}{\alpha}}$$

4) With the paths for r_t, w_t and n_t, one can solve the program of patient and impatient households to obtain their desired asset holdings at date $t-1$, respectively A_{t-1}^p and a_{t-1}. Total assets in period $t-1$ (and hence at the beginning of period t) are:

$$K_t^s = A_{t-1}^p + n_t a_{t-1}$$

5) We may then iterate over the path of the real interest rate r_t. If K_t^s and K_t^d. are different, then one goes back to step 2).

When K_t^s and K_t^d are equal for $t_0 \leq t \leq t + T$, up to the approximation criterion, stop: all market clears and firms and households optimality conditions are satisfied given the paths for f_t, s_t and z_t.

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